Spin Qubits in Semiconducting Nanostructures

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$$: Swiss NSF, Nano Basel, Quantum ETH/Basel, EU

Outline

1. Quantum dots, spin qubits, quantum gates, decoherence, hole spins in Si/Ge nanowires, scalable systems, surface code, long-distance coupler...

2. Topological quantum computing in nanowires with Majorana fermions, parafermions,..., hybrid spin-Majorana qubits;

Prospects for Spin-Based Quantum Computing in Quantum Dots
Kloeffel and Loss, Annu. Rev. Condens. Matter Phys. 4, 51 (2013)

Quantum Memories at Finite Temperature
Brown, Loss, Pachos, Self, and Wootton, Rev. Mod. Phys. 88, 045005 (2016)

Quantum Computing stands for…

…new Physics: 'Quantum coherence of matter'

…new technologies: Quantum communication
Quantum control
Quantum metrology
Quantum computer
…

…Information is (quantum) physical ➔
is mathematical proof probabilistic ?

… 'Quantum mind/bio': 'To be and not to be' (?)

Front-Runners for Quantum Computers

- spin qubits in semiconductors ‘small & fast’
- superconducting devices
- trapped ions

more advanced but not so ‘small & fast’

- topological quantum computing?
  ‘semi-superconductor hybrids’

‘exotic’
Majorana
Para- or
Fibonacci
fermions?

Front-Runners for Quantum Computers

- spin qubits in semiconductors 'small & fast'

semiconducting nanostructures

- topological quantum computing?
  'semi-superconductor hybrids'

'exotic'
Majorana
Para- or
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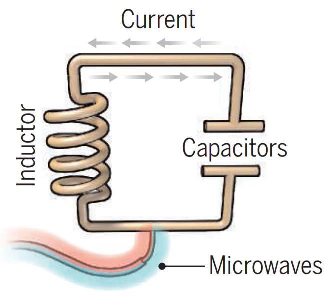
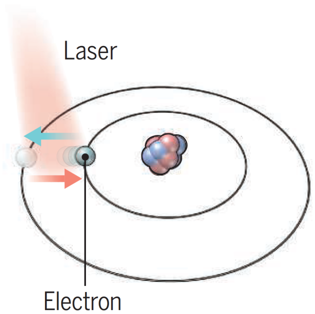
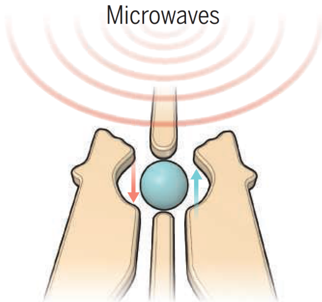
fermions?

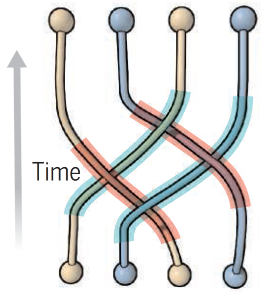
A bit of the action

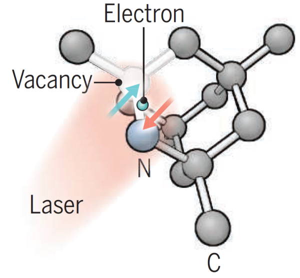
In the race to build a quantum computer, companies are pursuing many types of quantum bits, or qubits, each with its own strengths and weaknesses.











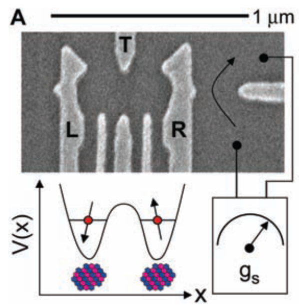
	Superconducting loops	Trapped ions	Silicon quantum dots	Topological qubits	Diamond vacancies
	A resistance-free current oscillates back and forth around a circuit loop. An injected microwave signal excites the current into superposition states.	Electrically charged atoms, or ions, have quantum energies that depend on the location of electrons. Tuned lasers cool and trap the ions, and put them in superposition states.	These "artificial atoms" are made by adding an electron to a small piece of pure silicon. Microwaves control the electron's quantum state.	Quasiparticles can be seen in the behavior of electrons channeled through semiconductor structures. Their braided paths can encode quantum information.	A nitrogen atom and a vacancy add an electron to a diamond lattice. Its quantum spin state, along with those of nearby carbon nuclei, can be controlled with light.
Longevity (seconds)	0.00005	>1000	0.03	N/A	10
Logic success rate	99.4%	99.9%	~99%	N/A	99.2%
Number entangled	9	14	2	N/A	6
Company support	Google, IBM, Quantum Circuits	ionQ	Intel	Microsoft, Bell Labs	Quantum Diamond Technologies
Pros	Fast working. Build on existing semiconductor industry.	Very stable. Highest achieved gate fidelities.	Stable. Build on existing semiconductor industry.	Greatly reduce errors.	Can operate at room temperature.
Cons	Collapse easily and must be kept cold.	Slow operation. Many lasers are needed.	Only a few entangled. Must be kept cold.	Existence not yet confirmed.	Difficult to entangle.

Note: Longevity is the record coherence time for a single qubit superposition state, logic success rate is the highest reported gate fidelity for logic operations on two qubits, and number entangled is the maximum number of qubits entangled and capable of performing two-qubit operations.

Quantum Computing in semiconductors: Which path to go?

Quantum computing

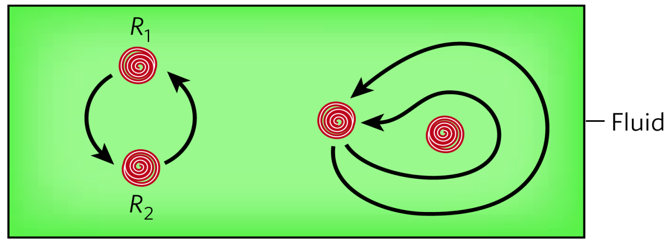
‘Conventional’



Loss and DiVincenzo, 1998

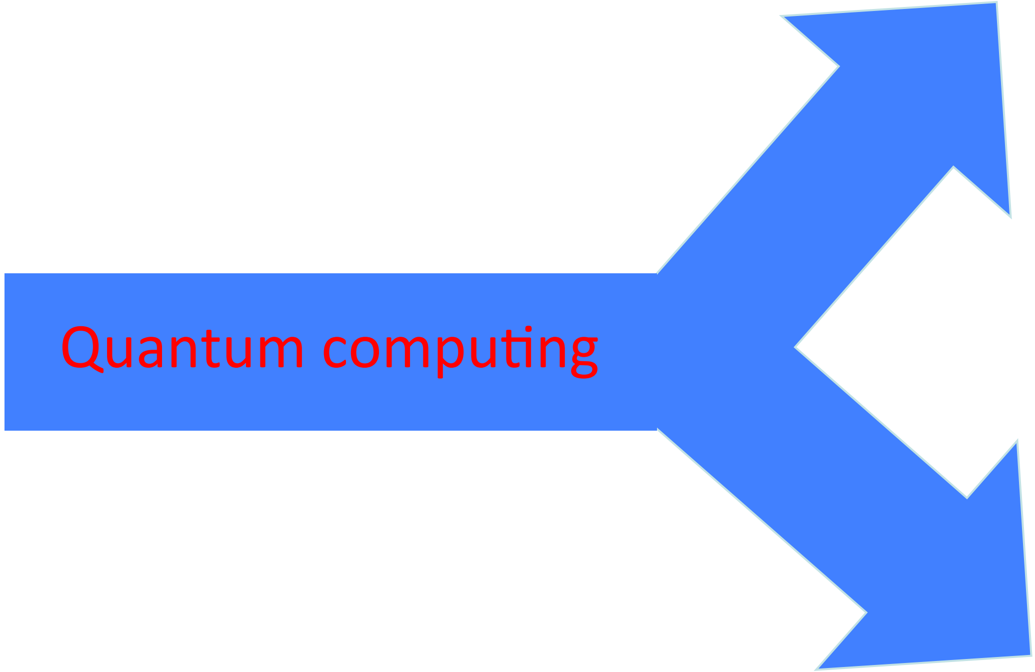
Kouwenhoven, Marcus, Petta,Tarucha, Vandersypen, Yacoby, Lukin, Bluhm, Dzurak, De Franceschi, Morello, Simmons, Eriksson,…

Topological

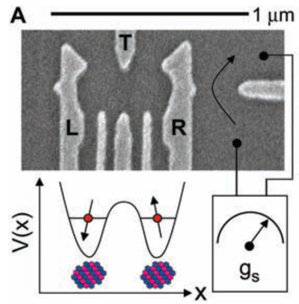


Kitaev 2003

Quantum Computing in semiconductors: Which path to go?



‘Conventional’



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Topological

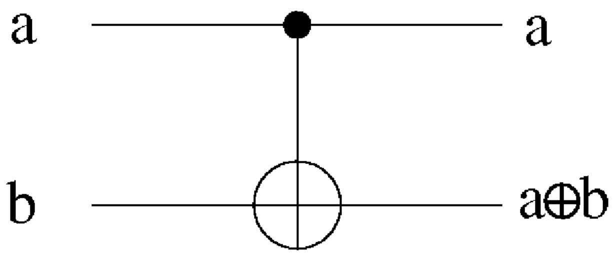
See later!

Kitaev 2003

Quantum Information

Classical digital computer

network of ‘Boolean logic gates‘, e.g. XOR (CNOT)



b \ a	0	1
0	0	1
1	1	0

- bits: $a, b = 0{,}1$
- physical implementation: e.g. 2 voltage levels
- ‘gate‘: electronic circuit

Quantum computer

- qubits $|a\rangle, |b\rangle \mathrel{\hat{=}} \alpha|0\rangle + \beta|1\rangle, \quad |\alpha|^2 + |\beta|^2 = 1$
- physical implementation: quantum 2-level-system: $|\uparrow\rangle \equiv |0\rangle, \quad |\downarrow\rangle \equiv |1\rangle$
- ‘quantum gate‘: unitary transformation (is reversible!)

Quantum Computing (basics)

• basic unit: **qubit** → any state of a quantum two-level system

$$|\Psi\rangle = a|1\rangle + b|0\rangle$$

"natural" candidate: **electron spin**

• quantum computation:

1) prepare N qubits (input)
2) apply unitary transformation in 2^N-dim. Hilbert space
→ computation
3) measure result (output)

• quantum computation faster than classical:

- factoring algorithm (Shor 1994): exp N → N^2
- database search (Grover 1996): N → $N^{1/2}$
- quantum simulations

...

What a quantum computer could do (faster):

…search large database (→ biology, climate, physics…)

…break `RSA-Encryption' (banking, industry, military,…)

…simulate physical und chemical processes (or models*)
(→ energy, catalysts, C-capture, material science, drug design,…)

…machine learning & cloud computing

…play quantum games

…and many unforseen applications (hopefully)

Intense search for new quantum algorithms !

*) See e.g. Wecker et al., Phys. Rev. A 92, 062318 (2015)
'Solving the 2D Hubbard model on a quantum computer'

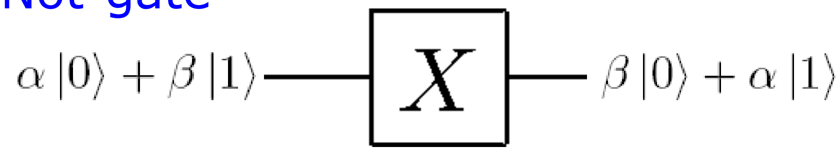
Quantum Computing with Quantum Gates

Barenco et al., PRA **52**, 3457 (1995)

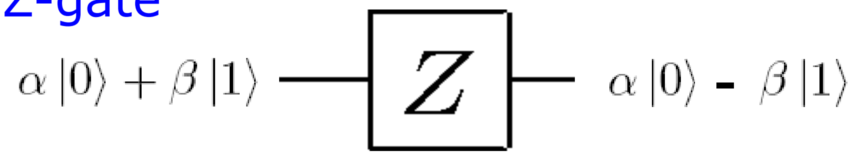
Single-qubit operations and a two-qubit gate that generates entanglement are sufficient for **universal quantum computation**:

Single-qubit gates

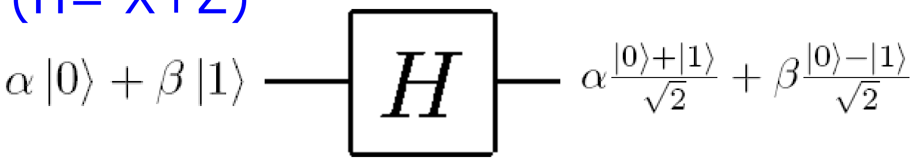
Not-gate



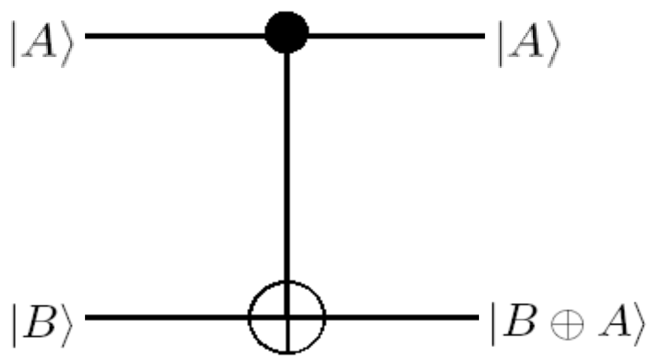
Z-gate



Hadamard-gate
(H= X+Z)



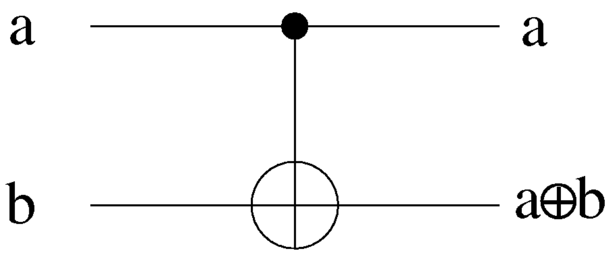
2-qubit gate:
CNOT (XOR) gate



⇔ entanglement

Quantum Gates

- quantum gate: unitary transformation acting on a few qubits at a time (universal set of quantum gates: all unitary operations on n qubits [U(2^n)] can be expressed as a composition of these gates)
- XOR together with one-qubit gates is a universal set for quantum computation (Barenco et al. 1995)



b \ a	0	1
0	0	1
1	1	0

- action of the quantum XOR gate:

two-particle state $|\psi\rangle = \alpha|00\rangle + \beta|01\rangle + \gamma|10\rangle + \delta|11\rangle$

$$|00\rangle \mapsto |00\rangle,\ |01\rangle \mapsto |01\rangle,\ |10\rangle \mapsto |11\rangle,\ |11\rangle \mapsto |10\rangle$$

$$U_{XOR} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix} = \exp\left(-\tfrac{i}{\hbar}\int dt H(t)\right)$$

Quantum Computing (basics)

- basic unit: **qubit** → any state of a quantum two-level system

$$|\Psi\rangle = a|1\rangle + b|0\rangle$$

"natural" candidate: **electron spin**

What should be used to implement qubits?

- quantum computation:

1) prepare N qubits (input)
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- quantum computation faster than classical:

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  ...

Historical remarks:
Electron qubit: `spin better than charge‘

due to longer relaxation/decoherence* times

$$\tau_\phi^{spin} >> \tau_\phi^{charge}$$

10ns -1s

1 – 10 ns

GaAs

Awschalom et al., ’97
Tarucha et al., ‘02
Kouwenhoven/Vandersypen et al., ’03-’09
Abstreiter et al., ’04/08; Warburton et al., ‘09
Zumbuhl et al., '08
Marcus, Yacoby et al.,’05- ’15 (T_2 ~ 270…850 μs)

‘mesoscopics’
Fujisawa et al. ‘03
Marcus et al. ‘01

➔ natural choice for qubit: spin ½ of electron

*) theory: $T_2 \sim T_1$ for single spin in GaAs dot (‘everything optimized’)

Magnetic moment of single spin (Bohr magneton) is very weak:

Advantage: spin couples weakly to environment

➔ spin has long decoherence time (0.001-1000 μs)

Disadvantage: spin couples weakly to “observer”

➔ spin is difficult to control

Instead: control spin via charge, made possible by Pauli exclusion principle which “locks spin to charge”

➔ manipulation & detection of spin-dynamics via charge (orbital) degrees of freedom of electron

D. Loss & D. DiVincenzo, PRA **57** (1998) 120

PHYSICAL REVIEW A VOLUME 57, NUMBER 1 JANUARY 1998

Quantum computation with quantum dots

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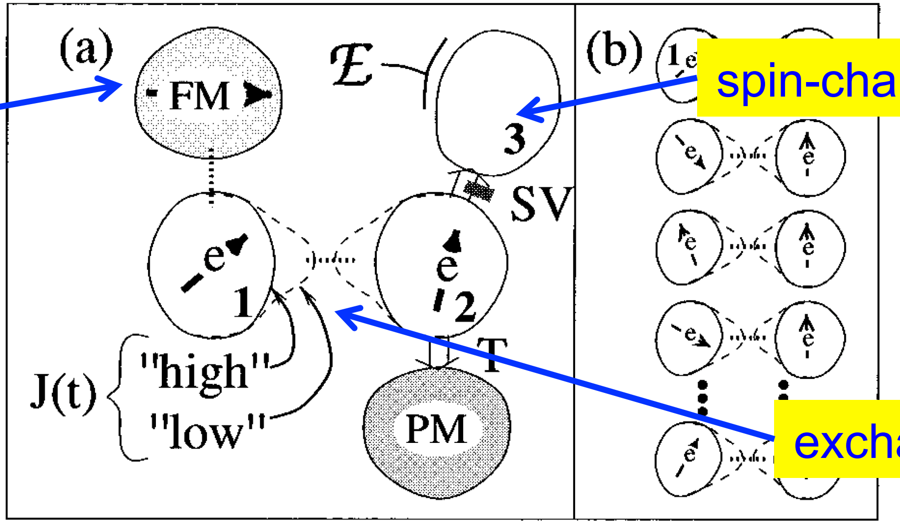
(Received 9 January 1997; revised manuscript received 22 July 1997)

spin rotation by exchange



spin-charge conversion for read-out

‘all-electrical scheme’

exchange coupling

$$U_{XOR} = e^{i\frac{\pi}{2}S_1^z}\, e^{-i\frac{\pi}{2}S_2^z}\, U_{SW}^{1/2}\, e^{i\pi S_1^z}\, U_{SW}^{1/2}$$

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Times Cited:
~ 6100
(Google Scholar)

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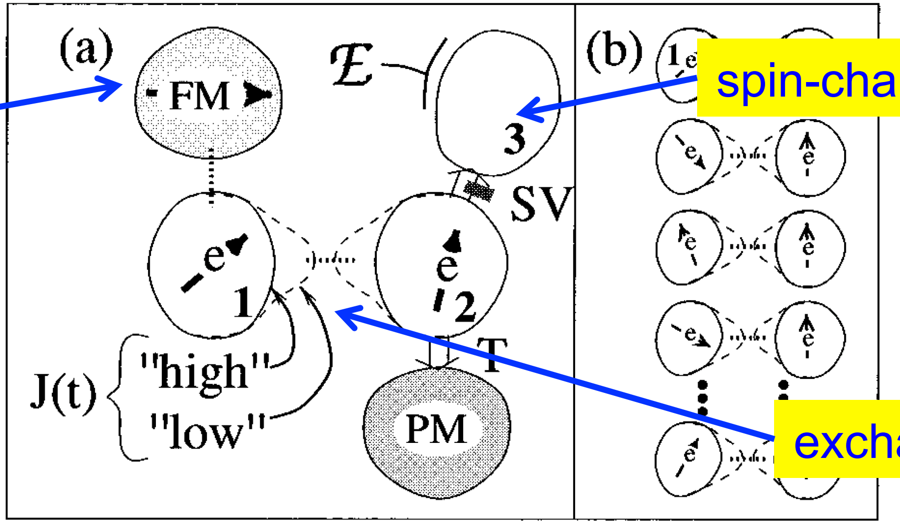

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Electric fields vs. Magnetic fields

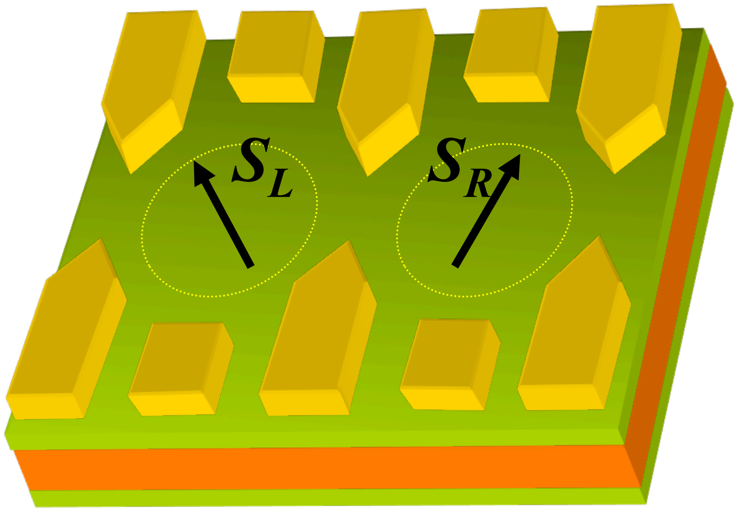
- Strong electric fields easy to produce (gates, STM-tips, etc)
- Fast switching of electric fields (picoseconds)
- Easy to apply electric fields locally and on nanoscale

- Strong magnetic (ac) fields hard to produce
- Slow switching of magnetic fields (nanoseconds)
- Hard to apply magnetic fields locally and on nanoscale

nanoscience

Quantum Processor for Spin-Qubits

DL & DiVincenzo, PRA **57** (1998)



2 quantum dots, each with
1 electron-spin (= qubit)

Key idea:
all-electrical control of spins
➔ scalable nanotechnology

Simple effective Hamiltonian:

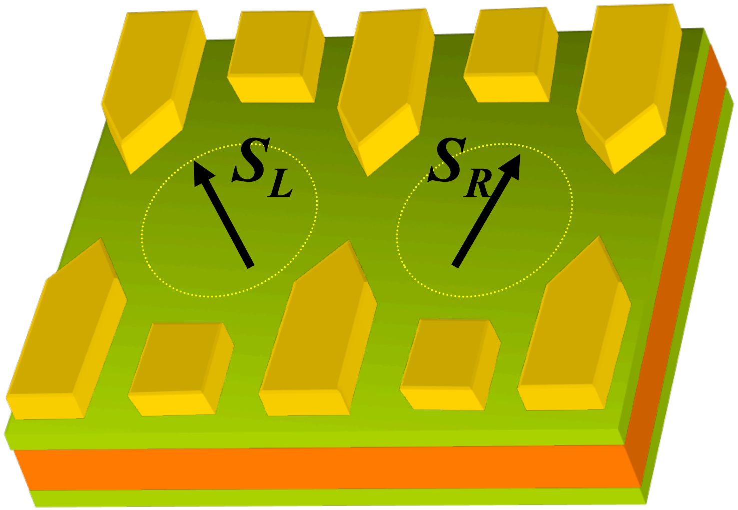
$$H(t) = J(t)\mathbf{S}_1 \cdot \mathbf{S}_2 + \mathbf{b}_1(t)\cdot \mathbf{S}_1 + \mathbf{b}_2(t)\cdot \mathbf{S}_2$$

Exchange coupling

Zeeman couplings

Quantum Processor for Spin-Qubits

DL & DiVincenzo, PRA **57** (1998)



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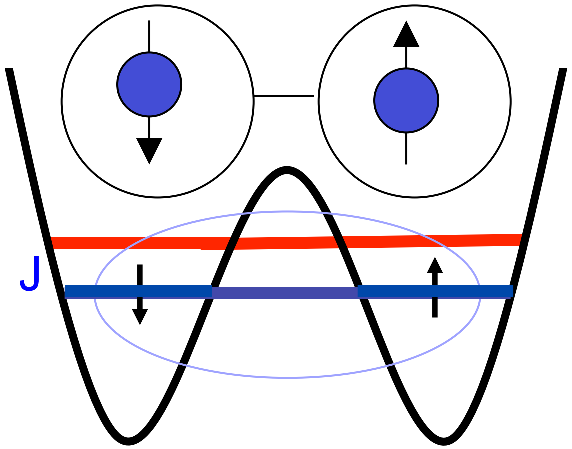


artificial hydrogen molecule ➔ exchange splitting $J \sim t^2/U$

➔ ‘CNOT quantum gate‘

Quantum Dot Molecular Physics

- two coupled dots = artificial ´´H_2 – molecule"
- use approximative methods from molecular physics:

→ Heitler-London (valence orbits) Burkard ea, PRB 59,`99

→ Hund-Mullikan (molecular orbits) Burkard ea, PRB 59,`99

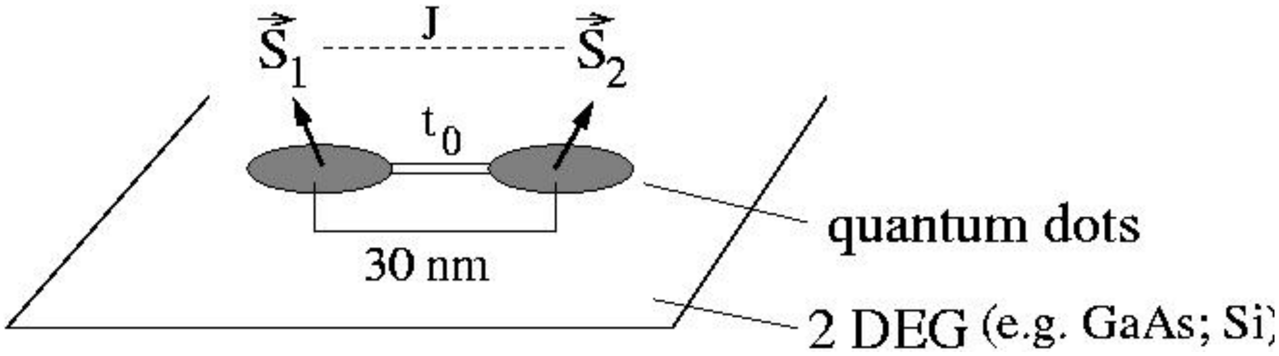
→ large scale numerics: Das Sarma & Hu ´01, Leburton ea ´01, Landman ea `01

- scale:

	artificial atom	real atom
energy	$\approx 1\,\mathrm{meV}$	$1\,\mathrm{Ry} = 13.6\,\mathrm{eV}$
length a_B	$\approx 100 - 500\,\text{Å}$	$0.5\,\text{Å}$

- magnetic length $l_B \approx 100$ Å at $B \approx 1$ T → molecular properties of quantum dots are very **sensitive to magnetic fields B**
- time dependent B field: → $J(t) = J(B(t))$
  electrical gate V(t): → $J(t) = J(V(t))$

quantum gate = two coupled dots



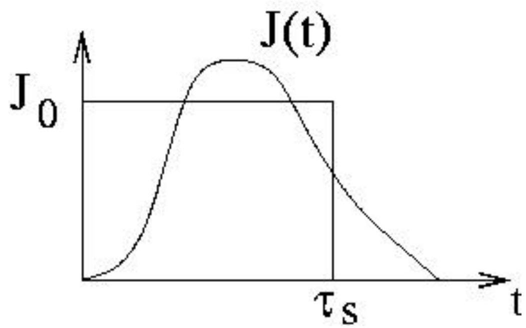
- idea: Hubbard physics: $J(t) \approx 4\, t_0(t)^2/U$ exchange

  $t_0 = t_0(t)$: tunable tunneling barrier

- e.g. swap and square-root-of-swap $U_{SW}^{1/2}$:

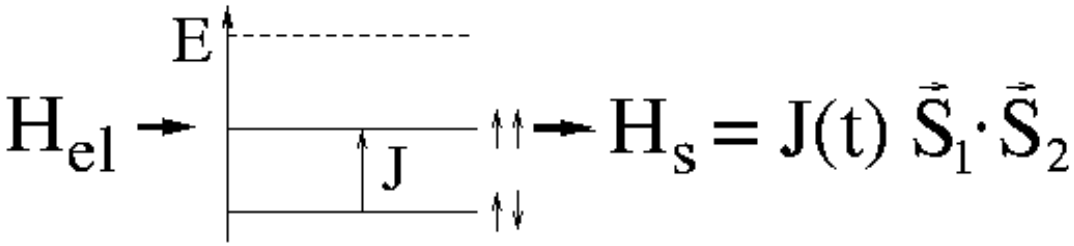
$$\int_0^t J(t')dt'/\hbar \approx J_0 \tau_S/\hbar = \pi/2(\mathrm{mod}\,\pi)$$

note: τ_s = 100 ps << T_2 = 1 ms (GaAs)

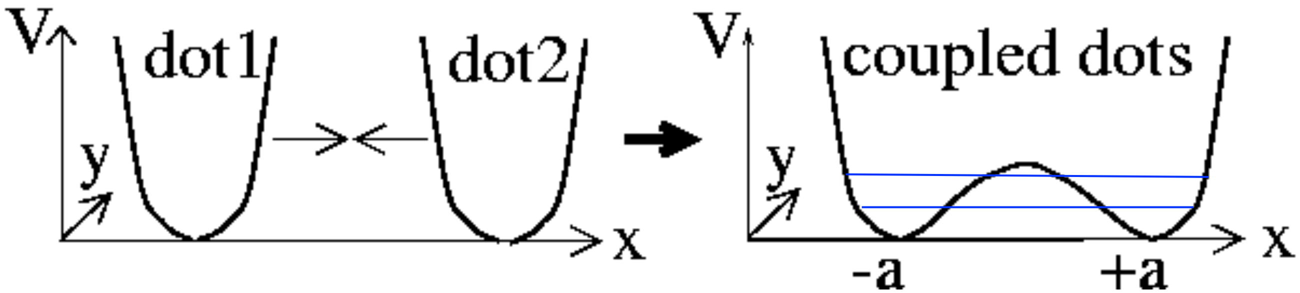


Electronic Model

- how do we find the exchange coupling *J*?



- 2D potential for electrons:



- Hamiltonian for one electron per dot:

$$
\begin{aligned}
H &= \sum_{i=1,2} h_i + \frac{e^2}{\epsilon|\vec{r}_1 - \vec{r}_2|} \\
h_i &= \frac{1}{2m}\left(\vec{p}_i - \frac{e}{c}\vec{A}(\vec{r}_i)\right)^2 + e x_i E + \frac{m\omega^2}{2}\left(\frac{1}{4a^2}\left(x_i^2 - a^2\right)^2 + y_i^2\right) \\
\vec{A}(\vec{r}) &= \frac{B}{2}(-y, x, 0); \qquad \vec{r} = (x, y, 0)
\end{aligned}
$$

I. Heitler-London Method

- single-dot problem in a magnetic field has exact solution (Fock '28,Darwin '30) → $\varphi(\vec{r})$

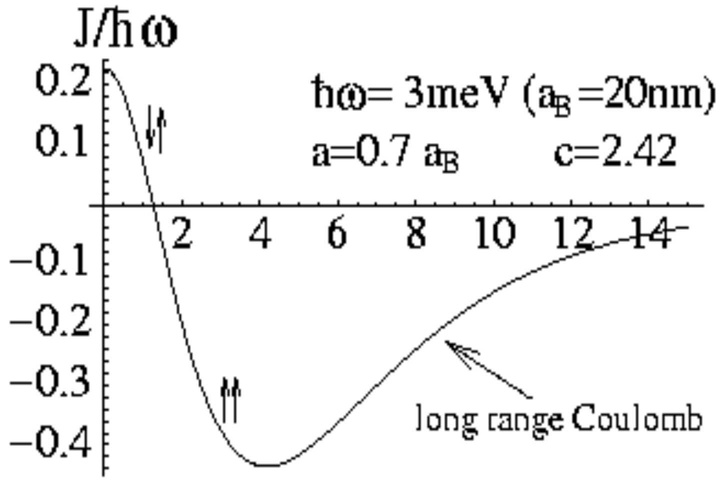
two-particle trial wavefunction (Heitler-London)

$$\begin{aligned} \Psi_{\pm} &= N\left[\varphi_{-a}(\vec{r}_1)\varphi_{+a}(\vec{r}_2) \pm \varphi_{-a}(\vec{r}_2)\varphi_{+a}(\vec{r}_1)\right] \\ J &= \langle \Psi_- | H_{el} | \Psi_- \rangle - \langle \Psi_+ | H_{el} | \Psi_+ \rangle \end{aligned}$$

- results: $d = a/a_B$, $b^2 = 1 + \omega_L^2/\omega_0^2$, $c \sim (e^2/\epsilon a_B)/\hbar\omega_0$, $\omega_L = eB/2m$

(Burkard,Loss,DiVincenzo '99)

$$\begin{aligned} J = \; & \frac{\hbar\omega_0}{\sinh\left(2d^2(2b-1/b)\right)}\Big[c\sqrt{b}\Big(e^{-bd^2}\mathrm{I}_0\left(bd^2\right) \\ & - e^{d^2(b-1/b)}\mathrm{I}_0(d^2(b-1/b))\Big) + \frac{3}{4b}\left(1+bd^2\right)\Big] \end{aligned}$$



- Theorem: $J > 0$ for 2 electrons and $B = 0$.

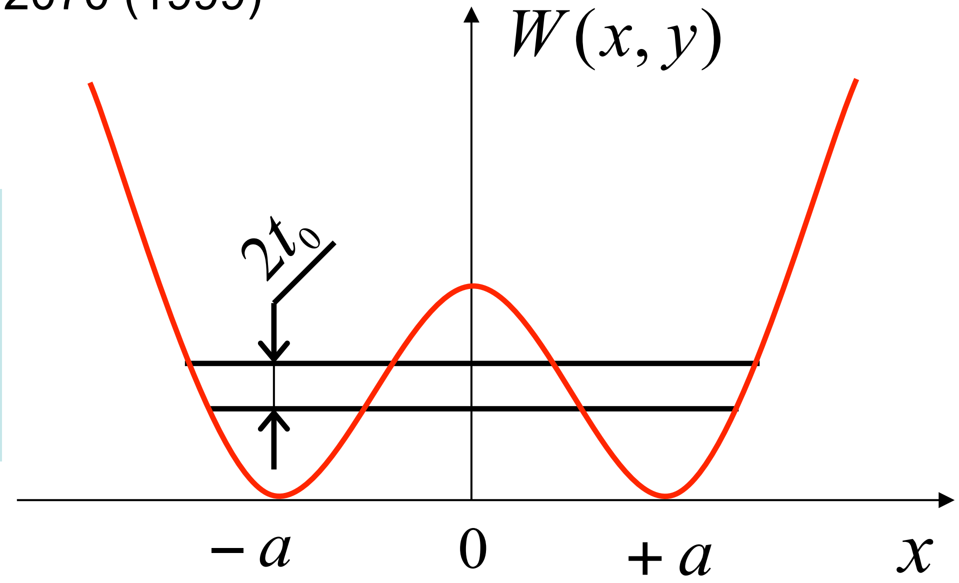
(see also numerics by X. Hu et al., PRB ’00, include higher orbitals)

II. Hund-Mullikan calculation

Burkard, Loss, DiVincenzo, PRB **59**, 2070 (1999)

- confinement is approximated by a quartic potential, with typically $\hbar\omega_0$=3 meV

$$W(x,y)=\frac{m\omega_0^2}{2}\left(\frac{\left(x^2-a^2\right)^2}{4a^2}+y^2\right)$$



- separates into two harmonic wells, if

$$a >> a_B \equiv \sqrt{\hbar/m\omega_0}$$

- Hamiltonian (neglecting the Zeeman splitting for GaAs):

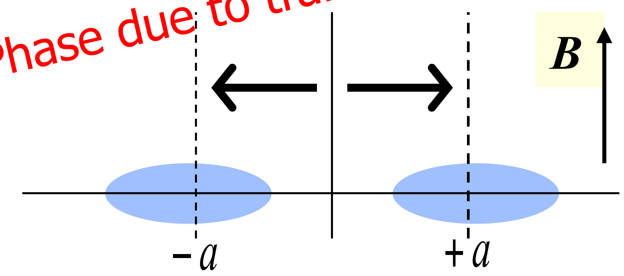
$$H_d=\sum_{i=1,2}\left[\frac{1}{2m}\left(\mathbf{p}_i-\frac{e}{c}\mathbf{A}(\mathbf{r}_i)\right)^2+W(\mathbf{r}_i)\right]+\frac{e^2}{\kappa\left|\mathbf{r}_1-\mathbf{r}_2\right|}$$

- gauge: $\mathbf{A}=\left(-yB/2,\, xB/2,0\right)\quad\Rightarrow\quad B\|z$

• Fock-Darwin states (Fock '28;Darwin '30), translated by $\pm a$ in presence of magnetic field B (Burkard,Loss,DiVincenzo '99):

Phase due to translation

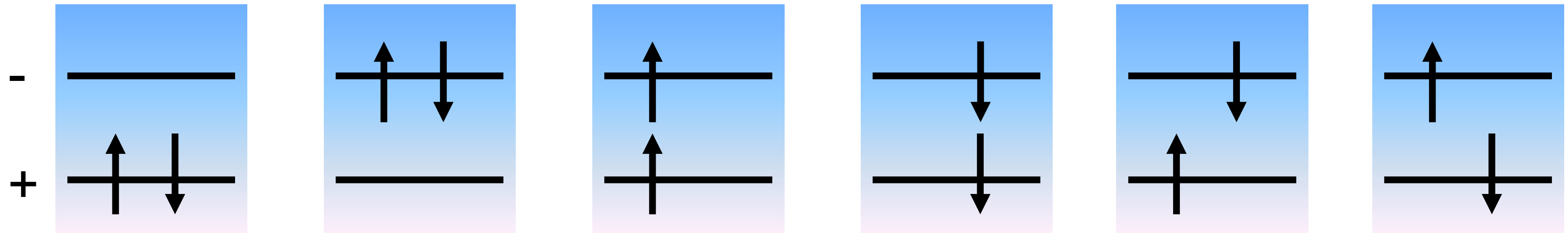
$$\varphi_{\pm a}(x,y) = \frac{1}{\lambda\sqrt{\pi}} \exp\left[-\frac{(x \mp a)^2 + y^2}{2\lambda^2} \mp \frac{iya}{2l^2} \right]$$



$$l = \sqrt{\frac{\hbar c}{|e|B}} \qquad \lambda = \sqrt{\frac{\hbar}{m\omega}} \qquad \omega = \sqrt{\omega_0^2 + \omega_L^2} \qquad \omega_L = \frac{|e|B}{2mc}$$

$$d_{\pm,\sigma}: \quad \psi_{\pm,\sigma} = \chi_\sigma \left(\varphi_{-a} \pm \varphi_{+a}\right) / \sqrt{2(1 \pm S)} \qquad S = \langle \varphi_{\pm a} | \varphi_{\mp a} \rangle$$

• two-particle states: ➔ *6 possible configurations*



J. Schliemann, D. Loss, and A. H. MacDonald, Phys. Rev. B **63**, 085311 (2001)
V. Golovach and D. Loss, Europhys. Lett. 62, 83 (2003)

Lowest energy eigenstates of DD:

$$|S1\rangle = \frac{1}{\sqrt{2}}\left(d^\dagger_{-\uparrow}d^\dagger_{+\downarrow} - d^\dagger_{-\downarrow}d^\dagger_{+\uparrow}\right)|0\rangle$$

$$|S2\rangle = \frac{1}{\sqrt{1+\phi^2}}\left(\phi\, d^\dagger_{+\uparrow}d^\dagger_{+\downarrow} + d^\dagger_{-\uparrow}d^\dagger_{-\downarrow}\right)|0\rangle$$

$$|T_+\rangle = d^\dagger_{-\uparrow}d^\dagger_{+\uparrow}|0\rangle, \quad |T_-\rangle = d^\dagger_{-\downarrow}d^\dagger_{+\downarrow}|0\rangle$$

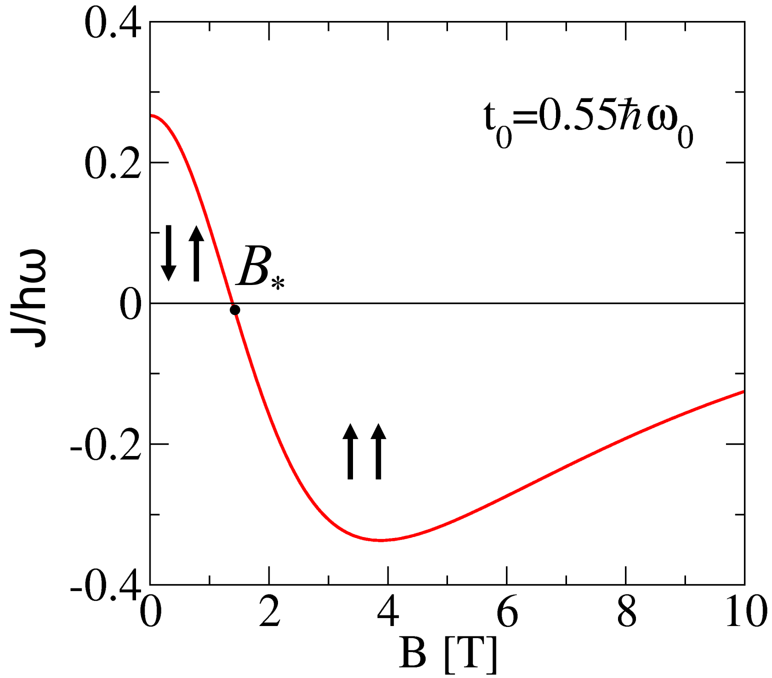
$$|T_0\rangle = \frac{1}{\sqrt{2}}\left(d^\dagger_{-\uparrow}d^\dagger_{+\downarrow} + d^\dagger_{-\downarrow}d^\dagger_{+\uparrow}\right)|0\rangle$$

$$|S\rangle = \frac{1}{\sqrt{1+\phi^2}}\left(d^\dagger_{+\uparrow}d^\dagger_{+\downarrow} - \phi\, d^\dagger_{-\uparrow}d^\dagger_{-\downarrow}\right)|0\rangle$$

$$\phi = \sqrt{1+\left(\frac{4t_H}{U_H}\right)^2} - \frac{4t_H}{U_H}$$

Exchange:

$$J = \mathrm{v} - \frac{U_H}{2} + \frac{1}{2}\sqrt{U_H^2 + 16t_H^2}$$



concurrence: $c[|S\rangle] = \dfrac{2\phi}{1+\phi^2}$

double occupancy: $D[|S\rangle] = \dfrac{(1-\phi)^2}{2(1+\phi^2)^2}$

Burkard et al. '99; Schliemann et al. '00, Golovach et al. '03

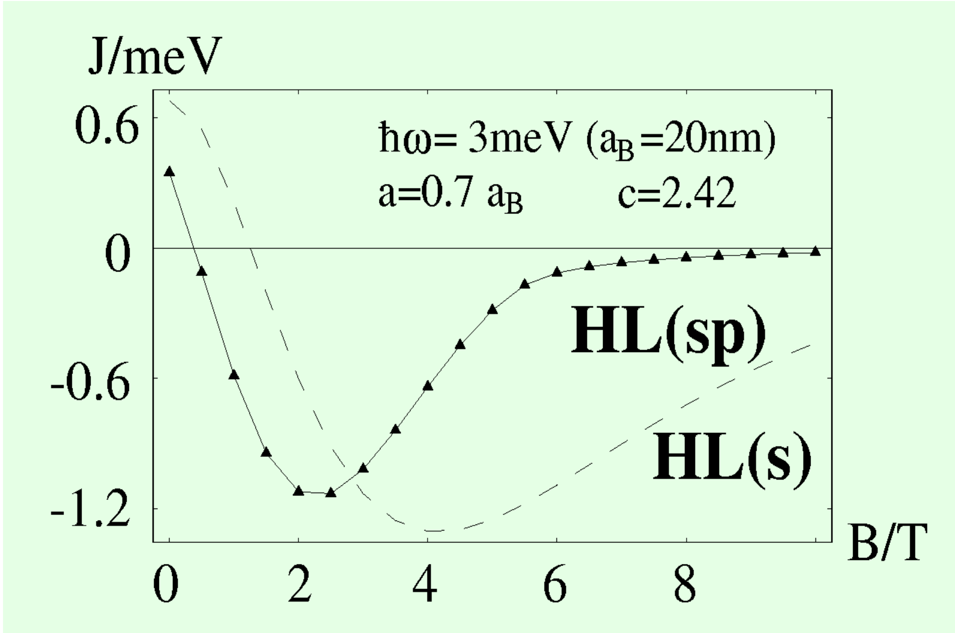
Lateral Coupling (GaAs dots)

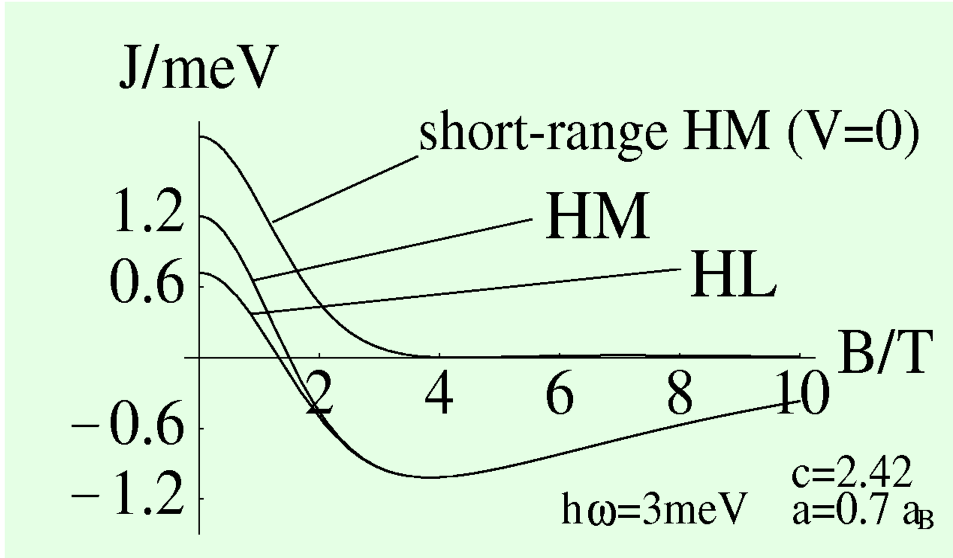
- **extended** Hubbard physics:

$$J = \frac{4t^2}{U} + V(B)$$

- note: HM→HL for increasing on-site Coulomb repulsion, i.e.

$$U(B,c) \sim c\sqrt{B} \nearrow$$





Sources of spin decoherence in GaAs quantum dots:

• spin-orbit interaction (band structure effects):
couples lattice vibrations with spin ➔ spin-phonon interaction, but weak in quantum dots due to 1. low momentum, 2. no 1st order s-o terms due to confinement (Khaetskii&Nazarov, '00; Golovach et al., '04-'10)

• spin-orbit intercation ➔ gate errors (XOR); but they can be minimized (Bonesteel et al., Burkard et al., '02, '03)

• dipole-dipole interaction: weak

• hyperfine interaction with nuclear spins: dominant decoherence source (Burkard, DL, DiVincenzo, PRB '99; Coish &DL, 2004-10), but absent e.g. in Si/Ge based dots!

Swichting Rate

Determine $N_{Op} \approx \tau_{\phi} / \tau_s$ for GaAs

- calculate *J*(*v*) **statically** and then take *J*(*t*) = *J*(*v*(*t*)) for time-dependent *v*(*t*), where *v* = *V*, *B*, *a*, *E* is control parameter
- sufficient criterion for this to work [$\bar{J} = (1/\tau_S) \int_0^{\tau_S} dt\, J(t)$]

  $1/\tau_S \approx |\dot{v}/v| \ll \bar{J}/\hbar$ **adiabaticity condition**

- compatible with $J\tau_S = n\pi$, $n = 1, 3, 5, \ldots$ (needed for XOR)
- self-consistency of calculation of *J*: $J \ll \Delta\epsilon$
- thus: $1/\tau_S \ll \bar{J}/\hbar \ll \Delta\epsilon/\hbar$ $, \pi U^2 / 8t_0$ (no double occupancy)
- numbers: $J \approx 0.2\,\mathrm{meV} \rightarrow$ $\tau_S \gtrsim 50\,\mathrm{ps}$ ➔ very fast gate
- decoherence of spin ca. 100 μs

➡ $N_{Op} \approx \tau_{\varphi} / \tau_s \approx 10^6$ sufficient for upscaling

Quantum XOR (CNOT) via Hamiltonian

$$U(t) = T\exp\left\{-\frac{i}{\hbar}\int_0^t H(t')\,dt'\right\},\ H \neq 0 \text{ during } \tau_s$$

can show that: (Loss+DiVincenzo, PRA **57** (120), 1998)

$$U_{\mathrm{XOR}} \leftrightarrow \int^t H_{\mathrm{XOR}} = \pi S_1^z S_2^z - \frac{\pi}{2}(S_1^z + S_2^z) \qquad s = \frac{1}{2}$$

H_{XOR} is **pure** Ising: not very physical (for **real** spin) !
instead use **Heisenberg** $H = J\mathbf{S}_1 \cdot \mathbf{S}_2$ for U_{XOR}
and **Zeeman** $H_B = \mathbf{B}_1 \cdot \mathbf{S}_1 + \mathbf{B}_2 \cdot \mathbf{S}_2$ for single-qubit operations

e.g. swap gate: qubit 1$\longleftrightarrow$qubit 2, choose $\int^t J(t)/\hbar \approx J_0\tau_s/\hbar = \pi \pmod{2\pi}$

$$\Rightarrow U(t) = e^{i\pi/4}U_{\mathrm{sw}} = e^{i\pi/4}\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

basis: $\{|\uparrow\uparrow\rangle, |\uparrow\downarrow\rangle, |\downarrow\uparrow\rangle, |\downarrow\downarrow\rangle\}$

$$U_{\mathrm{XOR}} = e^{i\frac{\pi}{2}S_1^z}\ e^{-i\frac{\pi}{2}S_2^z}\ U_{\mathrm{sw}}^{\frac{1}{2}}\ e^{i\pi S_1^z}\ U_{\mathrm{sw}}^{\frac{1}{2}}$$ (DL+DDV '97)

Entanglement with 'sqrt-of-swap'

$$U_{XOR} = e^{i(\pi/2)S_1^z} e^{-i(\pi/2)S_2^z} U_{SW}^{1/2} e^{+i\pi S_1^z} U_{SW}^{1/2},$$

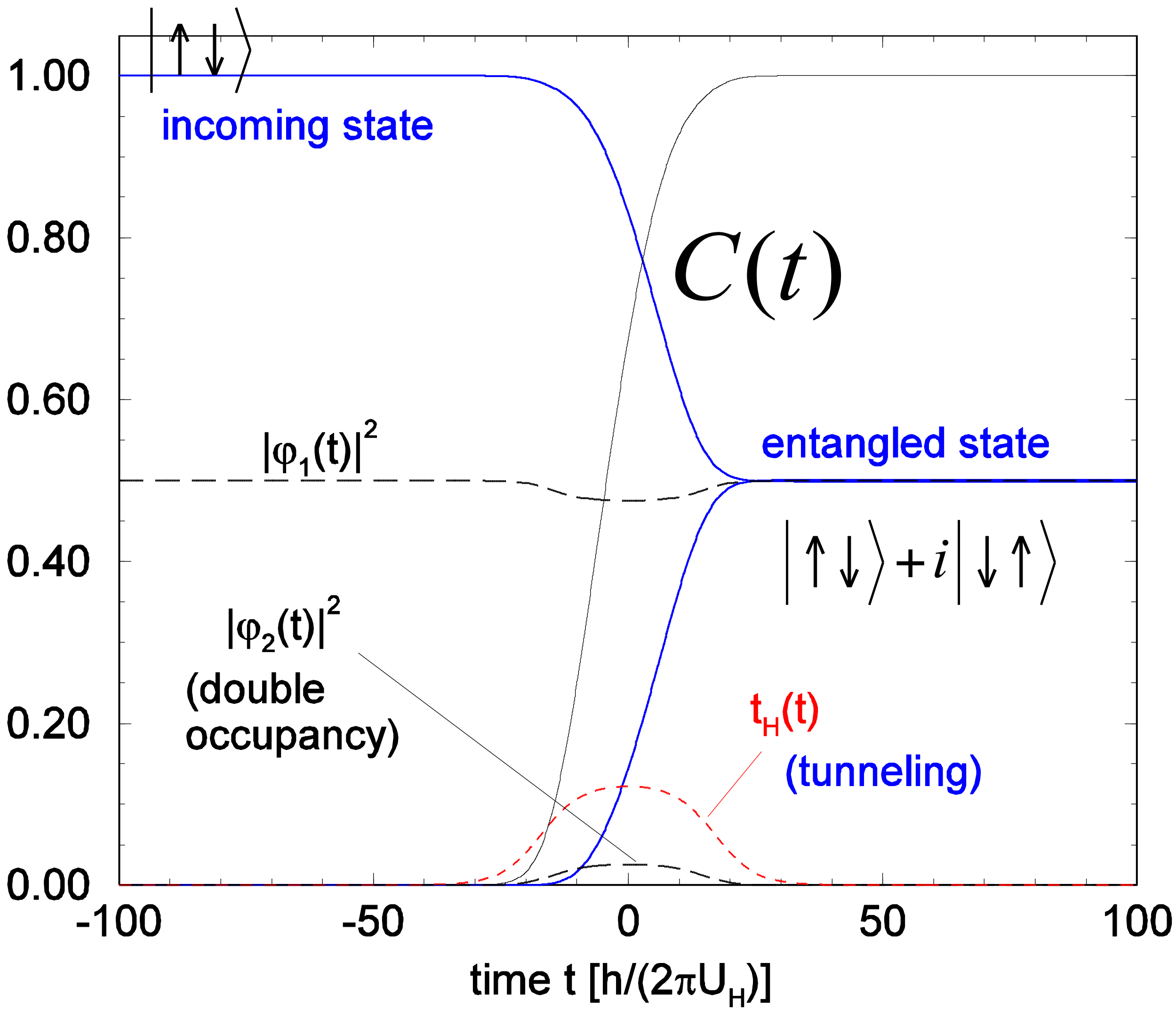
Square-root-of-swap: $|\uparrow\downarrow\rangle \xrightarrow{U_{SW}^{1/2}} |\uparrow\downarrow\rangle + i\,|\downarrow\uparrow\rangle$

$= |\uparrow\rangle_1 \times |\downarrow\rangle_2$

= entangler: product state ➔ entangled state

➔ Entanglement is crucial for quantum computing!

Dynamics of Entanglement for the square-root-of-swap



The square-root of a swap is obtained by halfing the duration of the tunneling pulse. The result is a fully entangled two-qubit state having only a vanishingly small amplitude for double-occupancies of one of the dots.
During the process the indistinguishable character of the electrons and their fermionic statistics are essential.

J. Schliemann, D. Loss, and A. H. MacDonald, Phys. Rev. B **63**, 085311 (2001)

Quantum XOR gate (DL & DDV `97)

$$U_{XOR} = e^{i\frac{\pi}{2}S_1^z} e^{-i\frac{\pi}{2}S_2^z} U_{SW}^{1/2} e^{+i\pi S_1^z} U_{SW}^{1/2},$$

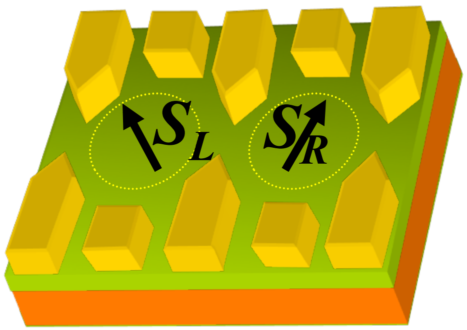
	$\lvert\uparrow\uparrow\rangle$	$\lvert\uparrow\downarrow\rangle$	$\lvert\downarrow\uparrow\rangle$	$\lvert\downarrow\downarrow\rangle$
$U_{SW}^{1/2}$	$\Downarrow$	$\Downarrow$	$\Downarrow$	$\Downarrow$
	$\lvert\uparrow\uparrow\rangle$	$\frac{e^{-i\pi/4}}{\sqrt{2}}(\lvert\uparrow\downarrow\rangle+i\lvert\downarrow\uparrow\rangle)$	$\frac{e^{-i\pi/4}}{\sqrt{2}}(i\lvert\uparrow\downarrow\rangle+\lvert\downarrow\uparrow\rangle)$	$\lvert\downarrow\downarrow\rangle$
$e^{i\pi S_1^z}$	$\Downarrow$	$\Downarrow$	$\Downarrow$	$\Downarrow$
	$i\lvert\uparrow\uparrow\rangle$	$\frac{ie^{-i\pi/4}}{\sqrt{2}}(\lvert\uparrow\downarrow\rangle-i\lvert\downarrow\uparrow\rangle)$	$\frac{ie^{-i\pi/4}}{\sqrt{2}}(i\lvert\uparrow\downarrow\rangle-\lvert\downarrow\uparrow\rangle)$	$-i\lvert\downarrow\downarrow\rangle$
$U_{SW}^{1/2}$	$\Downarrow$	$\Downarrow$	$\Downarrow$	$\Downarrow$
	$i\lvert\uparrow\uparrow\rangle$	$\lvert\uparrow\downarrow\rangle$	$-\lvert\downarrow\uparrow\rangle$	$-i\lvert\downarrow\downarrow\rangle$
$e^{-i\frac{\pi}{2}S_2^z}$	$\Downarrow$	$\Downarrow$	$\Downarrow$	$\Downarrow$
$\times e^{i\frac{\pi}{2}S_1^z}$	$i\lvert\uparrow\uparrow\rangle$	$i\lvert\uparrow\downarrow\rangle$	$i\lvert\downarrow\uparrow\rangle$	$-i\lvert\downarrow\downarrow\rangle$

How to make entanglement 'visible'

Loss & DiVincenzo, 1998

$$|\downarrow\uparrow\rangle - i\,|\uparrow\downarrow\rangle$$

$|\downarrow\uparrow\rangle$

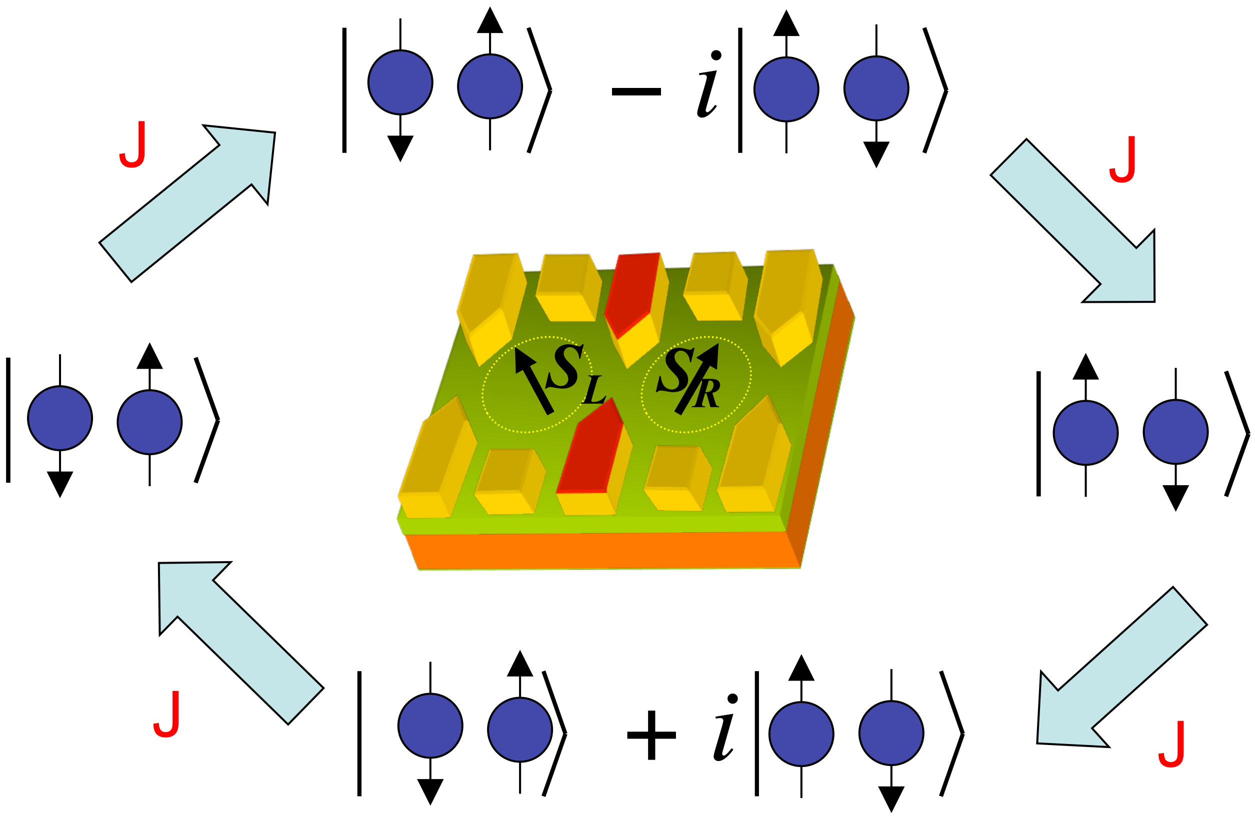


$|\uparrow\downarrow\rangle$

$$|\downarrow\uparrow\rangle + i\,|\uparrow\downarrow\rangle$$

How to make entanglement ‘visible’

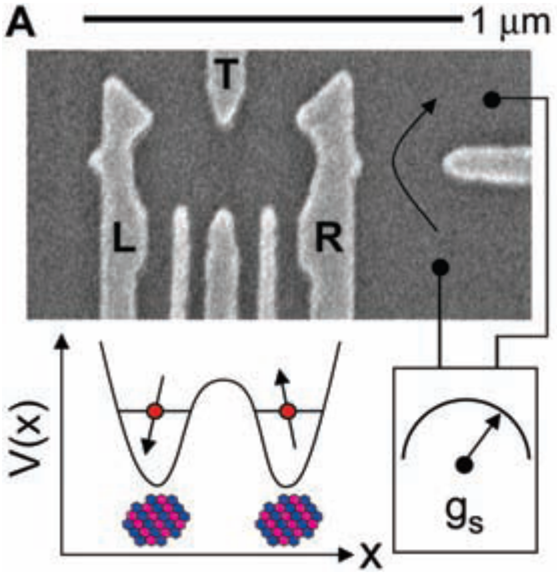
Loss & DiVincenzo, 1998

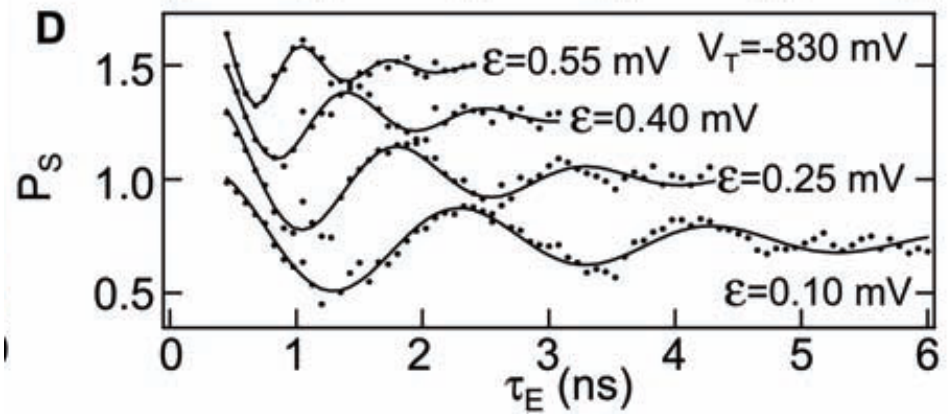


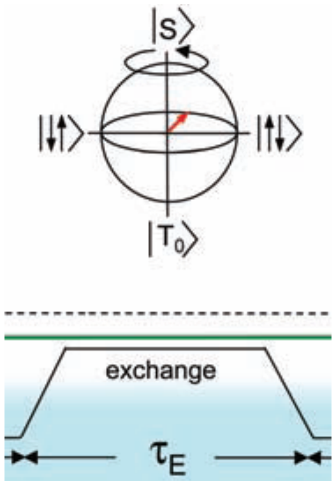
➔ entanglement oscillates !

Experiment: Entanglement oscillations

Petta et al., Science 2005



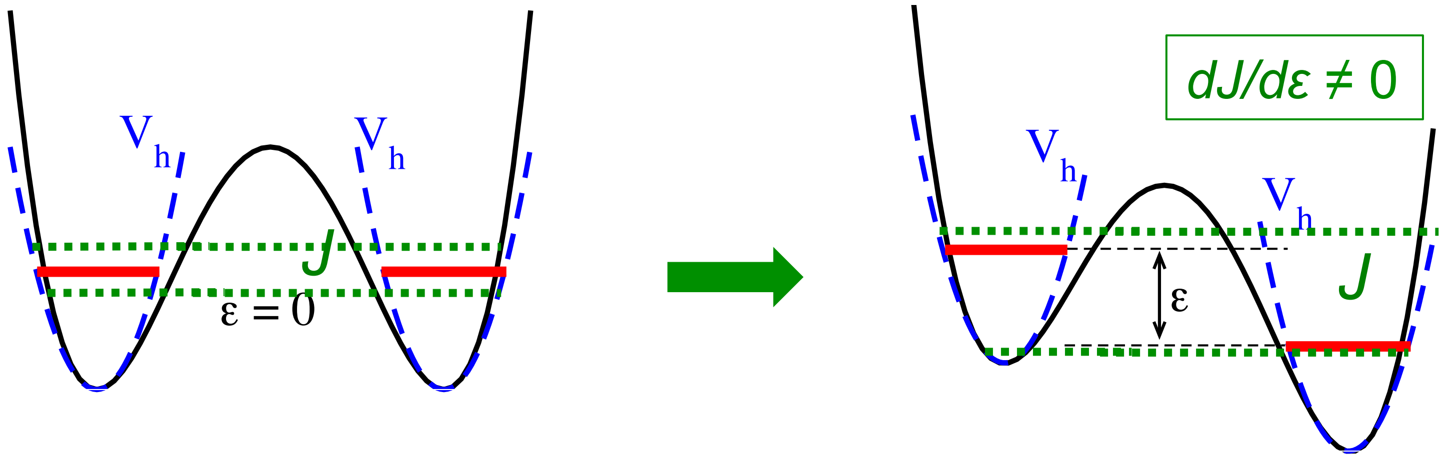




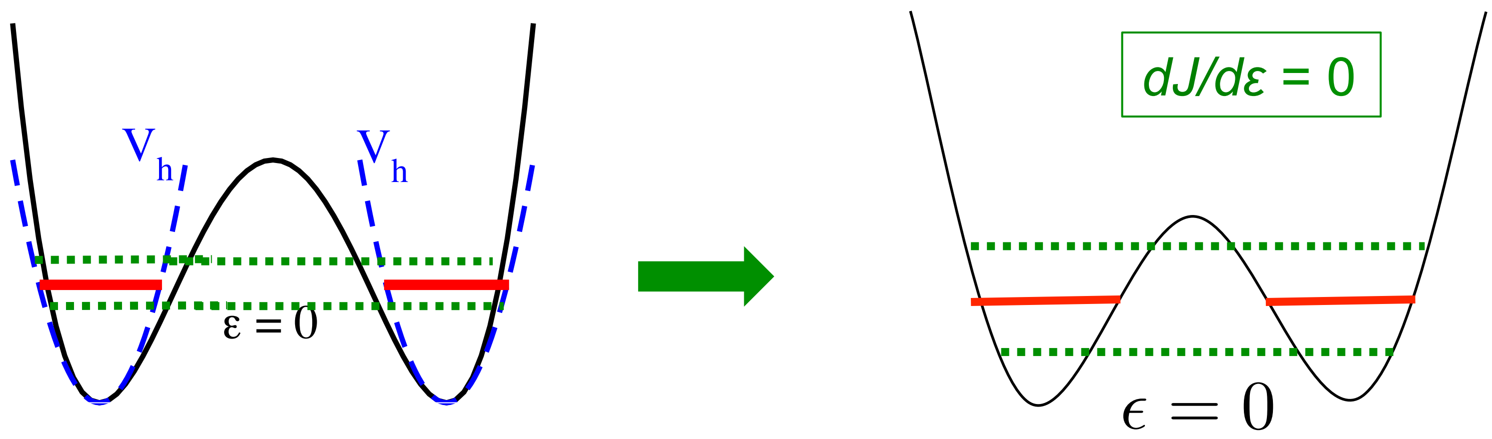
ultra-fast 'clock speed' :
entanglement generated in 180 ps !

Switching of exchange *J*

1. *Asymmetric* via bias ε



2. *Symmetric* via barrier height



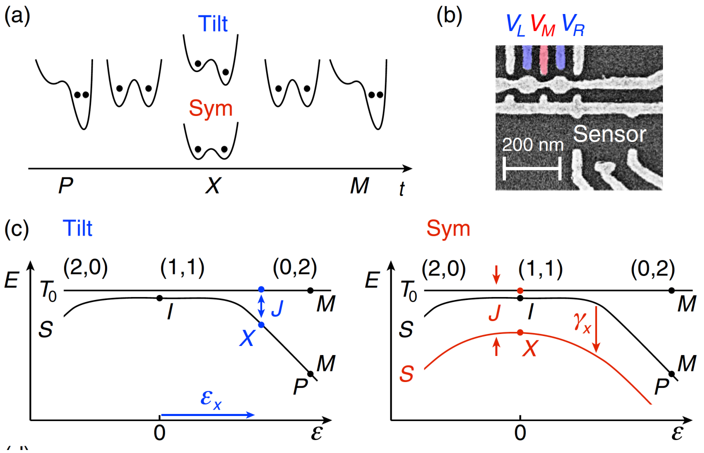
Burkard, Loss, and DiVincenzo, PRB (1999)

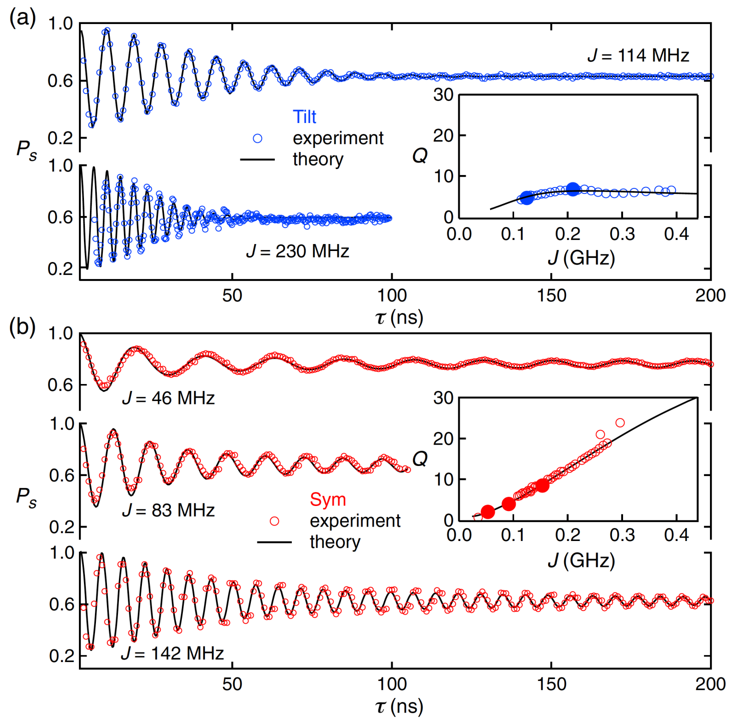
PRL **116**, 116801 (2016) PHYSICAL REVIEW LETTERS week ending 18 MARCH 2016

Noise Suppression Using Symmetric Exchange Gates in Spin Qubits

Frederico Martins,[1] Filip K. Malinowski,[1] Peter D. Nissen,[1] Edwin Barnes,[2,3] Saeed Fallahi,[4] Geoffrey C. Gardner,[5] Michael J. Manfra,[4,5,6] Charles M. Marcus,[1] and Ferdinand Kuemmeth[1,*]
[1]*Center for Quantum Devices, Niels Bohr Institute, University of Copenhagen, 2100 Copenhagen, Denmark*

Sqrt-of-Swap gate via J





asymmetric

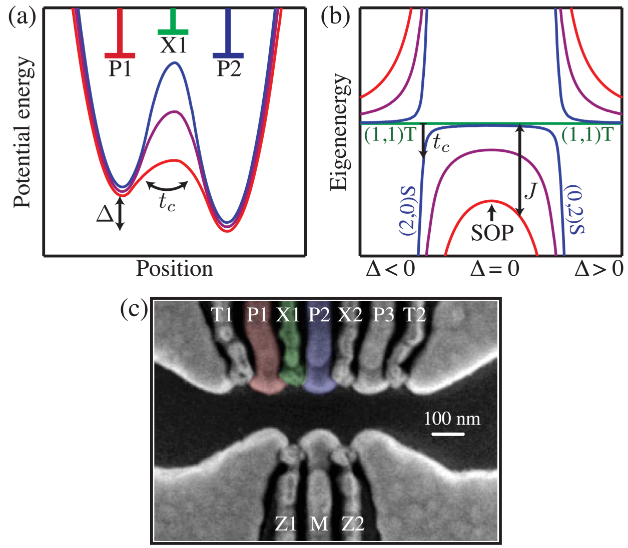
symmetric

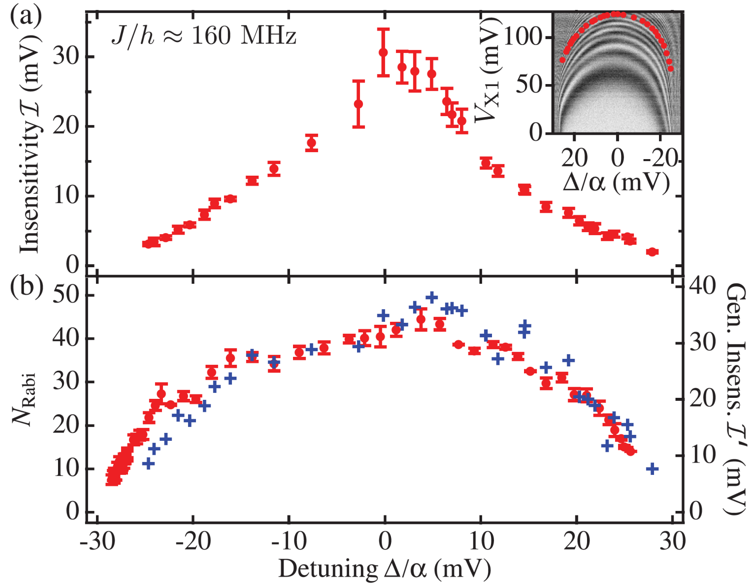
FIG. 3. (a) Tilt-induced exchange oscillations (i.e., $\gamma_x = 0$ mV) for $\varepsilon_x = 79.5$ and 82 mV, generating oscillation frequencies indicated by J. (b) Same as (a) but for the symmetric mode of operation ($\varepsilon_x = 13.5$ mV), with $\gamma_x = 100$, 120, and 140 mV.

Reduced Sensitivity to Charge Noise in Semiconductor Spin Qubits via Symmetric Operation

M. D. Reed, B. M. Maune, R. W. Andrews, M. G. Borselli, K. Eng, M. P. Jura, A. A. Kiselev, T. D. Ladd, S. T. Merkel, I. Milosavljevic, E. J. Pritchett, M. T. Rakher, R. S. Ross, A. E. Schmitz, A. Smith, J. A. Wright, M. F. Gyure, and A. T. Hunter*
HRL Laboratories, LLC, 3011 Malibu Canyon Road, Malibu, California 90265, USA
(Received 5 August 2015; published 16 March 2016)

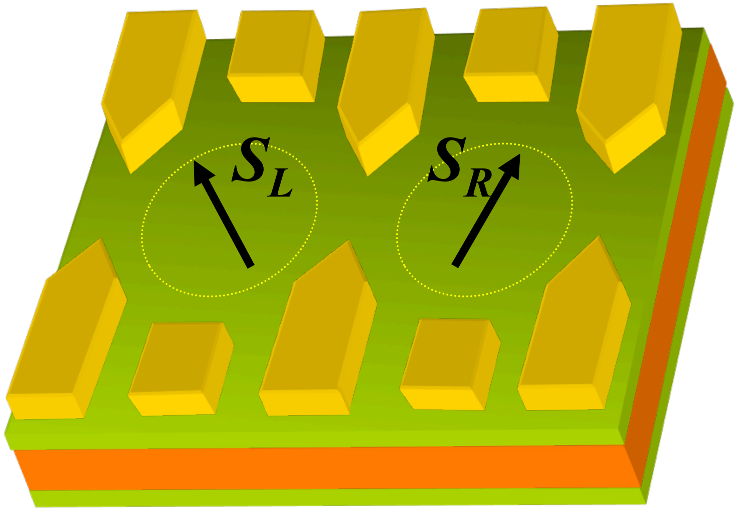
Si/Ge quantum dots





Quantum Processor for Spin-Qubits

DL & DiVincenzo, PRA **57** (1998)



$$H(t) = J(t)\mathbf{S}_1 \cdot \mathbf{S}_2 + \mathbf{b}_1(t)\cdot \mathbf{S}_1 + \mathbf{b}_2(t)\cdot \mathbf{S}_2$$

➔ CNOT (XOR) gate

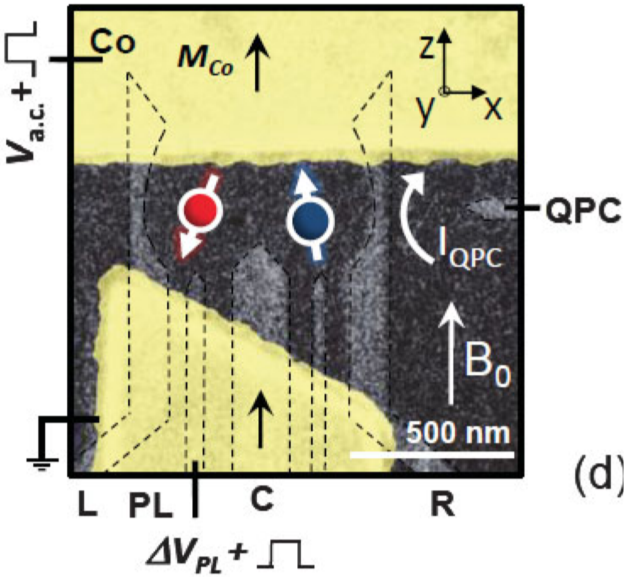
$$\boxed{U_{XOR} = e^{i\frac{\pi}{2}S_1^z}\, e^{-i\frac{\pi}{2}S_2^z}\, U_{SW}^{1/2}\, e^{i\pi S_1^z}\, U_{SW}^{1/2}}$$

$$U_{SW} : \uparrow\downarrow \;\Rightarrow\; \downarrow\uparrow$$

sqrt-of-swap: $U_{SW}^{1/2} : \uparrow\downarrow \;\Rightarrow\; \uparrow\downarrow + e^{i\alpha}\downarrow\uparrow$

Quantum Processor for Spin-Qubits

DL & DiVincenzo, PRA **57** (1998)



Brunner *et al.*
Tarucha group
PRL 2011

"Two-Qubit Gate of Combined Single-Spin Rotation and Interdot Spin Exchange in a Double Quantum Dot"

$$H(t) = J(t)\mathbf{S}_1 \cdot \mathbf{S}_2 + \mathbf{b}_1(t) \cdot \mathbf{S}_1 + \mathbf{b}_2(t) \cdot \mathbf{S}_2$$

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Petta *et al.*, Science, 2005: 180 ps
Brunner *et al.*, PRL 2011: 5 ns

Serial vs. Parallel gate

I. Serial gate: LD, PRA 57, 120 (1998)

$$H(t) = J(t)\mathbf{S}_1 \cdot \mathbf{S}_2 + \mathbf{b}_1(t)\cdot \mathbf{S}_1 + \mathbf{b}_2(t)\cdot \mathbf{S}_2$$ controlled such that

$$H(t) = J(t)\mathbf{S}_1 \cdot \mathbf{S}_2$$ or $$H(t) = \mathbf{b}_1(t)\cdot \mathbf{S}_1 + \mathbf{b}_1(t)\cdot \mathbf{S}_2$$

$$U_{SW}^{1/2} = \exp(i\int_0^{\tau_s} dt J(t)\mathbf{S}_1 \cdot \mathbf{S}_2) \;, \quad if \int_0^{\tau_s} dt J(t) = \pi/2 + 2\pi n$$

$$U_{XOR} = e^{-i(\pi/2)S_2^y}[e^{i(\pi/2)S_1^z} e^{-i(\pi/2)S_2^z} U_{SW}^{1/2} e^{+i\pi S_1^z} U_{SW}^{1/2}] e^{i(\pi/2)S_2^y}$$

➔ need 7 pulses (5 for CPF)

Serial vs. Parallel gate

II. Parallel gate: Burkard et al., PRB 60, 11404 (1999)

$$H(t) = J(t)\mathbf{S}_1 \cdot \mathbf{S}_2 + \mathbf{b}_1(t)\cdot\mathbf{S}_1 + \mathbf{b}_2(t)\cdot\mathbf{S}_2$$

$$U_{CPF} = \exp(i\int_0^{\tau_s} dt H(t))$$

only 1 pulse for CPF !

$$if \quad \int J = \pi/2, \; and \; \int b^z_{1/2} = \pi(1 \pm \sqrt{3})/4$$

$$U_{XOR} = e^{-i(\pi/2)S_2^y} U_{CPF} e^{i(\pi/2)S_2^y}$$

➔ need only 3 pulses

Implementation scheme: Meunier *et al.*, PRB 83, 121403 (2011)

Single-Qubit Operations or *How to Flip a Spin*?

1. Electron Spin Resonance (ESR)

An ac magnetic field is applied perpendicular to a static magnetic field, with a frequency that matches the Zeeman splitting

2. Electric-Dipole-Induced Spin Resonance (EDSR)

Exploits spin-orbit interaction and ac electric field

3. Electrically Driven ESR in a Slanting Magnetic Field

Exploits a magnetic field gradient and ac electric field

4. Electrically Driven ESR in an exchange field of auxilliary spin

Exploits the exchange field, magnetic field gradient, and ac electric field

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Exploits a magnetic field gradient and **ac electric field**

4. Electrically Driven ESR in an exchange field of auxilliary spin

Exploits the exchange field, magnetic field gradient, and **ac electric field**

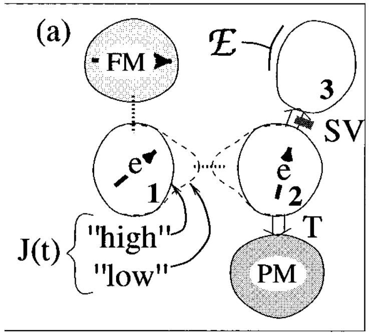
Fast

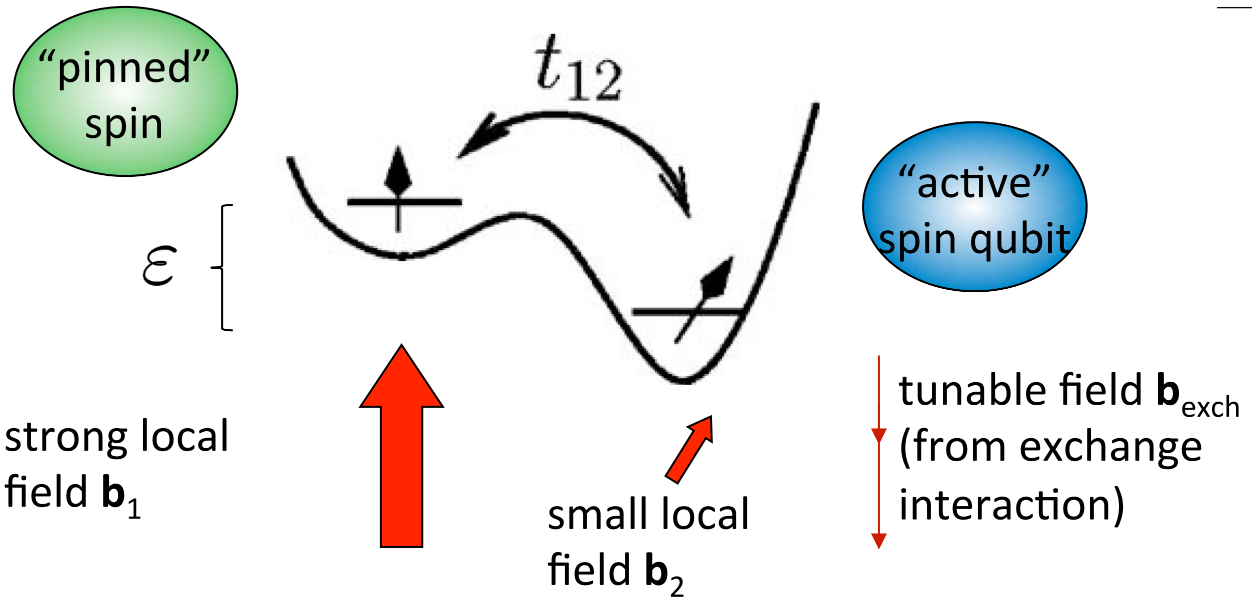
GHz

Ultra-fast single-qubit gates via ***exchange***

Alternative to ESR/EDSR: double dot with pulsed J-gate!

DL and DiVincenzo, PRA 57 (1998)
Coish and DL, PRB 75, 161302 (2007)
Chesi, Wang, Yoneda, Otsuka, Tarucha, and DL, PRB 90, 235311 (2014)



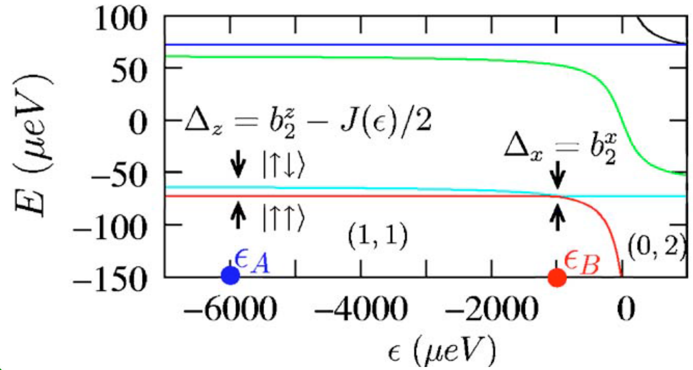


Single-qubit gates with high fidelity and ultrafast ~ 1 ns

Single-spin rotation via exchange: Two regimes

I. $\boxed{b_1 \gg b_2}$

Advantage: hybridization of logical states and (1,1) charge configuration can be made very small; but difficult to reach



Coish & DL, PRB 2007

II. $\boxed{b_1 \simeq b_2 \simeq B}$

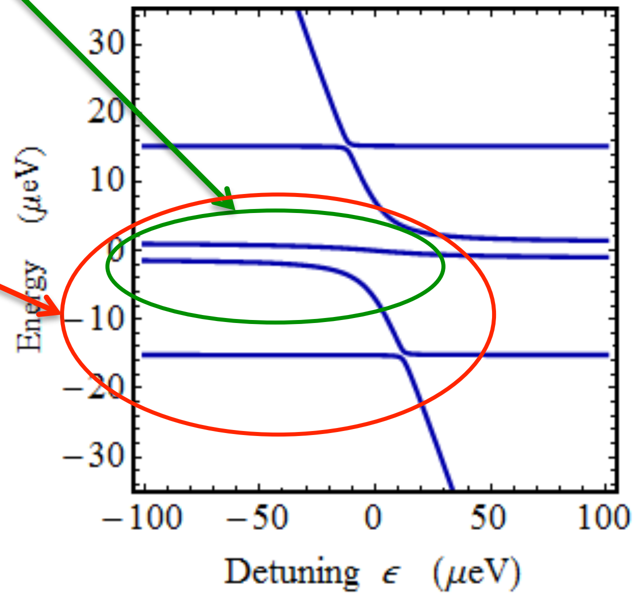
This is a more typical situation in exp.:

$$B \gtrsim 1\ \mathrm{T}$$

(due to saturation field of micromagnet)

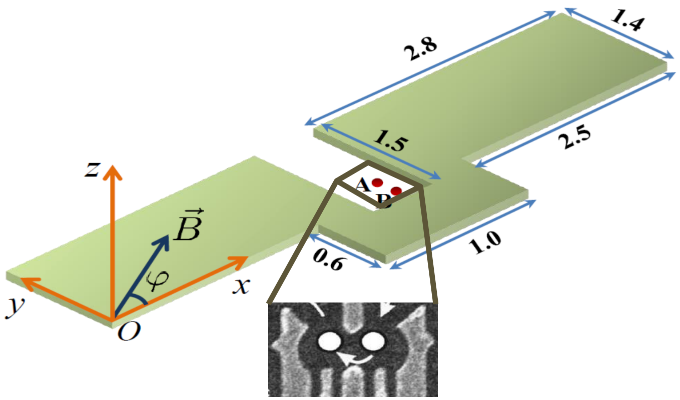
$$\Delta b = b_1 - b_2 \simeq 10 - 50\ \mathrm{mT}$$

Ultra-short gate times: 1 ns, with very high fidelity for GaAs double dots



Chesi et al., PRB 2014

Single-spin manipulation in double dots with micromagnet

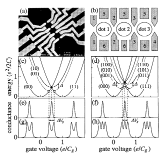


Chesi et al., PRB **90**, 235311 (2014)

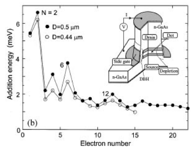
- Single-qubit gates implemented via exchange (as for two-qubit gates)
- Ultra-short gate times: 1 ns, with very high fidelity for GaAs double dots
- Noise sources: Nuclear and charge noise present but not a problem

Most Advanced: Spin qubits in **GaAs** quantum dots

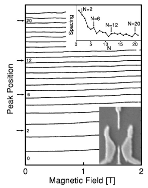
Kloeffel & DL, Annu. Rev. Condens. Matter Phys. 4, 51 (2013)



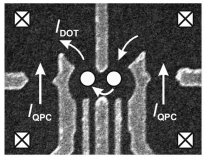
Westervelt **Gossard** 1995



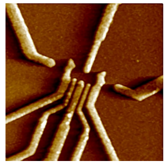
Kouwenhoven Tarucha 1996



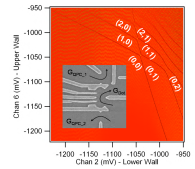
Sachrajda 2000



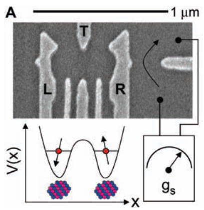
Kouwenhoven Tarucha 2003-13



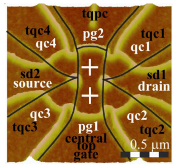
Vandersypen, Koppens, 2003



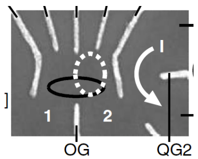
Marcus 2004



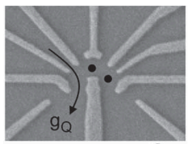
Petta, Marcus, Yacoby 2005



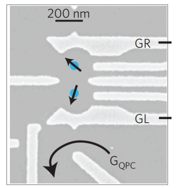
Ensslin, Ihn 2006



Zumbuhl, Kastner 2008

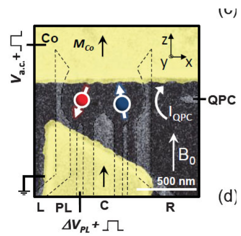


Petta 2010



Bluhm, Dial, Yacoby 2010-13

T_2 ~ 300 μs

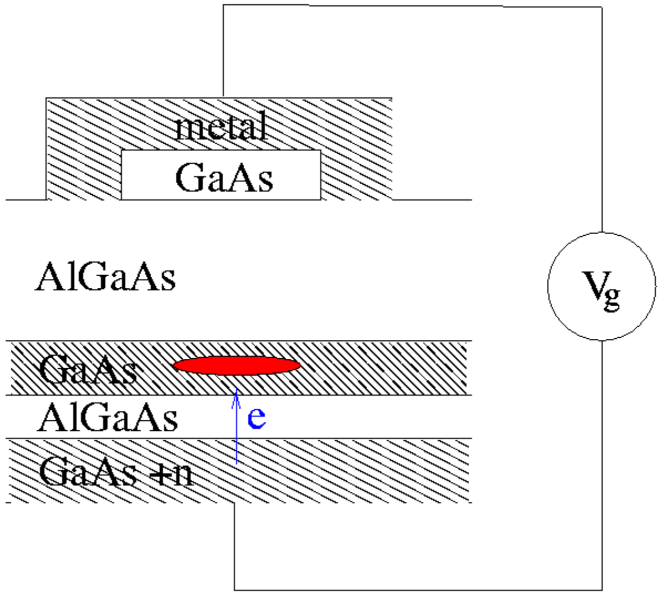


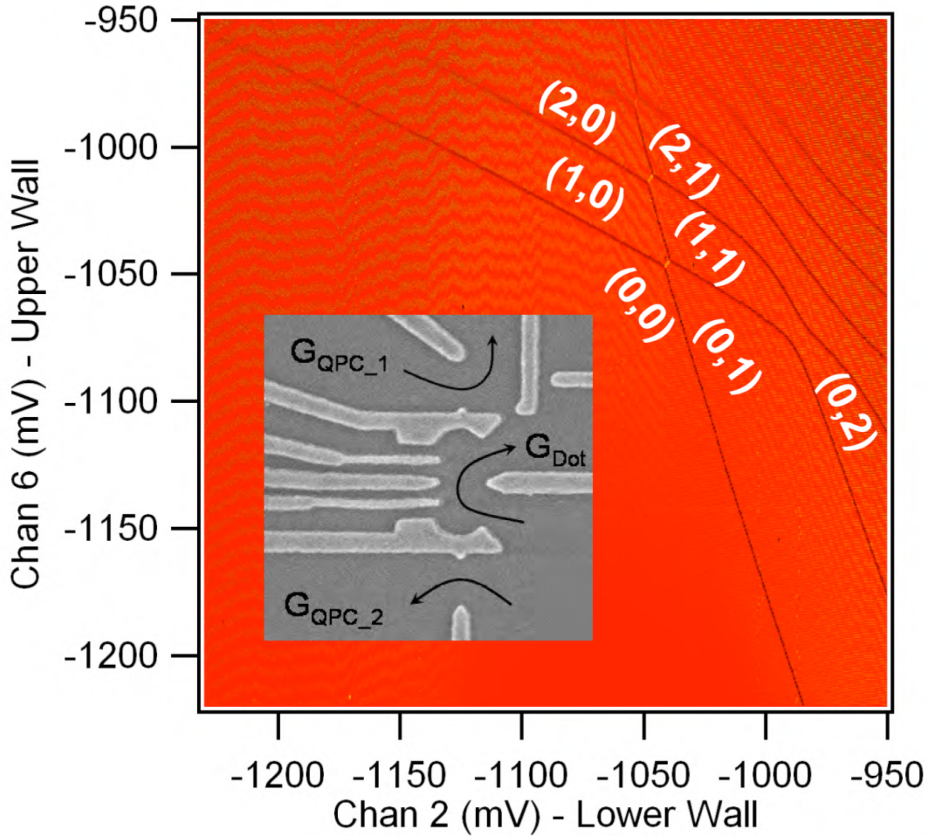
Brunner, Pioro-Ladriere, Tarucha 2011

… and many more …

GaAs/AlGaAs Heterostruktur
2DEG 90 nm depth, $n_s = 2.9 \times 10^{11}$ cm^{-2}

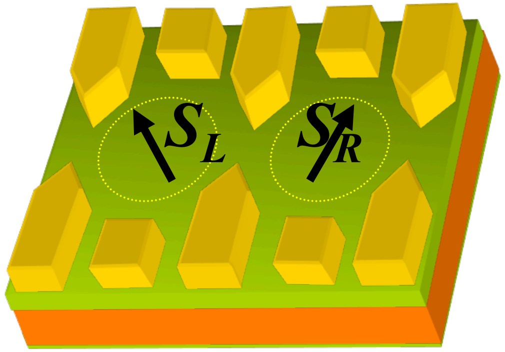
Temp.: 100 mK





C. Marcus *et al.*, PRL 2004

Spin-Qubits from Electrons



simplest spin-qubit:
spin-1/2 of 1 electron $|0\rangle = \uparrow,\ |1\rangle = \downarrow$

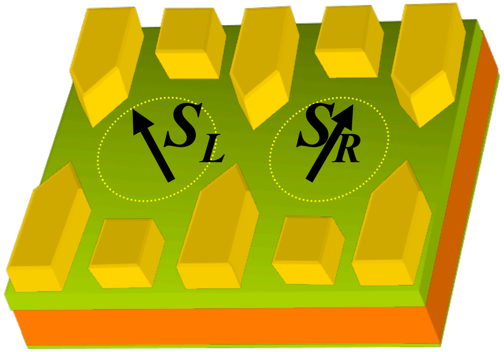
Many more choices for spin qubits:

- 'exchange-only qubits' DiVincenzo, Burkard *et al.* `00; Sachrajda `12; Marcus `13;
  3 electrons: $|0\rangle = S\uparrow,\ |1\rangle = T_+\downarrow - T_0\uparrow$ Doherty; Taylor; Rashba & Halperin, `13
- 'singlet-triplet' qubits Levy `02, Taylor *et al.* `05, Klinovaja *et al.*`12
  2 electrons: $|0\rangle = S,\ |1\rangle = T_0$
- 'spin-cluster qubits' Meier, Levy & DL, `03
  N electrons: AF spin chains, ladders, clusters,...
- `spin-orbit qubits' Golovach, Borhani &DL, `07; Kouwenhoven *et al.*, `11;

. hole spins: Bulaev & DL, `05; Marcus *et al.*, `11; Kloeffel, Trif & DL, `11-'16 (Si/Ge NW)

- molecular magnets Leuenberger & DL, '01; Affronte *et al.*, '06,
  Lehmann *et al.*, '07; Trif *et al.*, '08, '10

Spin-Qubits from Electrons



simplest spin-qubit:
spin-1/2 of 1 electron $|0\rangle = \uparrow, \ |1\rangle = \downarrow$

Many more choices for spin qubits:

- 'exchange-only qubits' DiVincenzo, Burkard *et al.* `00; Sachrajda `12; Marcus `13;
  3 electrons: $|0\rangle = S\uparrow, |1\rangle = T_+\downarrow - T_0\uparrow$ Doherty; Taylor; Rashba & Halperin, `13
- 'singlet-triplet' qubits Levy `02, Taylor *et al.* `05, Klinovaja *et al.*`12
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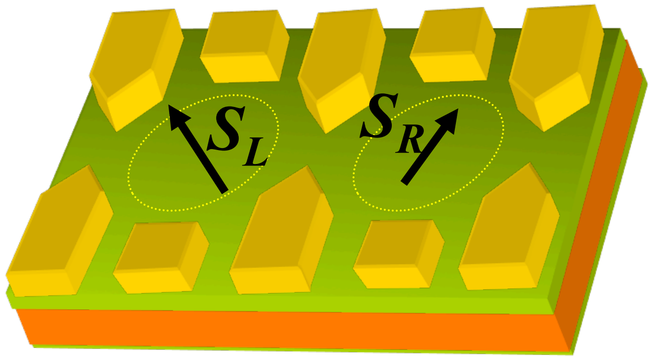
- molecular magnets Leuenberger & DL, '01; Affronte *et al.*, '06,
  Lehmann *et al.*, '07; Trif *et al.*, '08, '10

s

Most popular spin qubits (in GaAs)



LD spin qubit:
spin-1/2 of 1 electron

Loss and DiVincenzo , *Phys. Rev. A* **57**, p120 (1998)

$$|0\rangle = \uparrow, \ |1\rangle = \downarrow$$

'singlet-triplet' qubits :

2 electrons:

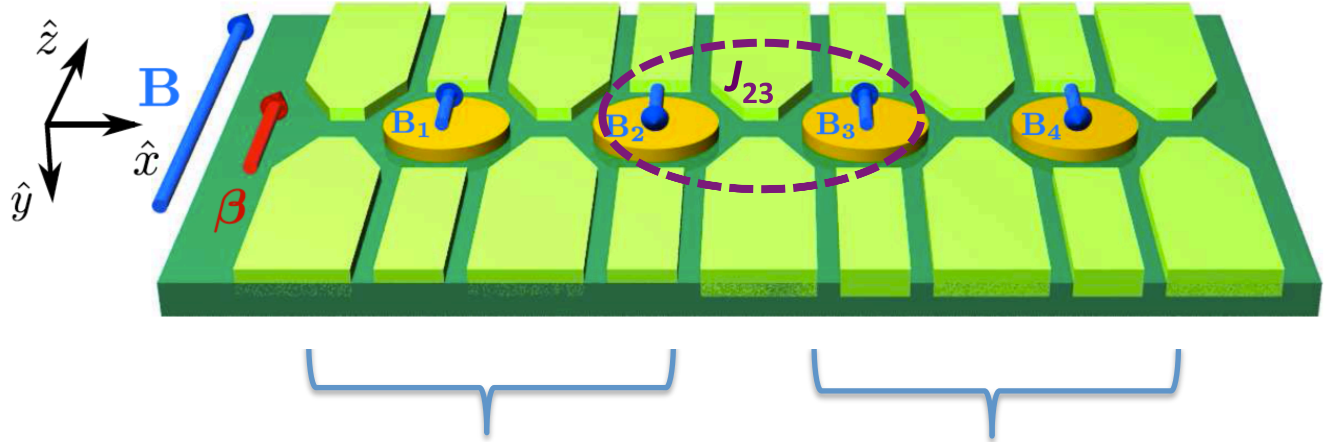
Levy (2002), Taylor *et al.* (2005)

$$|0\rangle = S, \ |1\rangle = T_0$$

Prospects for Spin-Based Quantum Computing in Quantum Dots
C. Kloeffel and D. Loss, Annu. Rev. Condens. Matter Phys. 4, 51 (2013);

Singlet-Triplet (ST) Qubit

s



four dots = two ST-qubits

Computational basis:

$$S^z = 0$$

$$[11] = (+ - + -)$$
$$[10] = (+ - - +)$$
$$[01] = (- + + -)$$
$$[00] = (- + - +)$$

Note: Decoherence time is very long $T_2 \sim 250\ \mu s$

Bluhm et al., Nat. Phys. 7, 109 (2011)

…from one to many quantum dots...

12 quantum dots - 4 RX spin qubits

Marcus & Kuemmeth et al., 2015/16

Qubit 1 Qubit 2 Qubit 3 Qubit 4

J_{12}^{Q1} J_{23}^{Q1} J_{eff}^{Q1-Q2} J_{12}^{Q2} J_{23}^{Q2} J_{eff}^{Q2-Q3} J_{12}^{Q3} J_{23}^{Q3} J_{eff}^{Q3-Q4} J_{12}^{Q4} J_{23}^{Q4}

Quadruple-quantum-dot

Baart, Jovanovic, Reichl, Wegscheider, and Vandersypen, arXiv:1606.00292

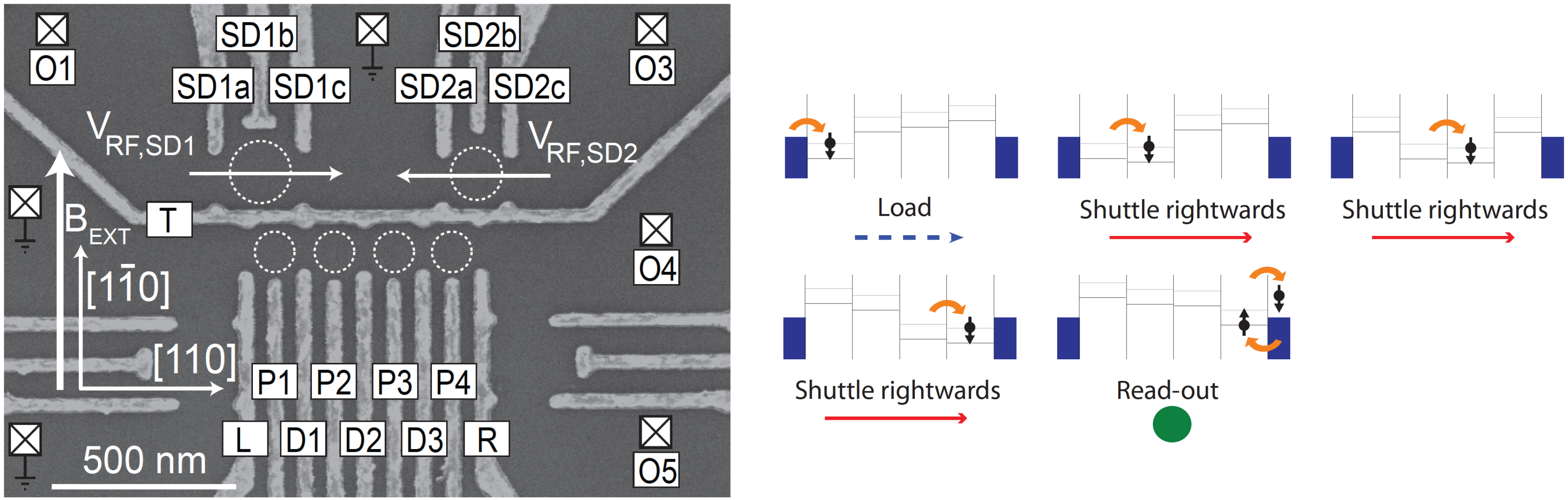


FIG. 1: (a) SEM image of a sample nominally identical to the one used for the measurements. Dotted circles indicate quantum dots, squares indicate Fermi reservoirs in the 2DEG, which are connected to ohmic contacts. The gates that are not labeled are grounded. The reflectance of SD2, $V_{RF,SD2}$, is monitored. (b) Charge stability diagram of the quadruple dot. The occupancy of each dot is denoted by (l, m, n, p) corresponding to the number of electrons in dot 1 (leftmost), 2, 3 and 4 (rightmost) respectively. The fading of charge transition lines from dot 2 and 3 can be explained in a similar way as in Ref. 17 (black dotted lines indicate their positions) and becomes less prominent for a slow scan (see Supplementary Information II). The pulse sequence for loading and read-out is indicated in the charge stability diagram via arrows, see also panel b. The black rectangle corresponds to the hot spot in dot 4 where spins relax on a sub-microsecond timescale; this hot spot is only used for the measurements of Fig. 3. (c) Read from left to right and top to bottom. The system is initialized by loading one electron from the left reservoir. Next, we shuttle the electron to dot 2, 3 and 4 sequentially and finally read out the spin state using spin-selective tunneling.

PHYSICAL REVIEW APPLIED **6,** 054013 (2016)

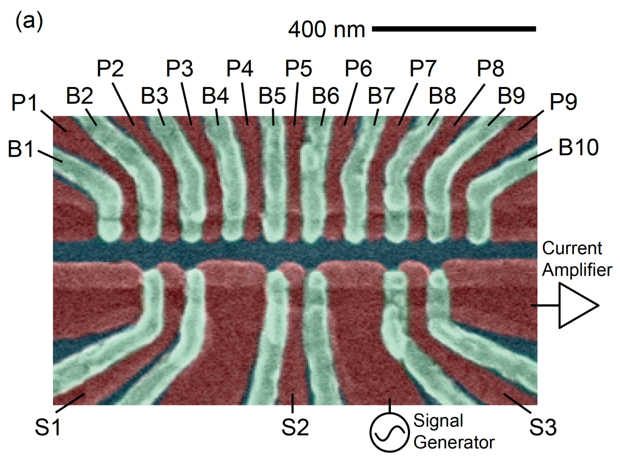
Scalable Gate Architecture for a One-Dimensional Array of Semiconductor Spin Qubits

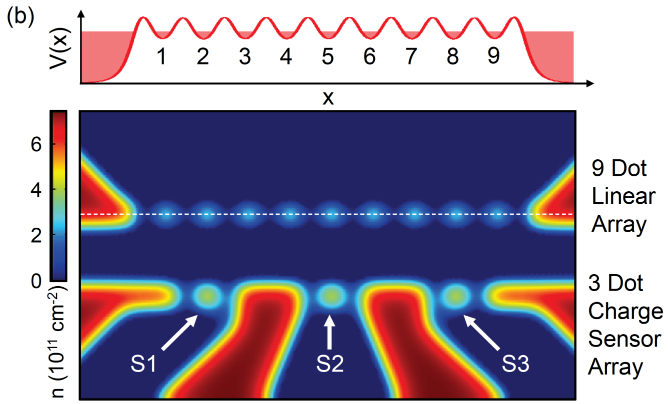
D. M. Zajac,[1] T. M. Hazard,[1] X. Mi,[1] E. Nielsen,[2] and J. R. Petta[1]

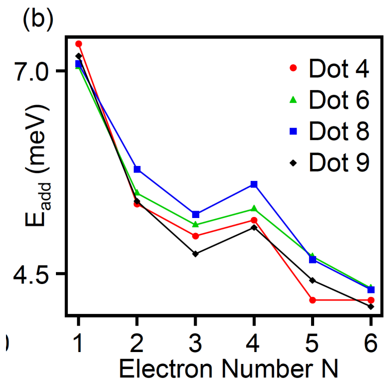
[1]*Department of Physics, Princeton University, Princeton, New Jersey 08544, USA*

[2]*Sandia National Laboratories, Albuquerque, New Mexico 87185, USA*

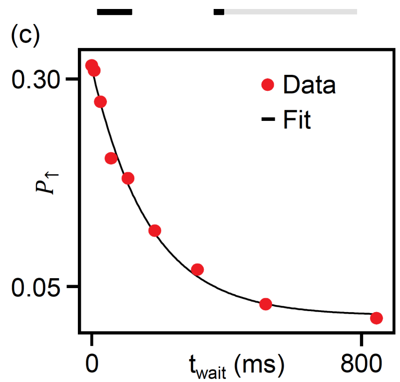
12 (=9+3) quantum dots in Si/SiGe heterostructure







Dot	α (meV/mV)	E_c (meV)	E_{orb} (meV)
1	0.14	6.6	2.7
2	0.13	6.1	2.6
3	0.11	5.6	2.1
4	0.14	7.3	3.3
5	0.14	7.2	3.3
6	0.14	7.1	3.0
7	0.14	7.7	3.5
8	0.14	7.1	3.4
9	0.13	7.2	3.4



A logical qubit in a linear array of semiconductor quantum dots

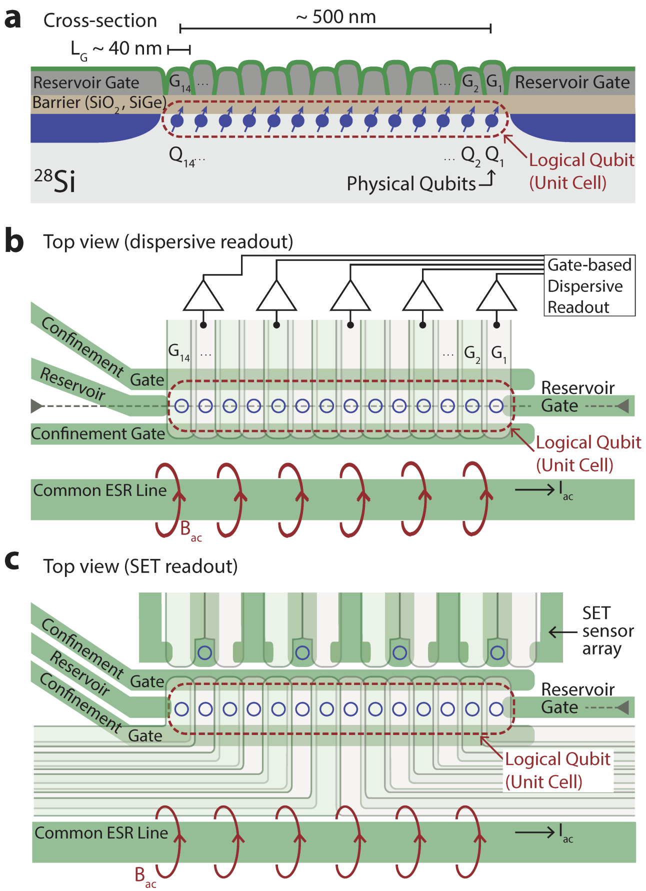
Cody Jones,* Mark F. Gyure, and Thaddeus D. Ladd

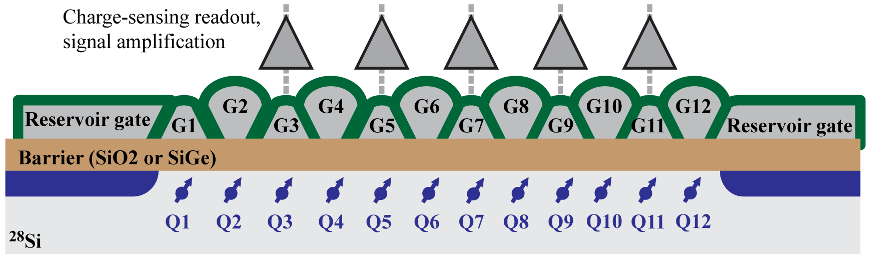
HRL Laboratories, LLC, 3011 Malibu Canyon Road, Malibu, CA 90265, USA

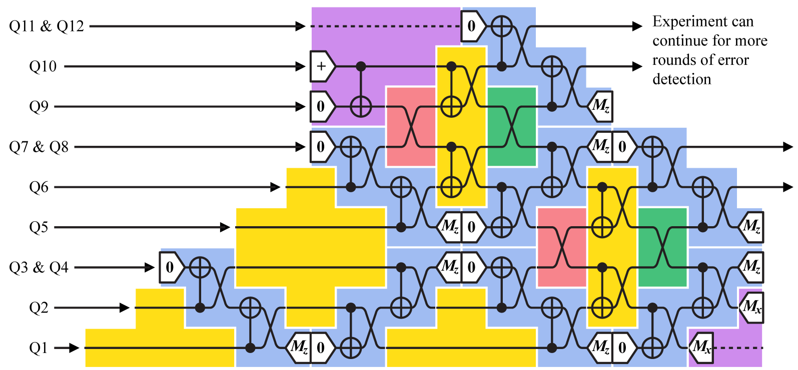
Michael A. Fogarty, Andrea Morello, and Andrew S. Dzurak

Centre for Quantum Computation and Communication Technology,
School of Electrical Engineering and Telecommunications,
The University of New South Wales, Sydney, New South Wales 2052, Australia

arXiv:1608.06335

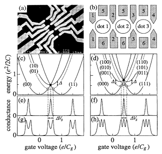




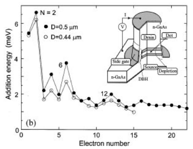


Spin qubits in **GaAs** quantum dots

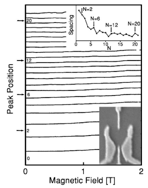
Kloeffel & DL, Annu. Rev. Condens. Matter Phys. 4, 51 (2013)



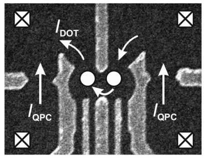
Westervelt
Gossard 1995



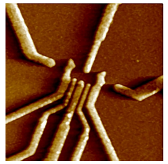
Kouwenhoven
Tarucha 1996



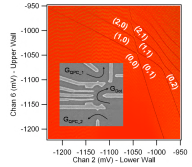
Sachrajda
2000



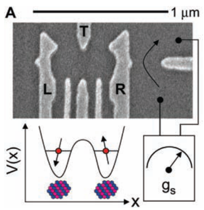
Kouwenhoven
Tarucha 2003-13



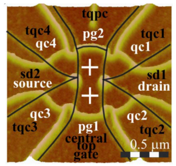
Vandersypen,
Koppens, 2003



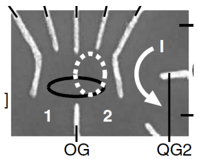
Marcus 2004



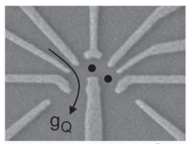
Petta, Marcus, Yacoby 2005



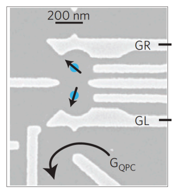
Ensslin, Ihn
2006



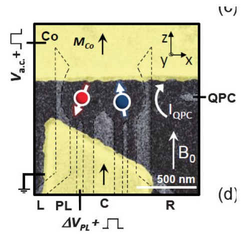
Zumbuhl,
Kastner
2008



Petta 2010



Bluhm, Dial,
Yacoby 2010-13
***T_2* ~ 300 μs**



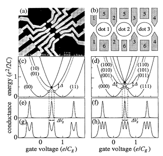
Brunner,
Pioro-Ladriere,
Tarucha 2011

… and many more …

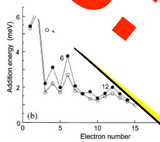
Spin qubits in **GaAs** quantum dots

& DL, Annu. Rev. Condens. Matter Phys. 4, 51 (2013)

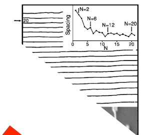
GaAs: Nuclear Spin 'Problem'?

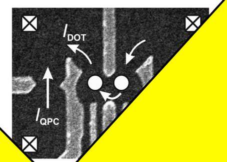


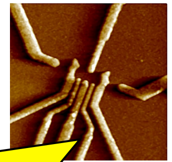
Westervelt
Gossard 1995



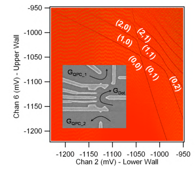
Kou
Taruch



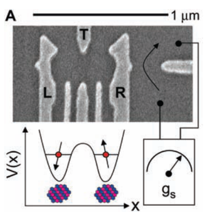




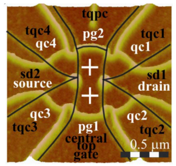
n,
ns, 2003



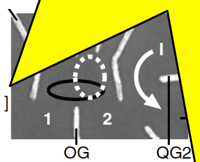
Marcus 2004



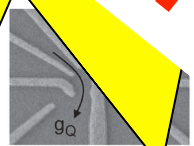
Petta, Marcus,
Yacoby 2005



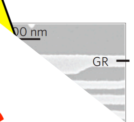
Ensslin, Ihn
2006



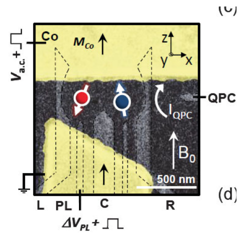
Zumbuhl,
Kastner
2008



Petta 2010



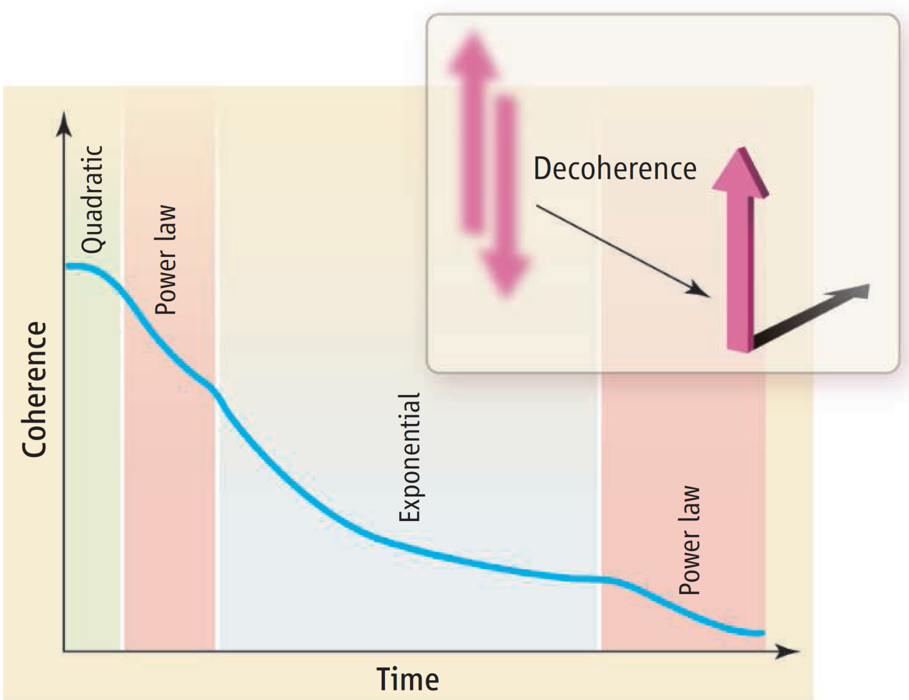
Bluhm, Dial,
Yacoby 2010-
T_2 ~ 300 µs



er,
riere,
2011

… and many more …

Decoherence due to Nuclear Spins in GaAs: rich and complex subject!



Fischer and DL, Science 324, 1277 (2009)

Reviews: Coish and Baugh, Phys. Stat. Sol. B 246:2203 (2009)
Kloeffel & DL, Annu. Rev. Condens. Matter Phys. 4, 51 (2013)

Strategies:

A) Use GaAs or InAs (still ‘best’ for electro-optical control) and deal with nuclear spins:

- echo techniques and/or dynamical narrowing
  Bluhm & Yacoby et al., Nat. Phys. 7, 109 (2011)
  Kuemmeth & Marcus et al., arXiv:1601.06677: T_2 ~1ms
- ordered nuclear state ➔ exp. evidence in GaAs wire
  Zumbuhl & Yacoby, PRL 112, 066801 (2014); T_2 =?
- brute force: >99.9999% polarization @ 1mK & 10T
  Chesi & DL, PRL `08

B) Avoid nuclear spin problem: use holes and/or other materials such as ^{13}C, Si, Ge,...

Many candidate materials for spin qubits

- GaAs quantum dots: **most advanced**
- InAs, InSb, InP nanowires
- InAs quantum dots (self-assembled, optics)
- hole spins in semiconductors

- Carbon: nanotubes, graphene, diamond (NV)
- Si and SiGe quantum dots
- Si:P donors
- **SiGe nanowires (holes!)**

- molecular magnets: Cu_3-rings, Co_3, Mn_{12}, V_{15}, polyoxometalates, …

etc.

Various kinds of spin qubits in semiconductors

Type	Material	f (Hz)	$T_2^\star$ (ns)	T_2(ns)	$Q \equiv T_2^\star / t_\pi$	N Qubit	
Spin-1/2	GaAs	$\leq 1.2 \times 10^8$	$[\mathbf{10}; \mathbf{30}]$		$\leq \mathbf{7}$	$1/1$	[1]
Bare S-T	GaAs	$\leq 5 \times 10^6$	$\mathbf{10}$	2×10^3	$\leq \mathbf{1}$	$2/1$	[2]
S-T (DNP)	GaAs	$\sim 2 \times 10^8$	$\mathbf{94}$	3×10^4	$\sim \mathbf{40}$	$2/1$	[3]
S-T (H est)	GaAs	$\sim 6 \times 10^7$	$\mathbf{2 \times 10^3}$	100	$\sim \mathbf{240}$	$2/1$	[4]
Exchange	GaAs	$\sim 3 \times 10^{10}$	$\mathbf{25}$	2×10^3	$\sim \mathbf{1500}$	$3/1$	[5]
Spin-1/2	Nat. SiGe	$\sim 5 \times 10^6$	$\sim \mathbf{9 \times 10^2}$	3.7×10^4	$\sim \mathbf{9}$	$1/1$	[6]
Spin-1/2	Pur. Si	$\sim 3 \times 10^5$	$\leq \mathbf{1.2 \times 10^5}$	1.2×10^6	$\leq \mathbf{80}$	$1/1$	[7]
e- spin	Nat. Si	$\sim 3 \times 10^6$	$\mathbf{55}$	2×10^5	$\leq \mathbf{1}$	$1/1$	[8]
P spin	Nat. Si	$\sim 2 \times 10^4$	$\sim \mathbf{3 \times 10^6}$	6×10^7	$\sim \mathbf{100}$	$1/1$	[9]
e- spin	Pur. Si.	$\sim 2 \times 10^5$	$\sim \mathbf{3 \times 10^5}$	1×10^6	$\sim \mathbf{108}$	$1/1$	[10]
P spin	Pur. Si.	$?$	$\sim \mathbf{6 \times 10^8}$	2×10^9	$> \mathbf{1000}$	$1/1$	[10]
Hybrid	SiGe	$\sim 1 \times 10^{10}$	$\sim \mathbf{11}$	~ 40	$\sim \mathbf{250}$	$2/1$	[11]

[1] J. Yoneda, PRL. **113**, 267601 (2014). [2] J. R. Petta, Science **327**, 669 (2010). [3] H. Bluhm, PRL **105**, 216803 (2010). [4] M. D. Shulman, Nat. Commun. **5**, 5156 (2014). [5] J. Medford, Nat. Nanotechnol. **8**, 654 (2013). [6] E. Kawakami, Nat. Nanotechnol. **9**, 666 (2014). [7] M. Veldhorst, Nature **526**, 410 (2015). [8] J. J. Pla, Nature **489**, 541 (2012). [9] J. J. Pla, Nature **496**, 334 (2013). [10] J. T. Muhonen, Nat. Nanotechnol. **9**, 986 (2014). [11] D. Kim, Nature **511**, 70 (2014).

Matthieu Delbecq, 2016

Electron *vs.* Hole Spin Qubits

Band Diagram Near Γ Point (k = 0)

Typical bulk spectrum
of a semiconductor:

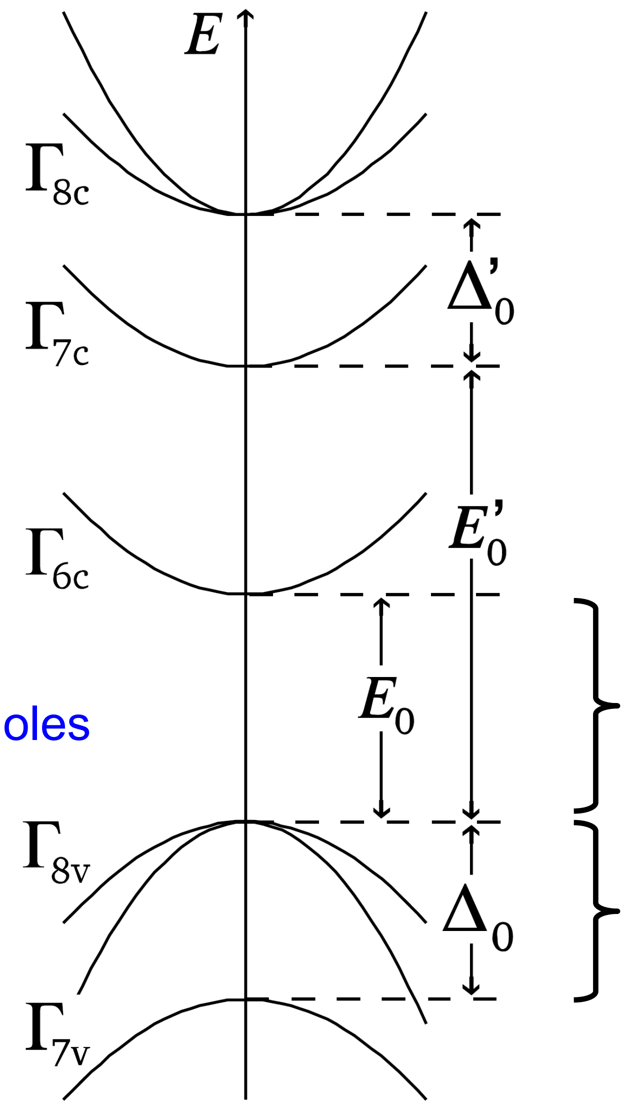
Higher bands

Conduction band
s-type Bloch functions

Spin 3/2, Heavy holes and light holes

Valence band
p-type Bloch functions

Split-Off Band



R. Winkler (Springer, Berlin, 2003)

GaAs:

band gap

1.5 eV

0.3 eV

Hole Spin Qubits in GaAs and InAs

PRL **95,** 076805 (2005) PHYSICAL REVIEW LETTERS week ending 12 AUGUST 2005

Spin Relaxation and Decoherence of Holes in Quantum Dots

Denis V. Bulaev and Daniel Loss

Department of Physics and Astronomy, University of Basel, Klingelbergstrasse 82, CH-4056 Basel, Switzerland

(Received 8 March 2005; published 11 August 2005)

We investigate heavy-hole spin relaxation and decoherence in quantum dots in perpendicular magnetic fields. We show that at low temperatures the spin decoherence time is 2 times longer than the spin relaxation time. We find that the spin relaxation time for heavy holes can be comparable to or even longer than that for electrons in strongly two-dimensional quantum dots. We discuss the difference in the magnetic-field dependence of the spin relaxation rate due to Rashba or Dresselhaus spin-orbit coupling for systems with positive (i.e., GaAs quantum dots) or negative (i.e., InAs quantum dots) g factor.

$$H_{\mathrm{so}}^{\mathrm{HH}} = i\alpha(\sigma_+ P_-^3 - \sigma_- P_+^3)$$ Rashba SOI for HH

$$- \beta(\sigma_+ P_- P_+ P_- + \sigma_- P_+ P_- P_+)$$ Dresselhaus SOI for HH

EDSR for HHs in III-V materials: Bulaev and DL, PRL 98, 097202 (2007)
ultrafast Rabi oscillations

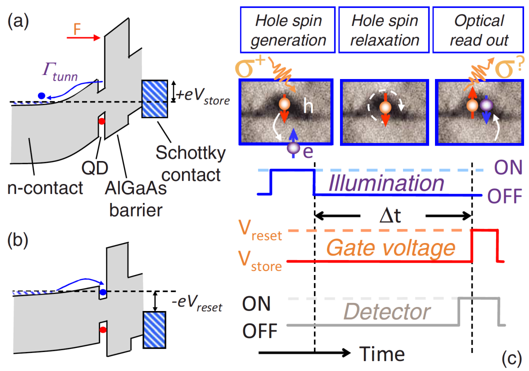
PHYSICAL REVIEW B **76**, 241306(R) (2007)

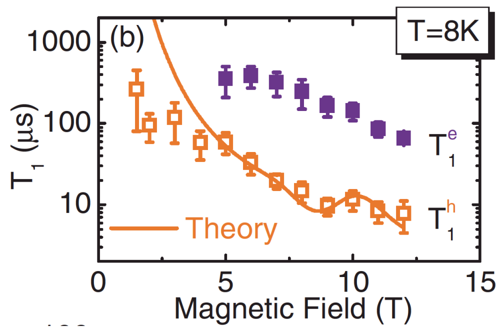
Observation of extremely slow hole spin relaxation in self-assembled quantum dots

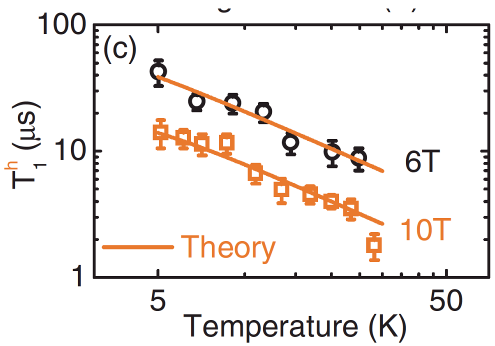
D. Heiss, S. Schaeck, H. Huebl, M. Bichler, G. Abstreiter, and J. J. Finley*
Walter Schottky Institut, Technische Universität München, Am Coulombwall 3, D-85748 Garching, Germany

D. V. Bulaev and Daniel Loss
Department of Physics and Astronomy, University of Basel, Klingelbergstrasse 82, CH-4056 Basel, Switzerland

(Received 24 October 2007; published 21 December 2007)







PHYSICAL REVIEW B **76**, 241306(R) (2007)

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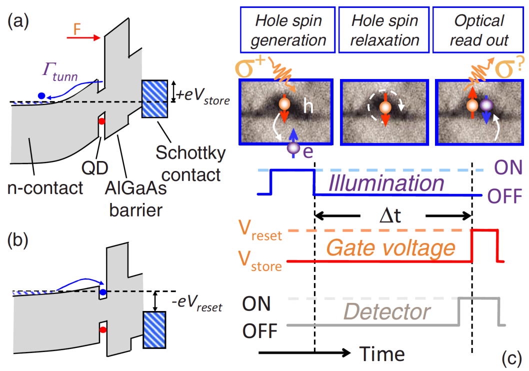
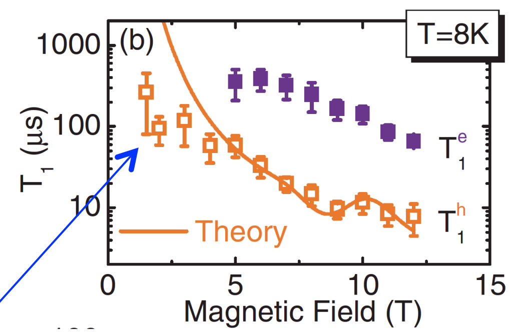
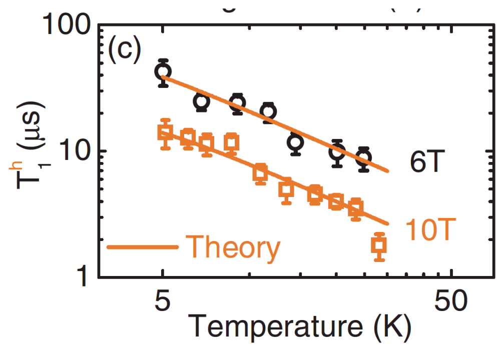
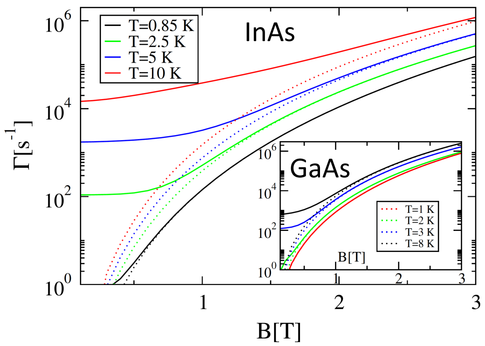
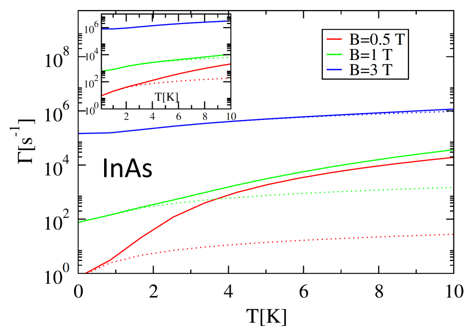










2-Phonon processes lead to saturation

M.Trif, P. Simon, and DL, PRL 103, 106601 (2009)

nature materials

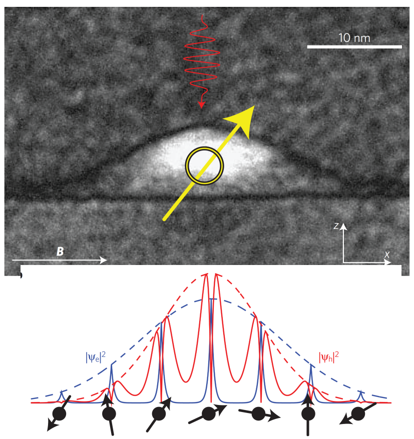
ARTICLES

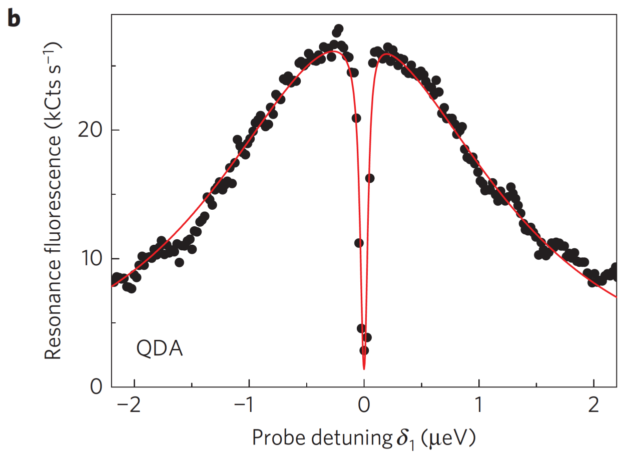
PUBLISHED ONLINE: 25 JULY 2016 | DOI: 10.1038/NMAT4704

Nature Mat., 2016

Decoupling a hole spin qubit from the nuclear spins

Jonathan H. Prechtel[1], Andreas V. Kuhlmann[1], Julien Houel[1,2], Arne Ludwig[3], Sascha R. Valentin[3], Andreas D. Wieck[3] and Richard J. Warburton[1]*





Hyperfine interaction of holes is nearly Ising (transverse < 1%)

InGaAs self-assembled dots

b, $\Gamma_r = 0.68$ μeV, $T_2 > 1$ μs, $T_1 \gg T_2$.

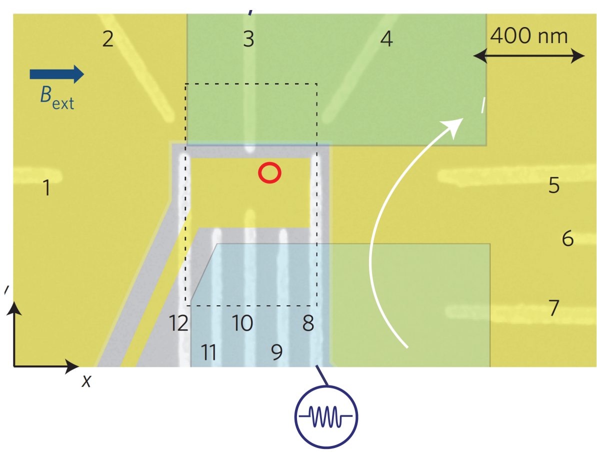
Theory: Fischer et al., PRB 78, 155329 (2008); Fischer and DL, PRL 105, 266603 (2010)

LETTERS
PUBLISHED ONLINE: 10 AUGUST 2014 | DOI: 10.1038/NNANO.2014.153

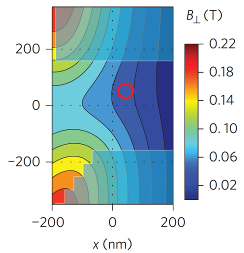
nature nanotechnology

Electrical control of a long-lived spin qubit in a Si/SiGe quantum dot

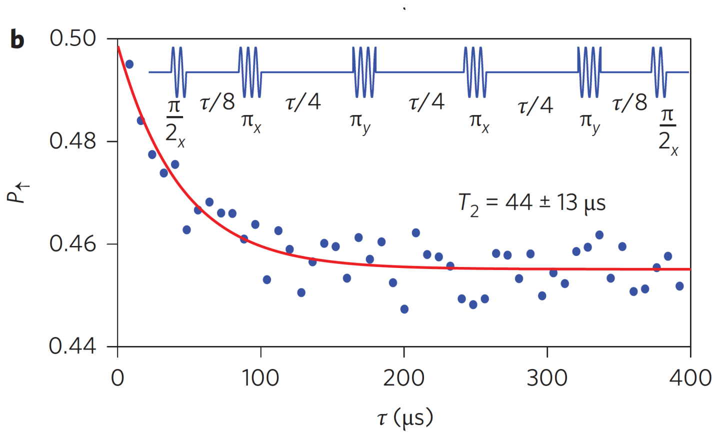
E. Kawakami[1‡], P. Scarlino[1‡], D. R. Ward[2], F. R. Braakman[1†], D. E. Savage[2], M. G. Lagally[2], Mark Friesen[2], S. N. Coppersmith[2], M. A. Eriksson[2] and L. M. K. Vandersypen[1*]



Universal qubit control



micromagnet



nature nanotechnology

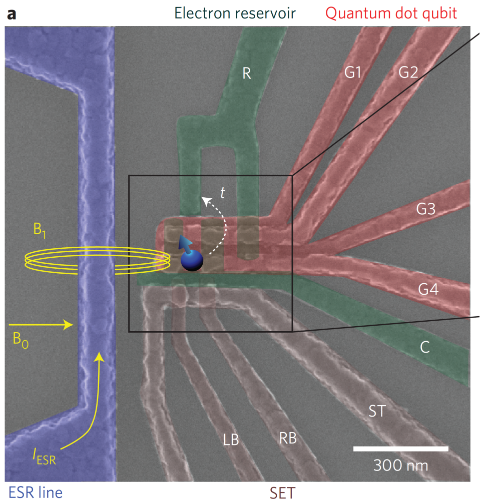
LETTERS

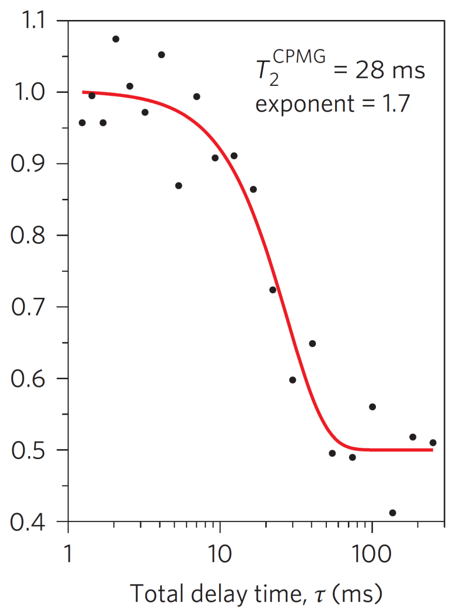
PUBLISHED ONLINE: 12 OCTOBER 2014 | DOI: 10.1038/NNANO.2014.216

An addressable quantum dot qubit with fault-tolerant control-fidelity

M. Veldhorst[1]*, J. C. C. Hwang[1], C. H. Yang[1], A. W. Leenstra[2], B. de Ronde[2], J. P. Dehollain[1], J. T. Muhonen[1], F. E. Hudson[1], K. M. Itoh[3], A. Morello[1] and A. S. Dzurak[1]*

ESR coil





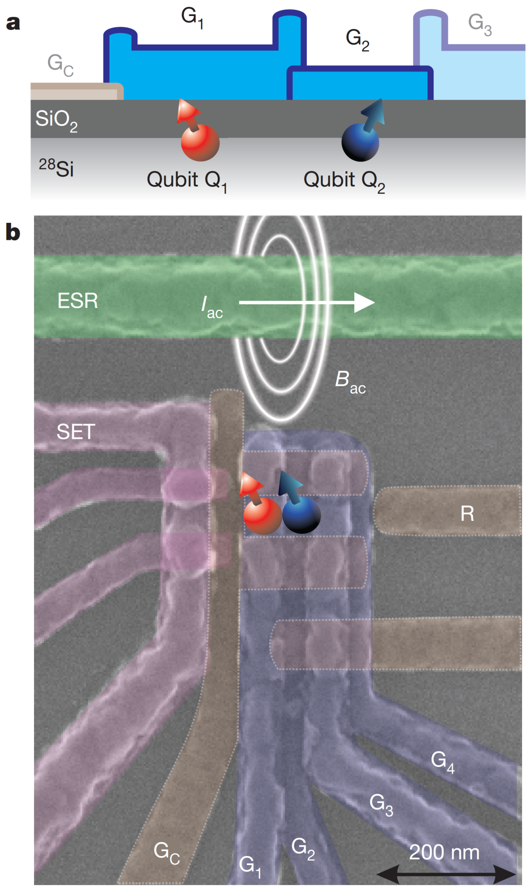
LETTER

Dzurak group: Nature 526, 410 (2015)

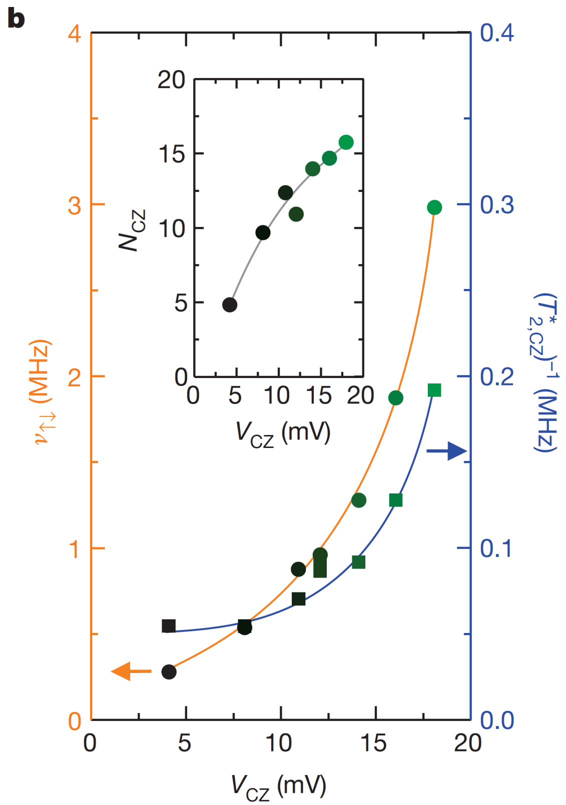
doi:10.1038/nature15263

A two-qubit logic gate in silicon

M. Veldhorst[1], C. H. Yang[1], J. C. C. Hwang[1], W. Huang[1], J. P. Dehollain[1], J. T. Muhonen[1], S. Simmons[1], A. Laucht[1], F. E. Hudson[1], K. M. Itoh[2], A. Morello[1] & A. S. Dzurak[1]



Controlled phase gate (CZ)



LETTER

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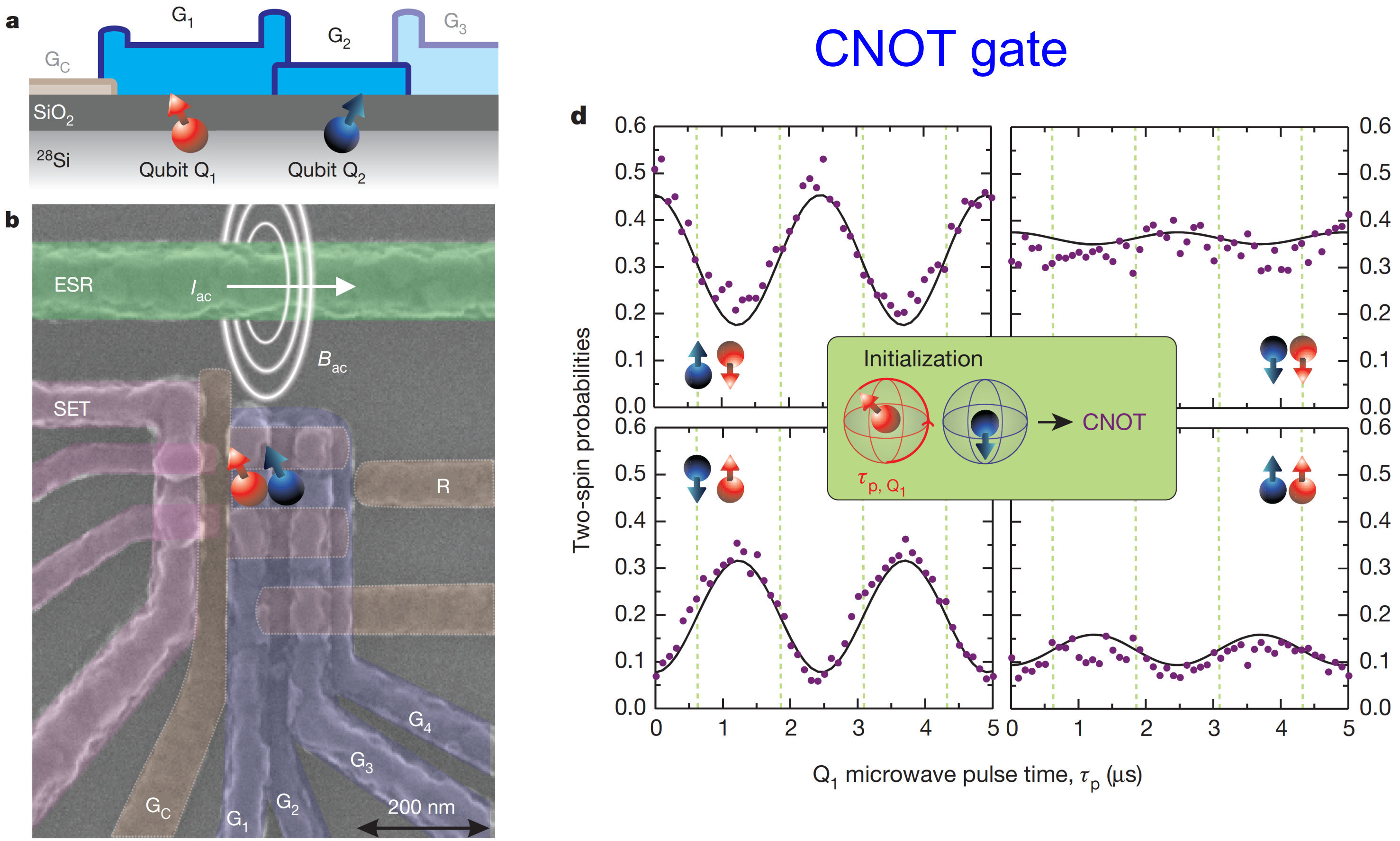
CNOT gate



LETTER

Dzurak group: Nature 526, 410 (2015)

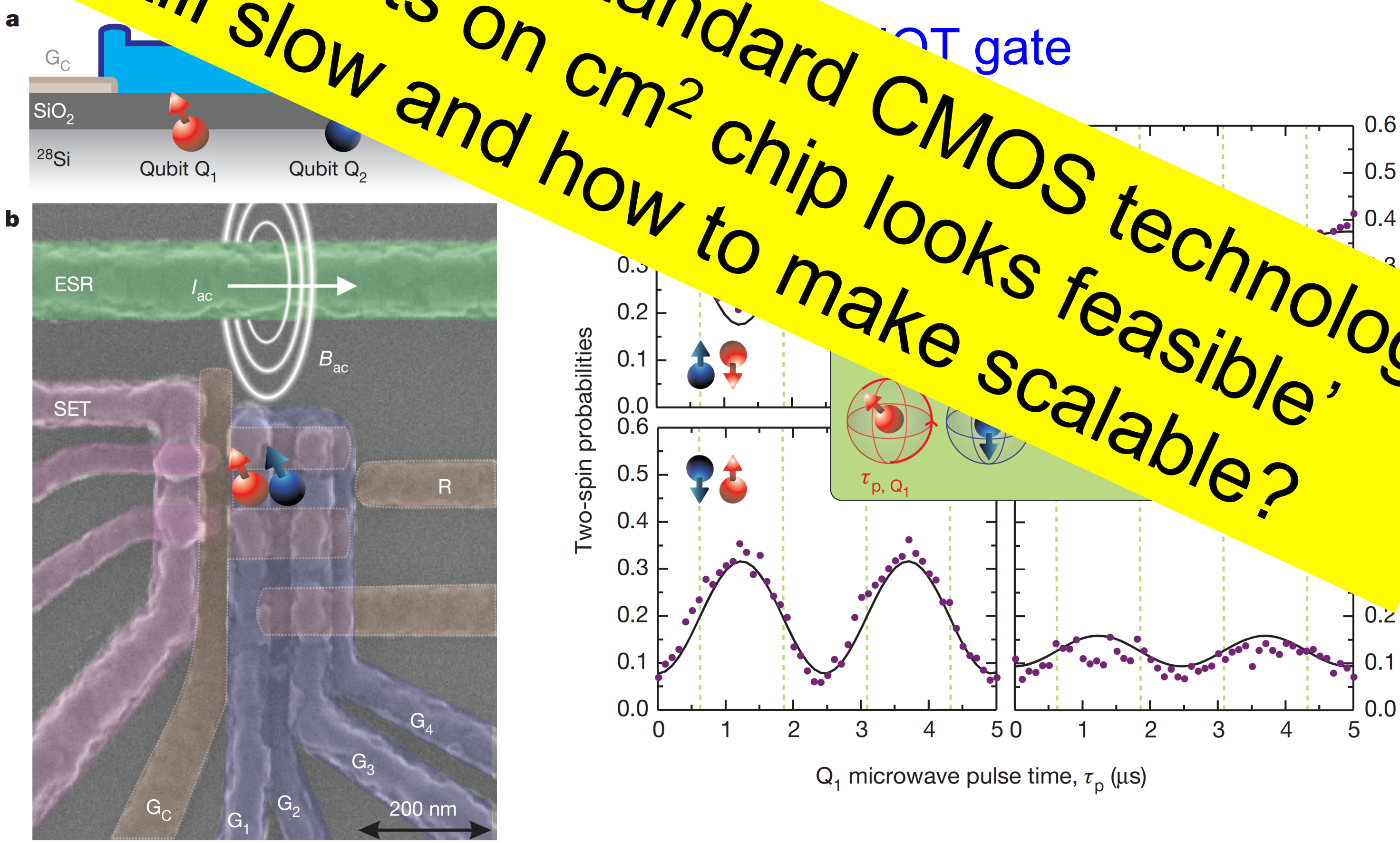
doi:10.1038/nature15263

A two ... ic gate in silicon

M. Veldhors ... J. P. Dehollain[1], J. T. Muhonen[1], S. Simmons[1],
A. Laucht[1] ... urak[1]

compatible with standard CMOS technology
➔ `10^9 qubits on cm^2 chip looks feasible'
But: still slow and how to make scalable?

...OT gate

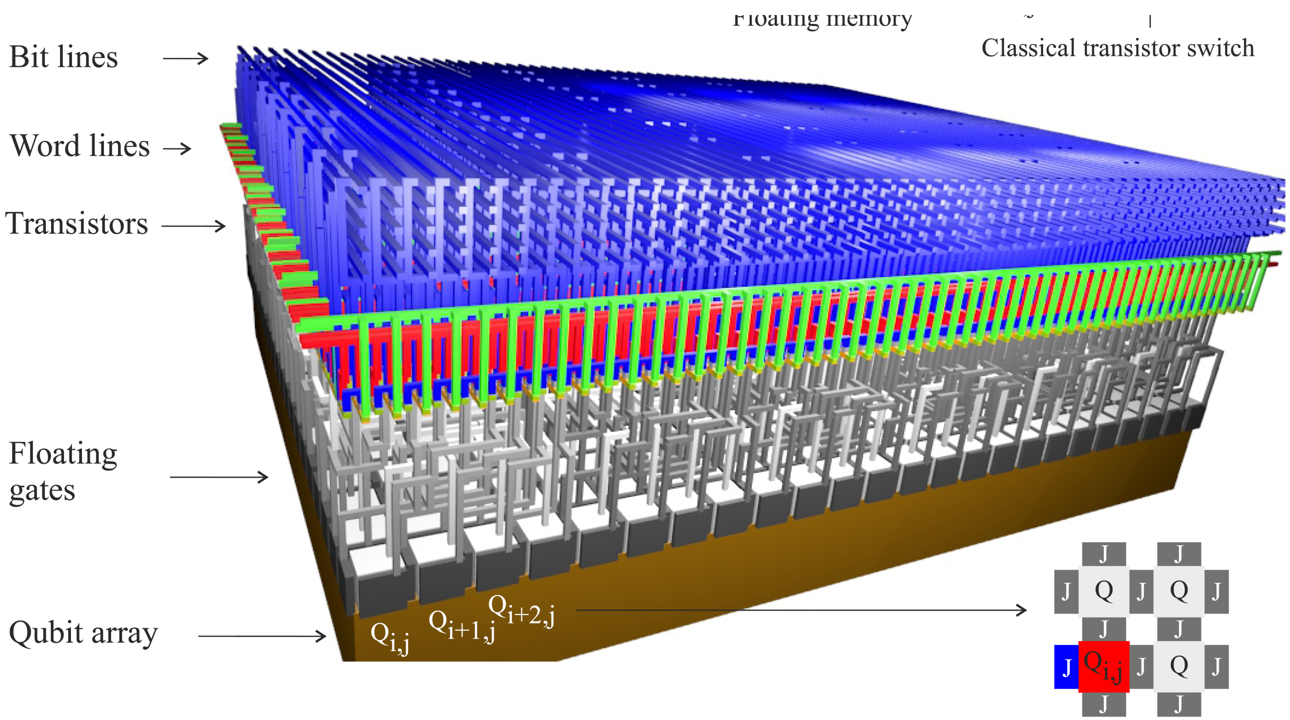


Silicon CMOS architecture for a spin-based quantum computer

M. Veldhorst,[1,2] H.G.J. Eenink,[2,3] C.H. Yang,[2] and A.S. Dzurak[2]

[1]Qutech, TU Delft, 2600 GA Delft, The Netherlands
[2]Centre for Quantum Computation and Communication Technology, School of Electrical Engineering and Telecommunications, The University of New South Wales, Sydney, NSW 2052, Australia
[3]NanoElectronics Group, MESA+ Institute for Nanotechnology, University of Twente, P.O. Box 217, 7500 AE Enschede, The Netherlands

(Dated: October 3, 2016)



goal: spin qubit surface code

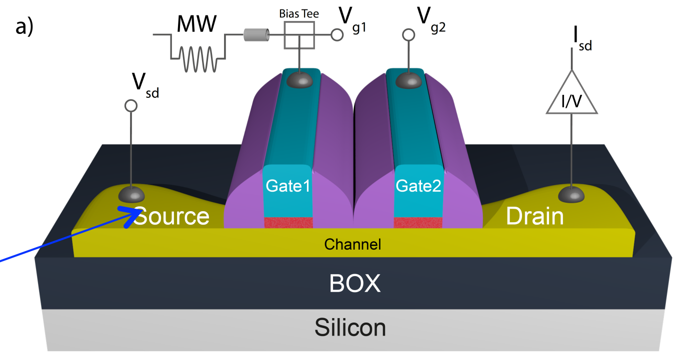
See, arXiv:1609.09700

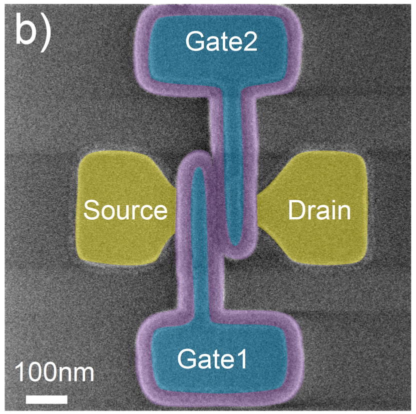
A CMOS silicon **hole** spin qubit

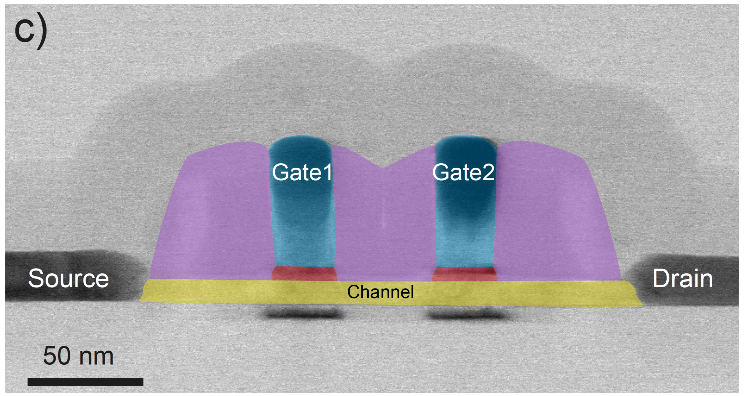
Sanquer & De Franceschi group, Maurand et al., arXiv:1605.07599

'quantumize' integrated bits:

➔ heavy hole spin in p-doped Si-on-Insulator nanowire







~10 holes per dot

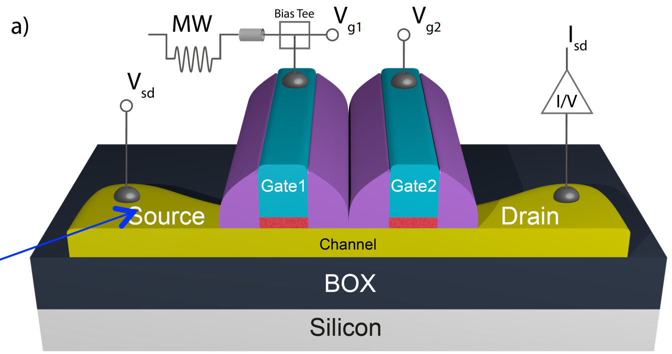
FIG. 1. CMOS qubit device. a, Simplified 3-dimensional schematic of a SOI nanowire field-effect transistor with two gates Gate 1 and Gate 2. Using a bias-T, Gate 1 is connected to a low-pass-filtered line, used to apply a static gate voltage Vg1, and to a 20-GHz bandwidth line, used to apply the high-frequency modulation necessary for qubit initialization, manipulation and readout. b, Colorized device top view obtained by scanning electron microscopy just after the fabrication of gates and spacers. c, Colorized transmission-electron-microscopy image of the device along a longitudinal cross-sectional plane.

CMOS silicon **hole** spin qubit: Fast EDSR

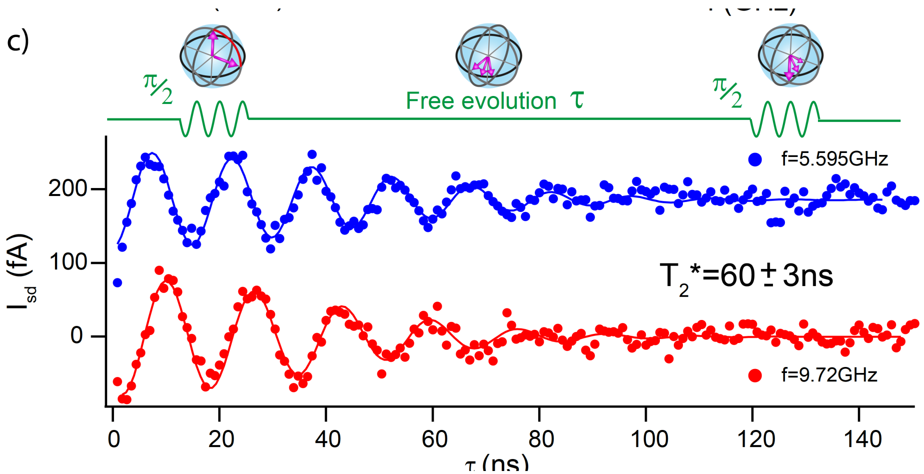
Sanquer & De Franceschi group, Maurand et al., arXiv:1605.07599

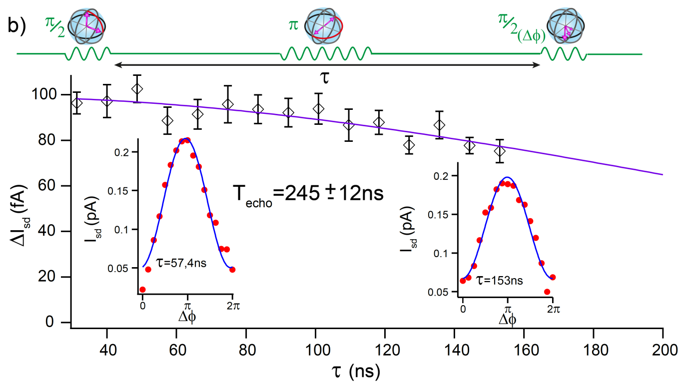
'quantumize' integrated bits:

➔ heavy hole spin in p-doped Si-on-Insulator nanowire



~10 holes per dot

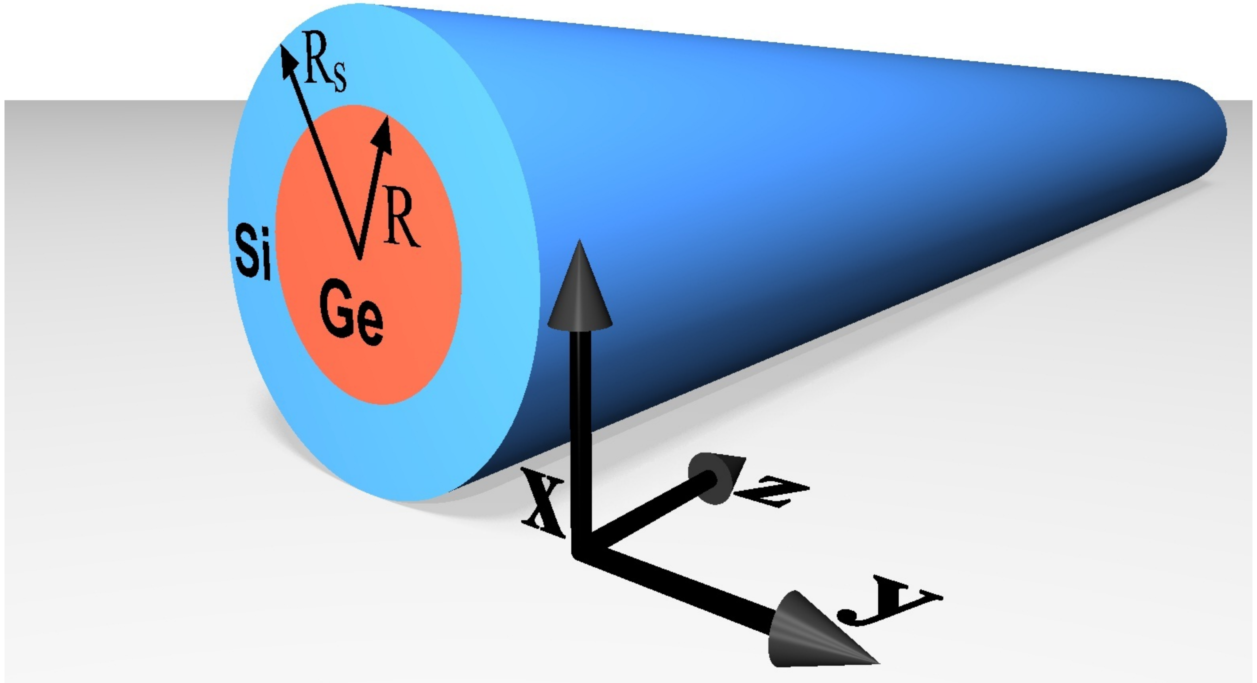


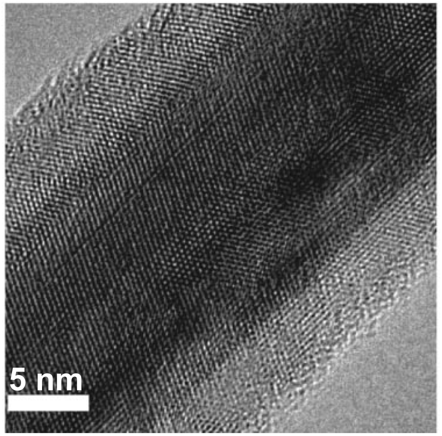


Fast Rabi oscillations ~85 MHz
and Ramsey fringe via EDSR of holes

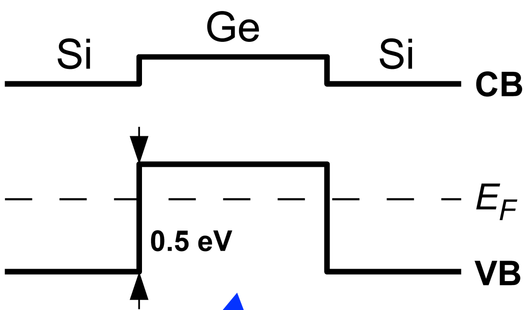
Two-axis qubit control and spin coherence times.

Hole Spins in Ge/Si Core/Shell Nanowires





HR-TEM, Lieber (2005)



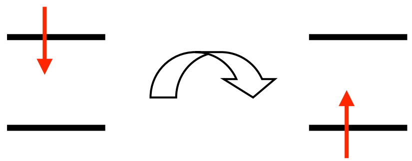
Strongly confined holes inside the Ge core

Lauhon *et al.*, Nature (2002)
Lu *et al.*, PNAS USA (2005)
Kloeffel, Trif, and Loss, PRB (2011)
Hu *et al.*, Nature Nano (2012)
Brauns *et al.*, PRB (2016)
…

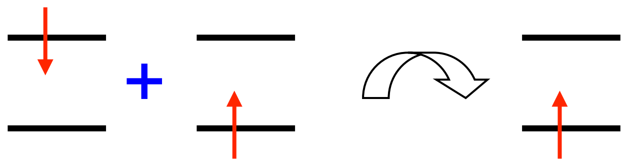
Spin decoherence in GaAs quantum dots

Two important sources of spin decay in GaAs:

1) Spin-orbit interaction (Dresselhaus & Rashba)
➔ interaction between spin and charge fluctuations



Relaxation with rate $1/ T_1$



Decoherence with rate $1/ T_2$
= decay of coherent superposition

2) Hyperfine interaction between electron spin and nuclear spins,
can lead to non-exponential decay

General spin Hamiltonian:

$$H = g\mu_B \mathbf{S}\cdot\mathbf{B} + \mathbf{S}\cdot\mathbf{h}(t)$$

where **h**(t) is a fluctuating (internal) field with < **h**(t) >=0

Relaxation (T_1) and decoherence (T_2) times in weak coupling approx.:

$$\frac{1}{T_1} = \int_{-\infty}^{\infty} dt\,\mathrm{Re}\left[\langle h_X(0)h_X(t)\rangle + \langle h_Y(0)h_Y(t)\rangle\right]e^{-iE_Z t/\hbar}$$

$$\frac{1}{T_2} = \frac{1}{2T_1} + \int_{-\infty}^{\infty} dt\,\mathrm{Re}\langle h_Z(0)h_Z(t)\rangle$$

[if $< h_i(t)\, h_j(t') > \sim\delta_{ij}$, $i,j=(X,Y,Z)$]

relaxation contribution

<< 'typically'

dephasing contribution

See e.g. Abragam

General spin Hamiltonian:

$$H = g\mu_B \mathbf{S}\cdot\mathbf{B} + \mathbf{S}\cdot\mathbf{h}(t)$$

where **h**(t) is a fluctuating (internal) field with < **h**(t) >=0

For SOI **linear** in momentum: $\boxed{\mathbf{h}(t)\cdot\mathbf{B} = 0}$ (unlike spin-boson model!)

$$\Rightarrow \quad \frac{1}{T_2} = \frac{1}{2T_1} + \cancelto{0}{\int_{-\infty}^{\infty} dt\, \mathrm{Re}\left\langle h_Z(0)\, h_Z(t)\right\rangle}$$

relaxation contribution

~~<< ‘typically‘~~

dephasing contribution

Golovach, Khaetskii, and DL, PRL 93, 016601 (2004)

Spin-Orbit Interaction in GaAs Quantum Dots (2DEG):

$$H_{SO} = \alpha\left(p_x\sigma_y - p_y\sigma_x\right) \quad \leftarrow \text{Rashba SOI}$$

$$- \beta\left(p_x\sigma_x - p_y\sigma_y\right) \quad \leftarrow \text{Dresselhaus SOI}$$

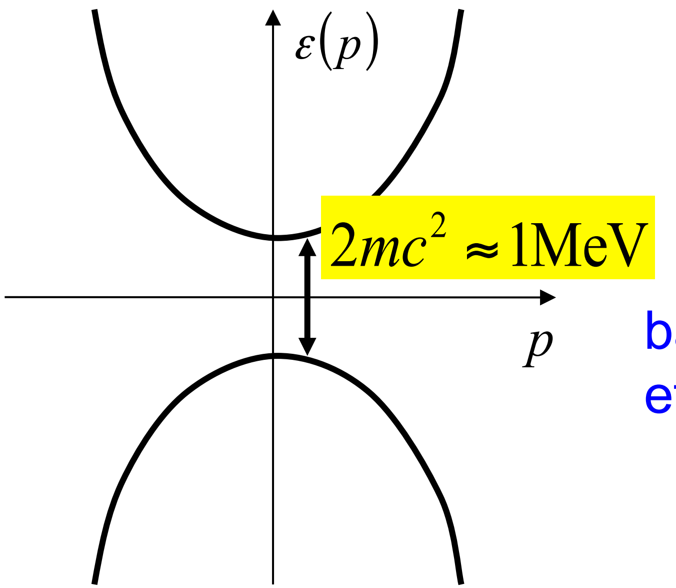
Basics on Spin-Orbit Interaction

Relativistic (Einstein) correction from Dirac equation:

$$\boxed{H_{so} = \frac{1}{2mc^2}\vec{s}\cdot\left(\nabla V \times \frac{\vec{p}}{m}\right)}$$

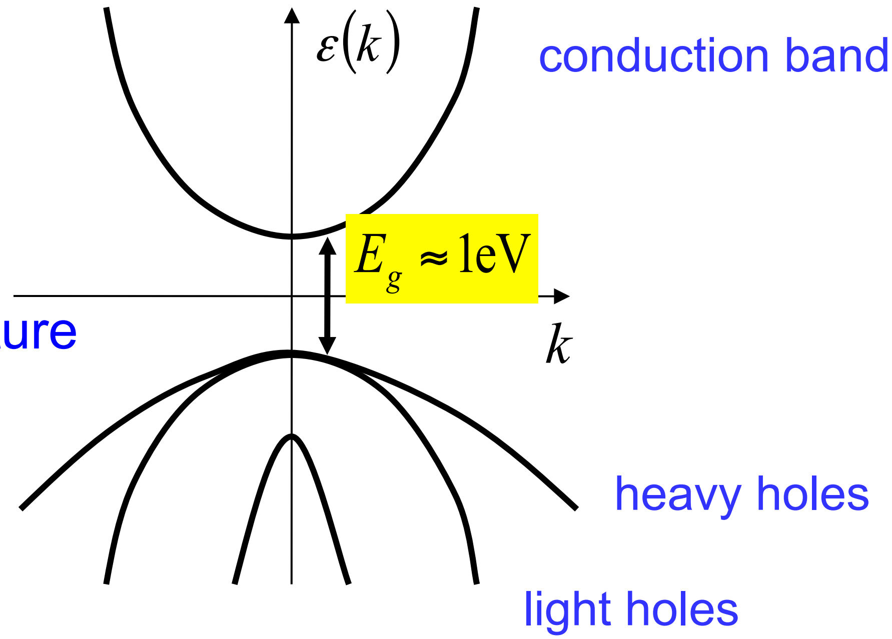
Thomas term (→ Rashba SOI)

free electrons



➔ bandstructure effects

semiconductor (zinc-blende)



Spin-Orbit Interaction in GaAs Quantum Dots (2DEG):

$$H_{SO} = \alpha\left(p_x\sigma_y - p_y\sigma_x\right)$$ ← Rashba SOI

$$-\beta\left(p_x\sigma_x - p_y\sigma_y\right)$$ ← Dresselhaus SOI

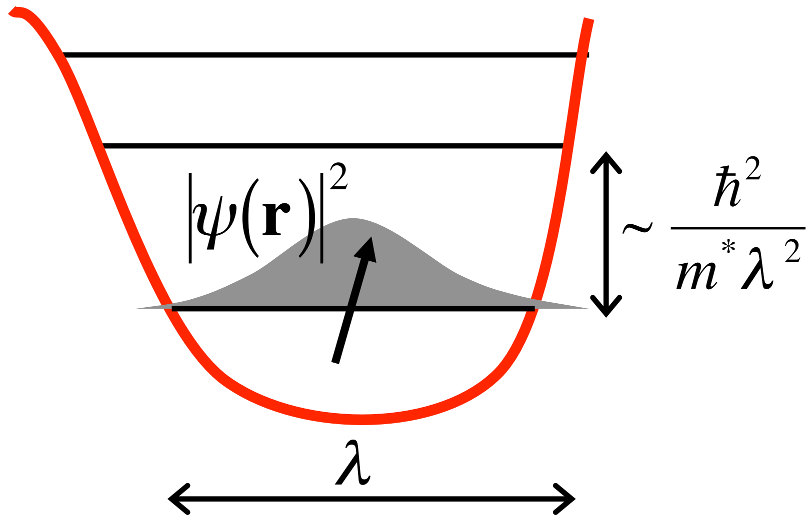
Model Hamiltonian:

piezoelectric & deformation
acoustic

$$H = H_{dot} + H_Z + H_{SO} + U_{el-ph}(t)$$

$$H_{dot} = \frac{p^2}{2m^*} + U(\mathbf{r}/\lambda)$$

$$H_Z = \frac{1}{2} g\mu_B \mathbf{B}\cdot\sigma$$



$$U_{el-ph}(t) = \ldots$$ ← any potential fluctuation, *e.g.*, phonons

Electron-phonon interaction (quasi-2D)

$$U_{el-ph} = \sum_{\mathbf{q},j} \frac{F(q_z) e^{i\mathbf{q}_{\parallel}\mathbf{r}}}{\sqrt{2\rho_c \omega_{qj}/\hbar}} \left(e\beta_{\mathbf{q}j} - iq\Xi_{\mathbf{q}j}\right)\left(b^{+}_{-\mathbf{q}j} + b_{\mathbf{q}j}\right)$$

- piezo-electric interaction:

$$\beta_{\mathbf{q}j} = \frac{2\pi}{q^2\kappa} \beta^{\mu\nu\varpi} q_{\mu} \left(q_{\nu} e^{(j)}_{\varpi}(\mathbf{q}) + q_{\varpi} e^{(j)}_{\nu}(\mathbf{q})\right) \qquad \mathbf{q} = \left(\mathbf{q}_{\parallel}, q_z\right)$$

- deformation potential interaction:

$$\Xi_{\mathbf{q}j} = \frac{1}{2q} \Xi^{\mu\nu} \left(q_{\mu} e^{(j)}_{\nu}(\mathbf{q}) + q_{\nu} e^{(j)}_{\mu}(\mathbf{q})\right)$$

for GaAs: $\Xi_{\mathbf{q}j} = \Xi_0 \delta_{j,1}$ and $\beta^{\mu\nu\varpi} = \begin{cases} h_{14}, & \mu\nu\varpi = xyz\,(cyclic) \\ 0, & \text{otherwise} \end{cases}$

Quantum well form-factor $F(q_z)$:

$$F(q_z) = \int dz\, e^{iq_z z} |\psi(z)|^2$$

• parabolic quantum well:

$$\psi(z) = \pi^{-1/4} d^{-1/2} e^{-z^2/2d^2} \quad \Rightarrow \quad F(q_z) = e^{-q_z^2 d^2/4}$$

• rectangular quantum well ($0<z<d$):

$$\psi(z) = \sqrt{\frac{2}{d}} \sin\frac{\pi z}{d} \quad \Rightarrow \quad F(q_z) = \frac{e^{iq_z d} - 1}{iq_z d} \frac{1}{1-(q_z d/2\pi)^2}$$

• triangular quantum well (Fang-Howard approx.):

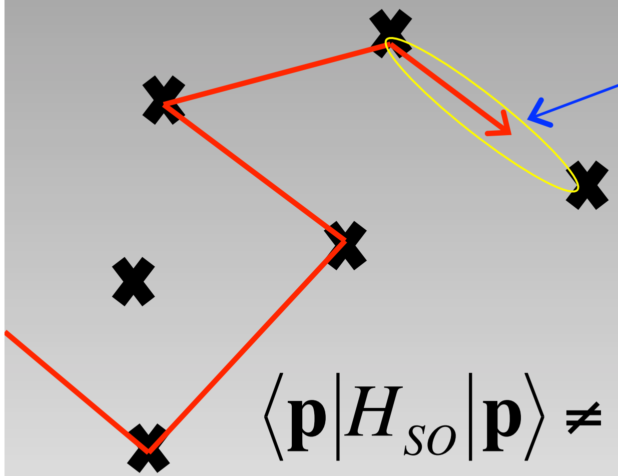
$$\psi(z) = \sqrt{\frac{b^3}{2}} z e^{-zb/2}, \quad b = \left(\frac{33e^2 m^* n_0}{8\hbar^2 \varepsilon \varepsilon_0}\right)^{1/3}, \quad \Rightarrow \quad F(q_z) = \frac{1}{(1-iq_z/b)^3}$$

Parameter regime:

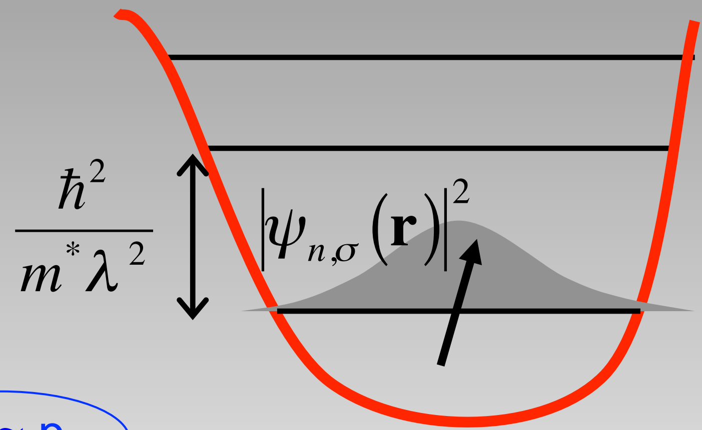
1. $\lambda << \lambda_{SO}, \quad \lambda_{SO} = \hbar / m^* \beta$ (typically: $\lambda \sim 100$ nm, and $\lambda_{SO} \sim 1\text{-}10\ \mu$m)

spin-orbit interaction in quantum dot is *weak*

2D bulk:



quantum dot:



$$\langle \mathbf{p} | H_{SO} | \mathbf{p} \rangle \neq 0$$

$$\langle \mathbf{p} | H_{SO} | \mathbf{p}' \rangle = 0$$

$H_{SO} \sim p_{x,y}$

$$\langle n | H_{SO} | n \rangle \propto \langle n | [H_0, x] | n \rangle = 0$$

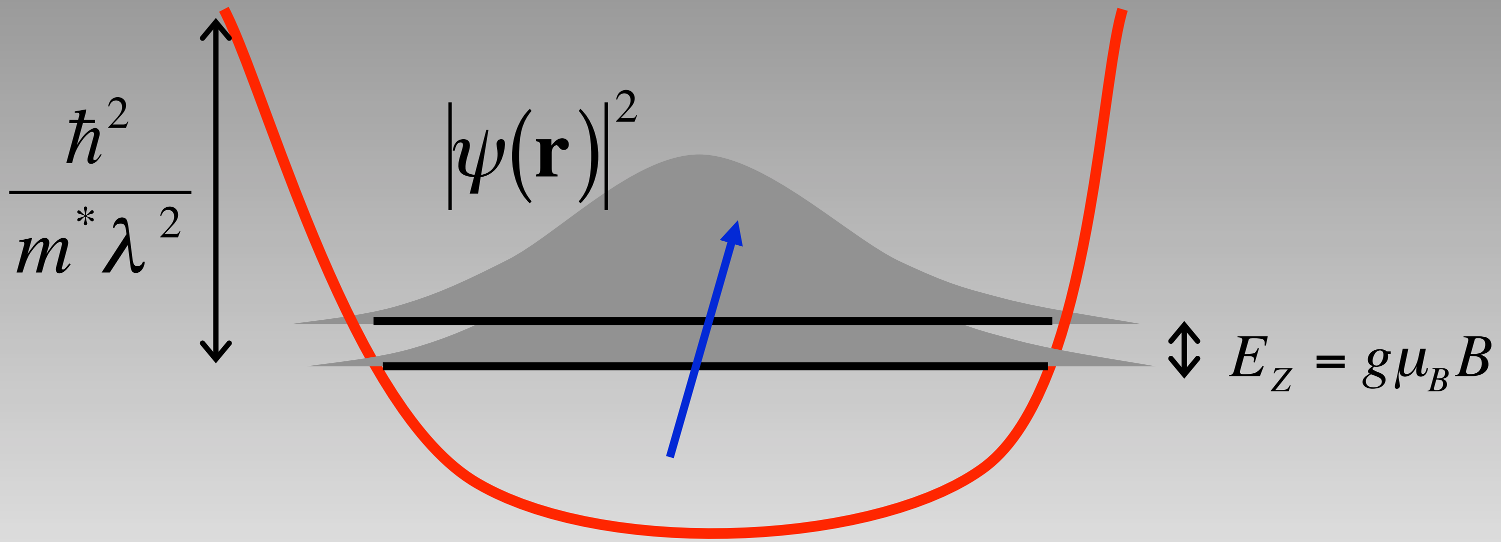
$$\langle n | H_{SO} | m \rangle \neq 0: \quad \frac{\langle n | H_{SO} | m \rangle}{E_n - E_m} \sim \frac{\lambda}{\lambda_{SO}} << 1$$

Parameter regime:

1. $\lambda << \lambda_{SO}, \quad \lambda_{SO} = \hbar / m^* \beta$ (typically: λ~100 nm, and λ_{SO}~1-10 μm)

2. $k_B T << \hbar^2 / m^* \lambda^2$ (typically: $h^2/m^*\lambda^2 \sim$ 1 meV ≈ 10 K)

the dot stays in its orbital ground state



Spin = Kramers *doublet* of ground state

Parameter regime:

1. $\lambda << \lambda_{SO}, \quad \lambda_{SO} = \hbar / m^* \beta$ (typically: λ~100 nm, and λ_{SO}~1-10 μm)
2. $k_B T << \hbar^2 / m^* \lambda^2$ (typically: $h^2/m^*\lambda^2 \sim 1$ meV $\approx$ 10 K)
3. $g\mu_B B << \hbar^2 / m^* \lambda^2$

In this regime, we find effective spin Hamiltonian (~ H_{SO}, U_{e-ph}):

$$H_{\text{eff}} = \frac{1}{2} g\mu_B \left(\mathbf{B} + \delta\mathbf{B}(t)\right) \cdot \sigma,$$

$$\delta\mathbf{B}(t) = 2\mathbf{B} \times \Omega(t),$$

→ no dephasing! i.e. $T_2 = 2T_1$

Golovach, Khaetskii & DL, PRL 93 (2004)

where $\Omega(t) = \langle \psi | \left[\left(\hat{L}_d^{-1} \xi\right), U_{el-ph}(t) \right] | \psi \rangle,$

$$\xi = \left(y'/\lambda_-, x'/\lambda_+, 0\right), \qquad 1/\lambda_\pm = m^*(\beta \pm \alpha)/\hbar, \qquad \begin{cases} x' = (x+y)/\sqrt{2} \\ y' = -(x-y)/\sqrt{2} \end{cases}$$

Bloch Equations (Born approx. in δB):

$$\boxed{\langle \dot{\mathbf{S}} \rangle = g\mu_B \mathbf{B} \times \langle \mathbf{S} \rangle - \Gamma \langle \mathbf{S} \rangle + \mathbf{Y}}$$

(spin: Kramers doublet)

$\left.\begin{array}{l} \tau_c = \lambda / s = 100 \text{ ps} \ll T_{1,2} \\ \text{\& super-Ohmic spectrum} \end{array}\right\}$ ➔ Born-Markov approx. ok

Decay tensor:

$$\boxed{\Gamma_{ij} \propto J_{ij}(w) = \frac{g^2 \mu_B^2}{2\hbar^2} \int_0^\infty \langle \delta B_i(0) \delta B_j(t) \rangle e^{-iwt} dt}$$

spectral function

decay: $\Gamma = \Gamma^r + \Gamma^d,$

relaxation: $\Gamma_{ij}^r = \delta_{ij}(\delta_{pq} - l_p l_q) J_{pq}^+(\omega) - (\delta_{ip} - l_i l_p) J_{pj}^+(\omega) - \delta_{ij} \varepsilon_{kpq} l_k I_{pq}^-(\omega) + \varepsilon_{ipq} l_p I_{qj}^-(\omega),$

dephasing: $\Gamma_{ij}^d = \delta_{ij} l_p l_q J_{pq}^+(0) - l_i l_p J_{pj}^+(0)$ ➔ **0**

$$J_{ij}^\pm(w) = \mathrm{Re}\left[J_{ij}(w) \pm J_{ij}(-w)\right], \quad I_{ij}^\pm(w) = \mathrm{Im}\left[J_{ij}(w) \pm J_{ij}(-w)\right]$$

Relaxation rate:

super-Ohmic: ~z^3

Bose function

$$\frac{1}{T_1} \propto \mathrm{Re}\, J_{XX}(z) = \frac{\omega^2 z^3 (2n_z + 1)}{(2\Lambda_+ m^* \omega_0^2)^2} \sum_j \frac{\hbar}{\pi \rho_c s_j^5} \int_0^{\pi/2} d\theta \sin^3\theta$$

$$\times e^{-(z\lambda\sin\theta)^2/2s_j^2} \left| F\left(\frac{|z|}{s_j}\cos\theta\right)\right|^2 \left(e^2 \overline{\beta_{j\theta}^2} + \frac{z^2}{s_j^2}\overline{\Xi_j^2}\right) \propto \lambda^2/\lambda^2{}_{SO}$$

$z \to \omega = g\mu_B B$

quantum well

piezo

deformation

Golovach, Khaetskii, Loss, PRL 93 (2004)

$$\frac{2}{\Lambda_\pm} = \frac{1-l_{x'}{}^2}{\lambda_-{}^2} + \frac{1-l_{y'}{}^2}{\lambda_+{}^2} \pm \sqrt{\left(\frac{1-l_{x'}{}^2}{\lambda_-{}^2} + \frac{1-l_{y'}{}^2}{\lambda_+{}^2}\right)^2 - \frac{4l_z{}^2}{\lambda_+{}^2\lambda_-{}^2}}$$

effective SO length

Relaxation rate:

super-Ohmic: ~z^3

Bose function

$$\frac{1}{T_1} \propto \mathrm{Re}\, J_{XX}(z) = \frac{\omega^2 z^3 (2n_z + 1)}{(2\Lambda_+ m^* \omega_0^2)^2} \sum_j \frac{\hbar}{\pi \rho_c s_j^5} \int_0^{\pi/2} d\theta \sin^3\theta$$

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$z \rightarrow \omega = g\mu_B B$ quantum well piezo deformation

$s_1 \approx 4.7 \times 10^3 m/s,\; s_2 = s_3 \approx 3.37 \times 10^3 m/s$ speed of sound

$$\sqrt{\overline{\Xi_j^2}} = \delta_{j,1} \Xi_0,\; \Xi_0 \approx 7eV,\; \sqrt{\overline{\beta_{1,\vartheta}^2}} = 3\sqrt{2}\pi h_{14} \kappa^{-1} \sin^2\vartheta \cos\vartheta,\; \sqrt{\overline{\beta_{2,\vartheta}^2}} = \sqrt{2}\pi h_{14}\kappa^{-1}\sin 2\vartheta,$$

$$\sqrt{\overline{\beta_{3,\vartheta}^2}} = 3\sqrt{2}\pi h_{14}\kappa^{-1}(3\cos^2\vartheta - 1)\sin\vartheta, \quad h_{14} \approx 0.16 C/m^2,\; \kappa \approx 13$$

$$\frac{2}{\Lambda_\pm} = \frac{1 - l_{x'}^2}{\lambda_-^2} + \frac{1 - l_{y'}^2}{\lambda_+^2} \pm \sqrt{\left(\frac{1 - l_{x'}^2}{\lambda_-^2} + \frac{1 - l_{y'}^2}{\lambda_+^2} \right)^2 - \frac{4 l_z^2}{\lambda_+^2 \lambda_-^2}}$$

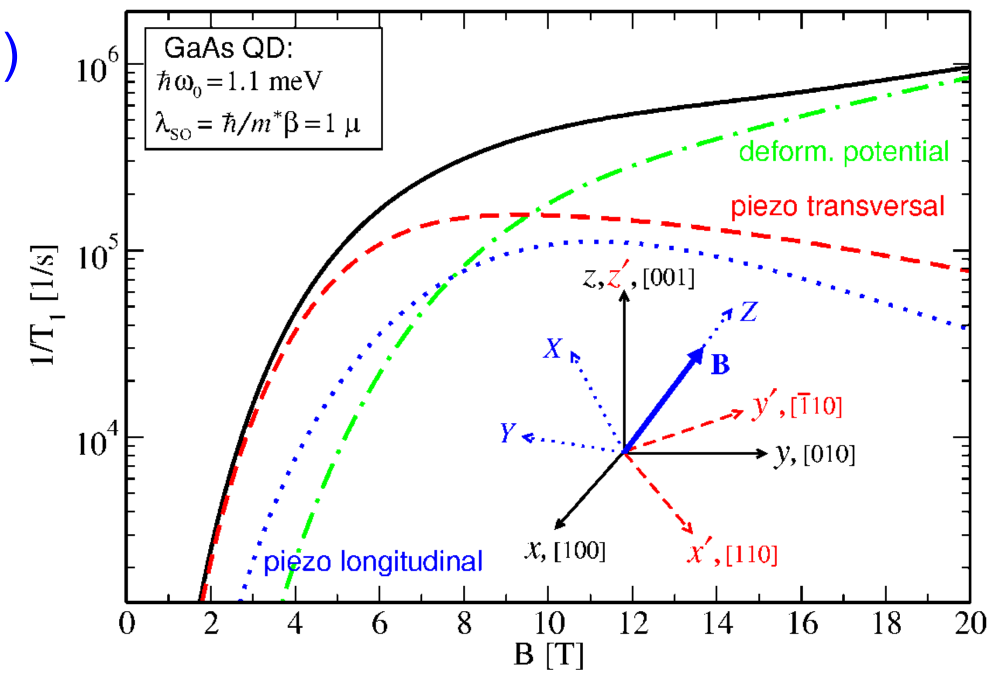
effective SO length

Spin relaxation rate $1/T_1$ for GaAs quantum dot

$$\frac{1}{T_1} \propto (g\mu_B B)^{2+2+1} \times \int_0^{\pi/2} d\theta \sin^k\theta\, e^{-(g\mu_B B\lambda\sin\theta)^2/2s_j^2}$$

$$\underbrace{\delta B^2 \propto B^2 \qquad \nu_{ph}(\omega) \propto \omega^2 \qquad H_{SO} \propto p_\alpha}$$

power-5 law for B< 3T (GaAs)



Golovach et al., PRL 93 (2004)

Numerical value of T_1 for GaAs parameters (13!):

$$\left(\hbar\omega_0, \lambda, d, \lambda_{SO} = \hbar/m^*\beta, \alpha, \kappa, \Xi_0, h_{14}, s_1, s_2 = s_3, \rho_c, m^*\right) =$$

$$\begin{pmatrix} 1.1\,\mathrm{meV}\ , 32\,\mathrm{nm}, 5\,\mathrm{nm}\ , 9\,\mu\mathrm{m}, 0, 13.1, 6.7\,\mathrm{eV}, 0.16\ \mathrm{C/m^2}, \\ 4.73\times10^5\,\mathrm{cm/s}, 3.35\times10^5\,\mathrm{cm/s}, 5.3\times10^3\,\mathrm{kg/m^3}, 0.067\,m_e \end{pmatrix}$$

Zumbuhl ea PRL 89 (276803) 2003

$$|g| = 0.43 \pm 0.04 - (0.0077 \pm 0.0020)B(T)$$

or with linear fit: $|g| = 0.29$ Hanson ea PRL 91 (196802) 2003

Theory: $\boxed{T_1 \approx 750\ \mu s, \quad \text{for B} = \ 8\text{T}}$ $\propto \lambda_{SO}{}^2 / \lambda^2$

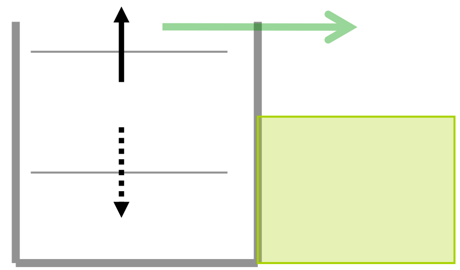
$T_1 = 550 - 1100\ \mu s$ due to uncertainties in *g* factor

$T_1 = 2.7\ \mathrm{ms}$ for λ_{SO}=17μm [Huibers ea PRL 83, 5090 (1999)]

Experiment: $\boxed{T_1^{\mathrm{exp.}} = 800\,\mu s @ 8T}$ Elzerman *et al.*, Nature 430, 431 (2004)

Read-out via spin-charge conversion:

DL&DiVincenzo 1998



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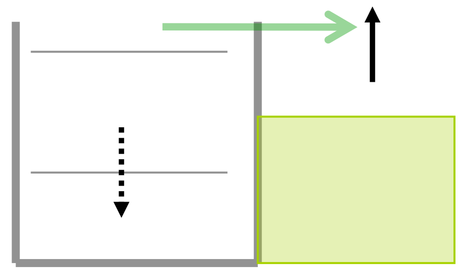

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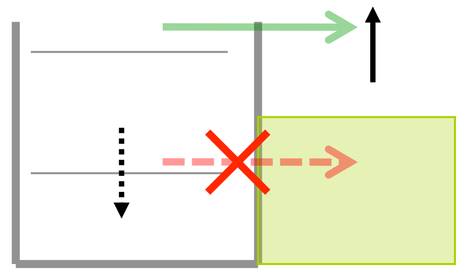
DL&DiVincenzo 1998



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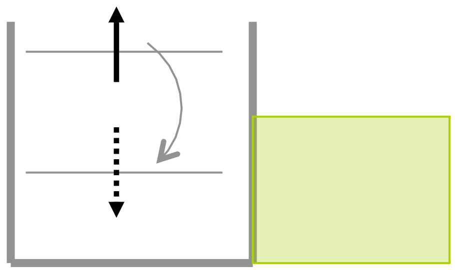
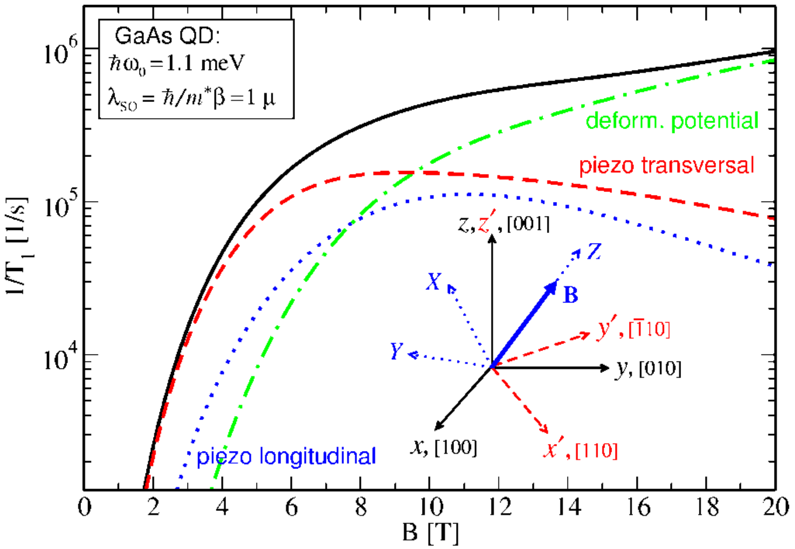


spin relaxation rates $1/T_1$:

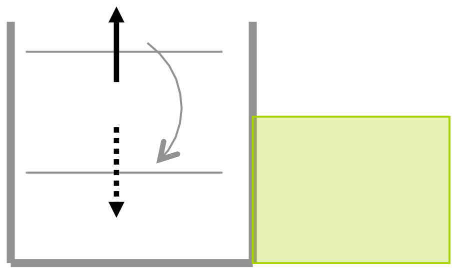
Theory: Basel, 2004



Golovach, Khaetskii, DL, PRL `04

Read-out via spin-charge conversion:

DL&DiVincenzo 1998



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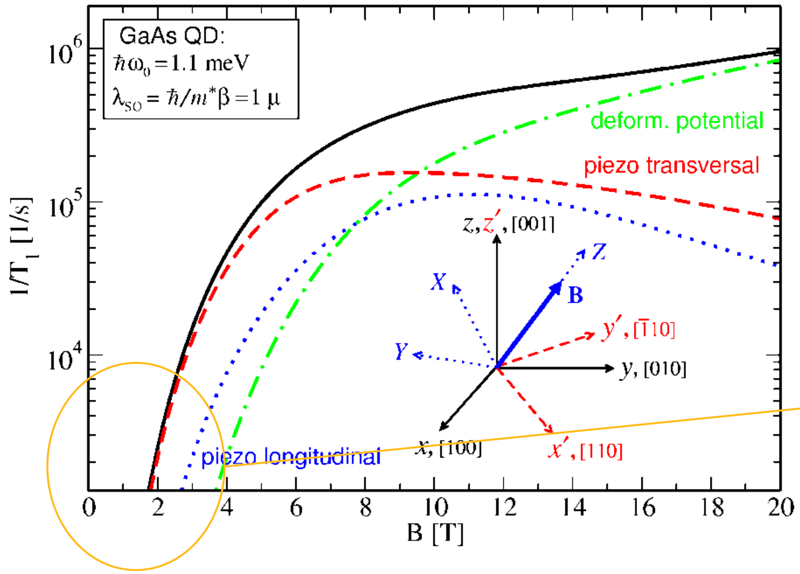
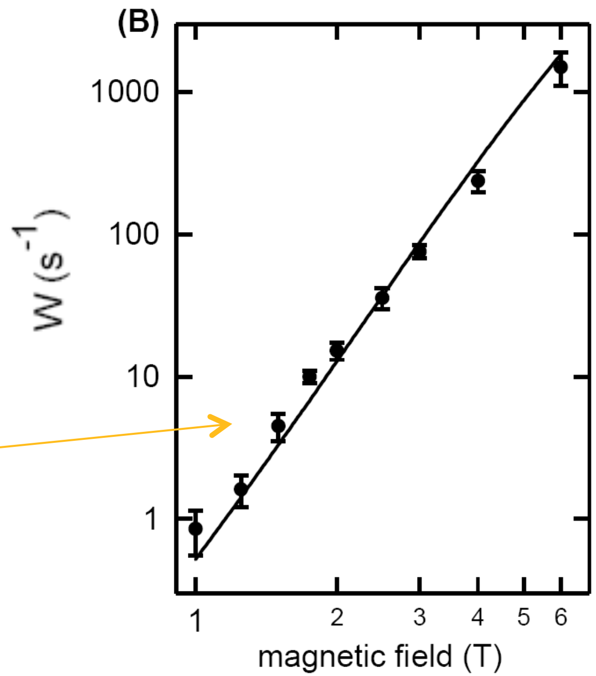


Experiment: MIT 2008



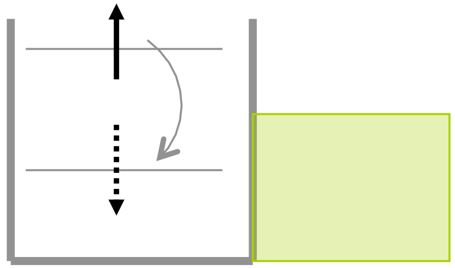
➔ prediction confirmed

Golovach, Khaetskii, DL, PRL `04

Amasha, Zumbuhl, et al., PRL `08

Read-out via spin-charge conversion:

DL&DiVincenzo 1998



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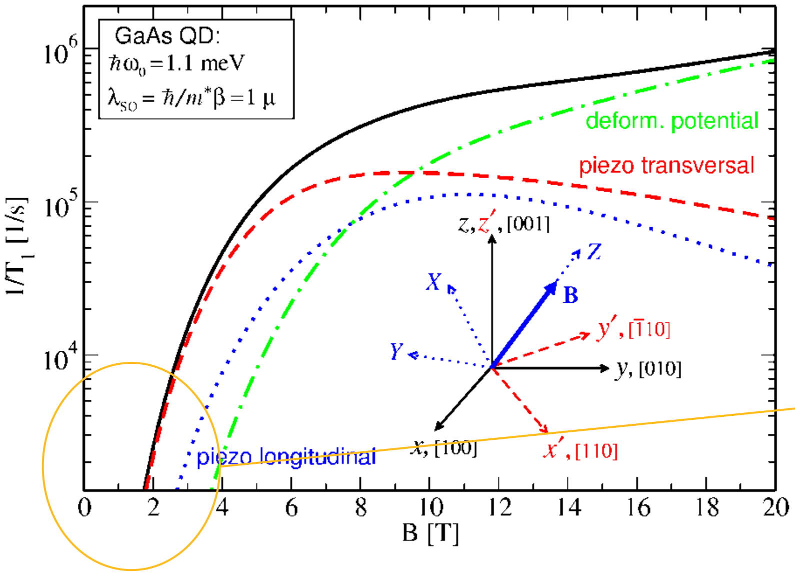

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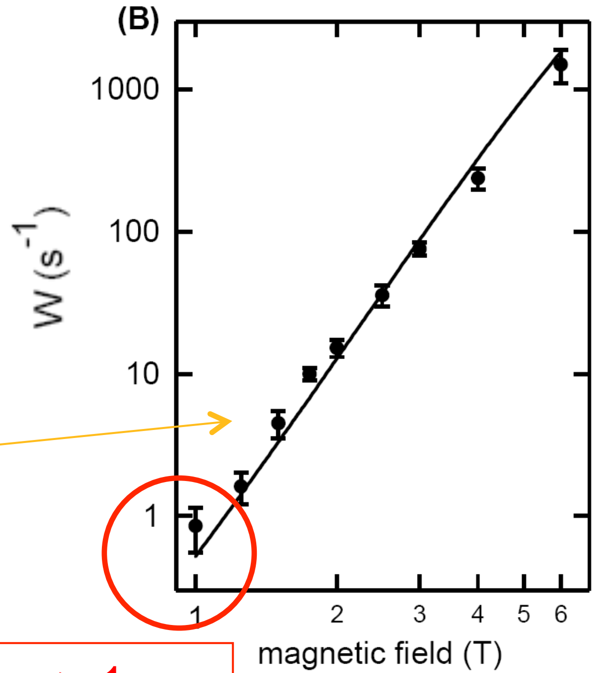
Golovach, Khaetskii, DL, PRL `04

Experiment: MIT 2008



> 1 sec

➔ record T_1 for GaAs

Amasha, Zumbuhl, et al., PRL `08

$1/T_{1,2}$ depends strongly on B-field direction ("magic angles"):

$$\boxed{\frac{1}{T_1} = \frac{f(\varphi,\theta,\alpha)}{T_1(\theta = \pi/2, \alpha = 0)}}$$

Golovach, Khaetskii, Loss
PRL 93, 016601 (2004)

"ellipsoid"

$$f(\varphi,\theta,\alpha) = \frac{1}{\beta^2}\left[\left(\alpha^2+\beta^2\right)\left(1+\cos^2\theta\right)+2\alpha\beta\sin^2\theta\sin 2\varphi\right]$$

Rashba and Dresselhaus interfere! *)

Special case:

$\alpha = \beta,\ \ \theta = \pi/2,\ \ \varphi = 3\pi/4$ → $T_1 \to \infty$

exact!

*) Schliemann, Egues, DL, PRL `03

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Rashba and Dresselhaus interfere! *)

Experiment ?!

Special case:

$$\alpha = \beta,\ \ \theta = \pi/2,\ \ \varphi = 3\pi/4 \quad \rightarrow \quad T_1 \to \infty$$

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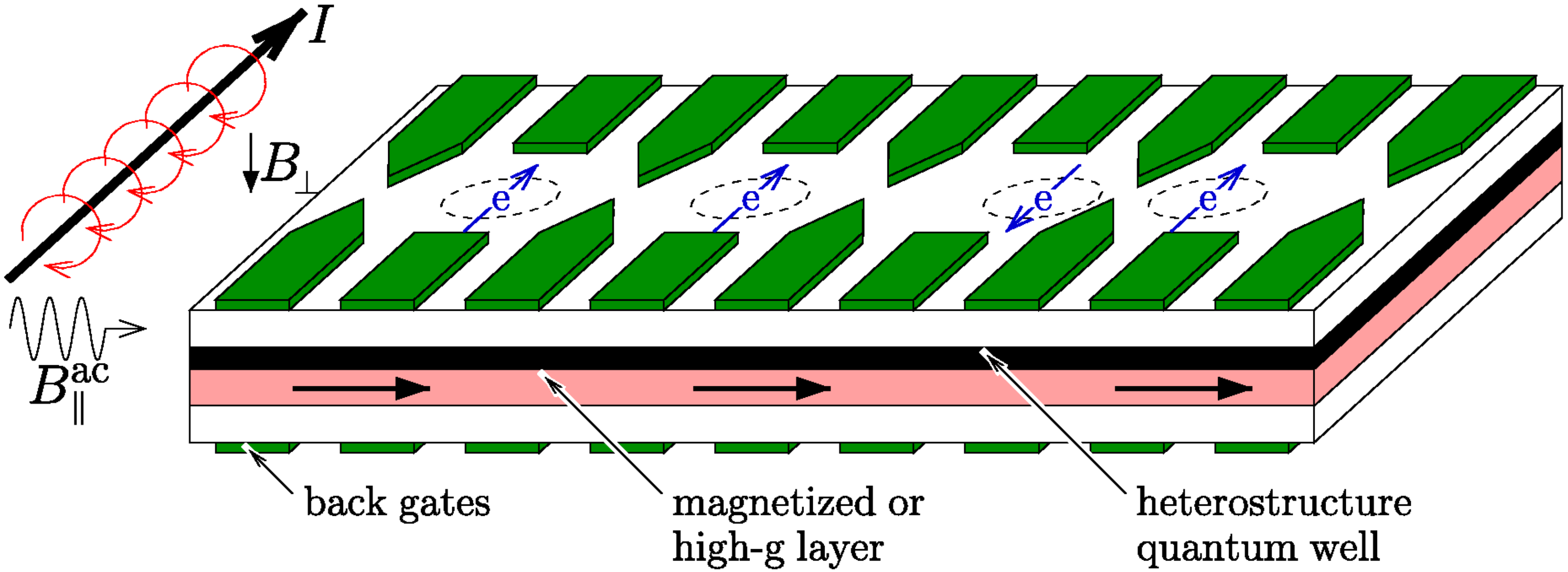
*) Schliemann, Egues, DL, PRL `03

Scaling-up of spin qubits ➔ quantum dot array

DL & DiVincenzo, PRA **57** (1998) 120



$$H = \sum_{\langle ij \rangle} J_{ij}(t)\mathbf{S}_i \cdot \mathbf{S}_j + \sum_i (g_i \mu_B \mathbf{B}_i)(t) \cdot \mathbf{S}_i$$

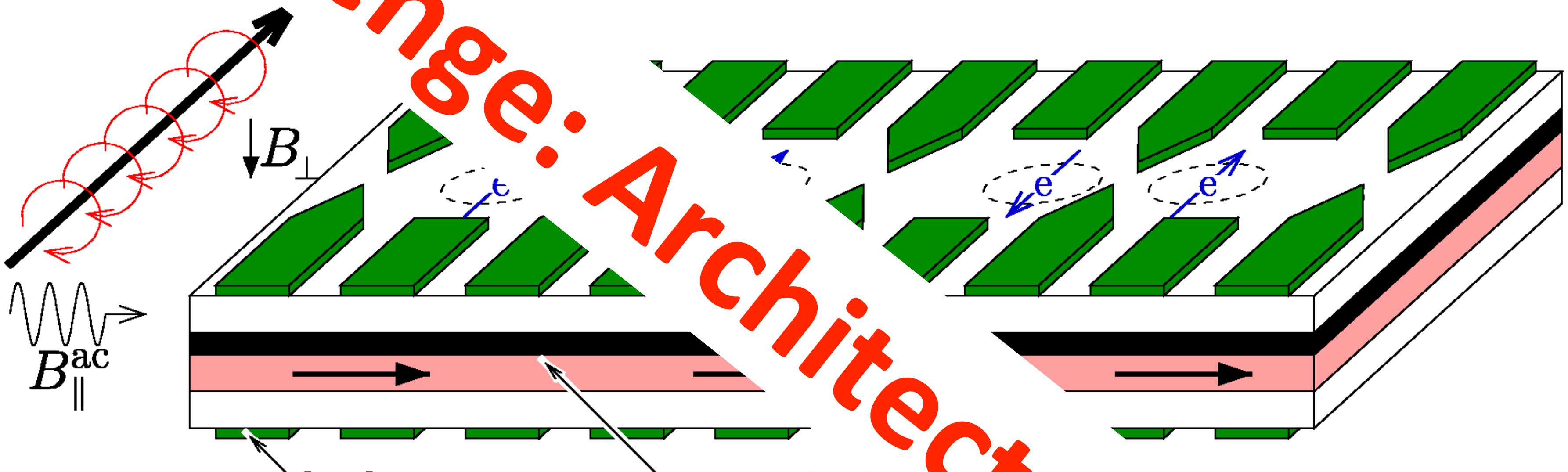
n.n. exchange

local Zeeman

Challenge: Architecture !?

…ng-up of spin qubits ➔ quantum dot array

… & DiVincenzo, PRA **57** (1998) 120



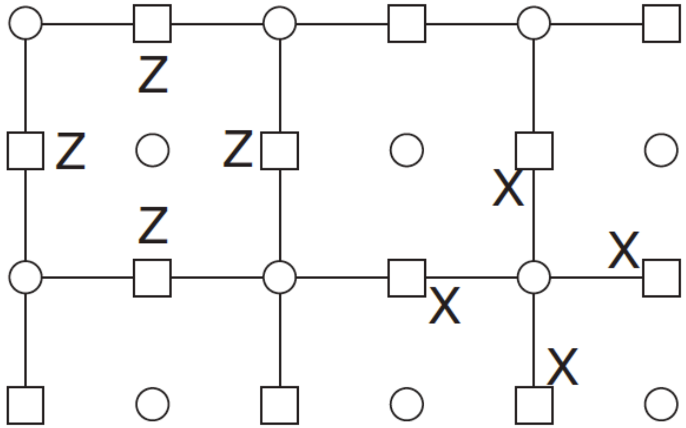
$$H = \sum_{\langle ij\rangle} J_{ij}(t)\,\mathbf{S}_i\cdot\mathbf{S}_j + \sum_i (g_i\mu \ldots)\cdot\mathbf{S}_i$$

n.n. exchange

local Zee…

Quantum Computing with **2D** Surface Code

Bravyi and Kitaev, arXiv:quant-ph/9811052
Raussendorf and Harrington, PRL 98,190504 (2007)



Surface code has error threshold of 1.1 % !

Wang, Fowler, and Hollenberg, PRA 83, 020302 (2011)

Quantum Computing with 2D Surface Code

Bravyi and Kitaev, arXiv:quant-ph/9811052
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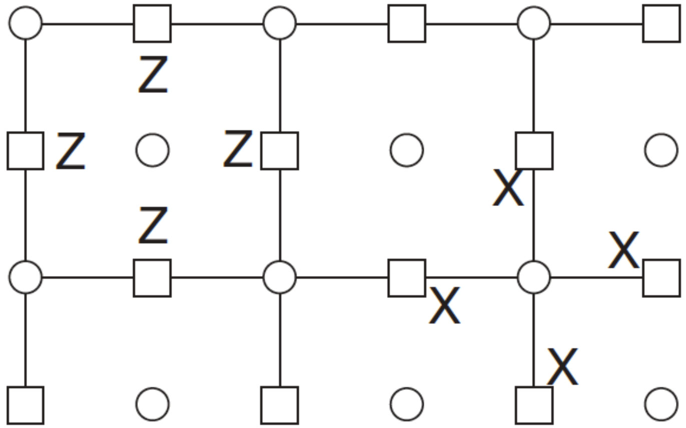


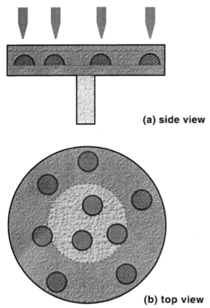
Surface code has error threshold of 1.1 % !

Wang, Fowler, and Hollenberg, PRA 83, 020302 (2011)

Need enough space to accommodate all the wirings
➔ need entanglement over 'long distances' of order μm

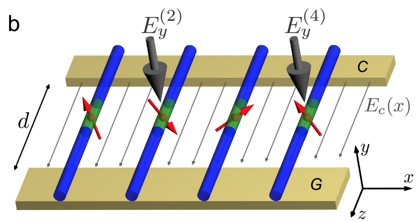
Long-distance coupling of spin qubits

Electromagnetic cavity & waves



Imamoglu et al.,
PRL 83, 4204 (1999)

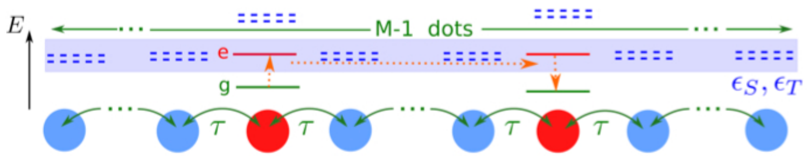
optical



microwaves

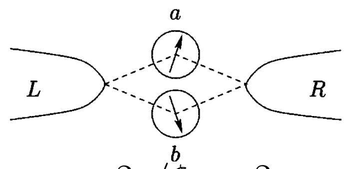
Trif, Golovach & DL, PRB 77, 045434 (2008)
Kloeffel et al., PRB 88, 241405 (2013)

PACT



Stano et al., PRB 92, 075302 (2015)

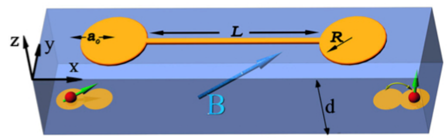
superconductor (CAR)



$$J(r) = J_0 e^{-2r/\xi} \sin^2(k_F r)/(k_F r)^2$$

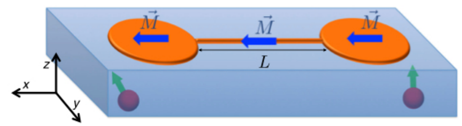
Choi, Bruder & DL, PRB 62, 13569 (2000)

Floating metallic gate



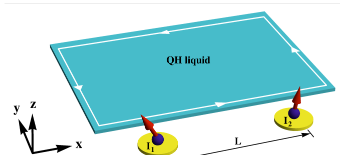
Trifunovic et al., PRX 2, 011006 (2012)

Dipolar ferromagnet



Trifunovic et al., PRX 3, 041023 (2013)

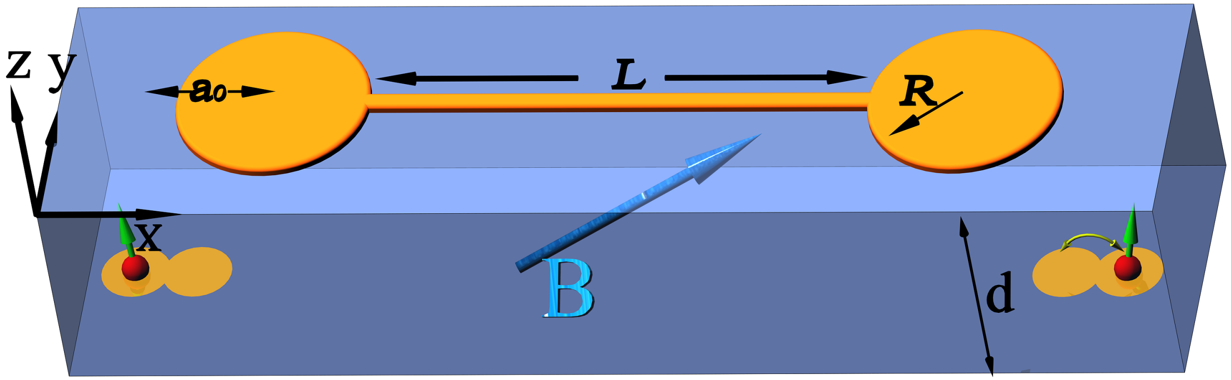
QHE edge states



Yang et al., PRB 93, 075301 (2016)

Long-distance entanglement via floating gates

Trifunovic, Dial, Trif, Wootton, Abebe, Yacoby, DL, Phys. Rev. X 2, 011006 (2012)

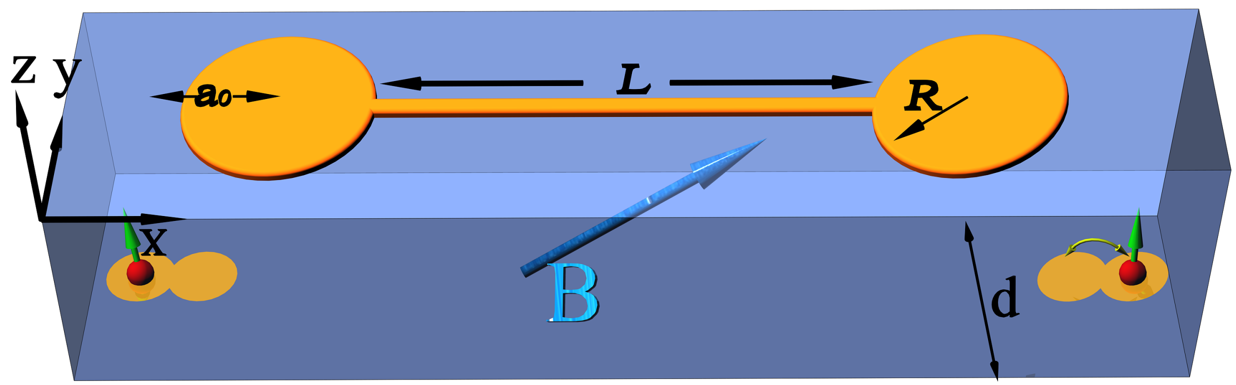


Electrostatics + spin orbit interaction ➔ effective exchange:

$$\boxed{H_{s-s} = J_x \sigma_1^x \sigma_2^x + J_y \sigma_1^y \sigma_2^y}$$

Long-distance entanglement via floating gates

Trifunovic, Dial, Trif, Wootton, Abebe, Yacoby, DL, Phys. Rev. X 2, 011006 (2012)



Electrostatics + spin orbit interaction ➔ effective exchange:

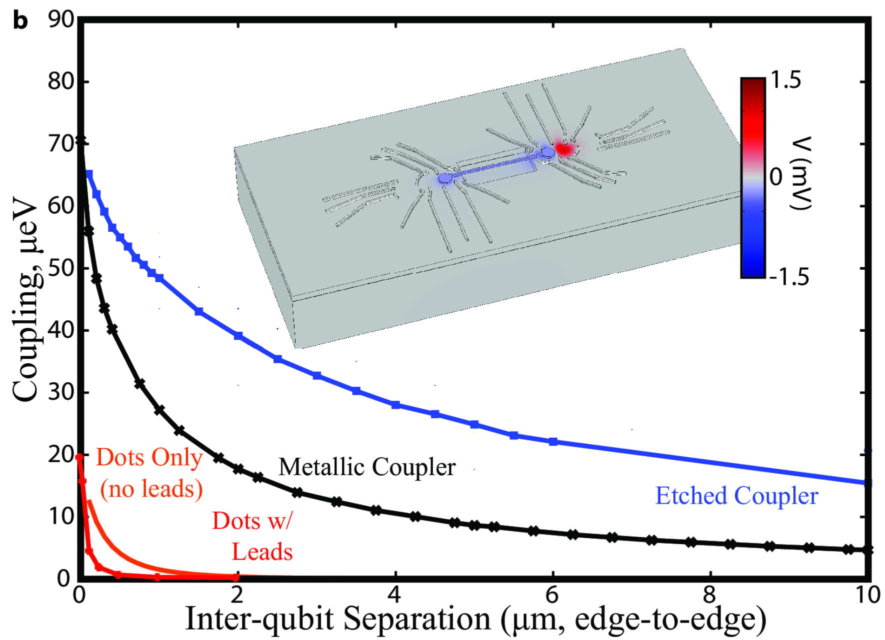
$$J \simeq \frac{\pi \alpha_q \alpha_C}{4} \left(\frac{\partial q_{ind}}{\partial \tilde{x}} \right)^2_{\boldsymbol{r}=0} \left(\frac{E_Z}{\omega_x} \right)^2 \left(\frac{\lambda}{\lambda_{SO}} \right)^2 \hbar \omega_x \sim 1\text{-}100\ \mu\text{eV}$$

$$\tau_{\text{switching}} = \hbar / J \approx 1\text{ns} \ - \ 10\text{ps}$$

Numerics

Trifunovic, Dial, Trif, Wootton, Abebe, Yacoby, DL, Phys. Rev. X 2, 011006 (2012)

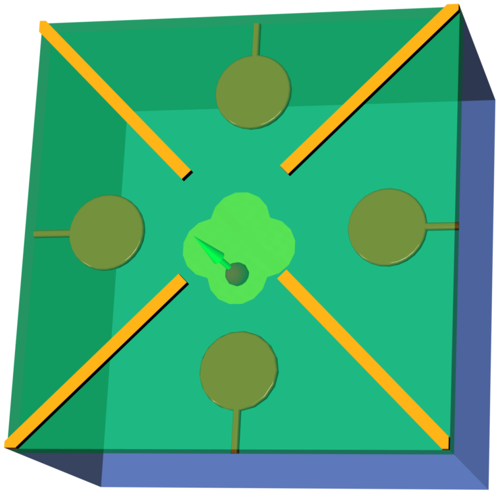
Long-range electrostatic coupling via metal or 2DEG channel

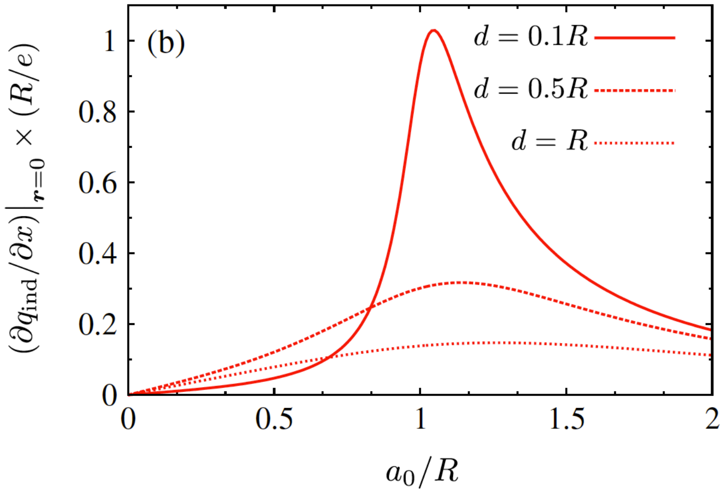


Note: gradient $dV(r)/dr$ matters [and not $V(r)$]!

Switching the coupling on/off

Trifunovic et al., PRX 2, 011006 (2012)

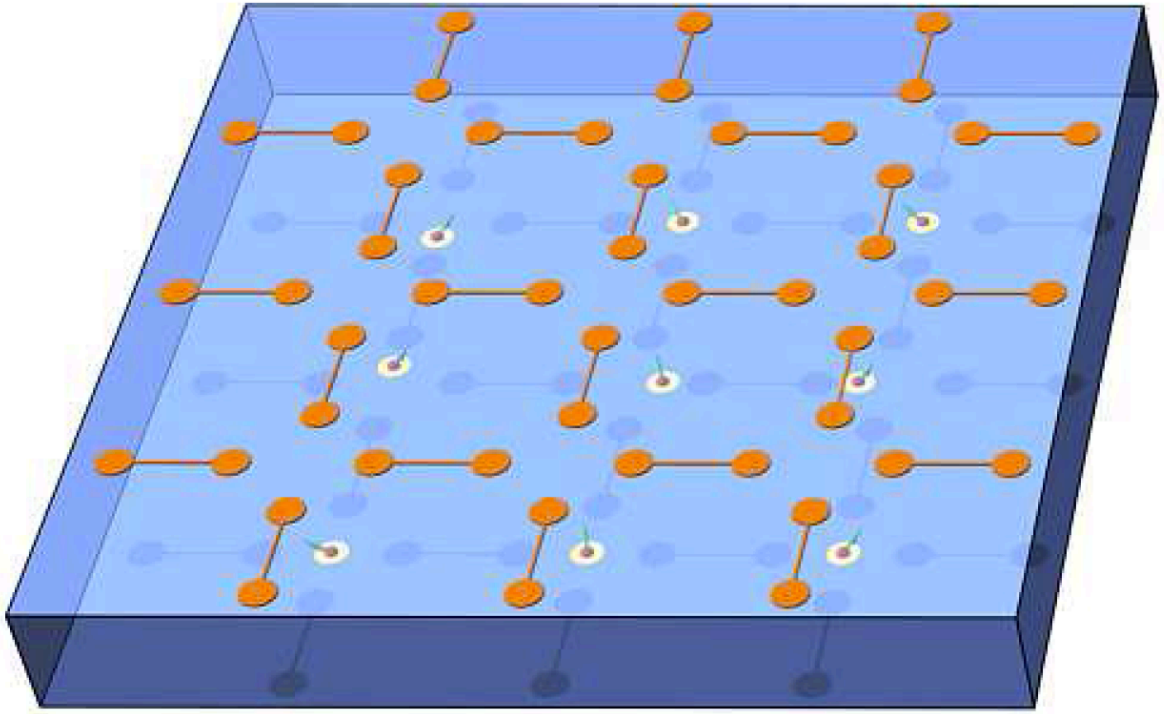




Control over the coupling is achieved by moving the electron to one of five possible spots

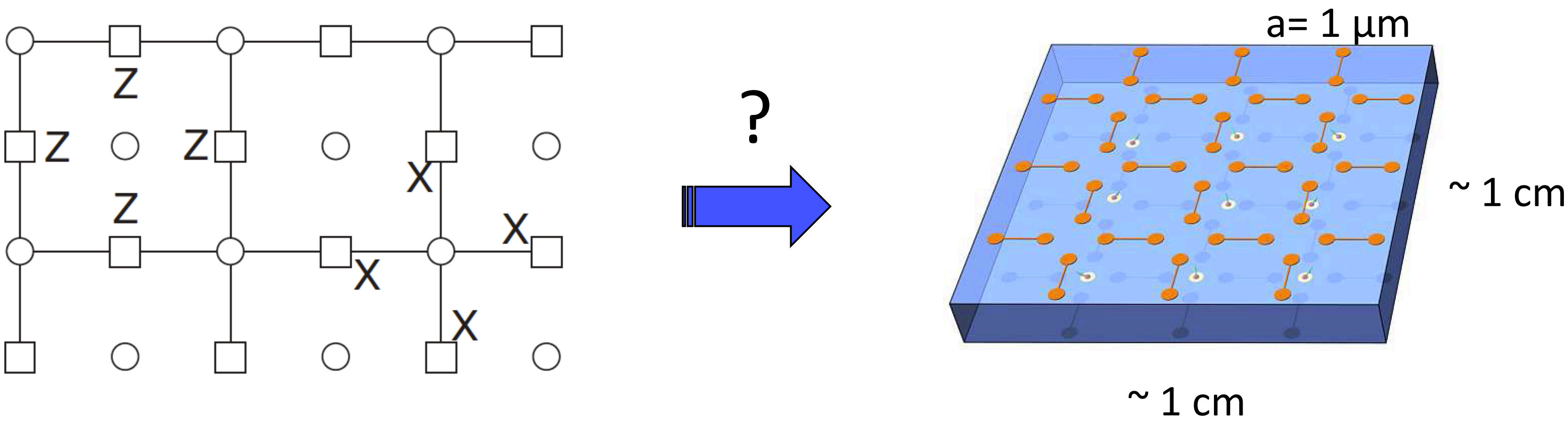
Note: in idle state the gate fluctuations (Nyquist noise) lead to standard but small spin decoherence

2D Network with Floating Gates



Trifunovic et al., PRX 2, 011006 (2012)

Quantum Chip Size in Surface Code Architecture

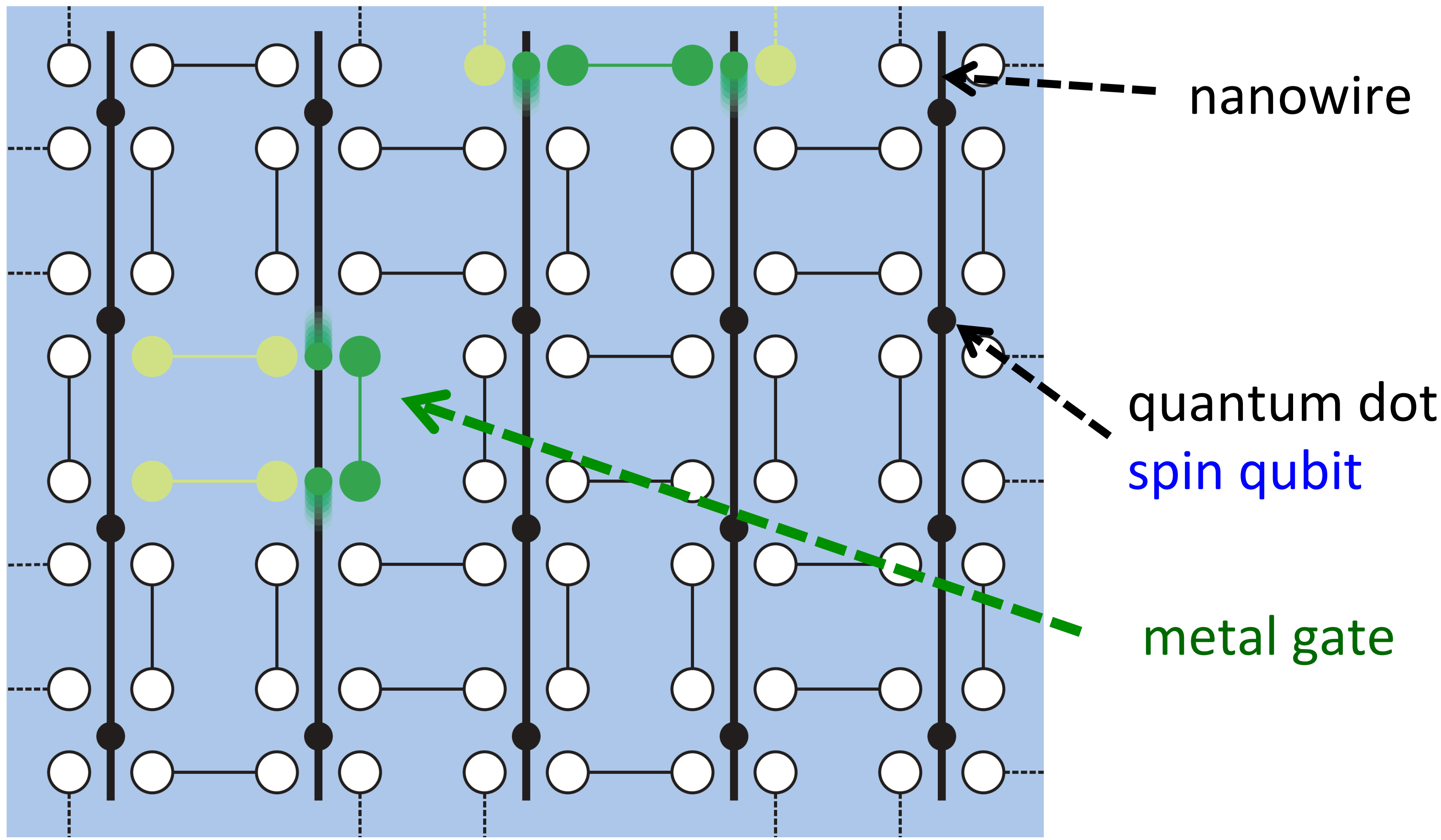


➔ powerful quantum computer of size 1 cm^2 with 10^8 spin qubits
GHz clock-speed ➔ can factor a 2000-bit number (RSA key) in 26h

error threshold of ca. 1 % – 18 % (perfect measurement)*

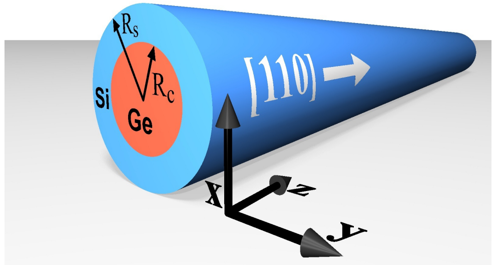
*Error correction: **James Wootton** and DL, PRL 109, 160503 (2012)

Scalable Quantum Computer with Nanowires



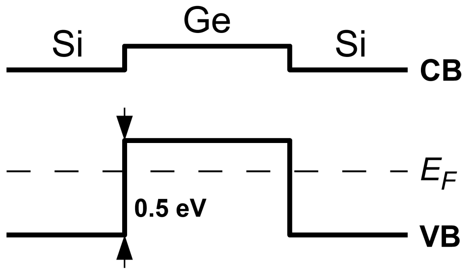
Trifunovic et al., PRX 2, 011006 (2012)

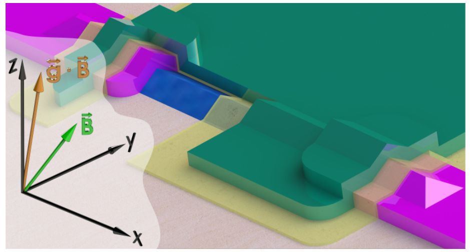
Hole Spin Qubits in Ge/Si Nanowires



core/shell Ge/Si

Lauhon *et al.*, Nature (2002)
Lu *et al.*, PNAS USA (2005)





Ge hut wire (triangular)
Watzinger et al.,
NanoLett., 16, 6879 (2016)

Strongly confined holes inside the Ge core

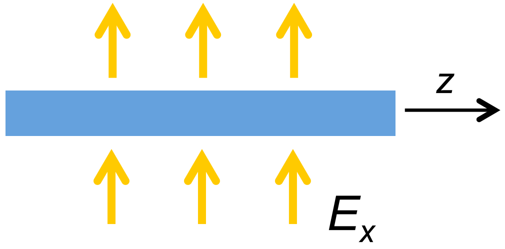
Remarkable experimental progress by many research groups:

Bakkers, Brauns, Fuhrer, Katsaros, Kuemmeth, Lieber, Lu, Marcus, McIntyre, Rastelli, Roddaro, Tutuc, van der Wiel, Watzinger, Zwanenburg,…

Theory: Kloeffel, Trif, and Loss, PRB 84, 195314 (2011)

Electric Field Effects

Kloeffel/Trif/Loss, PRB **84**, 195314 (2011)



Electric field E_x along x *perpendicular to wire*

Two key mechanisms for holes

Direct dipolar coupling

$$H_{\rm ed} = -eE_x x$$

First order effect,
from shift in potential energy

$\equiv H_{\rm DR}$

‘Direct Rashba SOI’
DRSOI

Analysis
$H_{\rm LK}$

Standard Rashba SOI

$$H_{\rm so} = \alpha E_x (k_y J_z - k_z J_y)$$

Third order effect,
from coupling to higher bands

$$\alpha \propto 1/(\text{band gap})^2$$

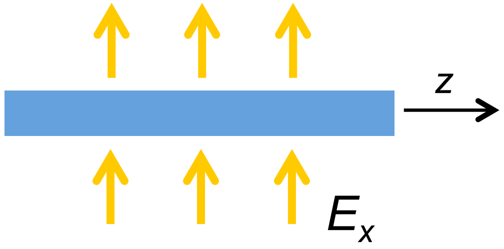
$\equiv H_{\rm R}$

RSOI

Note: mixing of HH and LH is crucial for DRSOI

Electric Field Effects

Kloeffel/Trif/Loss,
PRB **84**, 195314 (2011)



Electric field E_x along x *perpendicular to wire*

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‘Direct Rashba SOI’
DRSOI

Analysis
H_{LK}

Standard Rashba SOI

$$H_{\mathrm{so}} = \alpha E_x (k_y J_z - k_z J_y)$$

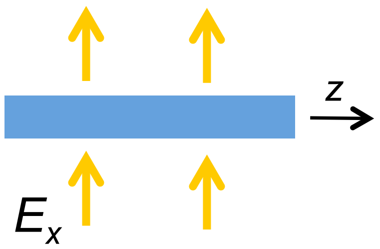
...ird order effect,
... to higher bands
α ... gap$)^2$

negligible!

RSOI

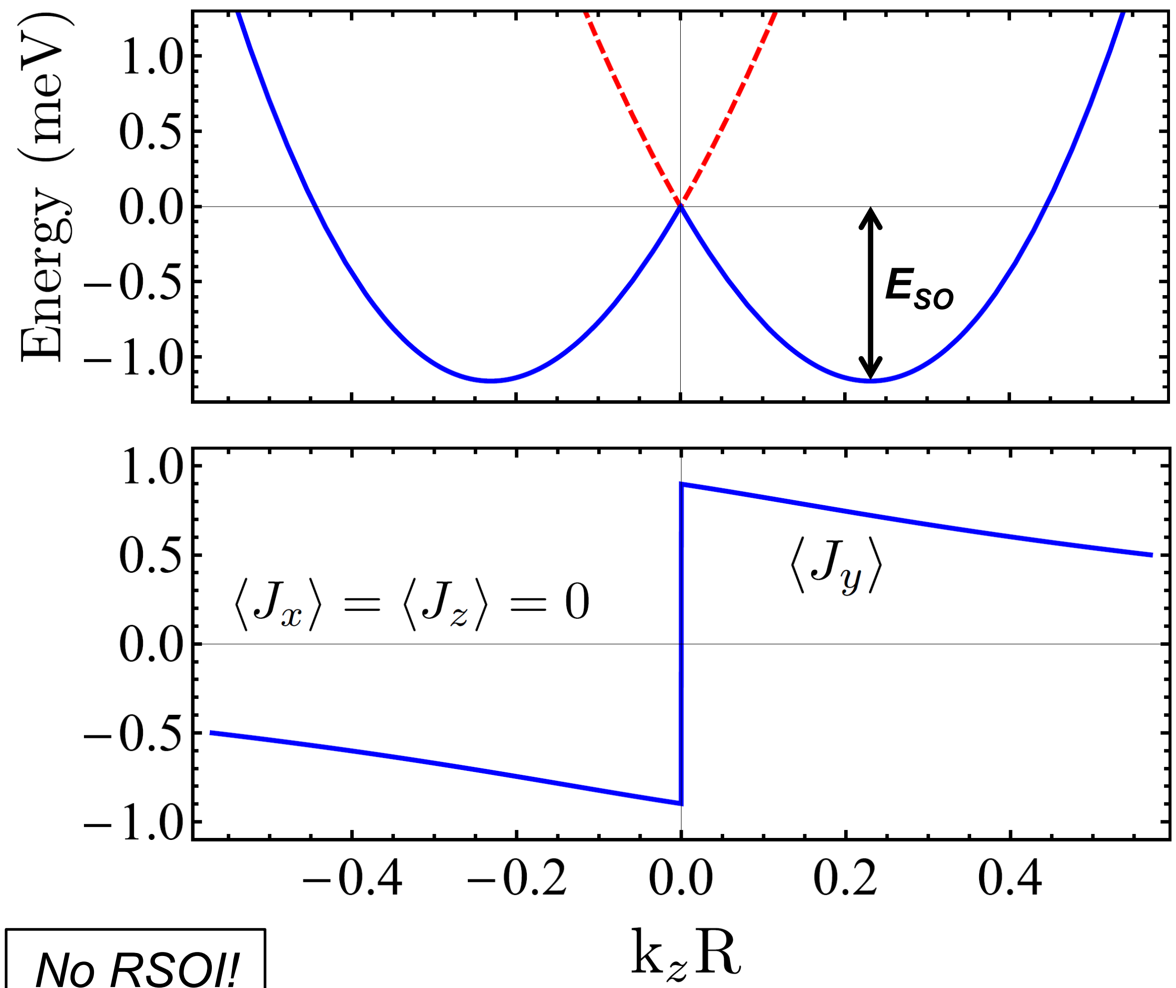
Note: mixing of HH and LH is crucial for DRSOI

Helical Hole States in Ge/Si NW

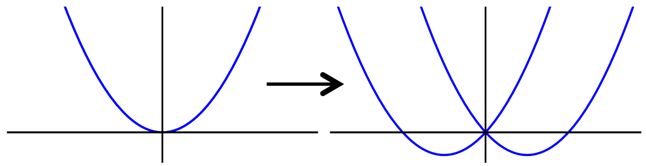


Typical wire:
R_c = 5.0 nm
R_s = 6.5 nm

E_x = 6 V/µm



Compare to RSOI in conduction band:



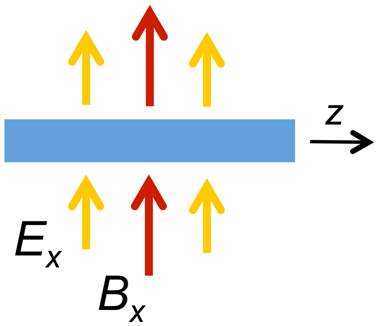
E_{SO} > 1.0 meV ➔10-100 times bigger than in InAs

Fasth *et al.*, PRL (2007)

Kloeffel/Trif/Loss, PRB **84**, 195314 (2011)

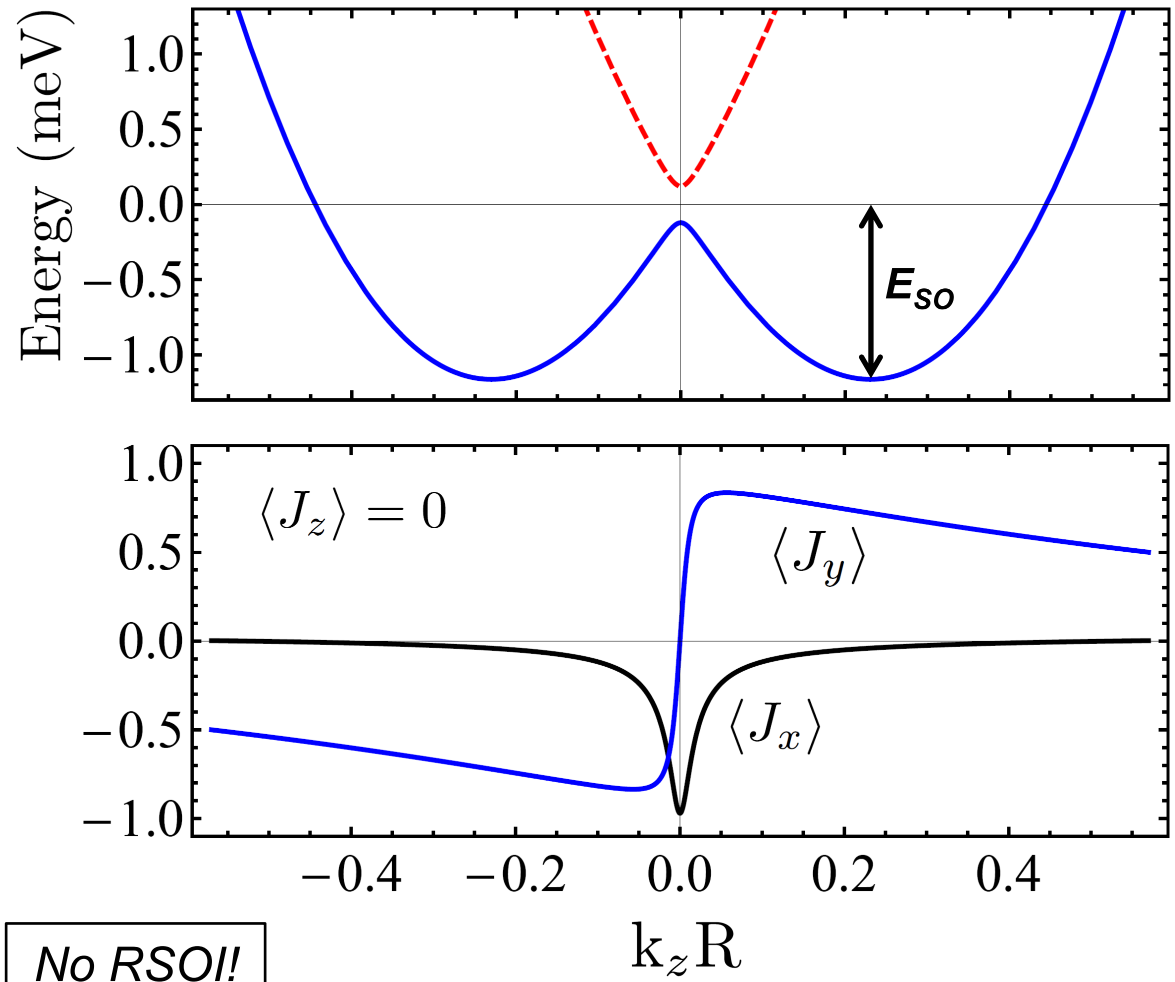
No RSOI!

Helical Hole States in Ge/Si NW

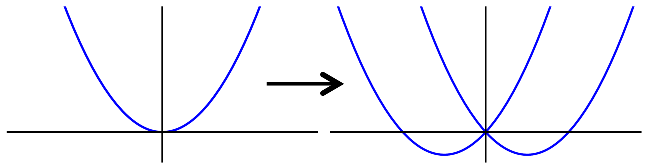


Typical wire:
R_c = 5.0 nm
R_s = 6.5 nm

E_x = 6 V/µm B_x = 0.8 T



Compare to RSOI in conduction band:

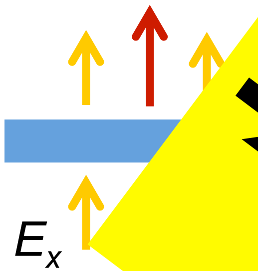


E_{SO} > 1.0 meV → 10-100 times bigger than in InAs

Fasth *et al.*, PRL (2007)

Kloeffel/Trif/Loss,
PRB **84**, 195314 (2011)

No RSOI!



Helical Hole States in Ge/Si NW

→ platform for Majorana fermions!

Maier/Klinovaja/Loss, PRB 90, 195421 (2014)

E_x

B_x

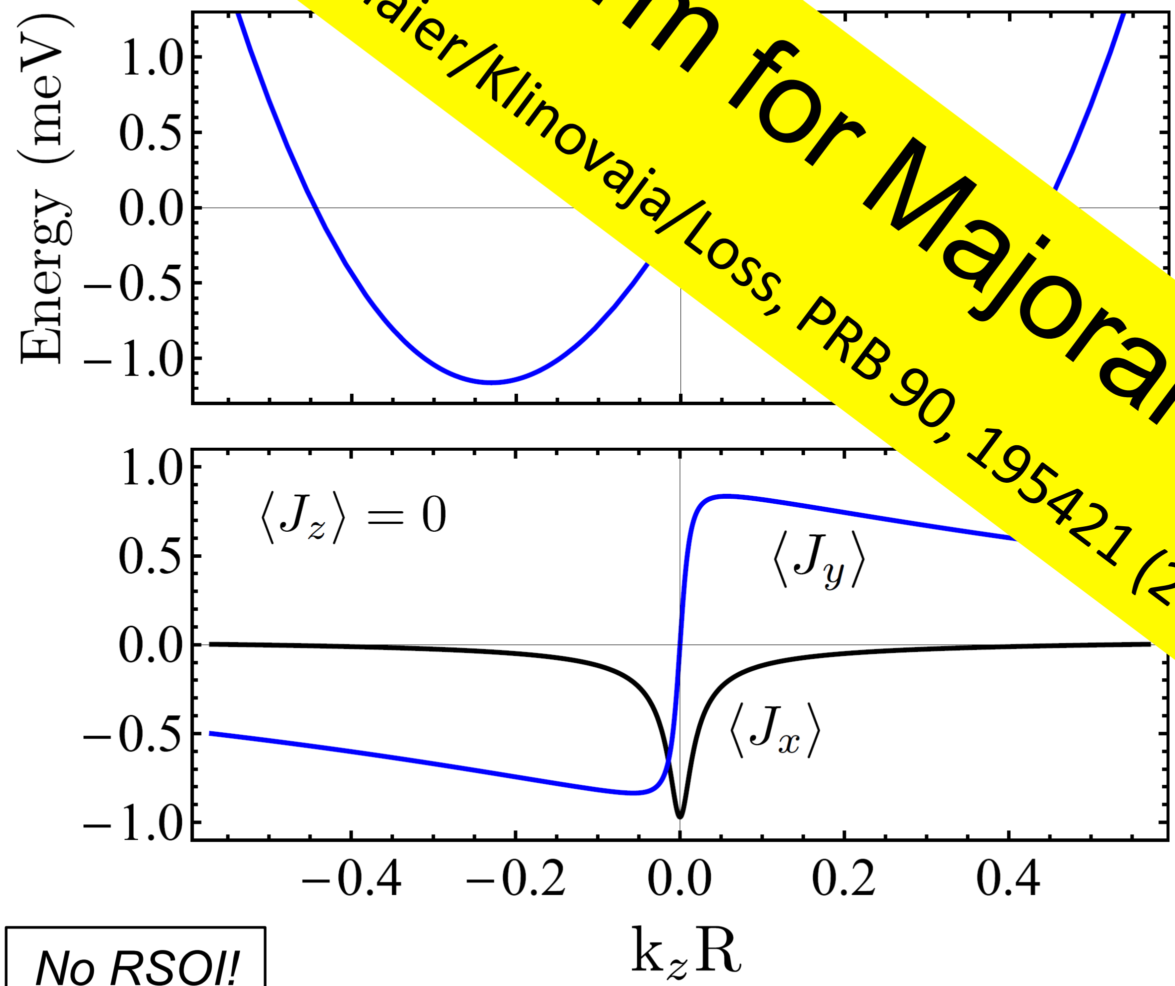
E_x

…pical wire:

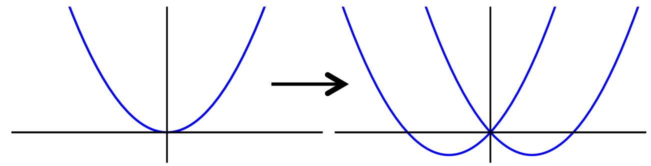
…5.0 nm

…nm

B_x = 0.8 T



Compare to RSOI in conduction band:



E_{SO} > 1.0 meV, ten times greater than in InAs

Fasth *et al.*, PRL (2007)

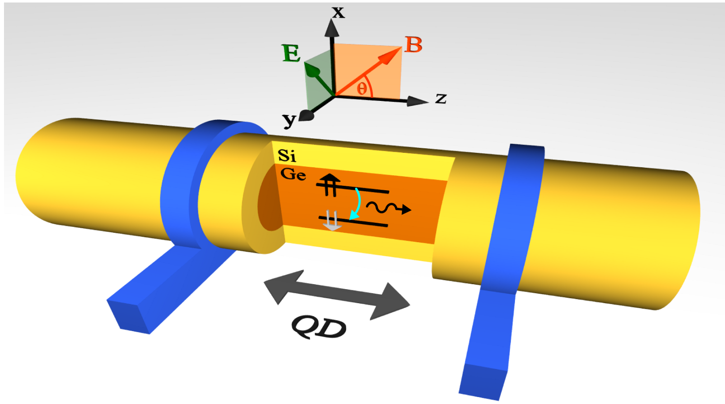
…ss,

…11)

No RSOI!

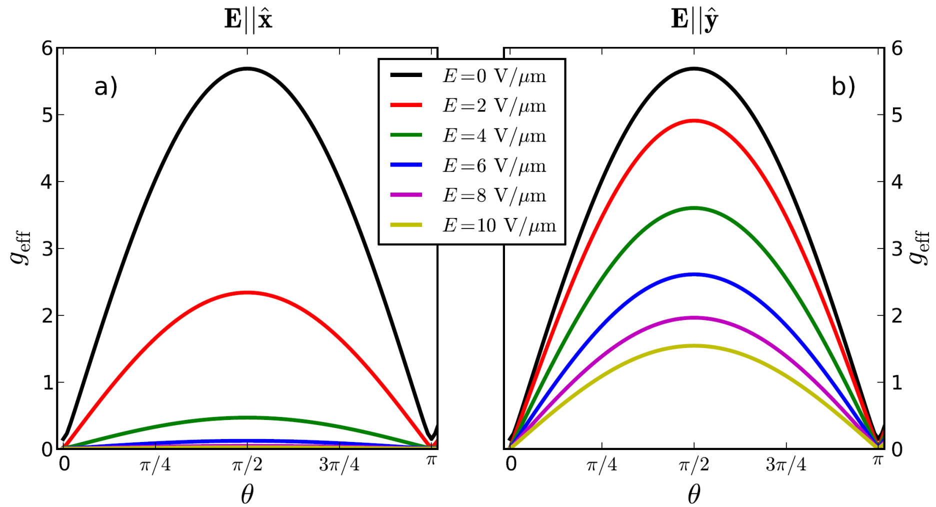
Hole Spin Qubits in Ge/Si Nanowire Quantum Dots

Maier, Kloeffel, and DL, PRB 87, 161305 (2013)



harmonic confinement in z

Highly anisotropic and electrically tunable *g* factor:

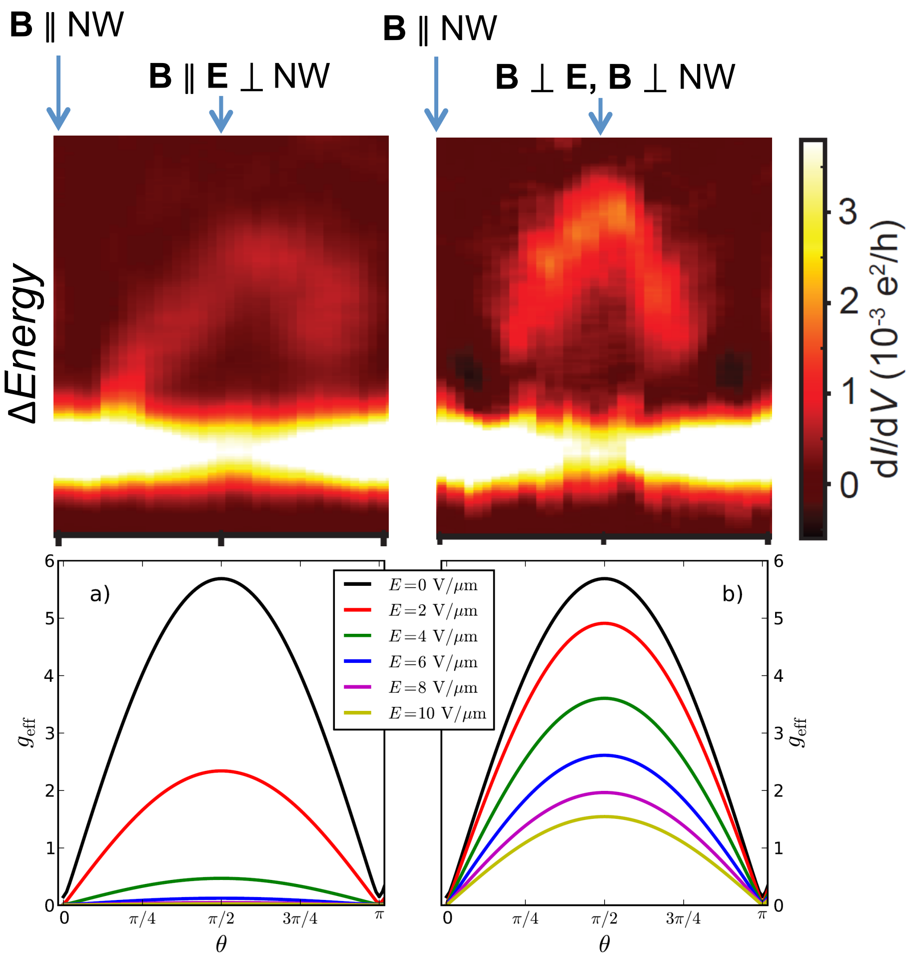


Core radius: 10 nm
Shell radius: 13 nm
Conf. length: 80 nm

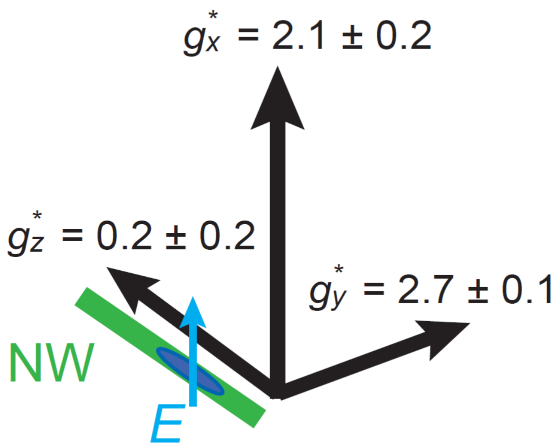
In good agreement with recent measurements by
Brauns, Ridderbos, Li, Bakkers, and Zwanenburg, PRB 93, 121408 (2016)

Experiment on Hole Spin Qubits in Ge/Si Nanowires

Brauns, Ridderbos, Li, Bakkers, and Zwanenburg, PRB **93**, 121408(R) (2016)



Theory: Maier *et al.*,
PRB **87**, 161305(R) (2013)



The measured g factor...

- ✓ ...is anisotropic with respect to both the wire and the electric field **E**
- ✓ ...is largest when **B** ⊥ NW and **B** ⊥ **E**
- ✓ ...is close to zero for **B** ∥ NW
- ✓ ...agrees very well with the theory

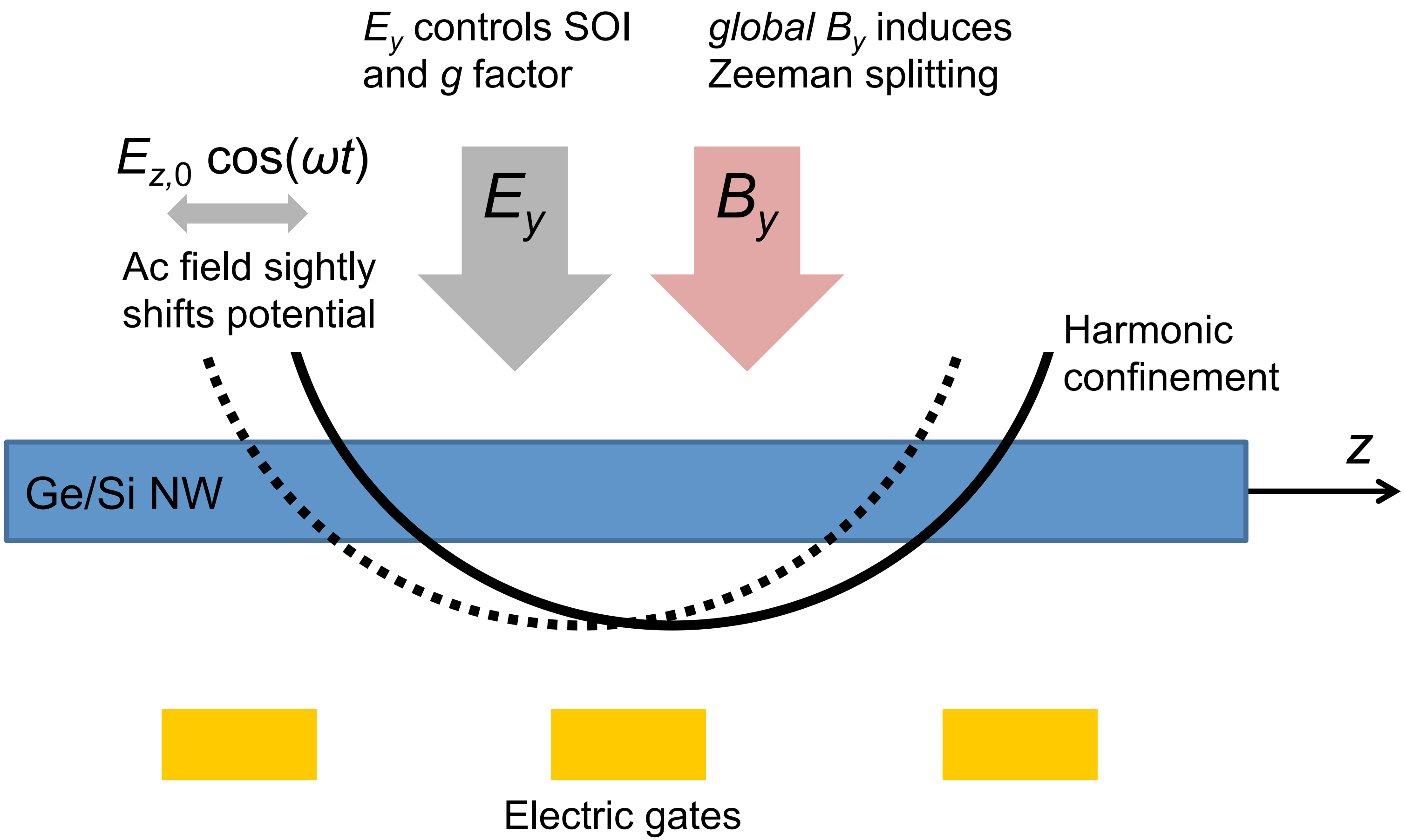
Single-hole regime not yet reached (~20 holes)

Kloeffel *et al.*, PRB **88**, 241405(R) (2013)

Setup with Ge/Si NW QDs

Ge/Si NWs typically have small core diameters ~ 10–20 nm

⟶ Elongated QDs

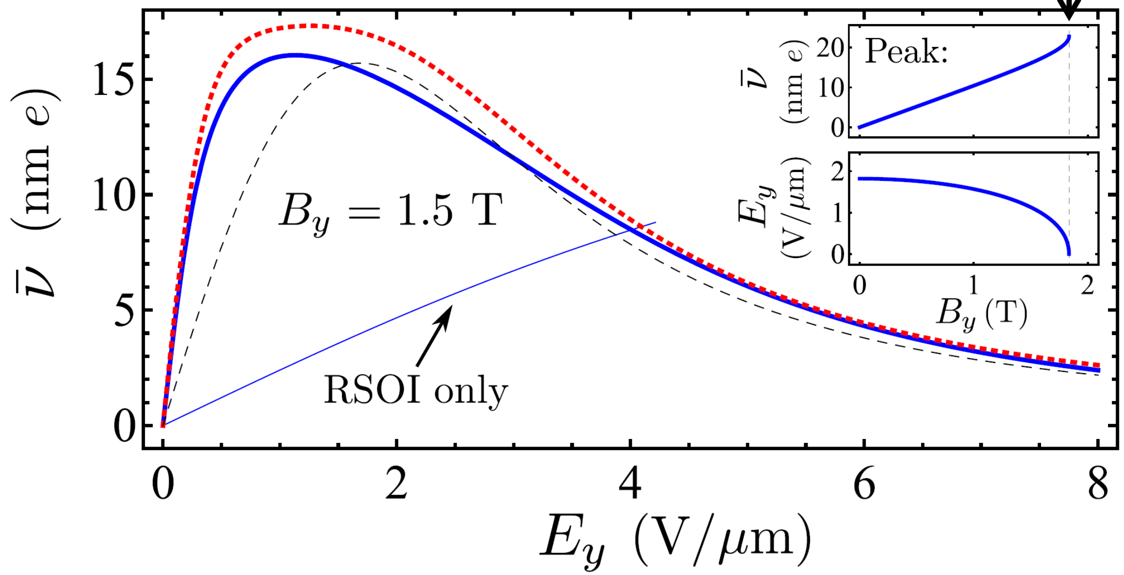


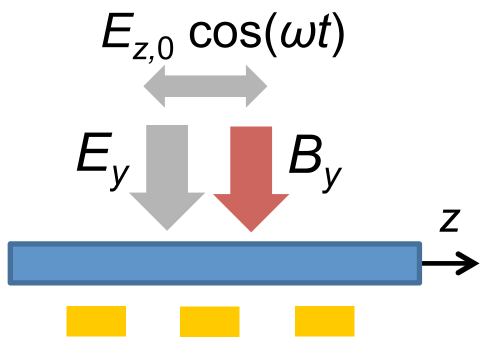
Kloeffel *et al.*, PRB **88**, 241405(R) (2013)

Single-Qubit Control via EDSR

Strong magnetic field, 1.5 T:

Level crossing at 1.8 T





R = 7.5 nm

R_s = 10 nm

l_g = 50 nm

Example: $E_{z,0} = 10^{-3}$–10^{-2} V/μm

→ $\boxed{T_{\text{flip}} \sim 100\text{–}10 \text{ ps}}$

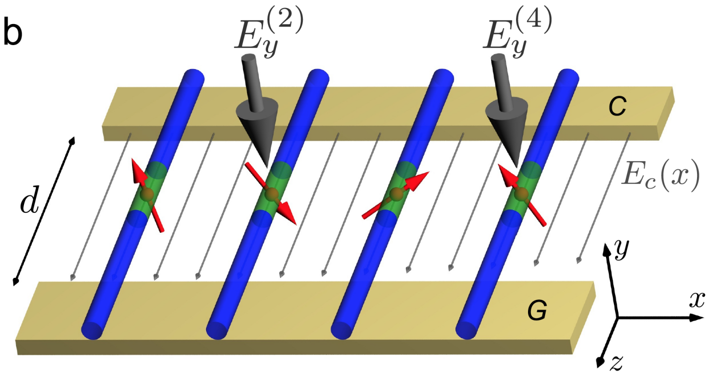
$$T_{\text{flip}} = \frac{\hbar\pi}{\bar{\nu}E_{z,0}}$$

DRSOI allows for ultrafast single-qubit gates via EDSR

Two-Qubit Gates via Circuit QED

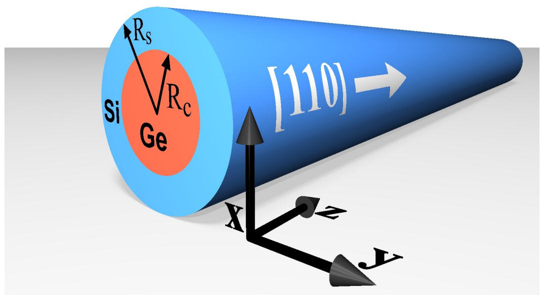
Kloeffel,Trif, Stano, and DL, PRB 88, 241405 (2013)

hole spin qubits coupled via cavity mode



Wallraff *et al*., Nature (2004)

Ge/Si nanowire

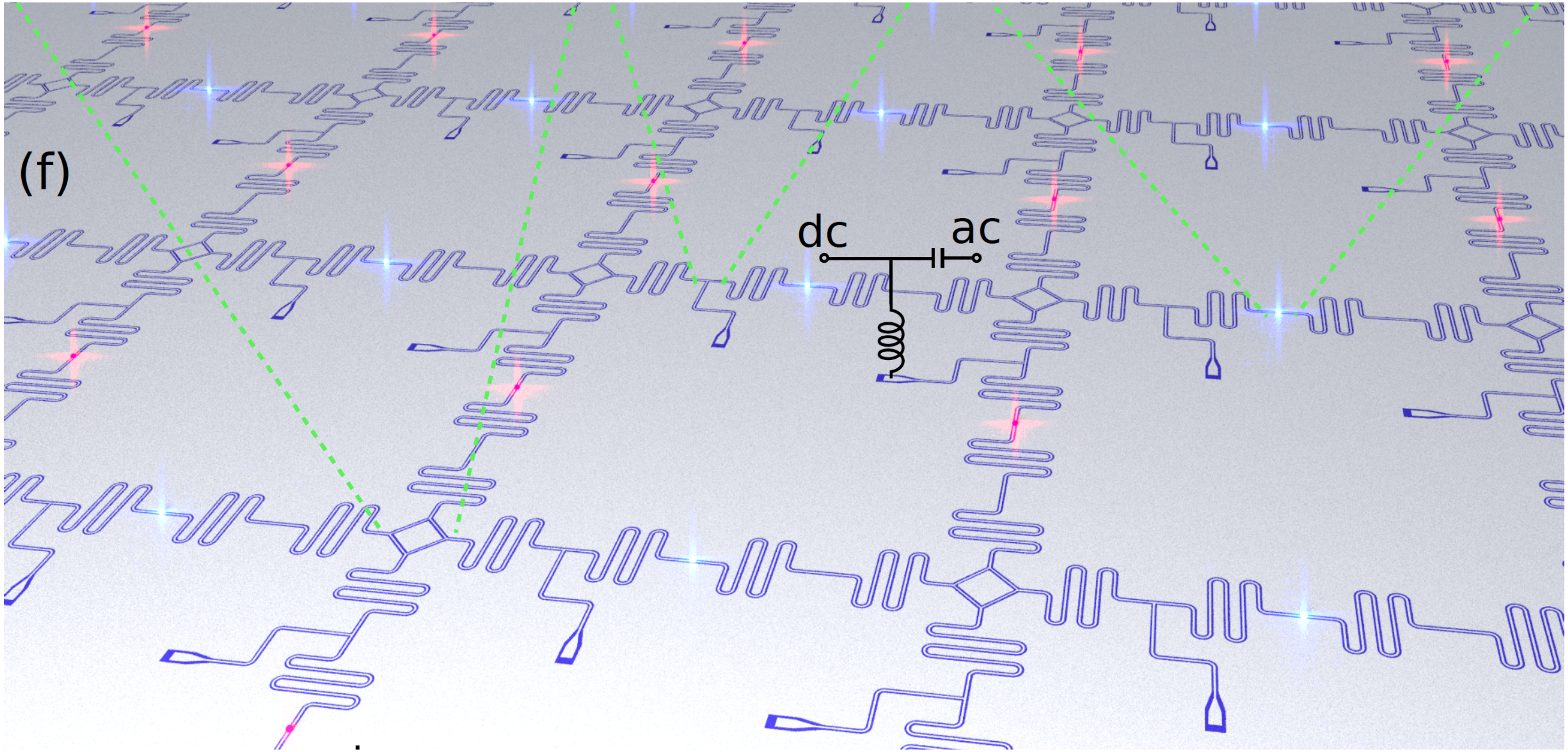


Ultra-fast gate operation times:

- single qubit : 40 - 100 ps
- two-qubit: 20 - 100 ns [limited by cavity field E_z ~ 3V/m]
- at reduced noise and large on/off ratio

Grid-bus Surface Code with Hole Spin Qubits

Nigg, Fuhrer, and Loss, 2016



Summary

- Spin qubits in quantum dots: electrons vs. holes spins
  GaAs, InAs,...vs. Si, Ge

- Surface code architecture via floating gates or microwave cavity

- Hole spin qubits in Si/Ge nanowires quantum dots
  Kloeffel,Trif, Stano, and DL, PRB 88, 241405 (2013)

- Topological quantum computing: hybrid spin-Majorana qubits
  Hoffman, Schrade, Klinovaja, and DL, PRB 94, 045316 (2016)

Prospects for Spin-Based Quantum Computing in Quantum Dots
C. Kloeffel and D. Loss, Annu. Rev. Condens. Matter Phys. 4, 51 (2013)