# Thermality from a Rindler quench: spacetime and laboratory

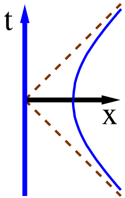

**Jorma Louko**

School of Mathematical Sciences, University of Nottingham



**RQIN 2019, NCKU Taiwan, 27 May – 1 June 2019**


JL CQG 35 (2018) 205006; Biermann et al (in development)



# Plan

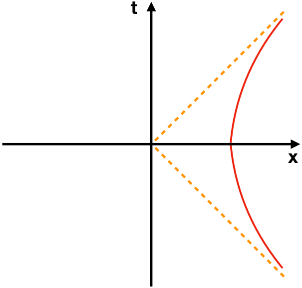
1. Linear motion Unruh effect: spacetime $\to$ lab?
2. Rindler quench

    **Motion without motion**
3. Lab simulation proposal

    **Rodríguez-Laguna, Tarruell, Lewenstein, Celi (2017)**
4. Analytic quench results

    **Encouraging for the experiment!**
5. Preview: Classical lab test proposal
6. Summary

# Linear motion Unruh effect: spacetime $\to$ lab




**Uniform linear acceleration:**

$$x = \sqrt{t^2 + 1/a^2}$$

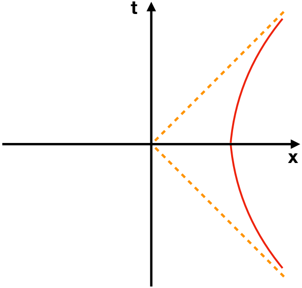
$a > 0$ proper acceleration

- Observer coupled to Minkowski vacuum experiences

$$T = \frac{a}{2\pi}$$ **Thermal!** (weak coupling, long time)

- Can cook a steak!

# Linear motion Unruh effect: spacetime $\rightarrow$ lab




**Uniform linear acceleration:**

$$x = \sqrt{t^2 + 1/a^2}$$

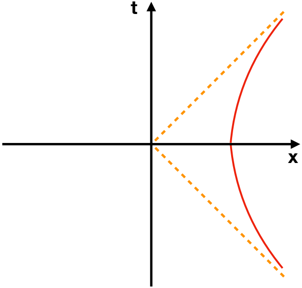
$a > 0$ proper acceleration

- Observer coupled to Minkowski vacuum experiences

$$T = \frac{a}{2\pi} \times \frac{\hbar}{c k_B}$$ **Thermal!** (weak coupling, long time)

- Can cook a steak! But $1\,\mathrm{K} \leftrightarrow 10^{20} \mathrm{m/s^2} \ldots$

# Linear motion Unruh effect: spacetime $\to$ lab




**Uniform linear acceleration:**

$$x = \sqrt{t^2 + 1/a^2}$$

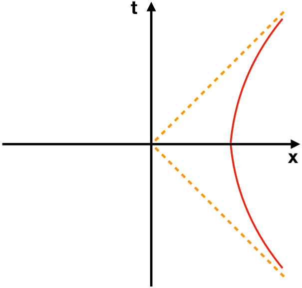
$a > 0$ proper acceleration

- Observer coupled to Minkowski vacuum experiences

$$T = \frac{a}{2\pi} \times \frac{\hbar}{ck_{\mathrm{B}}}$$ **Thermal!** (weak coupling, long time)

- Can cook a steak! But $1\,\mathrm{K} \leftrightarrow 10^{20}\mathrm{m/s^2}\ldots$
- **Analogue spacetime in condensed matter?**
  - **No relativistic time dilation!**
  - **Linear motion $\longleftrightarrow$ finite size???**

# Linear motion Unruh effect: spacetime $\to$ lab




**Uniform linear acceleration:**

$$x = \sqrt{t^2 + 1/a^2}$$

$a > 0$ proper acceleration

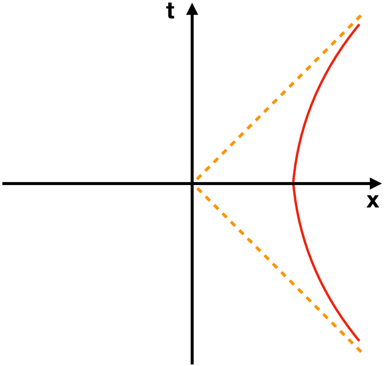
- Observer coupled to Minkowski vacuum experiences

$$T = \frac{a}{2\pi} \times \frac{\hbar}{ck_{\mathrm{B}}}$$ **Thermal!** (weak coupling, long time)

- Can cook a steak! But $1\,\mathrm{K} \leftrightarrow 10^{20}\mathrm{m/s^2}\ldots$
- **Analogue spacetime in condensed matter?**
  - **No relativistic time dilation!**
  - **Linear motion $\longleftrightarrow$ finite size???**

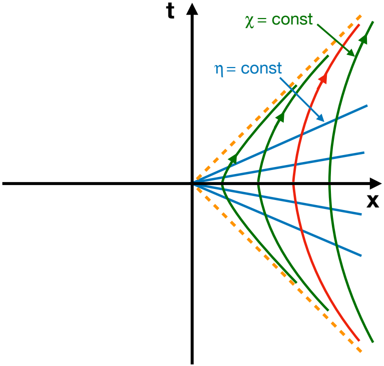
**Today: simulate motion, without motion!**

# Regularised Rindler




$$ds^2 = -dt^2 + dx^2$$

# Regularised Rindler



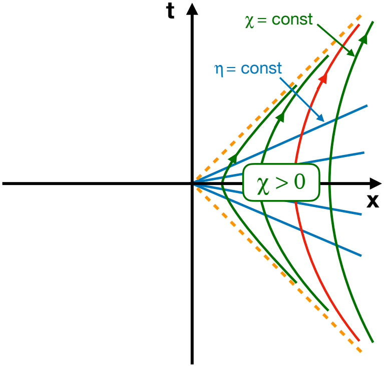

$$ds^2 = -dt^2 + dx^2$$
$$= -\chi^2 d\eta^2 + d\chi^2$$

$$t = \chi \sinh \eta$$
$$x = \chi \cosh \eta$$

$$\chi > 0$$

**Static:** $\partial_\eta$ boost Killing, timelike, future-pointing

# Regularised Rindler




$$ds^2 = -dt^2 + dx^2$$
$$= -\chi^2 d\eta^2 + d\chi^2$$

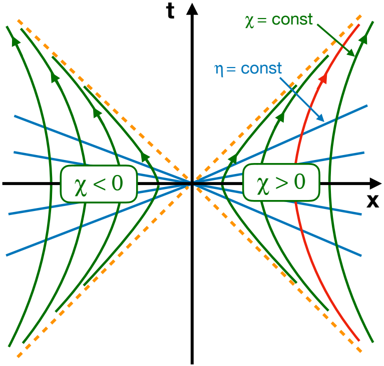
$$t = \chi \sinh \eta$$
$$x = \chi \cosh \eta$$

$\chi > 0$

**Static:** $\partial_\eta$ boost Killing, timelike, future-pointing

$\chi = 0$ **in the lab?**

# Regularised Rindler




$$ds^2 = -dt^2 + dx^2$$
$$= -\chi^2 d\eta^2 + d\chi^2$$

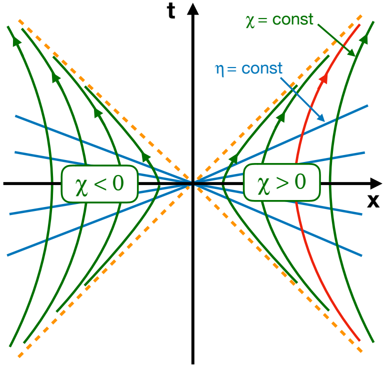
$$t = |\chi| \sinh \eta$$
$$x = |\chi| \cosh \eta$$

$$\chi \neq 0$$

**Static:** $\partial_\eta$ boost Killing, timelike, future-pointing

$\chi = 0$ **in the lab?**

# Regularised Rindler




$$ds^2 = -dt^2 + dx^2$$
$$= -\chi^2 d\eta^2 + d\chi^2$$

$$t = |\chi| \sinh \eta$$
$$x = |\chi| \cosh \eta \qquad \chi \neq 0$$

**Static:** $\partial_\eta$ boost Killing, timelike, future-pointing

$\chi = 0$ **in the lab?**

**Regularise:**

$$ds^2 = -\left(\chi^2 + b^2\right) d\eta^2 + d\chi^2 \qquad b > 0\text{: "regulator"}$$

- $|\chi|/b \gg 1$: asymptotically Rindler
- Negatively curved, $R \approx -2/b^2$ for $|\chi|/b \lesssim 1$
- $\partial_\eta$ timelike Killing: **static**
- $\chi =$ const: $a_\chi = |\chi|/(\chi^2 + b^2)$

# Rindler quench

New spacetime:

$$ds^2 = \begin{cases} -dt^2 + dx^2 & \text{for } t \leq 0 \\ -(\chi^2 + b^2)\,d\eta^2 + d\chi^2 & \text{for } \eta \geq 0 \end{cases}$$

At $t = 0 = \eta$, join so that $x = \chi$: **quench**

**Post-quench $\chi = \text{const}$ observer: Unruh effect?**

# Rindler quench

New spacetime:

$$ds^2 = \begin{cases} -dt^2 + dx^2 & \text{for } t \leq 0 \\ -(\chi^2 + b^2)\, d\eta^2 + d\chi^2 & \text{for } \eta \geq 0 \end{cases}$$

At $t = 0 = \eta$, join so that $x = \chi$: **quench**

**Post-quench $\chi =$ const observer: Unruh effect?**

**Pro:**

- Can prepare Minkowski vacuum $|0_{\mathrm{M}}\rangle$ pre-quench
- Asymptotically Rindler post-quench

**Con:**

- Quench injects energy!
- Timelike Killing vector $\partial_\eta$ everywhere future-pointing
  $\Rightarrow \chi > 0$ and $\chi < 0$ stay in causal contact: **entanglement?**
- Role of regulator $b$?

# Lab simulation proposal

Rodríguez-Laguna, Tarruell, Lewenstein, Celi (2017); Kosior, Lewenstein, Celi (2018)

**Ultracold fermionic atoms in planar optical lattice**

# Lab simulation proposal

Rodríguez-Laguna, Tarruell, Lewenstein, Celi (2017); Kosior, Lewenstein, Celi (2018)

## Ultracold fermionic atoms in planar optical lattice

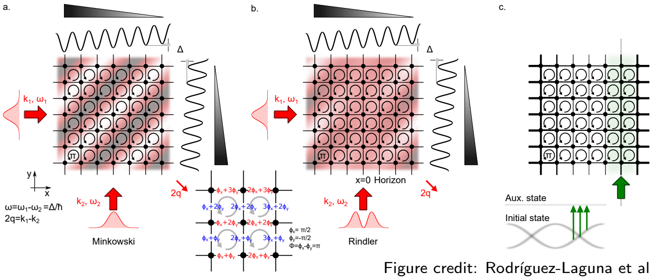



Figure credit: Rodríguez-Laguna et al

- Effective spacetime dimension 2+1 (or almost 1+1)
- Fluctuation dispersion relation adjustable "instantaneously"

  $v = \text{const} \quad \text{for } t_{\text{lab}} < 0; \qquad v \propto |x_{\text{lab}}| \quad \text{for } t_{\text{lab}} > 0$
- Lattice spacing $\longleftrightarrow$ $b$
- Post-quench: do one-particle excitation spectroscopy at $x_{\text{lab}} = \text{const} \quad \Rightarrow$ **simulates Unruh-DeWitt detector**

# Lab simulation proposal

Rodríguez-Laguna, Tarruell, Lewenstein, Celi (2017); Kosior, Lewenstein, Celi (2018)

**Ultracold fermionic atoms in planar optical lattice**

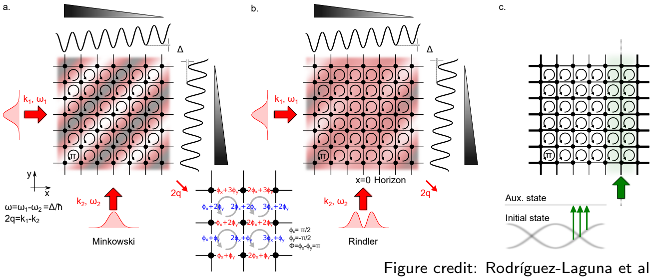
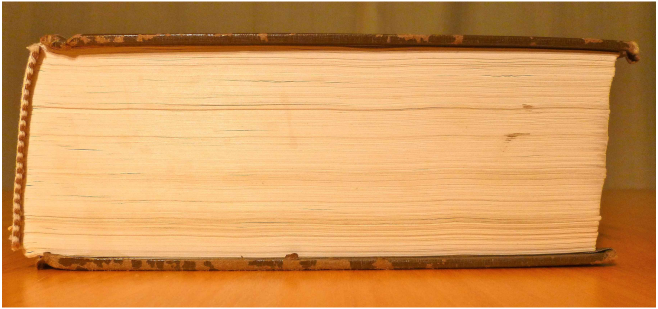



Figure credit: Rodríguez-Laguna et al

- Effective spacetime dimension 2+1 (or almost 1+1)
- Fluctuation dispersion relation adjustable "instantaneously"

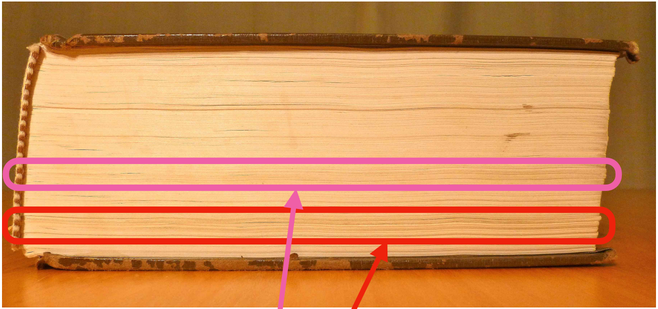
  $v = \text{const} \quad \text{for } t_{\text{lab}} < 0; \quad v \propto |x_{\text{lab}}| \quad \text{for } t_{\text{lab}} > 0$
- Lattice spacing $\longleftrightarrow$ $b$
- Post-quench: do one-particle excitation spectroscopy at $x_{\text{lab}} = \text{const}$ $\Rightarrow$ **simulates Unruh-DeWitt detector**

$\Rightarrow$ **(Numerical) evidence for post-quench Unruh effect**



I. S. Gradshteyn and I. M. Ryzhik

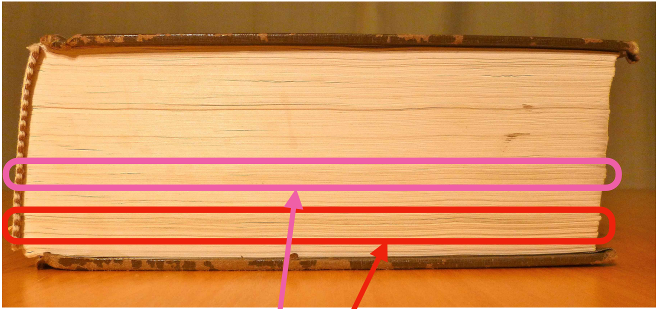
*Table of Integrals, Series, and Products*, 4th edition



**Bessel functions**

I. S. Gradshteyn and I. M. Ryzhik

*Table of Integrals, Series, and Products*, 4th edition



**Bessel functions**

$$ds^2 = \begin{cases} -dt^2 + dx^2 & \text{for } t \le 0 \\ -\left(\chi^2 + b^2\right) d\eta^2 + d\chi^2 & \text{for } \eta \ge 0 \end{cases}$$

**Bogoliubov coefficients are Bessel functions!**

# Analytic model results

Field: 1+1 massless scalar

- Narrow **finite** pulse of energy, rapidly escapes
- Post-pulse, Unruh-DeWitt detector at $\chi = \text{const}$ has **thermality** with

$$T_\chi = \frac{1}{2\pi\sqrt{\chi^2 + b^2}} \qquad \left(\text{cf.} \quad a_\chi = \frac{|\chi|}{\chi^2 + b^2}\right)$$

# Analytic model results

Field: 1+1 massless scalar

- Narrow **finite** pulse of energy, rapidly escapes
- Post-pulse, Unruh-DeWitt detector at $\chi = \text{const}$ has **thermality** with

$$T_\chi = \frac{1}{2\pi\sqrt{\chi^2 + b^2}} \qquad \left(\text{cf.} \quad a_\chi = \frac{|\chi|}{\chi^2 + b^2}\right)$$

**Strengthens the motivation to realise the experimental proposal!**

# Analytic model results

Field: 1+1 massless scalar

- Narrow **finite** pulse of energy, rapidly escapes
- Post-pulse, Unruh-DeWitt detector at $\chi = \text{const}$ has **thermality** with

$$T_\chi = \frac{1}{2\pi\sqrt{\chi^2 + b^2}} \qquad \left(\text{cf.} \quad a_\chi = \frac{|\chi|}{\chi^2 + b^2}\right)$$

**Strengthens the motivation to realise the experimental proposal!**

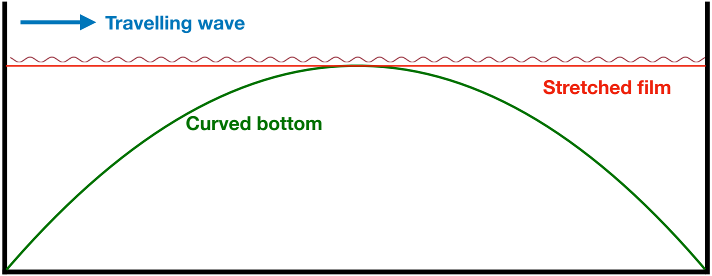
**Open:**

- Entanglement between $\chi \gg b$ and $\chi \ll -b$ (cf. talk by Mann)
- Fermions?
- 2+1 ?

# Preview: classical lab test?

Biermann, Erne, Gooding, Louko, Unruh, Weinfurtner (in development)

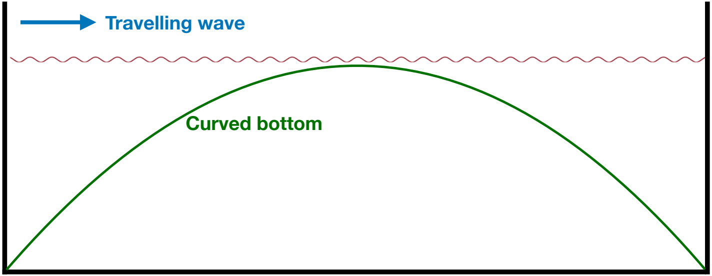
Dispersion relation quench in classical fluid surface waves



# Preview: classical lab test?

Biermann, Erne, Gooding, Louko, Unruh, Weinfurtner (in development)

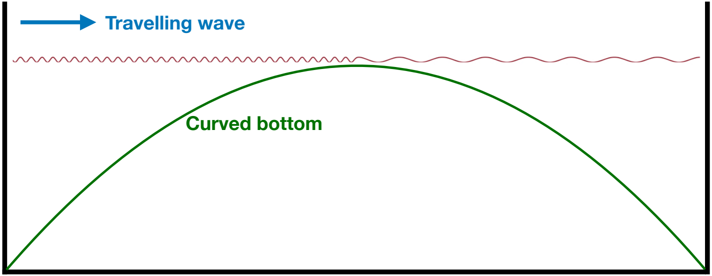
Dispersion relation quench in classical fluid surface waves



# Preview: classical lab test?

Biermann, Erne, Gooding, Louko, Unruh, Weinfurtner (in development)

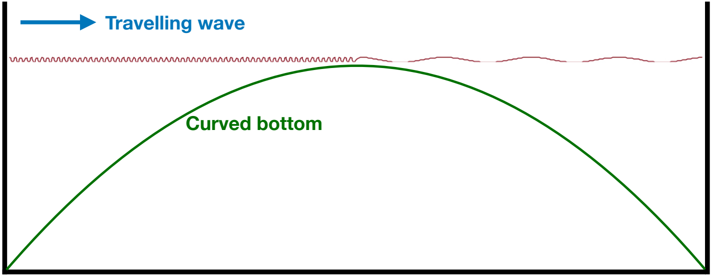
Dispersion relation quench in classical fluid surface waves



# Preview: classical lab test?

Biermann, Erne, Gooding, Louko, Unruh, Weinfurtner (in development)

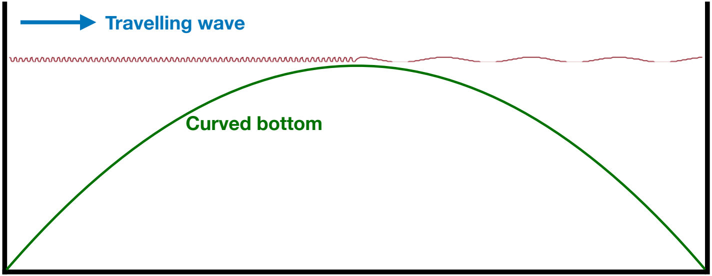
Dispersion relation quench in classical fluid surface waves



# Preview: classical lab test?

Biermann, Erne, Gooding, Louko, Unruh, Weinfurtner (in development)

Dispersion relation quench in classical fluid surface waves




**Stay tuned!**

# Summary

## Celi's team (2017):

Propose an analogue spacetime implementation of the Unruh effect with ultracold fermionic atoms in an optical lattice

- **Quench in the dispersion relation**
- Unruh temperature in post-quench measurements by one-particle excitation spectroscopy

## We:

- Provided supporting analytic evidence:
  **Similar bosonic quench is mild**
  $\Rightarrow$ **strengthens the motivation to realise the proposal**
- Outlined a laboratory test of the proposal's classical aspects

# Summary

## Celi's team (2017):

Propose an analogue spacetime implementation of the Unruh effect with ultracold fermionic atoms in an optical lattice

- **Quench in the dispersion relation**
- Unruh temperature in post-quench measurements by one-particle excitation spectroscopy

## We:

- Provided supporting analytic evidence:
  **Similar bosonic quench is mild**
  $\Rightarrow$ **strengthens the motivation to realise the proposal**
- Outlined a laboratory test of the proposal's classical aspects

**Thank you**