A brief guide to device-independent quantum information

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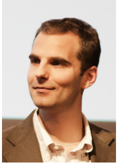
National Cheng Kung University, Taiwan

8th International Workshop on Solid-state Quantum Computing &
Mini-School on Quantum Information Science,
Taipei, Taiwan, 10-15th December 2016

Coworkers

N.Gisin

N.Brunner



S.Pironio

J.-D. Bancal

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Thanks to · · ·

Coworkers and funding agencies

N.Gisin

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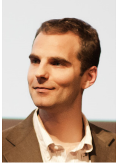

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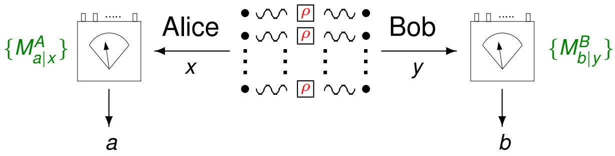
科 技 部

Ministry of Science and Technology

Two extreme levels of trusts in quantum experiments



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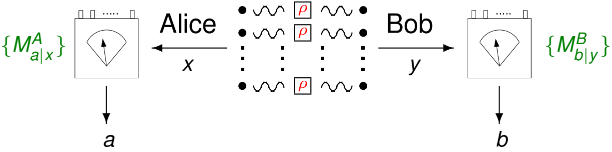
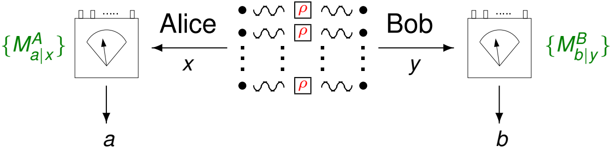


Full level of trust

"Conventional" approach

Two extreme levels of trusts in quantum experiments



Full level of trust
"Conventional" approach
$\left\{\langle M^A_{a|x} \otimes M^B_{b|y}\rangle_\rho\right\}$, ρ
Hermitian (witness) operator

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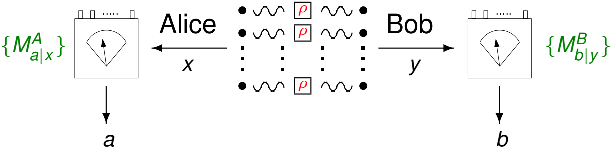

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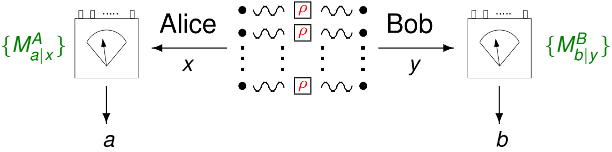

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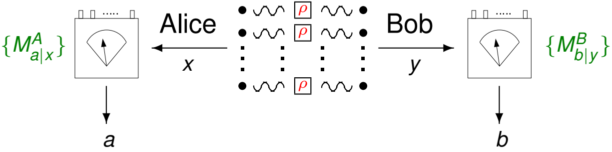

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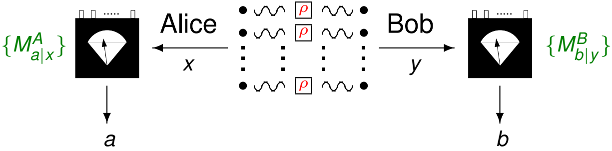
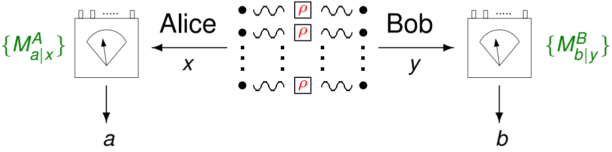


No trust "at all"

Device-independent

Correlation $P(a, b|x, y)$

Two extreme levels of trusts in quantum experiments



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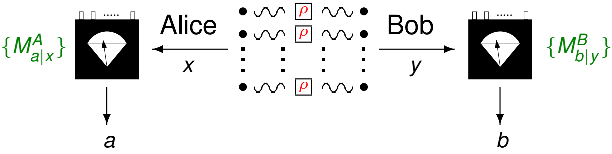

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Bell-like inequalities

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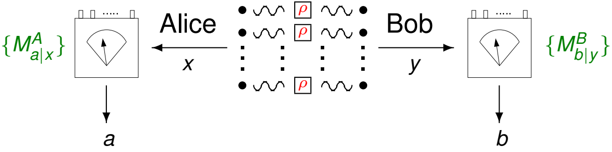

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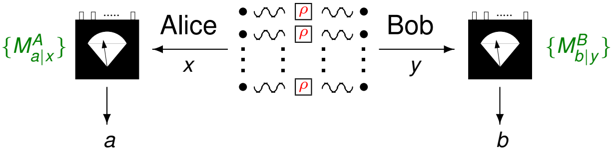

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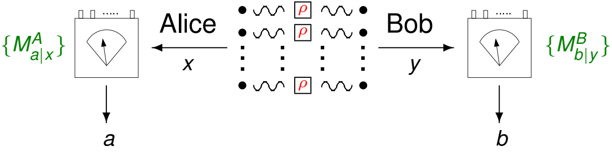

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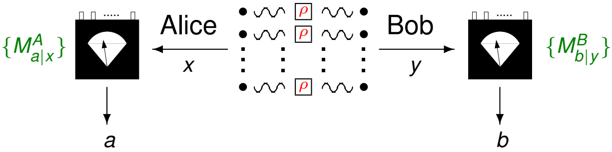

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Device-independent
Correlation $P(a, b|x, y)$
Bell-like inequalities

Entanglement based quantum key distributions I

Bennett-Brassard-Mermin 92 protocol

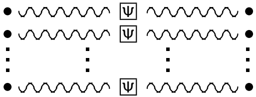
Alice

Bob

Entanglement based quantum key distributions I

Bennett-Brassard-Mermin 92 protocol

Alice



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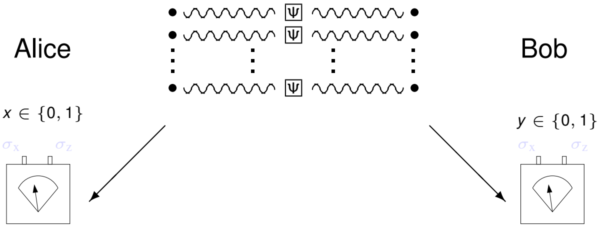

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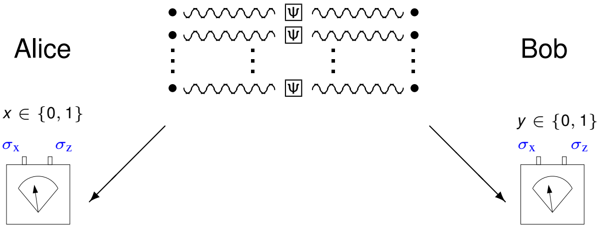

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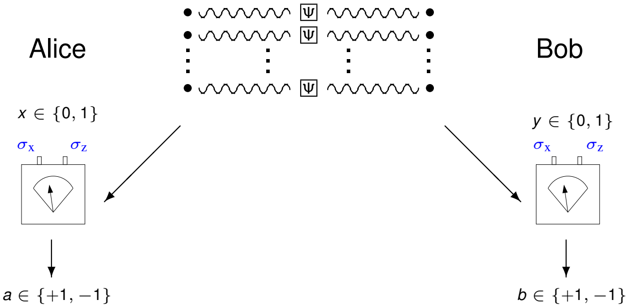

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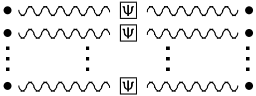
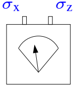


Alice

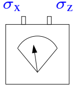
$x \in \{0, 1\}$



$a \in \{+1, -1\}$

Bob

$y \in \{0, 1\}$



$b \in \{+1, -1\}$

k	$x = 0$	$x = 1$	$y = 0$	$y = 1$
1	+1			+1
2		+1		-1
3	-1		+1	
⋮	⋮	⋮	⋮	⋮

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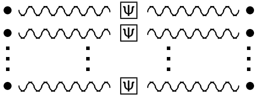

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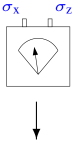

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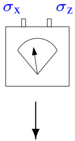
⋮	⋮	⋮	⋮	⋮

$y \in \{0,1\}$



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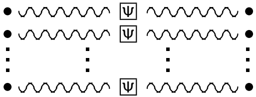

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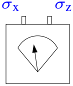

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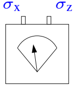

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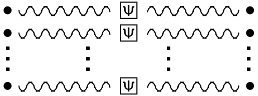

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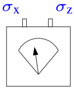

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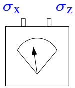

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Joint probability: $P(a, b|x, y) = \begin{cases} \frac{1}{2} & \text{if } x = y \text{ and } a = -b \\ \frac{1}{4} & \text{if } x \neq y \end{cases}$

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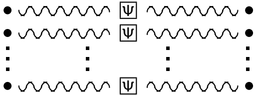

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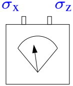

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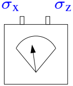

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Entanglement based quantum key distributions II

Bennett-Brassard-Mermin 92 protocol

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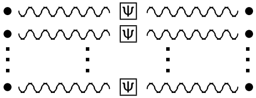

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- If local subsystem is 2-dimensional (qubit) $\Rightarrow |\Psi\rangle$ is a Bell state
- The same correlations can be achieved with:

$$\rho = \frac{1}{4}\left(|00\rangle\langle 00|_x + |11\rangle\langle 11|_x\right) \otimes \left(|00\rangle\langle 00|_z + |11\rangle\langle 11|_z\right)$$

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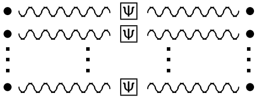

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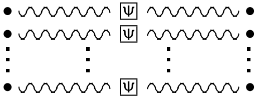

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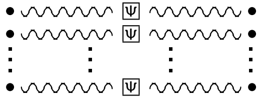

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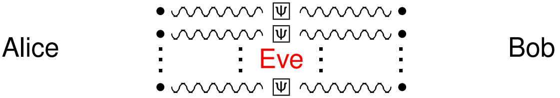

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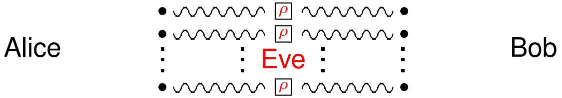

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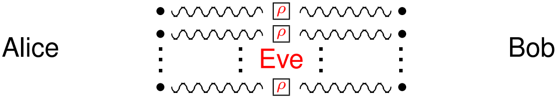

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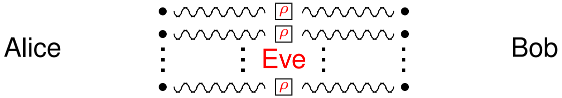
Why we cannot take dimension knowledge for granted?



"In theory, there is no difference between theory and practice. But, in practice, there is." by Jan L. A. van de Snepscheut

On the assumption of Hilbert space dimension

- Polarization of a photon ⟹ qubit!
- A photon has many other degrees of freedom: e.g., frequency, spatial mode, time bin.
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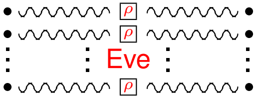
Why we cannot take dimension knowledge for granted?

Alice  Bob

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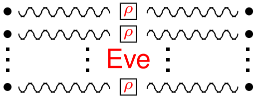

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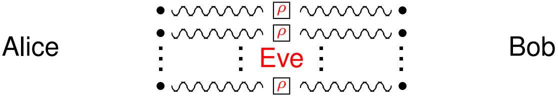

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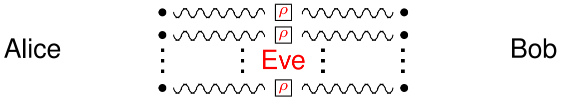

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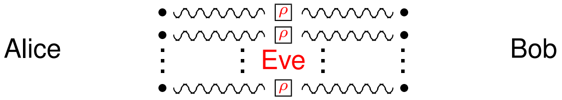

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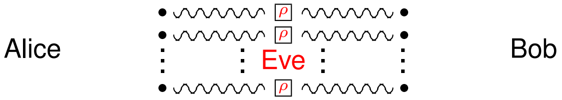

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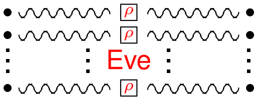
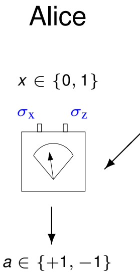
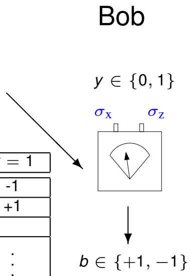
The black box scenario





k	$x = 0$	$x = 1$	$y = 0$	$y = 1$
1	+1			-1
2		-1		+1
3	+1		-1	
⋮	⋮	⋮	⋮	⋮



- Security analysis independent of assumption on dimension of ρ and detailed functioning of devices??
- Security from measurement statistics $P(a, b|x, y)$?
- Device-independent (DI) way to verify entanglement?

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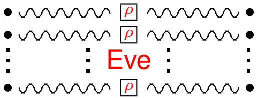

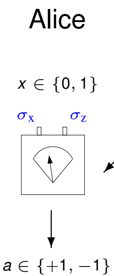

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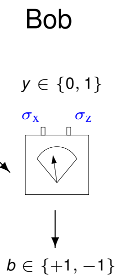

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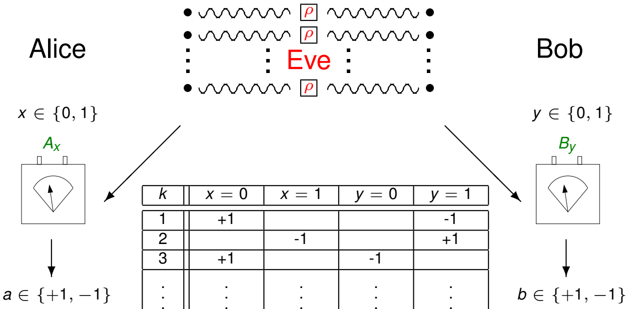

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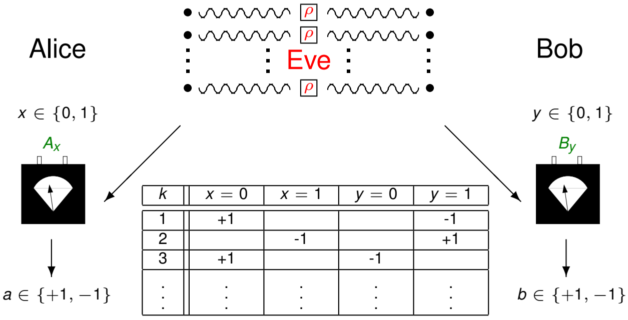

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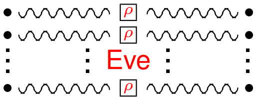

Alice

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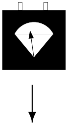
A_x

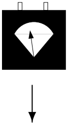


$a \in \{+1, -1\}$

Bob

$y \in \{0, 1\}$

B_y



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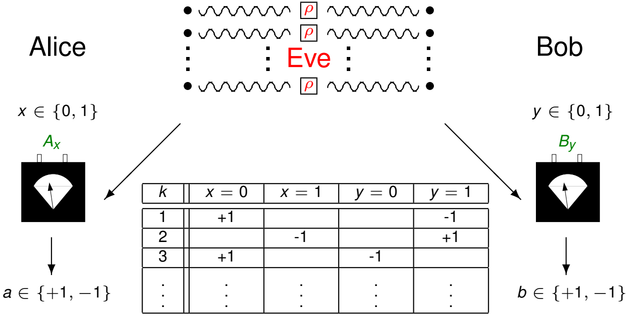

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Bell inequalities — from foundation to application

- Bell inequalities are constraints that have to be satisfied by local-hidden-variable models (LHVM)

$$P(a,b|x,y) = \sum_{\lambda} p_{\lambda}\, P(a|x,\lambda)\, P(b|y,\lambda)$$

- $\vec{P} = \{P(a,b|x,y)\}_{x,y,a,b}$ allowed by LHVM form a convex polytope.

- Message #1: Quantum correlations [cf. Born's rule] $P(a,b|x,y) = \mathrm{tr}(\rho\, M^{A}_{a|x} \otimes M^{B}_{b|y})$ can violate Bell inequalities.

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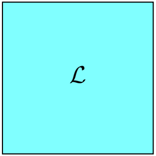

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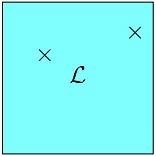

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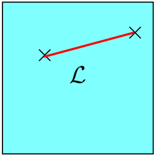

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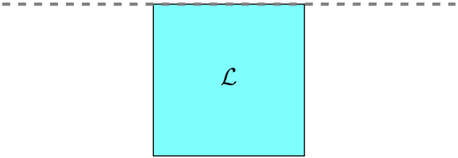

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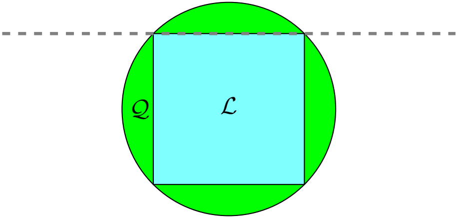

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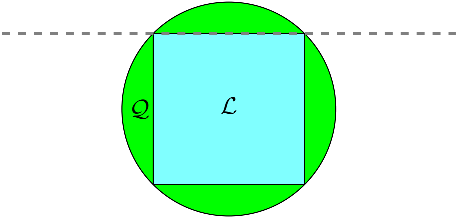

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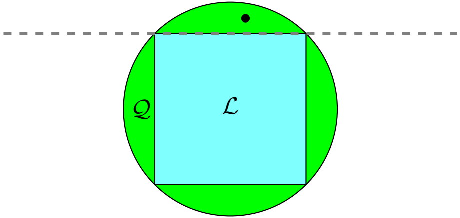

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Example: The Clauser-Horne-Shimony-Holt-Bell inequality

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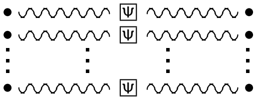
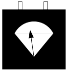
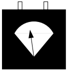


Bob

$x \in \{1, 2\}$

$y \in \{1, 2\}$





Joint conditional probability:

$$P(a, b|x, y)$$

$a \in \{+1, -1\}$

$b \in \{+1, -1\}$

Correlator/ Correlation function:

$$E_{xy} = \sum_{a,b} a\, b\, P(a, b|x, y)$$
$$= \sum_{a} P(a = b|x, y) - P(a \neq b|x, y)$$

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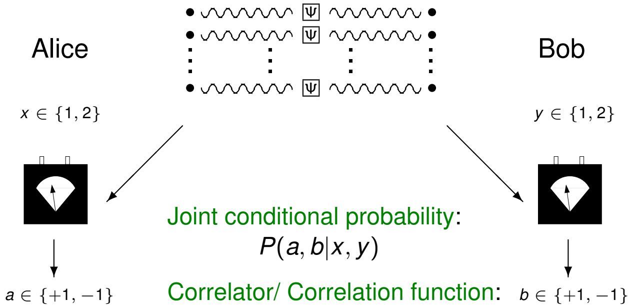

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$$\mathcal{S}_{\text{CHSH}} = E_{11} + E_{12} + E_{21} - E_{22}$$

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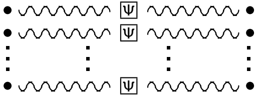

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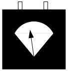

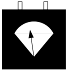

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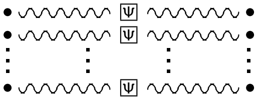
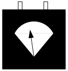
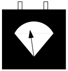


Alice — $x \in \{1,2\}$



$a \in \{+1,-1\}$

Bob — $y \in \{1,2\}$



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$$\mathcal{S}_{\text{CHSH}} \overset{\mathcal{Q}}{\leq} 2\sqrt{2} \quad \text{if } \Psi = \Psi^-, \; y = 1,2 \to -\tfrac{1}{\sqrt{2}}(\hat{x}+\hat{z}), \tfrac{1}{\sqrt{2}}(\hat{x}-\hat{z}),$$
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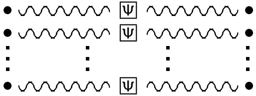

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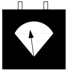

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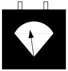

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Example: The Clauser-Horne-Shimony-Holt-Bell inequality

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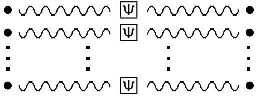

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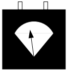

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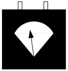

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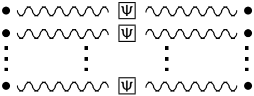

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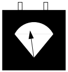

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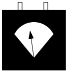

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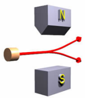

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* Stern Gerlach magnet picture from http://www.upscale.utoronto.ca/PVB/Harrison/SternGerlach/SternGerlach.html

Bell inequality as a device-independent entanglement witness

- Message #2: With local measurements, entanglement is necessary to produce Bell inequality violation.

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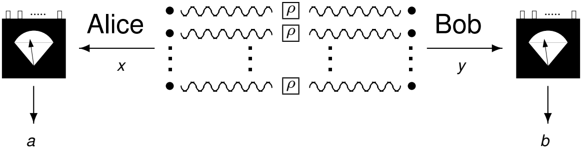

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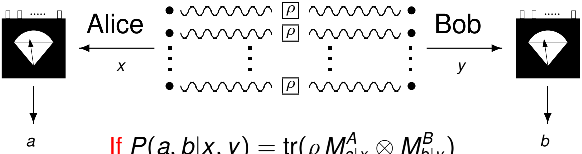

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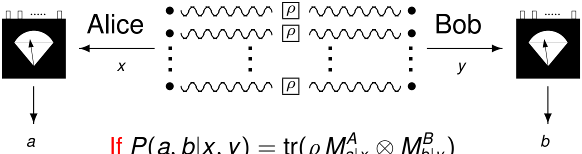

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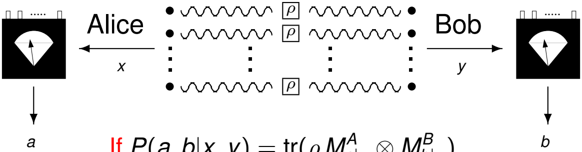

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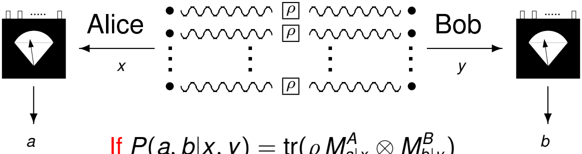

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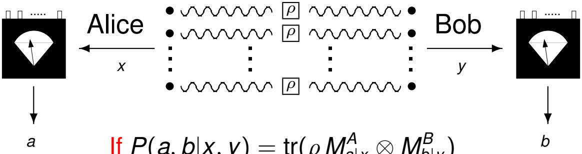

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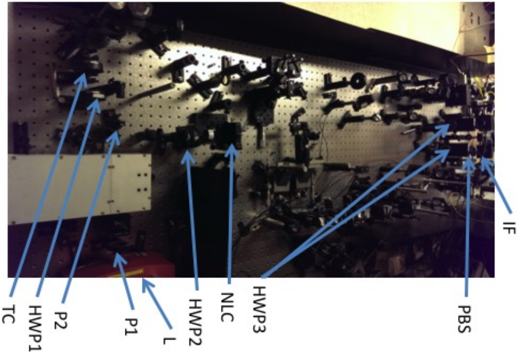

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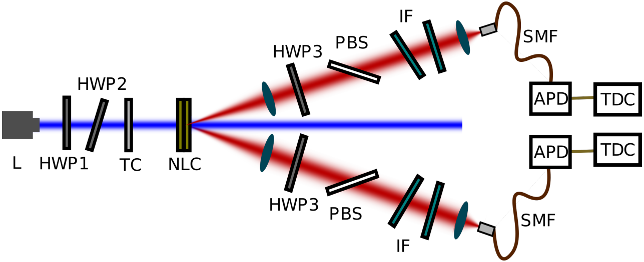

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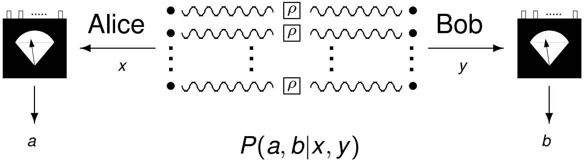

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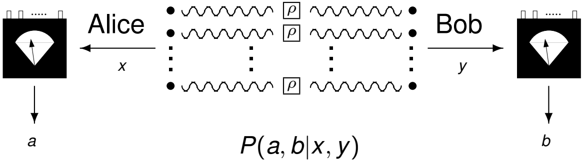

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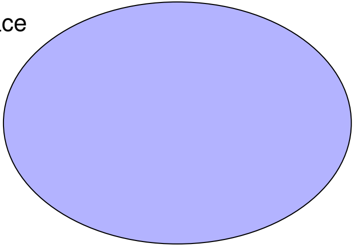

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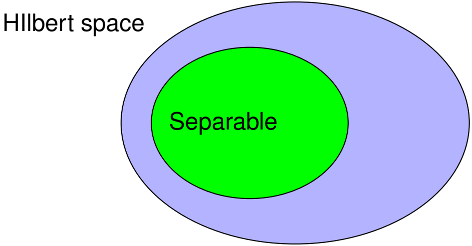

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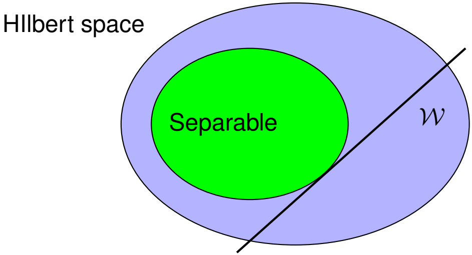

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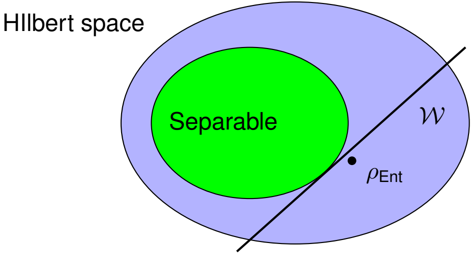

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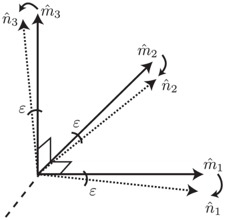

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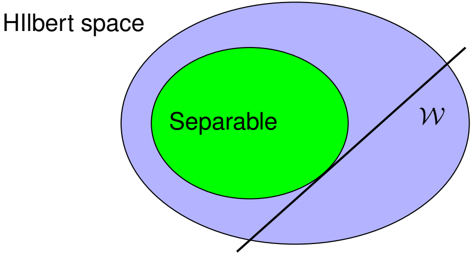

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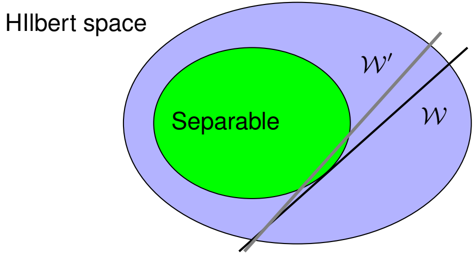

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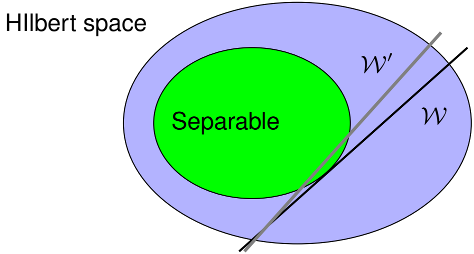

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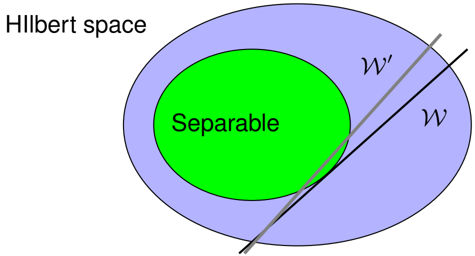

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- In the two-party case, Bell inequalities are the only DIEW.
- To detect full multipartite entanglement, i.e., states that cannot be written in the (biseparable) form

$$\rho_{\rm bs} = \sum_{k_1} q^{AB|C}_{k_1}\, \rho^{k_1}_{AB} \otimes \rho^{k_1}_{C} + \sum_{k_2} q^{AC|B}_{k_2}\, \rho^{k_2}_{AC} \otimes \rho^{k_2}_{B} + \sum_{k_3} q^{BC|A}_{k_3}\, \rho^{k_3}_{BC} \otimes \rho^{k_3}_{A},$$

  Bell violation is insufficient (cf. $|\Psi\rangle = |\Psi^-\rangle_{AB} \otimes |0\rangle_C$)
- Biseparable states must give biseparable correlations $\mathcal{Q}'_{2/1}$:

$$P(a,b,c|x,y,z) \stackrel{\mathcal{Q}'_{2/1}}{=} \sum_{\lambda_3} q^{AB|C}_{\lambda_3}\, P^{\mathcal{Q}}(a,b|x,y,\lambda_3)\, P(c|z,\lambda_3) + \sum_{\lambda_2} q^{AC|B}_{\lambda_2}\, P^{\mathcal{Q}}(a,c|x,z,\lambda_2)\, P(b|y,\lambda_2) + \sum_{\lambda_1} q^{A|BC}_{\lambda_1}\, P^{\mathcal{Q}}(b,c|y,z,\lambda_1)\, P(a|x,\lambda_1)$$

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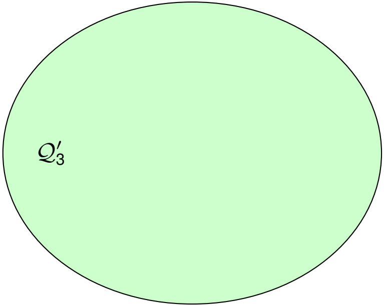

[4] Bancal, Gisin, YCL, Pironio, Phys. Rev. Lett., 2011; Pál & Vértesi, Phys. Rev. A, 2011; Bancal, Branciard, Brunner, Gisin, YCL, J. Phys. A, 2012.

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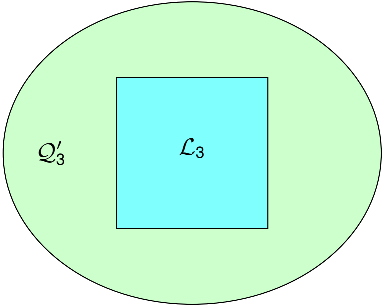

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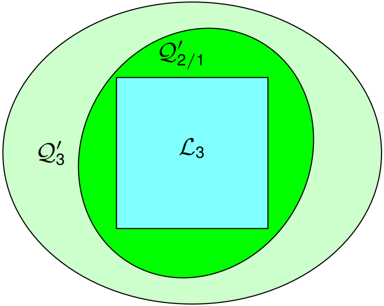

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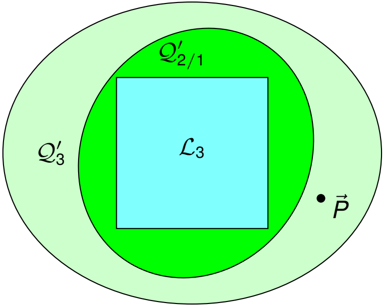

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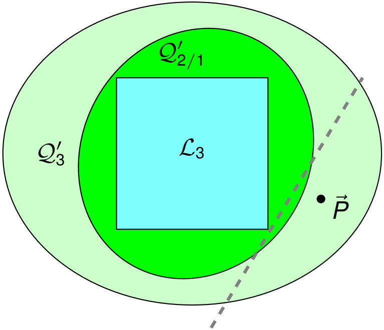

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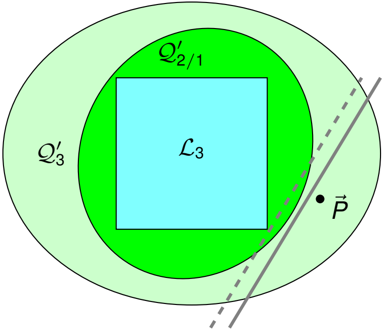

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Entanglement depth: The extent of many-body entanglement

- Entanglement depth[5]/ non-k-producibility[6]: the extent to which many-body entanglement is needed to prepare a (multi-partite) entangled state.

- A pure state $|\psi\rangle$ is k-producible if we can write:[7] $|\psi\rangle = |\varphi_1\rangle \otimes |\varphi_2\rangle \otimes \cdots \otimes |\varphi_m\rangle$ where the $|\varphi_i\rangle$ are states at most k-partite.

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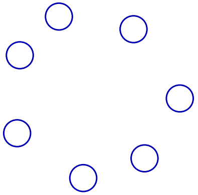

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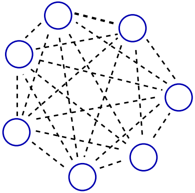

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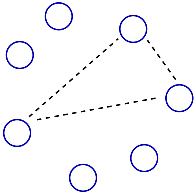

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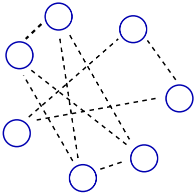

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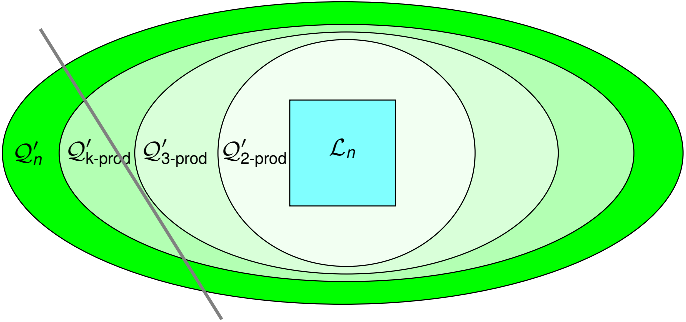
Entanglement depth: The extent of many-body entanglement II

- Message #4: Entanglement depth can be certified via the violation of Bell-like inequalities (device-independent witnesses for entanglement depth, DIWED).[8]



[8] YCL, Rosset, Bancal, Pütz, Barnea, Gisin, Phys. Rev. Lett., 2015; Curchod, YCL, Gisin, Phys. Rev. A, 2015.

Device-independent witnesses for entanglement depth I

- A family of n-partite, 2-setting, 2-outcome Bell inequalities:[14]

$$\mathcal{I}_n : \mathcal{S}_n = 2^{1-n} \sum_{\vec{x}\in\{0,1\}^n} E_n(\vec{x}) - E_n(\vec{1}_n) \overset{\text{LHV}}{\leq} 1$$

- A family of DIWED:

$$\mathcal{I}_n^k : 2^{1-n} \sum_{\vec{x}\in\{0,1\}^n} E_n(\vec{x}) - E_n(\vec{1}_n) \overset{k\text{-producible states}}{\leq} \mathcal{S}_k^{\mathcal{Q},*}.$$

k	2	3	4	5	6	7	8	∞
$\mathcal{S}_k^{\mathcal{Q},*}$	$\sqrt{2}$	$\frac{5}{3}$	1.8428	1.9746	2.0777	2.1610	2.2299	3

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$$\mathcal{I}_n^k : 2^{1-n} \sum_{\vec{x}\in\{0,1\}^n} E_n(\vec{x}) - E_n(\vec{1}_n) \overset{k\text{-producible states}}{\leq} \mathcal{S}_k^{\mathcal{Q},*}.$$

k	2	3	4	5	6	7	8	∞
$\mathcal{S}_k^{\mathcal{Q},*}$	$\sqrt{2}$	$\frac{5}{3}$	1.8428	1.9746	2.0777	2.1610	2.2299	3

[14] YCL, Rosset, Bancal, Pütz, Barnea, Gisin, Phys. Rev. Lett., 2015.

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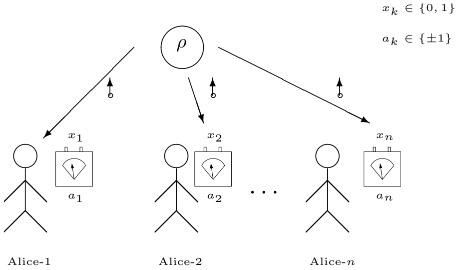

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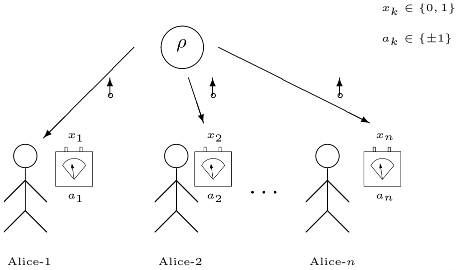

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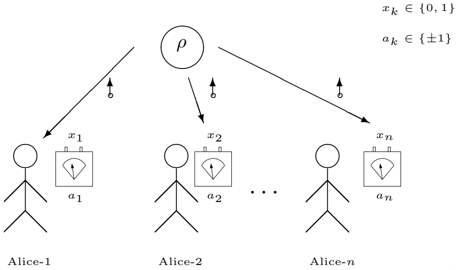
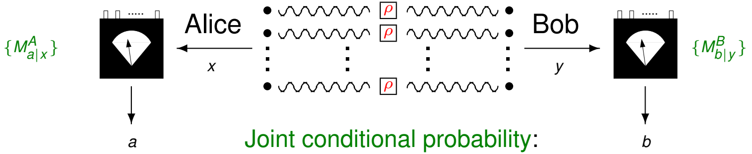


Bounding entanglement directly from correlations?



Joint conditional probability:

$$P(a,b|x,y) = \mathrm{tr}\left[\rho\, M^A_{a|x} \otimes M^B_{b|y}\right]$$

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- How do we quantify entanglement — using entanglement monotones — from Bell-inequality-violating correlations?
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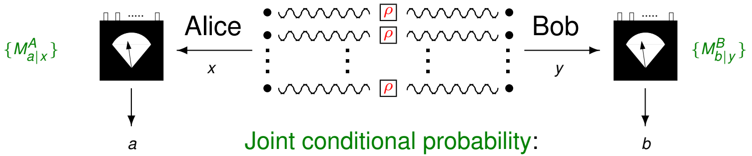

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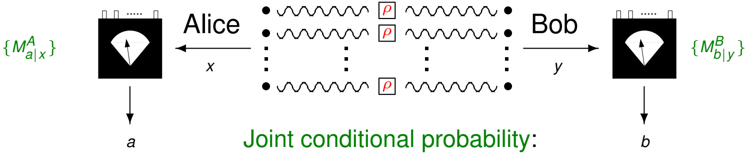

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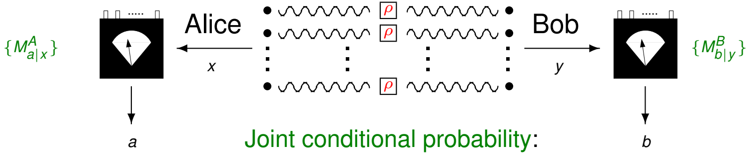

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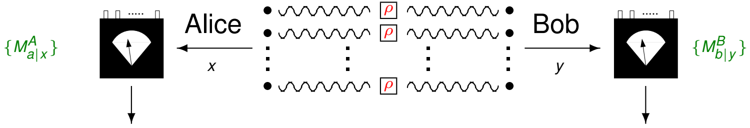

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$P(a, b|x, y)$ → Moment matrix $\chi(\rho)$

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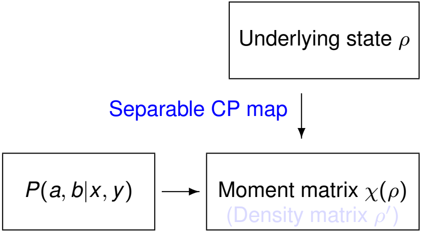

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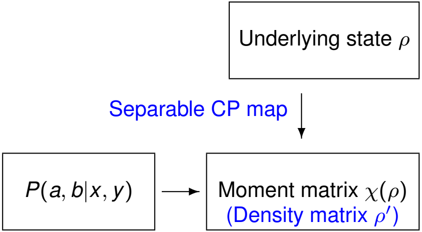

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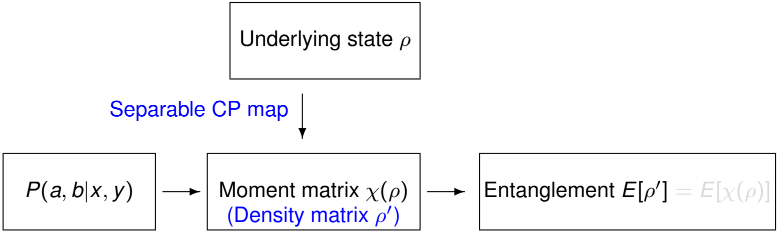

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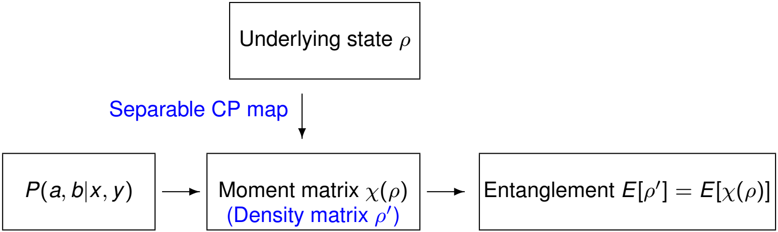

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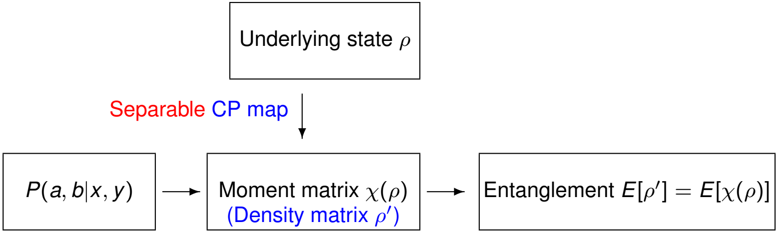

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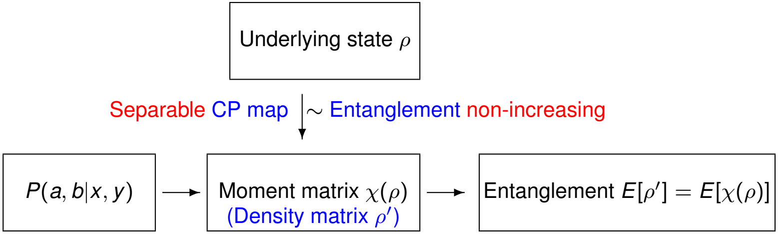

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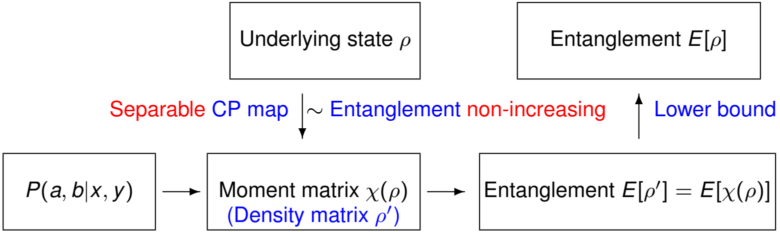

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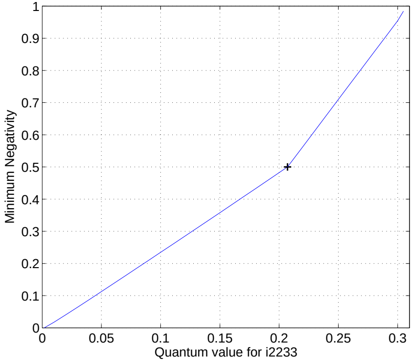

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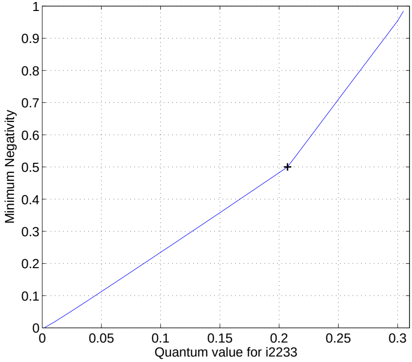

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$$N \leq \frac{d-1}{2}, d \leq \min\{d_1, d_2\}$$

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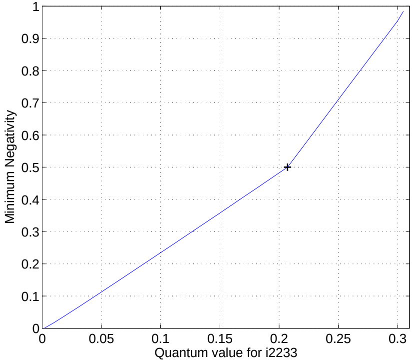

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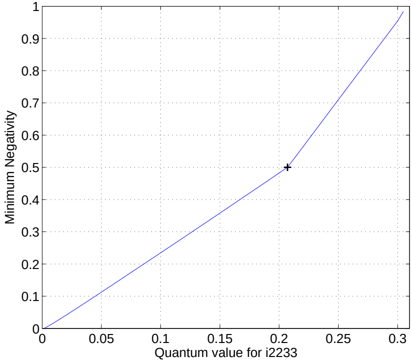
- Message #6: Strength of Bell-inequality violation may reveal dimension information - dimension witness.[11]



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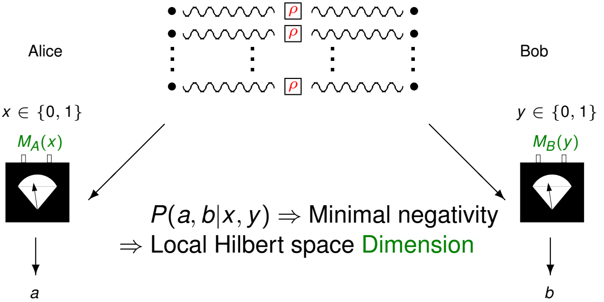

$P(a, b|x, y) \Rightarrow$ Minimal negativity
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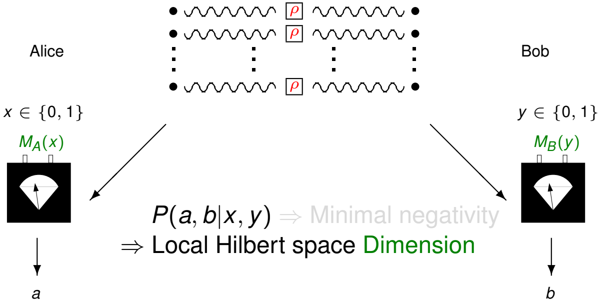

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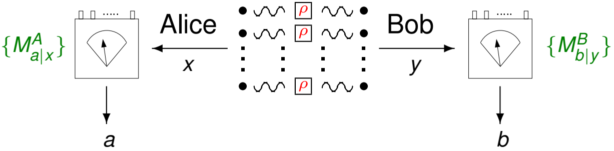
The various levels of trusts

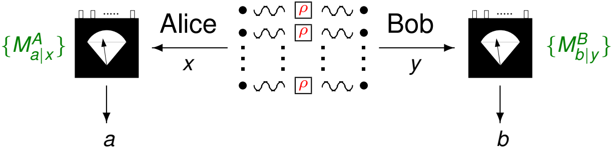


Full level of trust

"Conventional" approach

$\left\{\langle M^A_{a|x} \otimes M^B_{b|y} \rangle_\rho \right\}$, ρ

Hermitian (witness) operator



No trust "at all"

Device-independent

Correlation $P(a, b|x, y)$

Bell-like inequalities

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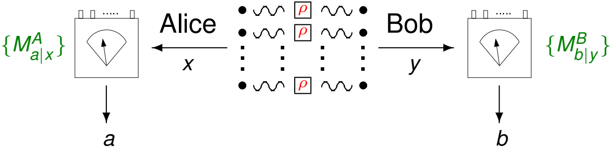

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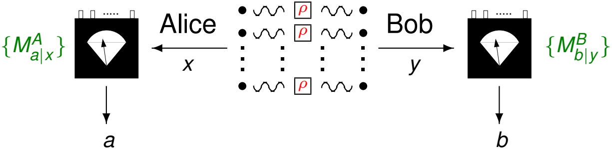

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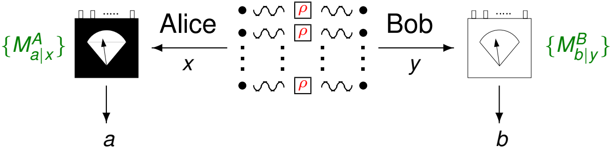

One side is trusted
1-sided device-independent
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Steering inequalities

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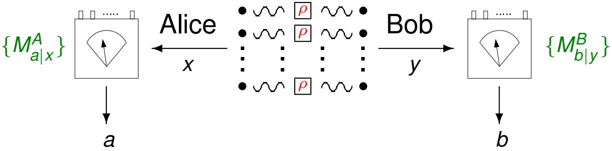

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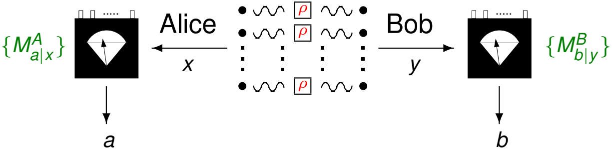

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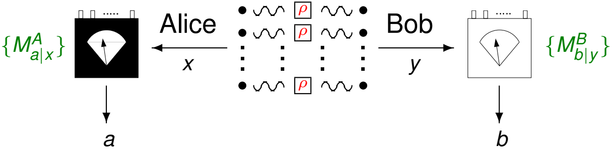

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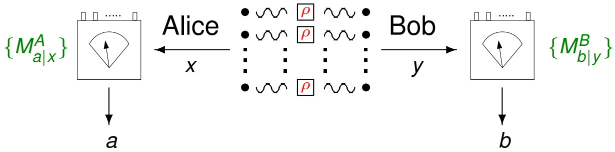
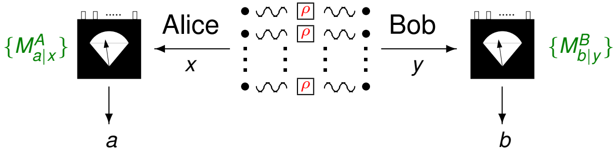
Steering inequalities

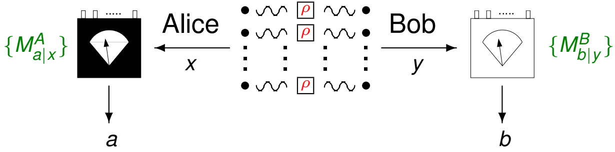
The various levels of trusts



Full level of trust
"Conventional" approach
$\left\{\langle M^A_{a|x} \otimes M^B_{b|y}\rangle_\rho\right\}$, ρ
Hermitian (witness) operator

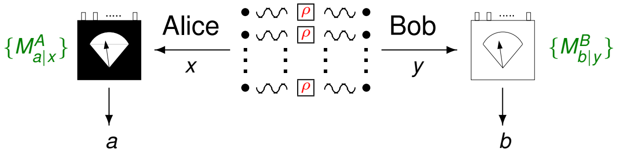


No trust "at all"
Device-independent
Correlation $P(a, b|x, y)$
Bell-like inequalities



One side is trusted
1-sided device-independent
$\left\{\langle M^B_{b|y}\rangle_{\rho_{a|x}}\right\}$, $\{\rho_{a|x}\}$
Steering inequalities

Einstein-Podolsky-Rosen-Schrödinger-steering



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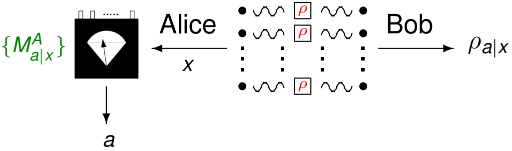
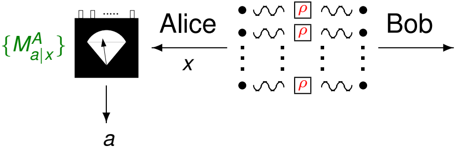


$$\rho_{a|x} = \mathrm{tr}_A(\rho\, M^A_{a|x} \otimes \mathbb{1}_B)$$

Einstein-Podolsky-Rosen-Schrödinger-steering



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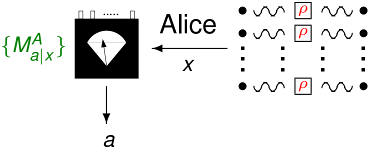

$\rho_{a|x} = \mathrm{tr}_A(\rho\, M^A_{a|x} \otimes \mathbf{1}_B)$

If $\rho = |\Psi^-\rangle\langle\Psi^-|$, then for $\begin{cases} \sigma_z \text{ measurement} \Rightarrow \rho_{a|x} \propto \{|0\rangle\langle 0|, |1\rangle\langle 1|\} \\ \sigma_x \text{ measurement} \Rightarrow \rho_{a|x} \propto \{|+\rangle\langle +|, |-\rangle\langle -|\} \end{cases}$

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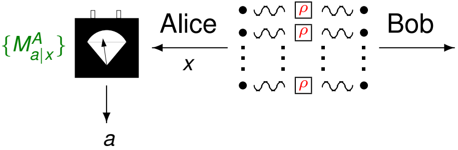

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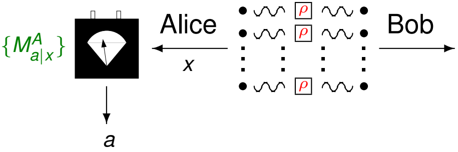

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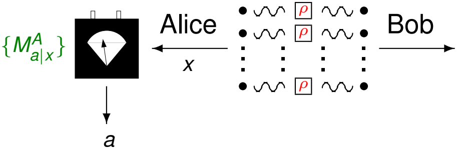

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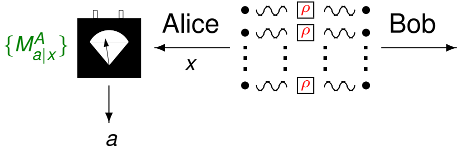

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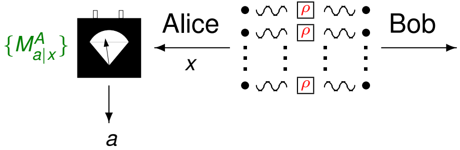

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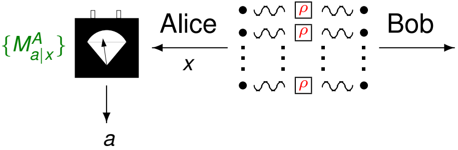
Schrödinger (1935): "It is rather discomforting . . . system to be steered or piloted into one or the other type . . . in spite of [her] having no access to it."

Einstein-Podolsky-Rosen-Schrödinger-steering



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Local-hidden-state (LHS) model: $\rho_{a|x} = \sum_\lambda P_\lambda D(a|x,\lambda)\hat{\sigma}_\lambda \quad \forall\, x, a$

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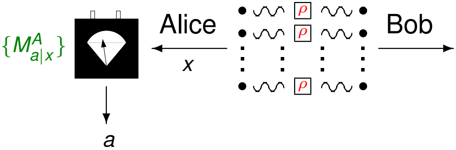

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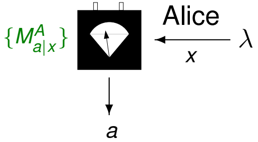

Bob $\rho_{a|x} = \mathrm{tr}_A(\rho\, M^A_{a|x} \otimes \mathbf{1}_B)$

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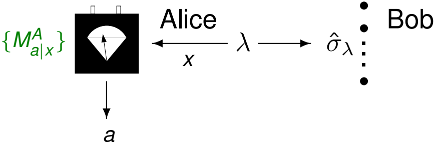

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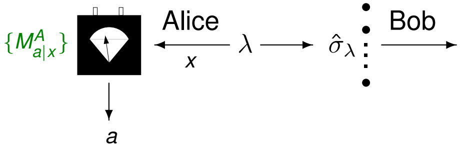

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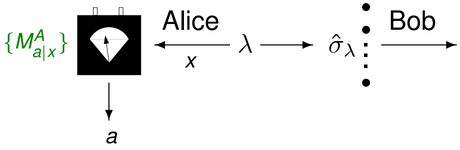

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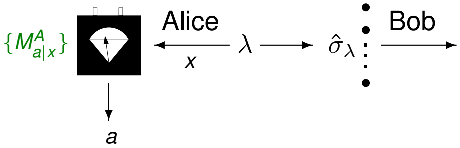
ρ admits a LHS model for all $\{M_{a|x}\}_{x,a}$: non-steerable (from Alice to Bob)

Einstein-Podolsky-Rosen-Schrödinger-steering



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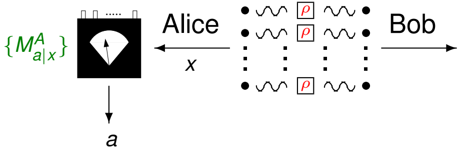
Quantum steering and its relevance



$$\rho_{a|x} = \mathrm{tr}_A(\rho\, M^A_{a|x} \otimes \mathbf{1}_B)$$

- Bell-inequality-violating ⇒ Steerable ⇒ Entangled.
- Steerability can be quantified: steerable weight[13] and steering robustness (SR).[14]
- SR ⇔ probability of success in certain quantum information processing tasks.[3]
- Steerable $\{\rho_{a|x}\}_{a,x} \Leftrightarrow \{M_{a|x}\}_{x,a}$ not jointly measurable.[15,16]

[13] Skrzypczyk, Navascués, Cavalcanti, Phys. Rev. Lett., 2014.
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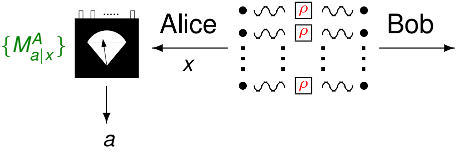

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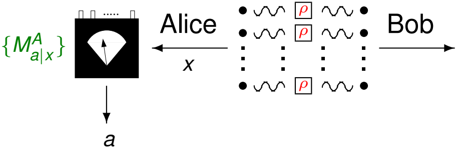

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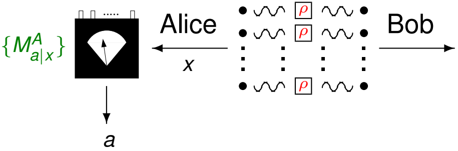

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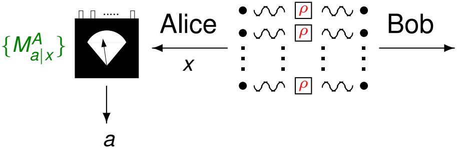

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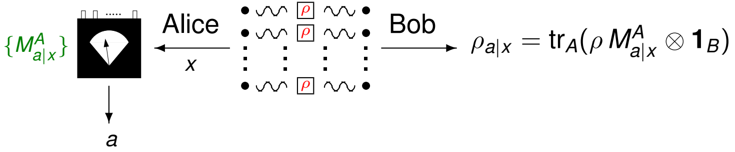

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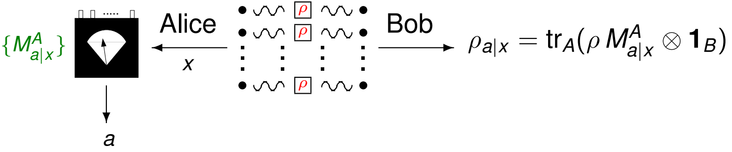
Incompatibility robustness vs steering robustness



- The incompatibility of $\{M^A_{a|x}\}_{x,a}$ can be quantified using incompatibility robustness (IR).
- It can be shown that IR($\{M^A_{a|x}\}_{x,a}$)$\geq$SR($\{\rho_{a|x}\}_{a,x}$).[17]
- Device-independent lower bound on SR($\{\rho_{a|x}\}_{a,x}$) can be computed via SDP $\Rightarrow$ device-independent lower bound on on IR($\{M^A_{a|x}\}_{x,a}$)
- Message #7: Steerability & measurement incompatibility can be estimated directly from observed $P(a,b|x,y)$

[17] Chen, Budroni, YCL, Chen, Phys. Rev. Lett., 2016; Cavalcanti & Skrzypczyk, Phys. Rev. A, 2016.

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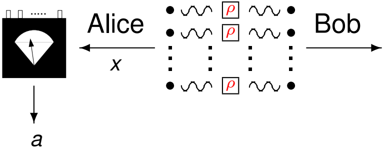

$$\{M^A_{a|x}\} \qquad \rho_{a|x} = \mathrm{tr}_A(\rho\, M^A_{a|x} \otimes \mathbf{1}_B)$$

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- It can be shown that $\mathrm{IR}(\{M^A_{a|x}\}_{x,a}) \geq \mathrm{SR}(\{\rho_{a|x}\}_{a,x})$.[17]
- Device-independent lower bound on $\mathrm{SR}(\{\rho_{a|x}\}_{a,x})$ can be computed via SDP $\Rightarrow$ device-independent lower bound on on $\mathrm{IR}(\{M^A_{a|x}\}_{x,a})$
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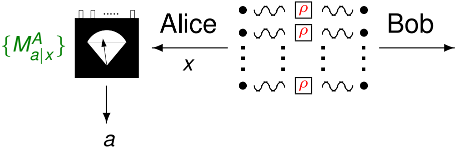
Incompatibility robustness vs steering robustness



$$\rho_{a|x} = \mathrm{tr}_A(\rho\, M^A_{a|x} \otimes \mathbf{1}_B)$$

- The incompatibility of $\{M^A_{a|x}\}_{x,a}$ can be quantified using incompatibility robustness (IR).
- It can be shown that $\mathrm{IR}(\{M^A_{a|x}\}_{x,a}) \geq \mathrm{SR}(\{\rho_{a|x}\}_{a,x})$.[17]
- Device-independent lower bound on $\mathrm{SR}(\{\rho_{a|x}\}_{a,x})$ can be computed via SDP $\Rightarrow$ device-independent lower bound on on $\mathrm{IR}(\{M^A_{a|x}\}_{x,a})$
- Message #7: Steerability & measurement incompatibility can be estimated directly from observed $P(a,b|x,y)$

[17] Chen, Budroni, YCL, Chen, Phys. Rev. Lett., 2016; Cavalcanti & Skrzypczyk, Phys. Rev. A, 2016.

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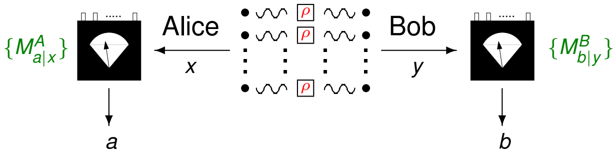

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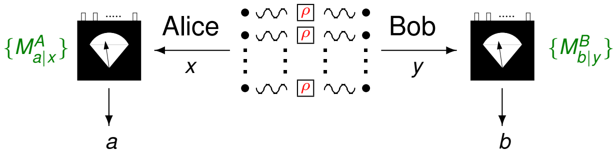

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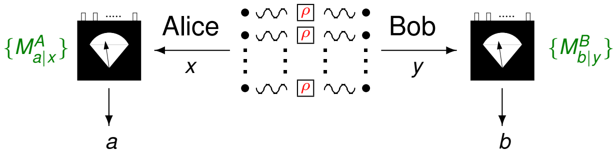

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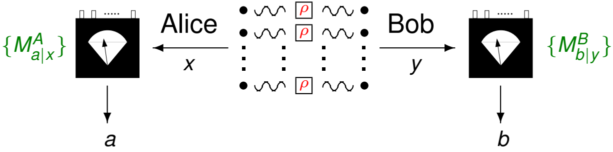
Self-testing of quantum devices: the idea



- Self-testing:[18] to certify directly from measurement statistics that quantum devices — preparation devices & measurement devices — function as expected.
- Given correlation $P(a, b|x, y)$ or observed Bell violation, bound "quality" of devices by some distance measures:

$$||\Lambda_A \otimes \Lambda_B(\rho) - |\psi_{\text{target}}\rangle\langle\psi_{\text{target}}|_{AB} \otimes |\varphi\rangle\langle\varphi|_{A'B'}|| \leq \epsilon,$$

[18] Mayers & Yao, Quant. Inf. Comput. , 2004.

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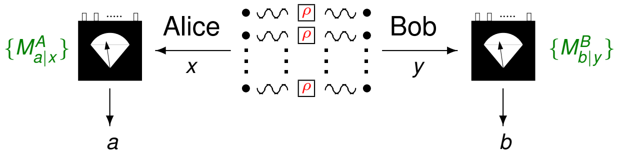

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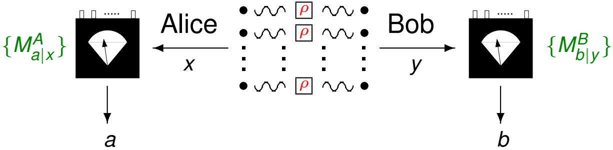

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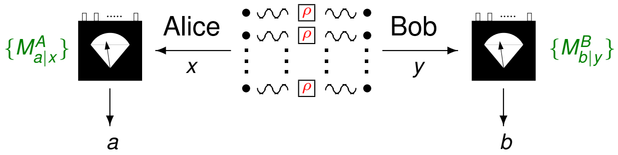

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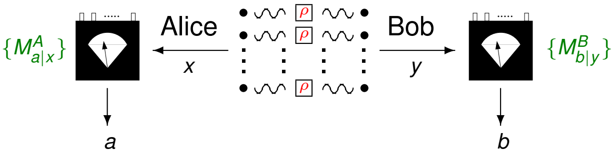

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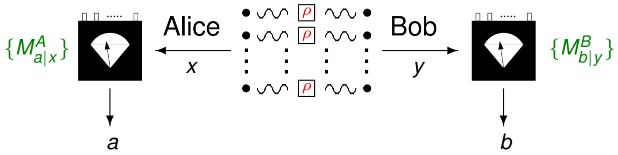

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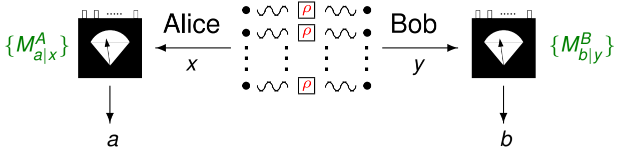
Self-testing of quantum devices: some latest developments



- Message #8: Self-testing of certain pure entangled states is possible when one observes near-maximal quantum violation of certain Bell inequalities.
- A general numerical technique — "SWAP"[19] technique (based on SDP) — can be applied to lower bound directly from observed correlations the fidelity with respect to the target state.

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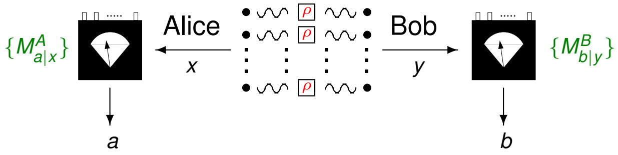

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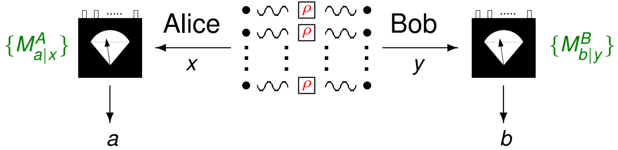

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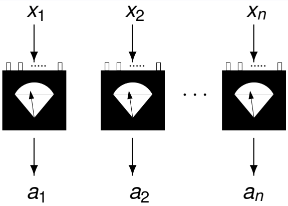

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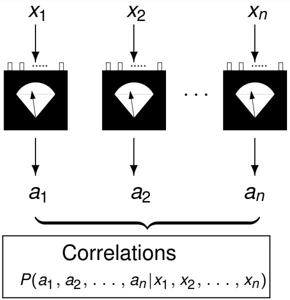

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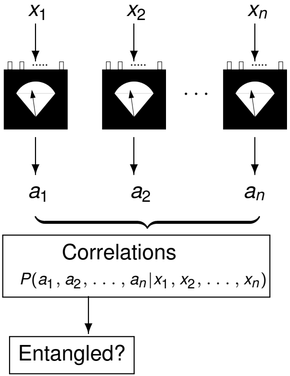

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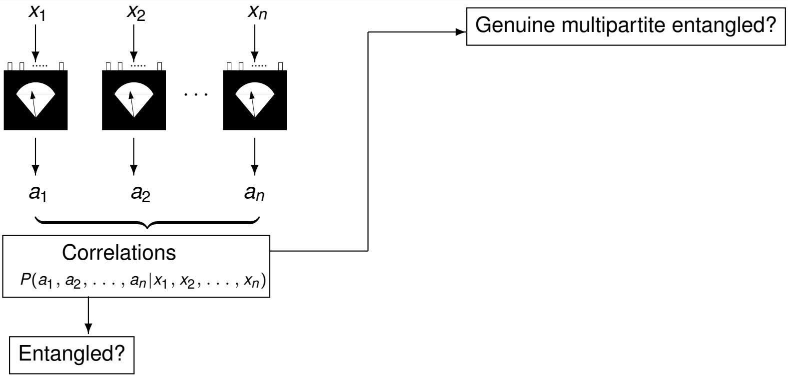

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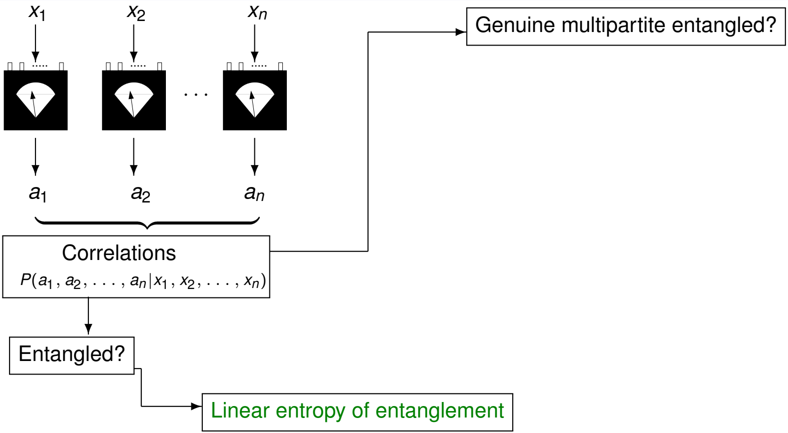

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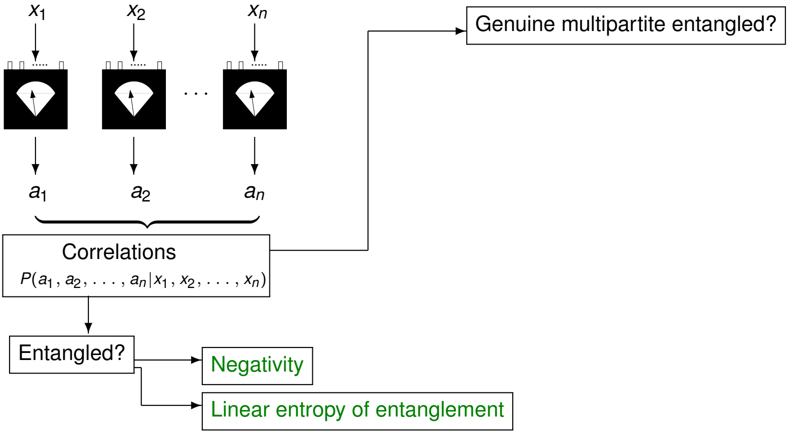

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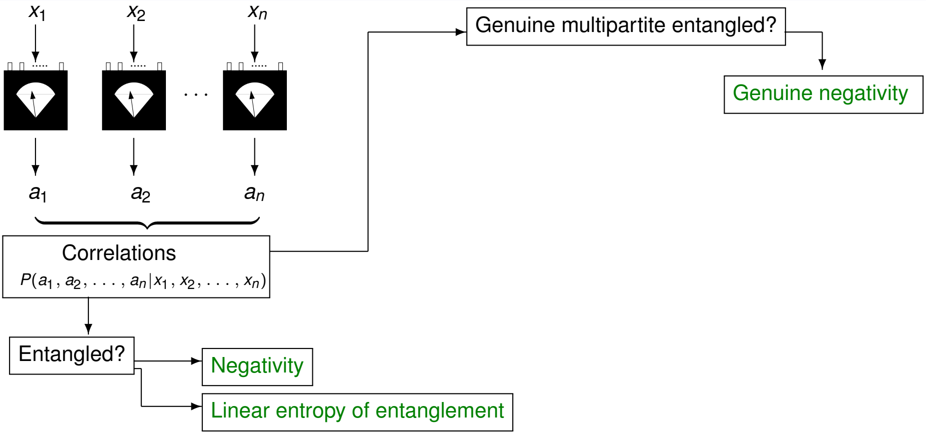

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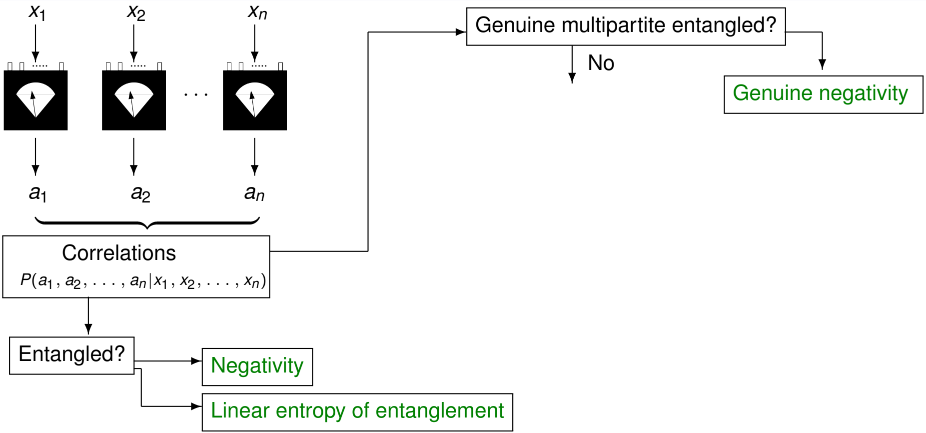

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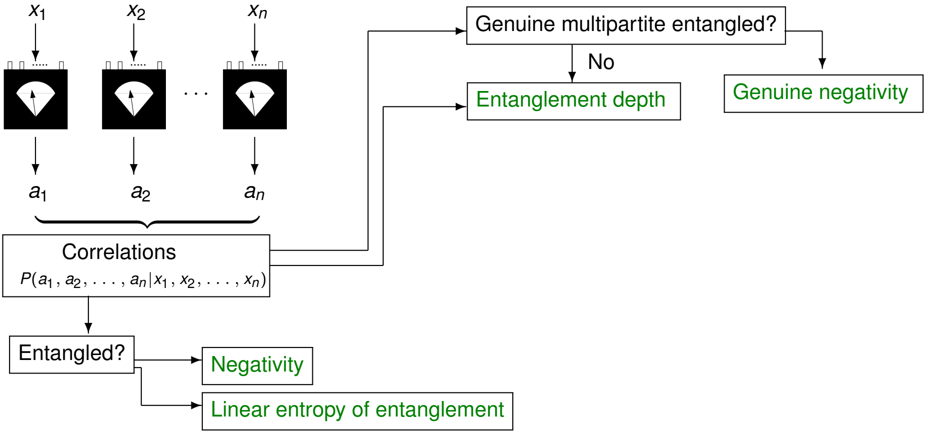

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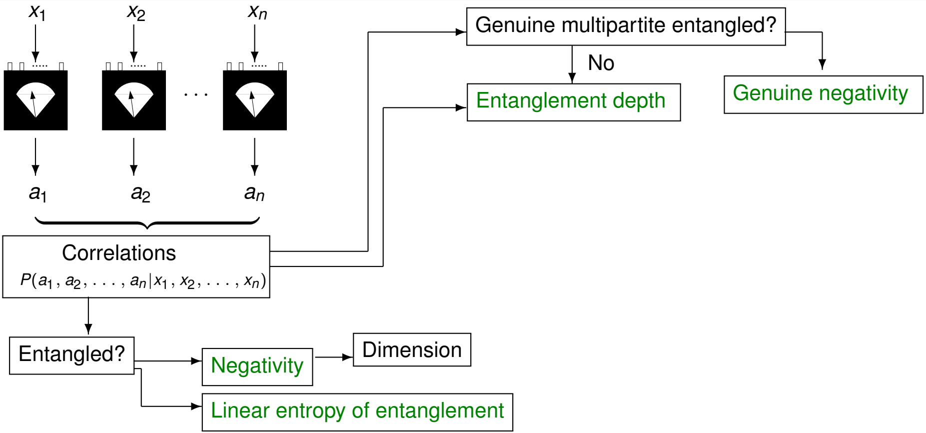

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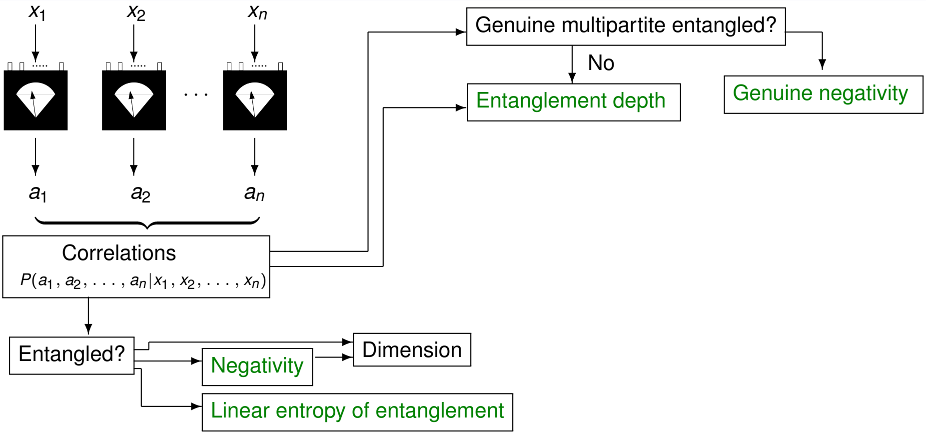

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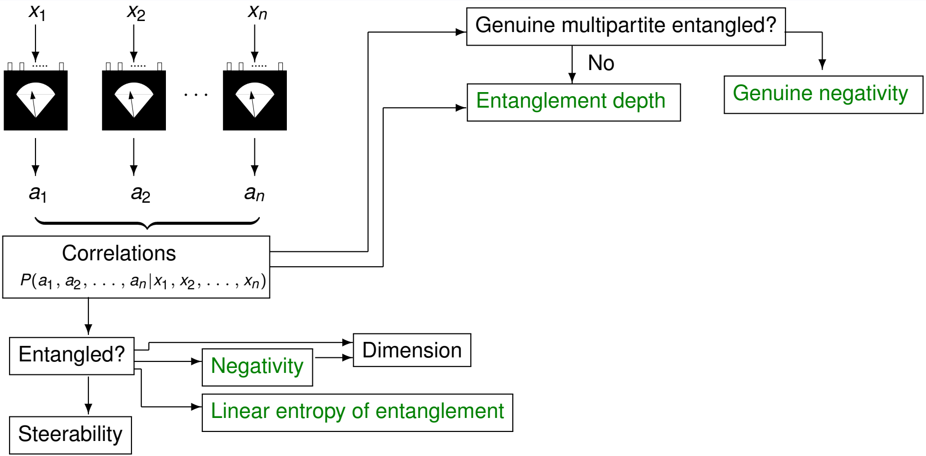

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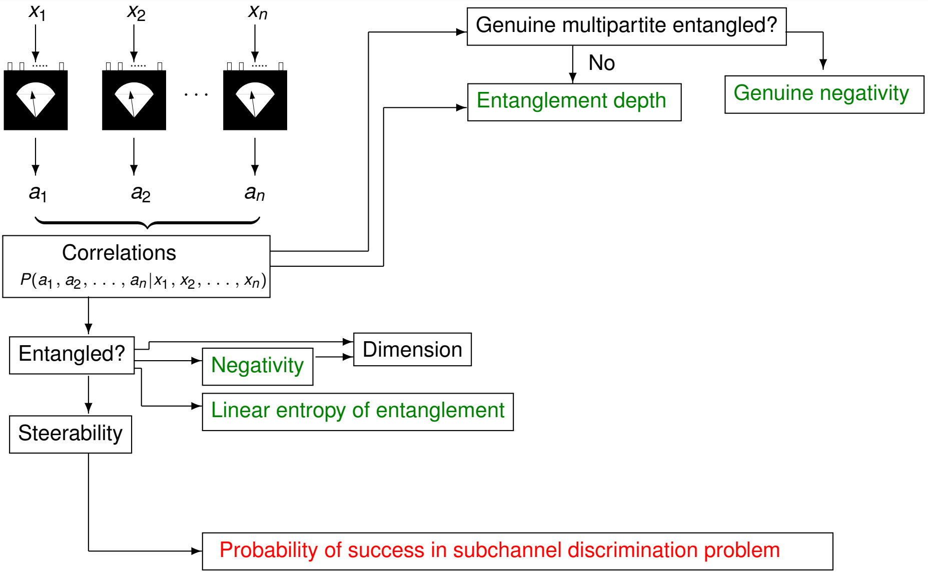

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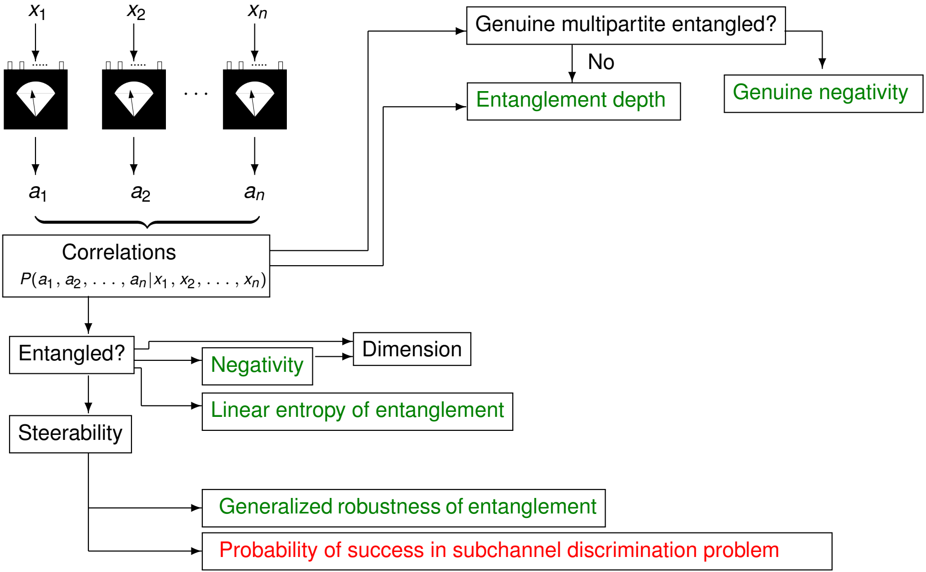

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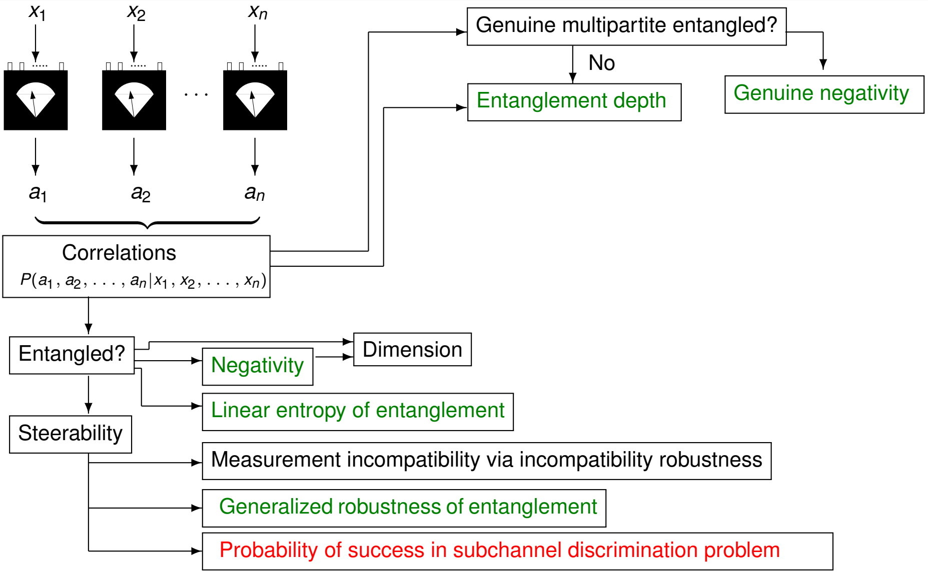

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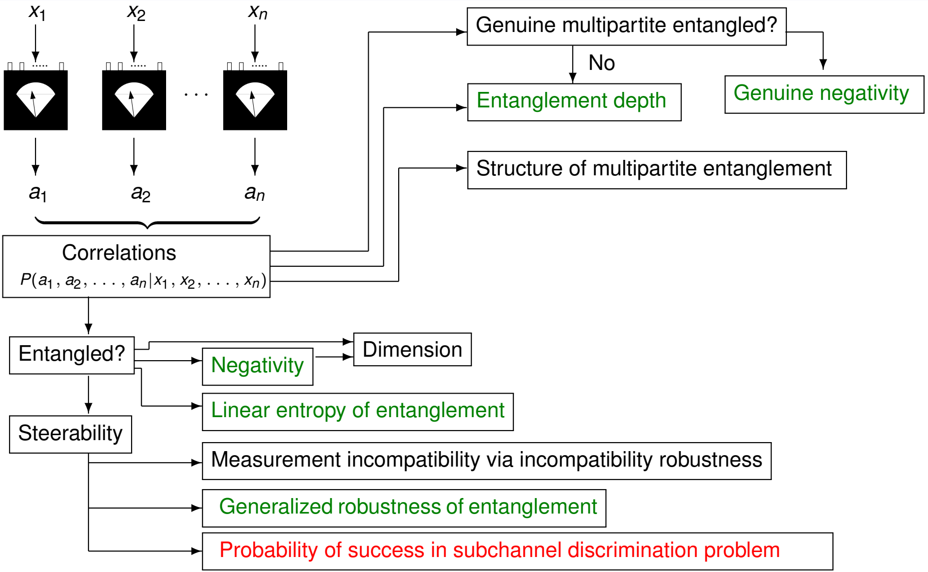

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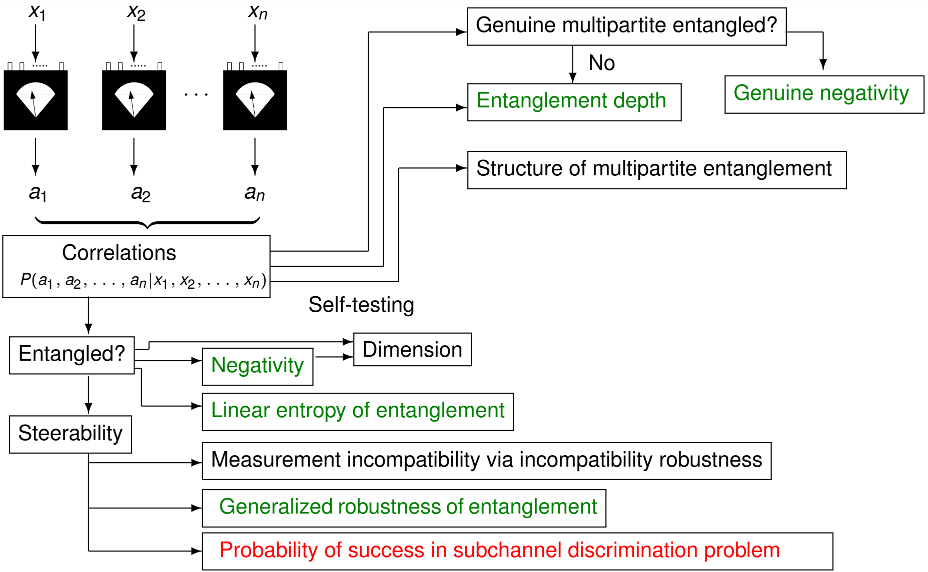

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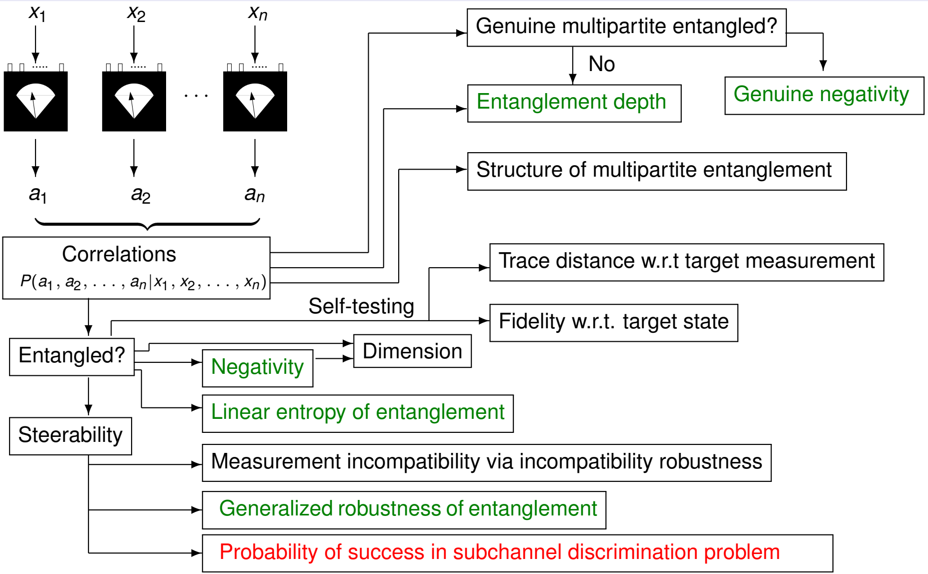

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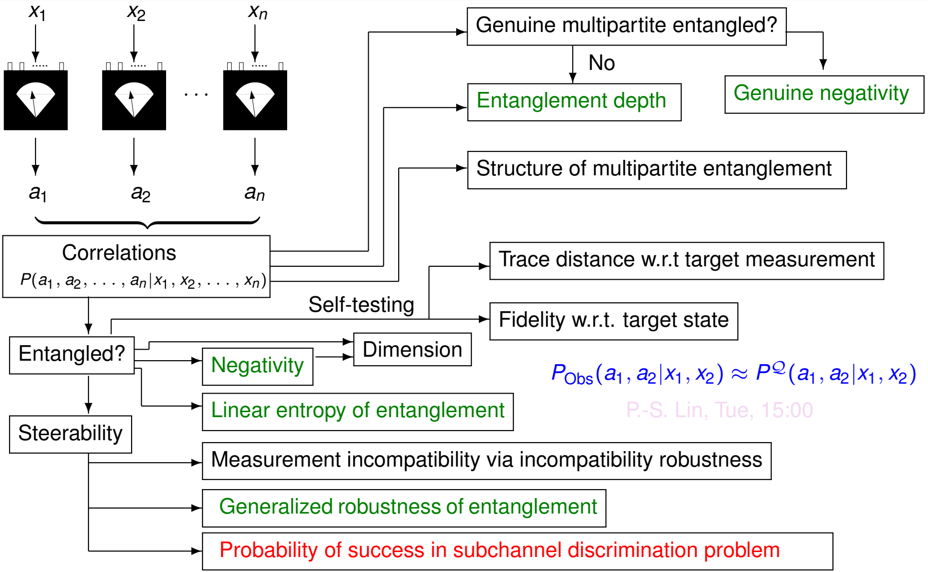

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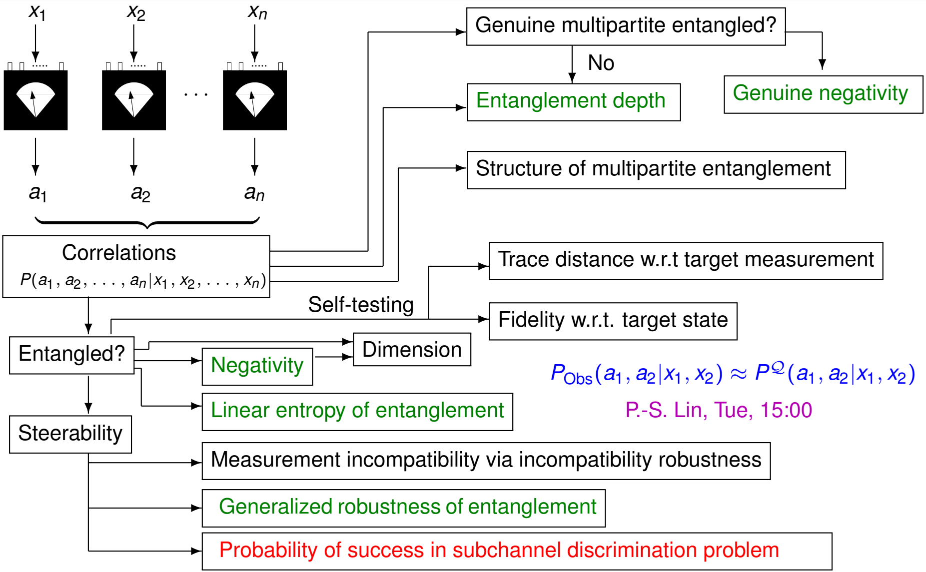

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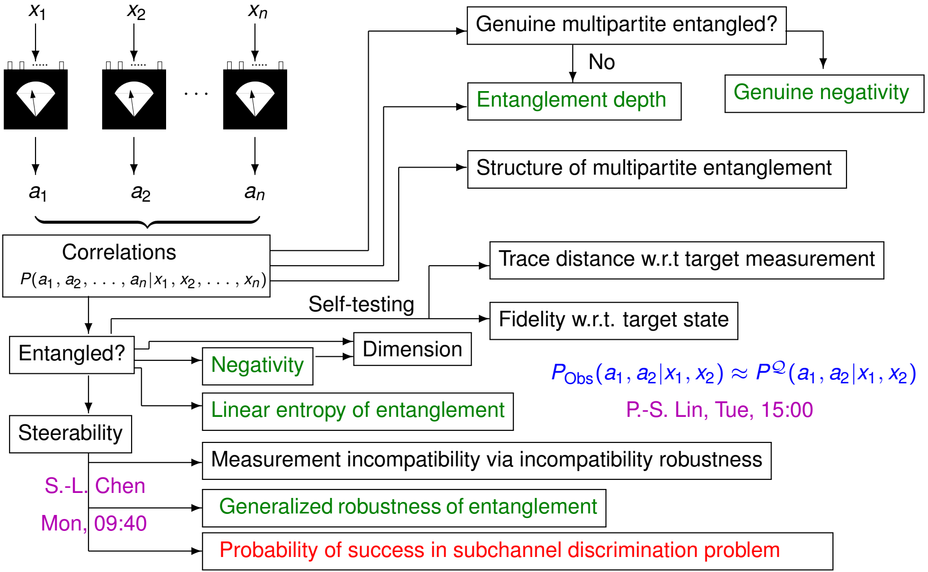

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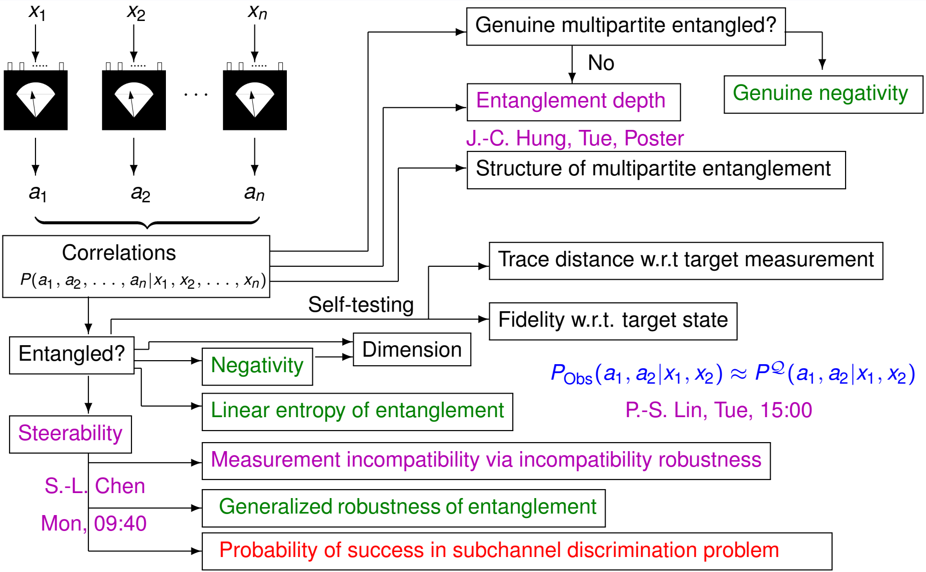

References

- N. Brunner, S. Pironio, A. Acín, A. A. Méthot & V. Scarani, Phys. Rev. Lett. (2008).
- J.-D. Bancal, N. Gisin, Y.-C. Liang & S. Pironio, Phys. Rev. Lett. (2011).
- T. Moroder, J.-D. Bancal, Y.-C. Liang, M. Hoffmann & O. Gühne, Phys. Rev. Lett. (2013).
- M. Navascués, G. de la Torre & T. Vértesi, Phys. Rev. X (2014).
- T. H. Yang, T. Vértesi, J.-D. Bancal, V. Scarani & M. Navascués, Phys. Rev. Lett. (2014).
- Y.-C. Liang, D. Rosset, J.-D. Bancal, G. Pütz, T. J. Barnea & N. Gisin, Phys. Rev. Lett. (2015).
- G. Tóth, T. Moroder & O. Gühne, Phys. Rev. Lett. (2015).
- S.-L. Chen, C. Budrondi, Y.-C. Liang & Y.-N. Chen, Phys. Rev. Lett. (2016).