8th International Workshop on Solid State
Quantum Computing (IWSSQC) 2016

Valley qubits for quantum computing and communication

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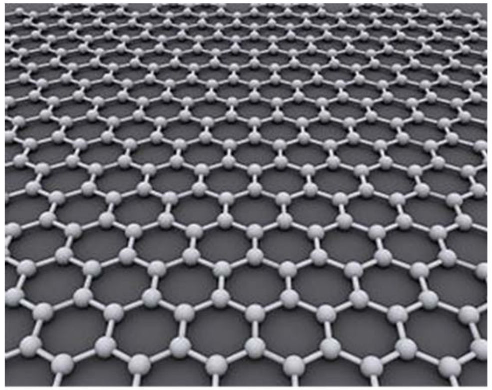
Outline



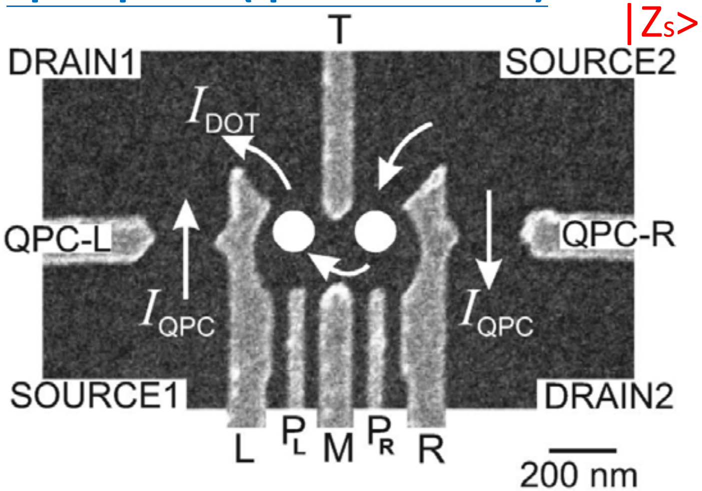
1. Introduction
   - ✓ Fundamentals of graphene
   - ✓ Valley pseudospin
2. Valley-based quantum computing **[PRB 84, 195463 (2011)]**
   - ✓ Encoding scheme: Valley pair qubit
   - ✓ Electrical qubit manipulation
3. Valley-based quantum communication **[PRB 86, 045456 (2012)]**
   - ✓ Quantum state transfer & Figure of merits
   - ✓ Setup: Valley pair qubit + cavities

Candidates for solid-state quantum information processing

Spin qubits (quantum dots)

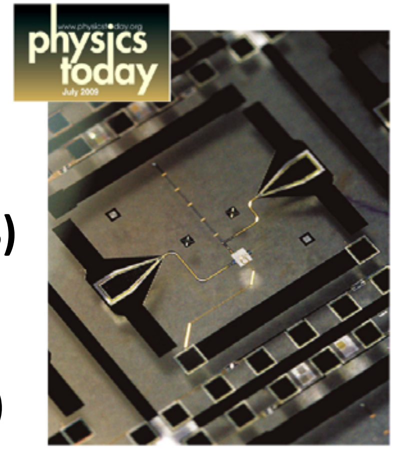
$|\uparrow\rangle=|0\rangle$; $|\downarrow\rangle=|1\rangle$

$|Z_S\rangle=|0\rangle$; $|Z_{T0}\rangle=|1\rangle$



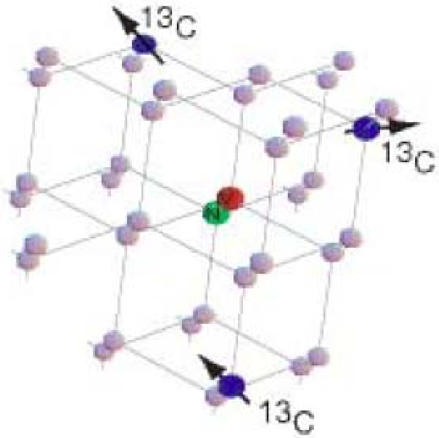
Daniel Loss & DiVincenzo, Phys. Rev. A **(1998)**
Kouwenhoven & Tarucha et al., Phys. Rev. B **(2003)**

Superconducting qubits



Devoret, et. al. (1998).
Nakamura, et.al. (1999)
Makhlin, et.al. (2001);
Martinis, et.al. (2002)

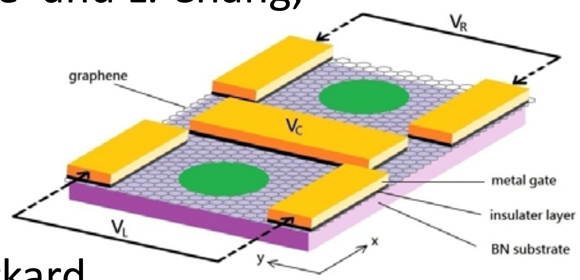
Nuclear spin qubits (NV center)



M. V. G. Dutt *et al.*, Science **(2007)**
G. D. Fuchs *et al.*, Nature Phys. **(2011)**

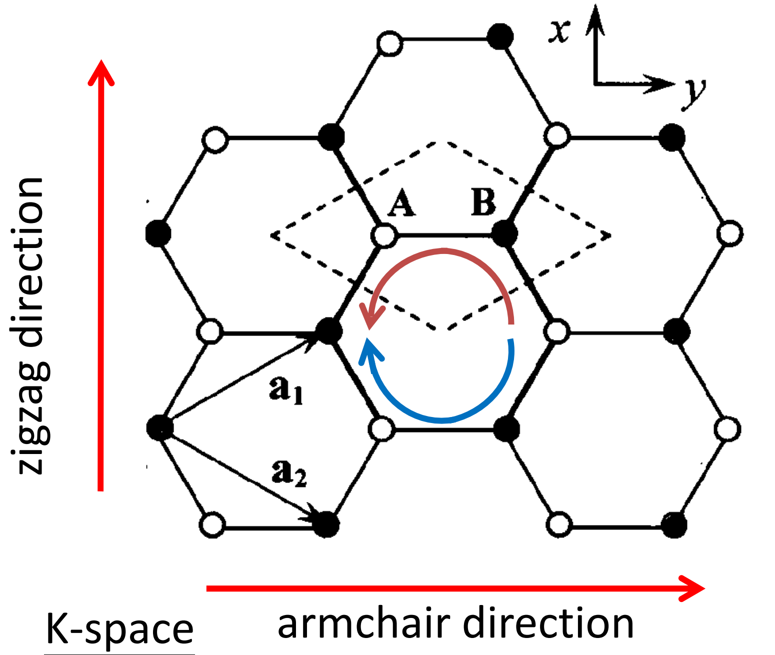
Valley (pseudospin) qubits (quantum dots)

G. Y. Wu and N. -Y. Lue and L. Chang, Phys. Rev. B **(2011)**

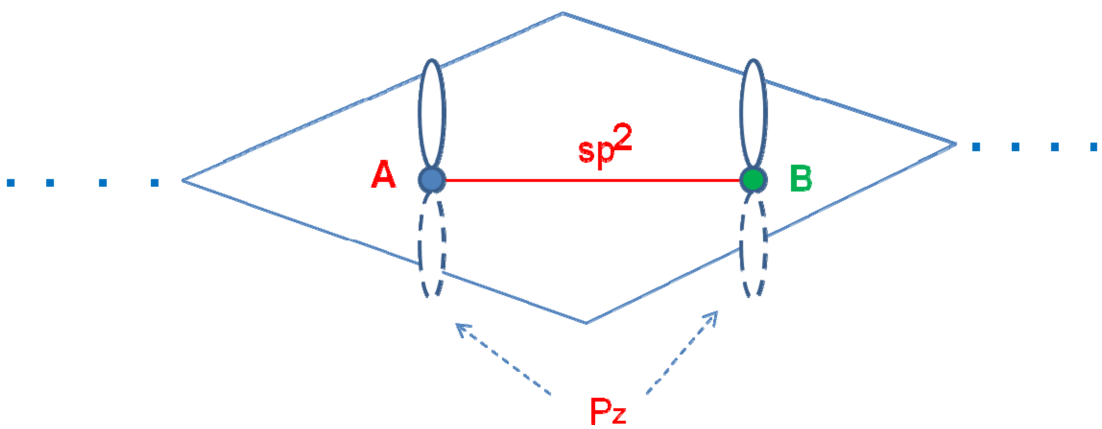


N. Rohling and G. Burkard, N. J. Phys. 14, 083008 **(2012)**

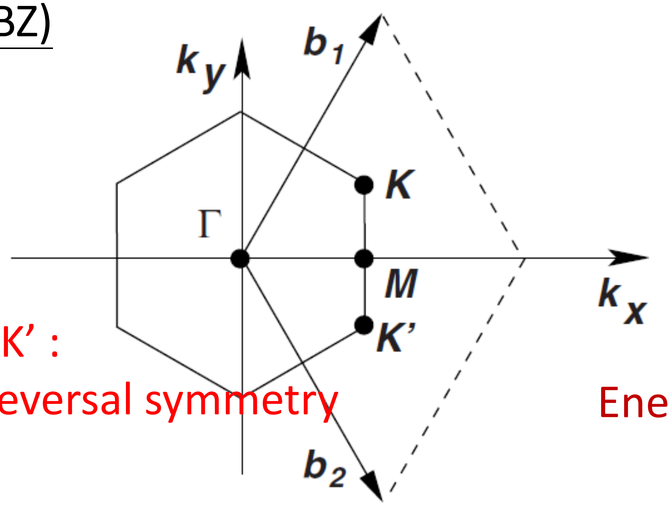
Graphene: fundamentals



Unit Cell

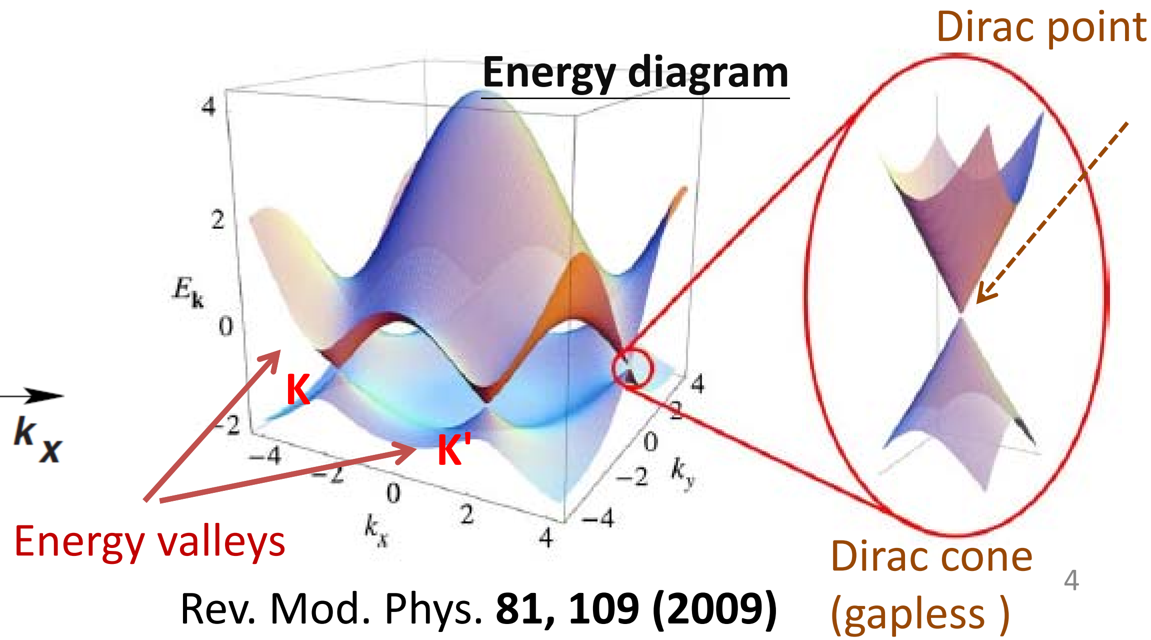


K-space (1st BZ)



K and K' : time reversal symmetry

Energy diagram

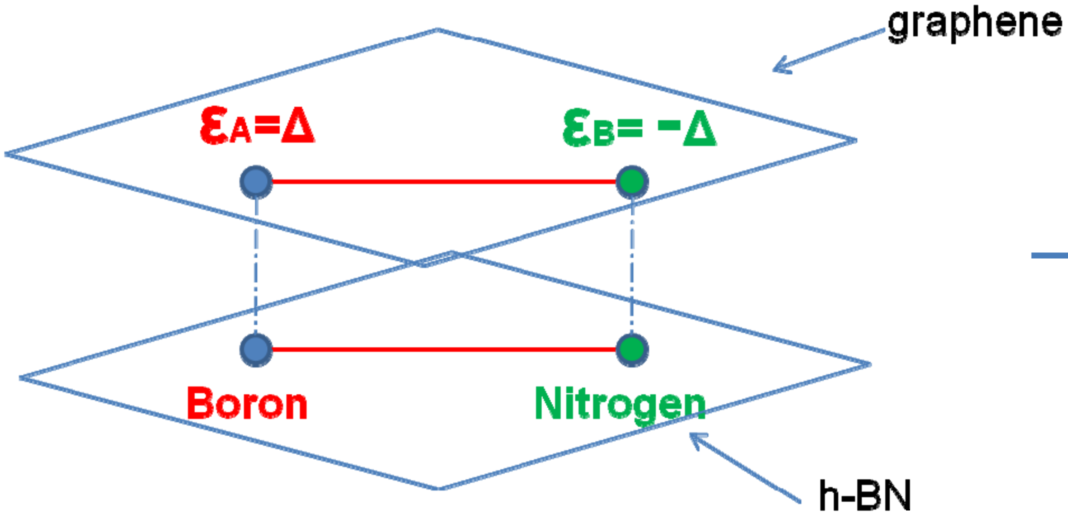


Rev. Mod. Phys. **81, 109 (2009)**

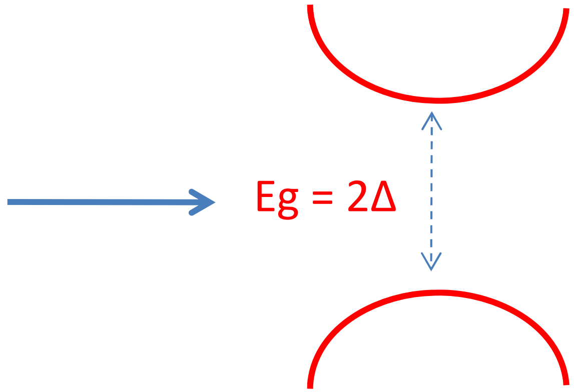
Introduction

Graphene: fundamentals

Unit Cell



On K or K' point



The asymmetry can introduce energy difference 2Δ on A and B site atoms.

asymmetry parameter :

$\Delta \neq 0$ for graphene/h-BN

(Giovannetti et al.)

This presentation focuses on monolayer graphene
but the work has been extended to bilayer graphene (PRB 88, 125422 (2013))

Graphene: theory

Tight-binding Hamiltonian

ε_A → Δ; t → $H_{BA}(\vec{k})$

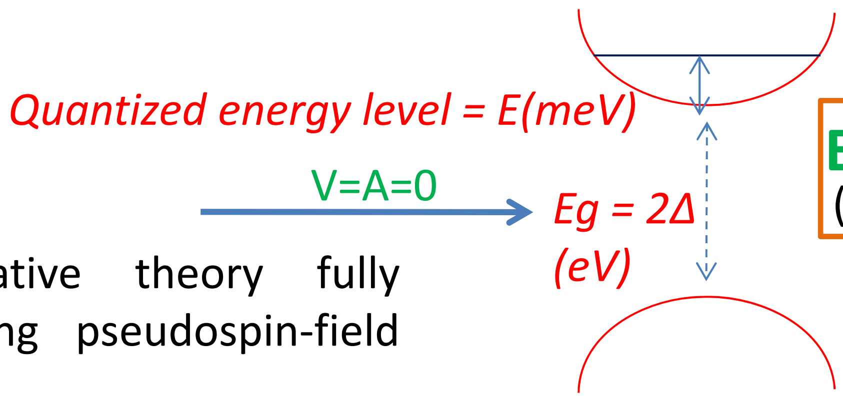
$$H_{tight-binding}\left(\vec{k}\right)=\begin{pmatrix}\Delta & H_{AB}\left(\vec{k}\right)\\ H_{BA}\left(\vec{k}\right) & -\Delta\end{pmatrix}\Rightarrow H_t\left(\vec{k}=\vec{K}+\delta\vec{k}\right)\approx H_t\left(\vec{K}\right)+\delta\vec{k}\cdot H_t'\left(\vec{K}\right)$$

Dirac-type Hamiltonian

(with asymmetrical parameter Δ; 2D *relativistic*)

$$\begin{pmatrix}\Delta+V(\vec{r}) & v_F\left(\hat{p}_-+eA_-(\vec{r})\right)\\ v_F\left(\hat{p}_++eA_+(\vec{r})\right) & -\Delta+V(\vec{r})\end{pmatrix}\begin{pmatrix}\varphi_A\\ \varphi_B\end{pmatrix}=E\begin{pmatrix}\varphi_A\\ \varphi_B\end{pmatrix}$$

V(r) = electrostatic potential
A(r) = vector potential



$E(k) = \pm(\Delta^2 + \hbar^2 v_F^2 k^2)^{1/2}$
(*relativistic type dispersion*)

$$m^* = \Delta / v_F^2$$

An alternative theory fully characterizing pseudospin-field interaction?

Δ: asymmetry parameter

Introduction

Valley Pseudospin

Effective Schrödinger theory (PRB 84, 195463 (2011))

Schrödinger Hamiltonian

$$H\phi = E\phi, \quad H = H^{(0)} + H^{(1)}$$

(RC)

nonrelativistic

$$H^{(0)} = \frac{\vec{\pi}^2}{2m^*} + V + \boxed{\boldsymbol{\tau}_v \boldsymbol{\mu}_{v0} \mathbf{B}_{normal}}$$

(Zeeman interaction)

$$\boldsymbol{\mu}_{v0} = \frac{e\hbar}{2m^*}(valley\ magnetic\ moment)$$

$V = potential\ energy$

$(due\ to\ in-plane\ \vec{\boldsymbol{\varepsilon}})$

relativistic (1st order)

$$H^{(1)} = -\frac{1}{2\Delta}\left(\frac{\vec{\pi}^2}{2m^*} + \tau_v \mu_{v0} B_{normal}\right)^2$$

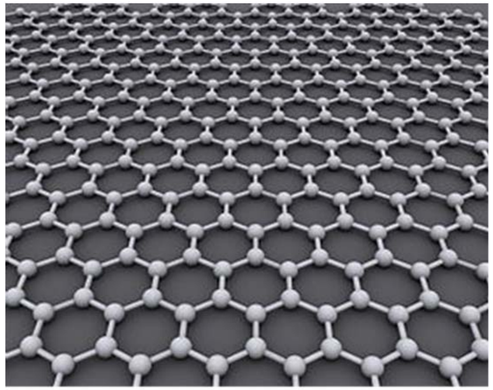
$$+ \boxed{\boldsymbol{\tau}_v \frac{\hbar}{4m^*\Delta} \vec{\boldsymbol{\varepsilon}} \times \vec{\pi} \cdot \hat{z}} \quad \textit{(VOI)}$$

$$- \frac{1}{8m^*\Delta}(\vec{p}^2 V)$$

Pseudospin (τ_v) / Spin ($\vec{\sigma}$) Analogy

	Pseudospin	Spin
B ≠ 0	$\mu_{v0} = e\hbar/2m^*$ *Zeeman:* $\tau_v \mu_{v0} B_{normal}$	$\mu_B = e\hbar/2m_e$ *Zeeman:* $\vec{\sigma}\mu_B \cdot \vec{B}$
ε ≠ 0	$\mathrm{VOI} \sim \tau_v \hat{z} \cdot \vec{\varepsilon} \times \vec{p}$ (Couple to K and K' valleys only)	$\mathrm{SOI} \sim \vec{\sigma} \cdot \vec{\varepsilon} \times \vec{p}$
State mixing	Valley operator $\tau_v = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$ Valley diagonal (conserving) contrasting phase |K> → exp(i ϕ) |K> |K'> → exp(-i ϕ) |K'>	Spin operators $\sigma_x = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ $\sigma_y = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}$ Spin mixing (decoherence) ↑ ↔ ↓

Outline



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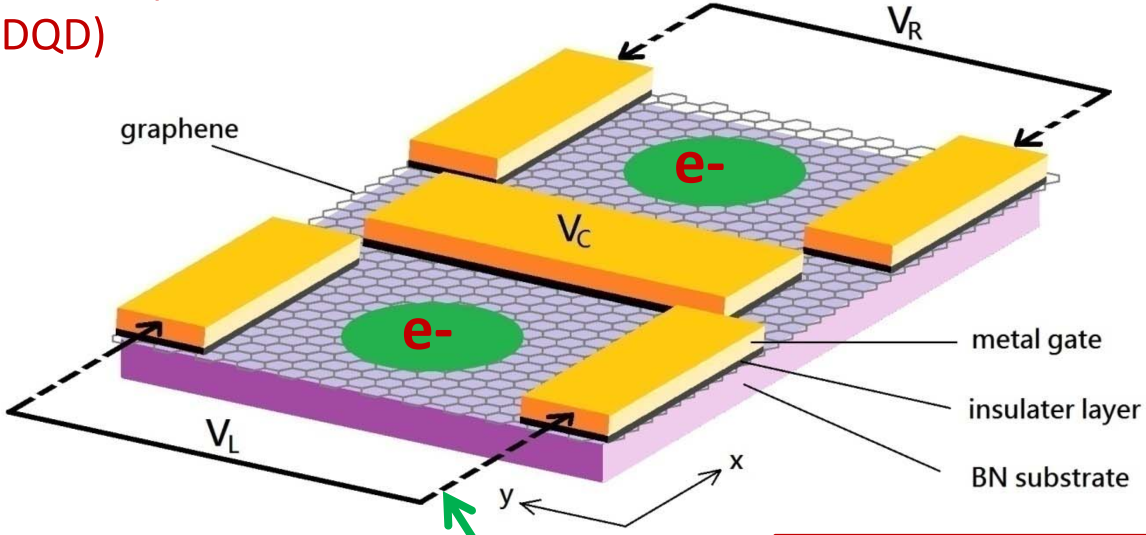
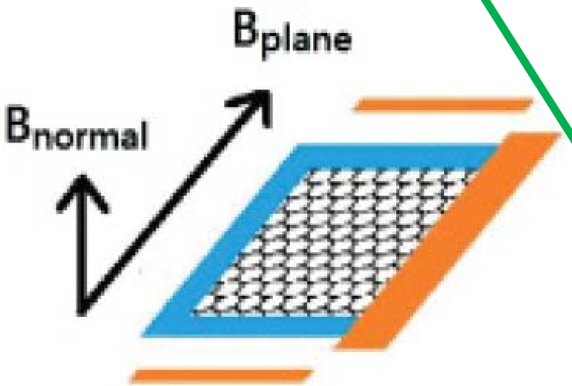
Double quantum dots (DQD)



Valley Qubit

PRB 84, 195463 (2011);
PRB 86, 045456 (2012)



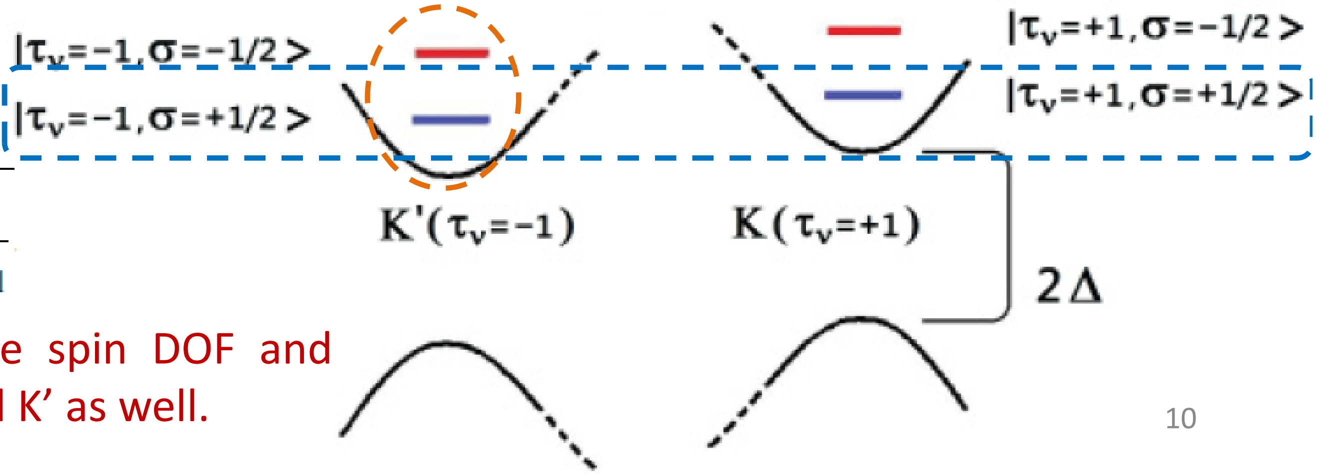
Spin / Valley

$$H_Z = -g^* s\mu_B|B_{\text{total}}| + \tau_v\mu_v|B_{\text{normal}}|$$

$$\frac{1}{2}g^*\mu_B|B_{total}| > \mu_v|B_{normal}|$$

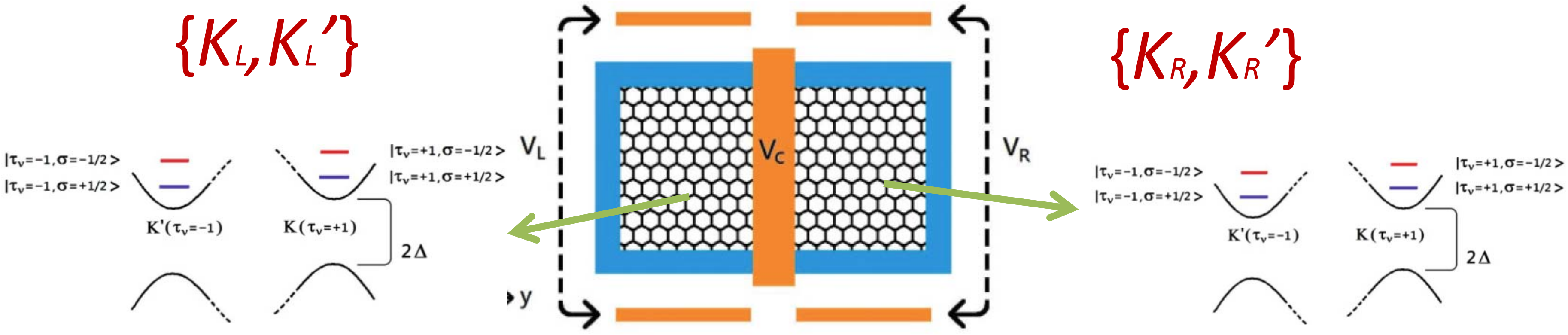
$$l_{\text{orbital}} \ll l_B$$

$$l_{\text{orbital}} \sim L, \quad l_B = \sqrt{\frac{\hbar}{eB_{\text{normal}}}}$$



The tilted B field freezes the spin DOF and break the degeneracy of K and K' as well.

Valley qubit pair: Two electrons in DQD



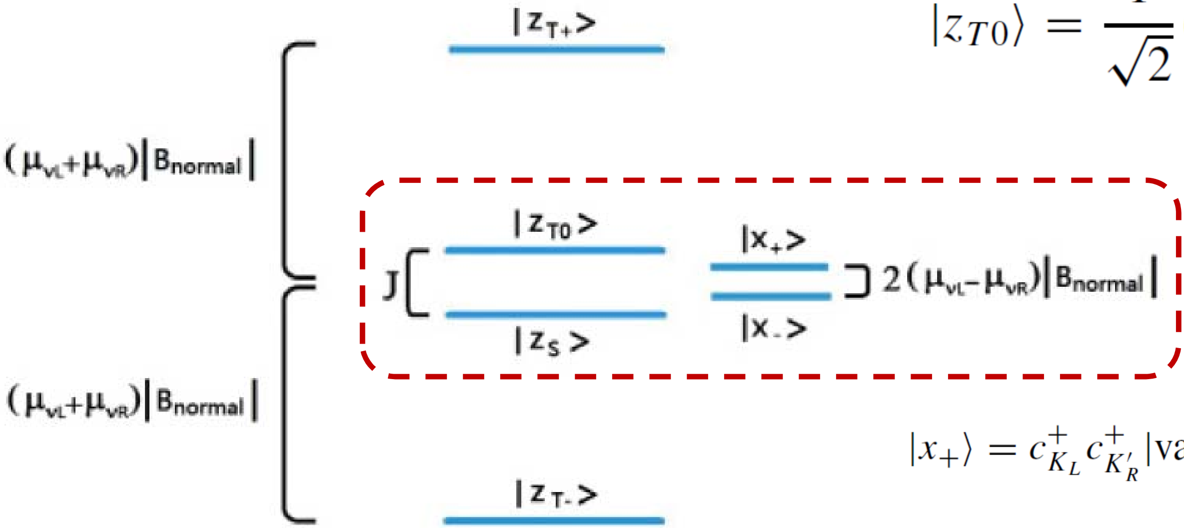
Heisenberg-type exchange coupling

$$H_J = \frac{1}{4} J \vec{\tau}_L \cdot \vec{\tau}_R, \quad J \sim 4t_{d\text{-}d}^2/U$$

Valley singlet and triplet

$$|z_S\rangle = \frac{1}{\sqrt{2}}(c^+_{K_L} c^+_{K'_R} - c^+_{K'_L} c^+_{K_R})|\text{vacuum}\rangle \Rightarrow |0\rangle$$

$$|z_{T0}\rangle = \frac{1}{\sqrt{2}}(c^+_{K_L} c^+_{K'_R} + c^+_{K'_L} c^+_{K_R})|\text{vacuum}\rangle \Rightarrow |1\rangle$$

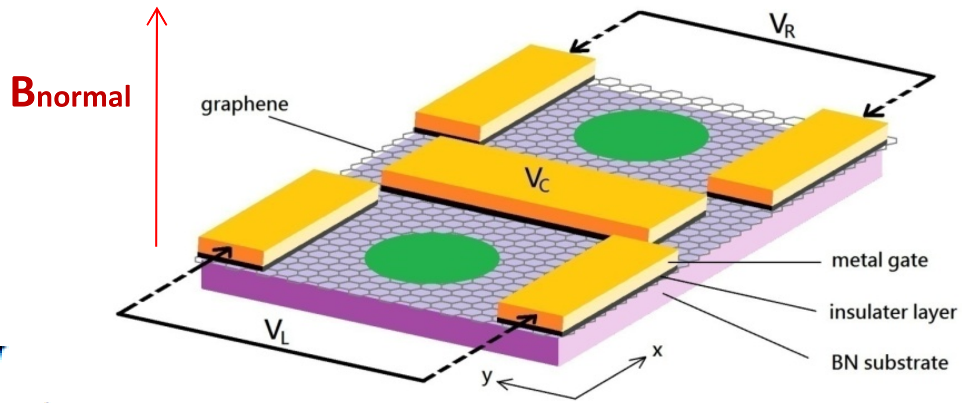


Qubit state space Γv

$$|x_+\rangle = c^+_{K_L} c^+_{K'_R}|\text{vacuum}\rangle, \quad |x_-\rangle = c^+_{K'_L} c^+_{K_R}|\text{vacuum}\rangle$$

Electrical Qubit Manipulation

(PRB 84, 195463 (2011); B 86, 045456 (2012))



Combing the above two field effects,

$$H_{\text{eff}} = (\mu_{vL} - \mu_{vR}) B_{\text{normal}} \tau_x + \frac{J}{2} \tau_z$$

Valley magnetic moment tuning

in QD_L , QD_R

control : V_L, V_R

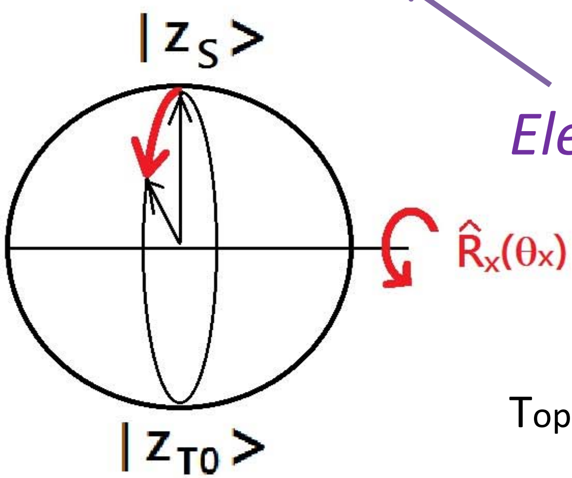
$$\delta E_Z^{(\text{dc})}(\tau_v) \approx -\frac{1}{8} \tau_v \mu_{v0} B_{\text{normal}} \left(k_{3x} x_\varepsilon^{(\text{dc})}\right) \frac{\hbar w_0}{\Delta}$$

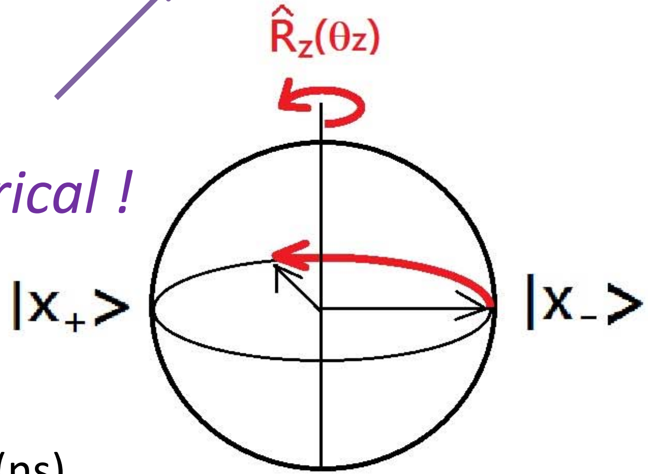
Exchange coupling:

control: V_C

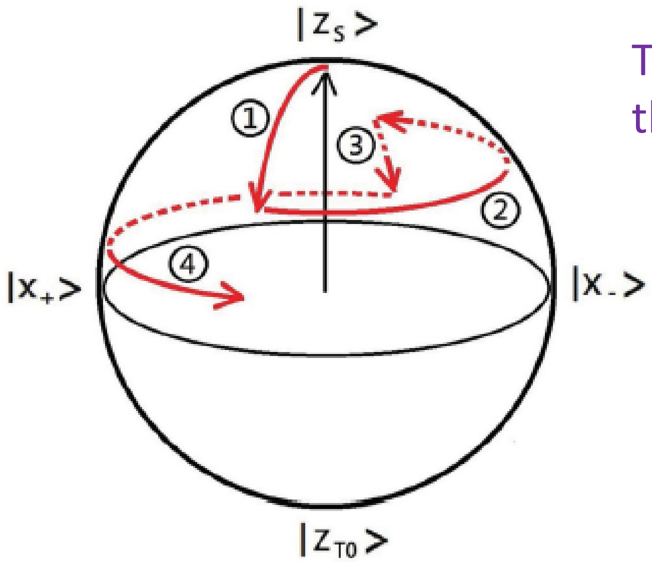
Electrical !

Bloch sphere (qubit) manipulation





Top ~O(ns)



The initial qubit state may be manipulated in the alternating sequence into target state.

$$|Z_S\rangle \xrightarrow{①} \hat{R}(\theta_x)|Z_S\rangle \xrightarrow{②} \hat{R}(\theta_z=\pi)\hat{R}(\theta_x)|Z_S\rangle$$

$$\xrightarrow{③} \hat{R}(-\theta_x)\hat{R}(\theta_z=\pi)\hat{R}(\theta_x)|Z_S\rangle$$

$$\xrightarrow{④} \hat{R}(\theta_z=\pi)\hat{R}(-\theta_x)\hat{R}(\theta_z=\pi)\hat{R}(\theta_x)|Z_S\rangle$$

$$\bullet\bullet\bullet \xrightarrow{\hat{R}(\theta_z^{(target)}+\pi/2)} |\text{target state}\rangle$$

Using the analogy: two-pseudospin qubit(S/T_0) ~ two-spin qubit(S/T_0)

(J. M. Taylor et al, *Fault-tolerant architecture for quantum computation using electrically controlled semiconductor spins*, Nature Phys. **1**, 177 (2005))

↓

initialization / readout / two-qubit qugate operation

↓

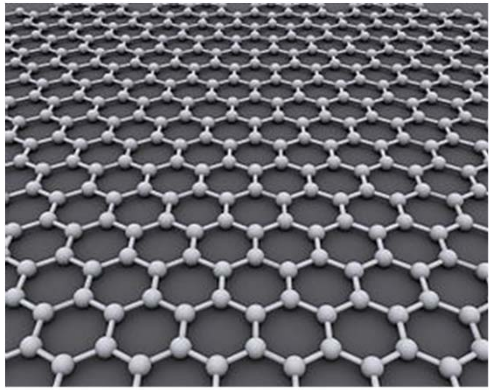
universal valley-based quantum computing

Qubit Coherence

Valley qubit	characteristics
Decoherence channel	phonon-mediated relaxation
Quantum dot size: L	350 A
QD potential depth: V_0	70meV
B_{normal}	100mT
Temperature	10K
Valley relaxation time	~O(ms)
qugate operation time (Ωx, Ωz)	~O(ns)

PRB 84, 195463 (2011)

Outline



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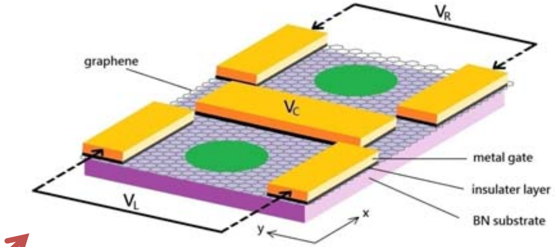
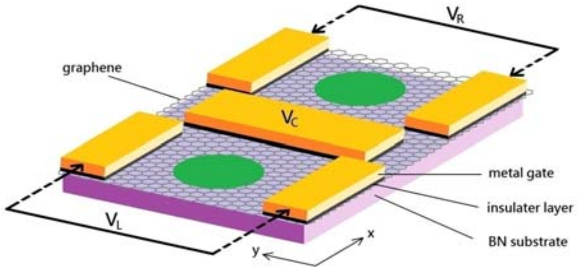
"Graphene + Photon" Quantum Network



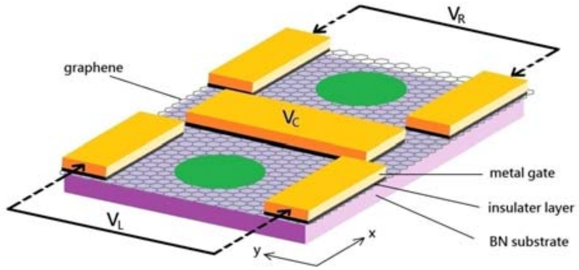
hv

hv

Quantum repeater/memory (graphene valley qubits)



Can we include the interaction with photon so the qubit can be transferred and stored as valley qubit from flying photon qubit?



quantum computer (graphene valley qubits)

quantum computer (graphene valley qubits)

PRB 86, 045456 (2012)

A faithful quantum state transfer

photon qubit **valley qubit**

e.g., α|σ+> + β|σ-> → α|K> + β|K'>

requires:

(1)Transition rules

|K(valence band)> + |σ+> → |K(conduction band)>
(with amplitude M)

|K'(valence band)> + |σ-> → |K'(conduction band)>
(with amplitude M')

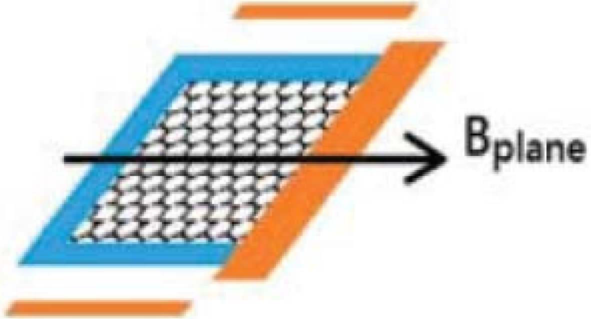
(2)Transition rules

|M|=|M'|

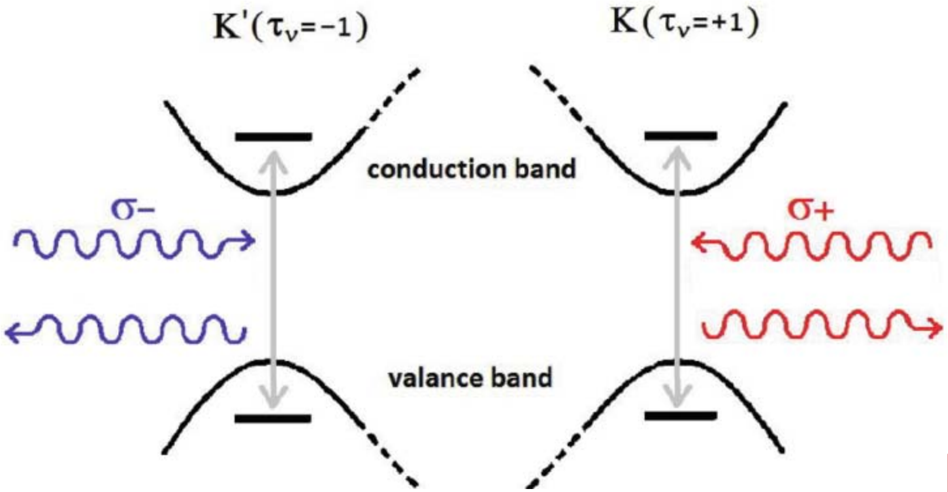
There should be a physical selection rule providing a set of persistent mapping between different physical states.

The mapping mechanism offered by graphene: Approximate selection rule

Bnormal =0



(symmetrical optical response)



$\sigma_{+(-)} = \sigma_x +(-)\, i\sigma_y$

Dirac theory: two-band model

$$\left(H_D^{(0)} + H_A\right)\phi_D = i\hbar\partial_t\phi_D,$$

$$H_D^{(0)} = \begin{pmatrix} \Delta(\vec{r}) + V(\vec{r}) & v_F\hat{p}_- \\ v_F\hat{p}_+ & -\Delta(\vec{r}) + V(\vec{r}) \end{pmatrix}, H_A = \begin{pmatrix} 0 & ev_F A_- \\ ev_F A_+ & 0 \end{pmatrix}$$

$$\phi_D = \begin{pmatrix} \varphi_A \\ \varphi_B \end{pmatrix},$$

Optical matrix elements

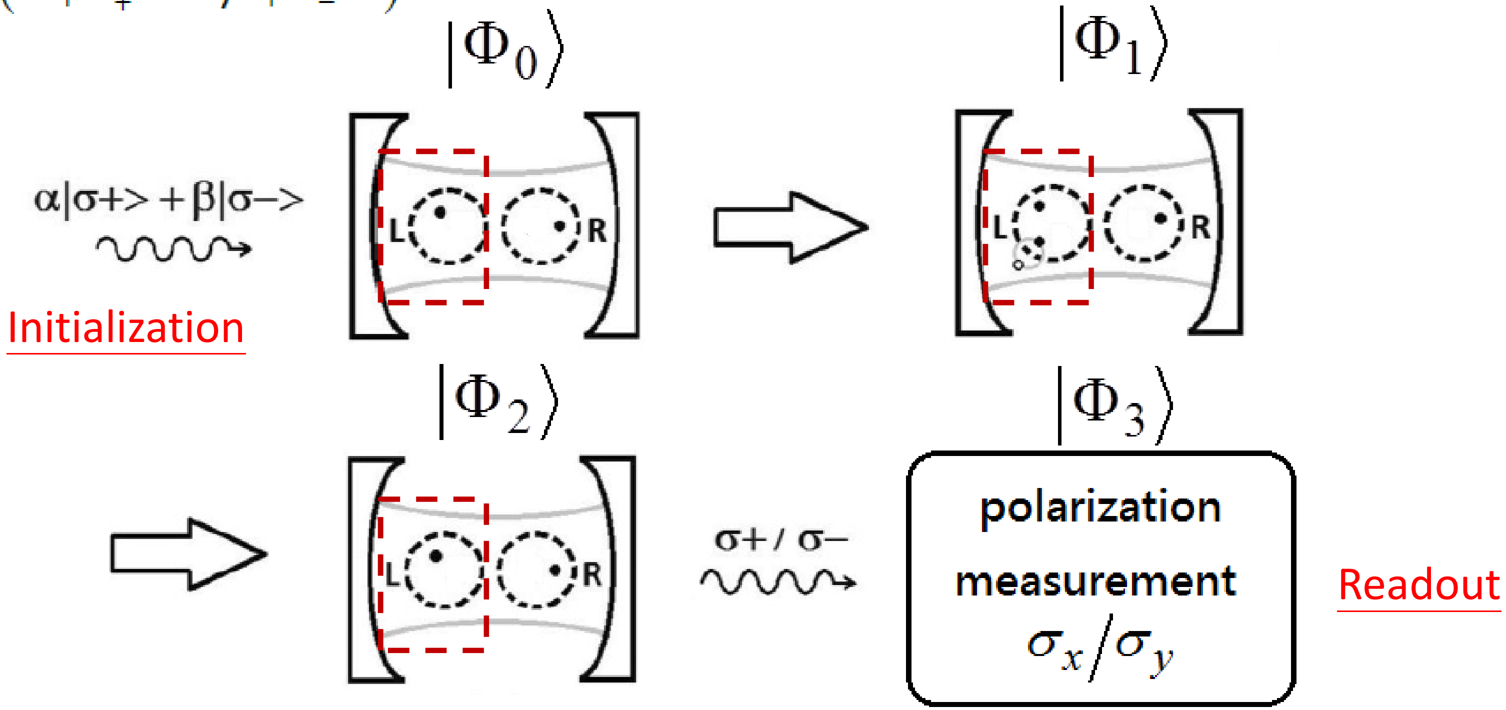
$$M_> = ev_F A_0 \langle \varphi_A^{(c)} | \varphi_B^{(v)} \rangle \rightarrow (\tau_v = +1, \sigma_+),\ (\tau_v = -1, \sigma_-)$$

$$M_< = ev_F A_0 \langle \varphi_B^{(c)} | \varphi_A^{(v)} \rangle \rightarrow (\tau_v = +1, \sigma_-),\ (\tau_v = -1, \sigma_+)$$

$$|M_<|/|M_>| \sim O(E/\Delta)$$

$$|\Phi_0 > = \frac{1}{\sqrt{2}}(|K_L K'_R > - |K'_L K_R >) \otimes (\alpha|\sigma_+ > + \beta|\sigma_- >)$$

$$|\Phi_1 > = \frac{1}{\sqrt{2}}\left(\beta|K'_{ex,L} K_L K'_R > - \alpha|K_{ex,L} K'_L K_R >\right)$$

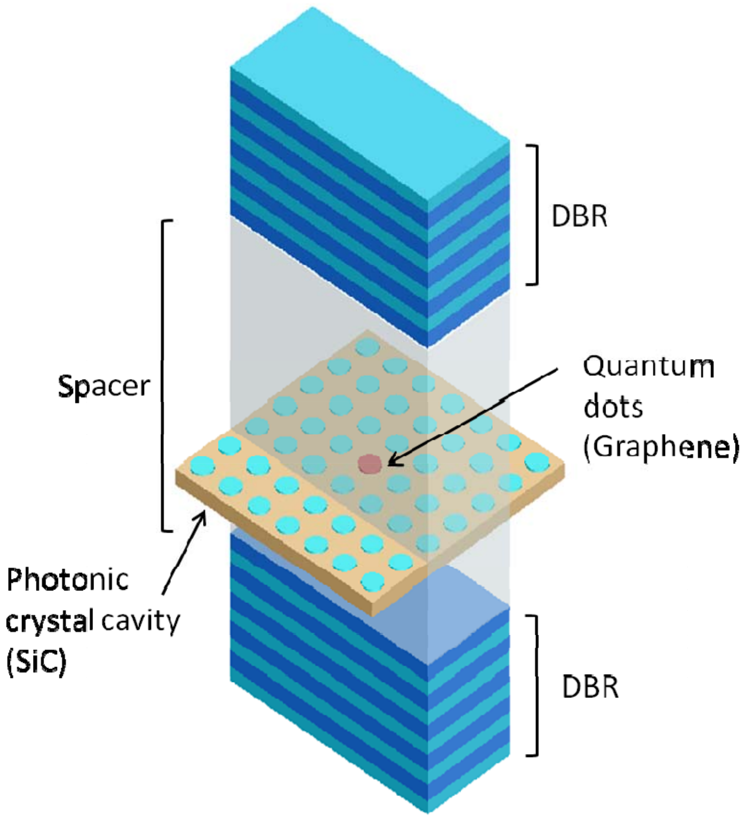


$$|\Phi_2 > = \beta|K_L K'_R > \otimes|\sigma_- > - \alpha|K'_L K_R > \otimes|\sigma_+ >$$

$$|\Psi_{ideal}\rangle \equiv \frac{1}{\sqrt{2}}[(\beta|K_L K'_R\rangle - \alpha|K'_L K_R\rangle)\otimes|\sigma_x\rangle - i(\beta|K_L K'_R\rangle + \alpha|K'_L K_R\rangle)\otimes|\sigma_y\rangle]$$

PRB 86, 045456 (2012)

To optimize the QST process: Valley pair qubit + double cavities



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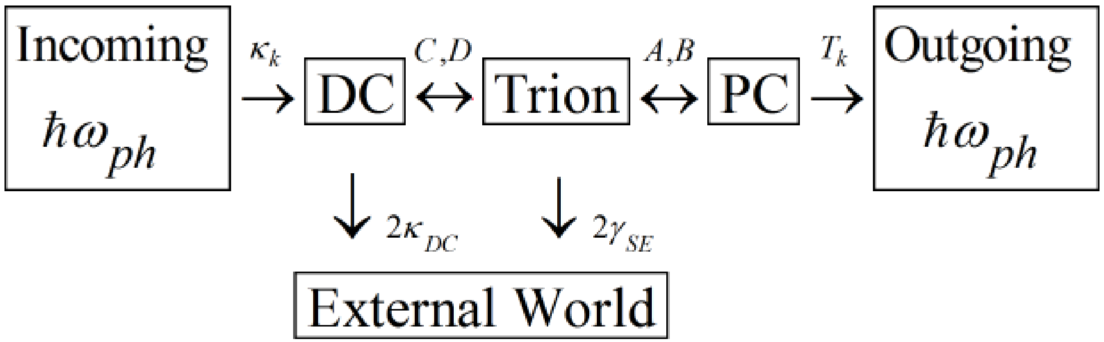


$$
\begin{aligned}
H = {} & H_{input} + H_{DC} + H_{trion} + H_{PC} + H_{output} + H_{reservoir} \\
& + H_{input-DC} + H_{DC-trion} + H_{trion-PC} + H_{PC-output} \\
& + H_{SE}.
\end{aligned}
$$

Figure of merits for QST

Yield

$$P = \sum_{\sigma,\tau,k_{2D}} \left|\phi_{\sigma\tau}^{output}\right|^2 = \sum_{k_{2D}} P_{k_{2D}}$$

Fidelity

$$F(\alpha,\beta) = \sum_{k_{2D}} F_{k_{2D}} P_{k_{2D}} / P,$$

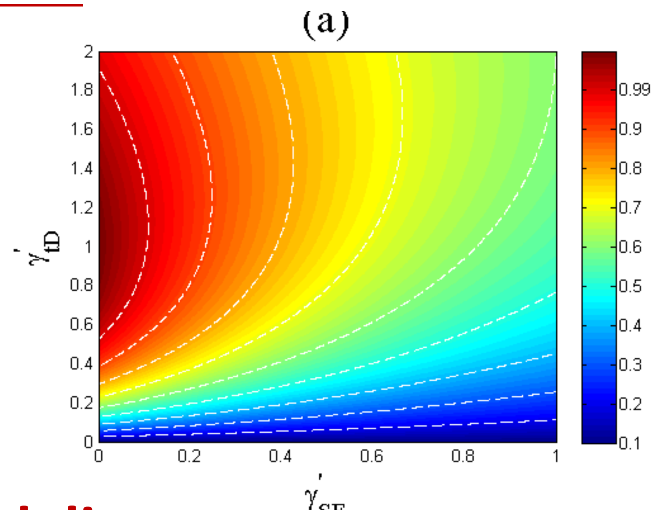
$$F_{k_{2D}} \equiv \left\langle \Psi_{ideal} \middle| \sum_{\sigma\tau,\sigma'\tau'} \phi_{\sigma\tau}^{output} \phi_{\sigma'\tau'}^{output*} \middle| \Psi_{ideal} \right\rangle / P_{k_{2D}}$$

(Submitted to PRB)

Optimal conditions (Submitted to PRB)

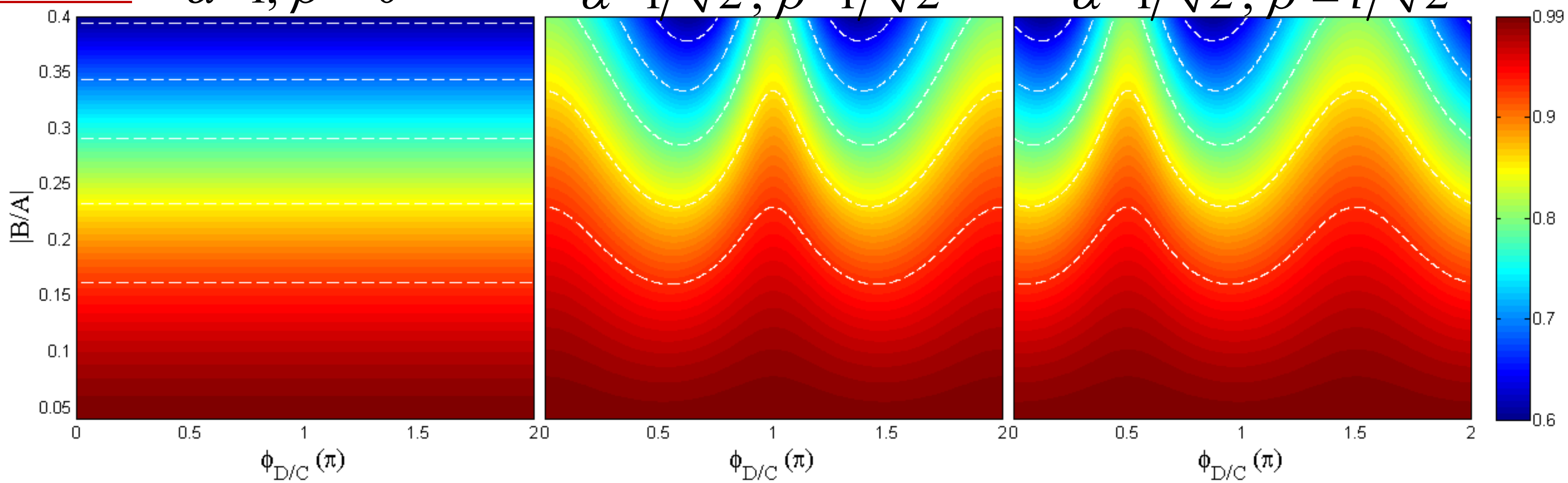
Setup parameters	Experimentally accessible conditions
ωph, Δωph	1.6x10^5 GHz, 5GHz
B/A = D/C	0.04
A, C	45, 30 GHz
γ_{SE}	1GHz
Q_{DBR}, Q_{PC}	550, 250
Yield, Fidelity	**~0.998**

Yield



Fidelity

$\alpha=1, \beta=0$ $\alpha=1/\sqrt{2}, \beta=1/\sqrt{2}$ $\alpha=1/\sqrt{2}, \beta=i/\sqrt{2}$



SUMMARY

- **Valley pseudospin and VOI**
  - Being of "relativistic" origin, the mechanism is similar to the SOI with difference -- valley-diagonal (state-mixing free).
- **Qubit manipulation (Bnormal ≠ 0)**
  - a dc or ac electric field can be applied to modulate the orbital magnetic moment of a confined electron, creating a magnetic moment gradient in the DQD structure, in the presence of a static, uniform magnetic field.
  - Along with exchange coupling, a fast, all-electric qubit manipulation may be performed by standard electric gate operation, in the time scale $\sim O(ns)$.
- **Quantum state transfer (Bnormal = 0)**
  - In the absence of a normal magnetic field, the optical excitation in gapped graphene is symmetric, and obeys the selection rule.
  - Optimized QST with experimentally accessible conditions give promising valley-photon state transfer.