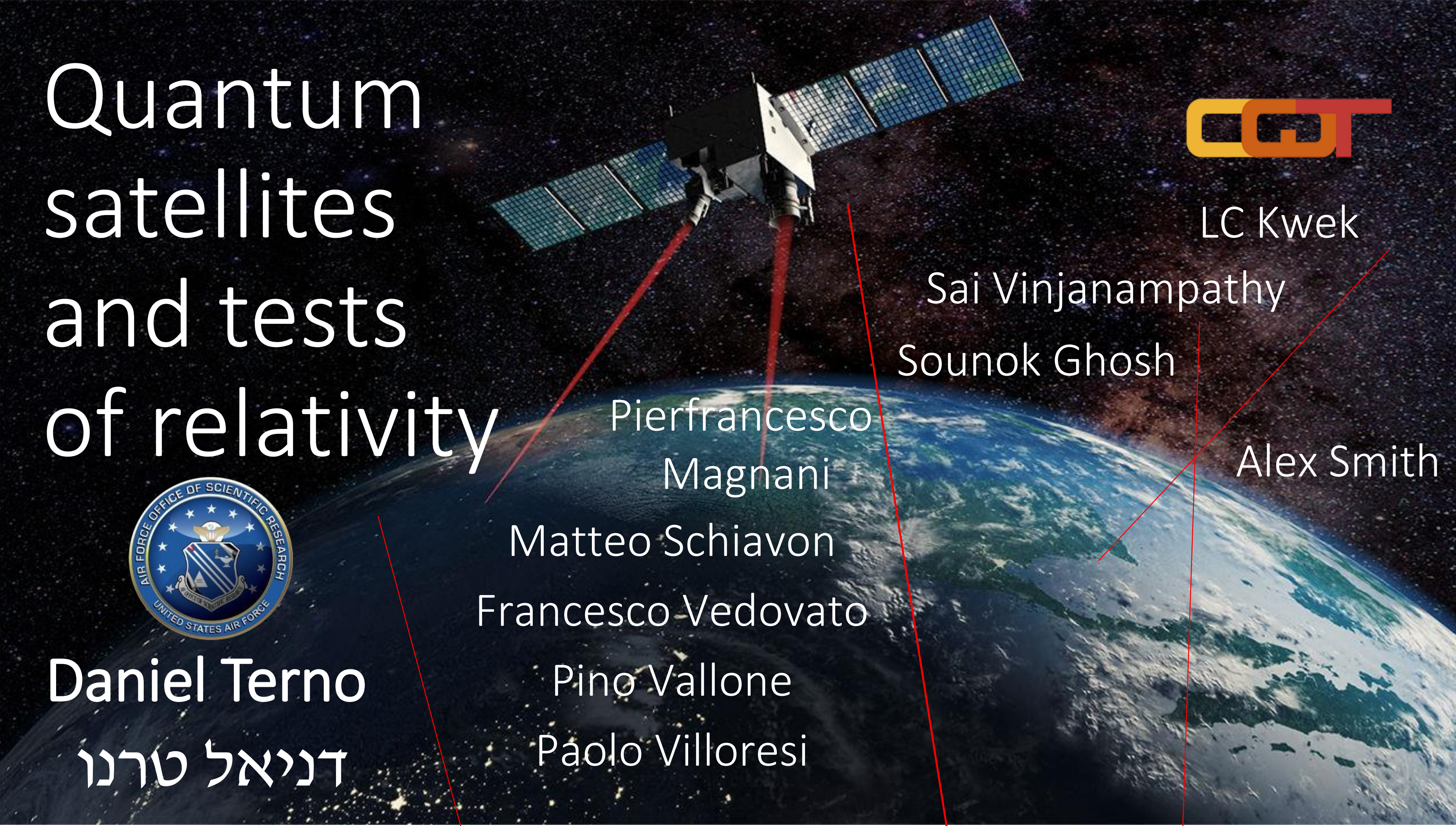

Quantum satellites and tests of relativity

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2019.05.31 RQI – N Tainan, Taiwan

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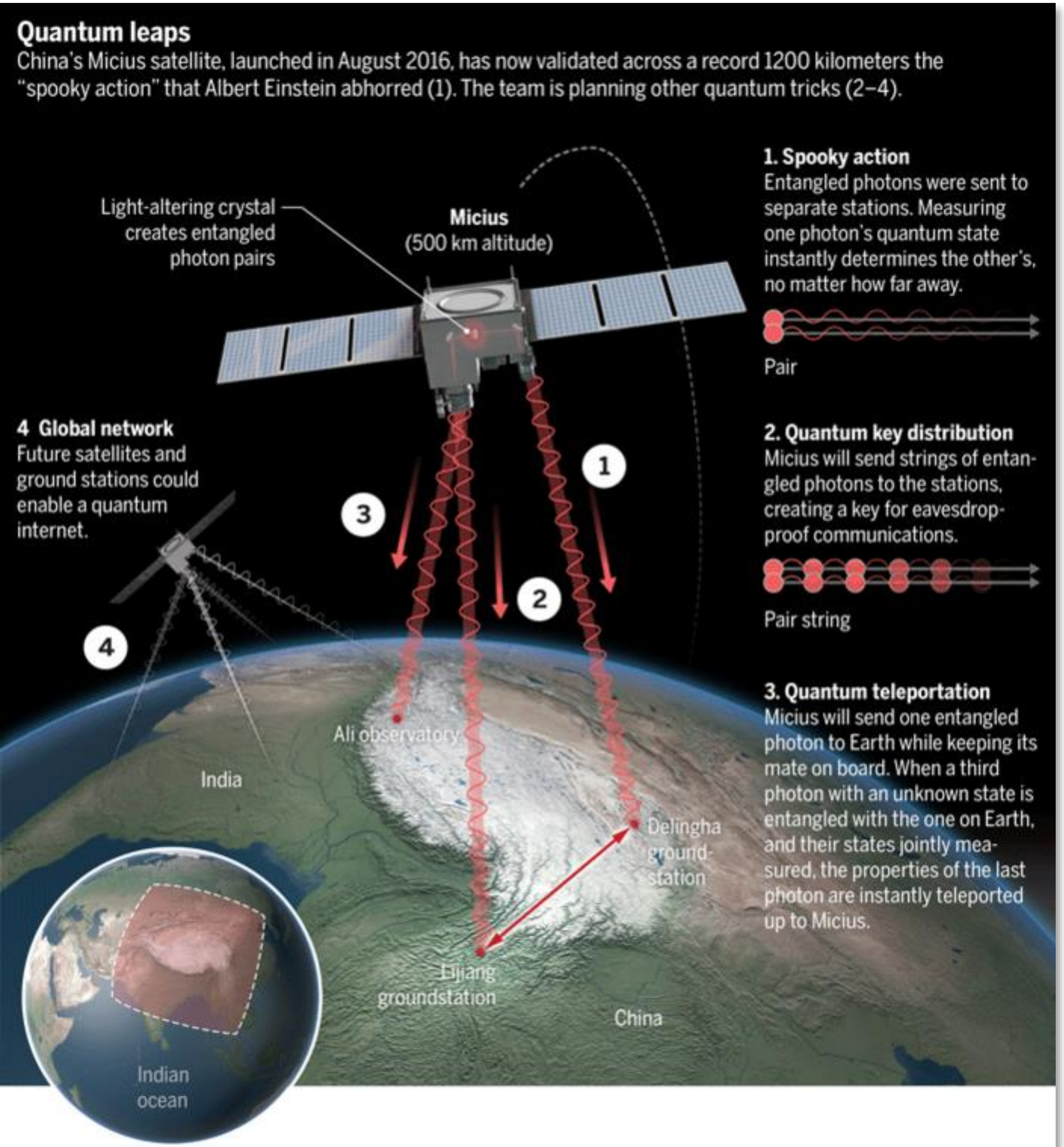
PHOTONS AS A GRAVITY PROBE

MACQUARIE UNIVERSITY
SYDNEY ~ AUSTRALIA

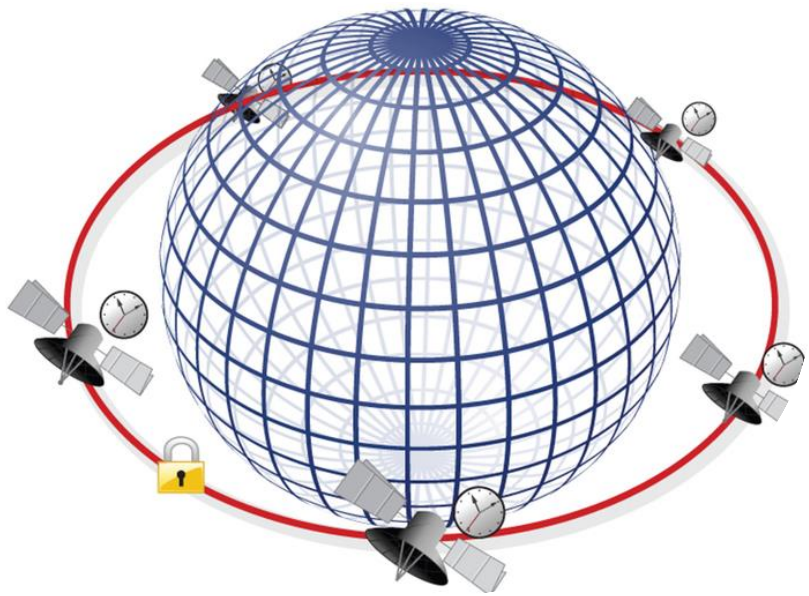
International Workshop on Relativistic Quantum Information (RQI-N)

May 28-30, 2010
Lecture Hall I, Science and Engineering Building, NDHU

Raison d'être: sat-based quantum technologies



✓ Space-based QKD



❖ Space-based entangled optical clock array

Outline

- $EEP=WEP_{1,2}+LLI+LPI_{1,2}$
- Interferometry & PPN
- Sat LPI_1 experiment
- Spin & WEP [+rotation]

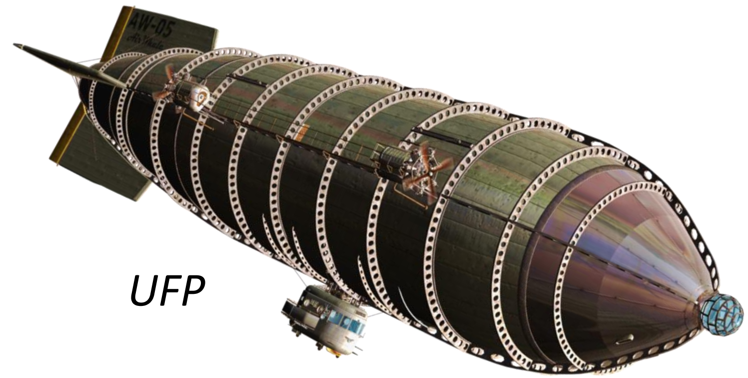
DRT, F. Vedovato, M. Schiavon, A. R. H. Smith, P. Magnani, G. Vallone, and P. Villoresi
Proposal for an optical test of the Einstein Equivalence Principle, arXiv: 1811.04835 (2018)

S.Ghosh, L.-C.Kwek, DRT, S. Vinjanampathy,
Detecting beyond standard model physics via weak-value magnetometry,
soon

Einstein Equivalence Principle

Weak Equivalence Principle

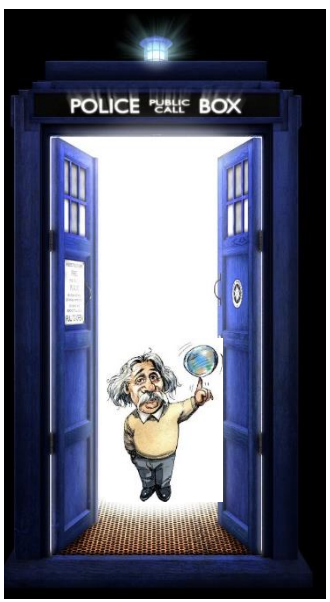
The trajectory of a freely falling test body is independent of its internal composition.



UFP

SM invariants

$$m_{\text{inertial}} = m_{\text{passive grav}}$$



=

Einstein's elevator:

IF all bodies fall with the same acceleration in an external gravitational field, then to an observer in a small freely falling lab in the same gravitational field, they appear unaccelerated.

Local Invariances

Local Lorentz Invariance:

outcomes of experiments are independent of the velocity of the laboratory where the experiment takes place.

Local Position Invariance:

the outcome of any local non-gravitational experiment is independent of (1) where and (2) when in the universe it is performed

where:

gravitational red shift

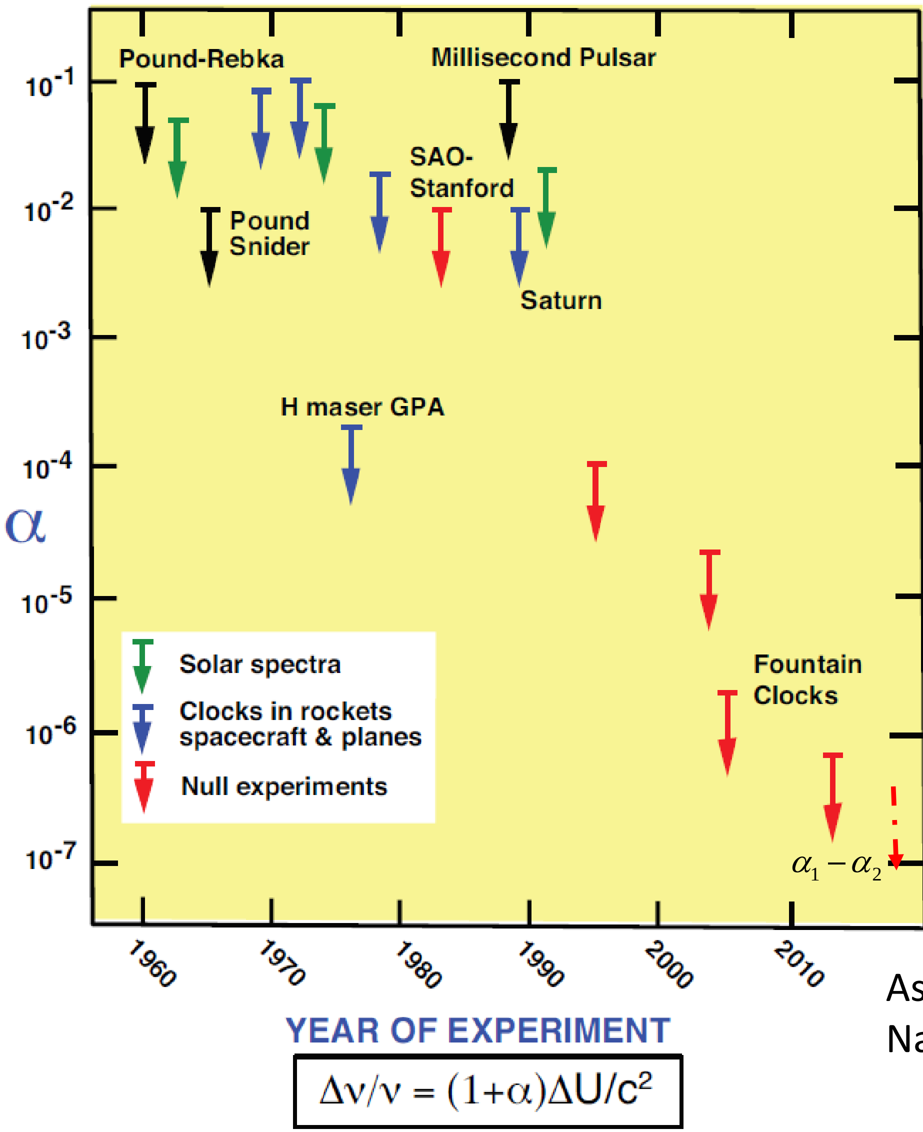
when:

variability of physical constants

$$\mathrm{EEP}=\mathrm{WEP}+\mathrm{LLI}+\mathrm{LPI}_{1,2}$$

Bounds: what do we know about the redshift?

Will, Liv. Rev. Rel. **17**, 4 (2014)



Parameterization of the equivalence principle violation

$$g_{00} = -1 + (1+\alpha)U/c^2 + \ldots$$

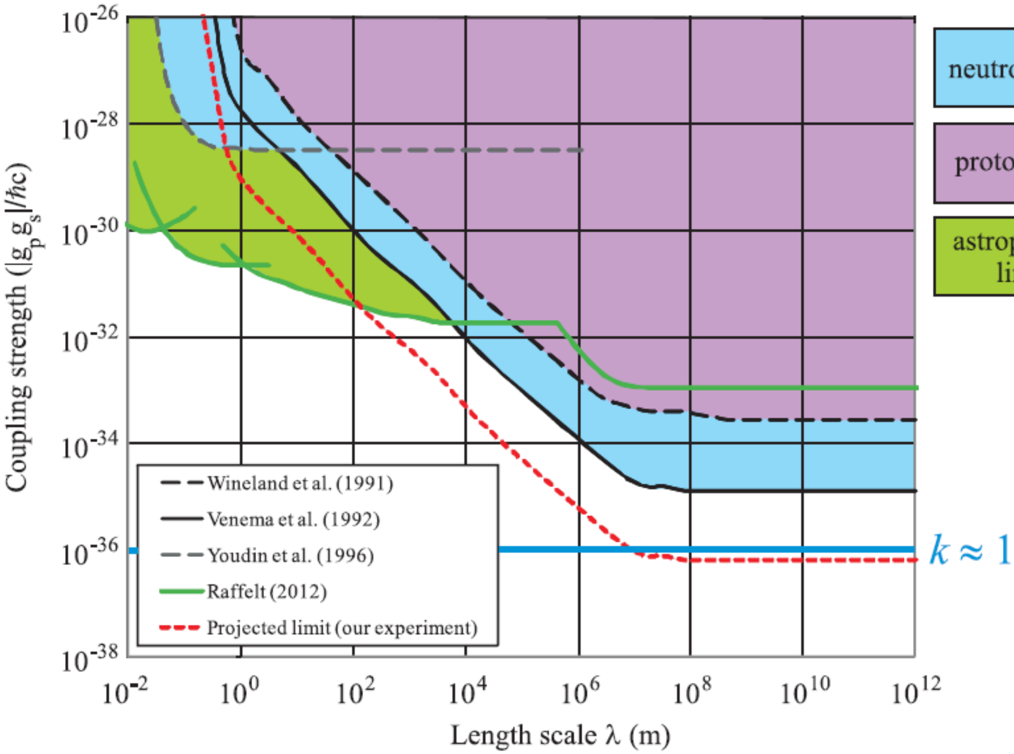
To be noted:

- the most precise tests compare fermions [EM is a messenger]
- <many> base the frequency standard on fermions

A wish:
In SME different sectors (may) couple differently, so it would be nice to have a single-source all-optical test

Ashby, Parker, Palta,
Nature Phys **14**, 802 (2018)

Bounds: what do we know about spin coupling?



Kimball et al, Ann. Phys. (Berlin) **525**, 514 (2013)

Parameterization of the direct spin gravity coupling

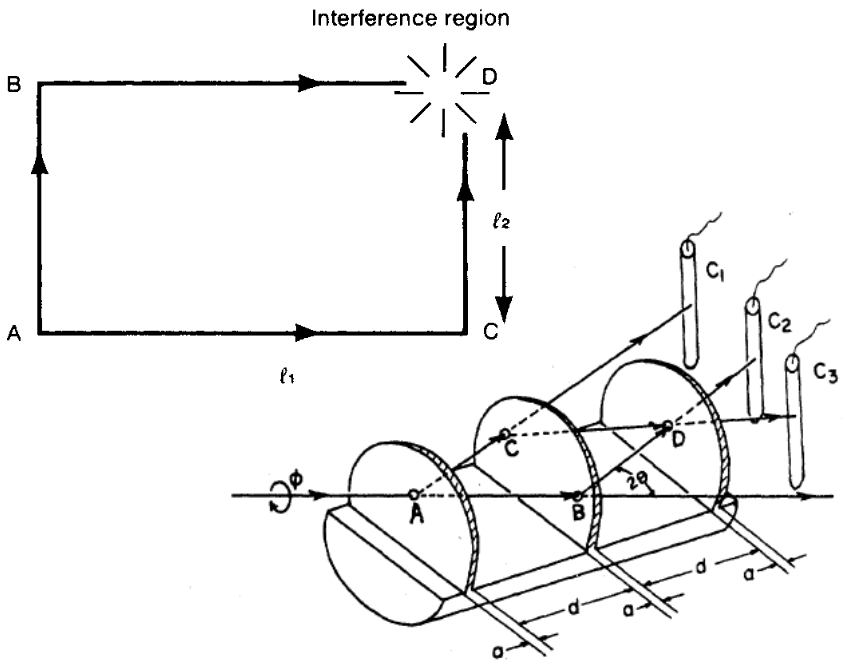
$$H_{\text{ext}} = \frac{\hbar k}{2c} \vec{a} \cdot \vec{\sigma}$$

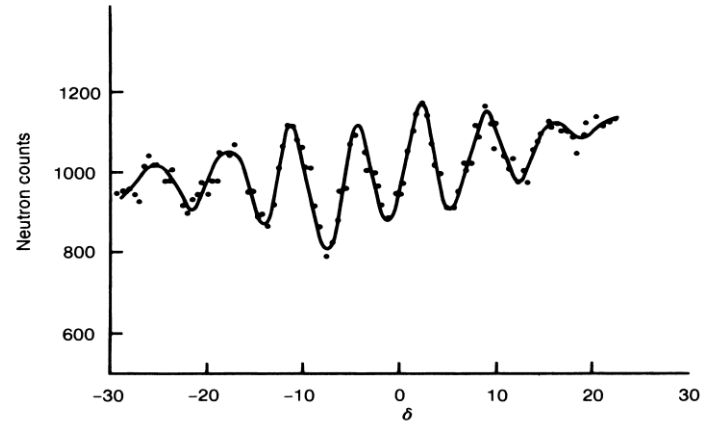
To be noted:

- Rotating frame effects

Idea: COW with photons

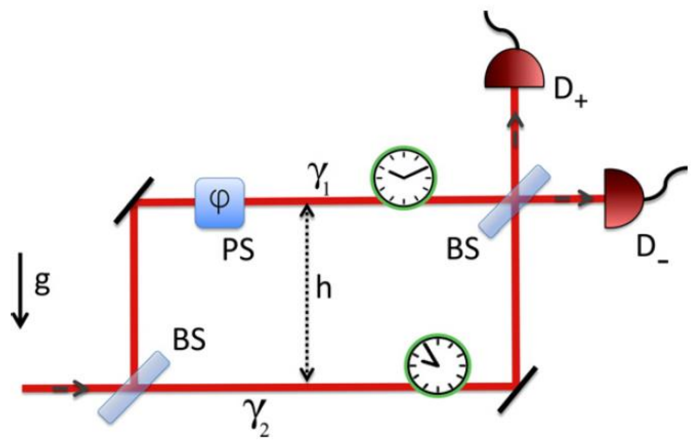
thermal neutrons, $\lambda = 1.42\text{Å}$





Colella, Overhauser, Werner 1975

- velocity ~ 2.9 km/sec
- size of the arms 3.1-3.5 cm.
- A, D are beam splitters
  (silicon slabs)
- B, C are mirrors
  (actually also beam splitters)



Zych, Costa, Pikovski, Brukner,
Nature Comm. **2**, 505 (2011)

Idea: COW with photons

Back of the envelope calculation:

- mass-energy equivalence,
- Newtonian photons

$$\Delta\psi = \frac{2\pi}{\lambda}\frac{ghq}{c^2}$$

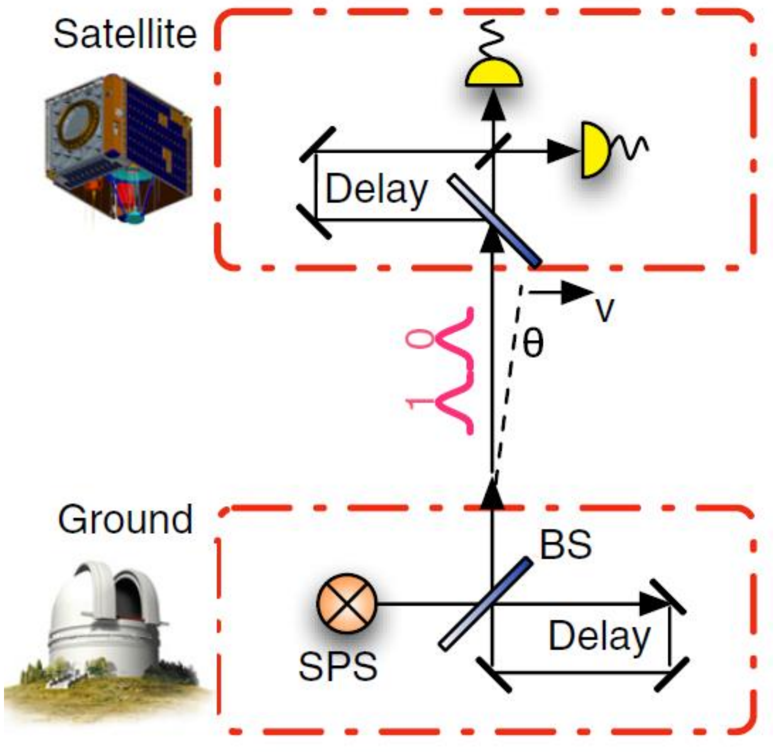
$h \sim 300$ km

$q = 6$ km

λ=800nm

$\Delta\psi \sim 2$ rad

Potential problems: factors of 2

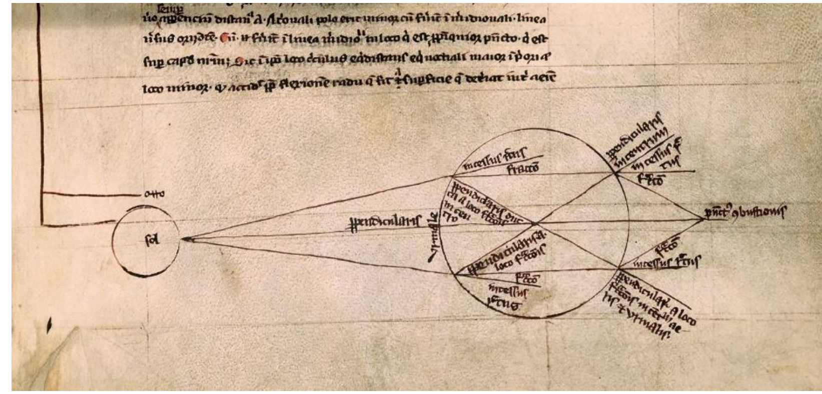


Rideout *et al.*,
Class. Quant. Grav. **29**, 224011 (2012).

Interferometry

Geometric optics

Shortwave asymptotics $A^{\mu} = a^{\mu} e^{i\psi}$



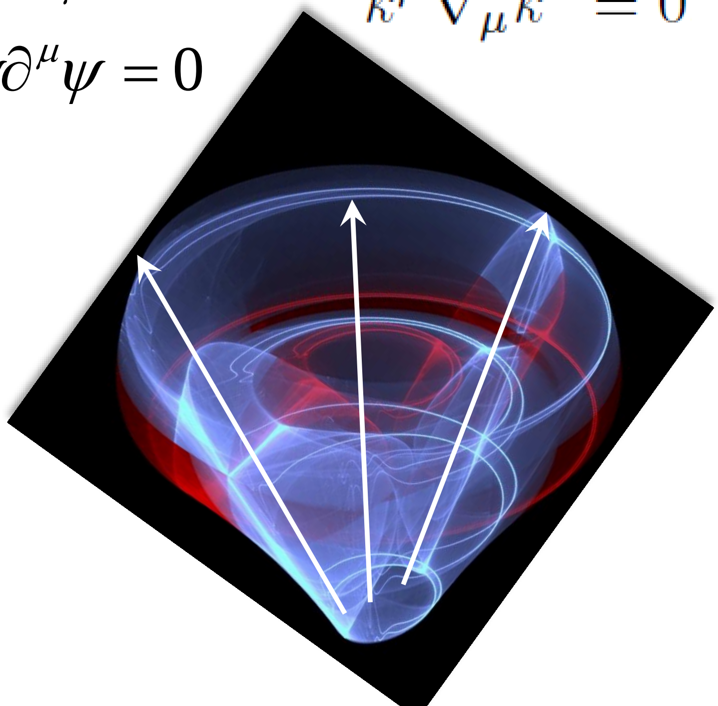
Substitute, gauge-fix, expand

$$k_{\mu} = \partial_{\mu} \psi$$

$$\partial_{\mu} \psi \partial^{\mu} \psi = 0$$

$$k^{\mu} \nabla_{\mu} k^{\nu} = 0$$

Leading order: eikonal equation



Trajectories: photons as massless point particles that move on the rays prescribed by geometric optics.

Hamilton-Jacobi

stationary spacetimes

Metric is time-independent.
Exists a timelike Killing vector ξ

Meaning:
Conservation of energy

Conserved frequency

$$k^{\mu}\xi_{\mu} = -\omega_{\infty}/c = \text{const} \qquad \rightarrow -k_0$$

Local frequency

Reference frame moves with u_F^{μ} $\qquad \omega_F = -k \cdot u_F$

Phase

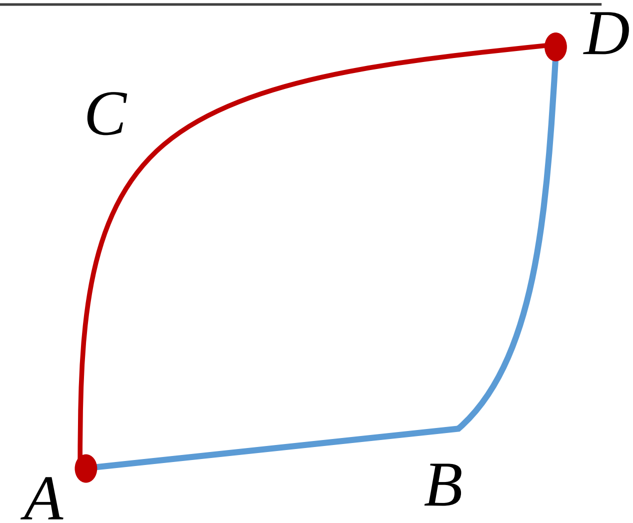
HJ equation separates, so the phase takes the usual form

$$\psi(t,\vec{x}) = -\omega_{\infty}(t-t_0) + \omega_{\infty} S(\vec{x},\vec{x}_0) \qquad \blacktriangleleft\ \partial_{\mu}\psi\partial^{\mu}\psi = 0$$

Physical optics

$$I_{12}\left(t_D,\vec{x}_D\right) \propto \cos\Delta\psi\left(t_D,\vec{x}_D\right)$$

$$\Delta\psi\left(t_D,\vec{x}_D\right) := \psi_{ABD}\left(t_D,\vec{x}_D\right) - \psi_{ACD}\left(t_D,\vec{x}_D\right)$$



Classical modes
Quantum detection/interpretation

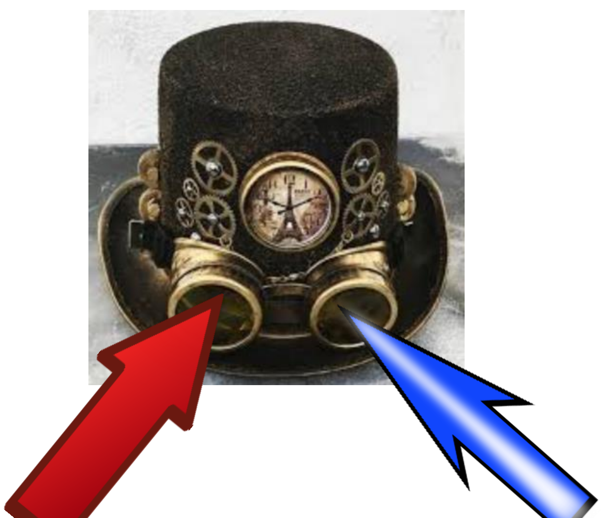
$$a \rightarrow \hat{a}$$

Phase difference, stationary spacetime

$$\Delta\psi(t,\vec{x}_D) = \omega_\infty S_{ABC}(\vec{x}_D,\vec{x}_A) - \omega_\infty S_{ACD}(\vec{x}_D,\vec{x}_A)$$

$$= \omega_\infty(t_{ABD} - t_{ACD}) = \omega_\infty \Delta t.$$

Two geodesics ▲



Phase difference

Segments, boundary conditions & Lorentz

$$\psi(t,x) = -\omega_n(t - t_{n-1}) + \omega_n \Delta t(\vec{x};\vec{x}_{n-1}) + \sum_{k=1}^{n} \phi_{k-1}$$

Lorentz transforms

The phase a scalar

$$\psi(x) = k^a x_a + \Phi_1$$

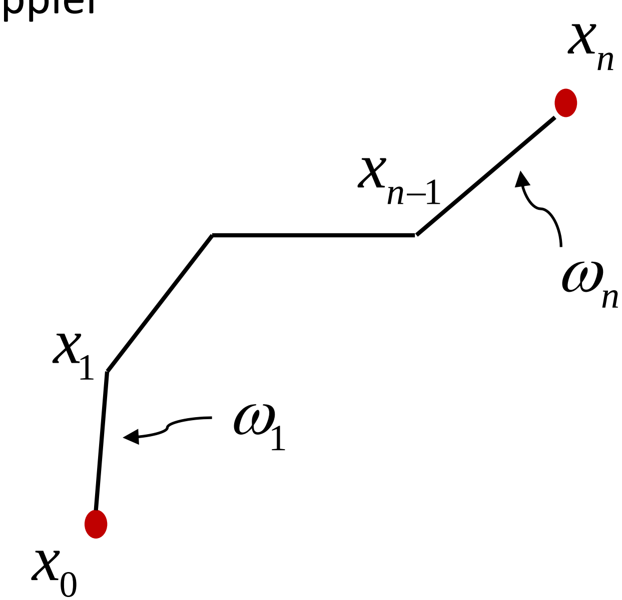
$$\psi(x) = \psi'(x') = k^{a'} x_{a'} + \Phi_1$$

Mirrors: boundary conditions, Doppler

Delay

(approximately) @ one point $\vec{x}_F$
for a proper time τ_F

$$\phi_F = \omega_F \tau_F$$



PPN

Parameterized post-Newtonian

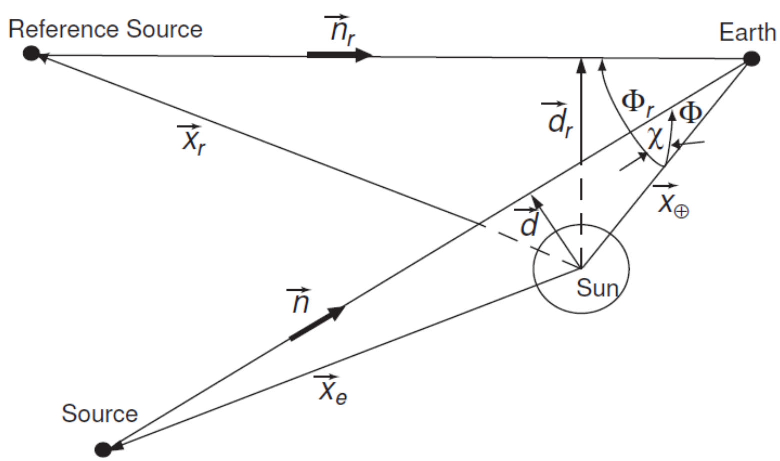
a systematic method for studying a system of slowly moving bodies bound together by weak gravitational forces

Newtonian limit:

$$g_{00} = -(1 + 2\varphi/c^2)$$

$$g_{ij} = \delta_{ij}$$

$$U := -\varphi/c^2$$



EoM for test particles including the post-Newtonian effects

Making sense of what to keep:

$$\frac{GM}{rc^2} \sim \frac{v^2}{c^2} \equiv \beta^2 \sim \varepsilon^2$$

PPN

for photons

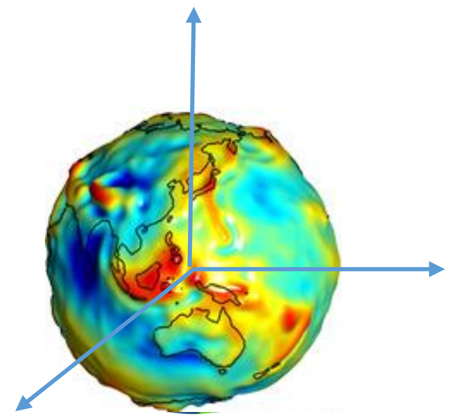
Frame:
Earth-centered, inertial

Earth:
Rigid, uniformly rotating
[spherical/elliptical]

Scale:

$$\frac{GM}{rc^2} = \frac{v^2}{c^2} \sim \varepsilon^2$$

$$\varepsilon = 2.6 \times 10^{-5}$$



$$U = \frac{GM}{r} Q \simeq \frac{GM}{r}$$

▲ Newton & co ▲

Metric:

$$ds^2 = -V^2(r)c^2dt^2 + \vec{R}\cdot d\vec{x}\, cdt + W^2(r)d\vec{x}\cdot d\vec{x}$$

$$V(r) = 1 - \epsilon^2 \frac{U}{c^2}, \qquad W(r) = 1 + \epsilon^2 \gamma \frac{U}{c^2}$$

$$\vec{R} = -\epsilon^3 2(1+\gamma) \frac{G}{c^3} \frac{\vec{J} \times \vec{x}}{r^3}$$

▲ Lense-Thirring aka frame dragging ▲

The Earth
is not round
▼

$$Q = 1 - \tfrac{1}{2} J_2 \frac{R^2}{r^2} (3\cos^2\theta - 1)$$

$$J_2 = 1.08 \times 10^{-3}$$

PPN

for photons

$$\vec{x}(t) =: \vec{x}_{(0)}(t) + \epsilon^2 \vec{x}_{(2)}(t)$$

EoM

$$\frac{d^2 \vec{x}^{\,\perp}_{(2)}}{dt^2} = (1+\gamma)\big(\vec{\nabla}U - \hat{n}(\hat{n}\cdot\vec{\nabla}U)\big)$$

$$\frac{dx^{\shortparallel}_{(2)}}{dt} = -(1+\gamma)\frac{U}{c},$$

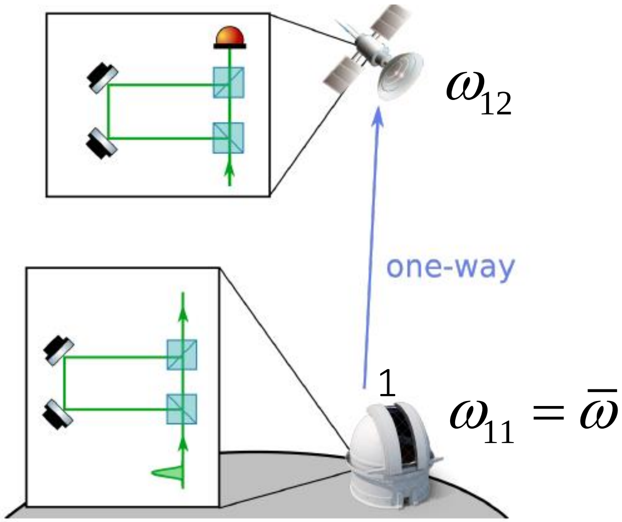
A useful consequence of massless particles

Time delay

$$c\Delta t = |\vec{x} - \vec{b}| + (1+\gamma)\frac{GM}{c^2}\ln\frac{r + \vec{x}\cdot\hat{n}}{b + \vec{b}\cdot\hat{n}}$$

Phase difference

PPN +SR

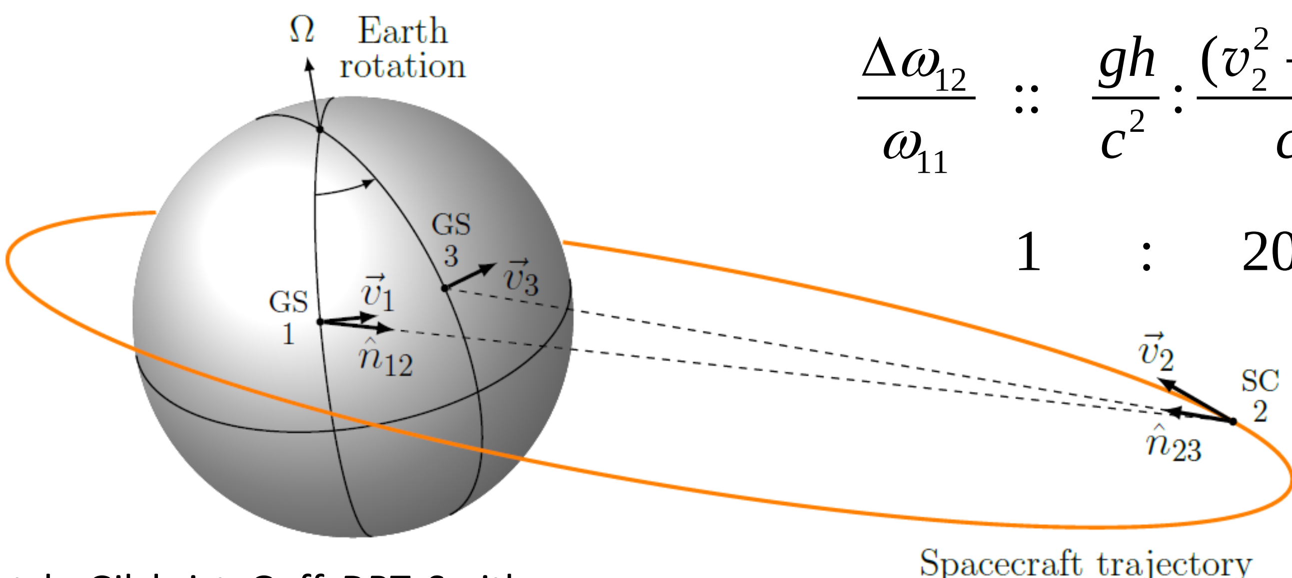


$$\Delta\varphi = (\omega_{12} - \omega_{11})\tau \qquad \text{Condition:} \quad \Delta t \ll \tau_c$$

Frequency shift for the pulse @ sat

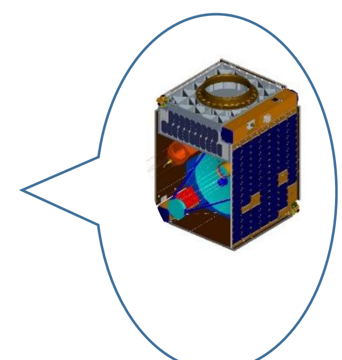
$$\frac{\omega_{12}}{\bar{\omega}} = \left(\frac{1 - U_E - \frac{1}{2}\beta_E^2}{1 - U_2 - \frac{1}{2}\beta_2^2}\right)\left(\frac{1 - \hat{n}_{12}\cdot\vec{\beta}_2}{1 - \hat{n}_{12}\cdot\vec{\beta}_1}\right) +$$

The Doppler problem of the optical COW



$$\frac{\Delta\omega_{12}}{\omega_{11}} :: \frac{gh}{c^2} : \frac{(v_2^2 - v_1^2)}{c^2} : \frac{(\vec{v}_2 - \vec{v}_1)\cdot\hat{n}_{12}}{c}$$

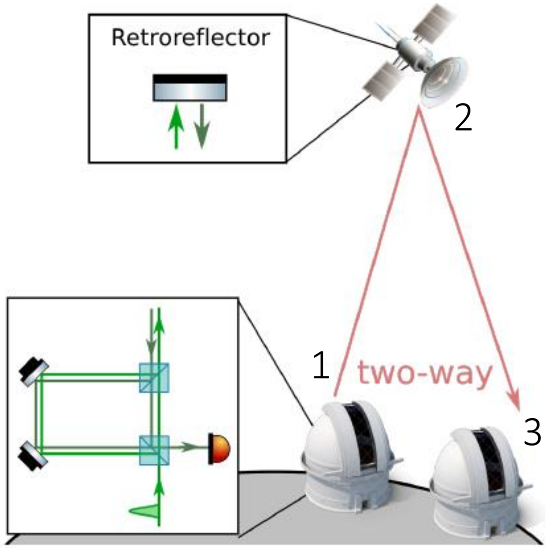
$$1 \quad : \quad 20 \quad : \quad 10^5$$



Brodutch, Gilchrist, Guff, DRT, Smith
PRD **84**, 121501(R) (2011)

Sat LPI_1 experiment

Doppler and qubits

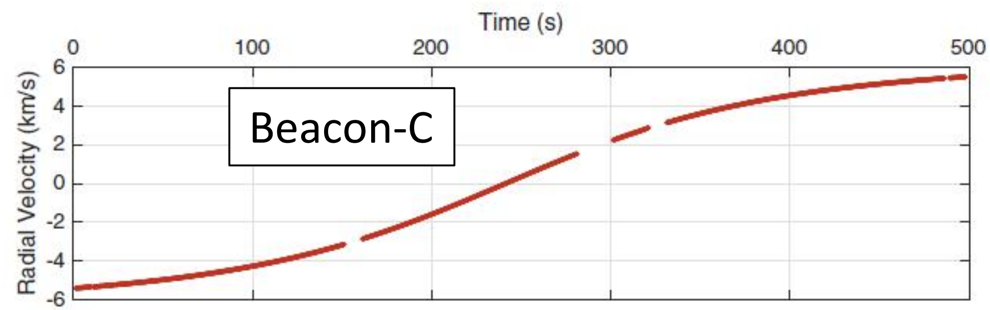


$$\frac{\omega_{13}}{\bar{\omega}} = \left(\frac{1-\hat{n}_{23}\cdot\vec{\beta}_3}{1-\hat{n}_{23}\cdot\vec{\beta}_2}\right)\left(\frac{1-\hat{n}_{12}\cdot\vec{\beta}_2}{1-\hat{n}_{12}\cdot\vec{\beta}_1}\right) +$$

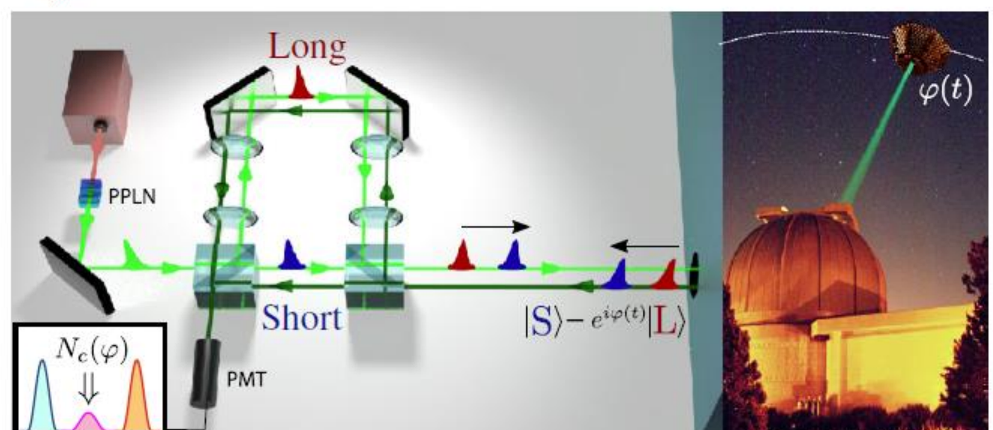
Interference at the single photon level along satellite-ground channels: successful simulation of quantum communication that is based on the 1st order Doppler

Vallone *et al.*, Phys. Rev. Lett. **116**, 253601 (2016)

Scheme of the experiment and satellite radial velocity

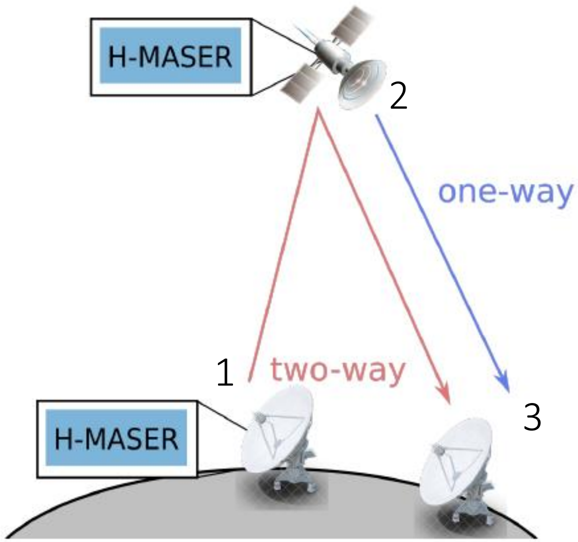


Bottom: the unbalanced MZI with the two 4f systems used for the generation of the state and the measurement of the interference. The light and dark green lines represent the beams outgoing to and ingoing from the telescope.



Inset: the [expected] detection pattern

removing Doppler



GP-A solution

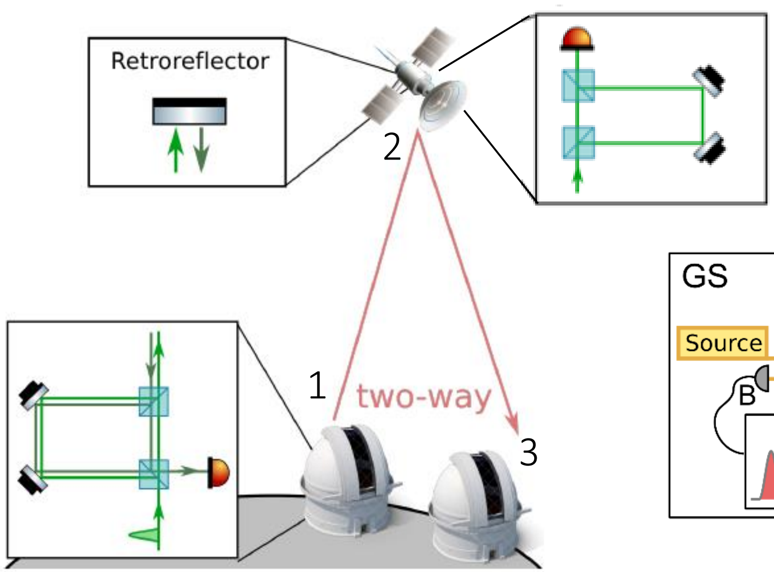
Two sources (1 & 2). Two detections at 3.

The 1st order Doppler is removed in

$$\Omega = \omega_{23} - \tfrac{1}{2}\,\omega_{13}$$

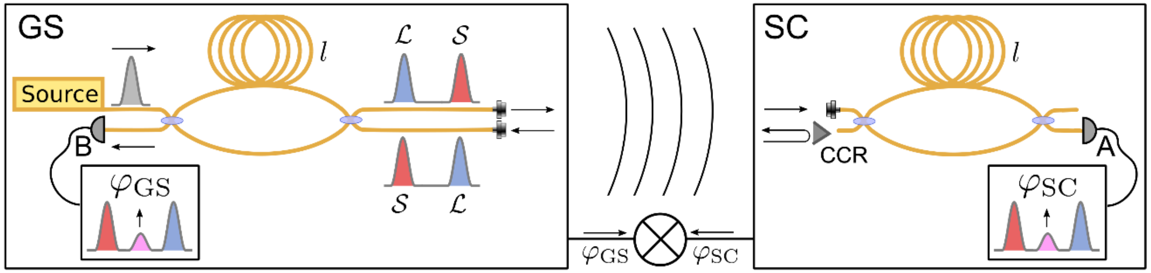
Vessot and Levine, *NASA technical report NASA-CR-161409* (1979)

A 1+2 way optical COW

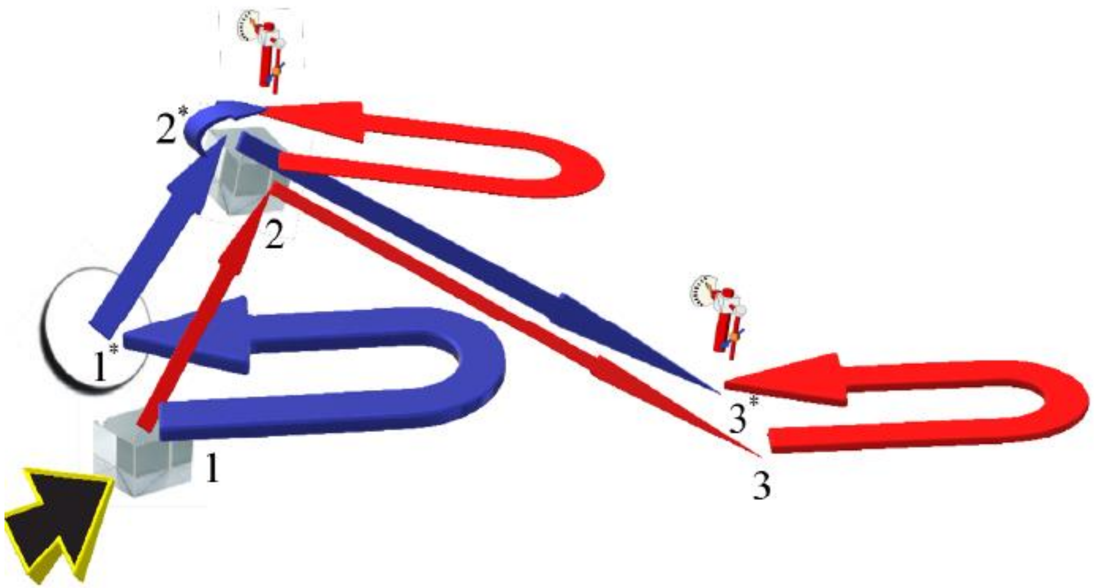


+Time-delay interferometry

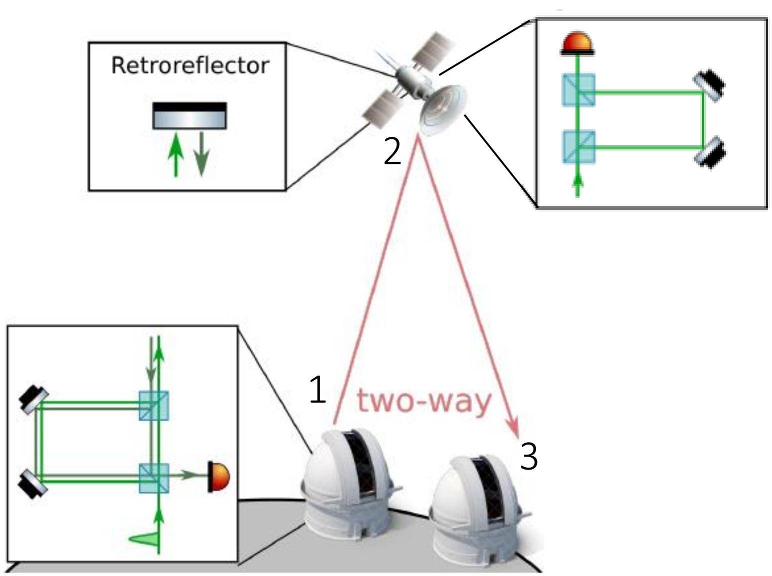
Tinto and Dhurandhar,
Living Rev. Relativity **17**, 6 (2014).



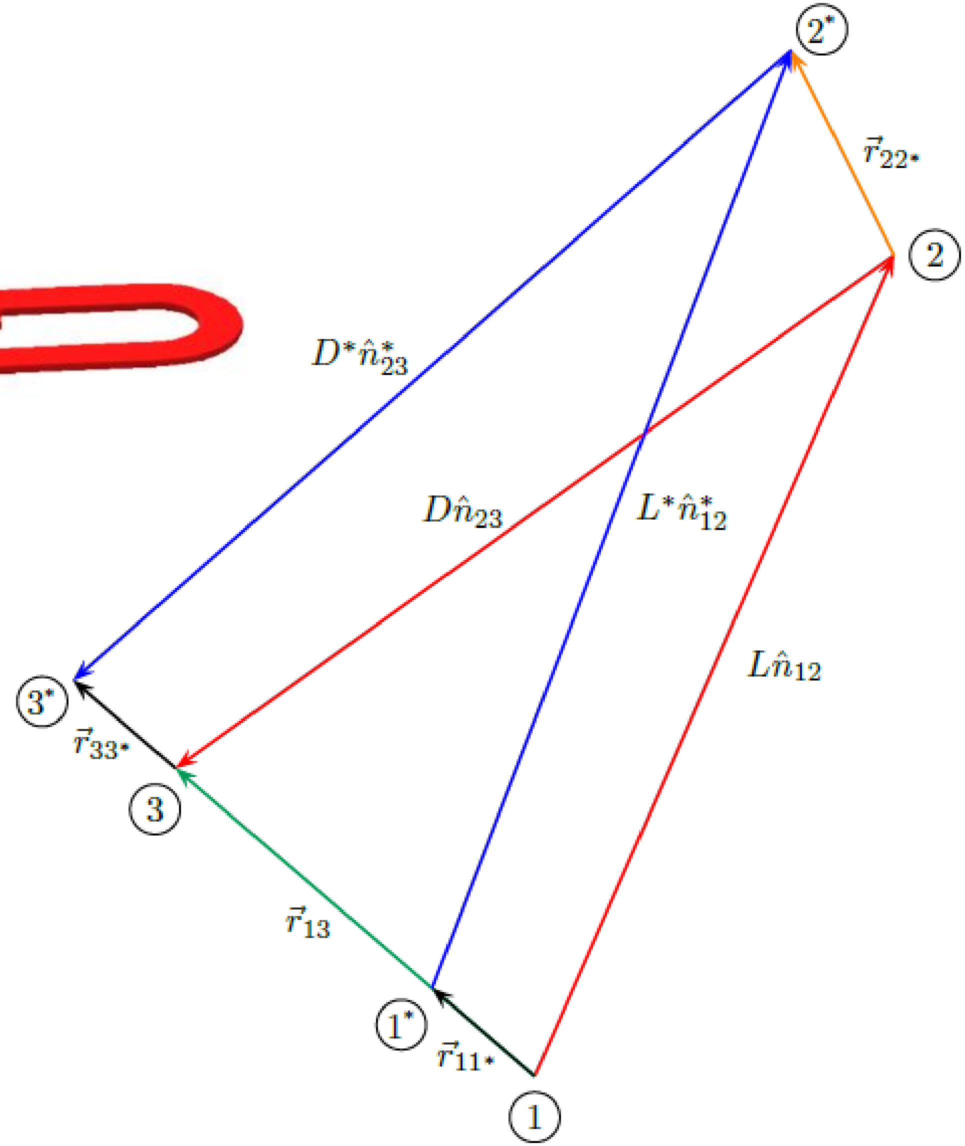
Geometry



2*
2
1*
1
3*
3

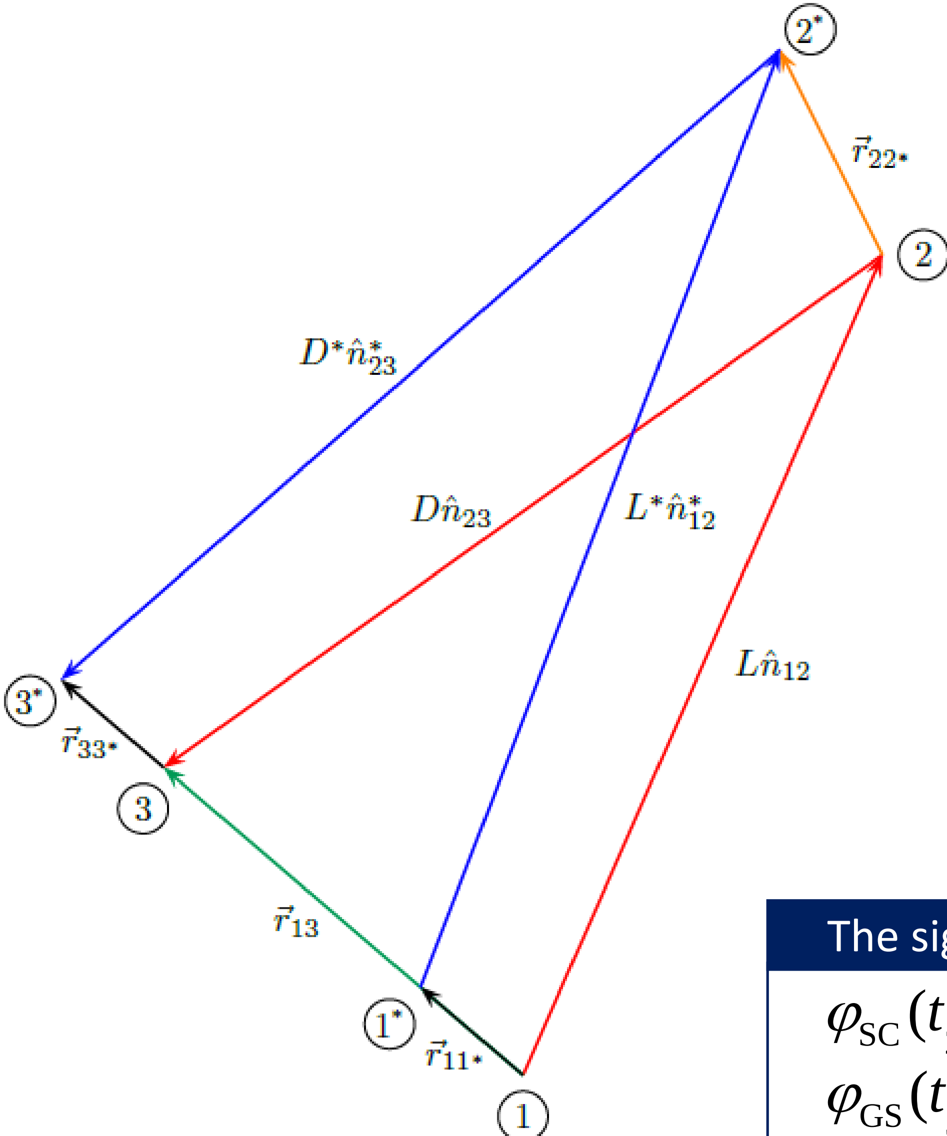


Retroreflector
2
1
two-way
3



2*
2
$\vec{r}_{22^*}$
$D^*\hat{n}^*_{23}$
$D\hat{n}_{23}$
$L^*\hat{n}^*_{12}$
$L\hat{n}_{12}$
3*
$\vec{r}_{33^*}$
3
$\vec{r}_{13}$
1*
$\vec{r}_{11^*}$
1

interferometry and PPN



Positions of the ground station and the satellite at different stages of the experiment.
Distances travelled by the beam 1 on the go-return trip are L and D, respectively.

Proper delay times: $\tau = nl/c$

Propagation time (0th order): $T = L/c$

Three useful frames: global, GS, SC

Another expansion parameter:

$$\mu = l/L \sim 10^{-4} \dots 10^{-5}$$

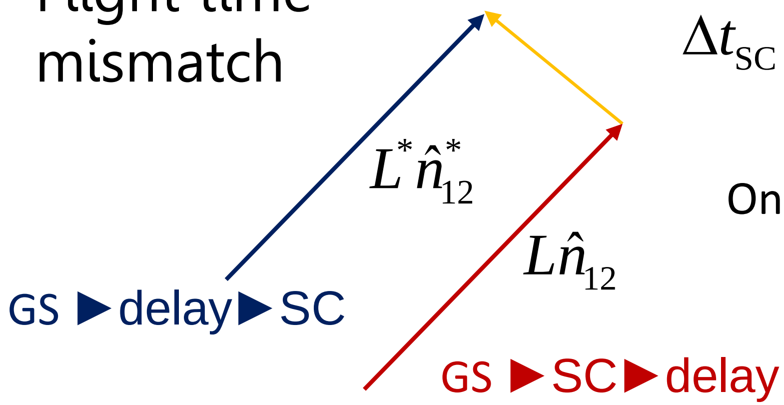
The signals
$\varphi_{SC}(t_{2^*})$
$\varphi_{GS}(t_{3^*})$

$$\varphi_{SC}(t_{2^*}) \approx (\omega_{12} - \omega_{11})\tau$$

$$\varphi_{GS}(t_{3^*}) \approx (\omega_{13} - \omega_{11})\tau$$

the signal

Flight time mismatch



$$\Delta t_{\mathrm{SC}} = (L^* - L)/c + \mathcal{O}\left(\varepsilon^2\right) = \tau\left((\beta_2 - \beta_1) + \mathcal{O}\left(\varepsilon^2\right)\right)$$

$$\beta_i := \hat{n}_{12} \cdot \vec{v}_i / c$$

On the ground:

$$\Delta t_{\mathrm{GS}} = 2\Delta t_{\mathrm{SC}} + \mathcal{O}\left(\varepsilon^2\right)$$

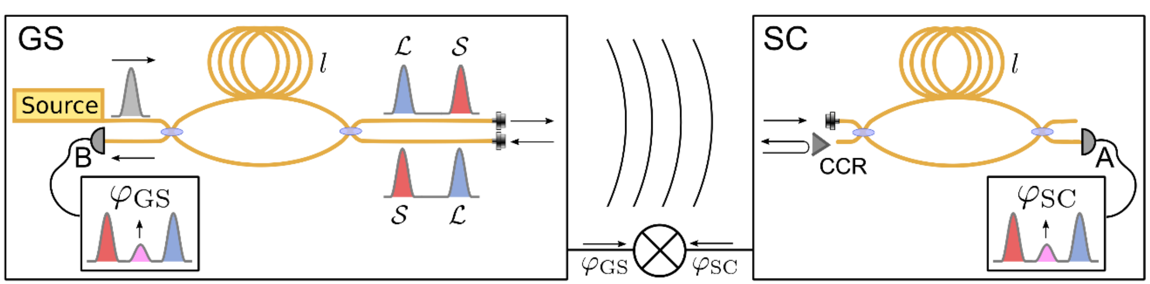
Condition: $\Delta t_{\mathrm{SC}} \ll \tau_{\mathrm{c}}$

Features

- ✓ Have to take into account the delay
- ✓ Subtraction of the signals removes the 1st order Doppler

$$S\left(t_{3^*}^{\mathrm{Earth}}, t_{2^*}^{\mathrm{sat}}\right) = \varphi_{\mathrm{SC}}\left(t_{2^*}^{\mathrm{sat}}\right) - \tfrac{1}{2}\varphi_{\mathrm{GS}}\left(t_{3^*}^{\mathrm{Earth}}, t_{2^*}^{\mathrm{sat}}\right)$$

Some real-life issues:



+free-space to single-mode fiber coupling
+ Turbulence
+ delay vs coherence times
+ Different imbalances

the signal

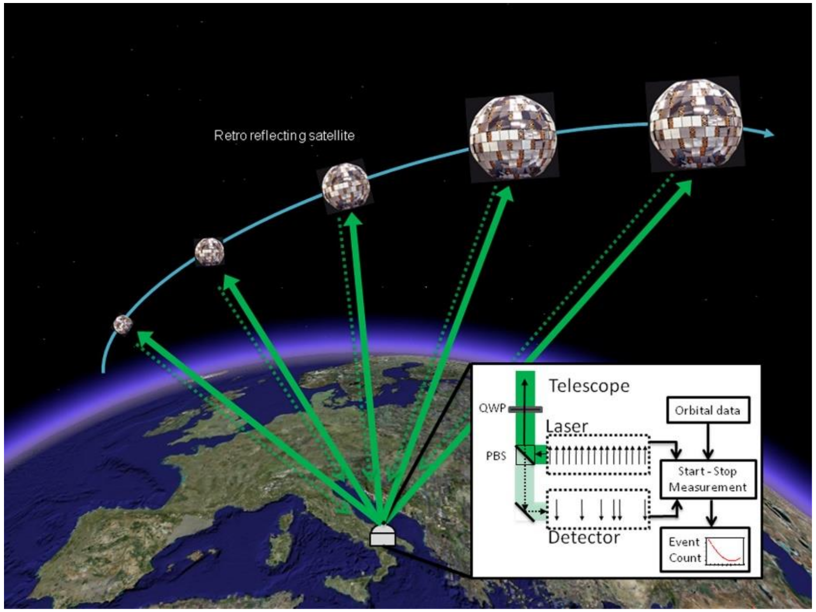
$$\frac{S}{\omega_{11}\tau} = \left(U_2 - U_1\right) + \tfrac{1}{2}\left(\vec{\beta}_1 - \vec{\beta}_2\right)^2 - (ð_1 - ð_2)^2 - T\hat{n}_{12}\cdot\vec{a}_1/c$$

$$\times(1+\alpha)$$

Looks [very much] like GP-A

Technically difficult, but possible

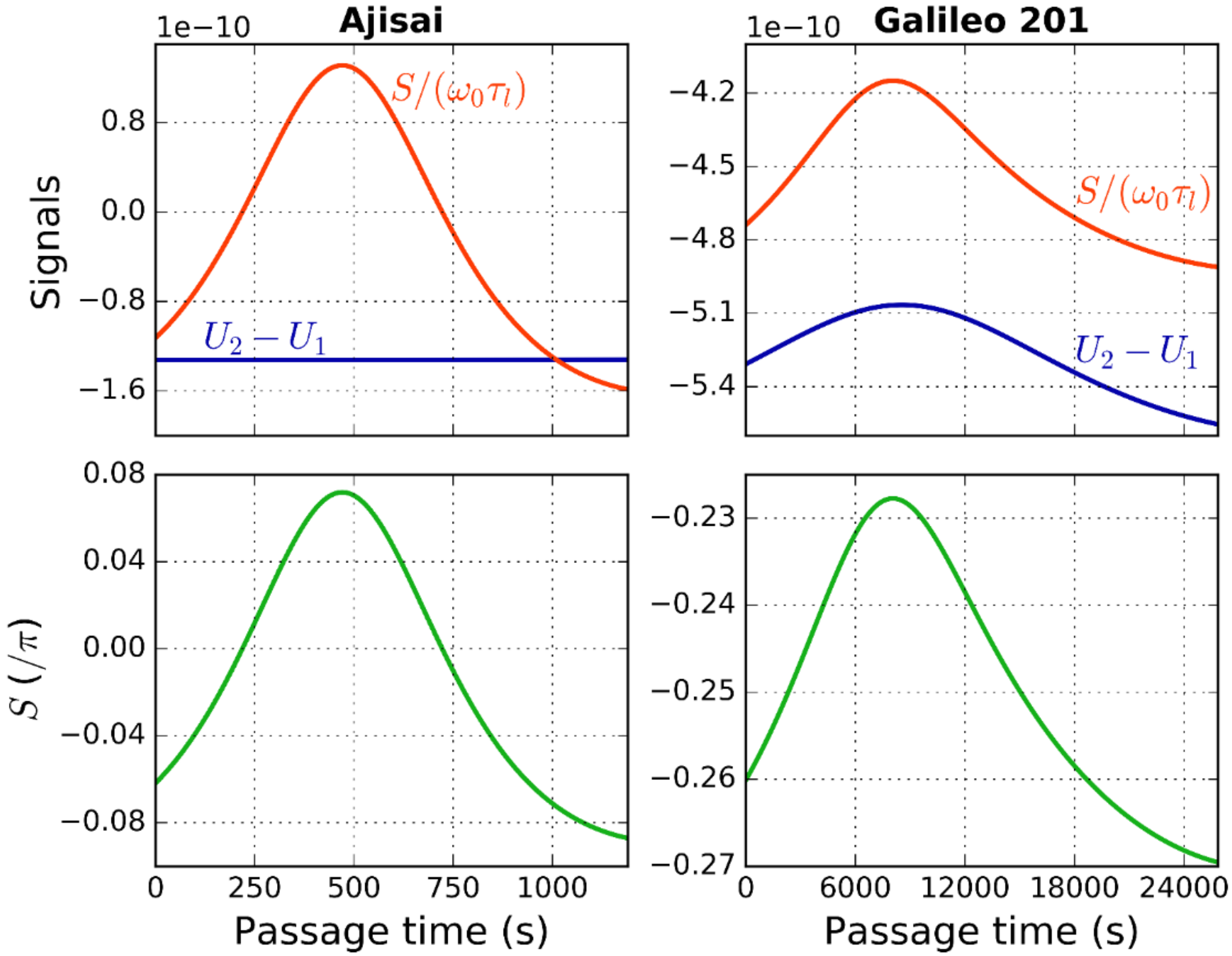
Simulations for the orbits of some satellites that are observable at Matera RLA

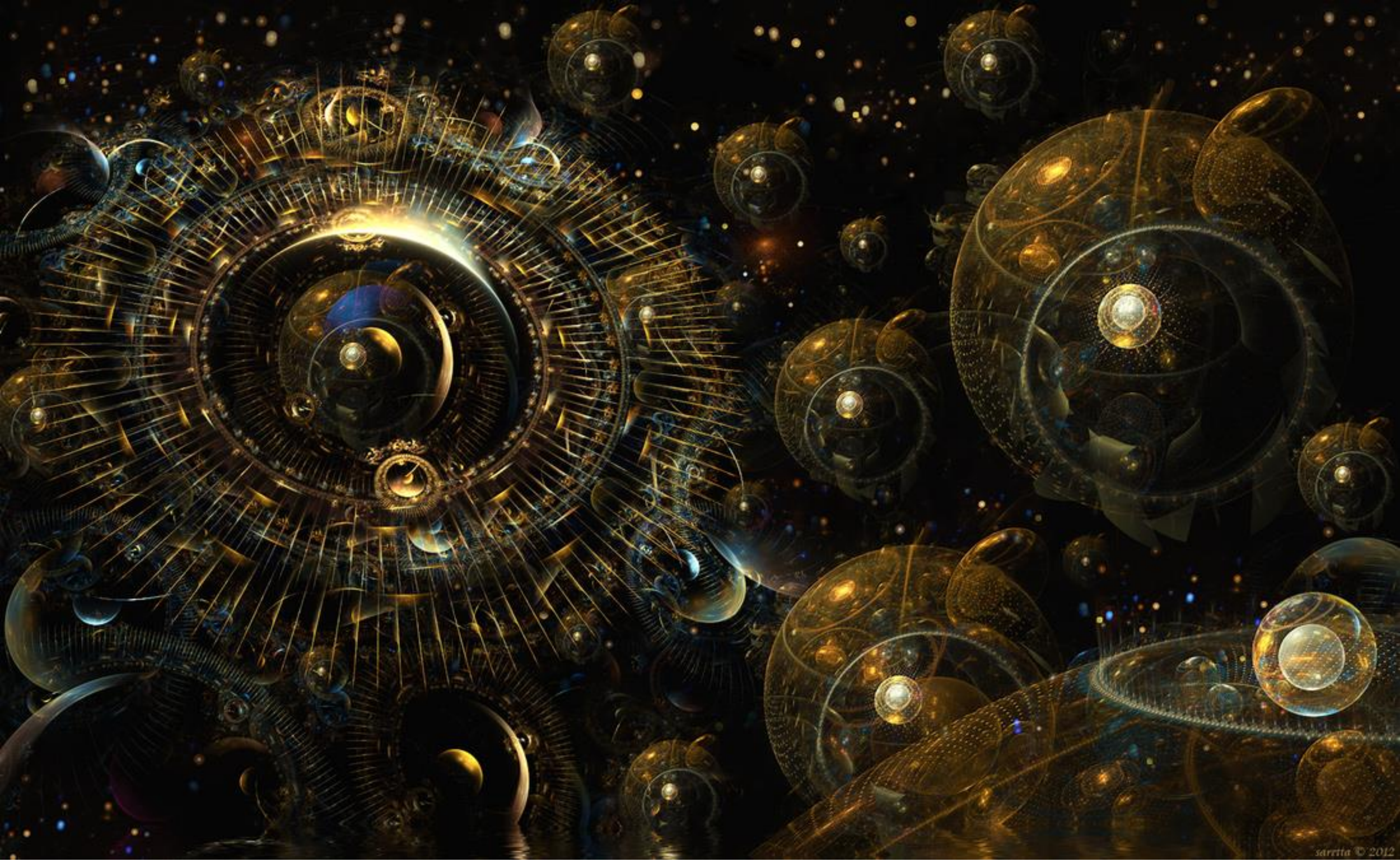


the simulation

as would be seen at MLRA

Ajisai: inclination 50°, eccentricity 0.001, altitude 1,490 km
Galileo 201: inclination 50°, eccentricity 0.158, altitude ranging from 17,000 to 26,210 km





Spin & WEP [+rotation]

(Non-relativistic) spin terms

Mundane & exotic

Spin in a non-inertial frame (linear acceleration and rotation)

Start with Dirac equation on curved background

Do [a/the] FW transform

(Pick the ``large'' part; drop the rest mass)

$$(i\hbar\gamma^\alpha D_\alpha - mc)\psi = 0,$$

$$H = H_{\rm cl} + H_{\rm rel} + H_\sigma + H_{\rm ext}$$

$$H_{\rm cl} = \frac{\vec{p}^{\,2}}{2m} + m\vec{a}\cdot\vec{x} - \vec{\omega}\cdot L,$$

$$H_\sigma = -\tfrac{1}{2}\hbar\vec{\omega}\cdot\vec{\sigma} + \frac{\hbar}{4mc^2}\vec{\sigma}\cdot(\vec{a}\times\vec{p}).$$

Hehl and Ni,
Phys. Rev. D **42**, 20145 (1990)

Exotic:
ad-hoc addition, controversial FW transform, or just SME parameters

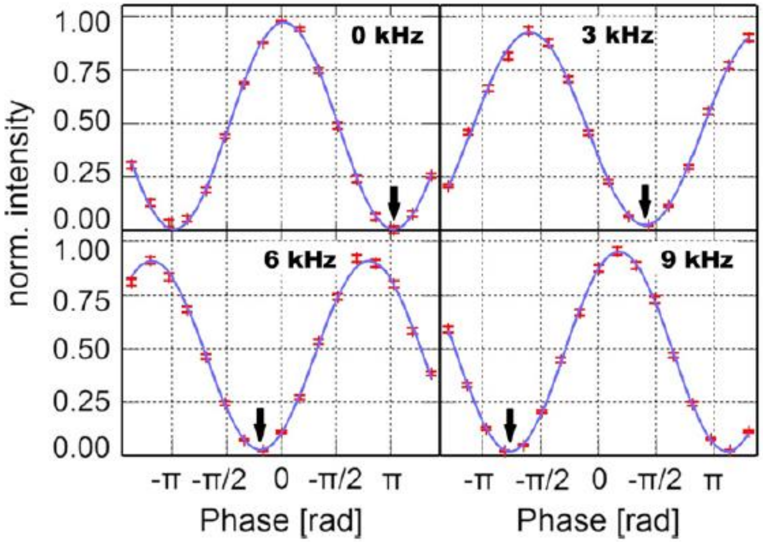
$$H_{\rm ext} = \frac{\hbar k}{2c}\vec{a}\cdot\vec{\sigma}$$

$k = 1$

Peres, Phys. Rev. D **18**, 2739 (1978).
Obukhov, Phys. Rev. Lett. **86**, 192 (2001).

Status

term	name	observation
$m\vec{a} \cdot \vec{x}$	Bose-Wroblewski	yes
$-\vec{\omega} \cdot L$	Page-Werner	yes
$-\frac{1}{2}\hbar\vec{\omega} \cdot \vec{\sigma}$	Mashhoon	may be
$\frac{\hbar}{4mc^2}\vec{\sigma} \cdot (\vec{a} \times \vec{p})$	Hehl-Ni	no



Polarization of neutrons in rotating magnetic field

Demirel, Sponar, and Hasegawa, New J. Phys. **17**, 023065 (2015).

Numbers

$$g\hbar/c = 2.15\times10^{-23}\,\mathrm{eV}$$

$$B_{eq} := \frac{g\hbar}{\mu_B} = 3.72\times10^{-19}\,\mathrm{Tl}$$

$$\omega_{\mathrm{Earth}} c / g = 2.2\times10^{3}$$

a hyperfine splitting in ^{133}Cs for $^2S_{1/2}$ is 3.80×10^{-5}eV

$$\frac{g}{c\Delta\omega_{\mathrm{HF}}} = 3.80\times10^{-19}$$

Stability of clocks:

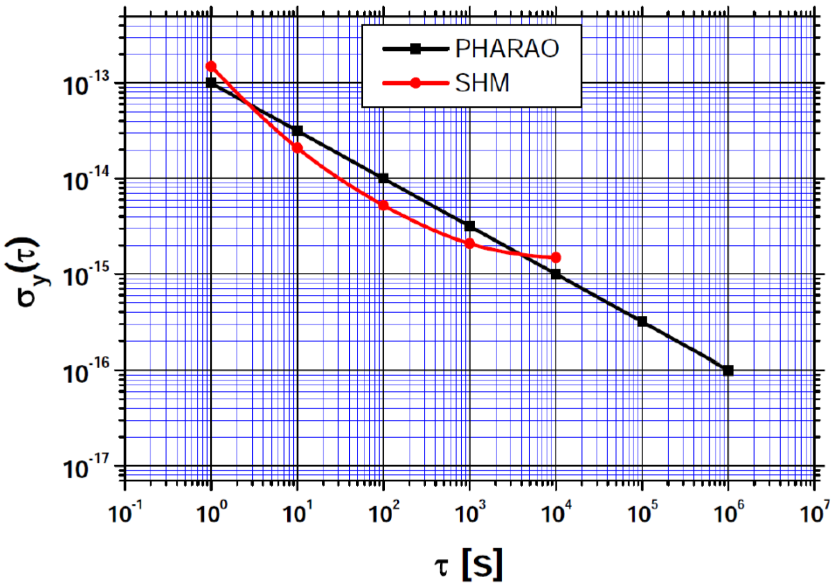
Atomic clocks (microwave) 10^{-16}
Atomic clocks (optical) 10^{-18}

Atomic clocks (quantum) 10^{-17}…10^{-20}

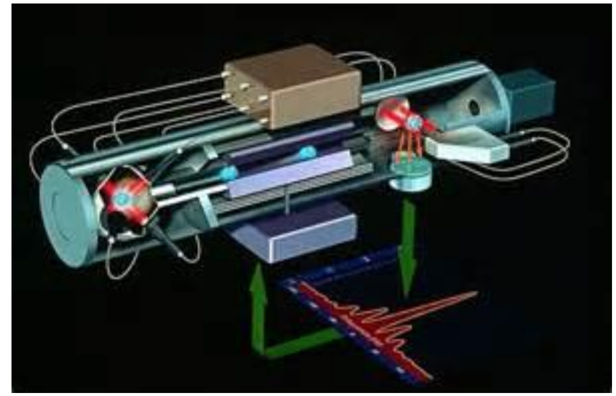
Hinkley *et al*,
Science **341**, 1215 (2013)

Kómár *et al.*,
Nat. Phys. **10**, 582 (2014)

Atomic clocks



Meynadier et al. Class. Quant. Grav. 35, 3 (2018)



ACES goal $10^{-16}...10^{-18}$

Dittus, Lammerzahl, Turischev (eds)
Lasers, Clocks and Drag-Free Control,
(Springer, 2008)

Transitions to be affected

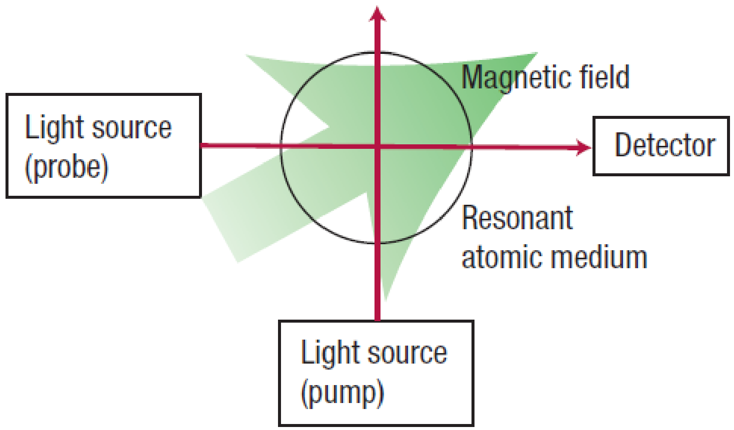
Non-zero ΔM_F

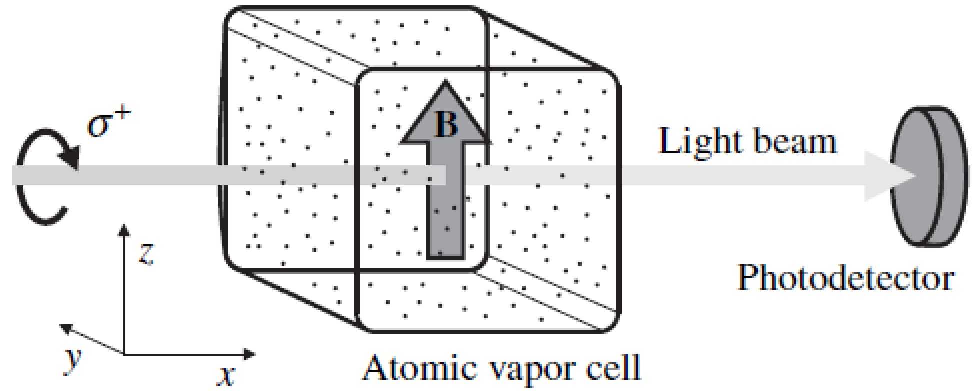
Cs standard: unaffected/insensetive

$$|F=4, M_F=0\rangle = \frac{1}{\sqrt{2}}\left(|M_S=\tfrac{1}{2}, M_I=-\tfrac{1}{2}\rangle + |M_S=-\tfrac{1}{2}, M_I=\tfrac{1}{2}\rangle\right)$$

$$|F=3, M_F=0\rangle = \frac{1}{\sqrt{2}}\left(|M_S=\tfrac{1}{2}, M_I=-\tfrac{1}{2}\rangle - |M_S=-\tfrac{1}{2}, M_I=\tfrac{1}{2}\rangle\right)$$

Optical magnetometry

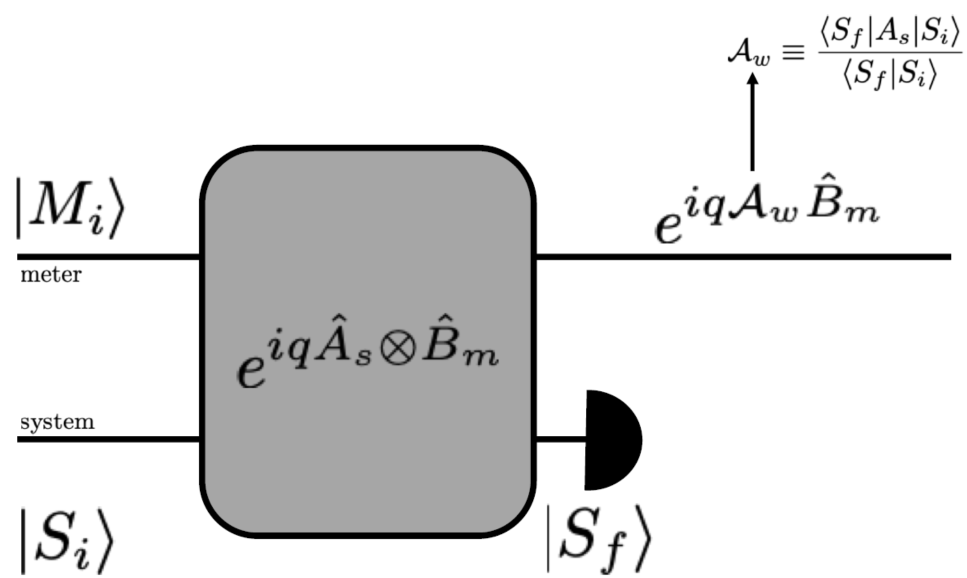




Achieved: ~10^{-15} Tl
Planned: 10^{-17} Tl

Budker & Romalis, Nat Phys. **3**, 228 (2007)
Budker & Kimbal (eds), *Optical Magnetometry,* (CUP, 2013)

+weak measurements



$$EEP = WEP + LLI + LPI_{1,2}$$

RQI:

how fundamental physics affects quantum info

how quantum info probes fundamental physics