Hidden teleportation power for entangled quantum states

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An ideal quantum teleportation is a process by which an unknown d-dimensional quantum state is teleported intact from Alice to another party, Bob, via the use of a shared maximally entangled resource and the communication of classical information [1]. If the shared state is not maximally entangled, the teleportation fidelity f, i.e., the maximal average fidelity between the state Alice wants to teleport and the state Bob receives is no longer unity. In particular, if Alice and Bob only share classical resources, the maximal average fidelity achievable will be $f_c = 2/(d+1)$ [2]. When a shared entangled state gives $f > f_c$, we say that it is useful for teleportation.

Imagine now that Alice and Bob share an entangled state ρ with $f < f_c$, one way to increase the teleportation fidelity is to apply a suitable local filtering operation [3] on ρ before performing the teleportation. This is a probabilistic process but with an appropriate choice of filters, and conditioned on the success of the local filtering operation, Alice and Bob may end up with a state $\rho_{filtered}$ that has a higher teleportation fidelity [4]. Henceforth, we say ρ has hidden teleportation power if it is useless for teleportation $f(\rho) < f_c$ but becomes useful for teleportation ($f(\rho_{filtered}) > f_c$) after successful local filtering.

Specifically, we investigate the problem for the following d-dimensional, one-parameter family of entangled states: $\rho(q) = q|\Phi^+\rangle\langle\Phi^+| + (1-q)|01\rangle\langle 01|$ where $|\Phi^+\rangle = \sum_{i=0}^{d-1}|ii\rangle/\sqrt{d}$ and $0 < q \leq 1$. Following [5], our task is to determine the optimal filter A on Alice's subsystem such that the overall teleportation fidelity:

$$K(p,q) = pf(\rho_{filtered}(q)) + (1-p)\frac{2}{d+1} \qquad (1)$$

is maximal, where p is the success probability of local filtering. Our analysis shows that the optimal local filter A, which is a function of q, is diagonal and reads as $A = \mathrm{diag}\left[\frac{(d-1)q}{d(1-q)}, 1, \ldots, 1\right]$.

Our results are summarized in Figure 1 where we use qutrit case as an example. In the left plot, the dashed line represents the classical threshold f_c— below this dashed line means that the entangled state is useless. The blue line represents the teleportation fidelity after successful local filtering, i.e., $f(\rho_{filtered}(q))$, while the black line shows the teleportation fidelity before filtering $[f(\rho(q))]$. The red line is the overall teleportation fidelity $K(p,q)$. The upper bound of $K(p,q)$ (green line)—obtained from Rain's [6] semidefinite program for positive-partial-transposition-preserving operation—and the red line clearly differ. This shows that these more general class of operations can allow for the increase in teleportation fidelity (in the qutrit case) more than that allowed by local operations assisted by classical communications.

The right plot shows the increase in the teleportation fidelity, i.e., $f(\rho_{filtered}(q)) - f(\rho(q))$, as a function of p in performing the filtering operation. We see that in the region where the success probability is small, we can increase the fidelity more, but after some threshold success probability, if we want to increase the success probability further, the amount of fidelity that we can increase gradually approaches zero, thereby showing a tradeoff between the two quantities.

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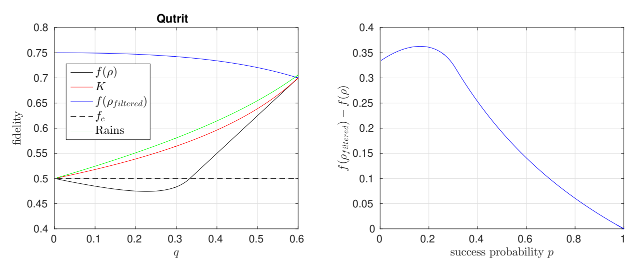

FIG. 1. Left: various teleportation fidelities vs the parameter q. The entangled two-qutrit states exhibit hidden teleportation power in the region $0 < q \leq \frac{1}{3}$. Right: improvement in teleportation fidelities vs the success probability in performing the local filtering operations.

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