Magnon Kerr effect in a strongly coupled cavity-magnon system

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Outline

- Introduction
  - Spin waves (magnons)

- Cavity quantum electrodynamics (QED) with magnons

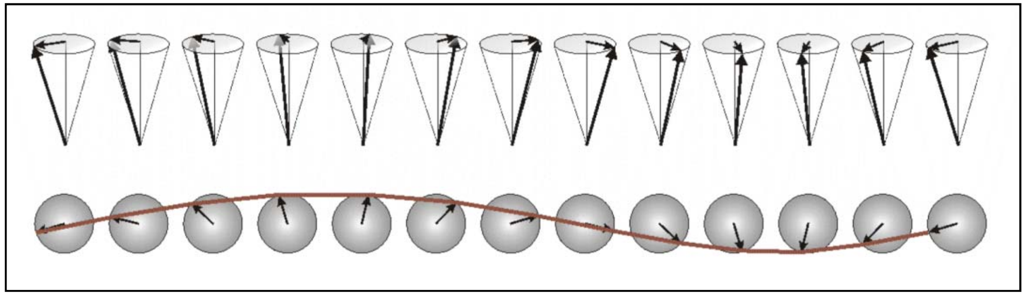
  D. Zhang *et al.*, npj Quantum Informtion 1, 15014 (2015).

- Magnon Kerr effect in a cavity QED system

  Y.P. Wang *et al.*, Phys. Re. B 94, 224410 (2016).

- Conclusions & Outlook

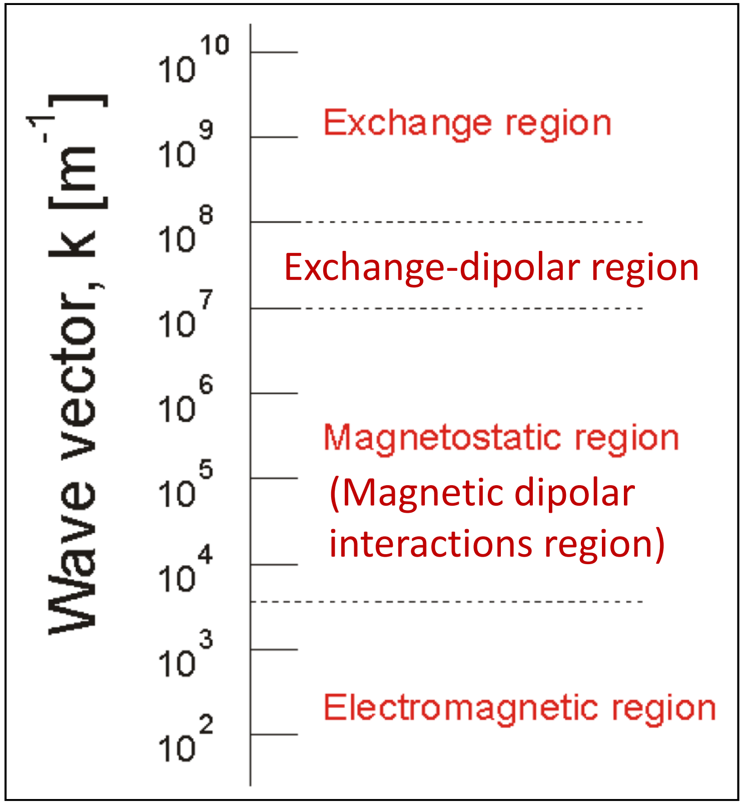
Spin waves (magnons)



Landau-Lifshitz equation

$$\frac{d}{dt}\vec{M} = -\gamma\mu_0\vec{M} \times \vec{H}_{\text{eff}} + T_{\text{D}}$$

+ Maxwell equations



M. Cottam and D. Tilley, *Introduction to surface and superlattice excitations* (IoP, 2004).

- **Robust extended spatial mode**
- **High spin density $10^{21} - 10^{22}$ cm^{-3}**

Quantum theory for spin waves in the long-wavelength limit

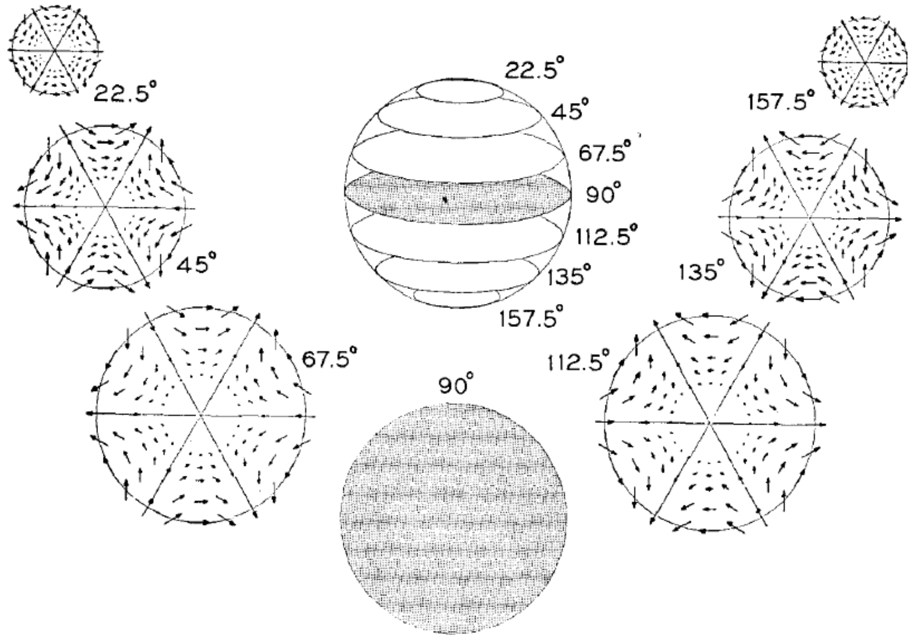
Magnetostatic wave

Magnetic dipolar interactions (dominating)

Exchange interactions (ignored)

$$\Lambda k^2 \ll 1,\ \Lambda = \text{exchange constant}$$

- Ferromagnetic resonance (**FMR**) mode
  (Kittel mode; uniform precession mode)
- Magnetostatic (**MS**) modes
  (Non-uniform precession modes)



L.R. Walker, J. Appl. Phys. 29, 318 (1958).

FMR mode in ferromagnetic sphere:

$$f_{\mathrm{m}} = \frac{\gamma_{\mathrm{e}}}{2\pi}\left|\vec{B}_0\right| + f_{\mathrm{m},0}$$

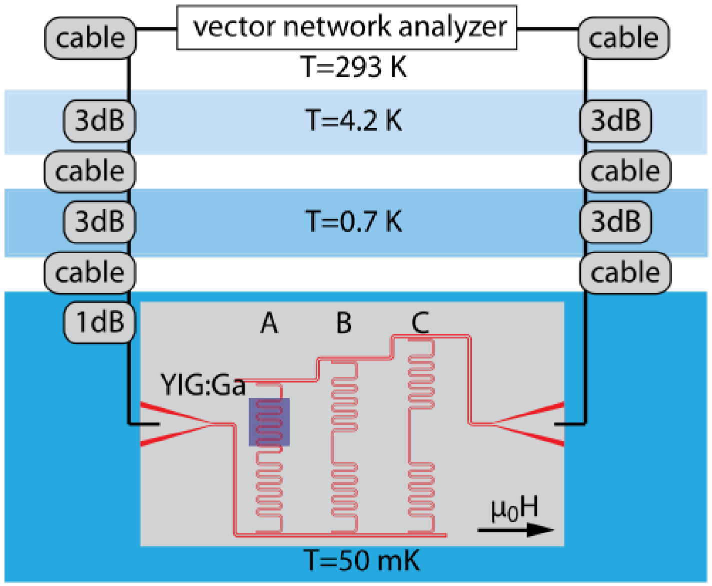
$\gamma_{\mathrm{e}}/2\pi = 28$ GHz/T, electron gyromagnetic ratio

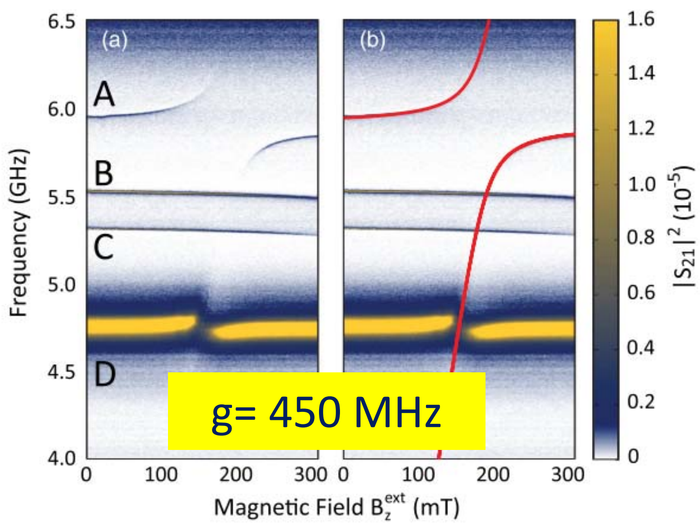
$f_{\mathrm{m},0} < 10$ MHz, determined by anisotropy field

YIG = yttrium iron garnet (Y3Fe5O12)

Quantum magnonics

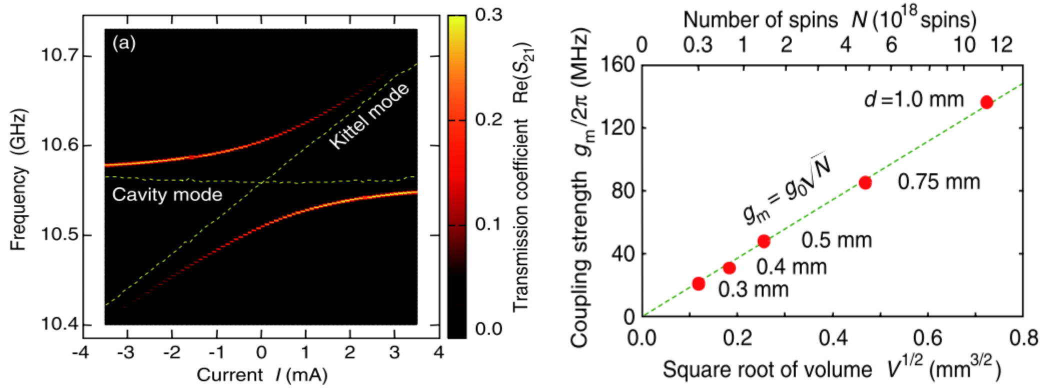
Recently reported experimental results on YIG sphere in cavity





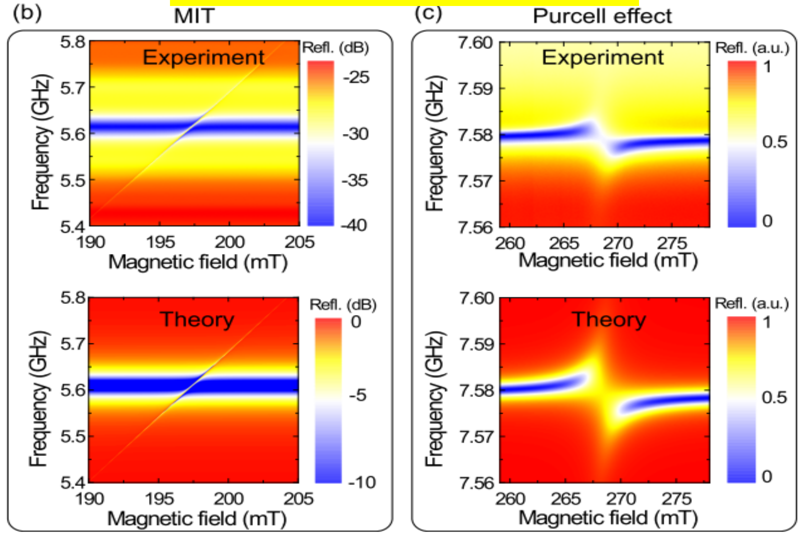
H. Huebl *et al.*, *Phys. Rev. Lett.* **111**, 127003 (2013).

Single photon level



Y. Tabuchi *et al.*, *Phys. Rev. Lett.* **113**, 083603 (2014).

Room temperature



X. Zhang *et al.*, *Phys. Rev. Lett.* **113**, 156401 (2014).

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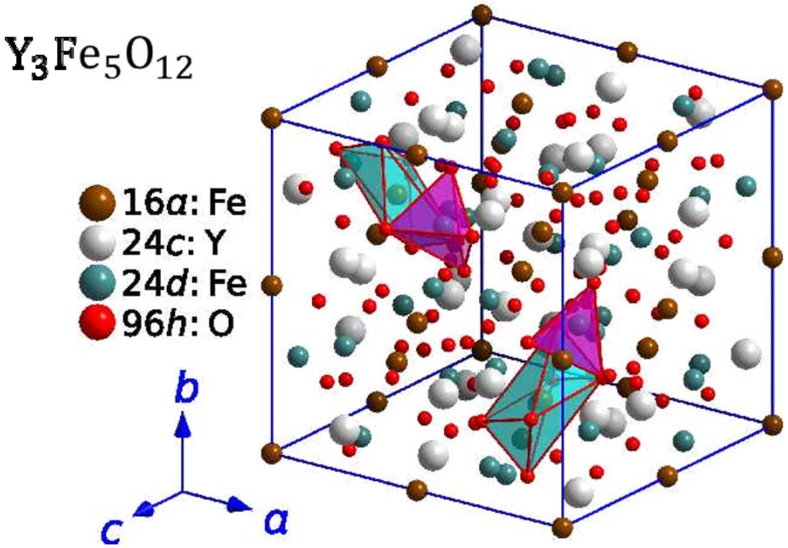
Yttrium-iron-garnet (YIG, $Y_3Fe_5O_{12}$)

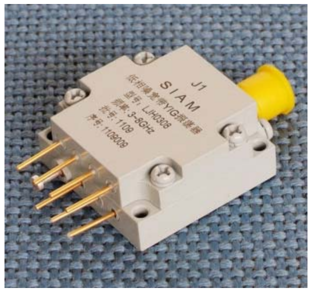
- Ferrimagnetic insulator
- Narrow FMR linewidth
- Transparent at infrared

- High Curie temperature: ~550 K
- Large spin density: 4.2×10^{21} cm^{-3}



Classic applications:

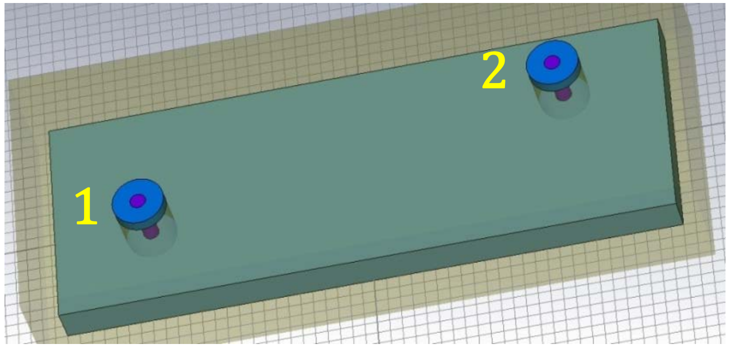
- Oscillator/ Filter
- Isolator/ Circulator

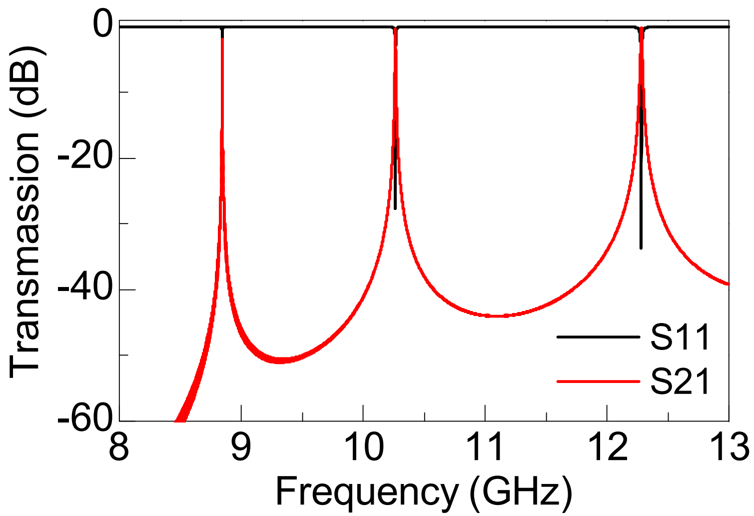


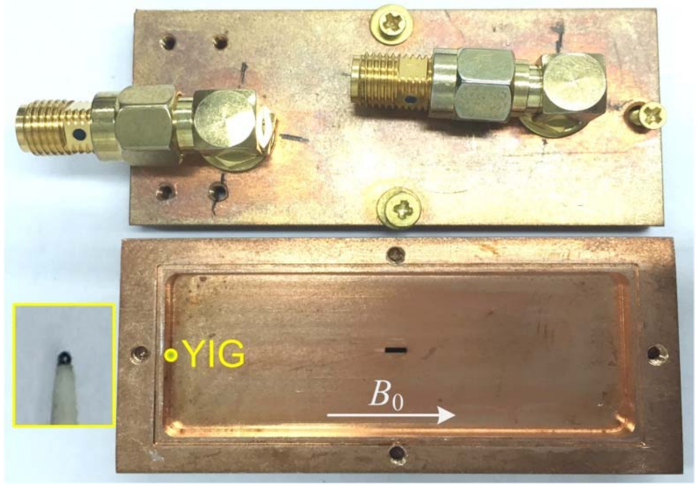
3D rectangular copper cavity

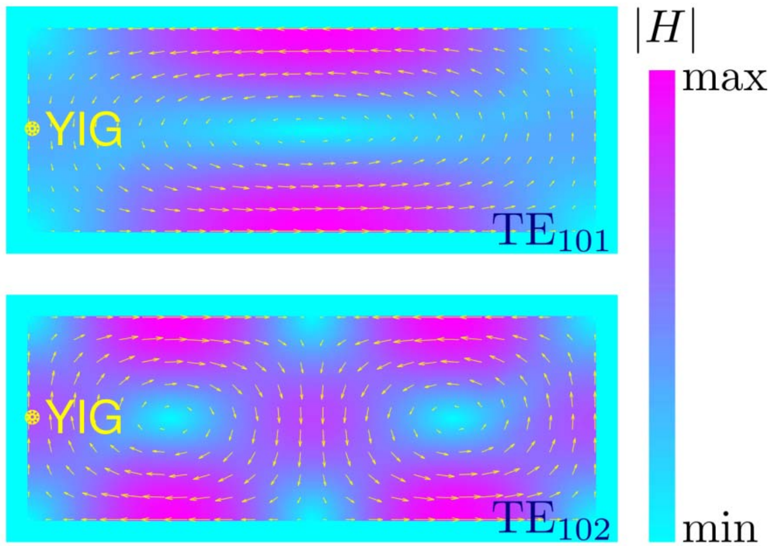
Two ports

Cavity size: 50×18×3 (mm^3)



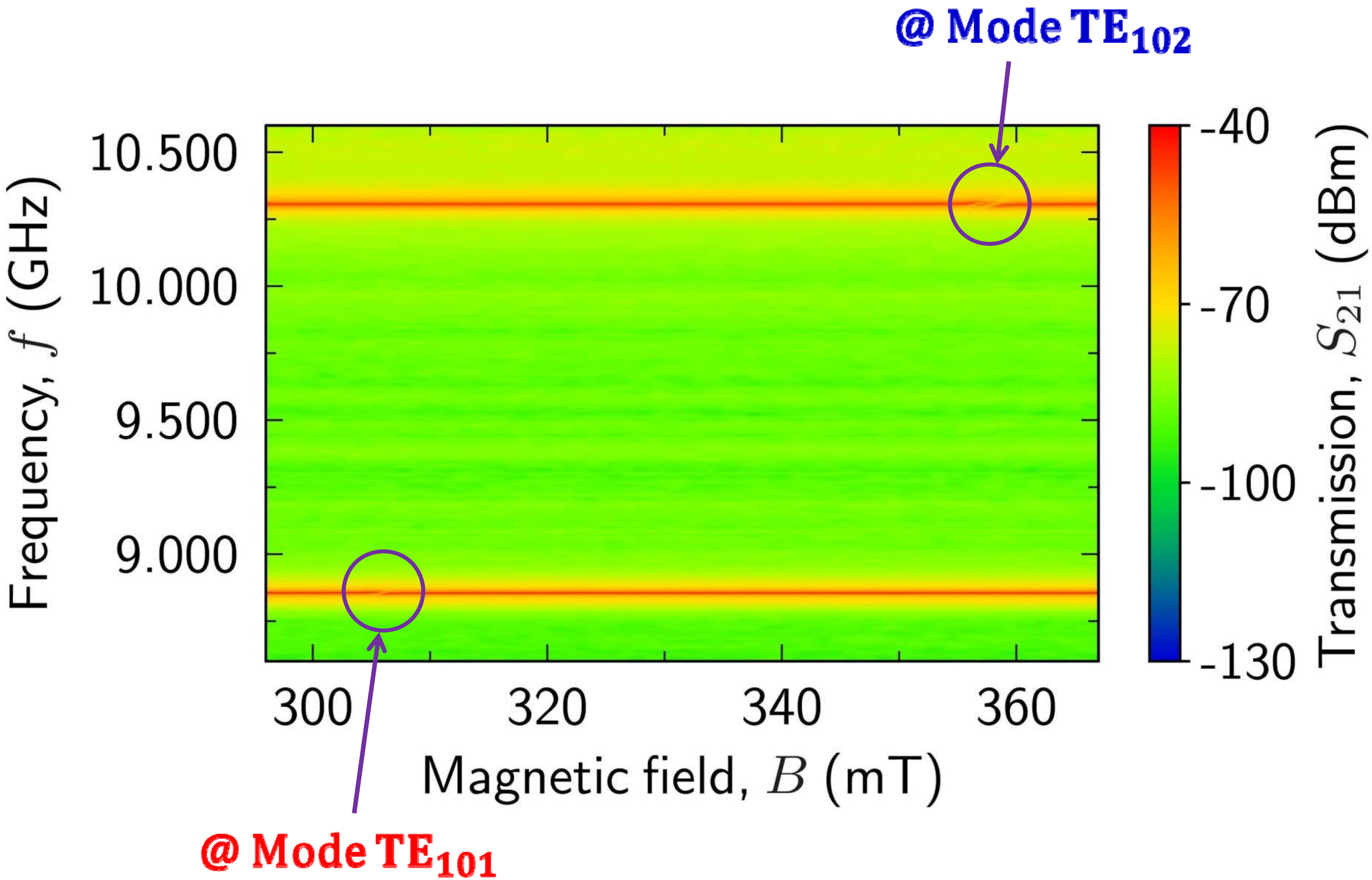






YIG diameter: 0.32 mm

S_{21} spectrum of cavity with a YIG sphere @ 22 mK



S_{21} @ cryogenic temperature (Cryo., 22 mK)

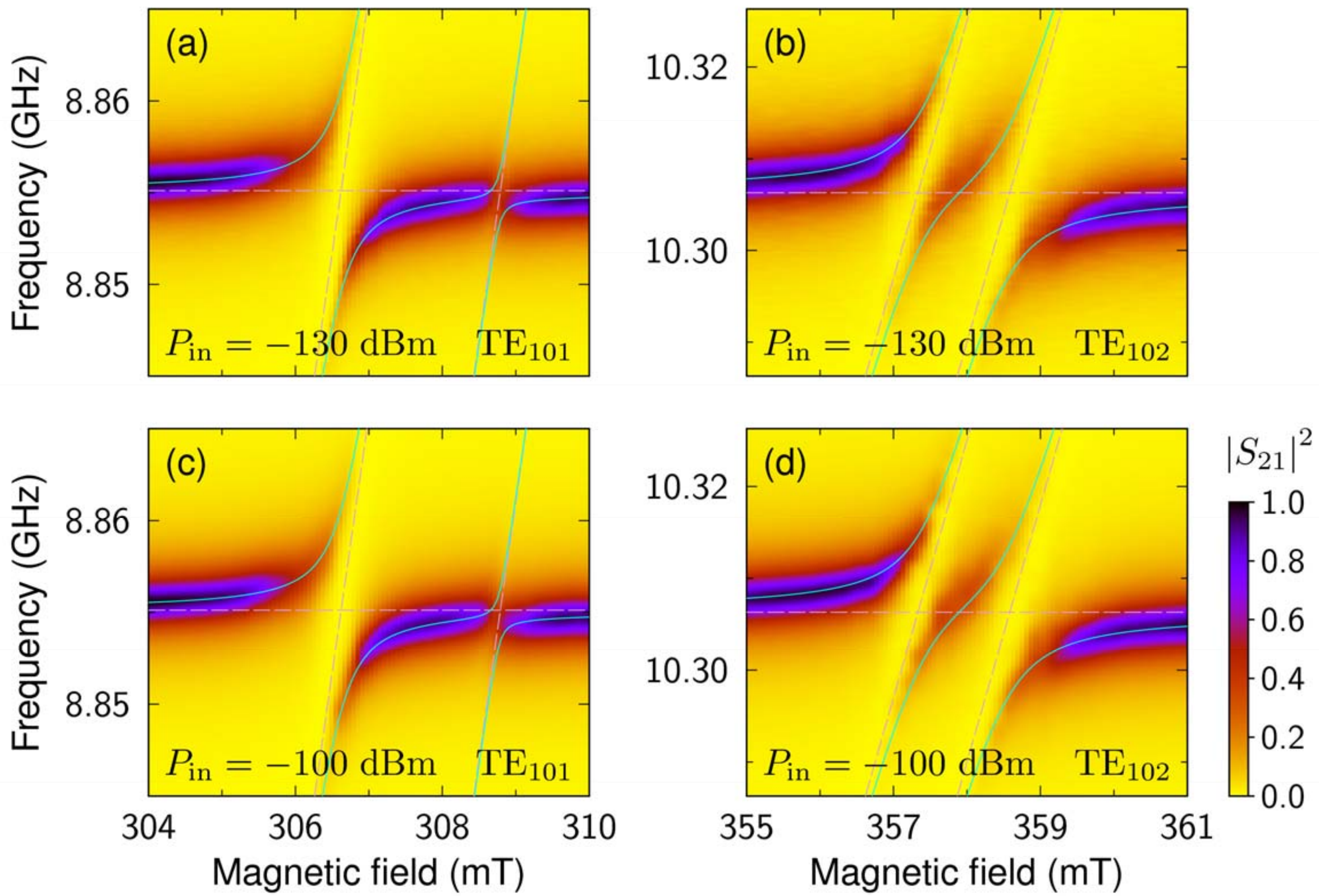
@ 22 mK average thermal photon number ~ 1×10^{-2}

Average photon numbers

0.8 (<1)

800

@ Mode TE_{101} **@ Mode TE_{102}**



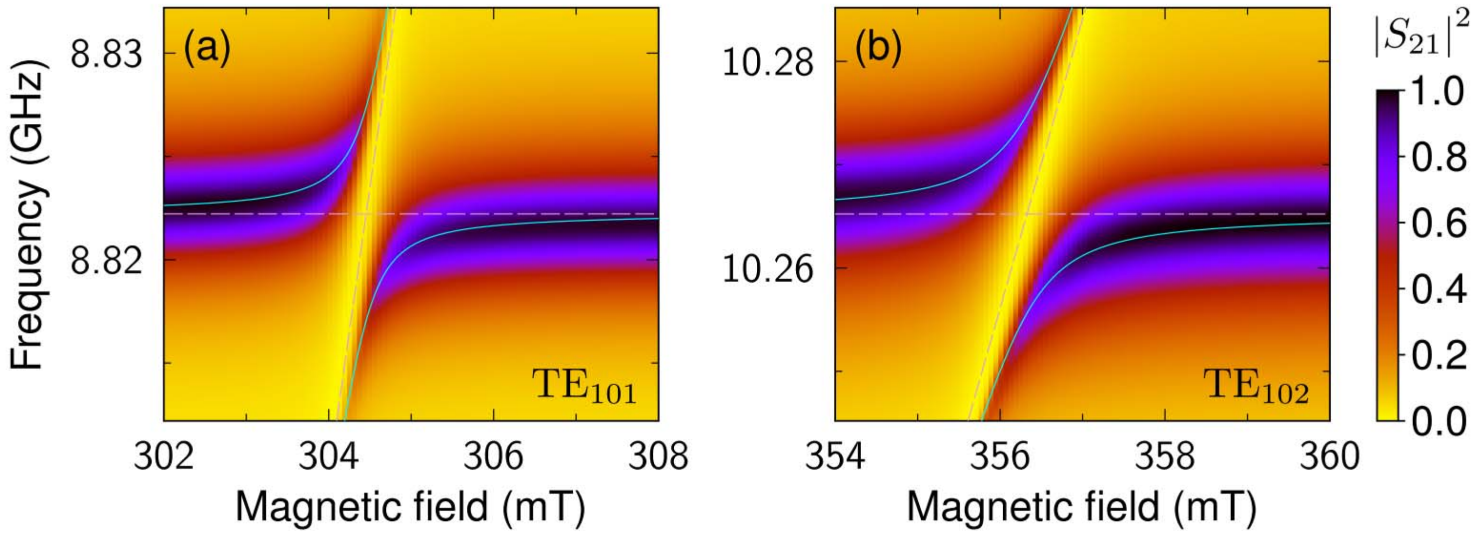
S_{21} @ room temperature (R. T., 300 K)

@ 300 K average thermal photon number ~ 700

Average photon numbers

1.8×10^{10}

@ Mode TE_{101} **@ Mode TE_{102}**



The MS mode related anti-crossing has disappeared.

S_{21} spectrum – Input-output theory

$$S_{21}(\omega) = \frac{2\sqrt{\kappa_\mathrm{i}\kappa_\mathrm{o}}}{i(\omega-\omega_\mathrm{c}) - \kappa_\mathrm{tot} + \sum(\omega)},$$

$$\sum(\omega) = \frac{\tilde{g}_\mathrm{FMR}^2}{i(\omega-\omega_\mathrm{FMR}) - \gamma_\mathrm{FMR}} + \frac{\tilde{g}_\mathrm{MS}^2}{i(\omega-\omega_\mathrm{MS}) - \gamma_\mathrm{MS}}$$

Coupling strength $\tilde{g}_m \ (m = \mathrm{FMR}, \mathrm{MS})$

Total cavity decay rate $\kappa_\mathrm{tot} = \kappa_\mathrm{i} + \kappa_\mathrm{o} + \kappa_\mathrm{int}$

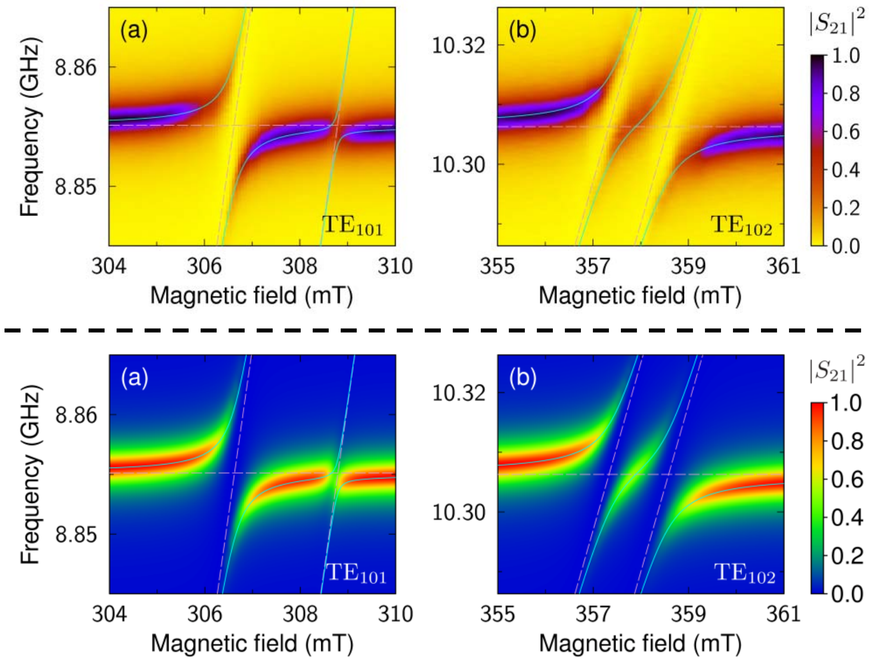
Magnon mode damping rate $\gamma_m \ (m = \mathrm{FMR}, \mathrm{MS})$

Strong coupling $\tilde{g}_\mathrm{m} > \kappa_\mathrm{tot}, \gamma_\mathrm{m}$

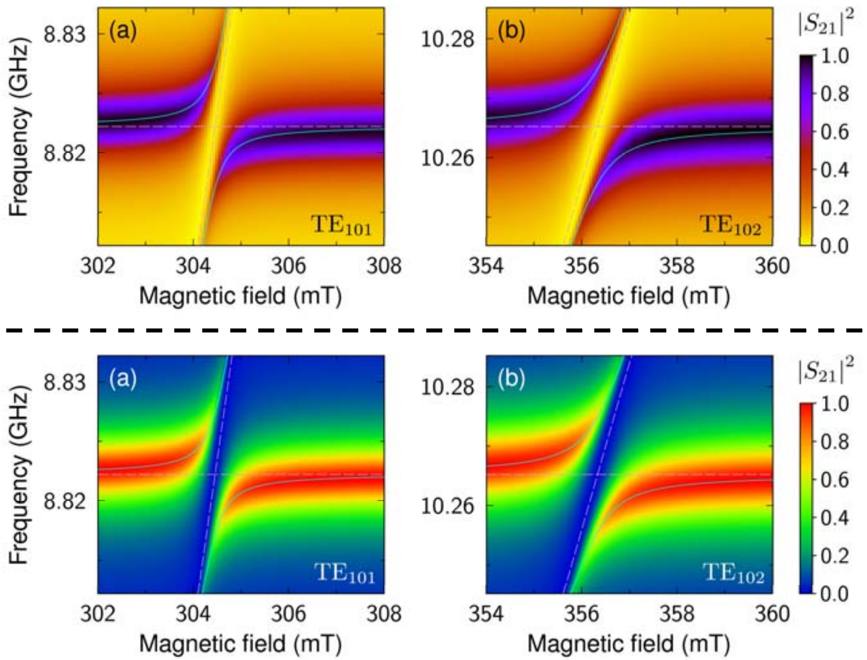
Cooperativity $C \equiv \tilde{g}_\mathrm{m}^2 / \kappa_\mathrm{tot}\gamma_\mathrm{m} > 1$

Parameters extracted by fitting with the experiment

Cryogenic temperature



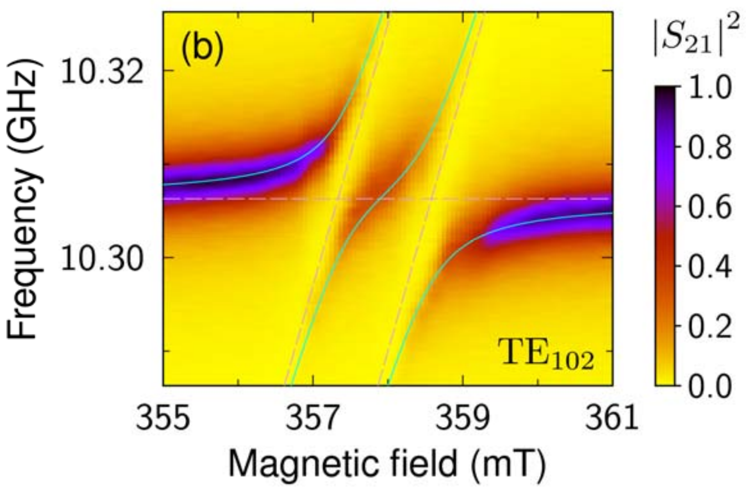
Room temperature



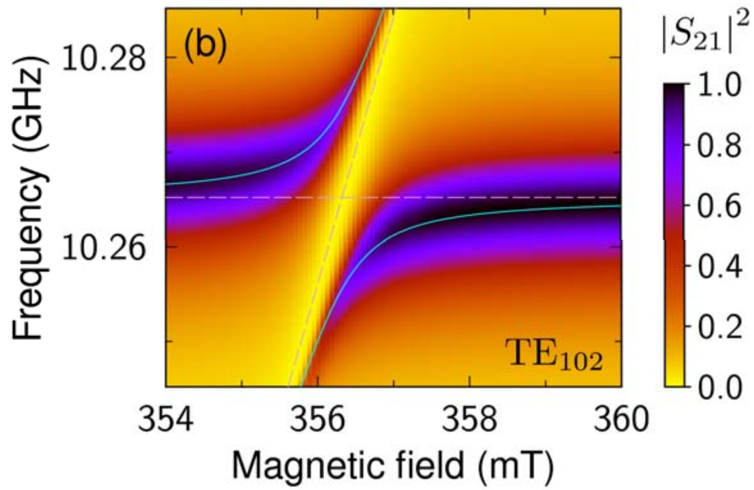
Temp.	Mode	$\tilde{g}_{FMR}, \tilde{g}_{MS}$ (/2π MHz)	κ_{tot} (/2π MHz)	$\gamma_{FMR}, \gamma_{MS}$ (/2π MHz)	C_{FMR}, C_{MS}
Cryo.	TE_{101}	5.1, 1.4	1.1	1.2, 2.7	22.1, 0.7
(22 mK)	TE_{102}	7.5, 8.3	2.4	1.3, 3.3	18.0, 8.7
R. T.	TE_{101}	5.2	2.5	1.3	8.3
(300 K)	TE_{102}	9.6	5.9	1.5	10.4

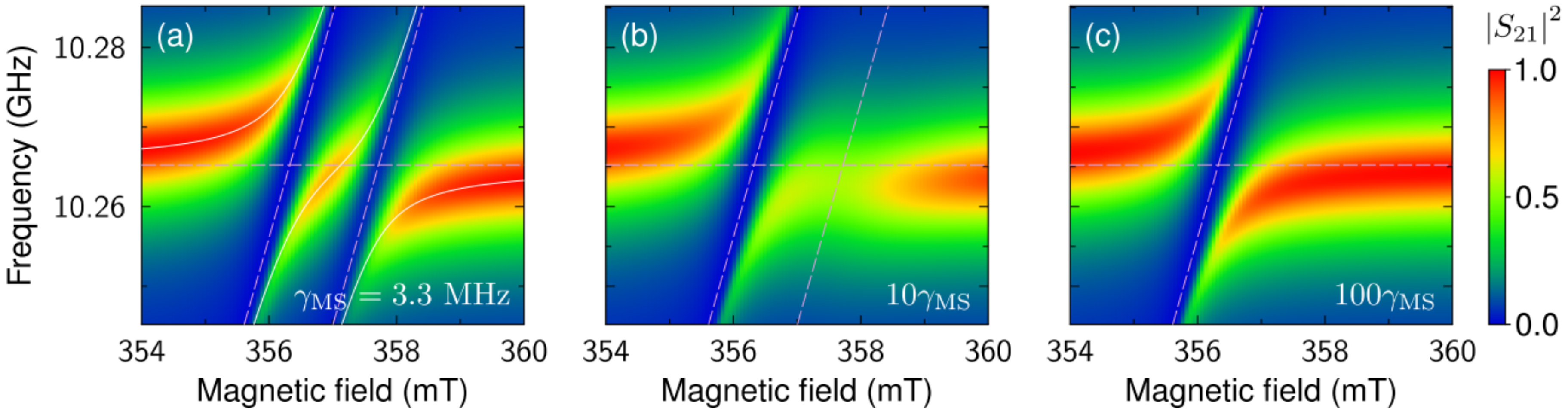
Comparison with simulations

Cryogenic temperature



Room temperature





For the MS mode, the damping rate is increased two orders from Cryo. to R.T.

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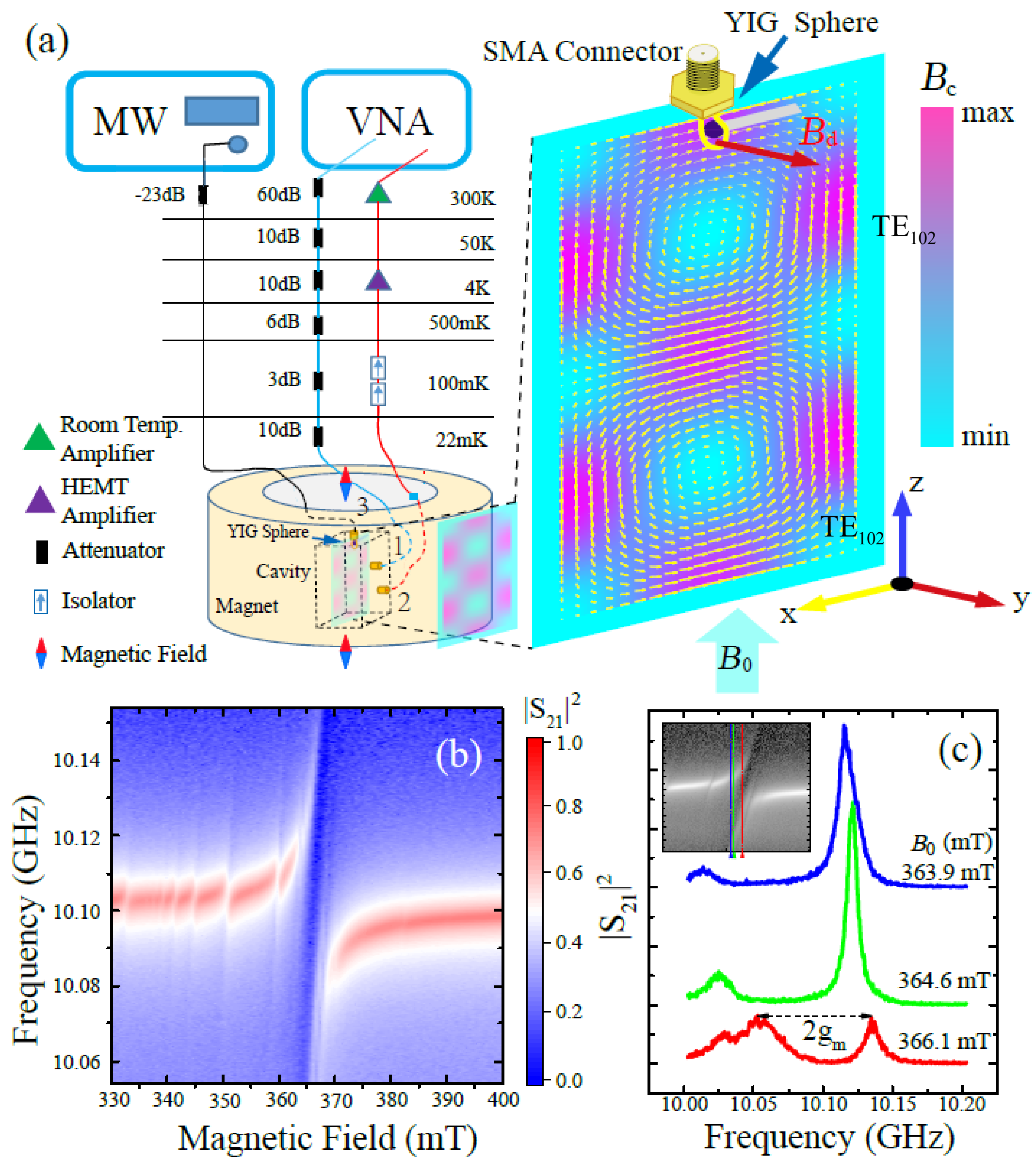
Experimental setup



Temperature: 22 mK

The YIG sphere is placed at the site where the magnetic field of the cavity mode TE102 is maximal (as before).

Newly added:
The YIG sphere can be directly pumped by a drive field.

Cavity QED system with magnon Kerr effect

Total Hamiltonian: $H = H_c + H_m + H_{int} + H_d.$

$$\text{Here } H_c = \omega_c a^\dagger a \qquad H_m = -\gamma B_0 S_z - \frac{\mu_0 \gamma^2 K_{an}}{M^2 V_m} S_z^2$$

Magnetocrystalline anisotropic energy

$$H_{int} = g_s(S^+ + S^-)(a^\dagger + a) \equiv 2g_s S_x(a^\dagger + a)$$

$$H_d = \Omega_s(S^+ + S^-)(e^{i\omega_d t} + e^{-i\omega_d t}) \equiv 4\Omega_s S_x \cos(\omega_d t)$$

Holstein-Primakoff transformation:

$$S^+ = \left(\sqrt{2S - b^\dagger b}\right) b, \quad S^- = b^\dagger \left(\sqrt{2S - b^\dagger b}\right), \qquad S_z = S - b^\dagger b$$

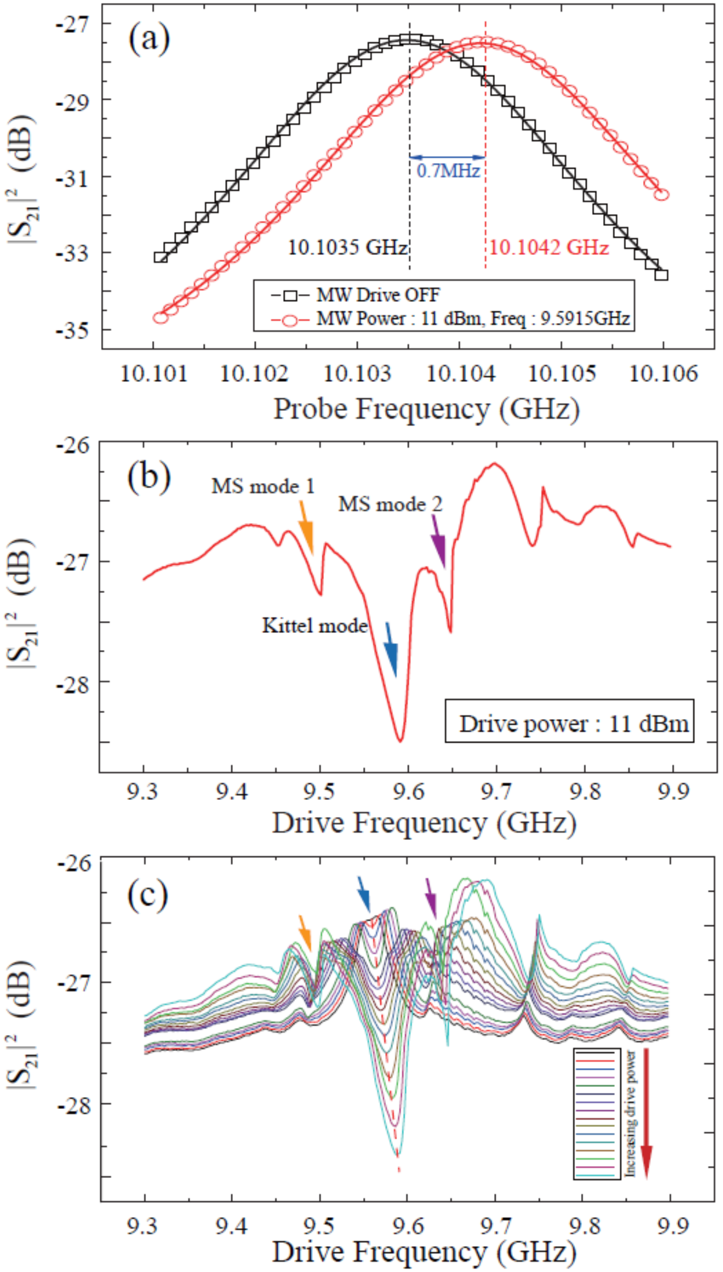
For the low-lying excitations with $\langle b^\dagger b \rangle / 2S \ll 1$, $S^+ \approx b\sqrt{2S}$, $S^- \approx b^\dagger \sqrt{2S}$.

In the rotating-wave approximation, the total Hamiltonian of the coupled hybrid system becomes

$$\begin{aligned} H = {} & \omega_c a^\dagger a + \omega_m b^\dagger b + K b^\dagger b b^\dagger b \\ & + g_m(a^\dagger b + a b^\dagger) + \Omega_d(b^\dagger e^{-i\omega_d t} + b e^{i\omega_d t}), \end{aligned}$$

$K = \mu_0 K_{an} \gamma^2 / (M^2 V_m)$ Kerr effect of magnons owing to the magnetocrystalline anisotropy.

Kerr-effect-induced frequency shifts



In the dispersive regime, when considerable magnons are generated by a drive field, the effective Hamiltonian of the system is

$$H_{\rm eff} = \left[\omega_{\rm c} + \frac{g_{\rm m}^2}{\Delta} + \frac{2g_{\rm m}^2}{\Delta^2}K\langle b^\dagger b\rangle\right]a^\dagger a + \left[\omega_{\rm m} - \frac{g_{\rm m}^2}{\Delta} + \left(1 - \frac{2g_{\rm m}^2}{\Delta^2}\right)K\langle b^\dagger b\rangle\right]b^\dagger b + \Omega_{\rm d}'(b^\dagger e^{-i\omega_{\rm d}t} + be^{i\omega_{\rm d}t}),$$

$$\Omega_{\rm d}' = \left[1 - \frac{1}{2(\omega_{\rm c} - \omega_{\rm d})}\left(\frac{g_{\rm m}^2}{\Delta} + \frac{2g_{\rm m}^2}{\Delta^2}K\langle b^\dagger b\rangle\right)\right]\Omega_d,$$

where $\Delta = \omega_{\rm c} - \omega_{\rm m}$.

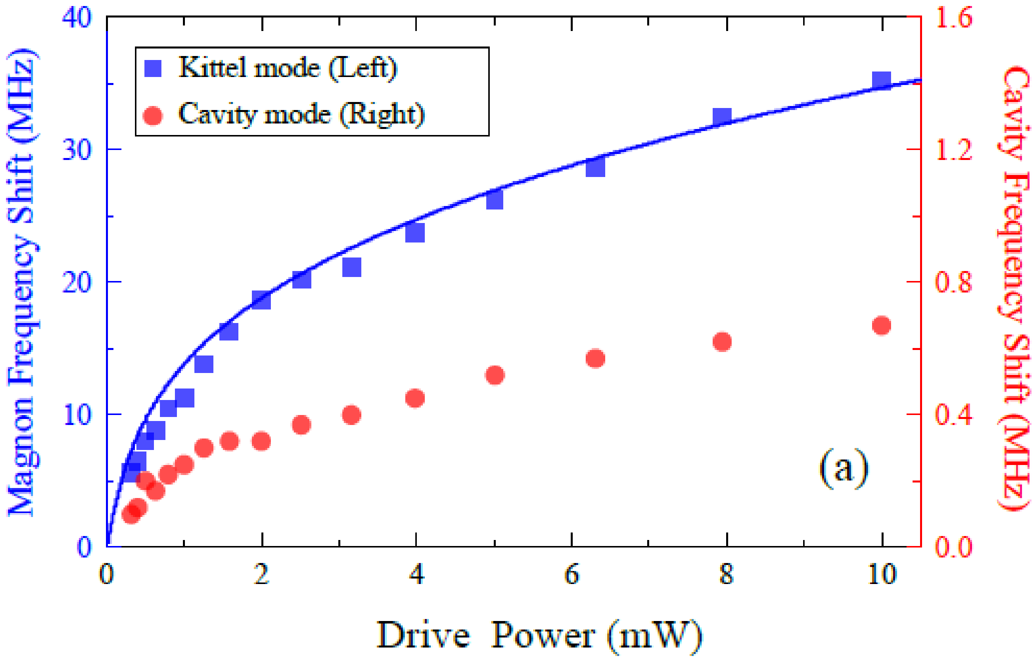
Kerr-effect-induced shift of the central cavity frequency:

$$\Delta_{\rm c} = (2g_{\rm m}^2/\Delta^2)K\langle b^\dagger b\rangle$$

Kerr-effect-induced shift of the magnon frequency:

$$\Delta_{\rm m} = (1 - 2g_{\rm m}^2/\Delta^2)K\langle b^\dagger b\rangle \approx K\langle b^\dagger b\rangle$$

Kerr-effect-induced frequency shifts vs. Drive power



Kerr-effect-induced shift of the central cavity frequency:

$$\Delta_c = (2g_m^2/\Delta^2)K\langle b^\dagger b\rangle$$

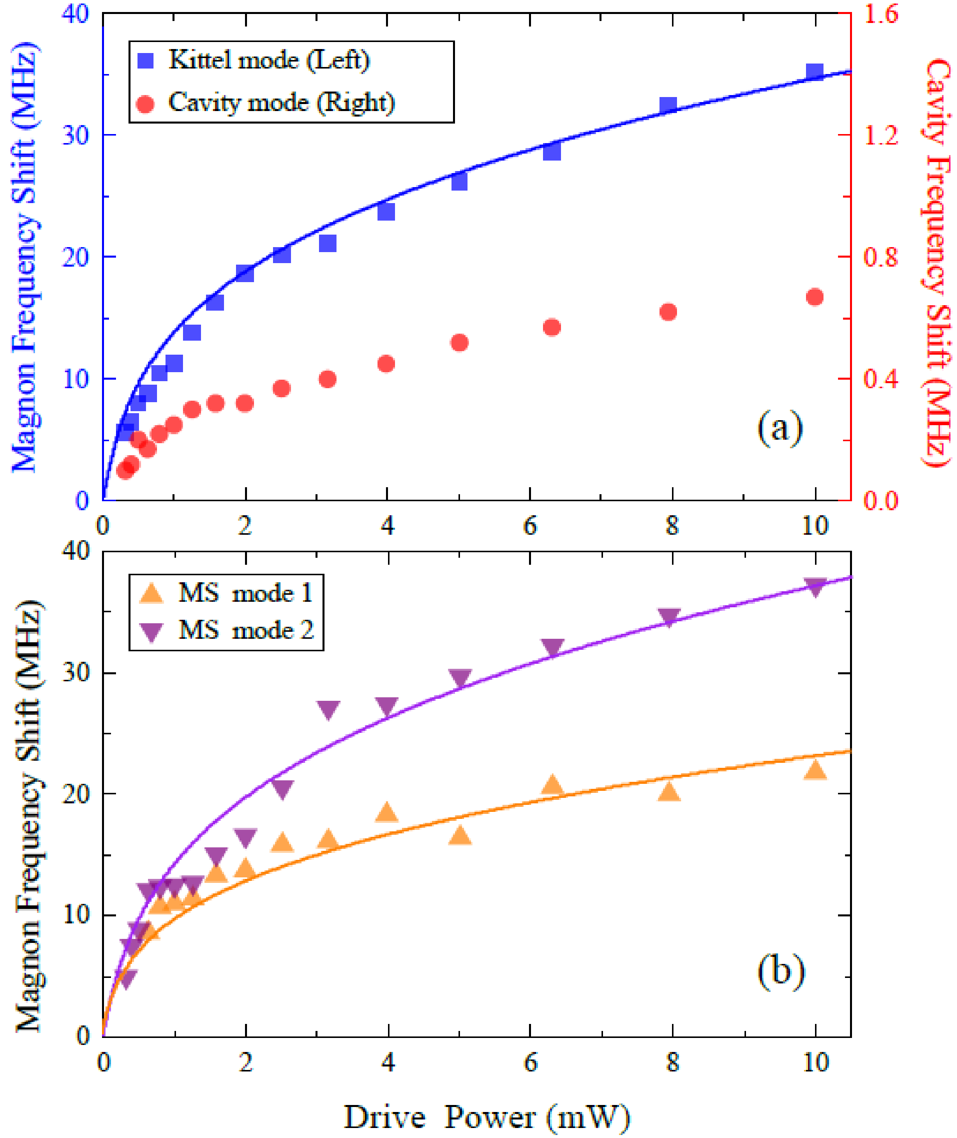
Kerr-effect-induced shift of the magnon frequency:

$$\Delta_m = (1 - 2g_m^2/\Delta^2)K\langle b^\dagger b\rangle \approx K\langle b^\dagger b\rangle$$

$$\Rightarrow \quad \Delta_c \propto \Delta_m$$

Therefore, it is predicted that the shift of the central cavity frequency should have a *similar behavior* as the frequency shift of the Kittle mode.

Magnon frequency shift vs. Drive power



Using a Langevin approach, we obtain *the relation between the magnon frequency shift and the drive power*:

$$\left[\Delta_{\mathrm{m}}^{2} + \left(\frac{\gamma_{\mathrm{m}}}{2}\right)^{2}\right]\Delta_{\mathrm{m}} - cP = 0$$

- The experimental results for the Kittel mode agree well with this relation (derived for an uniformly magnetized YIG sphere).
- The experimental results for the MS modes deviate from this relation, which confirms the deviations of the MS modes from homogeneous magnetization.

Conclusions

Experimental study of a cavity QED with magnons at both cryogenic and room temperatures.

- Robustness of the FMR mode against temperature.
- A drastic increase of the damping rate of the MS mode from cryogenic to room temperature.

Experimental demonstration of the magnon Kerr effect in a cavity QED system

- Kerr-effect-induced shift of the central cavity mode & Kerr-effect-induced magnon frequency shift.
- The experimental results for the Kittel mode agree well with the analytical relation between the magnon frequency shift and the drive power.

Outlook: Quantum magnonics & solid-state hybrids

- Ultrastrong coupling of magnons to photons
- Tim-domain properties of magnons
- Demonstrate non-classical states of magnon mode
- Coupling with qubits (single transmon qubit, U. of Tokyo/RIKEN group, Science, 2015).
- More nonlinear effects (bistability, chaos)
- BEC with magnons?

Main Collaborators:

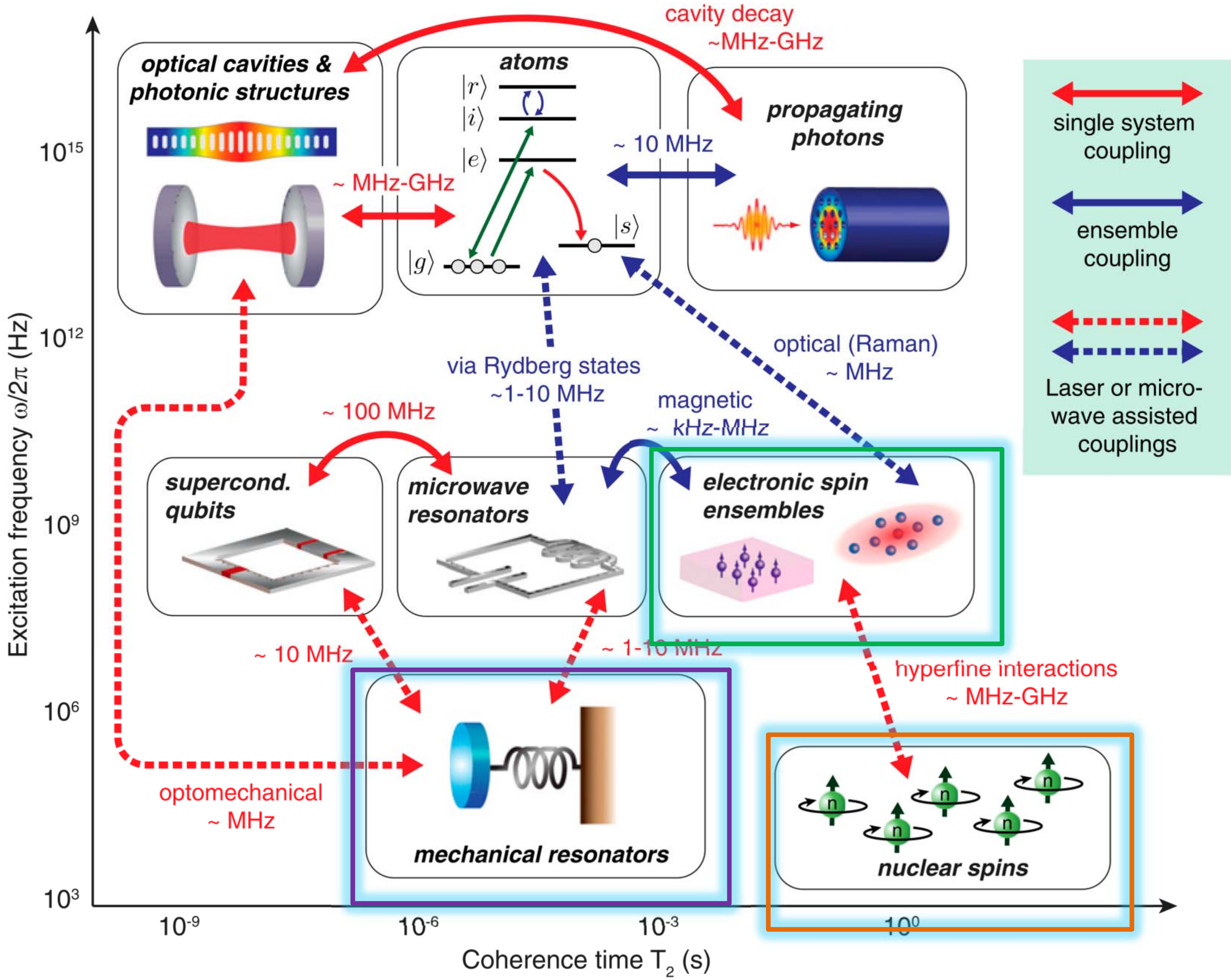
CSRC: Dengke Zhang, Yi-Pu Wang (experimental) & Guo-Qiang Zhang (theoretical)

Tie-Fu Li (Tsinghua & CSRC), Franco Nori (*RIKEN/Univ. of Michigan*)

& Can-Ming Hu (Univ. of Manitoba)

Thank you
for your attention!

Hybrid quantum systems



Kurizki *et al*., “Quantum Technologies with Hybrid Systems” *PNAS* , 2015;
Xiang *et al*., “Hybrid quantum circuits”, Rev. Mod. Phys. 2013.

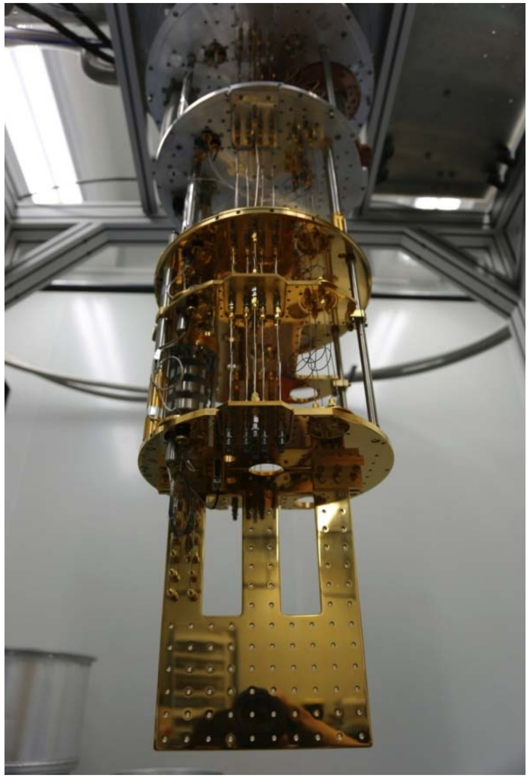
Magnetic resonance

	Resonant frequency
Nuclear magnetic resonance (NMR)	*Radio-frequency*
Paramagnetic materials	
Electron paramagnetic resonance (EPR)	*Microwave*
---Electron spin resonance (ESR)	
Ferromagnetic material	
Ferromagnetic resonance (FMR)	*Microwave*
Ferrimagnetic resonance (FiMR)	*Microwave — Far infrared*
Antiferromagnetic resonance (AFMR)	*Microwave — Far infrared*

Measurement setups

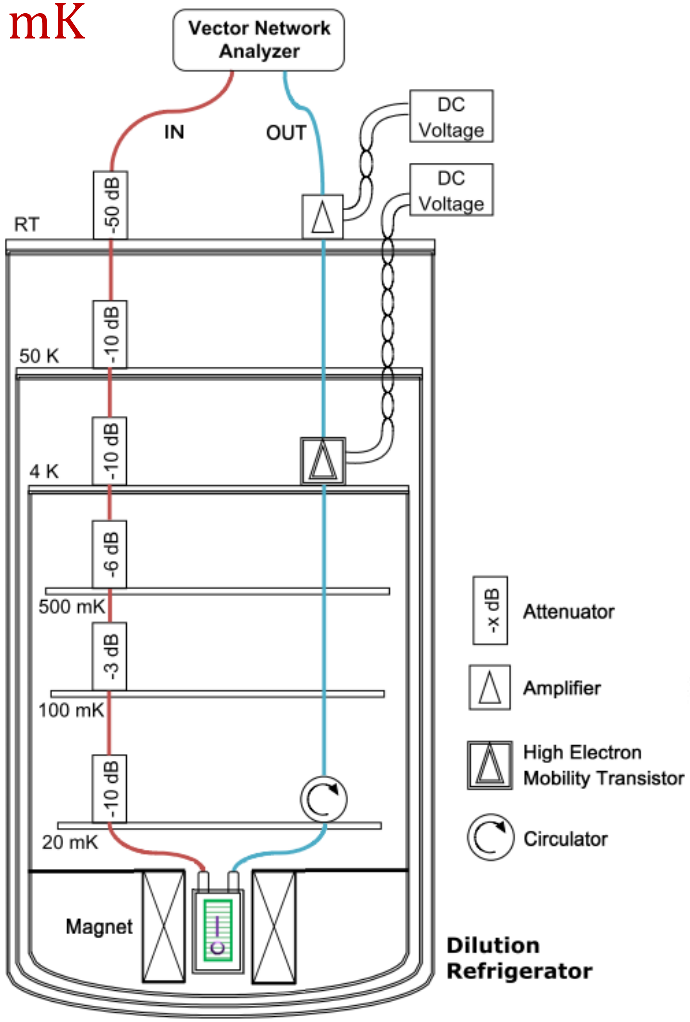
Cryogenic temperature (Cryo.) 22 mK

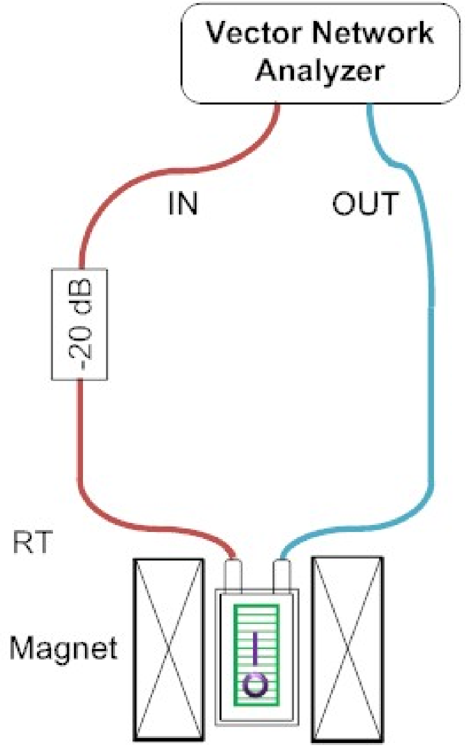
Room temperature (R. T.) 300 K



Bluefors cryo-free dilution refrigerator:

Base temperature 7mK



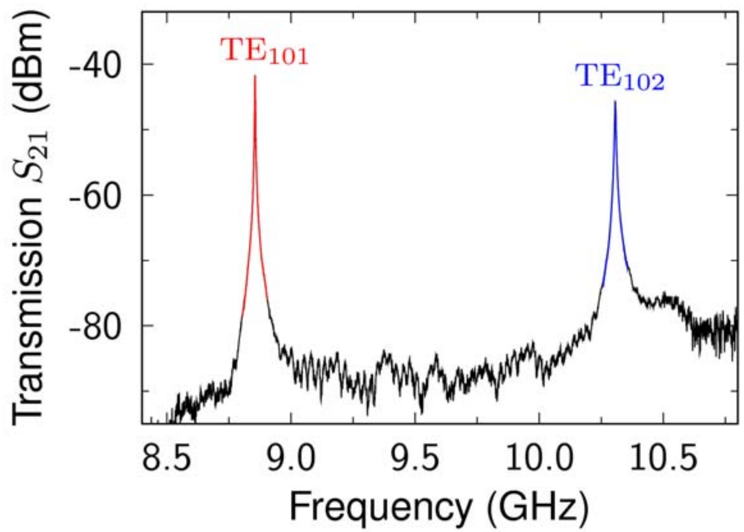


S_{21} spectra of cavity without a YIG sphere

$$\mathrm{TE}_{101}: \ \kappa_{i,1}/2\pi = 0.19\ \mathrm{MHz}; \ \kappa_{o,1}/2\pi = 0.20\ \mathrm{MHz}$$

$$\mathrm{TE}_{102}: \ \kappa_{i,2}/2\pi = 0.85\ \mathrm{MHz}; \ \kappa_{o,2}/2\pi = 0.99\ \mathrm{MHz}$$

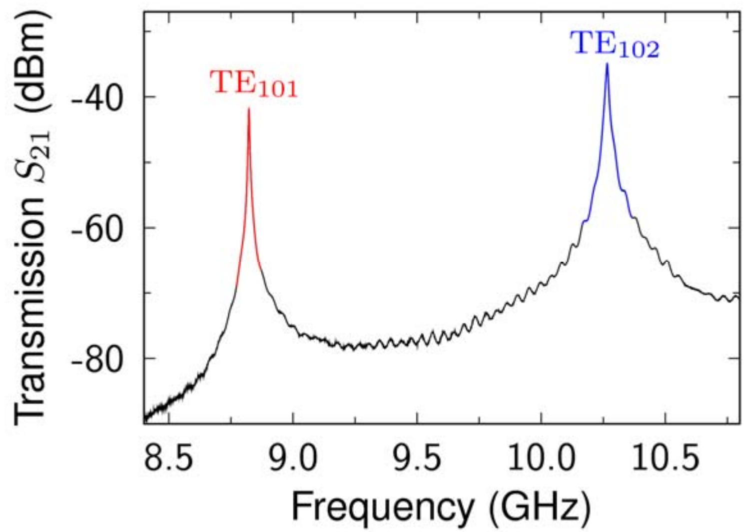
Cryogenic temperature (22 mK)



$$\kappa_{tot,1}/2\pi = 1.1\ \mathrm{MHz}; \ \kappa_{int,1}/2\pi = 0.71\ \mathrm{MHz}$$

$$\kappa_{tot,2}/2\pi = 2.4\ \mathrm{MHz}; \ \kappa_{int,2}/2\pi = 0.56\ \mathrm{MHz}$$

Room temperature



$$\kappa_{tot,1}/2\pi = 2.5\ \mathrm{MHz}; \ \kappa_{int,1}/2\pi = 2.11\ \mathrm{MHz}$$

$$\kappa_{tot,2}/2\pi = 5.9\ \mathrm{MHz}; \ \kappa_{int,2}/2\pi = 4.06\ \mathrm{MHz}$$

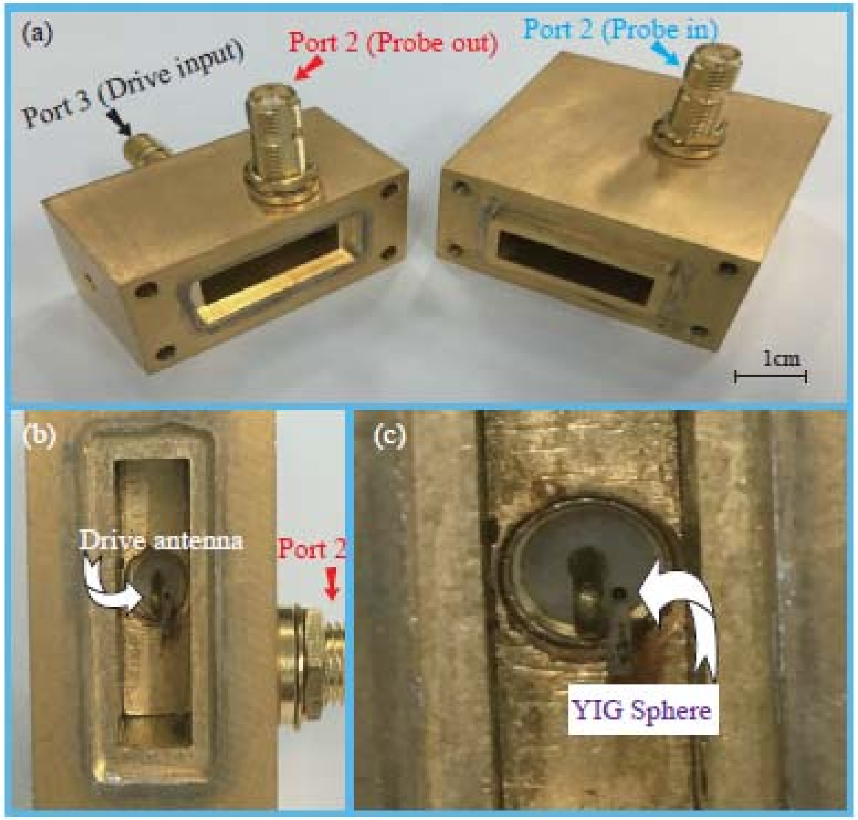
Experimental parameters

Temp.	Mode	Input power (dBm)	Input photon number	Thermal photon number
Cryo. (22 mK)	TE_{101}	-130 -100	0.8 800	$\sim 1 \times 10^{-2}$
	TE_{102}	-130 -100	0.7 656	$\sim 1 \times 10^{-2}$
R. T. (300 K)	TE_{101}	-20	1.8×10^{10}	705
	TE_{102}	-20	1.1×10^{10}	606

Quantum limit is reached at ~22 mK.

Question: Strong magnon-photon coupling was reached @ R. T. for the MS mode?

The 3D cavity containing a YIG sample



The 3D cavity has inner dimmensions:

$44.0 \times 20.0 \times 6.0 \text{ mm}^3$

The diameter of the YIG sphere: 1mm