Multipartite Entanglement in Black Hole Evaporation and Primordial Fluctuations

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Entanglement

- state is separable

pure state

$$|A, B\rangle = |A\rangle|B\rangle$$

- state is entangled

$$|A, B\rangle = |a_1\rangle|b_1\rangle + |a_2\rangle|b_2\rangle + \cdots$$

entangled state:
quantum mechanical non-locality

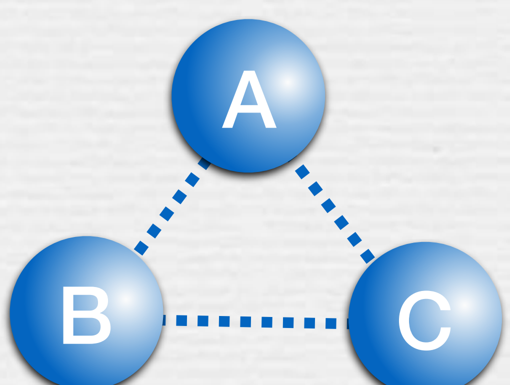
violation of Bell's inequality

2 qubit system

Bell state

$$|\Phi^+\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$$

3 qubit system



$$|\mathrm{W}\rangle = \frac{1}{\sqrt{3}}(|001\rangle + |010\rangle + |001\rangle)$$

$$|\mathrm{GHZ}\rangle = \frac{1}{\sqrt{2}}(|111\rangle + |000\rangle)$$

Multipartite Entanglement

3 qubit system

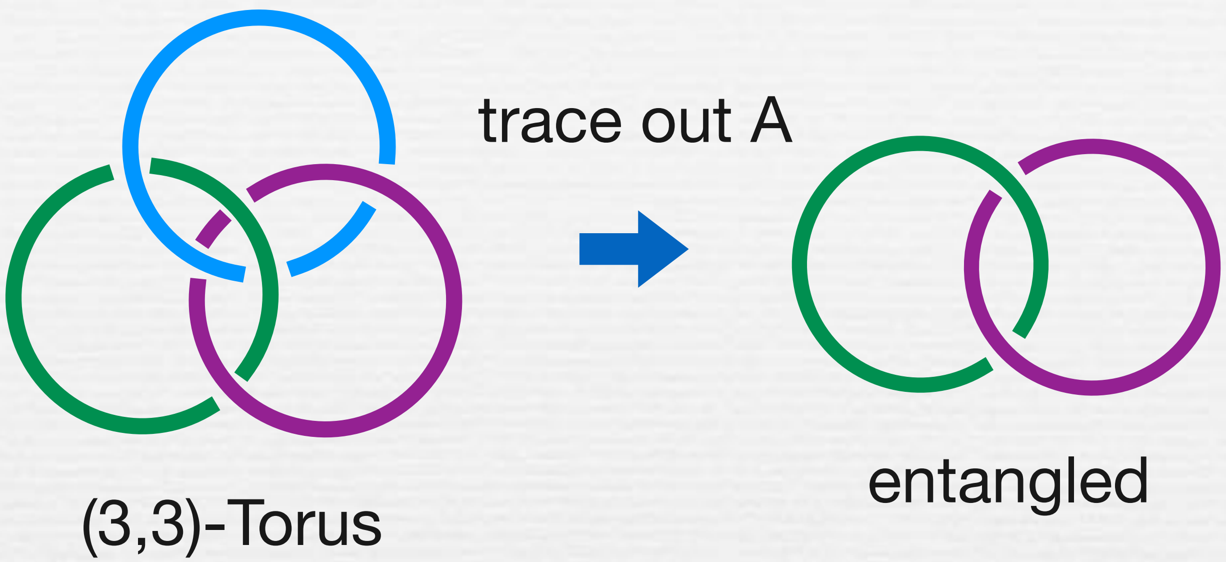
$$|\mathrm{W}\rangle = \frac{1}{\sqrt{3}}(|001\rangle + |010\rangle + |001\rangle)$$

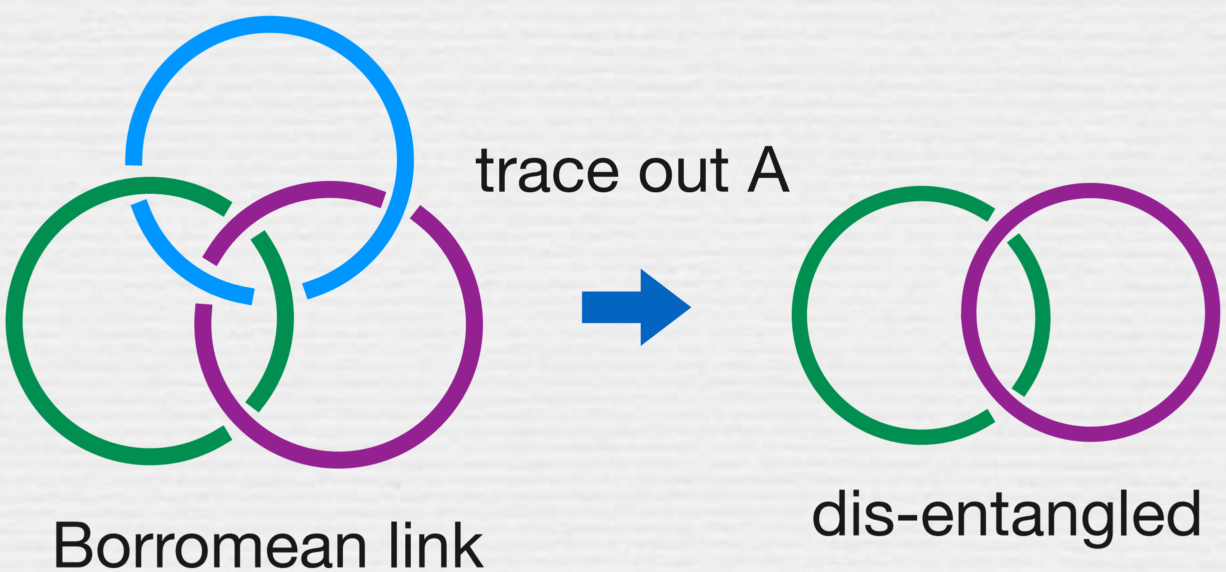
$\mathcal{N}(\mathrm{A} : \mathrm{BC}) = \sqrt{2}/3$ $\quad \mathcal{N}(\mathrm{B} : \mathrm{C}) = \frac{\sqrt{5}-1}{6}$

$$|\mathrm{GHZ}\rangle = \frac{1}{\sqrt{2}}(|111\rangle + |000\rangle)$$

$\mathcal{N}(\mathrm{A} : \mathrm{BC}) = 1/2$ $\quad \mathcal{N}(\mathrm{B} : \mathrm{C}) = 0$

analogy of link

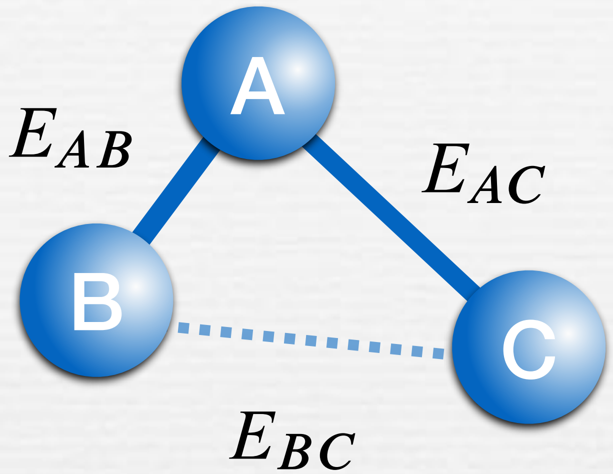




Way of entanglement sharing is restricted depending on structure of states: **entanglement monogamy**

Entanglement Monogamy

basic properties



E :entanglement measure

qubit system

square of concurrence, negativity

Gaussian system

square of negativity

monogamy relation of entanglement

$$E_{\mathrm{A|BC}} \geq E_{\mathrm{AB}} + E_{\mathrm{AC}}$$

V.Coffman, J.Kundu, W.K.Wootters 1998

T.J.Osbone, F.Verstraete 2006

G.Adesso, F.Illuminati 2006

trade off relation between E_{AB} and E_{AC}

- universal relation characterizing multi-partite entanglement
- may provide upper bound of E_{AB} and E_{AC}
- sharing of quantum information, no-cloning theorem

In this talk:

Emergence of multi-partite entangled state and separable state

Hawking radiation (BH evaporation)

Primordial quantum fluctuations (cosmic inflation)

- Quantum circuit model of BH evaporation
- Entanglement harvesting of quantum field in de Sitter space
- Summary

References

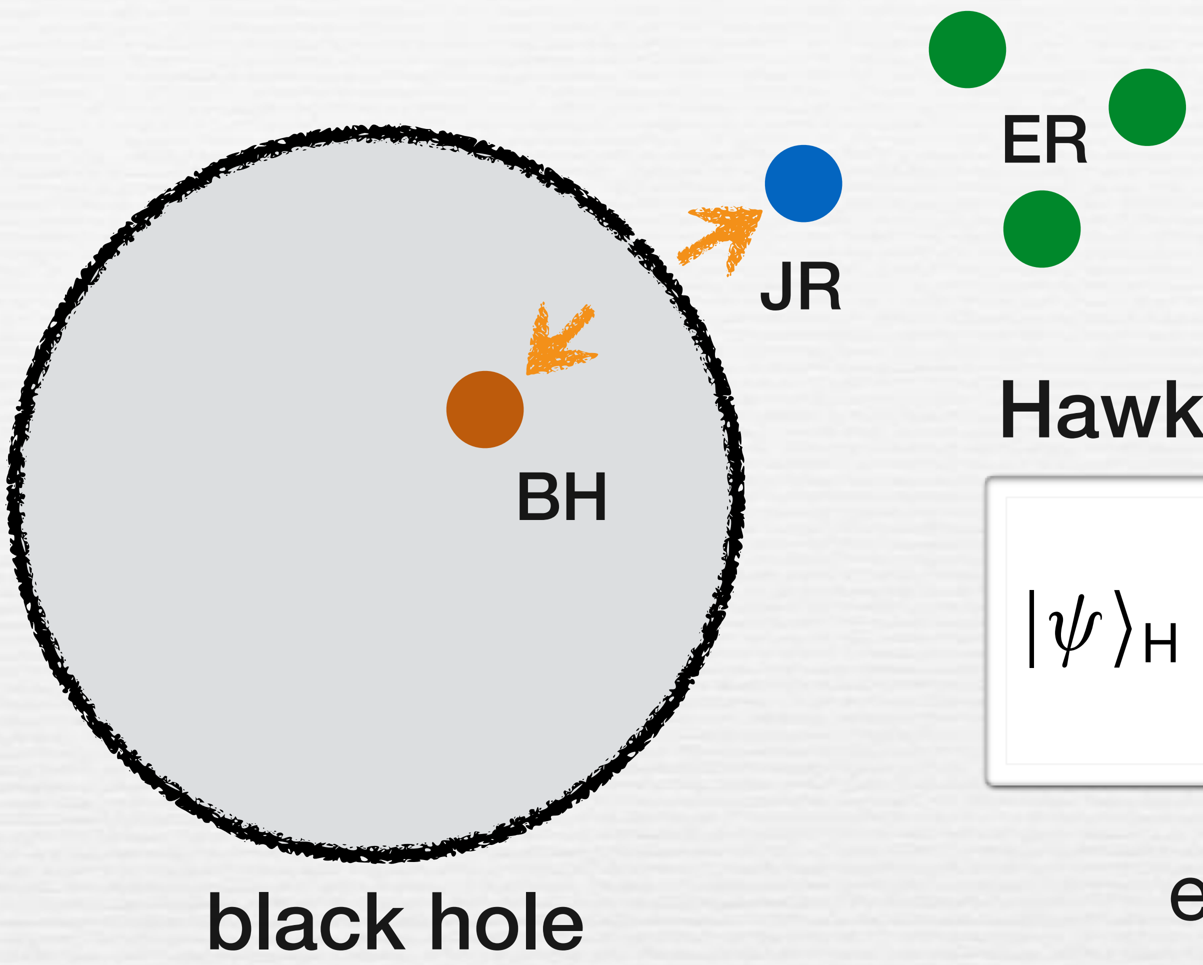
T.Tokuzumi, A.Matsumura, Y.Nambu
“Quantum circuit model of black hole evaporation”, CQG 35 (2018) 235013

A.Matsumura, Y.Nambu
“Large scale quantum entanglement in deSitter spacetime”, PRD 98 (2018) 025004

S.Kukita, Y.Nambu
“Harvesting large scale entanglement in deSitter space with multiple detectos”
Entropy 19 (2017) 449

Quantum Circuit Model of Black Hole Evaporation

Black Hole Evaporation and Monogamy



Hawking radiation

$$|\psi\rangle_{\mathrm{H}} = \prod_i \sum_{n_i=0}^{\infty} e^{-4\pi M \omega_i n_i} |n_i\rangle_{\mathrm{BH}} \otimes |n_i\rangle_{\mathrm{JR}}$$

entangled pair of particles

thermal property $T_H = \dfrac{1}{8\pi M}$

We want to know about

Final state of evaporating system ?

Possible scenario, remnant ?

information issue ?

Page's Theorem

A+B: pure state   If $N_A \ll N_B$ for a typical state, entanglement entropy

A	B

$$\langle S_A \rangle \approx \log N_A \qquad \rho_A \propto I_A$$

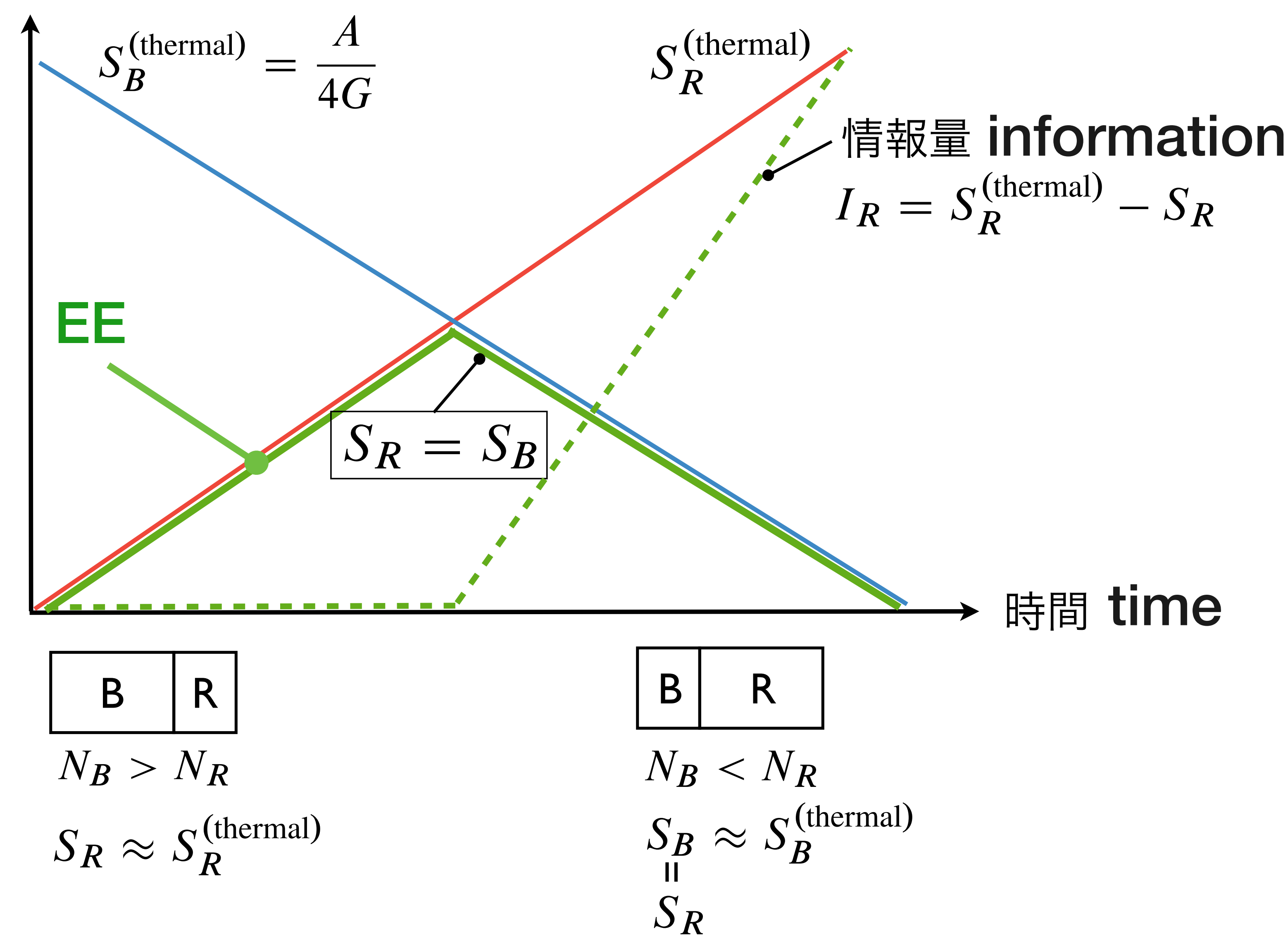
A and B are maximally entangled

Page Curve

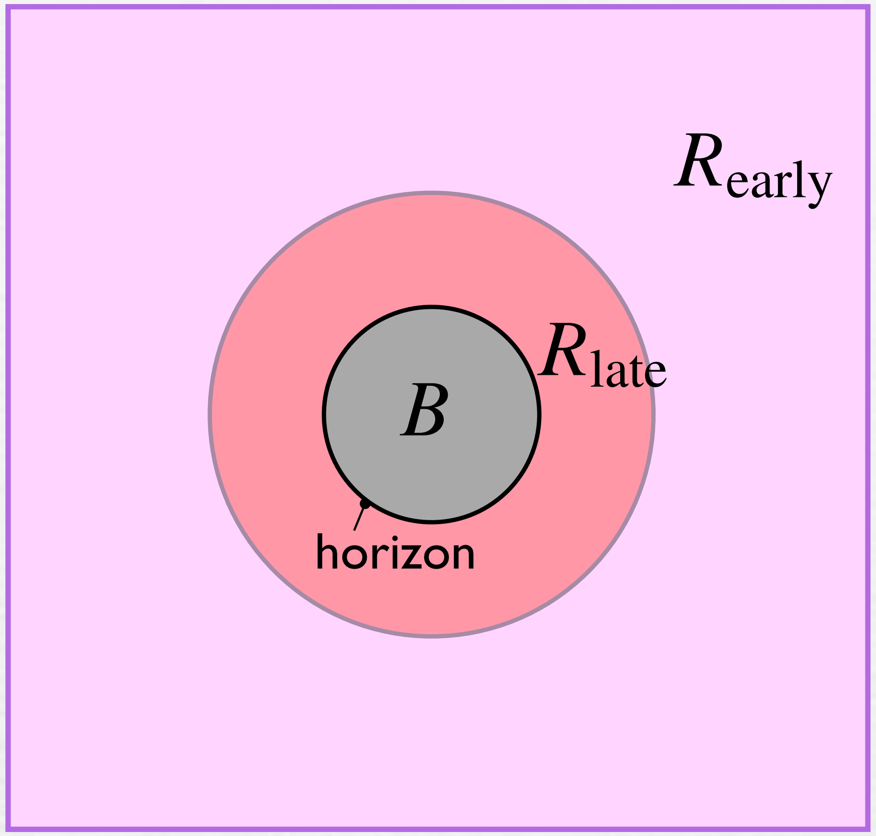
Without detail of models, we can follow evolution of EE during evaporation if we adopt unitarity

Page time

S_{EE}: max



Firewall? AMPS *Almheiri, Marolf, Polchinski and Sully 2013*



After Page time,

$$N_{R_{\rm early}} \gg N_B N_{R_{\rm late}}$$

B+$R_{\rm late}$ are maximally entangled with $R_{\rm early}$

$$\rightarrow \quad \rho_{B+R_{\rm late}} \propto I_{B+R_{\rm late}}$$

Correlation of field between B and $R_{\rm late}$ is lost and high energetic curtain emerges around the horizon

- Emergence of field separable (product) state

Effect of multipartite entanglement:
entanglement cannot be freely distributed

Does quantum state really evolve like this scenario during evaporation?

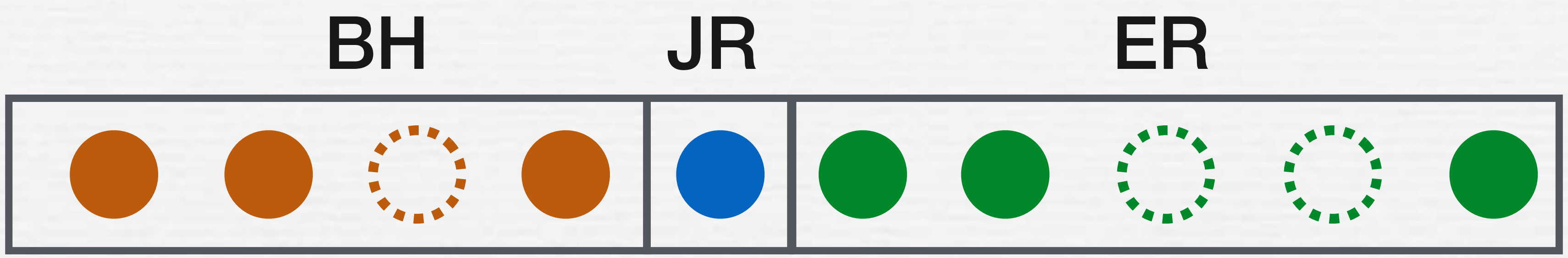
- We investigate this issue using a simple model of evaporation

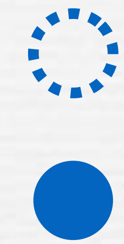
Quantum circuit model of BH evaporation

T.Tokuzumi, A. Matsumura & YN, 2018

Rule1 : the system is composed of n qubits

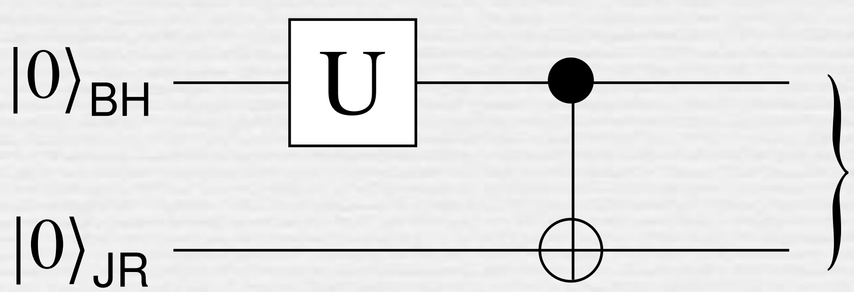
$$\mathcal{H}_{\rm tot} = \mathcal{H}_{\rm BH} \otimes \mathcal{H}_{\rm JR} \otimes \mathcal{H}_{\rm ER}.$$





$$|\psi\rangle = |{\rm BH}\rangle \otimes |{\rm JR}\rangle \otimes |{\rm ER}\rangle,$$

Rule2 : entanglement between Hawking particles is generated by CNOT-U gate



$$\cos\gamma\, |0\rangle_{\rm BH}|0\rangle_{\rm JR} + \sin\gamma\, |1\rangle_{\rm BH}|1\rangle_{\rm JR}$$

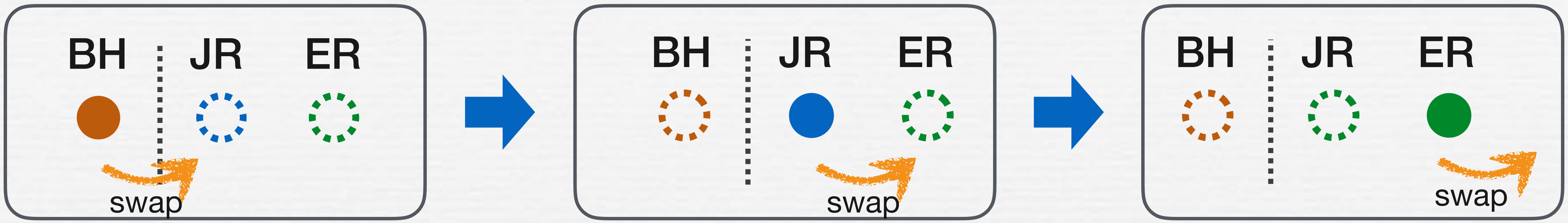
$$\longleftrightarrow \quad |0\rangle_{\rm BH}|0\rangle_{\rm JR} + \exp(-4\pi M\omega)\, |1\rangle_{\rm BH}|1\rangle_{\rm JR}$$

$$\tan\gamma = \exp\left(-4\pi M\omega\right)$$

Mass is evolved following formula of Bekenstein-Hawking entropy

$$M_n = M_0\sqrt{1 - \frac{n-1}{N_{\rm BH}}}$$

Rule3 : created particles are moved outward via SWAP operation



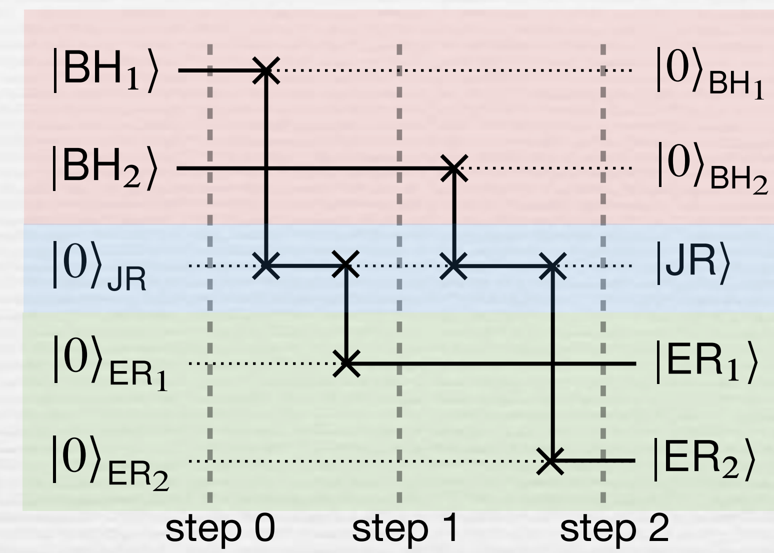
For initial state

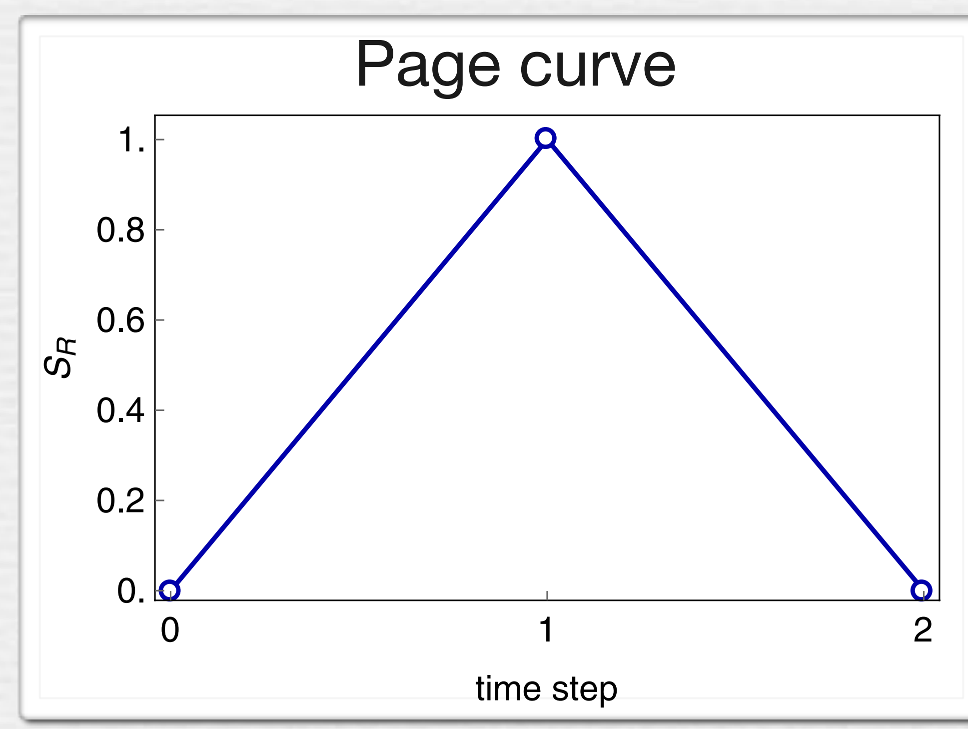
$$|\psi_0\rangle = \frac{1}{\sqrt{2}}\Big[|10\rangle + |01\rangle\Big]_{\mathrm{BH}_{12}} |0\rangle_{\mathrm{JR}} |00\rangle_{\mathrm{ER}_{12}}$$

$$|\psi_1\rangle = \frac{1}{\sqrt{2}} |0\rangle_{\mathrm{BH}_1} |0\rangle_{\mathrm{JR}} \Big[|01\rangle + |10\rangle\Big]_{\mathrm{BH}_2,\mathrm{ER}_1} |0\rangle_{\mathrm{ER}_2}$$

$$|\psi_2\rangle = \frac{1}{\sqrt{2}} |00\rangle_{\mathrm{BH}_{12}} |0\rangle_{\mathrm{JR}} \Big[|10\rangle + |01\rangle\Big]_{\mathrm{ER}_{12}}$$

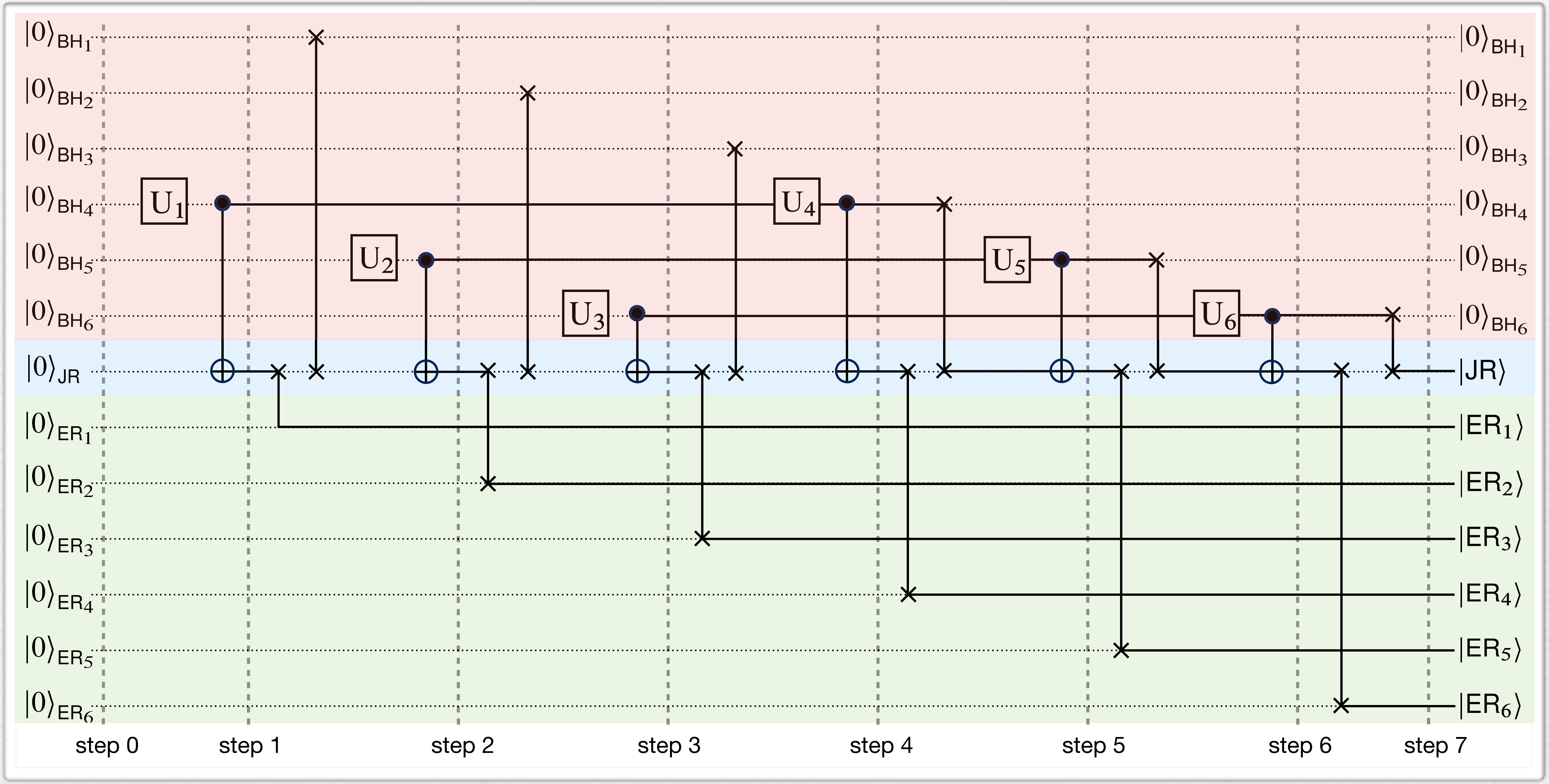
Entanglement of BH is transferred to Radiation by SWAP operations





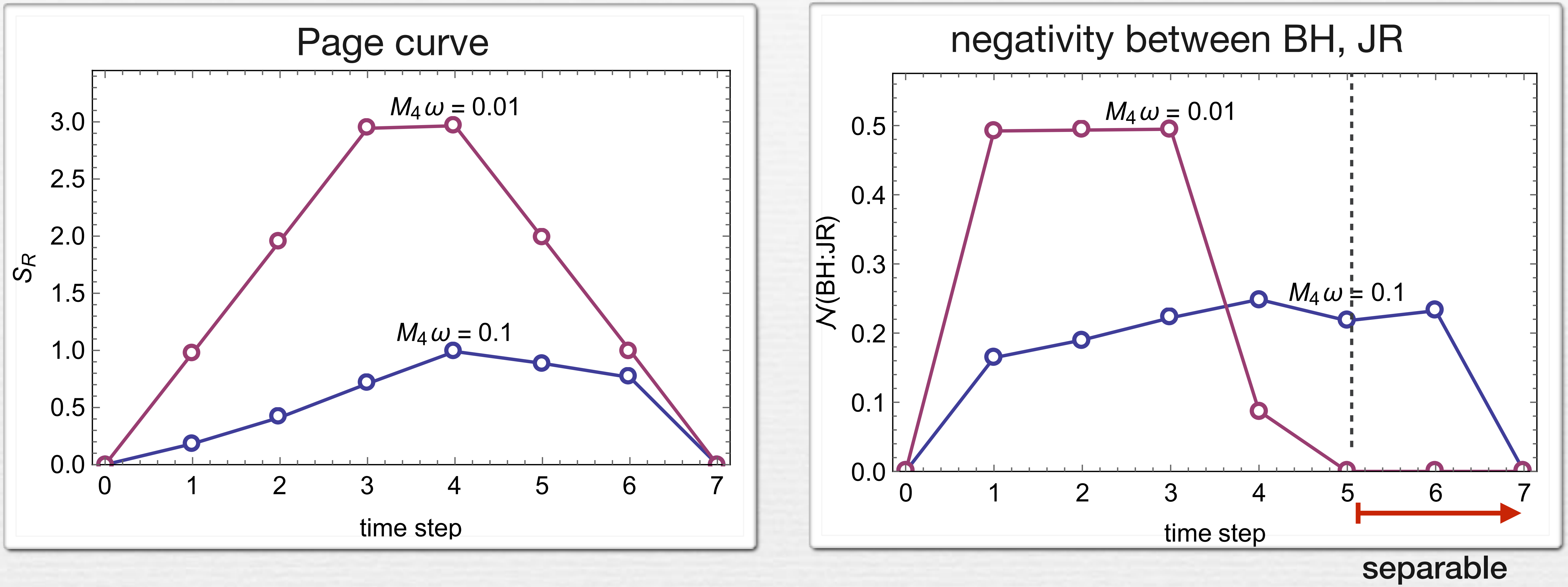
Our circuit model

13 qubits $|\text{BH}| = 2^6$ $|\text{JR}| = 2^1$ $|\text{ER}| = 2^6$



This circuit is just one example realizing basic features of Hawking radiation and black hole evaporation

Result and interpretation



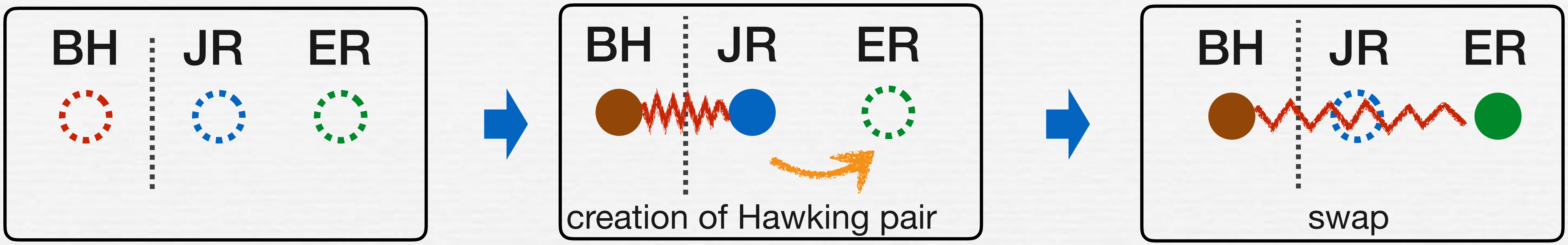
- Information contained in BH is transferred to Radiation

  shape of Page curve depends of mode frequency

- After Page time, BH:JR becomes separable for low frequency modes

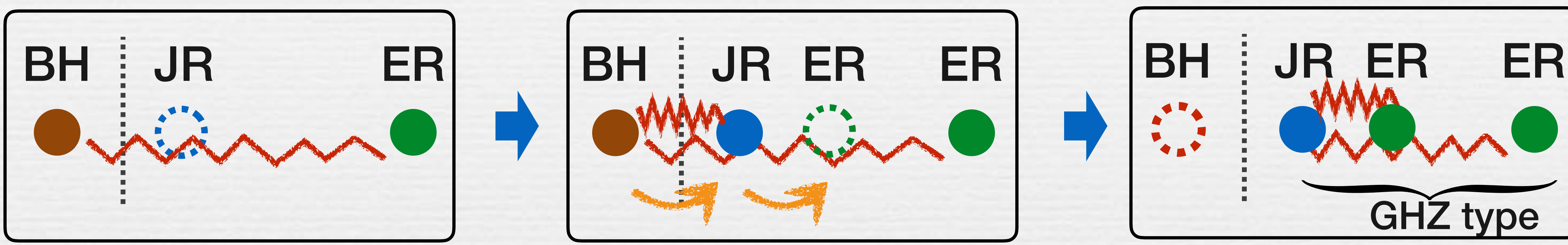
  “firewall” structure but classical correlation remains (separable state)

Before Page time



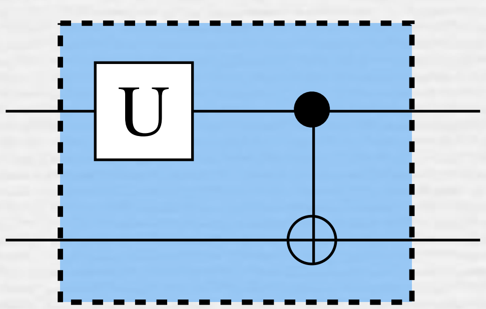
BH:ER pair is created as a entangled pair

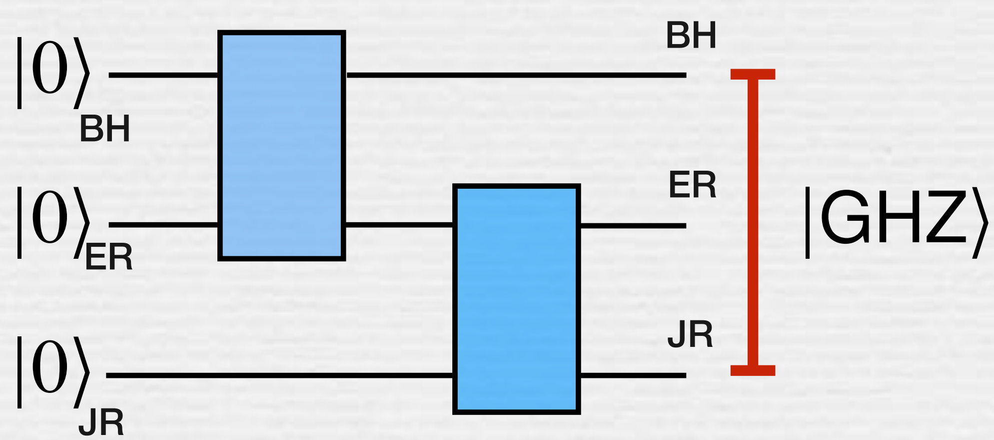
After Page time



After successive application of CNOT-U gate, GHZ type state is created

CNOT-U gates create GHZ type state



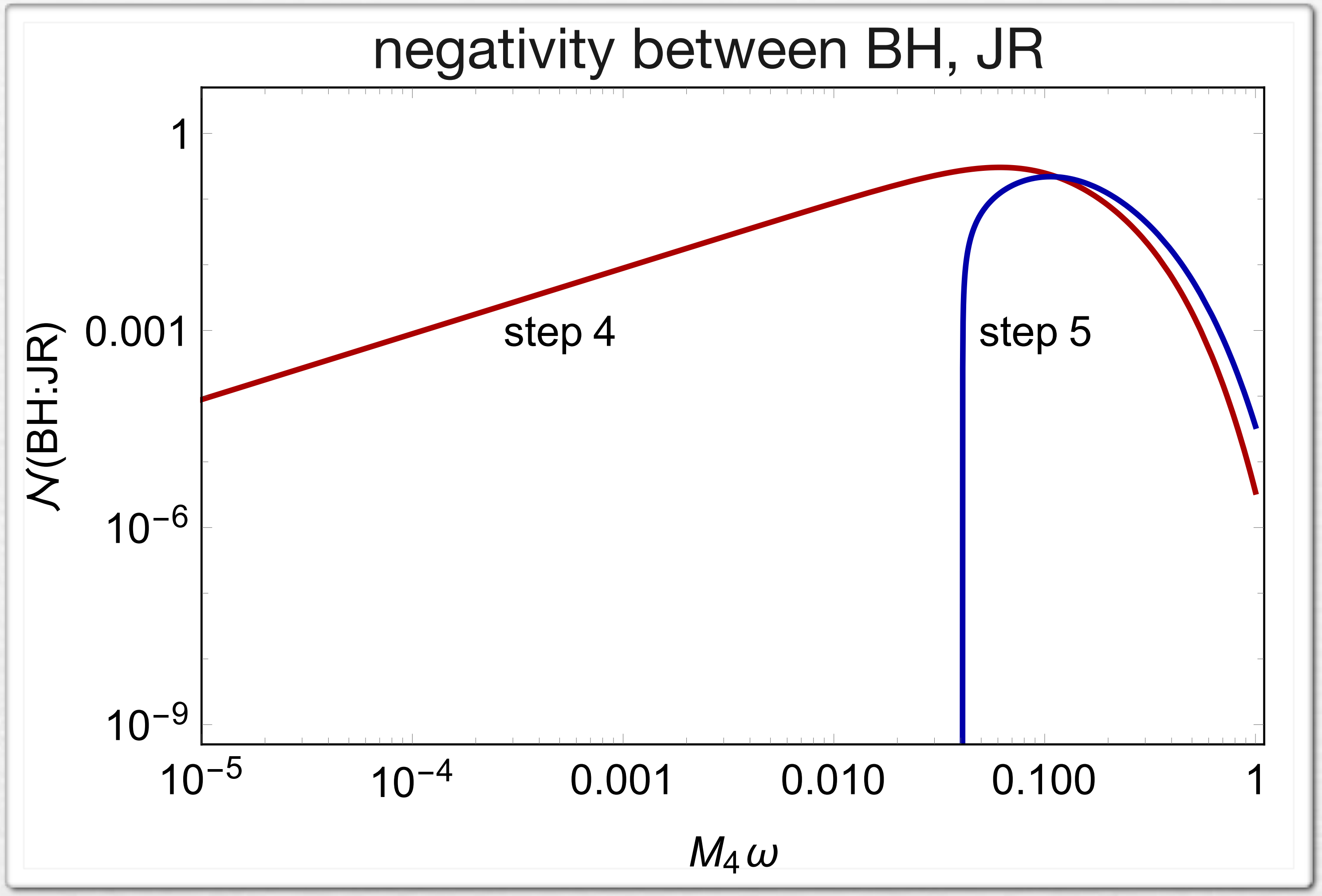


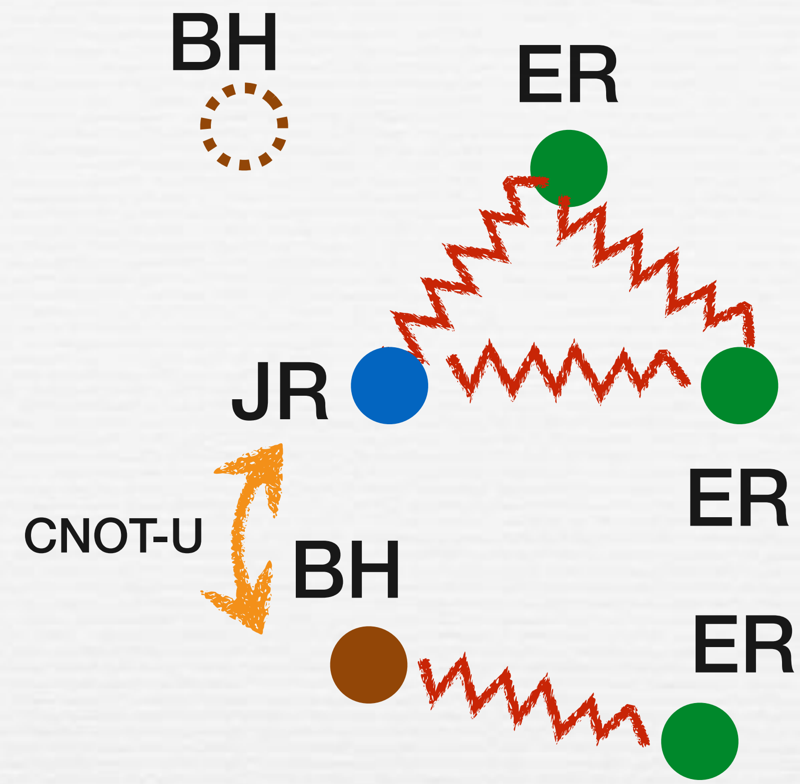
$T_H \to \infty$ U=H

exact GHZ state

(maximally entangled)

After Page time





CNOT-U gate tries to establish a new correlation between GHZ type state and previously generated Hawking pair

cannot establish new correlation due to monogamy of entanglement

BH: JR becomes separable for $M_4\omega \leq 0.041$

low frequency ⟷ maximally entangled state

Monogamous property of multipartite state is related to emergence of separable state (firewall)

Entanglement Harvesting in de Sitter space

Quantum Effect in Expanding Universe

- Quantum field (scalar field, EM field, GW)

$$\hat{\varphi}(\eta, \boldsymbol{x}) = \int \frac{d^3k}{(2\pi)^{3/2}} \hat{\varphi}_{\boldsymbol{k}}(\eta) e^{i\boldsymbol{k}\cdot\boldsymbol{x}}$$

$$\hat{\varphi}_{\boldsymbol{k}} = \frac{1}{\sqrt{2k}} \left(\hat{c}_{\boldsymbol{k}} + \hat{c}^{\dagger}_{-\boldsymbol{k}}\right)$$

$$\hat{p}_{\boldsymbol{k}} = i\sqrt{\frac{k}{2}} \left(\hat{c}^{\dagger}_{\boldsymbol{k}} - \hat{c}_{-\boldsymbol{k}}\right)$$

Hamiltonian in expanding universe

$$\hat{H} = \int d^3k \left[\frac{k}{2}\left(\hat{c}^{\dagger}_{\boldsymbol{k}}\hat{c}_{\boldsymbol{k}} + \hat{c}_{\boldsymbol{k}}\hat{c}^{\dagger}_{\boldsymbol{k}}\right) + i\frac{a'}{a}\left(\hat{c}^{\dagger}_{\boldsymbol{k}}\hat{c}^{\dagger}_{-\boldsymbol{k}} - \hat{c}_{\boldsymbol{k}}\hat{c}_{-\boldsymbol{k}}\right)\right]$$

squeezing op: generate entanglement between $\boldsymbol{k}, -\boldsymbol{k}$
scale factor: a

$$\begin{pmatrix} \hat{c}_{\boldsymbol{k}}(\eta) \\ \hat{c}^{\dagger}_{-\boldsymbol{k}}(\eta) \end{pmatrix} = \begin{pmatrix} \alpha_k & \beta_k \\ \beta^*_k & \alpha^*_k \end{pmatrix} \begin{pmatrix} \hat{c}_{\boldsymbol{k}}(\eta_0) \\ \hat{c}^{\dagger}_{-\boldsymbol{k}}(\eta_0) \end{pmatrix}$$

Bogoluibov tranformation

$$\alpha_k = \cosh r_k, \quad \beta_k = \sinh r_k$$

particle number

$$\langle \hat{c}^{\dagger}_{\boldsymbol{k}}(\eta)\hat{c}_{\boldsymbol{k}}(\eta)\rangle = |\beta_k|^2$$

$$|\psi\rangle = \frac{1}{\cosh r_k} \sum_{n=0}^{\infty} (\tanh r_k)^n |n_{\boldsymbol{k}}\rangle \otimes |n_{-\boldsymbol{k}}\rangle$$

vacuum state evolves to 2 mode squeezed state by accelerated expansion of the universe

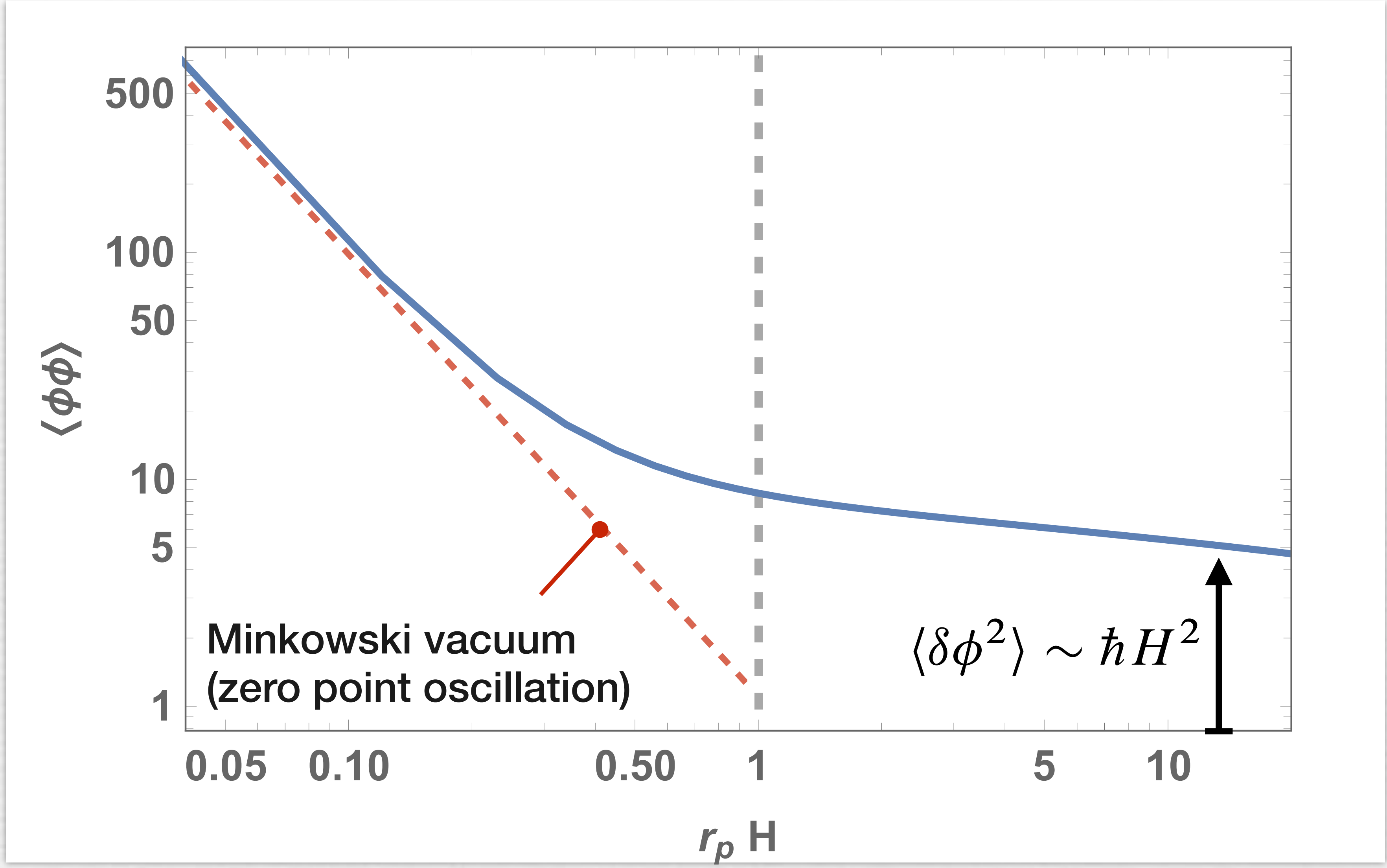
squeezing parameter

$r_k \sim 100$ for cosmic inflation

r→∞ EPR state

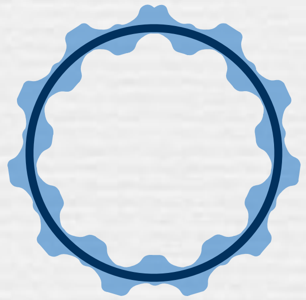
Correlation Function in de Sitter Space massless minimal scalar

$$\langle \delta\phi(\boldsymbol{x}_1)\delta\phi(\boldsymbol{x}_2)\rangle = \frac{\hbar}{4\pi^2 r_p^2} - \frac{\hbar H^2}{4\pi^2}\log\left(H r_p\right) + \frac{\hbar H^3 t}{4\pi^2}$$



curvature fluctuation

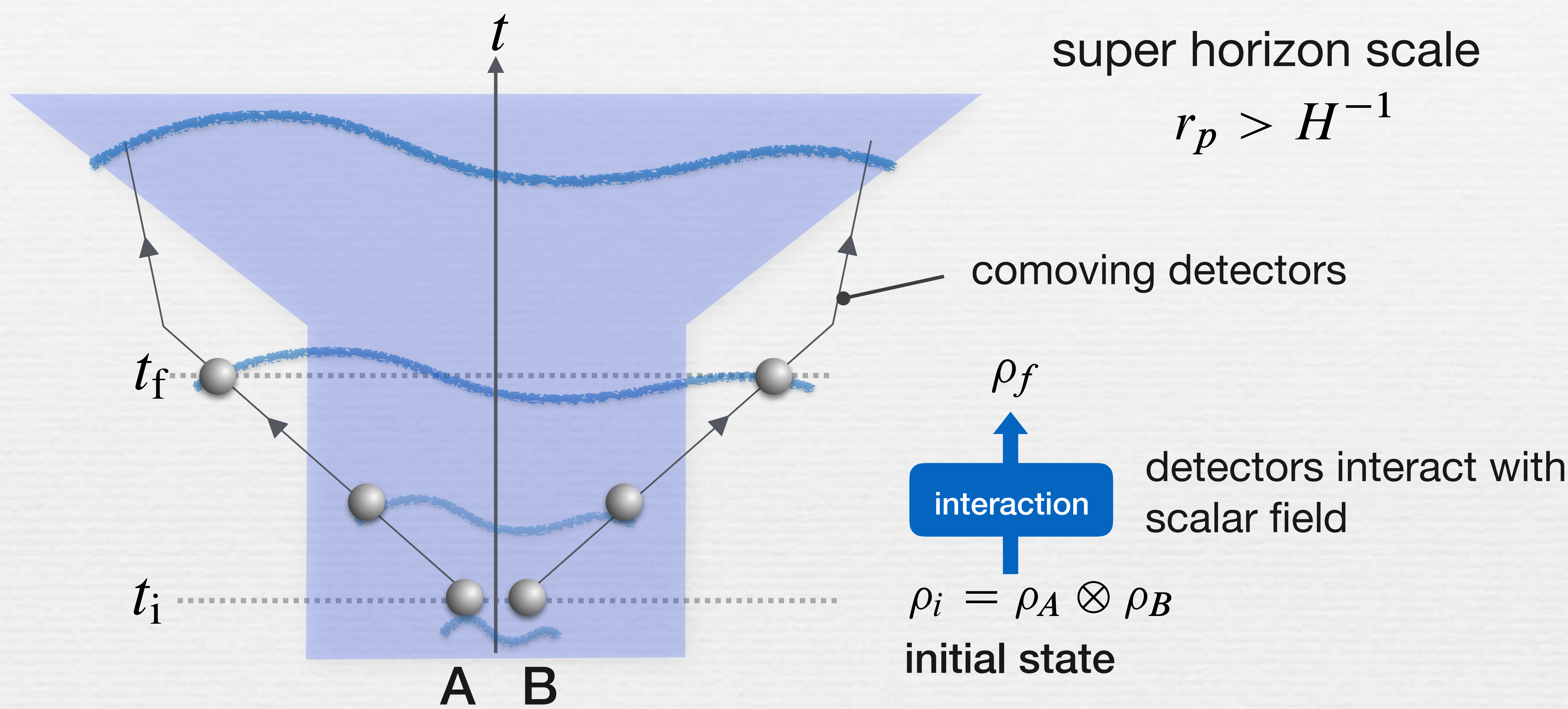
$$\langle |\hat{\mathcal{R}}_k|^2\rangle \sim \left(\frac{H}{\dot{\phi}}\right)^2 \times \langle |\delta\hat{\phi}_{\boldsymbol{k}}|^2\rangle$$



Cosmic inflation generate large scale quantum fluctuations beyond horizon scale of deSitter space $r_p > H^{-1}$ origin of primordial fluctuation

Large Scale Entanglement in Cosmic Inflation

and entanglement harvesting of quantum fluctuations



detection of field entanglement using operational procedure

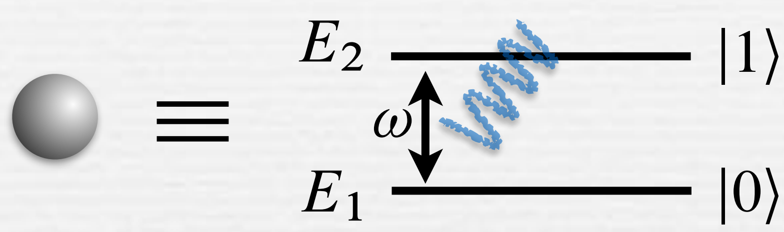
- theoretical justification of origin of primordial fluctuations
- experimental verification of quantumness of primordial fluctuations

Entanglement harvesting with quit detectors:

Detection of entanglement of quantum field

Read out entanglement of quantum field using entanglement between qubit detectors

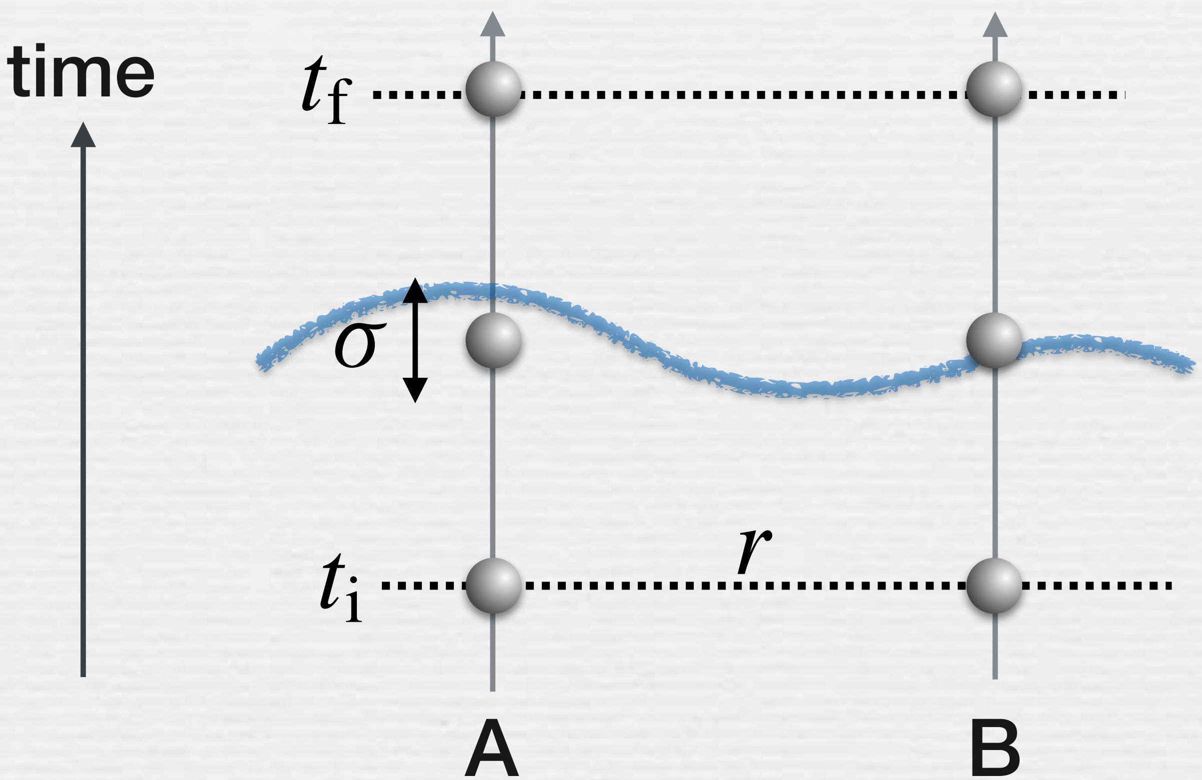
two level atom (qubit)



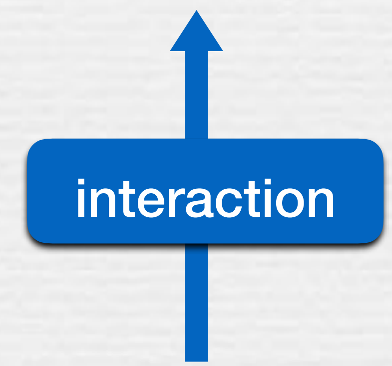
$$\hat{H}_{\rm int} = g(\hat{\sigma}_+ + \hat{\sigma}_-)\phi(x(\tau))$$

$$\hat{\sigma}_+ = |1\rangle\langle 0| \qquad \hat{\sigma}_- = |0\rangle\langle 1|$$

g: coupling with finite interval of time



$\rho_{\rm f}^{(d)}$ entangled



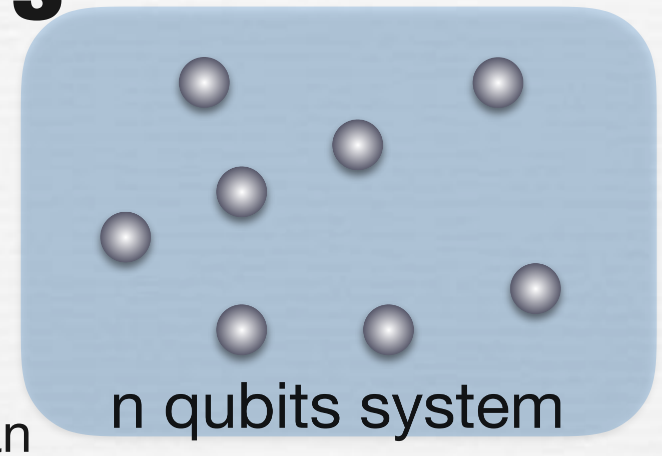
$\rho_{\rm i}^{(d)} = \rho_A \otimes \rho_B$

separable

Local operation cannot generate entanglement

→ a pair of detectors copies entanglement of quantum field

Entanglement harvesting with quit detectors

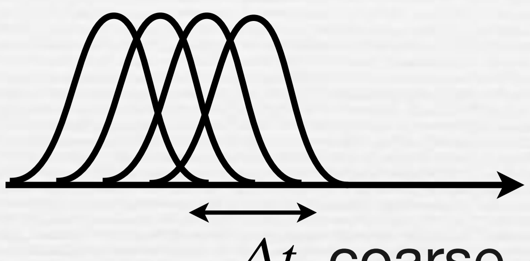


State of n-qubit system: ρ $\quad \dim \mathscr{H}(\rho) = 2^n$

Evolution of the state with time coarse-graining

GKLS master equation Gorni-Kossakowski-Lindblad-Sudarshan

$$\frac{d\rho}{dt} = -i\,[H_{\text{eff}}, \rho] + \underbrace{\left(2L\rho L^\dagger - \{L^\dagger L, \rho\}\right)}_{\mathscr{L}[\rho]}$$



- Markovian
- keeps complete positivity
- with dynamical time coarse-graining (without RWA)

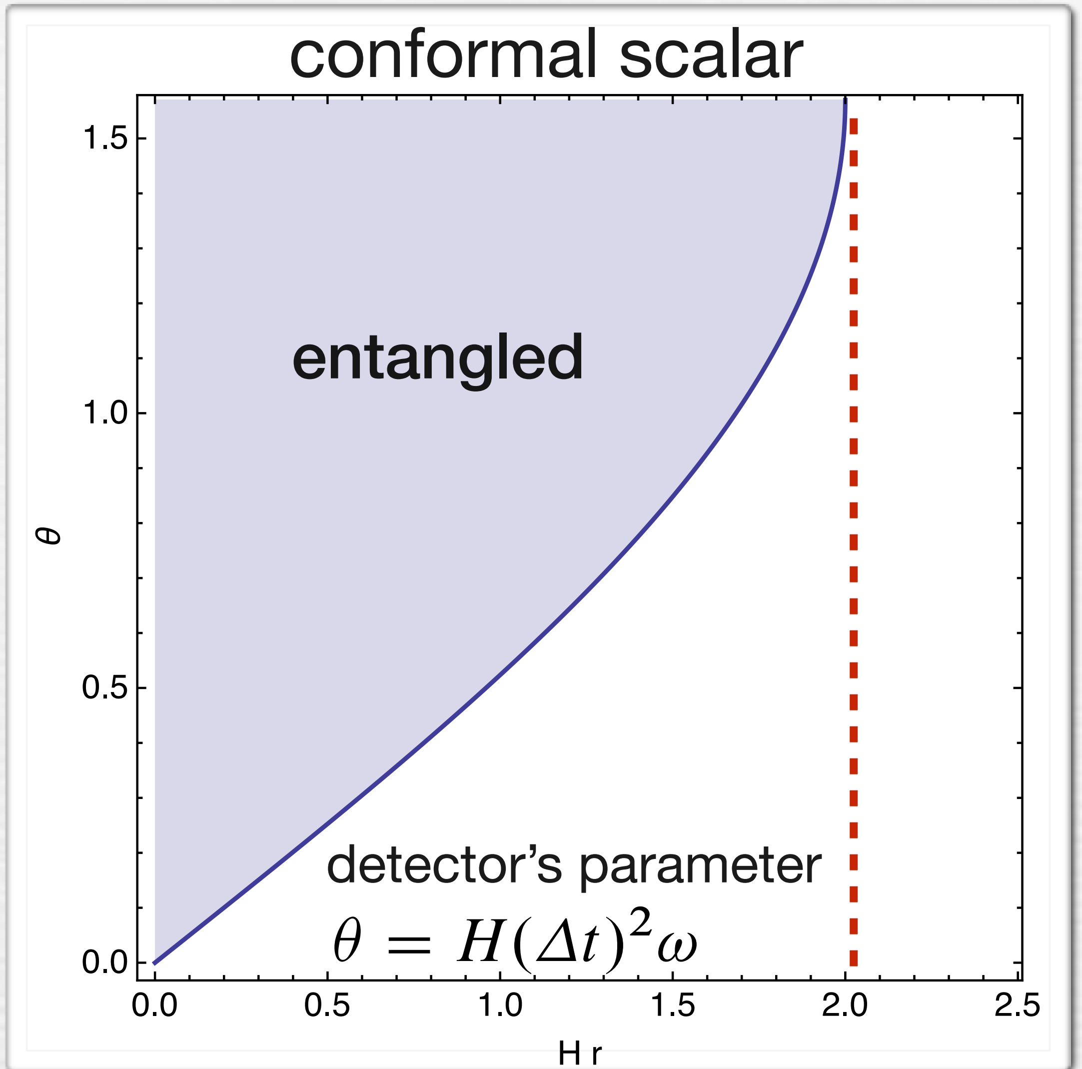
S. Kukita & YN, CQG 34 (2017), 235010

$$\mathscr{L}[\rho] = \frac{1}{2}\sum_{\alpha_1,\alpha_2}\sum_{j_1,j_2=\pm} C^{(\alpha_1\alpha_2)}_{j_1 j_2}\left[2\sigma^{(\alpha_2)}_{j_2}\rho\sigma^{(\alpha_1)}_{j_1} - \sigma^{(\alpha_1)}_{j_1}\sigma^{(\alpha_2)}_{j_2}\rho - \rho\sigma^{(\alpha_1)}_{j_1}\sigma^{(\alpha_2)}_{j_2}\right]$$

$$C^{(\alpha_1\alpha_2)}_{j_1 j_2} = \frac{2g^2 e^{-\omega^2\sigma^2} e^{i\omega\sigma j_+}}{\pi\sigma}\int_{-\infty}^{+\infty} dx\,dy\, e^{-\frac{1}{\sigma^2}\left[x-(\sigma+i\omega\sigma^2 j_+/2)\right]^2 - \frac{1}{\sigma^2}\left(y-i\omega\sigma^2 j_-/2\right)^2} D(r_c, t+x, y)$$

$$\Delta t = \sigma$$

Entanglement between 2 quit detectors

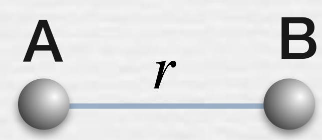


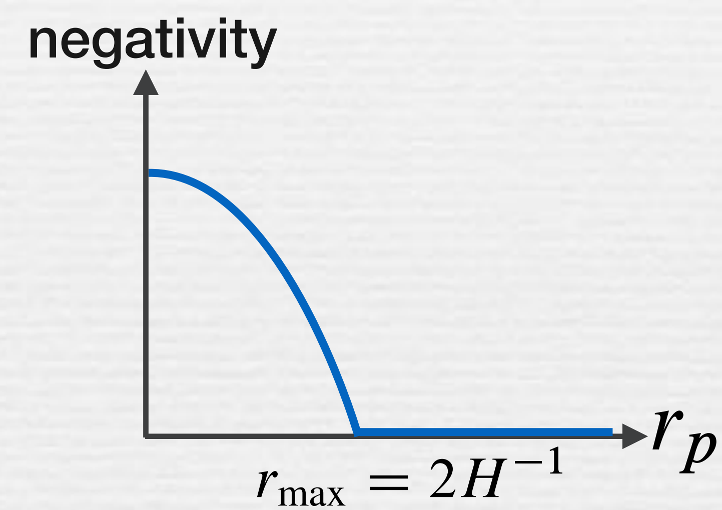
G.V.Steeg and N.C.Menicucci 2009
Y. Nambu and Y.Ohsumi 2011
E. Martin-Martinez and N.C.Menicucci 2016

$$\mathcal{N} = \left|C^{(r)}_{++}\right| - C^{(0)}_{-+}$$

non-local correlation $|00\rangle \to |11\rangle$

local noise $|00\rangle \to |10\rangle, |01\rangle$





- For large r, local noise destroys correlation and system becomes separable
- A pair of qubit detectors cannot access super horizon scale entanglement

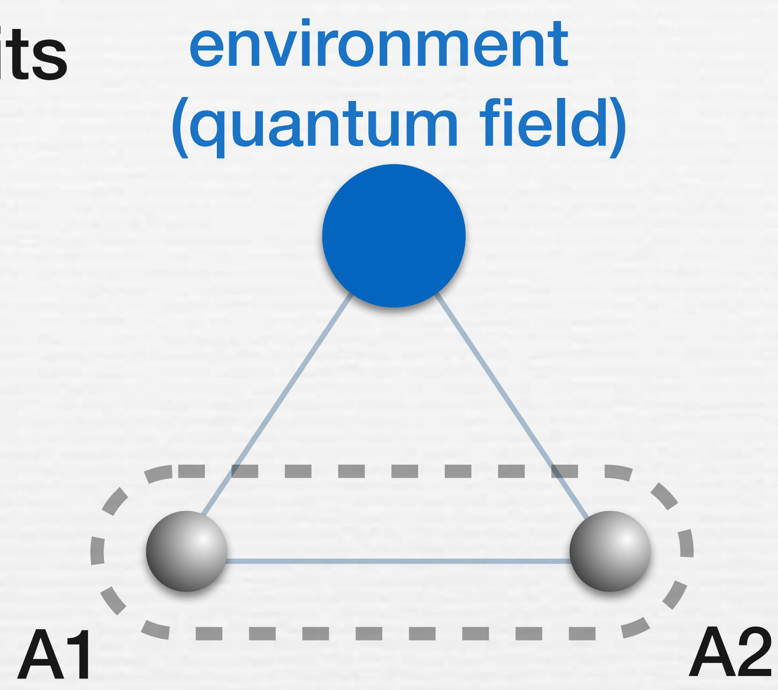
typical scale of entanglement $\sim H^{-1}$

Does this imply there is no quantum correlation in super horizon scale?

➡ We must check multi-partite entanglement effect on $r_{\max}$

Monogamy for Entanglement Harvesting

2 qubits



$$\mathcal{N} = \left| C^{(r)}_{++} \right| - C^{(0)}_{-+}$$

non-local correlation local noise

strength of entanglement between detectors and environment

We expect following monogamy inequality for total pure system

$$\tilde{E}_{\mathrm{env}\,A_1|A_2} \geq \tilde{E}_{\mathrm{env}|A_2} + E_{A_1|A_2}$$

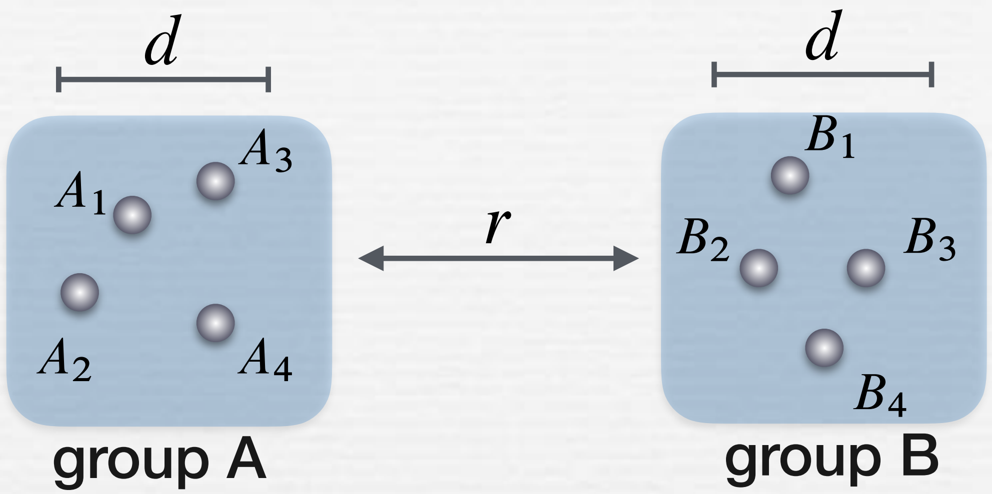
As entanglement between detectors and environment becomes maximal,

$$E_{A_1|A_2} \to 0$$

Dis-entanglement between detectors beyond horizon scale may be explained based on monogamous argument

Entanglement harvesting with (n+n) quit detectors

S. Kukita & YN, Entropy 19 (2017), 449



- negativity of n+n detector system
- analytic formula of negativity in terms of coefficients of the master eq.

State of 2n qubit system is obtained as solution of master equation

initial state $\rho_0 = \bigotimes_{j=1}^{n} |0_j\rangle\langle 0_j|$

under $\omega \Delta t \ll 1$

$$\rho \approx \rho_0 + \Delta t \left(2L\rho_0 L^\dagger - \{L^\dagger L, \rho_0\}\right)$$

$$\mathcal{N} = \left[mn\left|C^{(r)}_{++}\right|^2 + \left(\frac{m-n}{2}\right)^2 \left(C^{(d)}_{-+}\right)^2\right]^{1/2} - \left[C^{(0)}_{-+} + \left(\frac{m+n}{2} - 1\right) C^{(d)}_{-+}\right]$$

$$C^{(r)}_{-+} = 2g^2 e^{-\omega^2\sigma^2} \sigma D(r_c, t+\sigma, -i\omega\sigma^2)$$

local correlation $|00\rangle \to |10\rangle, |01\rangle$

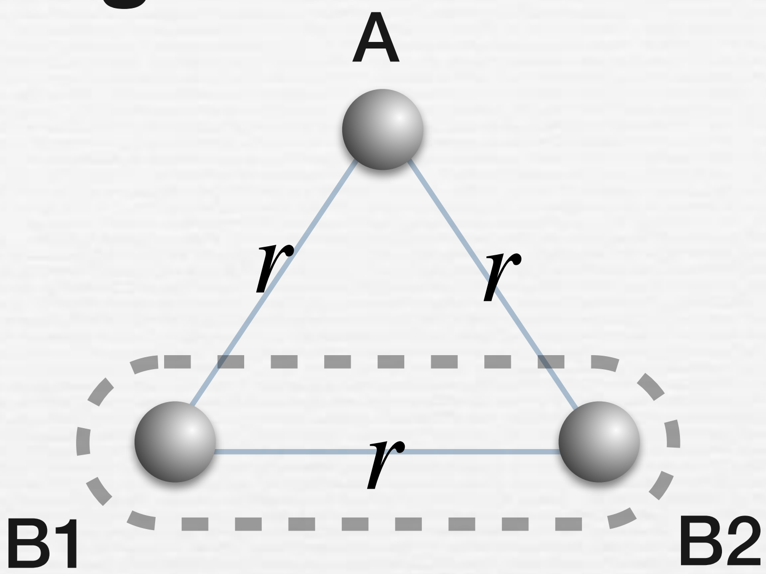
$$C^{(r)}_{++} = 2g^2 e^{-\omega^2\sigma^2} e^{2i\omega\sigma} \sigma D(r_c, t+\sigma+i\omega\sigma^2, 0).$$

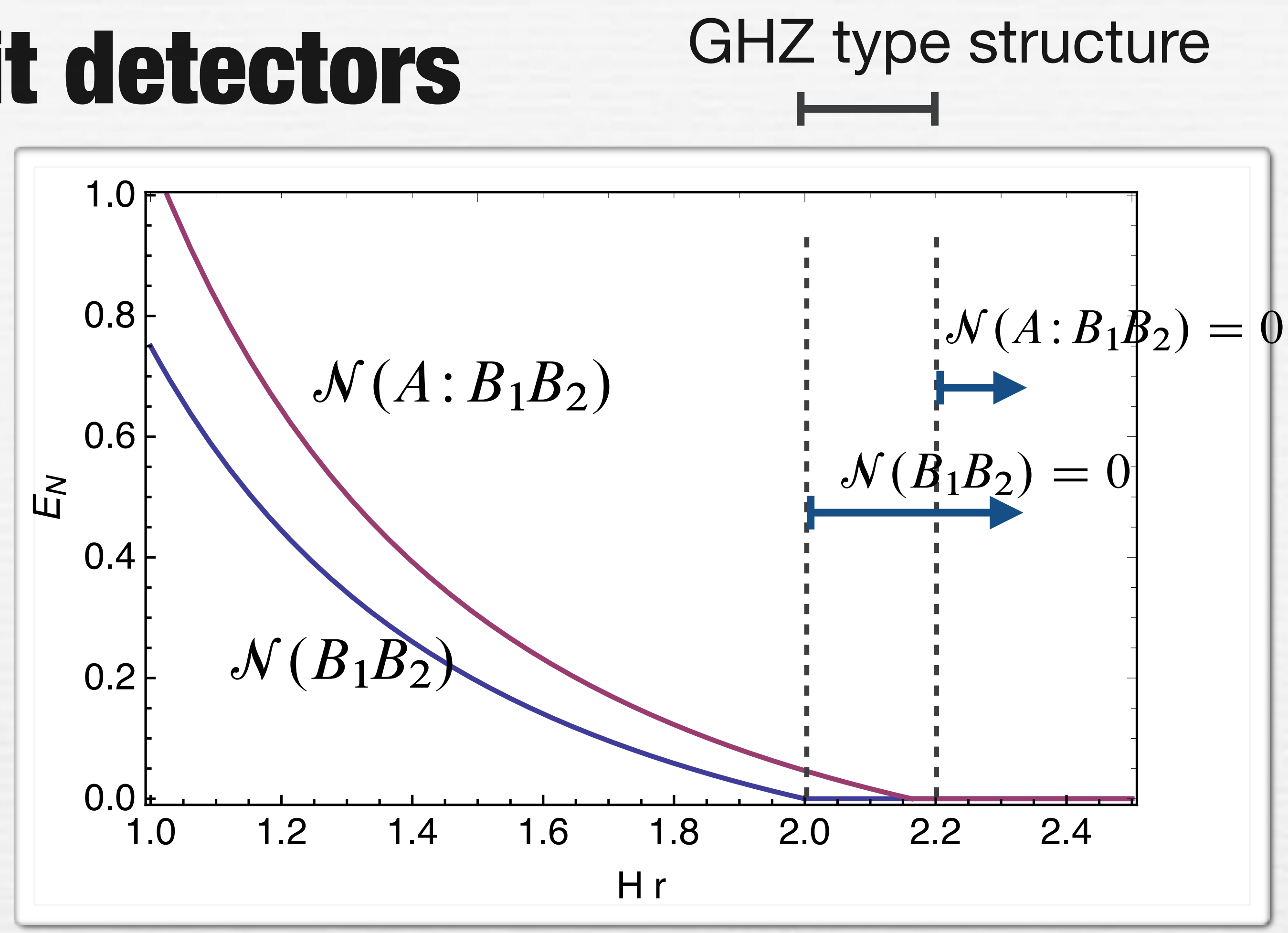
non-local correlation $|00\rangle \to |11\rangle$

σ : switching time scale

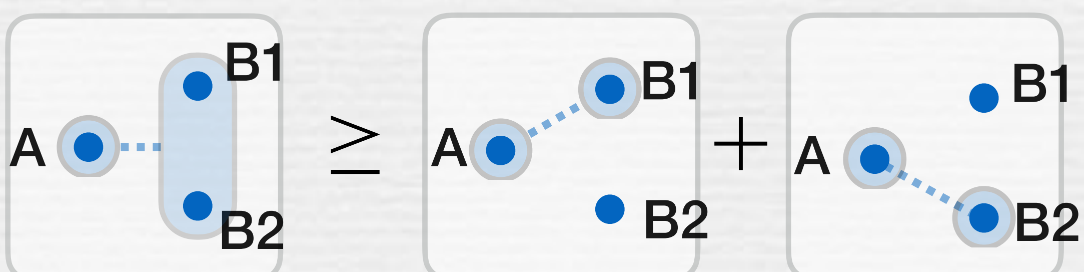
$$D(r_c, x, y) = \langle \phi(t_1, \boldsymbol{x}_{\alpha_1}) \phi(t_2, \boldsymbol{x}_{\alpha_2}) \rangle, \quad x = (t_1 + t_2)/2, \quad y = (t_1 - t_2)/2,$$

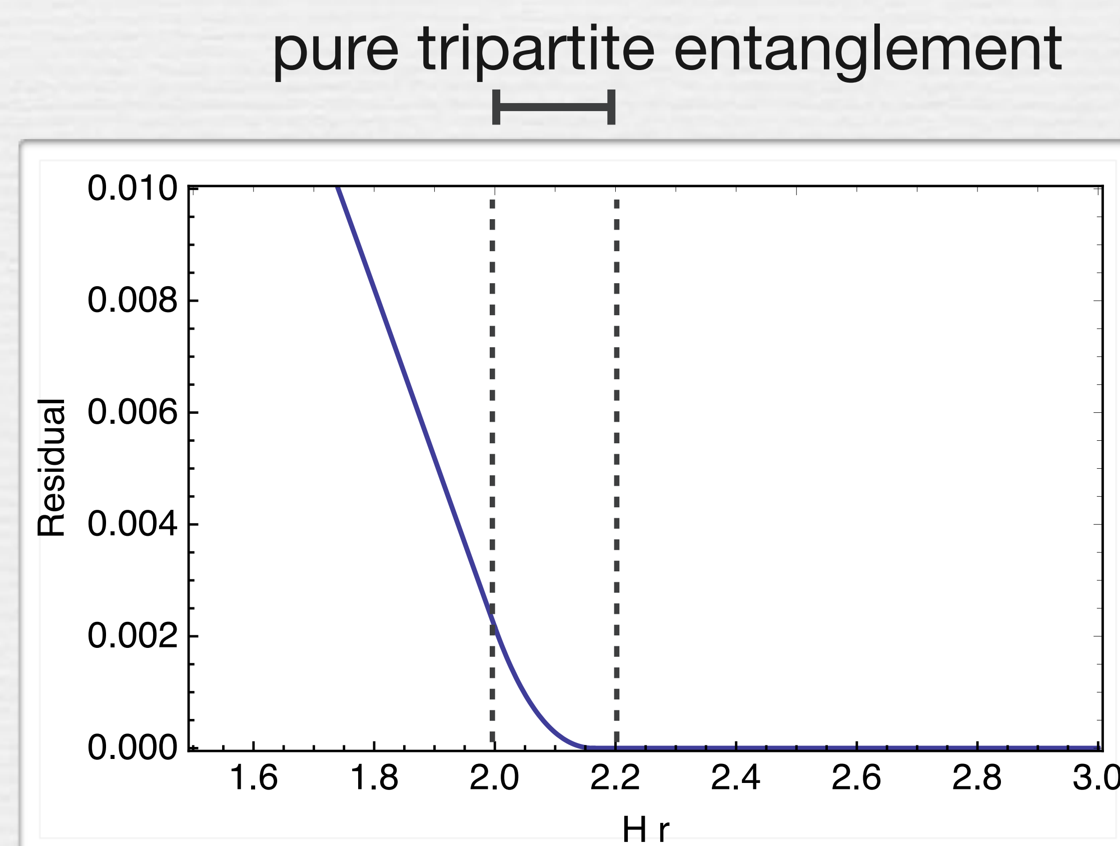
Entanglement between 3-quit detectors





For $2 \leq rH \leq 2.2$, GHZ type state appears





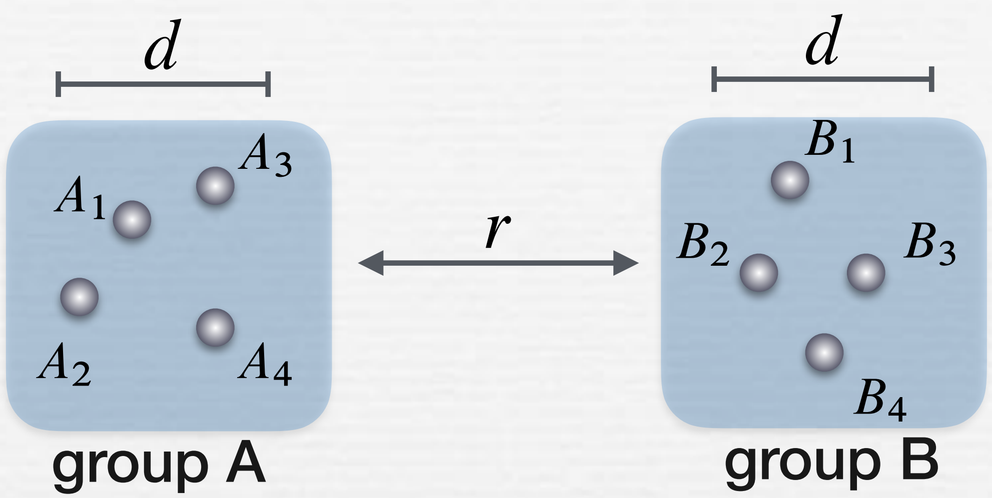
residual:=

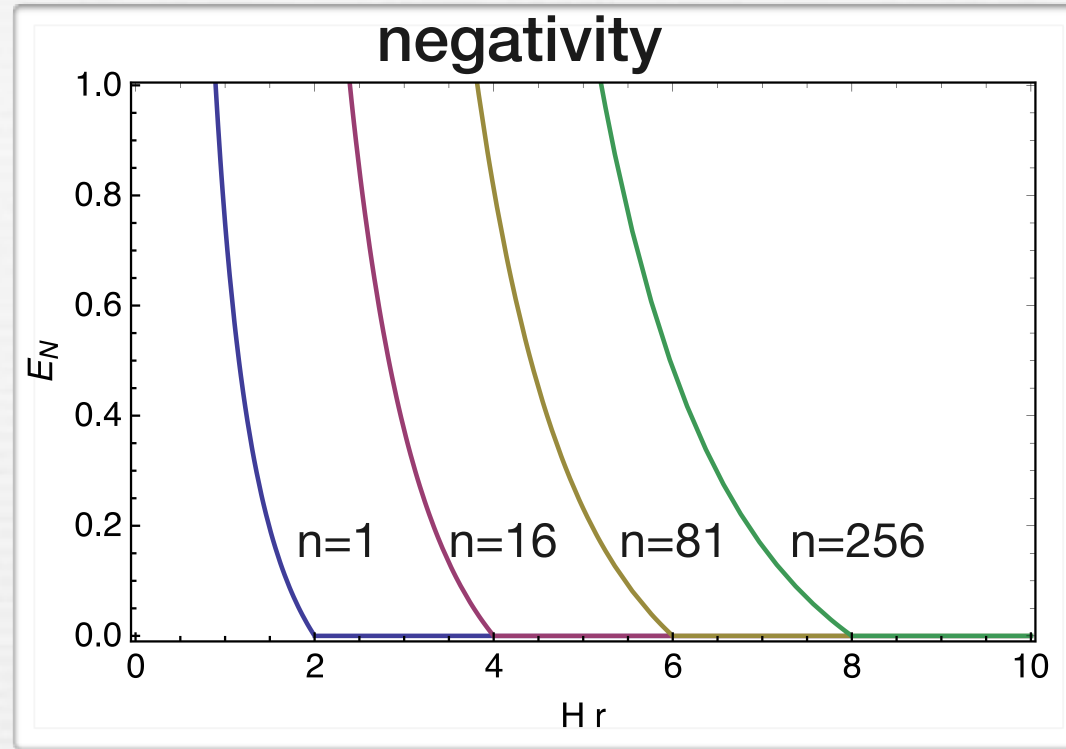
$$\mathcal{N}(A:B_1B_2)^2 - \mathcal{N}(A:B_1)^2 - \mathcal{N}(A:B_2)^2$$

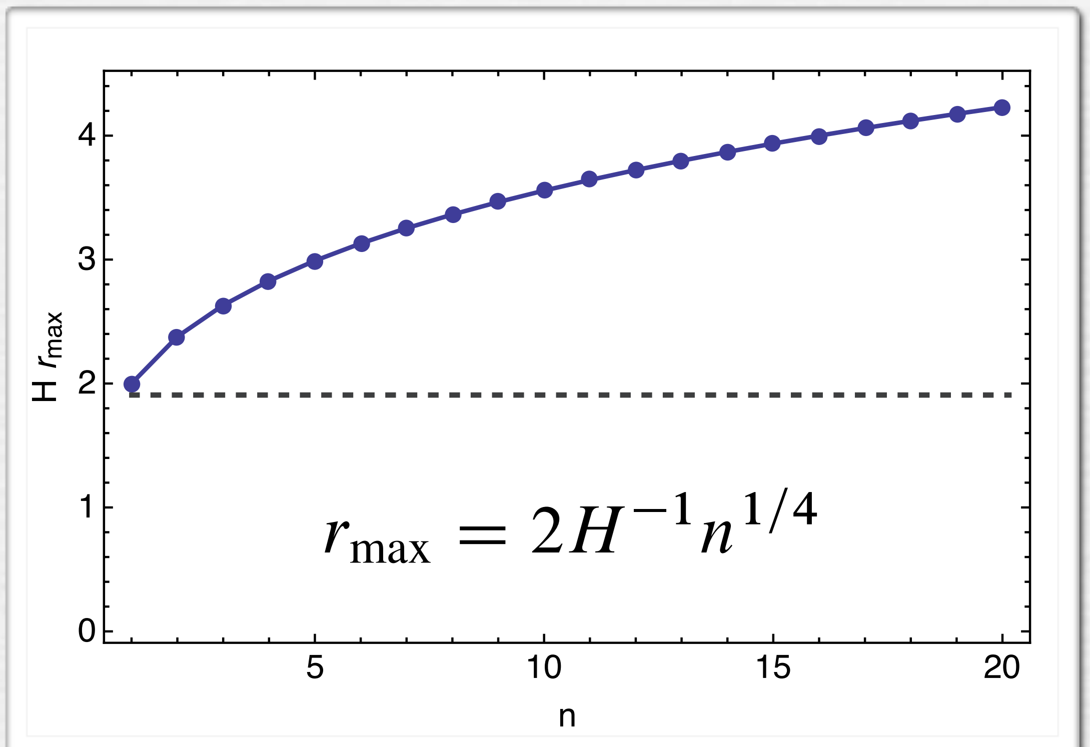
This quantity is positive by monogamy relation for qubits system

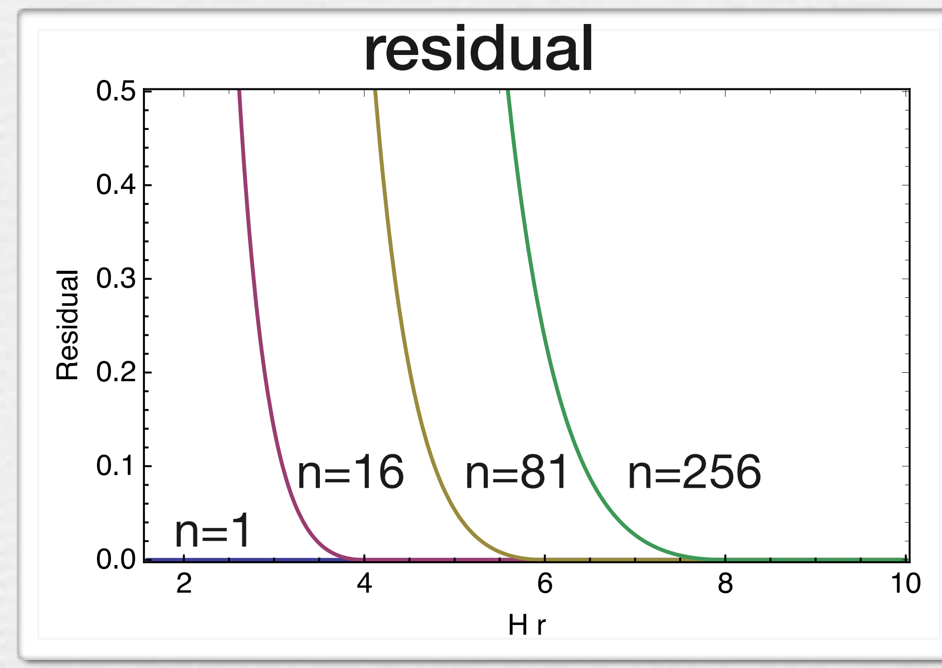
contribution of purely tripartite entanglement increases $r_{\max}$

Entanglement of (n+n)-quit detectors









- Effect of multipartite entanglement:
  increases non-local correlation and reduces local noise
- Detection of super horizon scale entanglement is possible
  with multiple detectors

Summary

Multipartite entanglement, monogamy and separable state

- Circuit model of BH evaporation

formation of GHZ type state

monogamy

emergence of separable state
between BH & R for low frequency

- Entanglement harvesting in de Sitter space

A pair of qubit detectors cannot reveal entanglement beyond
super horizon horizon

Local noise of de Sitter space kills quantum correlation
(but this is related to monogamy of entanglement)

This behavior can be understood from monogamy of
qubit-qubit-environment (quantum field)

For (n+n) detectors, it is possible to access entanglement of
quantum field beyond horizon scale

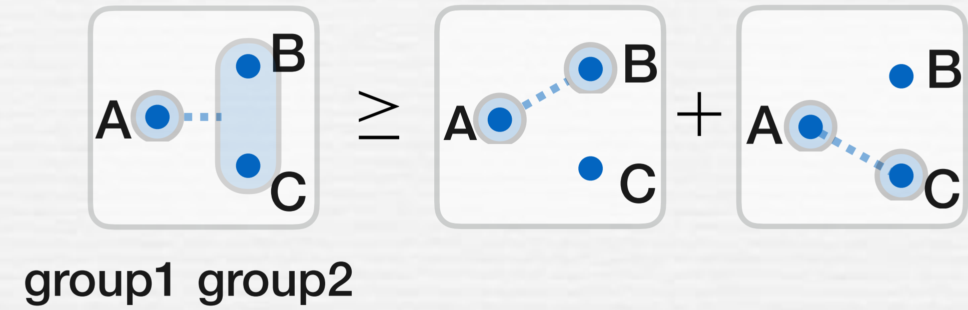
Multipartite entanglement is crucial to reduce local noise and
enhance non-local correlation

Back Up

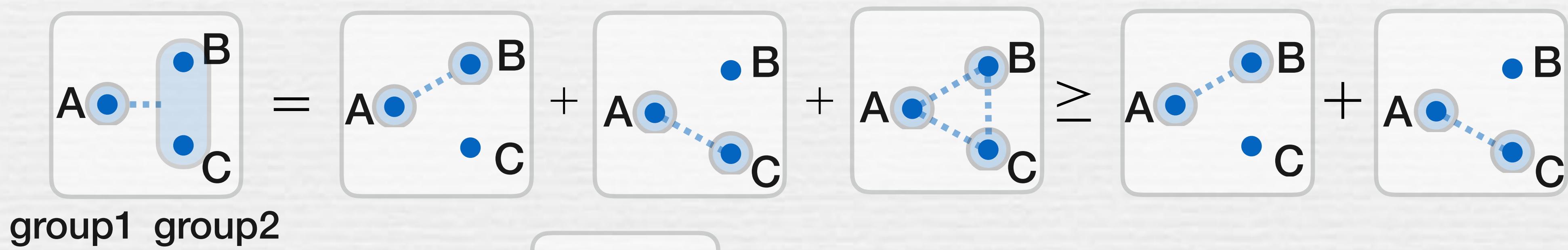
Structure of Multipartite Entanglement: Monogamy

Monogamy of entanglement: property of entanglement sharing
prohibits cloning of an unknown quantum state

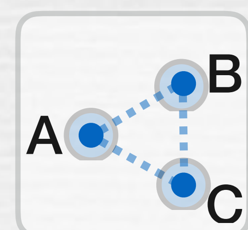
$$\mathcal{N}^2_{A|BC} \geq \mathcal{N}^2_{A|B} + \mathcal{N}^2_{A|C}$$



Residual: $\mathcal{N}^2_{A|BC} - \mathcal{N}^2_{A|B} - \mathcal{N}^2_{A|C}$ measure of multipartite entanglement

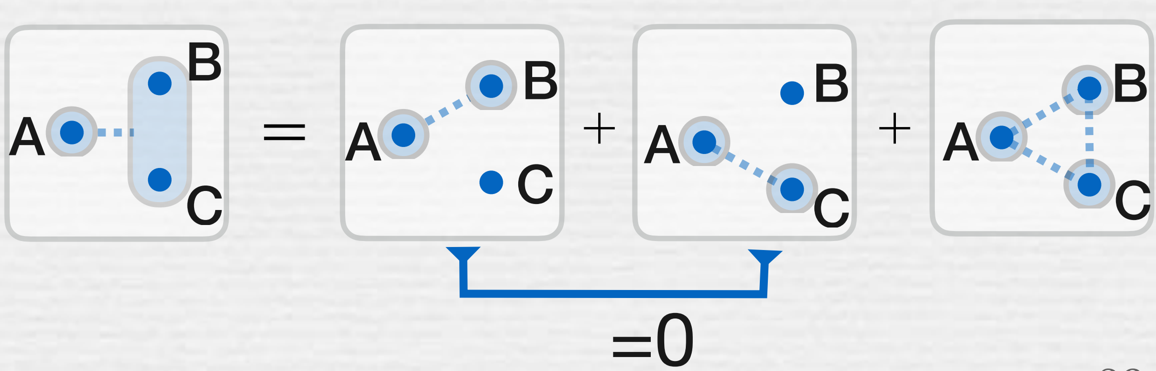


∴ **Residual =**



pure tripartite entanglement

For super horizon mode in de Sitter space,

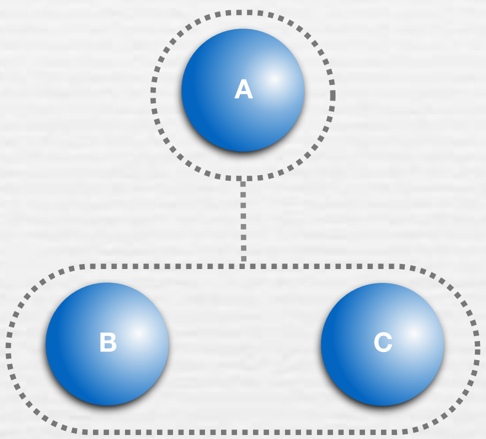


Multipartite effect is responsible
for large scale entanglement

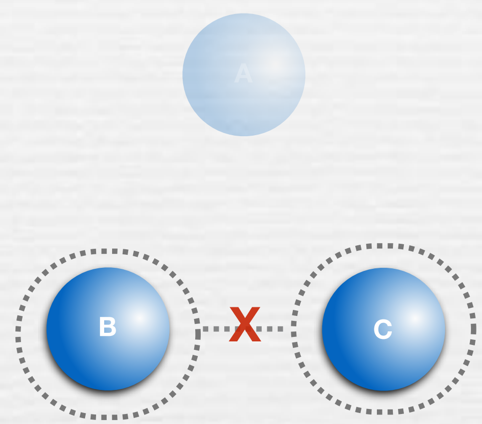
Monogamy and Separability

- Is it possible to say something about emergence of separable state just applying monogamy inequality?

$$|\mathrm{GHZ}\rangle = \frac{1}{\sqrt{2}}\left(|111\rangle + |000\rangle\right)$$



$$\mathcal{N}(\mathrm{A} : \mathrm{BC}) = 1/2$$

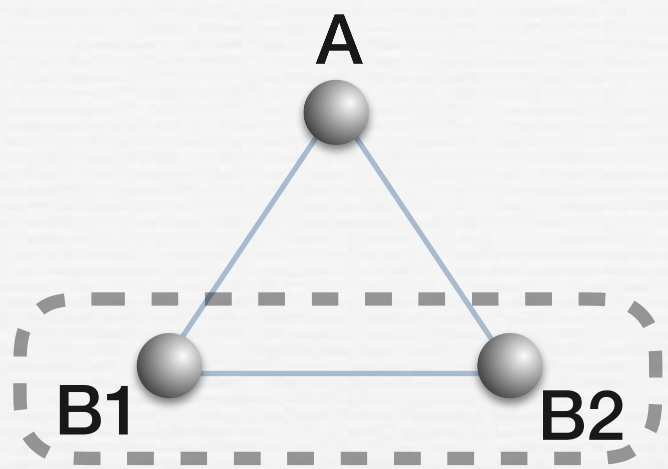


$$\mathcal{N}(\mathrm{B} : \mathrm{C}) = 0$$

$$\mathcal{N}^2_{B|CA} \geq \mathcal{N}^2_{BC} + \mathcal{N}^2_{AB} \qquad \mathcal{N}^2_{A|BC} \geq \mathcal{N}^2_{AB} + \mathcal{N}^2_{AC}$$

These inequalities are trivially satisfied and do not proved any useful information on relation between separability and strength of entanglement

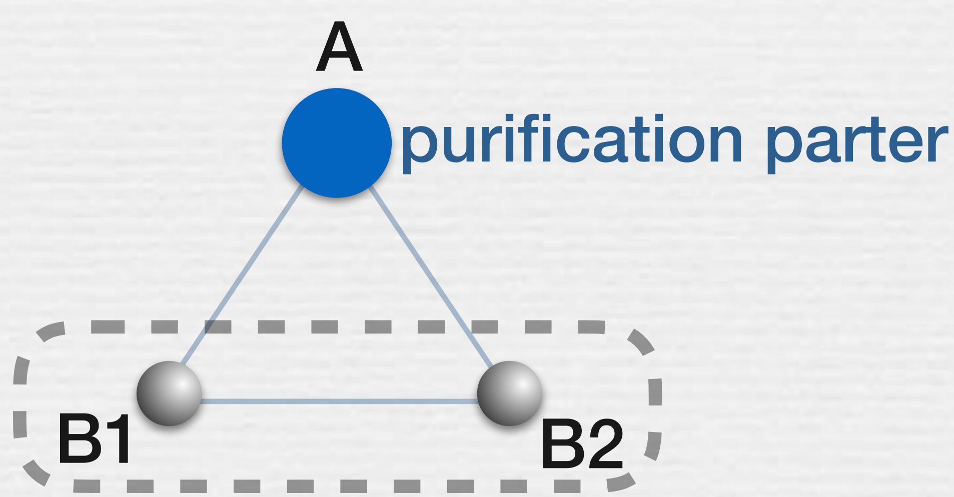
Standard monogamy relation:



$$\mathcal{N}^2_{A|B_1B_2} \geq \mathcal{N}^2_{A|B_1} + \mathcal{N}^2_{A|B_2}$$

This inequality does not bound strength of correlation between B1 and B2 as a function of correlation between A and (B1B2)

A new monogamy inequality: *S. Camalt 2017*



$$\tilde{E}_{\max} \geq \tilde{E}(B_1 : B_2) + E(A : B_1B_2)$$

For maximally entangled system B1B2, $\tilde{E}(B_1 : B_2) = \tilde{E}_{\max}$ and this inequality says $E(A : B_1 B_2) = 0$

relation usually used to illustrate entanglement monogamy