Feedback Control and Counting Statistics in Quantum Transport

Tobias Brandes (Institut für Theoretische Physik, TU Berlin)

- Quantum Transport
  - Example: particle counting.
  - Moments, cumulants, generalized density operators.
  - Quantum dots.
- Feedback control
  - Introduction.
  - Theoretical model.
  - A recent experiment with quantum dots.

Transport Master Equation

- Transitions of a system between states $n = 0, 1, 2, ...$
  - at rate γ_n: forward transition from n to $n + 1$.
  - at rate $\bar{\gamma}_n$: backward transition from n to $n - 1$.
- $p(n, t)$: probablity to find the system in state n at time t.

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- Temporal change

$$
\begin{aligned}
dp(n,t) &= -p(n,t)(\gamma_n + \bar{\gamma}_n)dt + p(n-1,t)\gamma_{n-1}dt + p(n+1,t)\bar{\gamma}_{n+1}dt \\
dp(0,t) &= -p(0,t)\gamma_0 dt + p(1,t)\bar{\gamma}_1 dt.
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\end{aligned}
$$

- In matrix form as $\dot{\mathbf{p}}(t) = M\mathbf{p}(t)$,

$$
M \equiv \begin{pmatrix} -\gamma_0 & \bar{\gamma}_1 & 0 & ... \\ \gamma_0 & -\gamma_1 - \bar{\gamma}_1 & \bar{\gamma}_2 & 0... \\ 0 & \gamma_1 & -\gamma_2 - \bar{\gamma}_2 & \bar{\gamma}_3.... \\ .. & ... & ... & ... \end{pmatrix}.
$$

- Finite state space ($n = 0, 1, ..., n_{\mathrm{max}}$), e.g. $n_{\mathrm{max}} = 2$:

$$\dot{\mathbf{p}}(t) = M\mathbf{p}(t), \quad M \equiv \begin{pmatrix} -\gamma_0 & \bar{\gamma}_1 & 0 \\ \gamma_0 & -\gamma_1 - \bar{\gamma}_1 & \bar{\gamma}_2 \\ 0 & \gamma_1 & -\bar{\gamma}_2 \end{pmatrix}.$$

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- Conservation of probability

$$\sum_n p(n,t) = 1 \rightsquigarrow \frac{d}{dt} \sum_n p(n,t) = 0.$$

- We can express this by defining $\mathbf{e}^T \equiv (1, 1, 1, 1, ...)$;

$$\rightsquigarrow \mathbf{e}^T \mathbf{p}(t) \;=\; 1, \quad \mathbf{e}^T M = 0.$$

Moment generating function

- Simple example: $\gamma_i = \gamma$, $\bar{\gamma}_i = 0$ (unidirectional).

$$
\begin{aligned}
\dot{p}(0,t) &= -\gamma p(0,t) \\
\dot{p}(n,t) &= -\gamma p(n,t) + \gamma p(n-1,t), \quad n = 1,2,...
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- Moment generating function $\mathcal{M}(\chi, t) \equiv \sum_{n=0}^{\infty} e^{in\chi} p(n,t)$.

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- Moment generating function $\mathcal{M}(\chi,t) \equiv \sum_{n=0}^{\infty} e^{in\chi} p(n,t)$.
- Due to normalisation $\mathcal{M}(0,t) = 1$. Summing over n yields

$$\begin{aligned}\sum_{n=0}^{\infty} e^{in\chi}\dot{p}(n,t) &= \sum_{n=0}^{\infty} e^{in\chi}\left[-\gamma p(n,t) + \gamma p(n-1,t)\right]\\ &= -\gamma \sum_{n=0}^{\infty} e^{in\chi} p(n,t) + \gamma e^{i\chi} \sum_{n=0}^{\infty} e^{i(n-1)\chi} p(n-1,t)\\ &= \gamma\left(-1 + e^{i\chi}\right) \sum_{n=0}^{\infty} e^{in\chi} p(n,t).\end{aligned}$$

- Differential equation for $\mathcal{M}(\chi, t)$,

$$\frac{\partial}{\partial t}\mathcal{M}(\chi, t) = \gamma(e^{i\chi} - 1)\mathcal{M}(\chi, t) \rightsquigarrow \mathcal{M}(\chi, t) = e^{(e^{i\chi}-1)\gamma t}\mathcal{M}(\chi, t = 0).$$

- Initial condition (other choices also possible)

$$p(n, 0) \quad = \quad \delta_{n,0} \rightsquigarrow \mathcal{M}(\chi, 0) = \sum_{n=0}^{\infty} e^{in\chi} p(n, 0) = 1.$$

- We thus find the moment generating function for the *Poisson process*

$$\boxed{\mathcal{M}(\chi, t) = e^{(e^{i\chi}-1)\gamma t}}$$

- From $\mathcal{M}(\chi, t)$, we obtain the probabilities $p(n, t)$:

$$\mathcal{M}(\chi, t) \equiv \sum_{m=0}^{\infty} e^{im\chi} p(m, t) \rightsquigarrow p(n, t) = \int_{-\pi}^{\pi} \frac{d\chi}{2\pi} e^{-in\chi} \mathcal{M}(\chi, t)$$

- Our example with $\mathcal{M}(\chi, t) = \exp\left[\left(e^{i\chi} - 1\right)\gamma t\right]$:

$$\boxed{p(n, t) = \frac{(\gamma t)^n}{n!} e^{-\gamma t} \quad \text{Mandel formula}}$$

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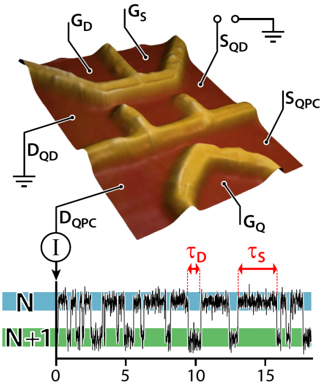
- Describes a **Poisson process** with parameter $\gamma t = \langle n \rangle$;

$$\langle n \rangle \equiv \sum_{n=0}^{\infty} n p(n, t) = \frac{\partial}{\partial i\chi} \mathcal{M}(\chi, t)\Big|_{\chi=0} = \gamma t.$$

Quantum Transport

Example: Full Counting Statistics of electrons

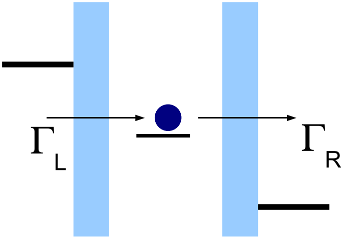


- Quantum dot coupled to quantum point contact.

Full Counting Statistics

- Probability $p(n, t)$ of n electrons after time t.

C. Flindt, C. Fricke, F. Hohls, T. Novotný, K. Netocný, T. Brandes, and R. J. Haug; PNAS **106**, 10116 (2009).



Transport process with internal states

- Unidirectional transport of electrons through single level dot.

$$
\begin{aligned}
\dot{p}_0(n,t) &= -\Gamma_{\rm L} p_0(n,t) + \Gamma_{\rm R} p_1(n-1,t) \\
\dot{p}_1(n,t) &= \Gamma_{\rm L} p_0(n,t) - \Gamma_{\rm R} p_1(n,t)
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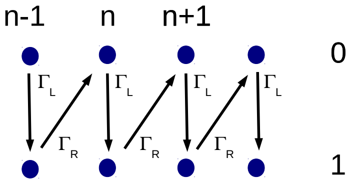
- Compact vector form $\rho \equiv (p_0, p_1)^T$;

$$
\begin{aligned}
\dot{\rho}(n,t) &= \mathcal{L}_0 \rho(n,t) + \mathcal{J} \rho(n-1,t) \\
\mathcal{L}_0 &\equiv \begin{pmatrix} -\Gamma_{\rm L} & 0 \\ \Gamma_{\rm L} & -\Gamma_{\rm R} \end{pmatrix}, \quad \mathcal{J} \equiv \begin{pmatrix} 0 & \Gamma_{\rm R} \\ 0 & 0 \end{pmatrix}.
\end{aligned}
$$



Transport process with internal states

- ***n*-resolved Master equation**

$$\dot{\rho}(n,t) = \mathcal{L}_0\rho(n,t) + \mathcal{J}\rho(n-1,t)$$

- **jump (super)–operator** $\mathcal{J}$. Liouvillian $\mathcal{L} \equiv \mathcal{L}_0 + \mathcal{J}$.

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- Statistics of internal (dot) states: summation over n;

$$\rho(t) \equiv \sum_{n=0}^{\infty} \rho(n,t) \equiv (p_0(t), p_1(t))^T \rightsquigarrow \dot{\rho}(t) = \mathcal{L}\rho(t).$$

- Statistics of 'external' (detector) states: Full Counting Statistics (FCS);

$$p(n,t) \equiv p_0(n,t) + p_1(n,t) \equiv \mathrm{Tr}\rho(n,t).$$

Transport process with internal states

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- **jump (super)–operator** $\mathcal{J}$. Liouvillian $\mathcal{L} \equiv \mathcal{L}_0 + \mathcal{J}$.

- Generalized density operator $\rho(\chi,t) \equiv \sum_{n=0}^{\infty} e^{in\chi}\rho(n,t)$.

$$\dot{\rho}(\chi,t) = \left(\mathcal{L}_0 + e^{i\chi}\mathcal{J}\right)\rho(\chi,t).$$

Transport process with internal states

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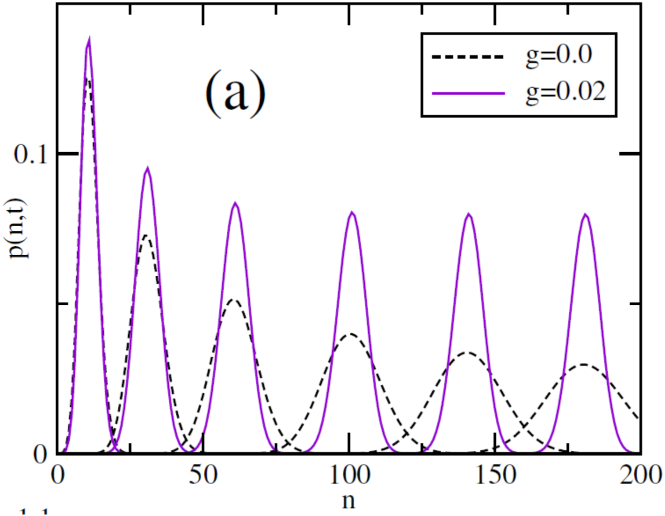
$$\dot{\rho}(\chi,t) = \left(\mathcal{L}_0 + e^{i\chi}\mathcal{J}\right)\rho(\chi,t).$$

- Full Counting Statistics (Bagrets, Nazarov 2003)
  - Eigenvalue of $\lambda_-(\chi)$ of $\mathcal{L}_0 + e^{i\chi}\mathcal{J}$ with $\lambda_-(0) = 0$.

$$p(n, t\to\infty) \sim \int_{-\pi}^{\pi} \frac{d\chi}{2\pi} e^{-in\chi} e^{t\lambda_-(\chi)}.$$

Full Counting Statistics



- Probability distribution function $p(n, t)$: diffusion on state space n.

Full Counting Statistics

Moments and cumulants of $p(n,t)$

- Moments

$$\mu_1(t) \equiv \sum_{n=0}^{\infty} np(n,t), \quad \mu_2(t) \equiv \sum_{n=0}^{\infty} n^2 p(n,t), \ldots$$

- Stationary current

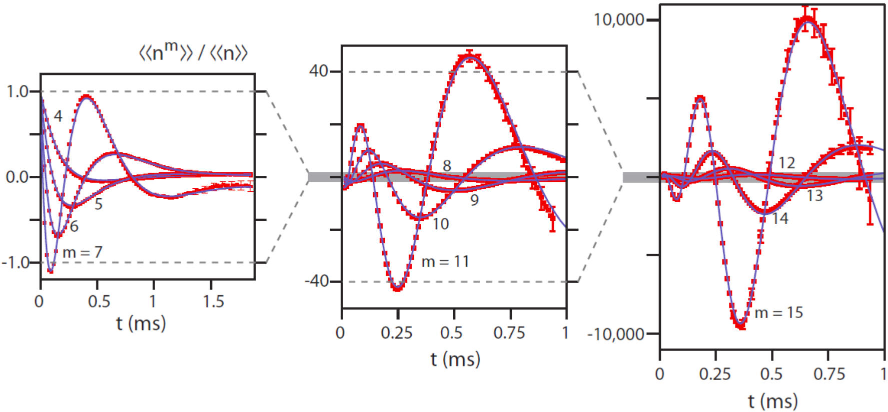
$$\boxed{I_{\rm st} \equiv \lim_{t\to\infty} \frac{\mu_1(t)}{t}.}$$

- Cumulants

$$\begin{aligned} C_1(t) &\equiv \mu_1(t), \quad C_2(t) \equiv \mu_2(t) - \mu_1^2(t) \\ C_3(t) &= \mu_3(t) - 3\mu_2(t)\mu_1(t) + 2\mu_1(t)^3 \\ &\ldots \end{aligned}$$

Full Counting Statistics

Universal oscillations in Full Counting Statistics



- High-order cumulants $C_k(t)$ as a function of time.

C. Flindt, C. Fricke, F. Hohls, T. Novotný, K. Netocný, T. Brandes, and R. J. Haug; PNAS **106**, 10116 (2009).

Full Counting Statistics

- Moment generating function

$$\mathcal{M}(\chi, t) \equiv \mathrm{Tr}\rho(\chi, t), \quad \mu_k(t) = \frac{\partial^k}{\partial(i\chi)^k}\mathcal{M}(\chi, t)\Big|_{\chi=0}.$$

- Cumulant generating function

$$\mathcal{F}(\chi, t) \equiv \ln \mathrm{Tr}\rho(\chi, t), \quad C_k(t) = \frac{\partial^k}{\partial(i\chi)^k}\mathcal{F}(\chi, t)\Big|_{\chi=0}.$$

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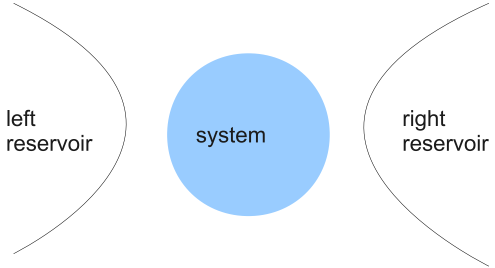
- 'Law of large numbers' (Gaussian statistics after appropriate rescaling)
  - Cumulants $C_k(t \to \infty)$ usually grow $\propto t$ (no feedback).
  - Compare with grandcanonical equilibrium potential Ω,temperature β^{-1}, chem. potential μ, fugacity $\zeta \equiv e^{\beta\mu}$: $\mathcal{F}(\chi) = -\beta[\Omega(\zeta e^{i\chi}) - \Omega(\zeta)]$, cumulants $\propto V$ at large volumes $V \to \infty$.

n-resolved Master equation

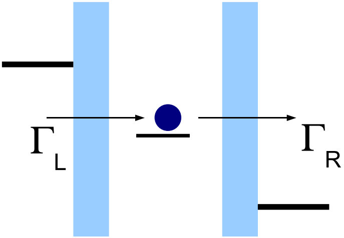
General setup

- Open system Hamiltonian. $\mathcal{H} = \mathcal{H}_S + \mathcal{H}_{\rm res} + \mathcal{H}_T$.
  - $\mathcal{H}_S$ system.
  - $\mathcal{H}_{\rm res}$ reservoir.
  - $\mathcal{H}_T$ system-reservoir coupling.
- Reduced density matrix $\rho(t)$, Liouvillian $\mathcal{L}$, Born-Markov approximation $\dot{\rho}(t) = \mathcal{L}\rho(t), \quad \mathcal{L} \equiv \mathcal{L}_0 + \mathcal{J}$



n-resolved Master equation

Example: single level quantum dot



$$\dot{\rho}(\chi, t) = \mathcal{L}(\chi)\rho(\chi, t), \quad \mathcal{L}(\chi) \equiv \sum_{\alpha=L,R} \Gamma_\alpha \begin{pmatrix} -f_\alpha & e^{i\chi_\alpha}(1-f_\alpha) \\ e^{-i\chi_\alpha} f_\alpha & -(1-f_\alpha) \end{pmatrix}.$$

- Generalized density operator $\rho(\chi, t) \equiv \sum_{n=0}^{\infty} e^{in\chi}\rho(n, t)$.

n-resolved Master equation

Quantum jumps

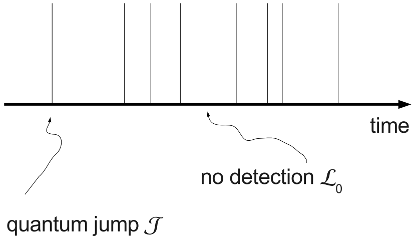
Jump-resolved ('n-resolved') Master equation

$$\rho(t) = \sum_{n=0}^{\infty} \rho^n(t) = \sum_{n=0}^{\infty} \int_0^t dt_n ... \int_0^{t_2} dt_1 \rho^c(t; t_n, ..., t_1)$$

$$\rho^c(t; t_n, ..., t_1) \equiv e^{\mathcal{L}_0 \cdot (t - t_n)} \mathcal{J} e^{\mathcal{L}_0 \cdot (t_n - t_{n-1})} \mathcal{J} ... \mathcal{J} e^{\mathcal{L}_0 \cdot t_1} \rho_0$$

- Non-unitary free time-evolution, interrupted by n quantum jumps at times t_i.

Textbook: G. Schaller, *Open Quantum Systems Far From Equilibibrium (2014)*.



n-resolved Master equation

S. A. Gurvitz and Ya. S. Prager (1996)

Wave function method

- Trace out reservoir $\rightsquigarrow$ reduced density operator.
- Conditioning on number n of tunnelled electrons.

$$|\Psi(t)\rangle = \left[b_0(t) + \sum_l b_{1l}(t) a_1^\dagger a_l + \sum_{l,r} b_{lr}(t) a_r^\dagger a_l + \sum_{l<l',r} b_{1ll'r}(t) a_1^\dagger a_r^\dagger a_l a_{l'} + \cdots \right] |0\rangle, \qquad (2.2)$$

$$\dot{\sigma}_{aa}^{(n)} = -2\Gamma_L \sigma_{aa}^{(n)} + \Gamma_R \sigma_{bb\uparrow}^{(n-1)} + \Gamma_R \sigma_{bb\downarrow}^{(n-1)}, \qquad (3.6a)$$

n-resolved Master equation

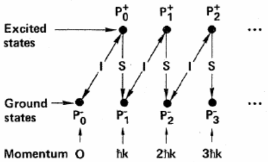
Borrowing from quantum optics: Resonance Fluorescence

- Count recoil events $n\hbar k$.

EW A VOLUME 23, NUMBER 3 MARCH 1981

Photon number statistics in resonance fluorescence

Richard J. Cook*



$$p(n) = \mathrm{Tr}[\sigma^n] . \tag{10}$$

The equations governing the time development of the partial density matricies are

$$\dot{\sigma}^n = i[\sigma^n(H+iB) - (H-iB)\sigma^n] + 2\beta L\sigma^{n-1}L^T , \tag{11}$$

Counting and two-point quantum measurement

Two-point measurement

- $\hat{N}$ particle number operator, eigenvalues $0, 1, 2, ...$
- $\rho(t)$ system state at time t.
- First measurement of $\hat{N}$ (with result n_0); $\rho(t=0) \equiv \sum_{n_0} \rho(n_0)$.

Counting and two-point quantum measurement

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- $\rho(t)$ system state at time t.
- First measurement of $\hat{N}$ (with result n_0); $\rho(t=0) \equiv \sum_{n_0} \rho(n_0)$.
- Spectral decomposition $e^{i\chi\hat{N}} = \sum_{m=0}^{\infty} e^{i\chi m} \hat{P}_m$ with projector $\hat{P}_m$.
- Second measurement at time t, $p(n,t) \equiv \sum_{n_0} \mathrm{Tr}\rho(t)\hat{P}_{n+n_0}$.

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- Spectral decomposition $e^{i\chi\hat{N}} = \sum_{m=0}^{\infty} e^{i\chi m}\hat{P}_m$ with projector $\hat{P}_m$.
- Second measurement at time t, $p(n,t) \equiv \sum_{n_0} \mathrm{Tr}\rho(t)\hat{P}_{n+n_0}$.
- Insert $\rho(t) = U(t)\rho(0)U^\dagger(t)$ and $e^{-i\chi n_0}\rho(n_0) = e^{-i\chi\hat{N}}\rho(n_0)$,

$$
\begin{aligned}
\rightsquigarrow \quad p(n,t) &= \sum_{n_0} \frac{1}{2\pi}\int_{-\pi}^{\pi} d\chi e^{-i\chi(n+n_0)} \mathrm{Tr}\rho(t)e^{i\chi\hat{N}} \\
&= \frac{1}{2\pi}\int_{-\pi}^{\pi} d\chi \sum_{n_0} e^{-i\chi n}\mathrm{Tr}e^{-i\chi n_0}U(t)\rho(n_0)U^\dagger(t)e^{i\chi\hat{N}} \\
&= \frac{1}{2\pi}\int_{-\pi}^{\pi} d\chi e^{-i\chi n} \underbrace{\mathrm{Tr}\rho(0)U^\dagger(t)e^{i\chi\hat{N}}U(t)e^{-i\chi\hat{N}}}_{\mathcal{M}(\chi,t)}.
\end{aligned}
$$

Counting and two-point quantum measurement

Influence functional for two-point measurement

- Total time evolution with $U(t) = e^{-i\mathcal{H}t}$.
- Moment generating function $\mathcal{M}(\chi, t) = \sum_{n=-\infty}^{\infty} e^{i\chi n} p(n, t)$,

$$\mathcal{M}(\chi, t) = \mathrm{Tr}\rho(0) e^{-i\frac{\chi}{2}\hat{N}} U^{\dagger}(t) e^{i\frac{\chi}{2}\hat{N}} e^{i\frac{\chi}{2}\hat{N}} U(t) e^{-i\frac{\chi}{2}\hat{N}}.$$

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$$\mathcal{M}(\chi, t) = \mathrm{Tr}\rho(0)e^{-i\frac{\chi}{2}\hat{N}}U^{\dagger}(t)e^{i\frac{\chi}{2}\hat{N}}e^{i\frac{\chi}{2}\hat{N}}U(t)e^{-i\frac{\chi}{2}\hat{N}}.$$

- Define χ-dependent generalized density operator

$$\rho(\chi, t) \equiv U_{\chi}(t)\rho(0)U^{\dagger}_{-\chi}(t) \quad U_{\chi}(t) \equiv e^{i\frac{\chi}{2}\hat{N}}U(t)e^{-i\frac{\chi}{2}\hat{N}}.$$

- Moment generating function as influence functional

$$\mathcal{M}(\chi, t) = \mathrm{Tr}\rho(\chi, t) \equiv \langle U^{\dagger}_{-\chi}(t)U_{\chi}(t)\rangle_0$$

  - Counting fields $\pm\frac{\chi}{2}$ with opposite signs on two branches of Keldysh contour. Levitov, Lesovik '93; Klich '03;...;Schönhammer '07; Esposito, Harbola, Mukamel '09.

Counting and two-point quantum measurement

Example: Fano-Anderson model

- χ-depending Hamiltonian in $U_\chi(t) = e^{-it\hat{\mathcal{H}}_\chi}$

$$\hat{\mathcal{H}}_\chi \equiv e^{i\frac{\chi}{2}\hat{N}}\hat{\mathcal{H}}e^{-i\frac{\chi}{2}\hat{N}}$$

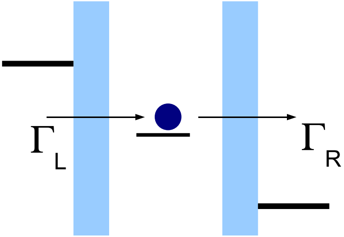
Counting and two-point quantum measurement

Example: Fano-Anderson model

- χ-depending Hamiltonian in $U_\chi(t) = e^{-it\hat{\mathcal{H}}_\chi}$

$$\boxed{\hat{\mathcal{H}}_\chi \equiv e^{i\frac{\chi}{2}\hat{N}}\hat{\mathcal{H}}e^{-i\frac{\chi}{2}\hat{N}}}$$

$$\begin{aligned}
\hat{\mathcal{H}} &\equiv \hat{\mathcal{H}}_{\rm res} + \hat{\mathcal{H}}_d + \hat{\mathcal{V}} \\
\hat{\mathcal{H}}_d &\equiv \varepsilon_0 d^\dagger d, \quad \hat{\mathcal{H}}_{\rm res} \equiv \sum_{k\alpha} \varepsilon_{k\alpha} c^\dagger_{k\alpha} c_{k\alpha} \\
\hat{\mathcal{V}} &\equiv \sum_{k\alpha}\left(V_{k\alpha} c^\dagger_{k\alpha} d + V^*_{k\alpha} d^\dagger c_{k\alpha}\right)
\end{aligned}$$



- Counting in right reservoir $\rightsquigarrow \hat{\mathcal{H}}_\chi \equiv \hat{\mathcal{H}}_{\rm res} + \hat{\mathcal{H}}_d + \hat{\mathcal{V}}_\chi$

$$\hat{\mathcal{V}}_\chi \equiv \sum_k \left(V_{kR} e^{i\frac{\chi}{2}} c^\dagger_{kR} d + V_{kL} c^\dagger_{kL} d + H.c.\right).$$