A tutorial on non-Markovian quantum processes

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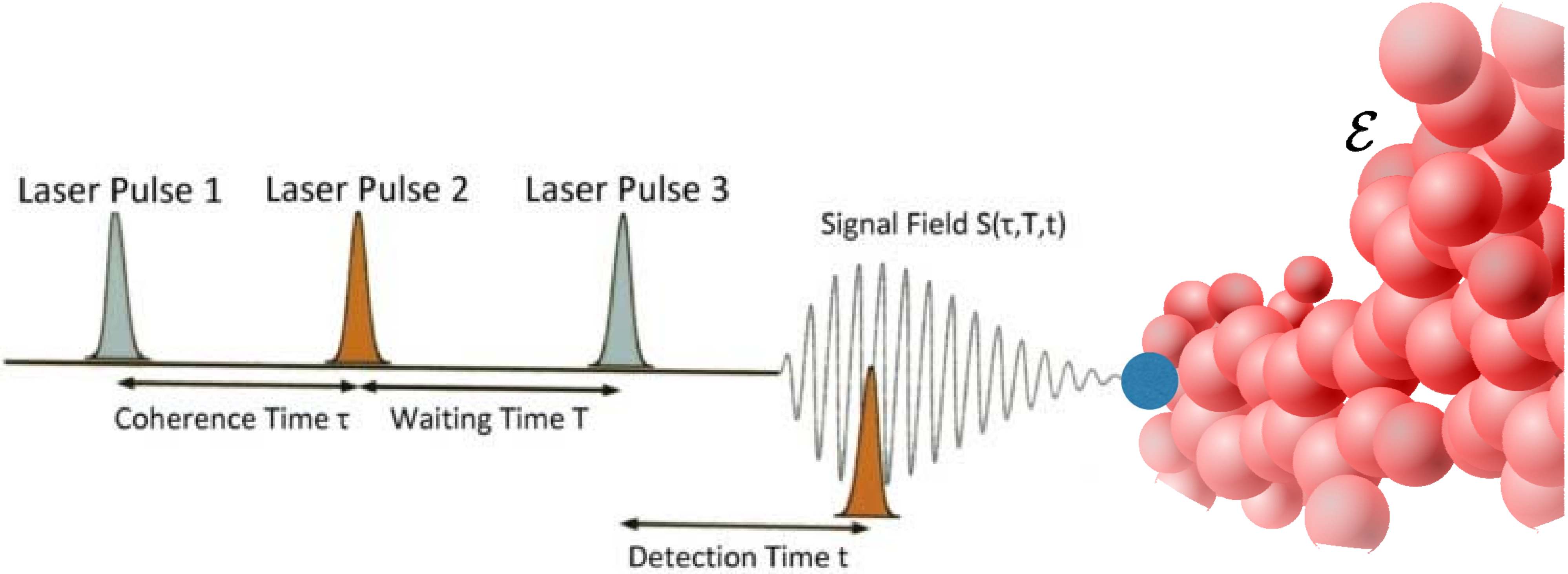
Open quantum systems



http://gaigroup.chem.upenn.edu/images/2DIR_pulse_sequence_small.jpg dynamics

quantum to classical

non-Markovian

coherence

environmental effects

memory kernel

heat

spectroscopy

system-bath correlations

memory effects

entanglement

Redfield

transport

density matrix

evolution

strong coupling

temperature

decoherence

energy exchange

master equation

weak coupling

dissipation

dephasing

Hierarchical Equation of Motion

positivity

spin-boson model

Markovian

Lindblad

Stochastic process

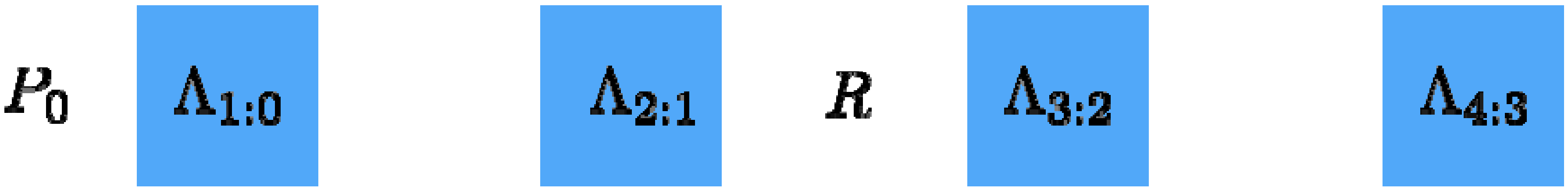
P_0 $\Lambda_{1:0}$

$$\Lambda_{1:0} \quad P_0 = P_1$$

$$\begin{pmatrix} \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \end{pmatrix} \begin{pmatrix} \cdot \\ \cdot \\ \cdot \end{pmatrix} = \begin{pmatrix} \cdot \\ \cdot \\ \cdot \end{pmatrix}$$

What is Markovian?

$$P_{k+1} = \Lambda_{k+1;k}[P_k]$$



$$\begin{aligned} P_2 &= \Lambda_{2:1} P_1 \\ &= \Lambda_{2:1} \Lambda_{1:0} P_0 \end{aligned}$$

$$\begin{aligned} R_4 &= \Lambda_{4:3} R_3 \\ &= \Lambda_{4:3} \Lambda_{3:2} R \end{aligned}$$

Master equation

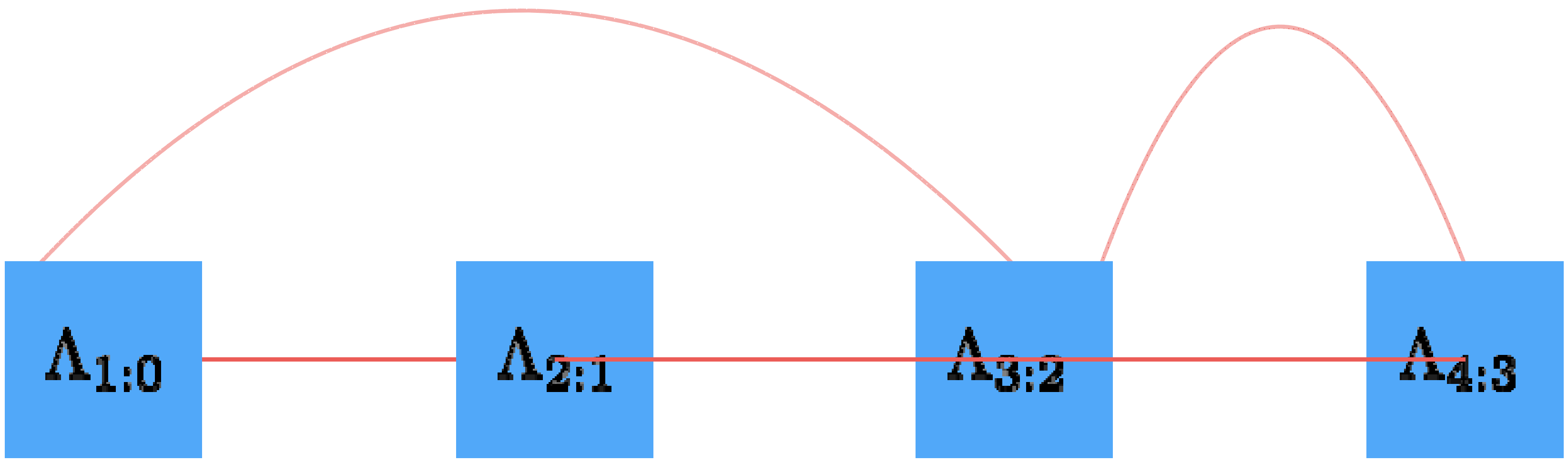
$$P_{t+\delta t} - P_t = \left(\Lambda_{t+\delta t:t} - \mathcal{I}\right) P_t$$

$$\frac{\partial}{\partial t} P_t = \left(\lim_{\delta t \to 0} \frac{\Lambda_{t+\delta t:t} - \mathcal{I}}{\delta t} \right) P_t$$

$$\frac{\partial}{\partial t} P_t = \mathcal{L}_t P_t$$

What is non-Markovian?

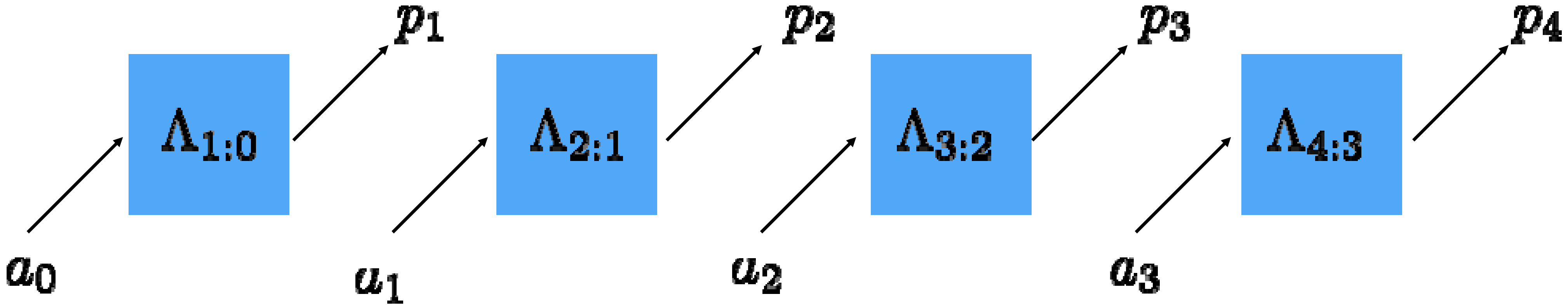
$$P_{k+1} = \Lambda_{k+1;k}[P_k | P_{k-1}, \ldots, P_0]$$



How do we deal with this?

Markovianise it

$$p_4 = t_L(a_3, a_2, a_1, a_0)$$
$$\Lambda_{k+1:k|\{a_{k-1} \dots a_0\}}[a_k] = P_{k+1}$$
$$corr(p_k, a_k) \neq 0$$

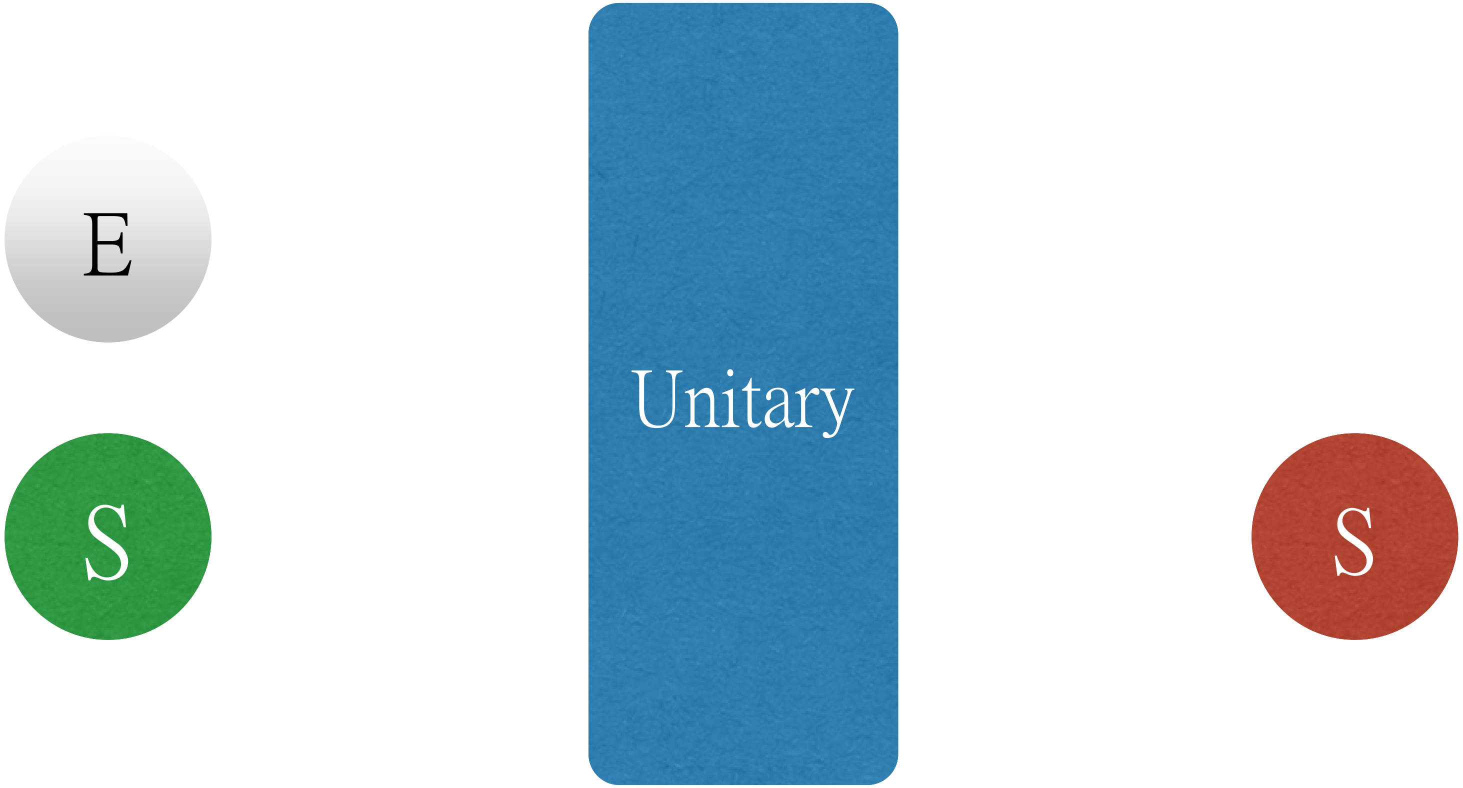


Quantum version of stochastic dynamics

Dynamical map

We can use the system to characterise the Blackbox.

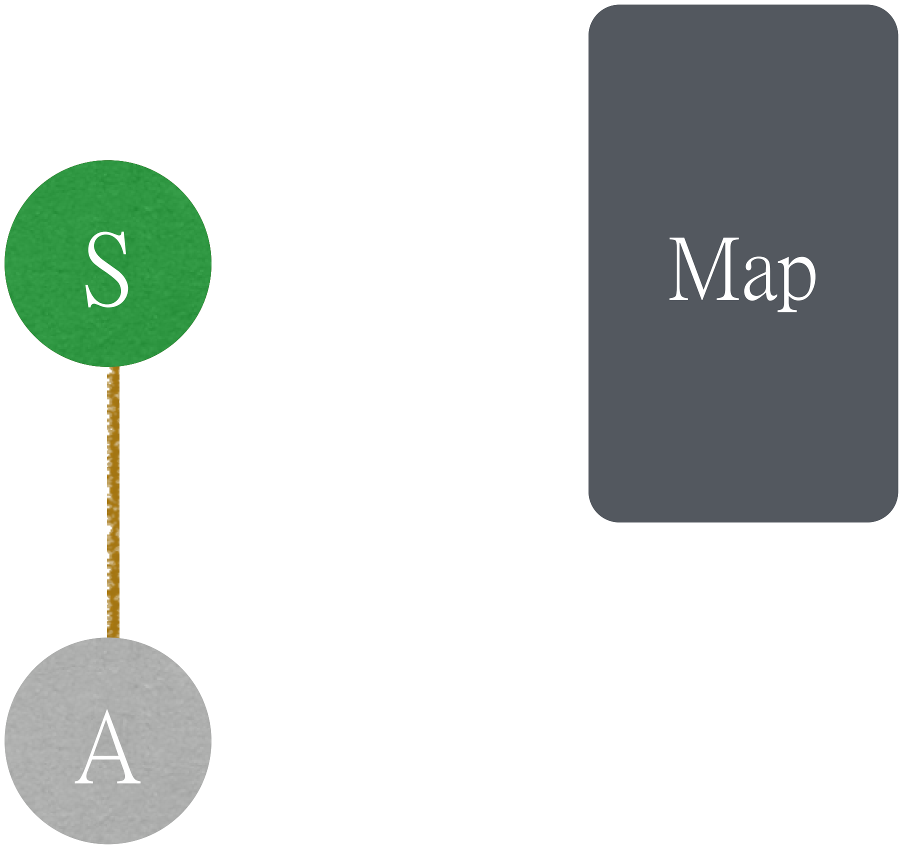
Contracted unitary dynamics



Properties of map

- Dynamical maps = Reduced unitary dynamics
- Linear $\Lambda(a\,\rho + b\,\sigma) = a\,\Lambda(\rho) + b\,\Lambda(\sigma)$
- Completely positive $\Lambda(\rho) = \sum_k A_k\,\rho A_k^\dagger$
- Contractivity $D(\rho,\sigma) \geq D(\Lambda[\rho], \Lambda[\sigma])$

Complete positivity



Charactering stochastic quantum processes

Quantum process tomography

$$\rho_k \longrightarrow \boxed{\text{Map}} \longrightarrow \rho'_{z+}$$

$$\varrho = \sum_k r_k \rho_k$$

$$\mathrm{tr}[D_k \rho_l] = \delta_{kl}$$

$$\Lambda = \sum_k \rho'_k \otimes D_k^T$$

$$\Lambda[\varrho] = \sum_k \rho'_k \, \mathrm{tr}[D_k \varrho]$$

$$\Lambda[\varrho] = \sum_k r_l \rho'_k \, \mathrm{tr}[D_k \rho_l] = \sum_k r_l \rho'_k$$

Modi, Rodríguez-Rosario, Aspuru-Guzik, Phys. Rev. A 86, 064102 (2012)

Quantum process tomography of a qubit

$$\rho_{z+} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \longrightarrow \boxed{\text{Map}} \longrightarrow \rho'_{z+}$$

$$\rho_{x+} = \frac{1}{2}\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} \longrightarrow \boxed{\text{Map}} \longrightarrow \rho'_{x+}$$

$$\rho_{y+} = \frac{1}{2}\begin{pmatrix} 1 & -i \\ i & 1 \end{pmatrix} \longrightarrow \boxed{\text{Map}} \longrightarrow \rho'_{y+}$$

$$\rho_{z-} = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \longrightarrow \boxed{\text{Map}} \longrightarrow \rho'_{z-}$$

Sequence of dynamical maps=Markovian process

Divisibility $\Lambda_{1:0}\,\Lambda_{2:1} = \Lambda_{2:0}$

$$\rho_{t+\delta t} - \rho_t = \left(\Lambda_{t+\delta t:t} - \mathcal{I}\right)\rho_t$$

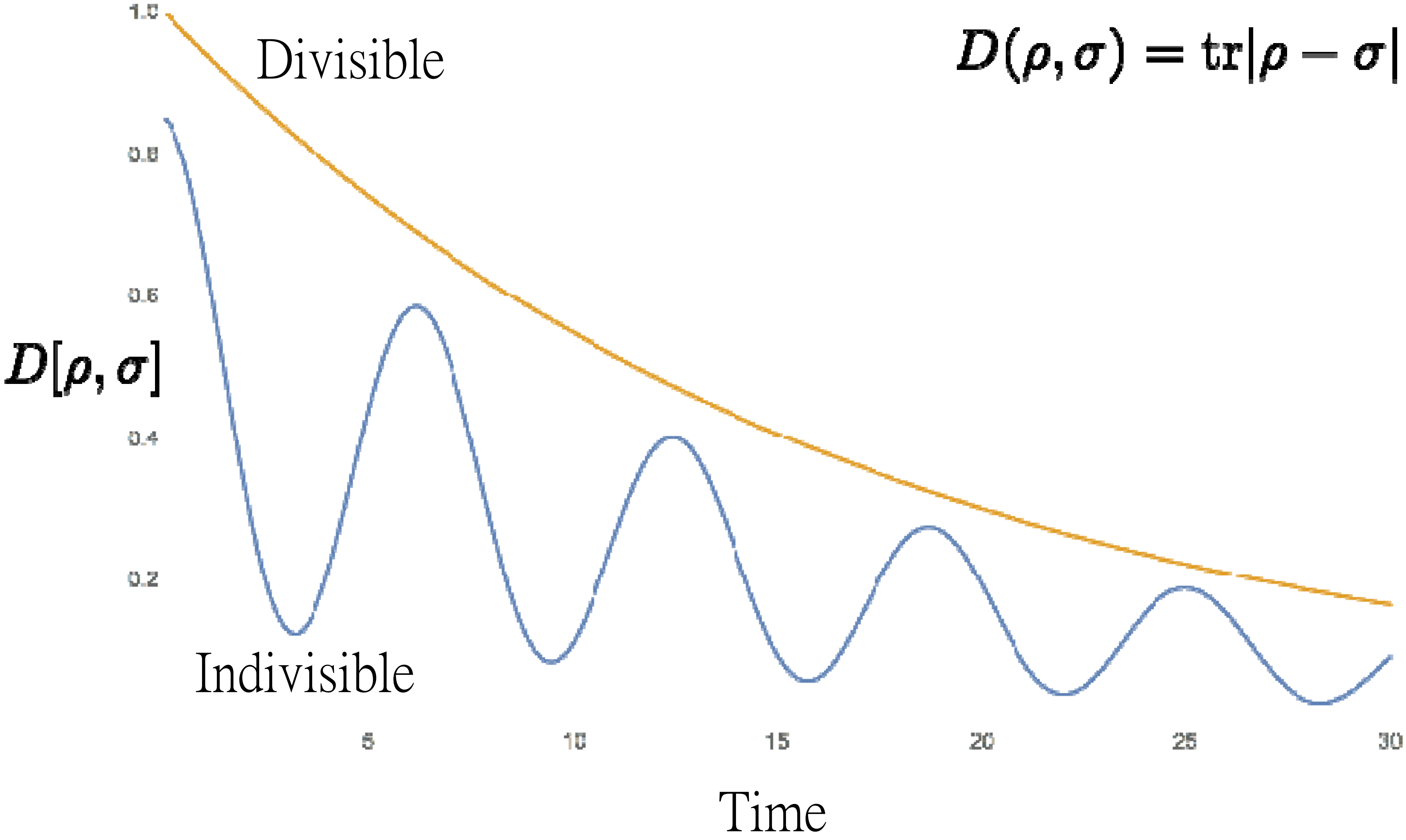
$$\frac{\partial}{\partial t}\rho_t = \left(\lim_{\delta t\to 0}\frac{\Lambda_{t+\delta t:t} - \mathcal{I}}{\delta t}\right)\rho_t$$

$$\frac{\partial}{\partial t}\rho_t = \mathcal{L}_t\rho_t$$

Lindblad, Redfield

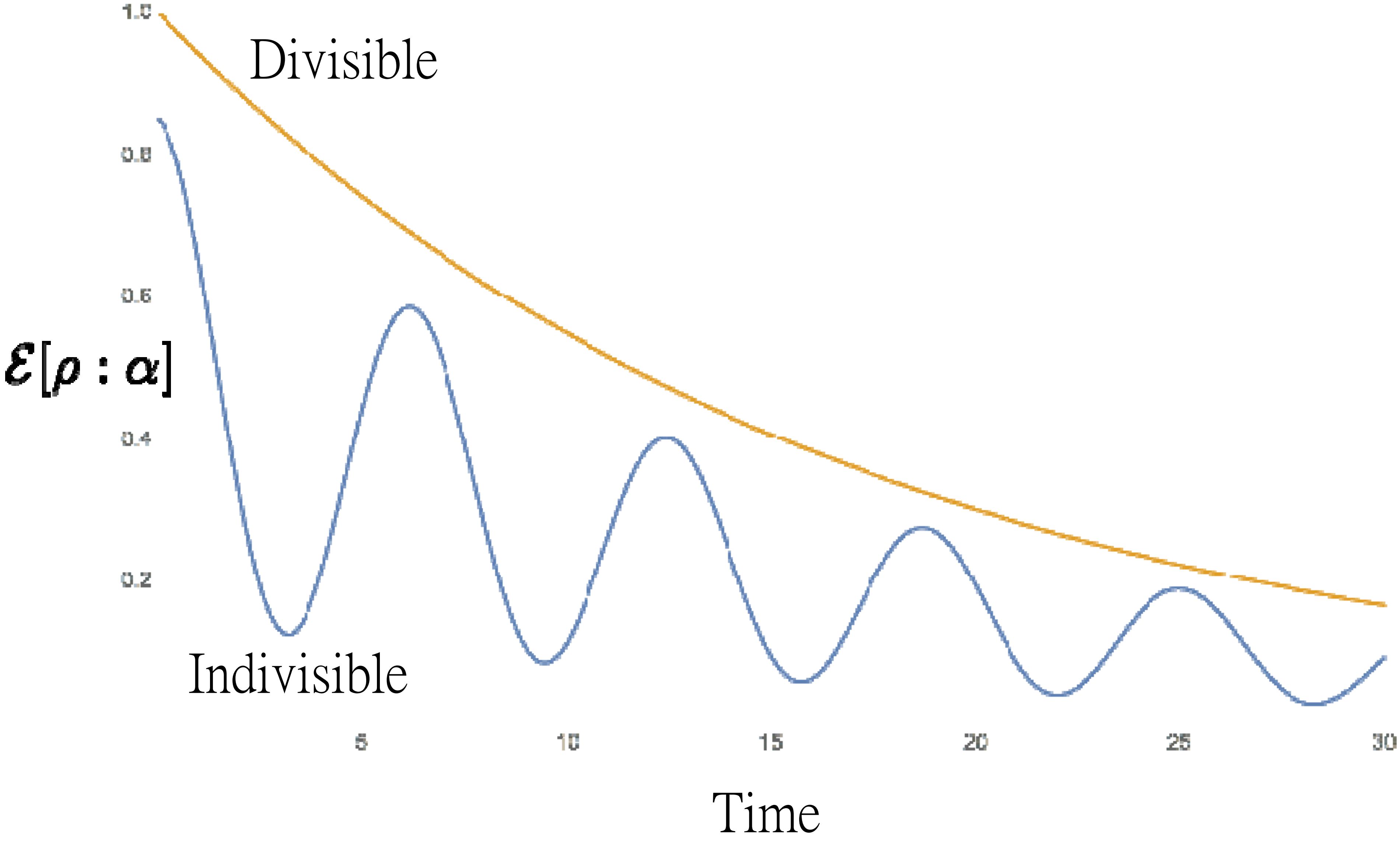
Witnesses

Witness NM through contractivity



Breuer, Laine, Piilo, PRL 103, 210401 (2009)

Check divisibility directly



Rivas, Huelga, Plenio, PRL 105, 050403 (2010)

Question

Suppose you have a system undergoing open dynamics. The entropy (purity) of the system is first increasing (decreasing) and later decreasing (increasing). Can we conclude that the dynamics are non-Markovian?

$$\Lambda_m[\rho] = \frac{\mathbb{I}}{d}$$

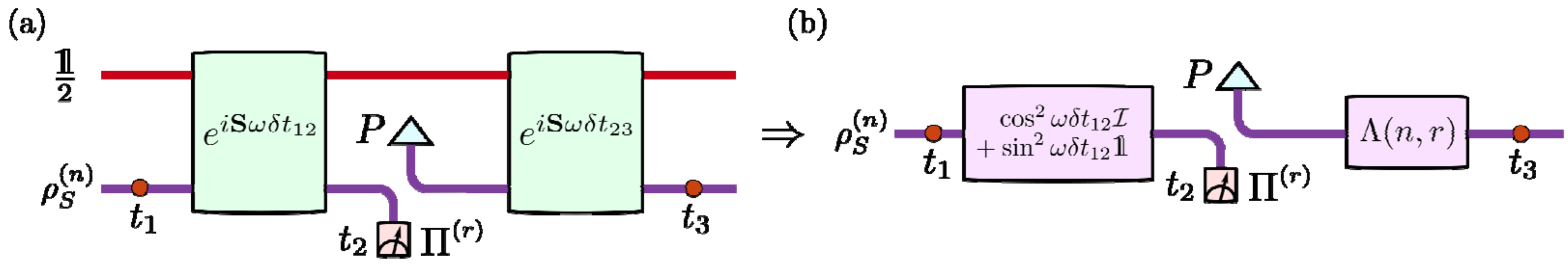
$$\Lambda_m \, \Lambda_p \, \Lambda_m \, \Lambda_p \, \Lambda_m[\rho_0] = \rho_5$$

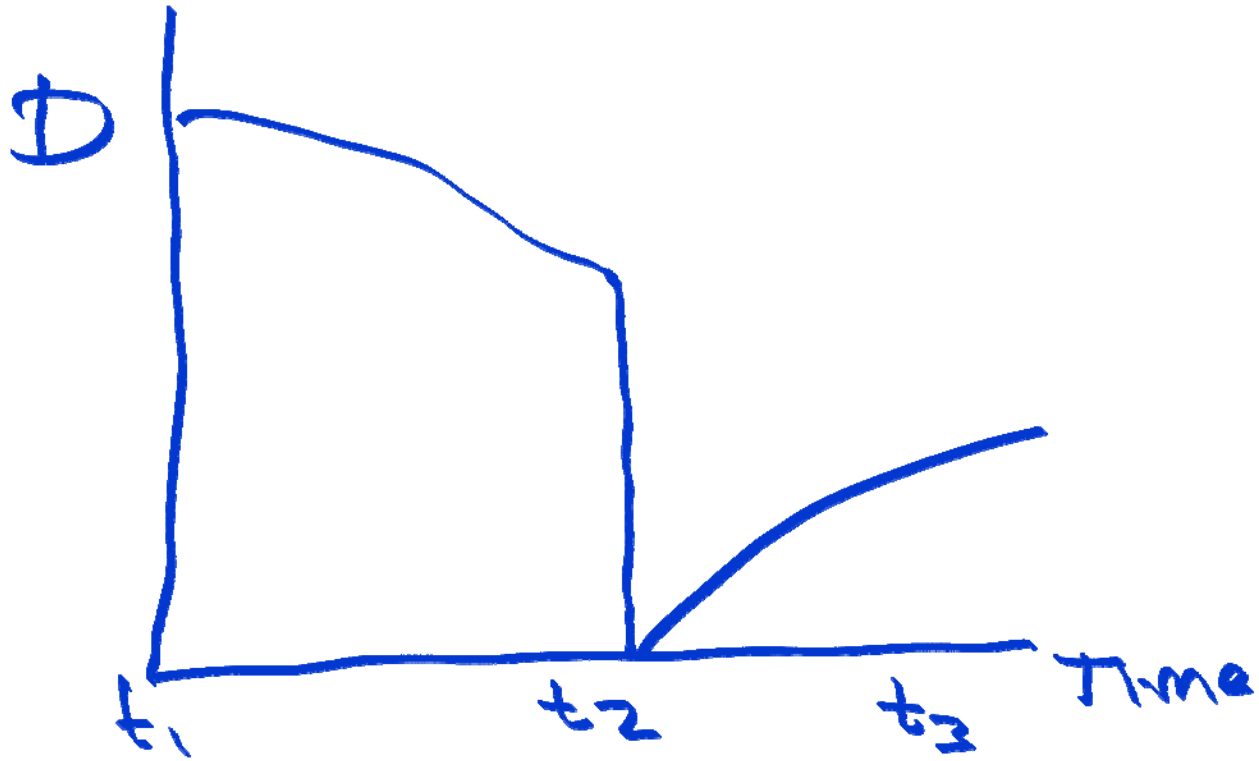
$$\{-\mathrm{tr}[\rho \log \rho], \, \log d, \, 0, \, \log d, \, 0, \log d\}$$

$$\Lambda_p[\rho] = |0\rangle \langle 0|$$

$$\left\{\mathrm{tr}[\rho^2], \, \frac{1}{d}, \, 1, \, \frac{1}{d}, \, 1 \, \frac{1}{d}\right\}$$

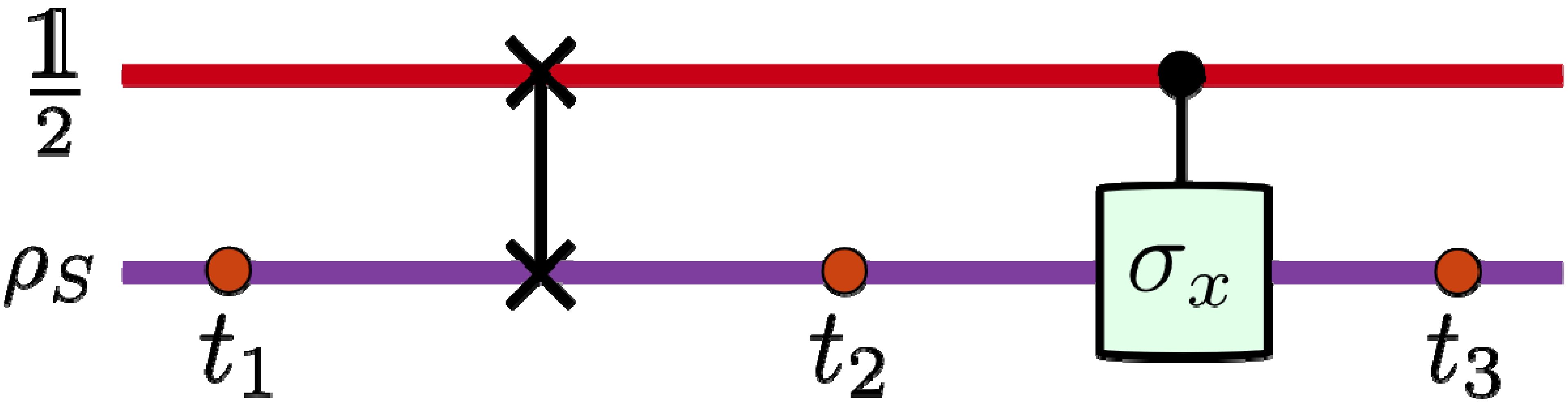
NM but monotonic in distance





Pollock, Rodríguez-Rosario, Frauenheim, Paternostro, Modi, arXiv:1512.00589 (2015)

Divisible but NM

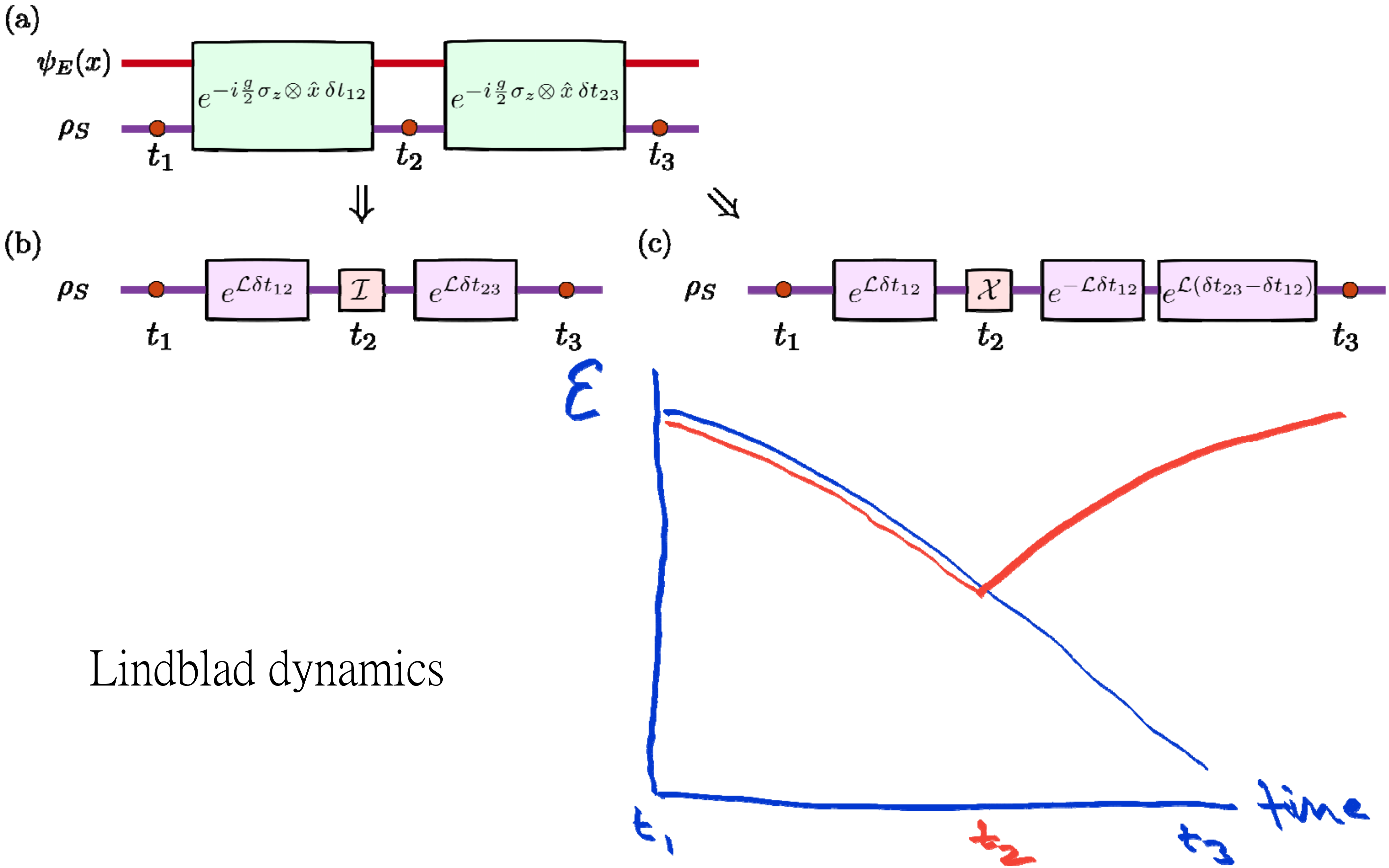


$$\Lambda_{2:1} = \frac{\mathbb{I}}{2} \otimes \mathbb{I} \qquad \Lambda_{3:2} = \frac{\mathbb{I}}{2} \otimes \mathbb{I}$$

$$\Lambda_{3:1} = \frac{\mathbb{I}}{2} \otimes \mathbb{I}$$

Pollock, Rodríguez-Rosario, Frauenheim, Paternostro, Modi, arXiv:1512.00589 (2015)

Divisible but NM



Lindblad dynamics

Pollock, Rodríguez-Rosario, Frauenheim, Paternostro, Modi, arXiv:1512.00589 (2015)

Full description?

Classical to quantum?

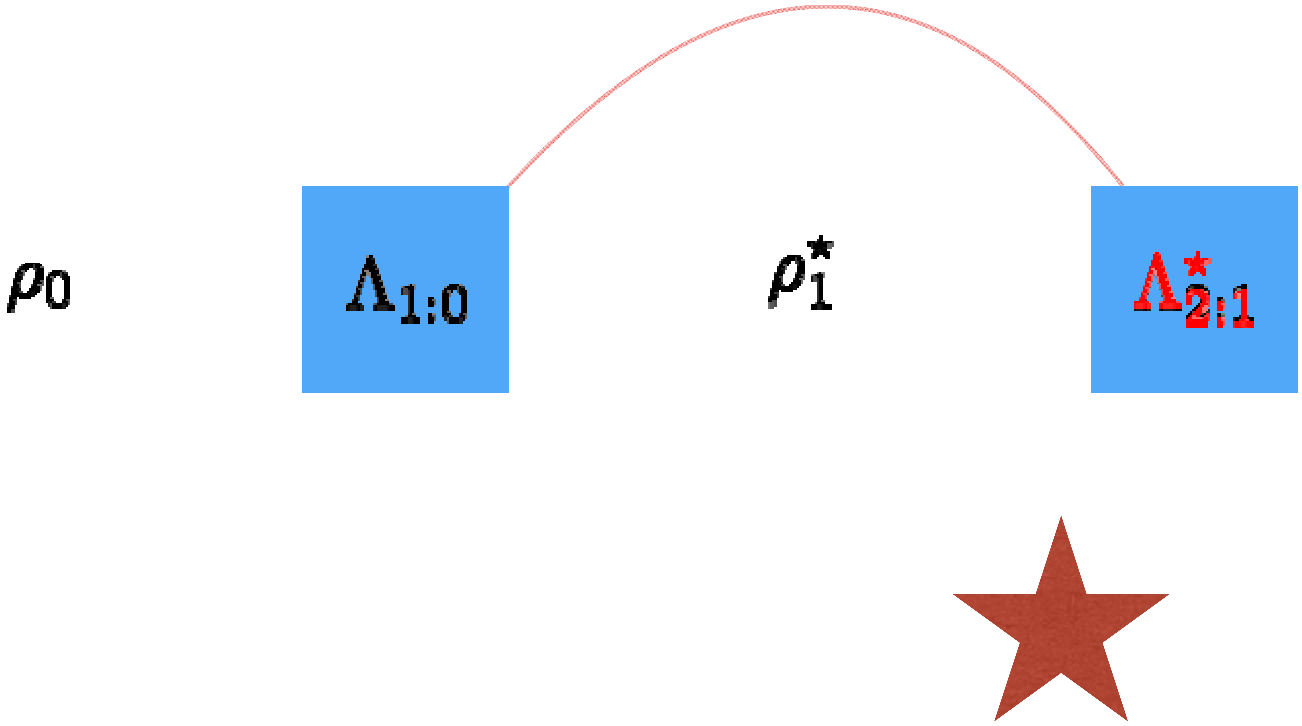
$$P_t \to \rho_t$$

$$P_{A|B} \to \rho_{A|B}$$

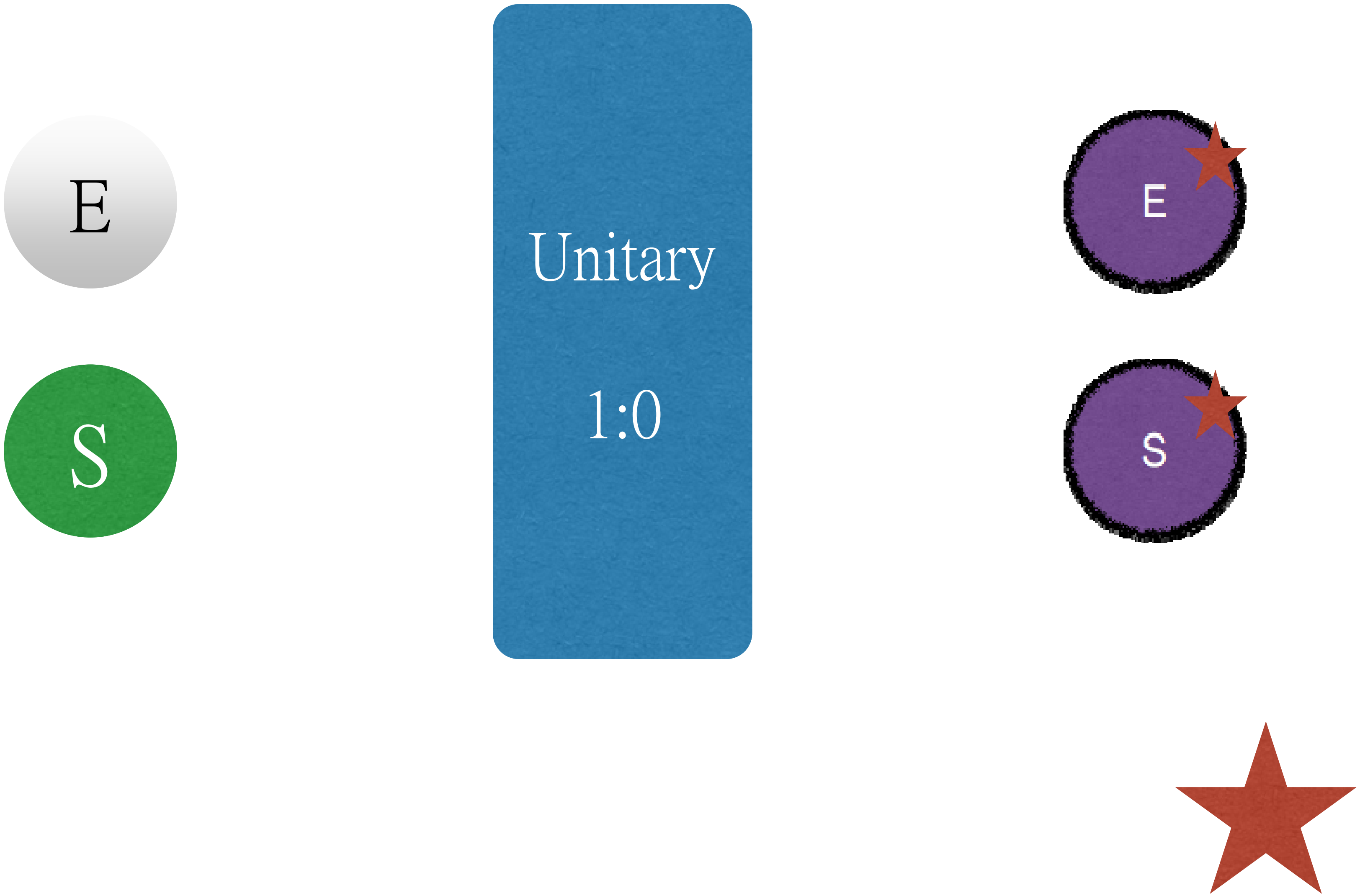
Convex **and** informationally complete

Convex **or** informationally complete

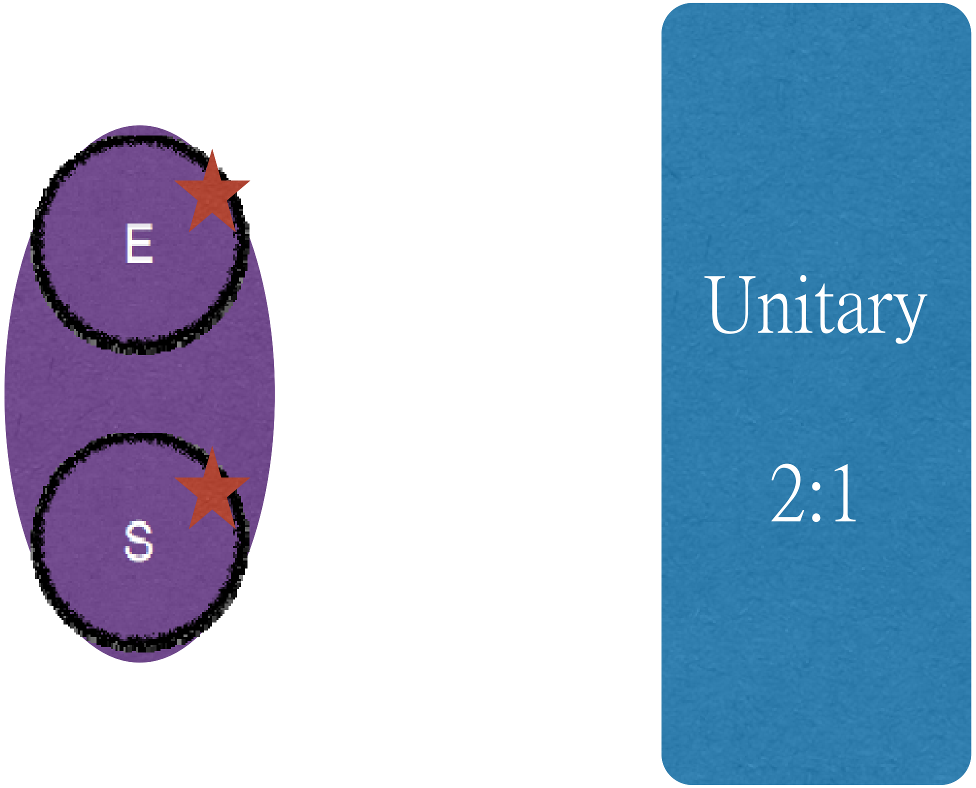
An illustration



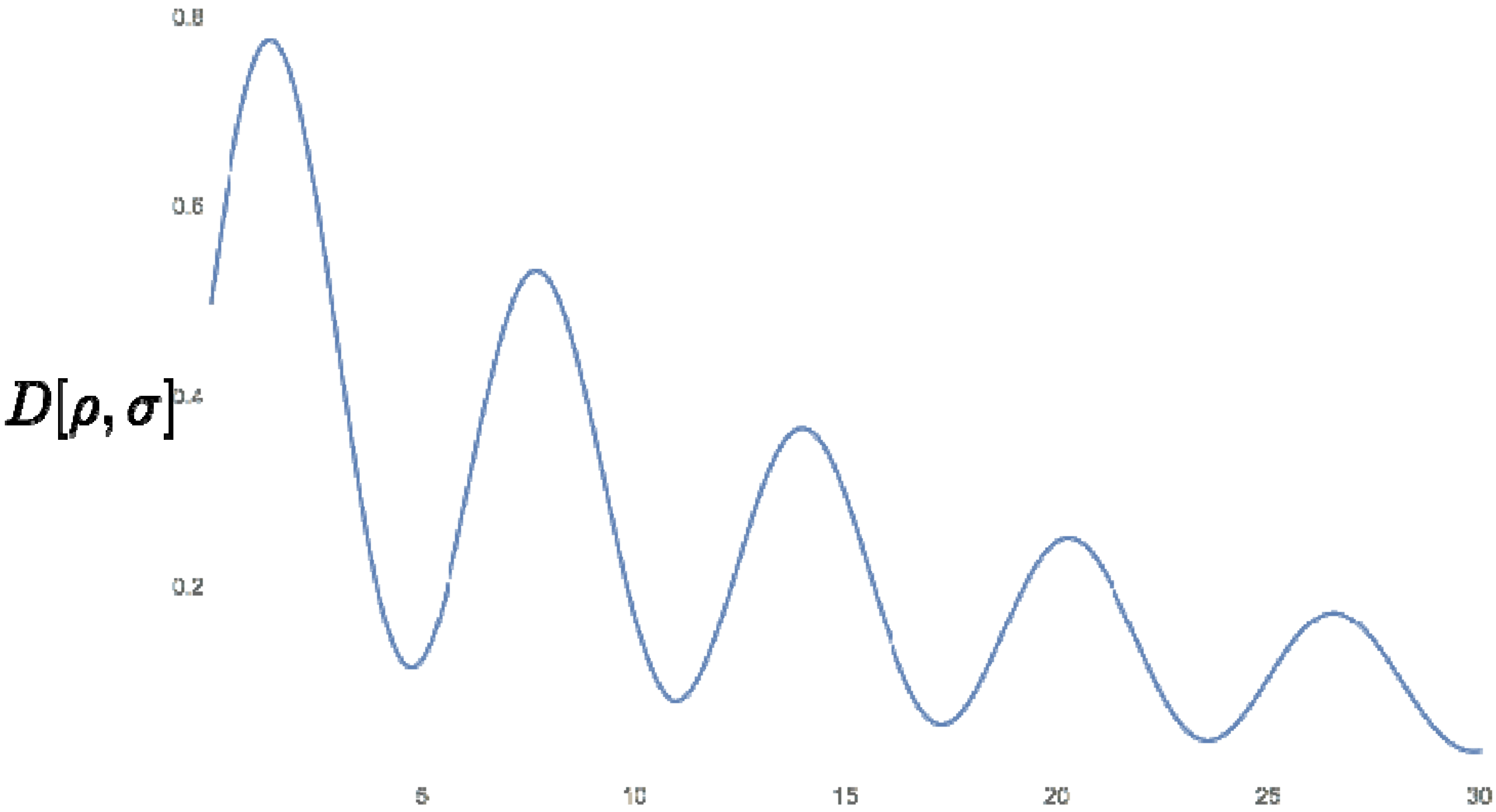
Contracted unitary dynamics



Contracted unitary dynamics

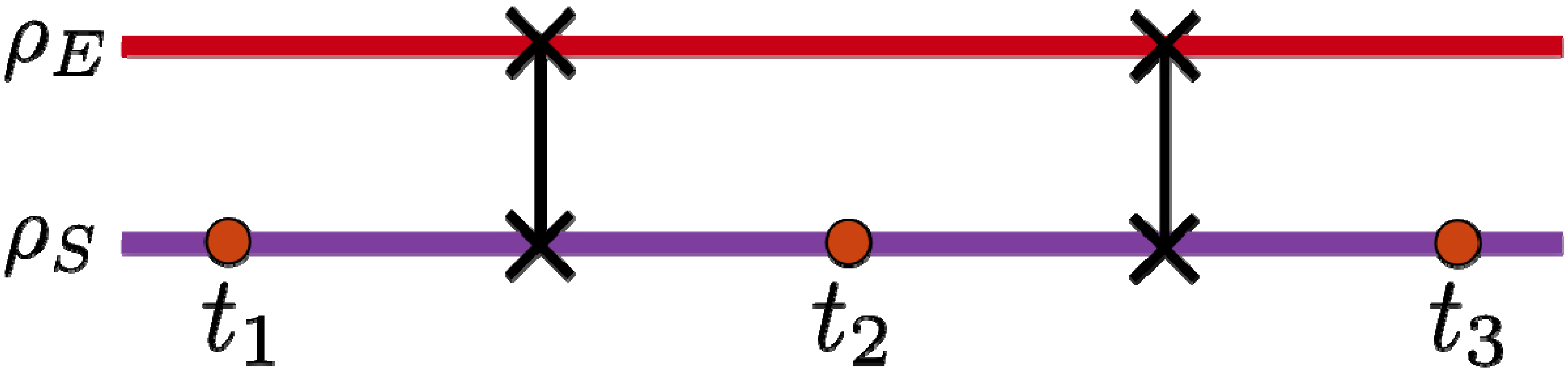


Initial correlations witnesses



Mazzola, Rodrıguez-Rosario, Modi, Paternostro, Phys. Rev. A 86, 010102(R) (2012)
Rodríguez-Rosario, Modi, Mazzola, Aspuru-Guzik, Europhys. Lett. 99 20010 (2012)

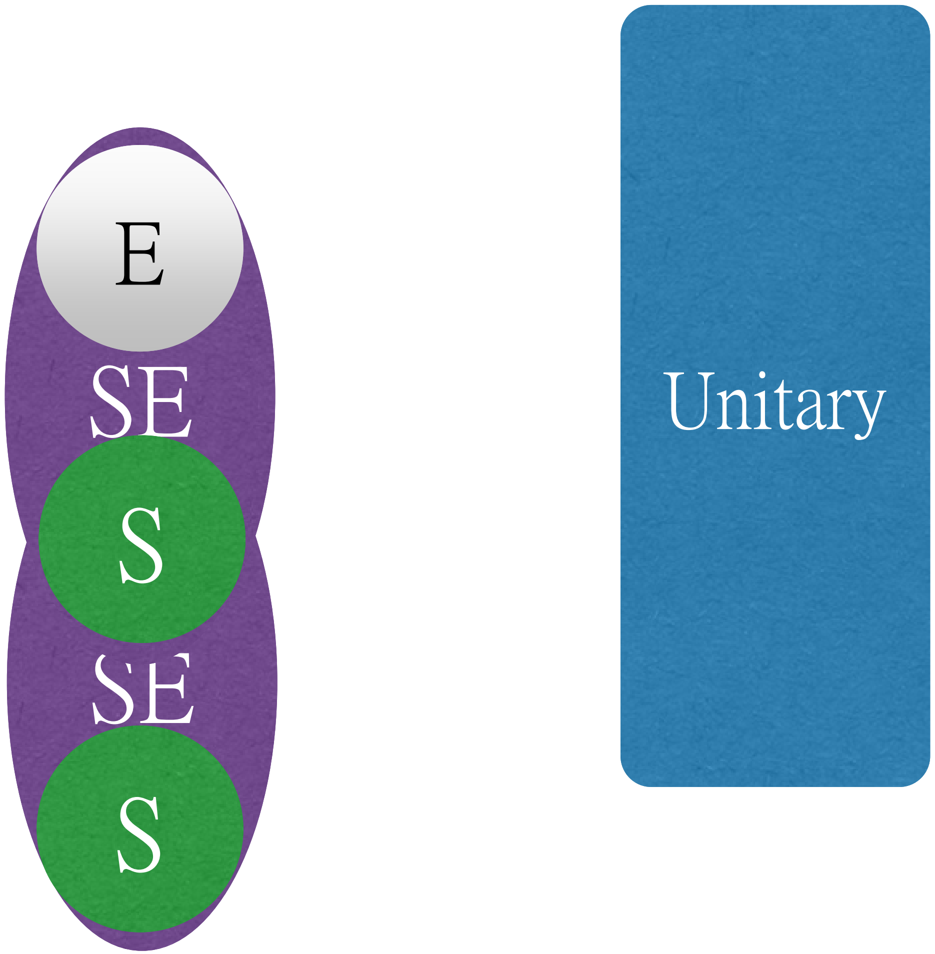
NM without correlations



Pollock, Rodríguez-Rosario, Frauenheim, Paternostro, Modi, arXiv:1512.00589 (2015)

Dealing with initial correlations

Initially correlated SE



Pechukas: We must give up **complete positivity** or **linearity**

This is the simplest non-Markovian case study.

Giving up CP?

Holevo bound

Masillo, Scolarici, Solombrino, J Math Phys 52, 012101 (2011)

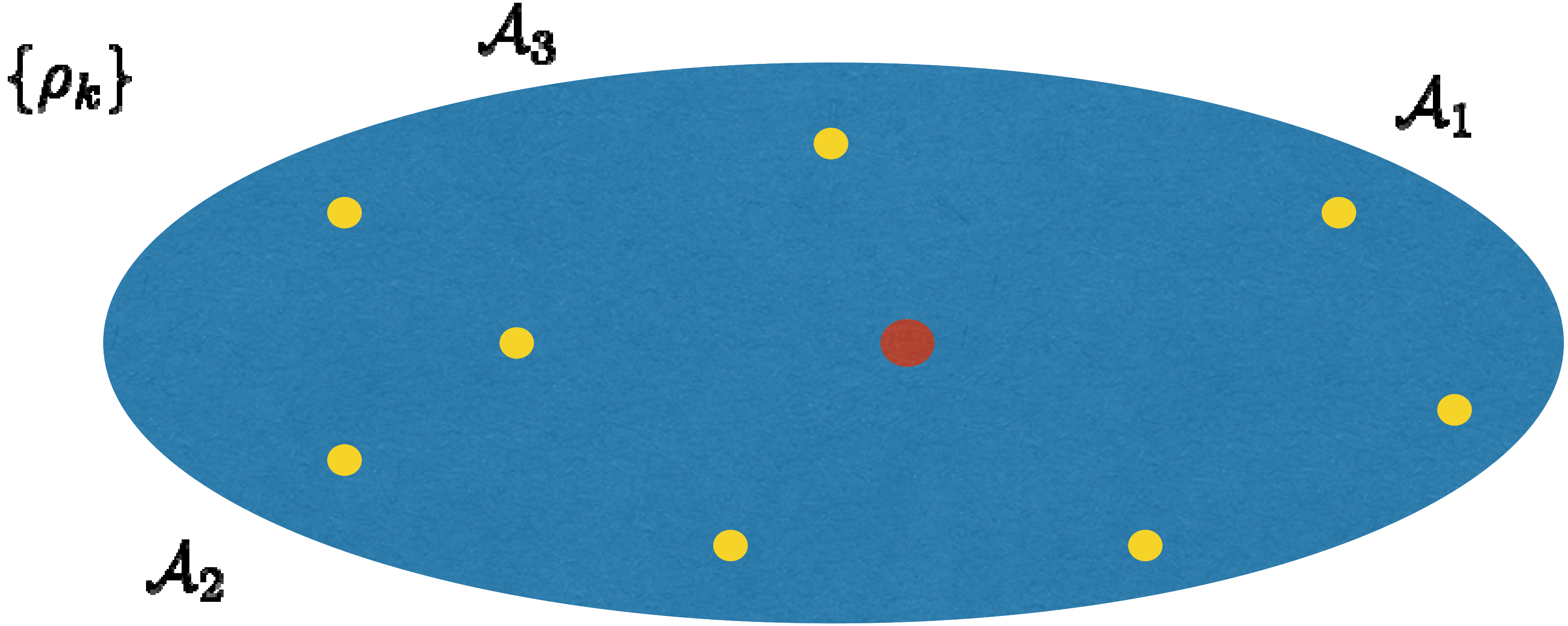
Data processing inequality

Buscemi, PRL. 113, 140502 (2014)

Entropy production

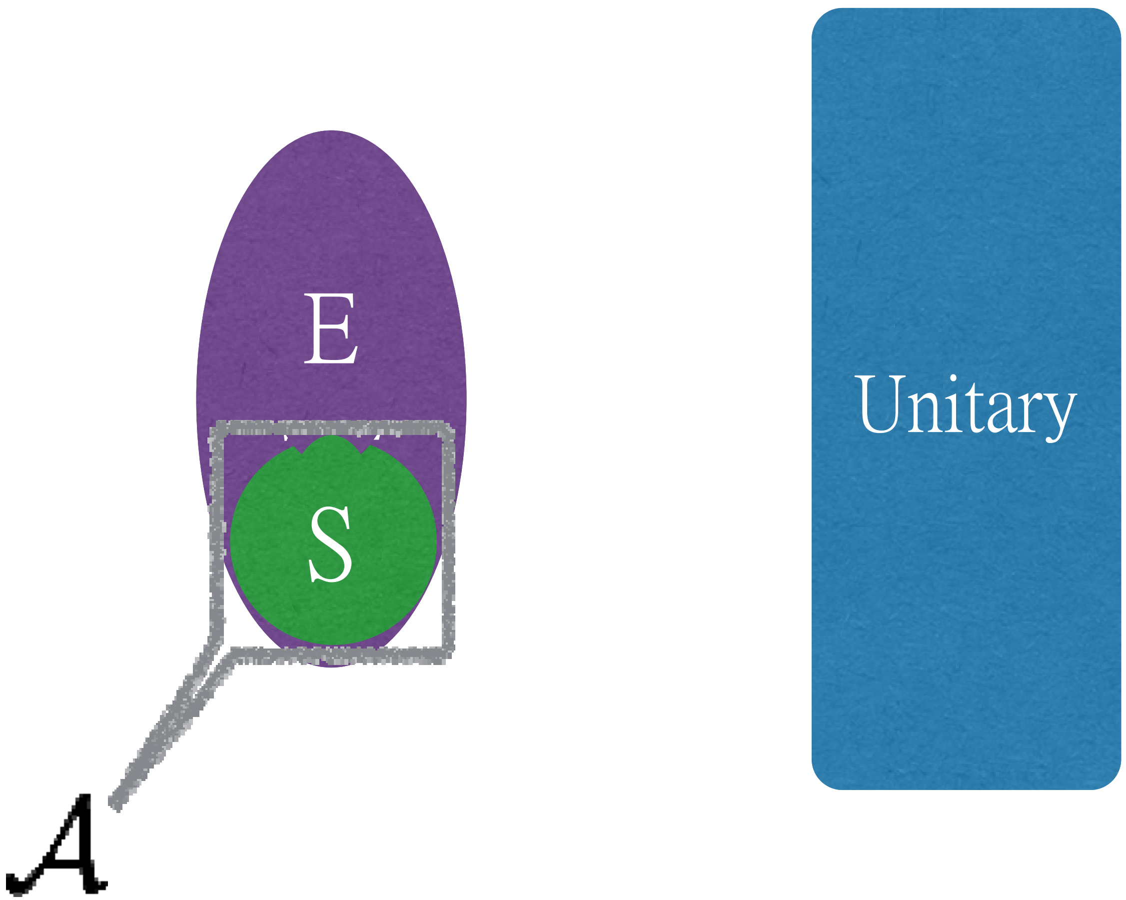
Argentieri, Benatti, Floreanini,... EPL. 107, 50007 (2014)

What can we say about initial state?



$$\rho_{\mathrm{avg}} = \sum_k p_k \rho_k$$

Initially correlated SE



Grad student presses buttons

We can give up states as inputs

Superchannel

Completely positive and linear

$$\mathcal{M}[\mathcal{A}] := \rho$$

Modi Sci. Rep. 2, 581 (2012)

Ringbauer, Wood, Modi, Gilchrist, White, Fedrizzi, PRL 114, 090402 (2015)

Using CP

Holevo bound [Masillo, Scolarici, Solombrino, J Math Phys 52, 012101 (2011)]

Data processing inequality [Buscemi, PRL. 113, 140502 (2014)]

Entropy production [Argentieri, Benatti, Floreanini,… EPL. 107, 50007 (2014)]

Vinjanampathy & Modi *PRA* 92, 052310 (2015)
Vinjanampathy & Modi, *Int. J. Quantum Inf*. 14, 1640033 (2016)

Characterising a generic quantum processes

What is missing?

- Can we characterise non-Markovian **quantum** processes?
  If so, how?

- Can we do this without making assumptions?

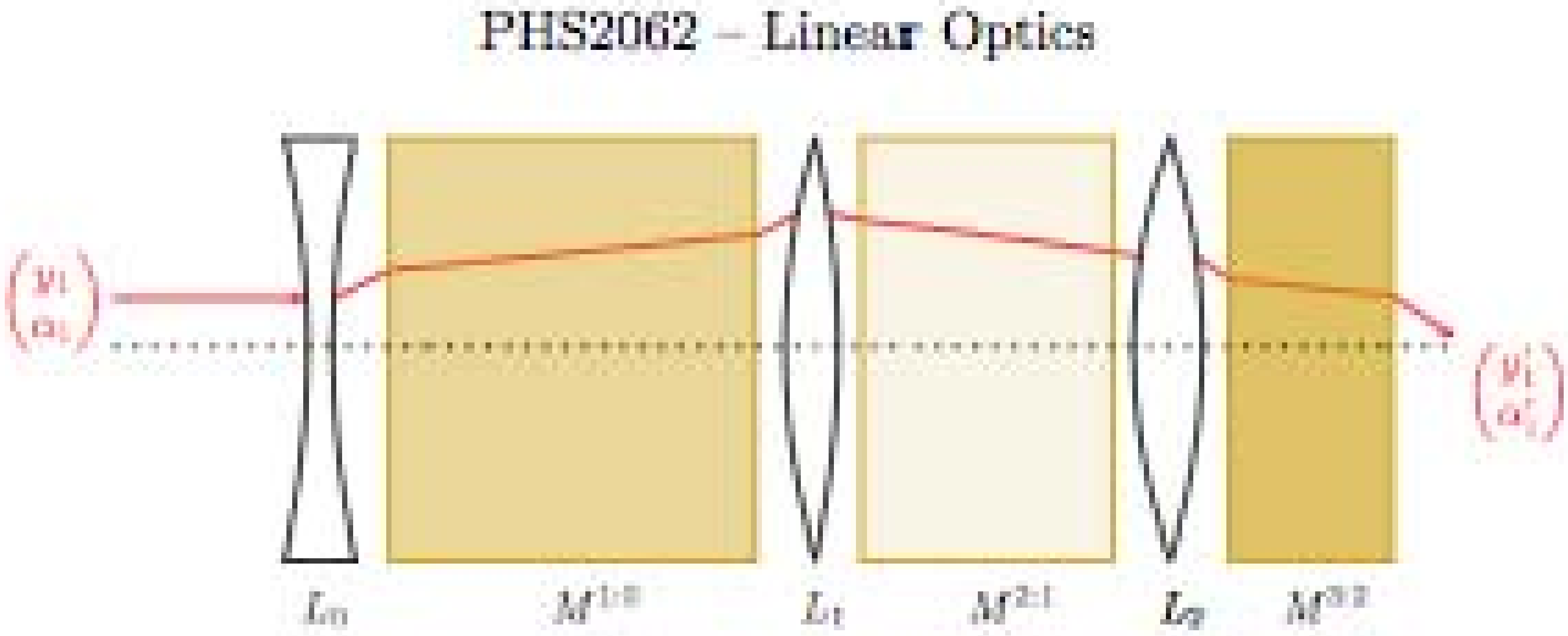
- Can have our cake and eat it?
  Keep positivity & linearity

- Can we do this practically?

Next time.

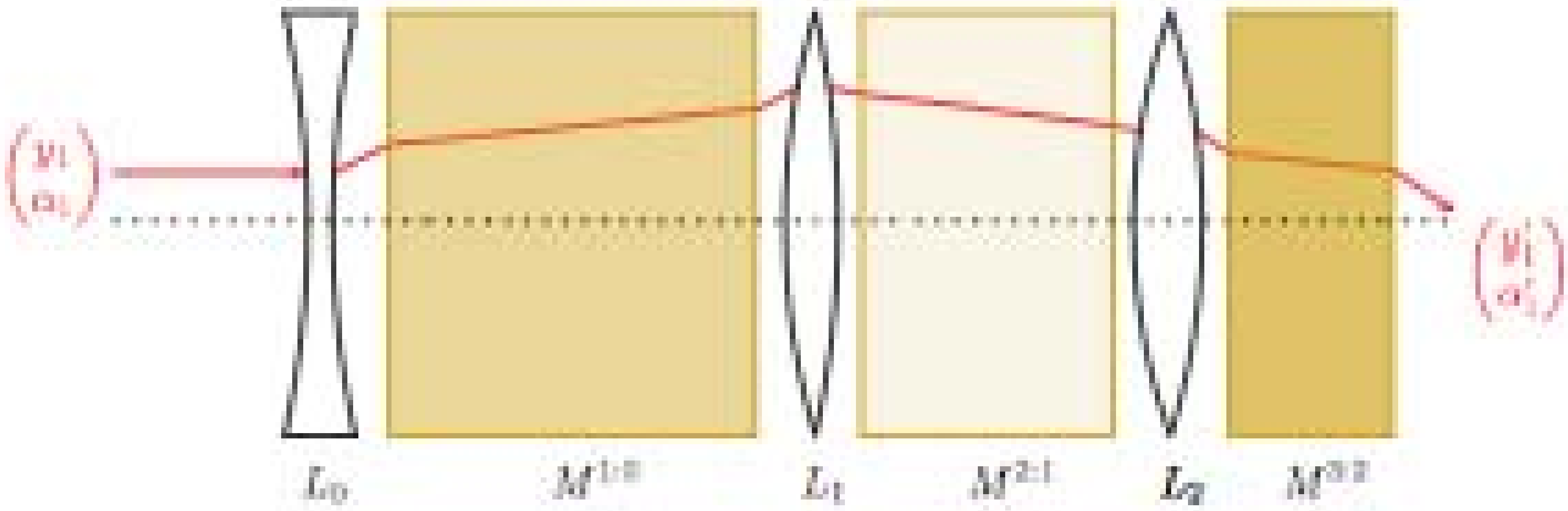
Thanks!!

Ray optics 101



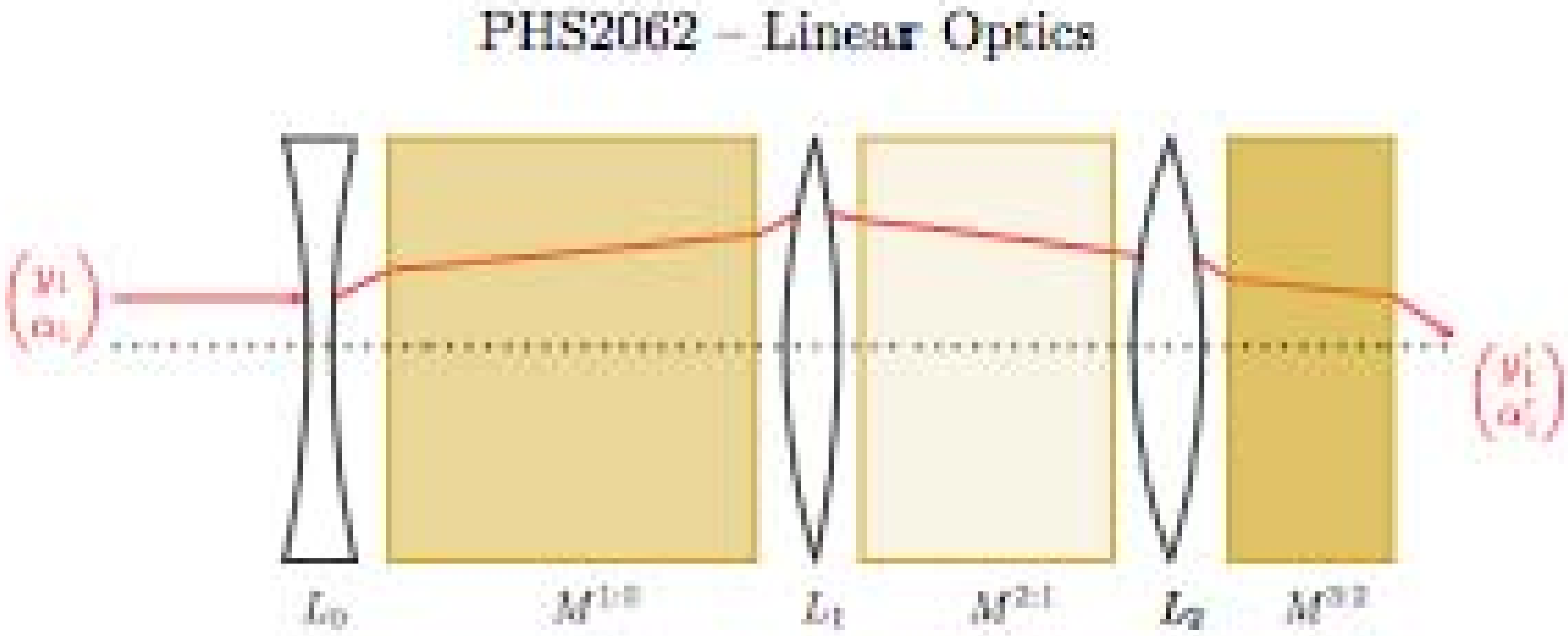
Ray optics 101

PHS2062 – Linear Optics



$$L = \begin{pmatrix} A & B \\ C & D \end{pmatrix}$$

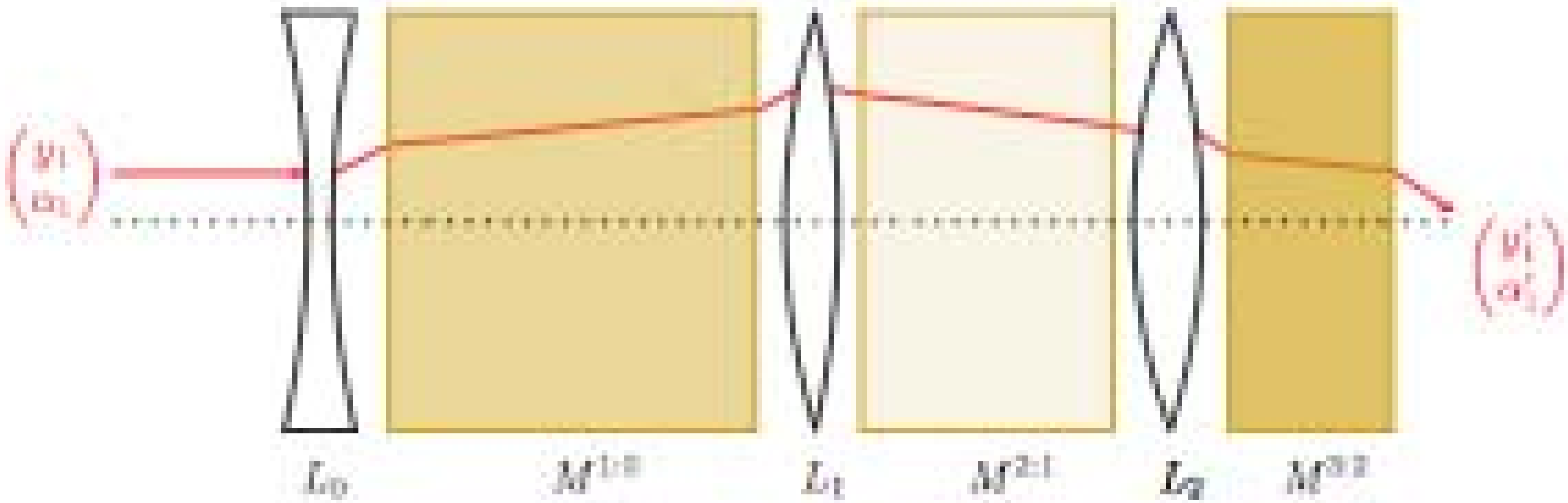
Ray optics 101



$$\Lambda = M^{3:2} \cdot L_2 \cdot M^{2:1} \cdot L_1 \cdot M^{1:0} \cdot L_0$$

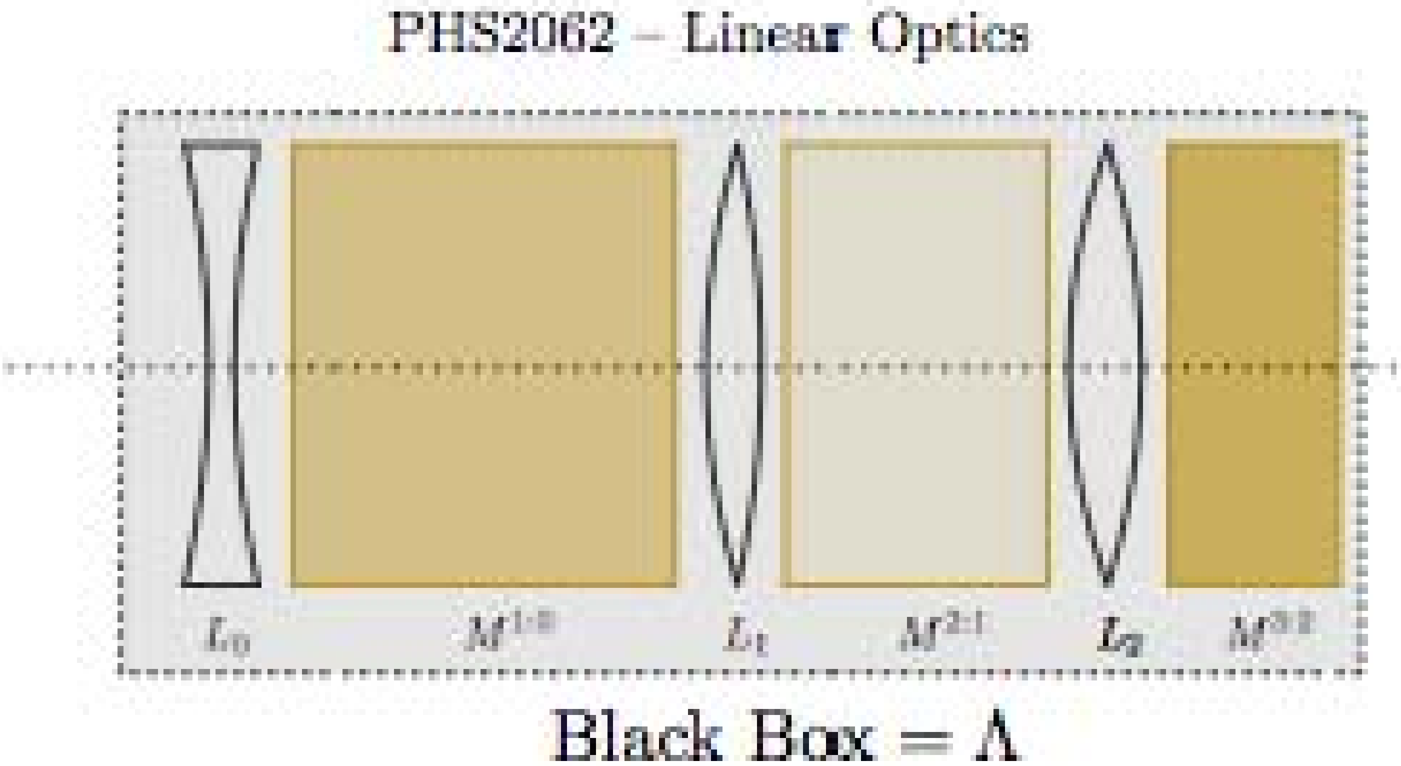
Ray optics 101

PHS2062 – Linear Optics

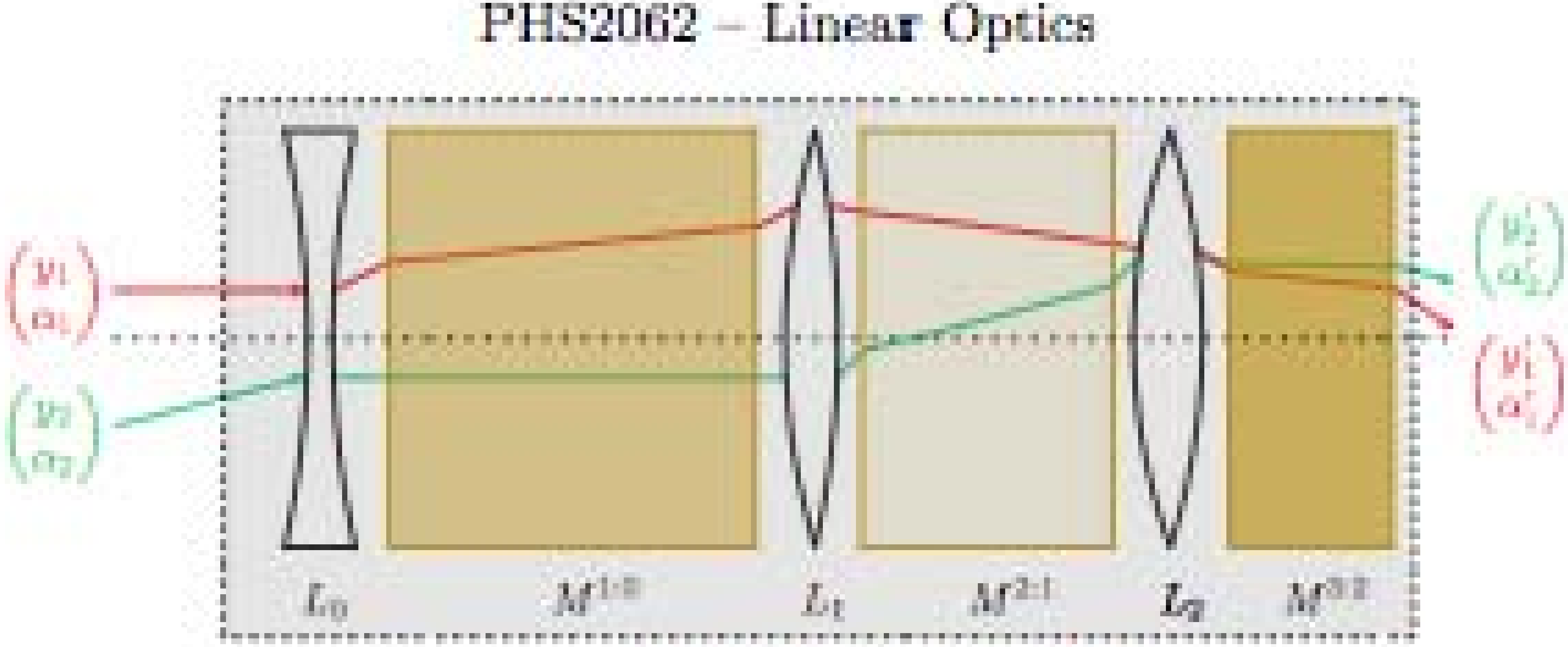


$$\begin{pmatrix} y_1' \\ \alpha_1' \end{pmatrix} = \Lambda \left[\begin{pmatrix} y_1 \\ \alpha_1 \end{pmatrix} \right]$$

Ray optics 101



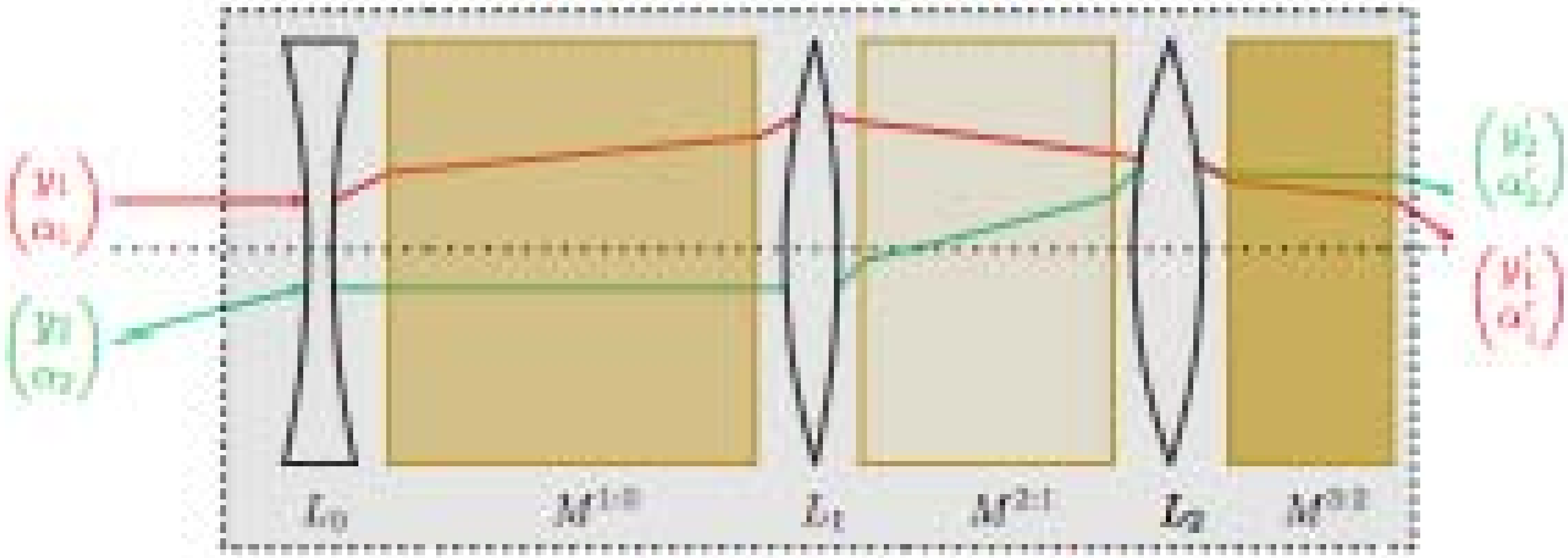
Ray optics 101



Black Box = Λ

Ray optics 101

PHS2062 – Linear Optics

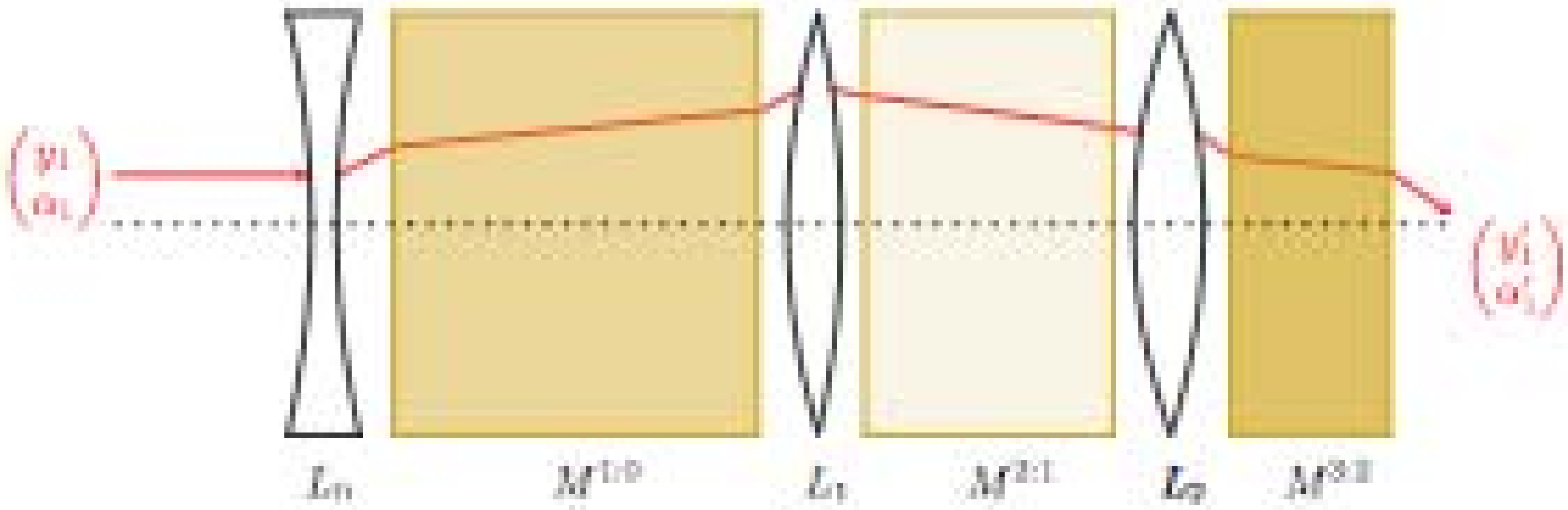


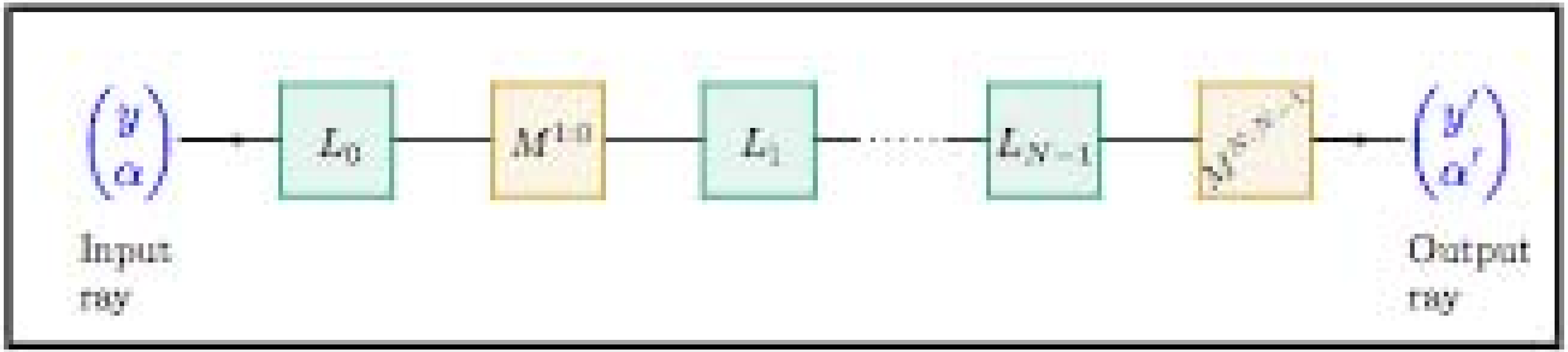
Black Box = Λ

$$\begin{pmatrix} y' \\ \alpha' \end{pmatrix} = \Lambda \left[\begin{pmatrix} y \\ \alpha \end{pmatrix} \right]$$

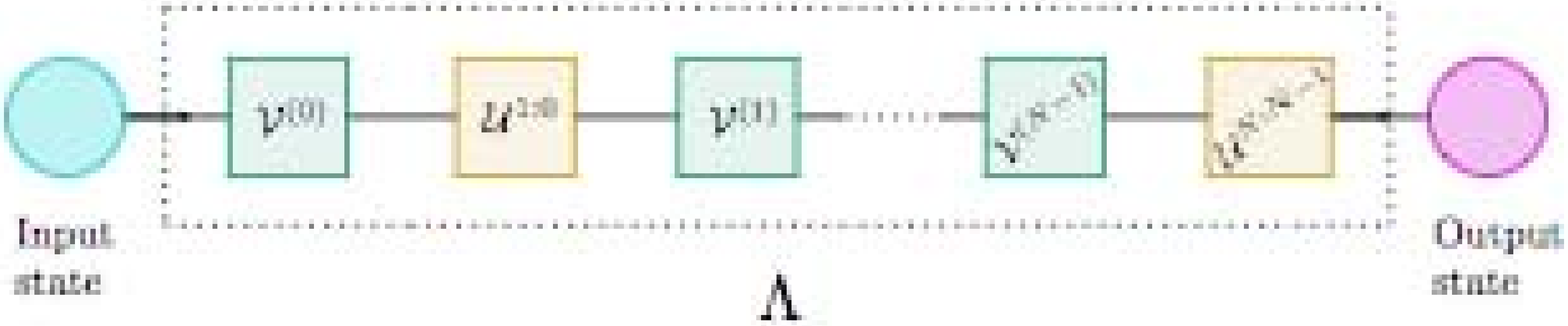
Closed Quantum Mechanics 101

PHS2062 – Linear Optics





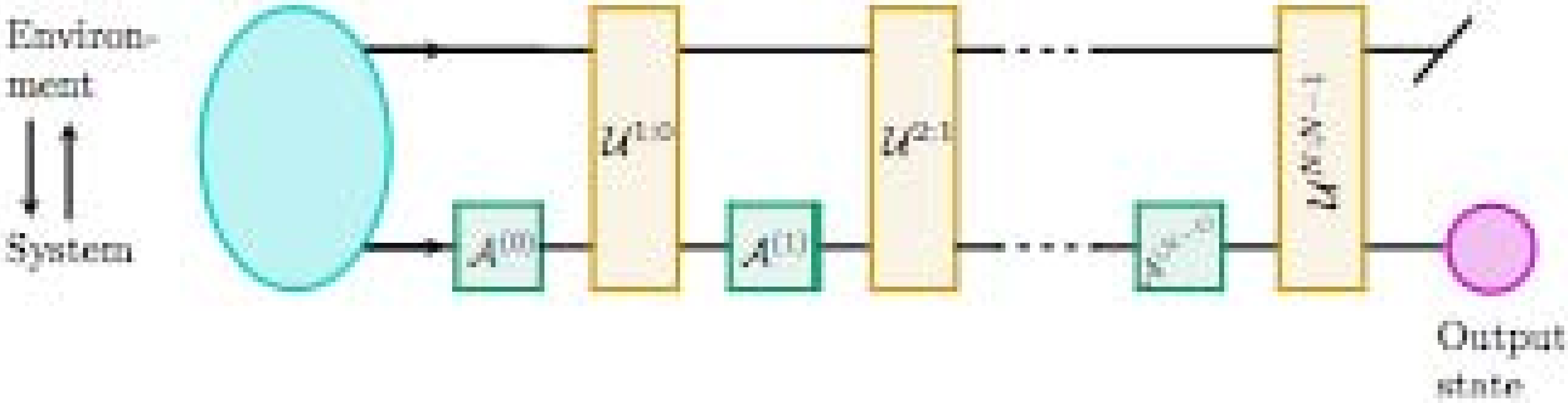
Closed Quantum Mechanics 101



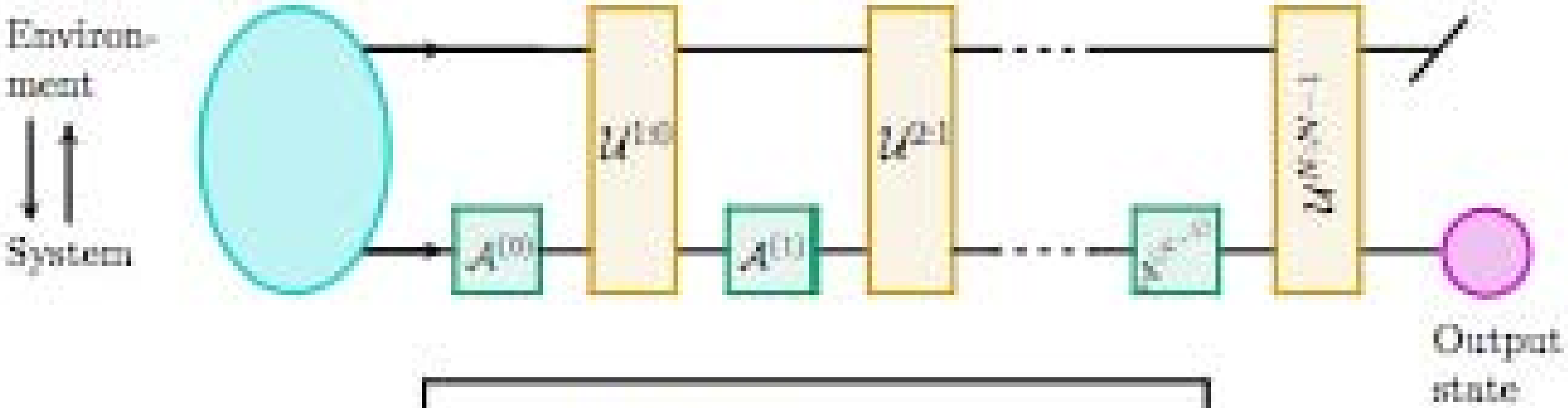
$\mathcal{V}^{(i)}$: Experimental controls

$\mathcal{U}^{k+1:k}$: Free unitary evolution (Schrödinger equation)

Open quantum mechanics

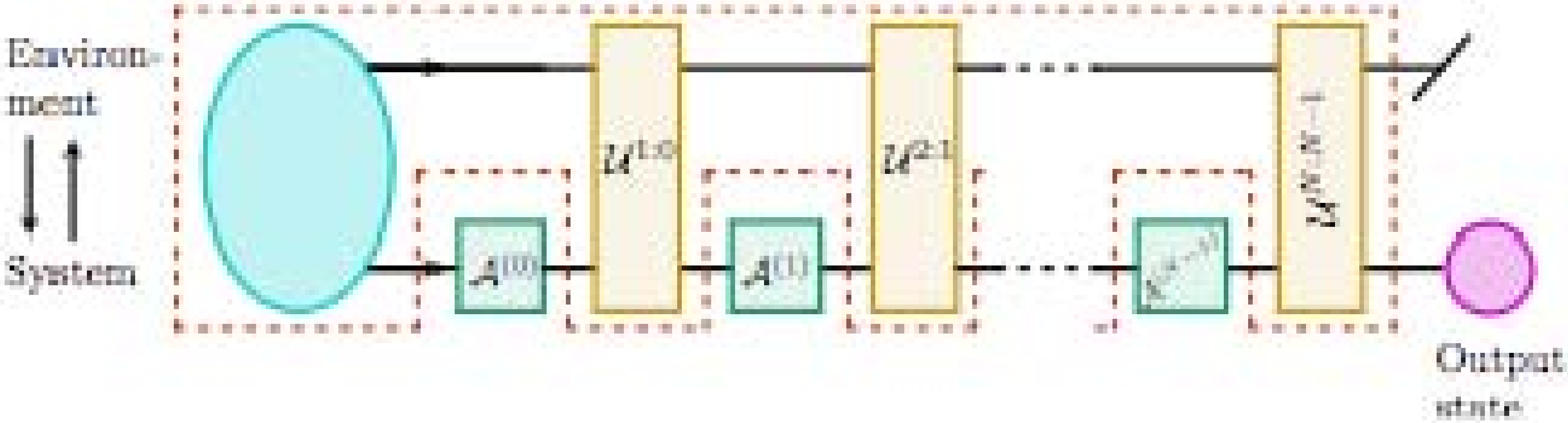


Open quantum mechanics

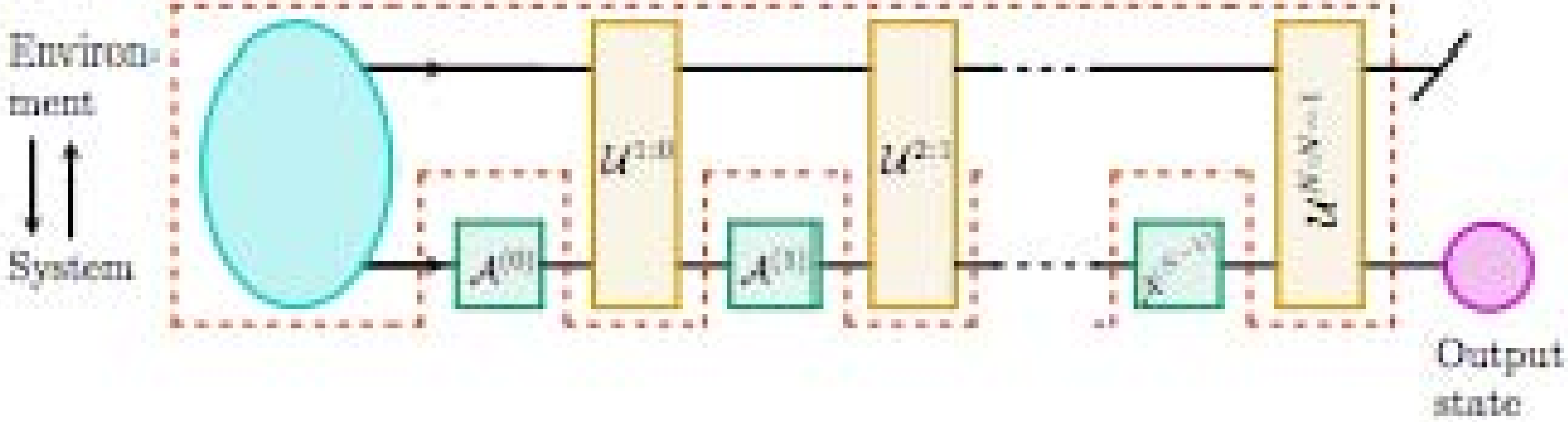


- Measurements
- Laser pulses
- RF pulses
- unitary operations
- ......

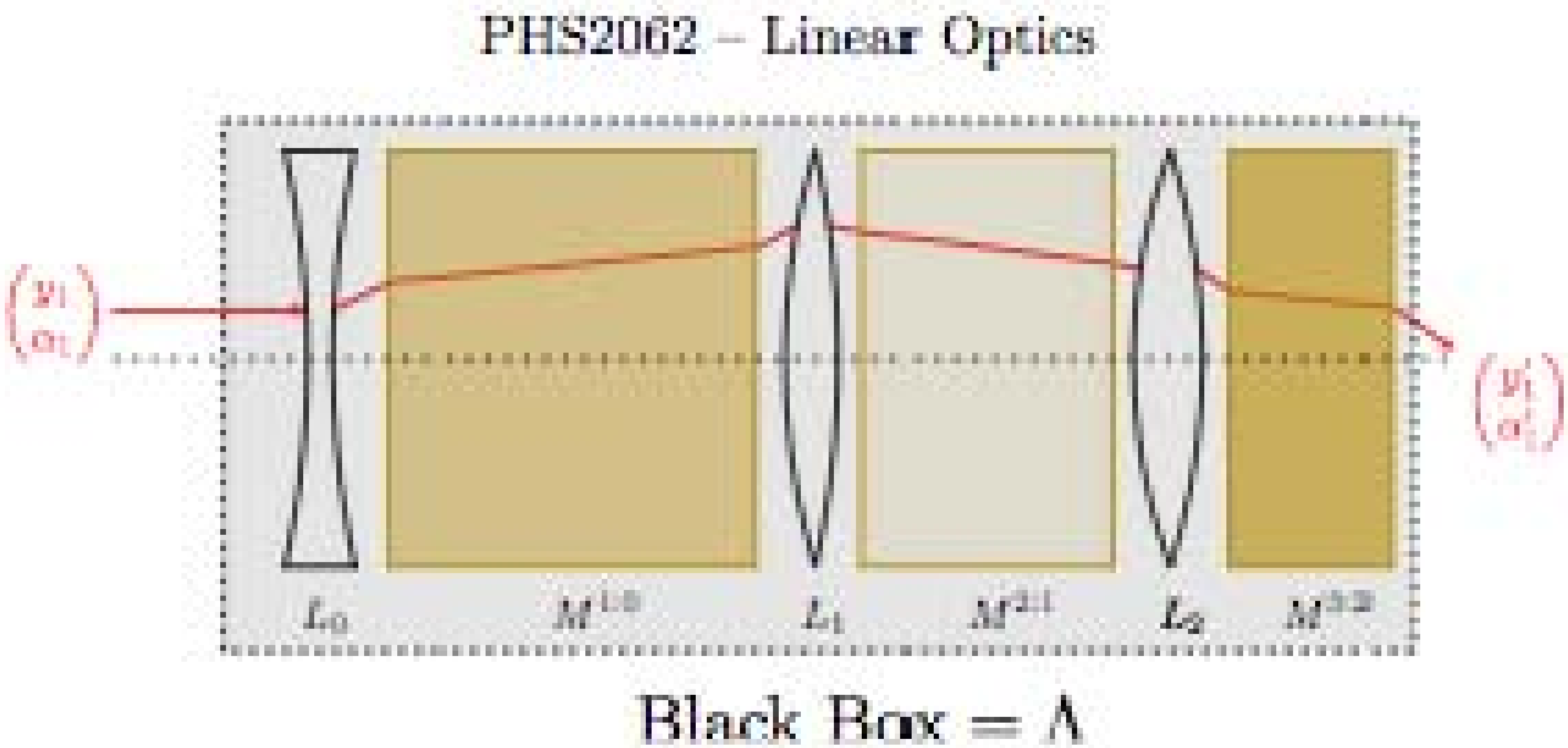
Open quantum mechanics



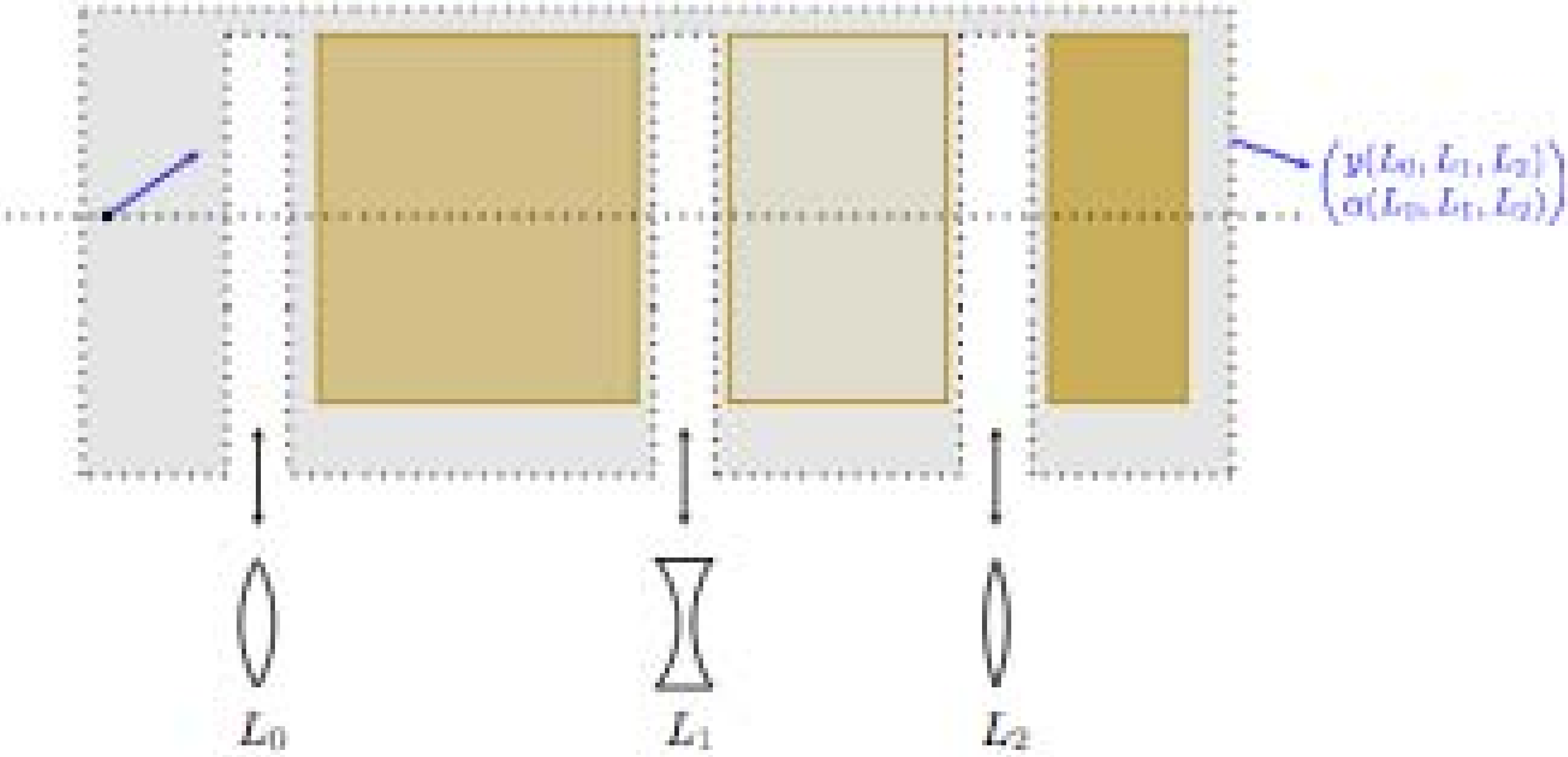
Open quantum mechanics



Open quantum mechanics



Open quantum mechanics



Open quantum evolution $\Rightarrow \mathcal{T}$

$$\rho_k^{SE} = \mathcal{U}_{k:k-1}\,\mathcal{A}_{k-1}\,\mathcal{U}_{k-1:k-2}\dots\mathcal{A}_1\,\mathcal{U}_{1:0}\,\mathcal{A}_0\,[\rho_0^{SE}]$$

$$\rho_k = \mathcal{T}_{0:k}[\mathbf{A}_{0:k}] = \sum_l (T_{k:0})_l\,\mathbf{A}_{0:k}\,(T_{0:k})_l^\dagger$$

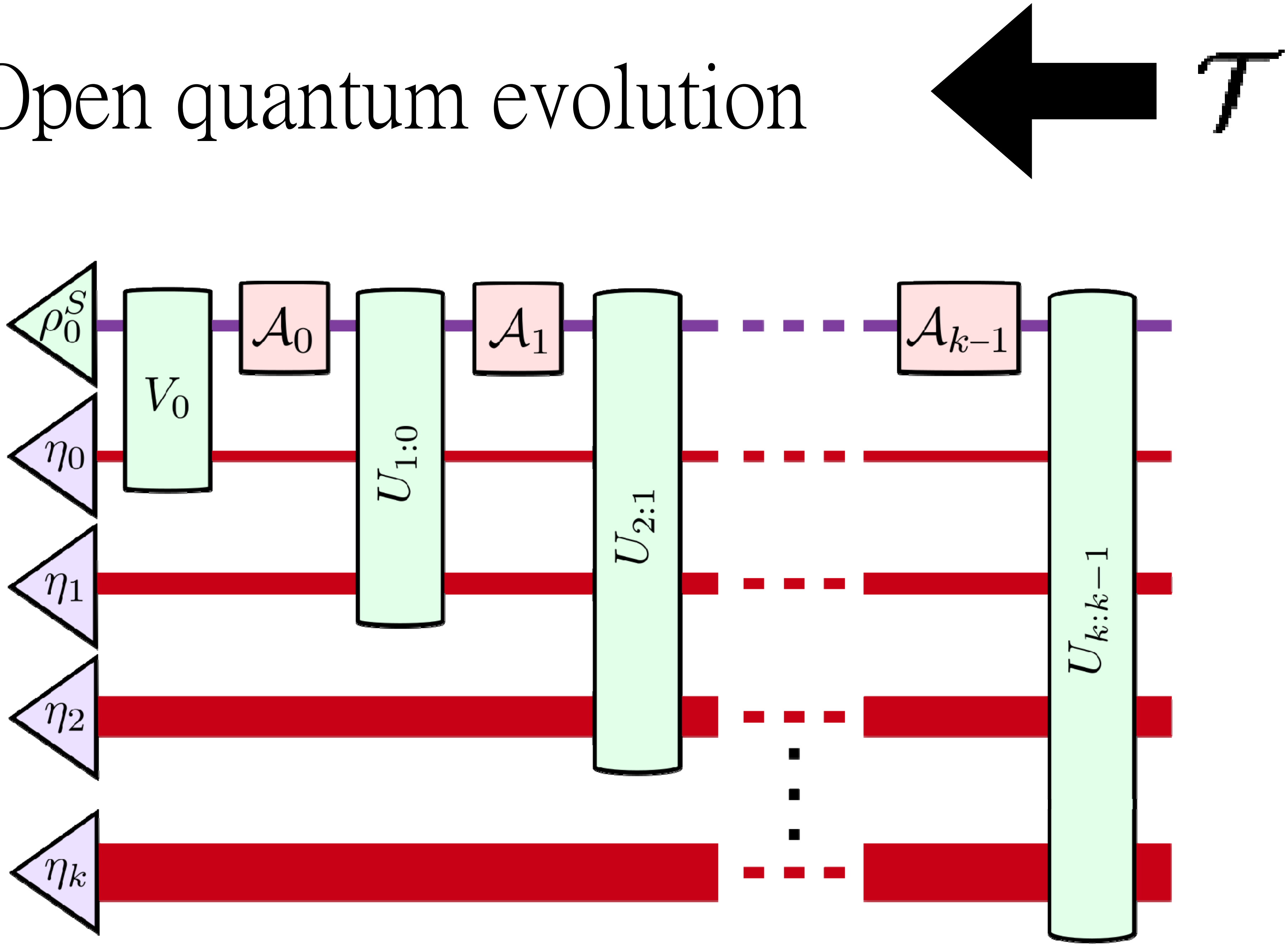
$$\mathbf{A}_{k-1:0} = [\mathcal{A}_{k-1};\mathcal{A}_{k-2};\dots;\mathcal{A}_1;\mathcal{A}_0]$$

Completely positive and linear

$$\mathcal{T}_{k':j'} \subset \mathcal{T}_{k:0}$$

Taking a middle page from the lab notebook

Open quantum evolution

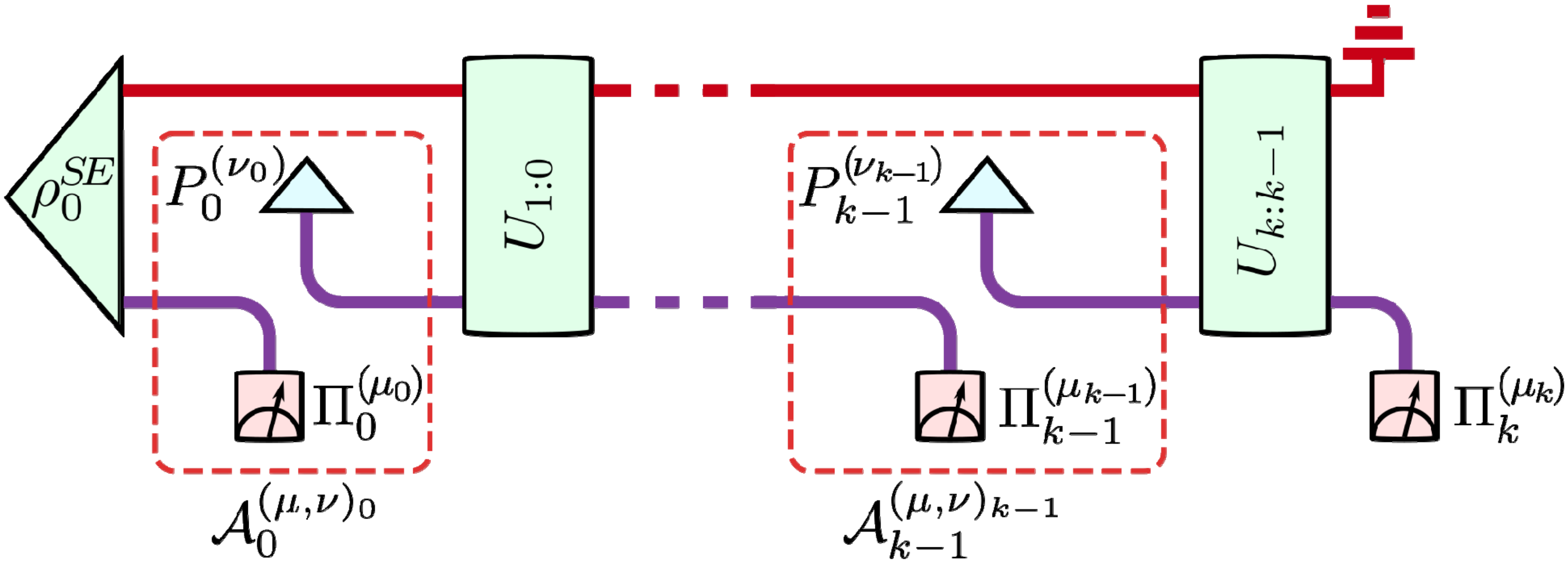


Universality

- Process tensor is the most general descriptor of quantum processes
- We have not assumed any model
- It has all of the desired properties:
  linear, complete positivity, containment
- It’s operationally motivated, meaning it is designed to work hand-in-hand with experiments

How do we find $\mathcal{T}$?

Linear expansion



$$\mathcal{A}[\rho] = \sum_{\mu,\nu} \alpha^{(\mu,\nu)} \mathcal{A}^{(\mu,\nu)}[\rho] = \sum_{\mu,\nu} \alpha^{(\mu,\nu)} P^{(\nu)} \mathrm{tr}[\Pi^{(\nu)} \rho]$$

SHAMELESS SELF-ADVERTISMENT

Reconstructing open quantum system dynamics with limited control

arxiv: 1610.02152

Simon Milz, Felix A. Pollock, and Kavan Modi

School of Physics and Astronomy, Monash University, Clayton, Victoria 3800, Australia

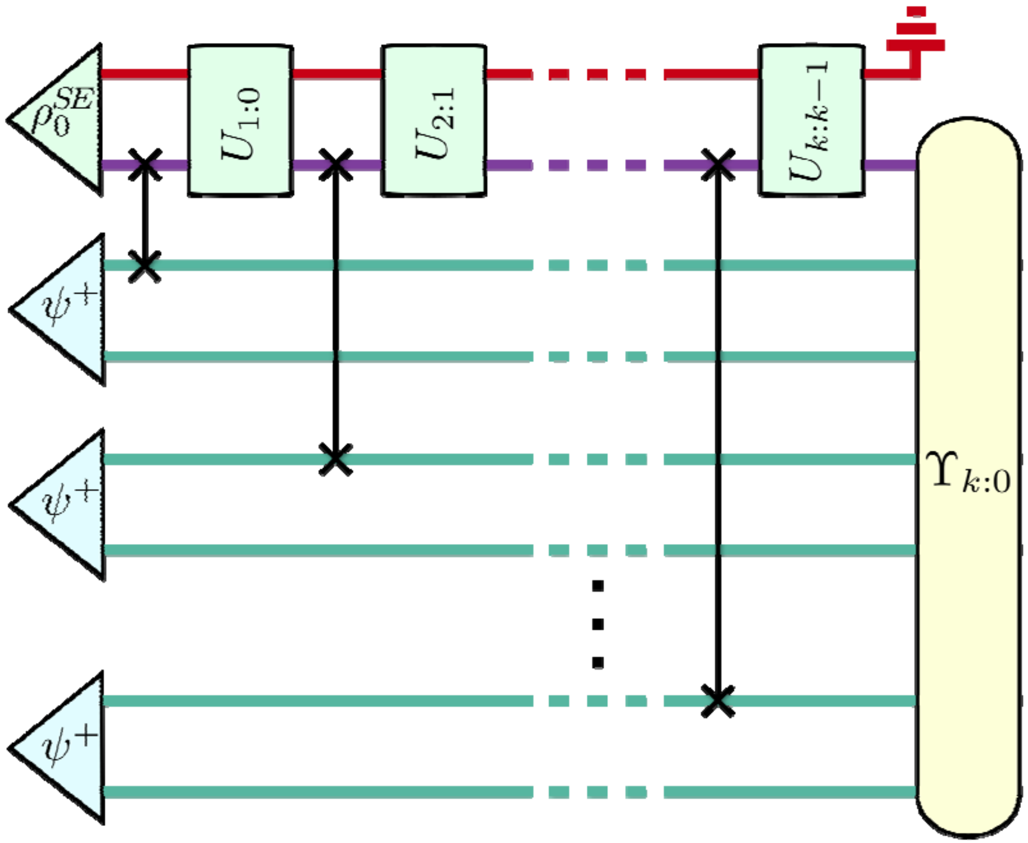
October 10, 2016

[quant-ph] 7 Oct 2016

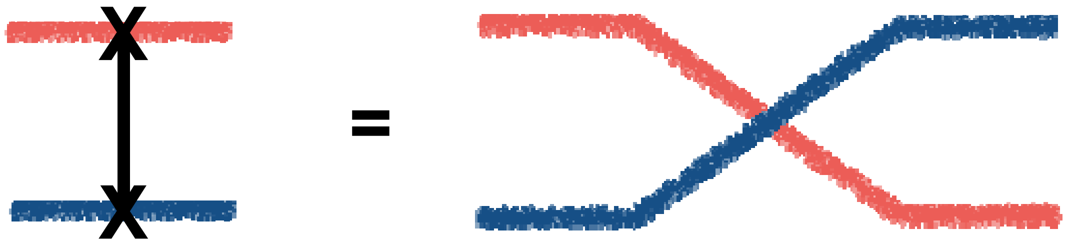
The dynamics of an open quantum system can be fully described and tomographically reconstructed if the experimenter has complete control over the system of interest. Most real-world experiments do not fulfill this assumption, and the amount of control is restricted by the experimental set-up. That is, the set of performable manipulations of the system is limited. For instance, imagine a set-up where unitary operations are easy to make, but only one measurement at the end of the experiment is allowed. In this paper, we provide a general reconstruction scheme that yields operationally well-defined dynamics for any conceivable kind of experimental situation. If one additional operation can be performed, these 'restricted' dynamics allow for the construction of witnesses for initial

states'. This description is well-defined and experimentally reconstructible for the case of Markovian dynamics [1–4], but loses its physical meaning for more general cases [5, 6]. To overcome this problem, the so-called superchannel formalism [7] was proposed, which is an operational framework for the description of general quantum mechanical processes. In this framework, open quantum system dynamics are modeled only in terms of parameters that are in principle controllable/measurable by the experimenter, namely the settings of the experimental set-up at the beginning of the experiment and the final state of the system after the experiment has concluded. Consequently, the superchannel is a linear and completely positive (CP) map that unambiguously maps each possible manipulation of the system – or, more generally, each input that is controllable by the experimenter – to a final out-

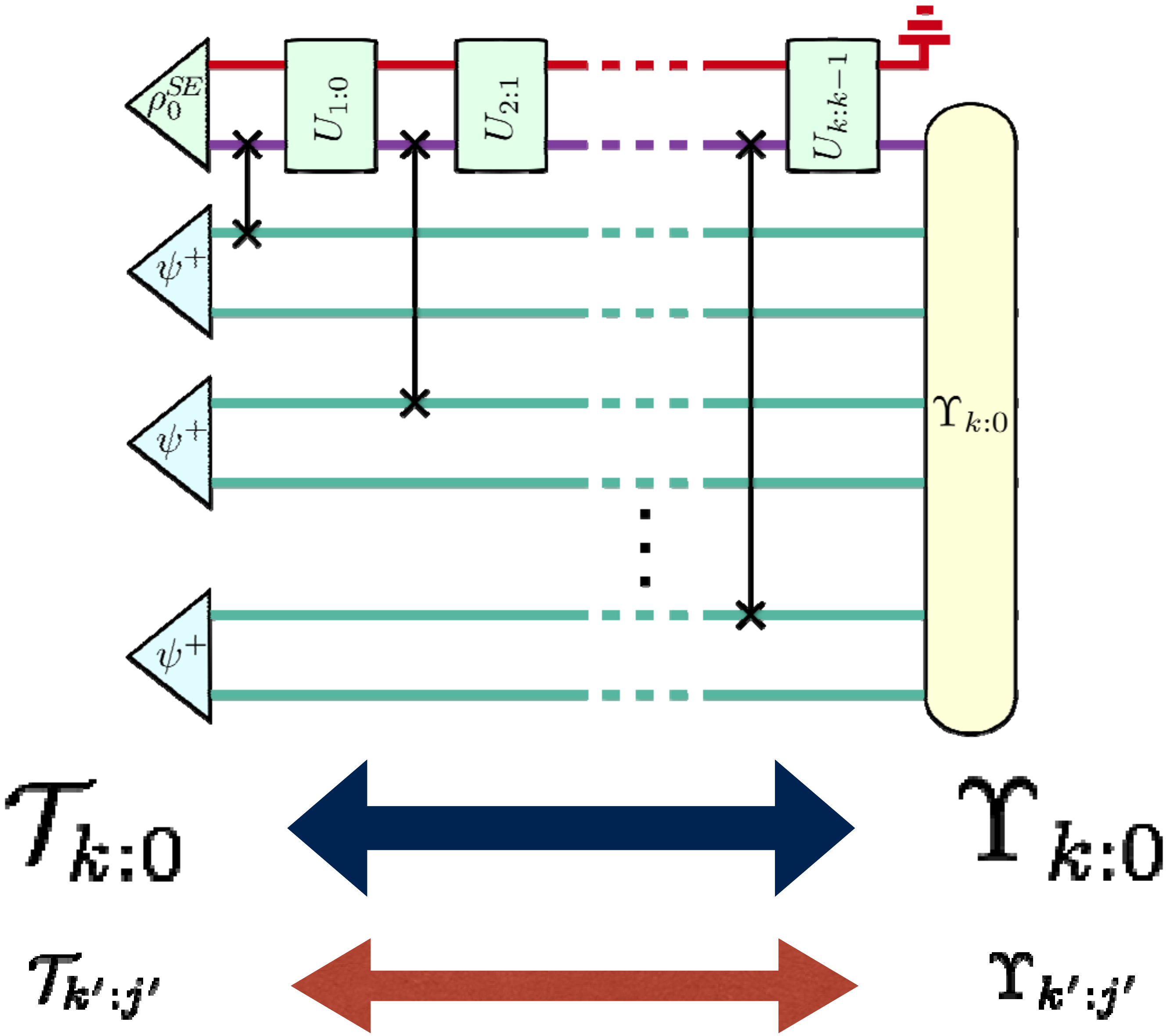
Encode into state



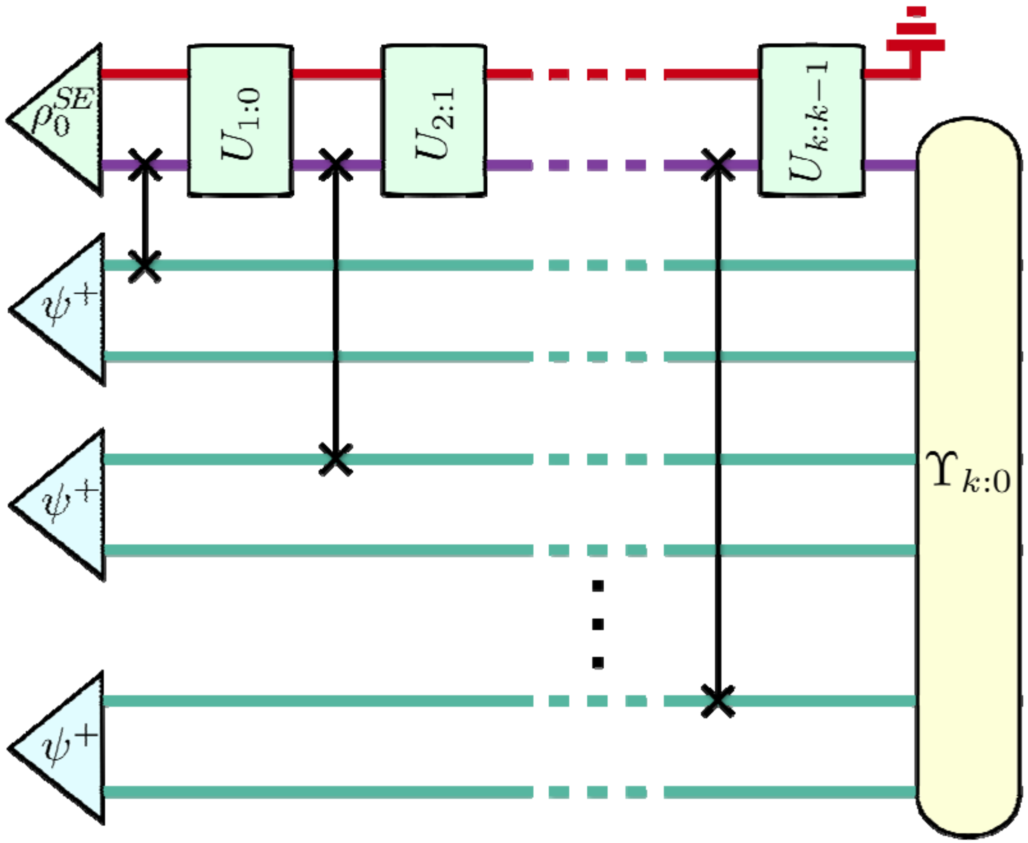
$$|\psi^+\rangle = \frac{1}{\sqrt{d}} \sum_x |xx\rangle$$



Encode into state



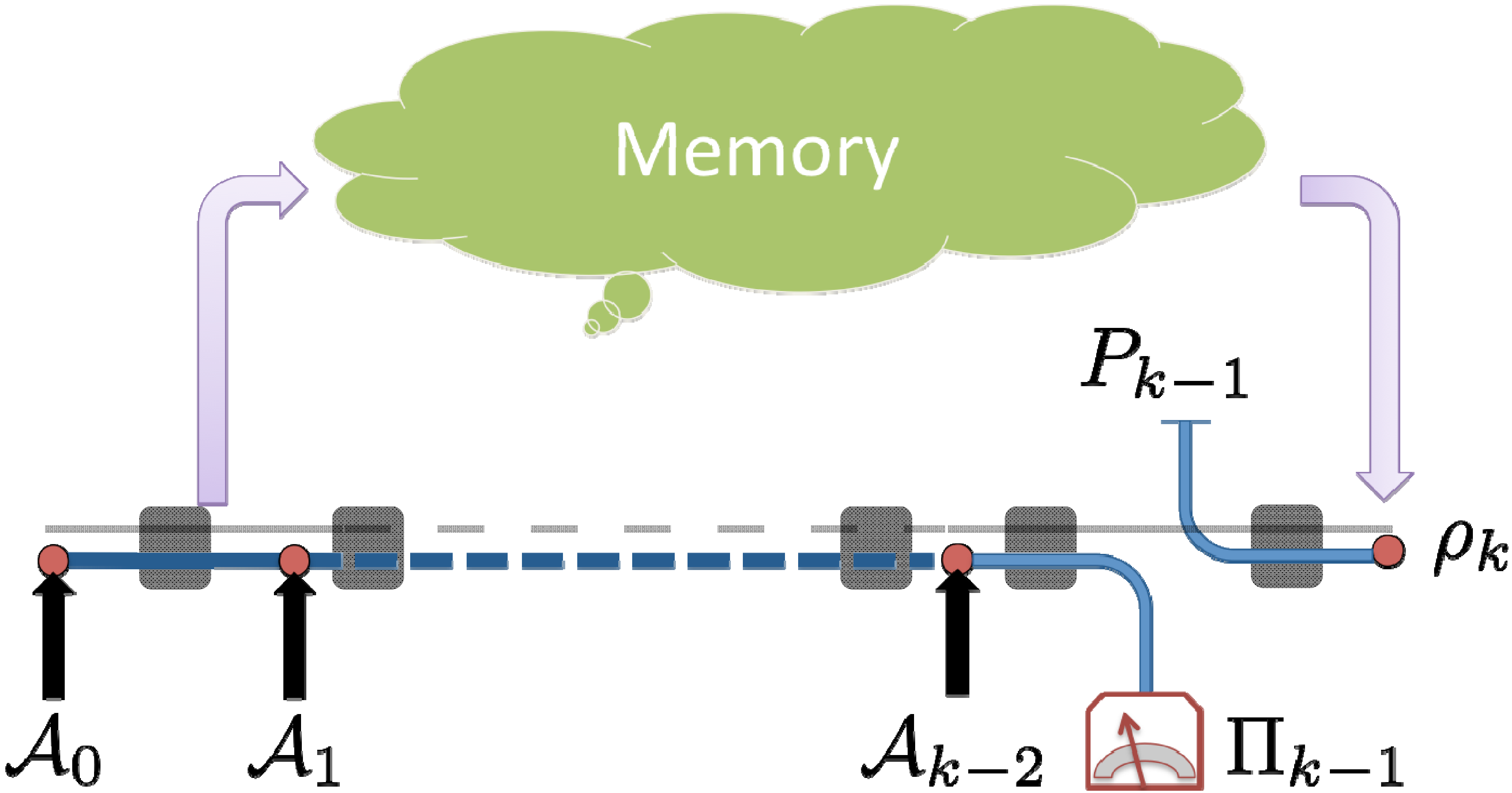
Encode into state

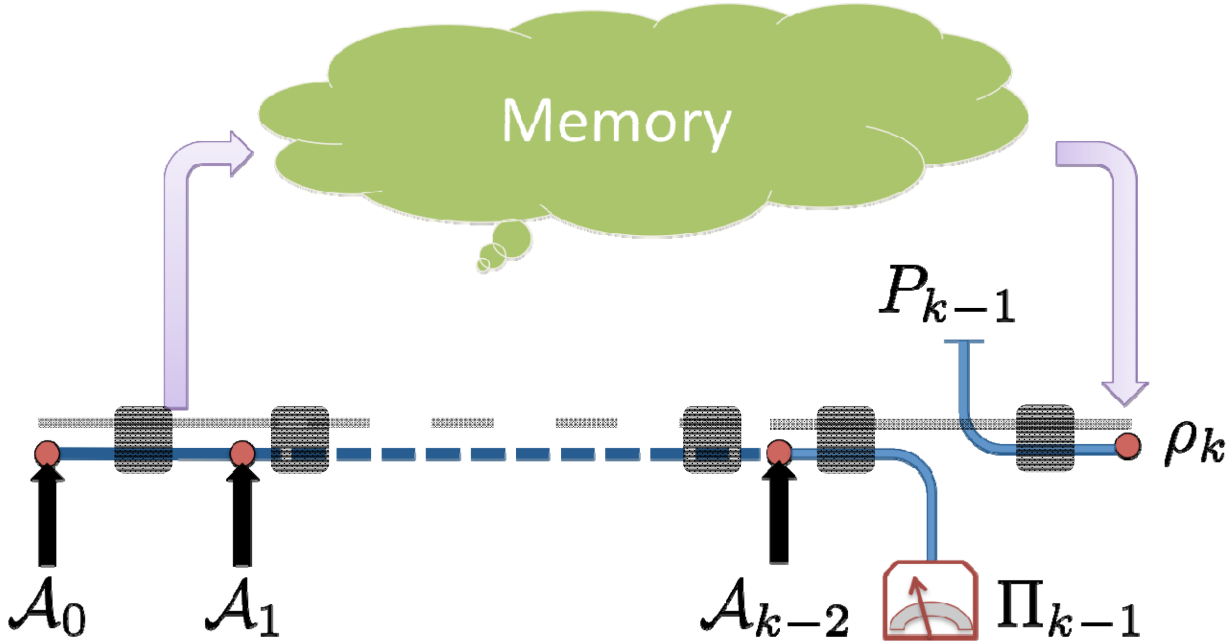


$\Upsilon_{k:0}$ is a matrix product density operator.

What is Markovian?

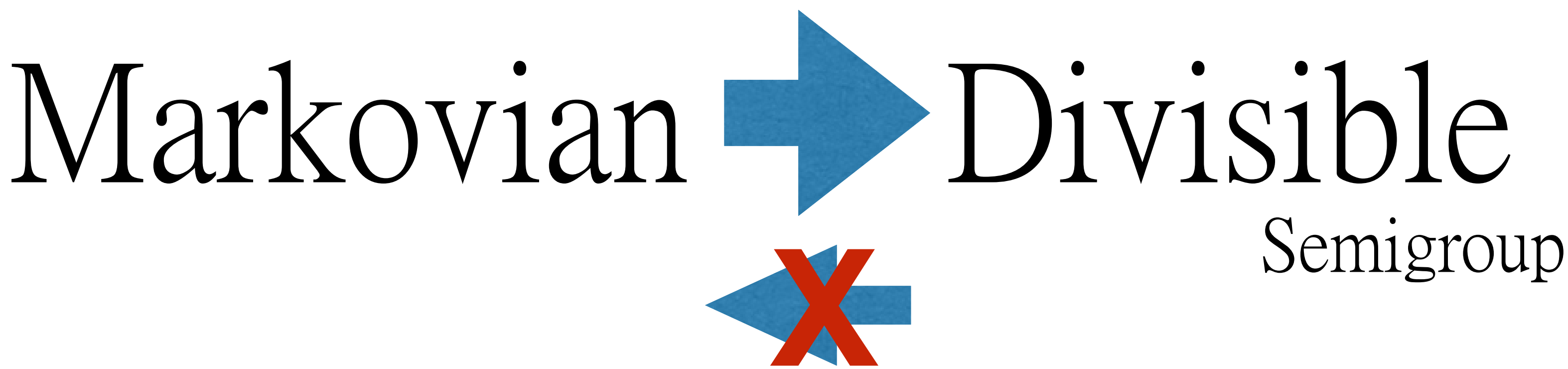
Causal break



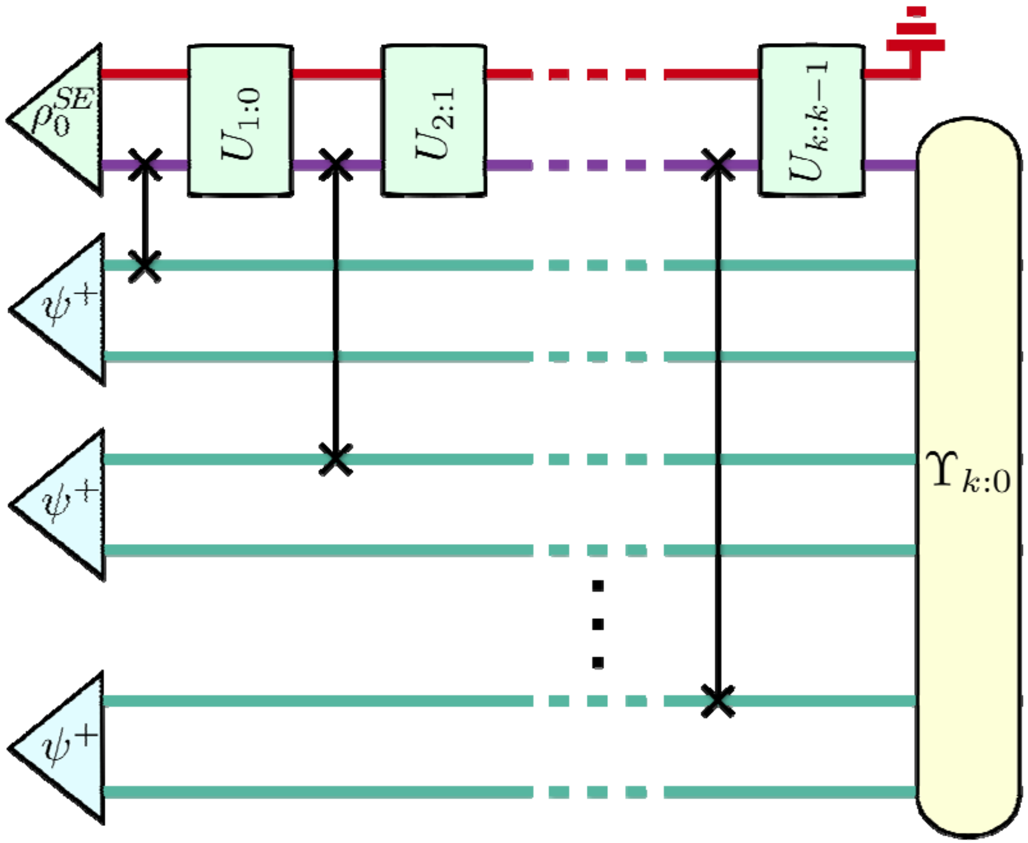


$$\rho_k(P_{k-1}|\Pi_{k-1};\mathbf{A}_{k-2:0}) = \rho_k(P_{k-1})$$

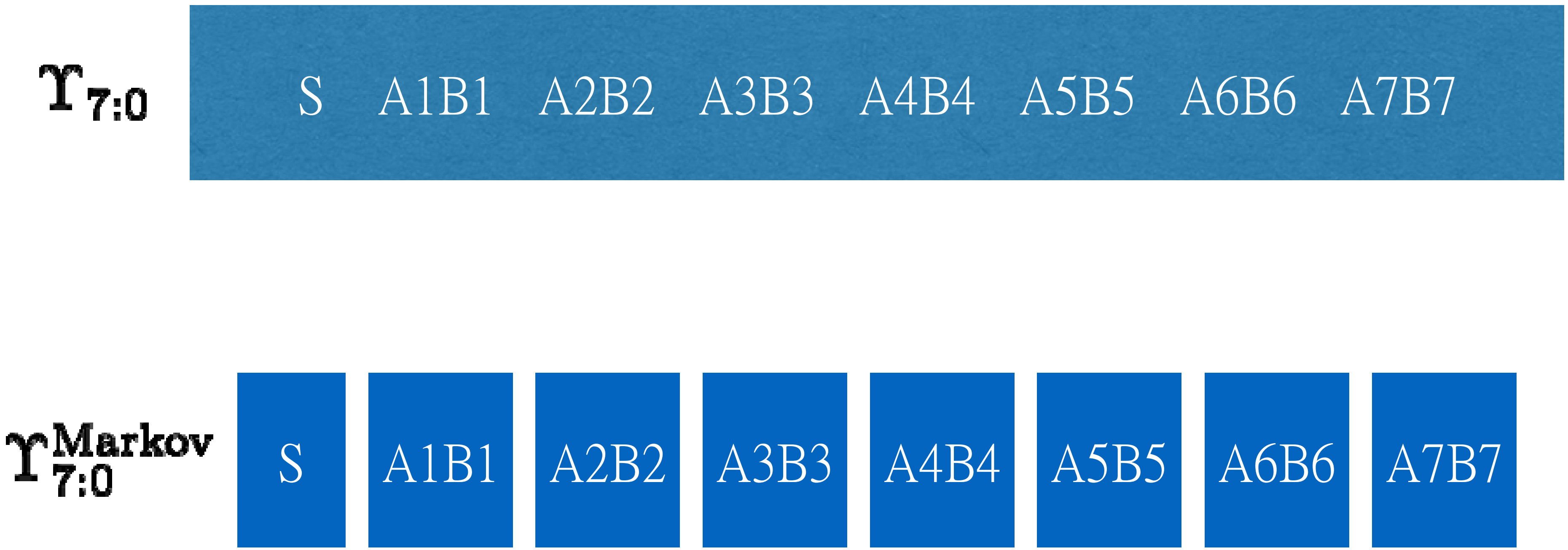
$$\rho_k(P_{k-1}|\Pi_{k-1};\mathbf{A}_{k-2;0}) \neq \rho_k(P_{k-1}|\Pi'_{k-1};\mathbf{A}'_{k-2;0})$$



Encode into state



Measuring non-Markovianity with Relative entropy



$$\mathcal{N} = R(\Upsilon_{7:0} \| \Upsilon_{7:0}^{\text{Markov}})$$

$$\text{Confusion probability} = e^{-n\mathcal{N}}$$

You have a Markovian model to describe a non-Markov process. How surprised are you when model gives wrong answer

What now?

- Existing experiments, e.g. ultrafast spectroscopy
- Bounds on energy transport
- Derive new master equations
- Typicality of non-Markovian process
- Quantum information theory / error correction
- Causal structures

Conclusions

arXiv:1512.00589

- We have a universal descriptor for arbitrary quantum processes

- We can encode the process into a many-body state, leading to an efficient characterisation

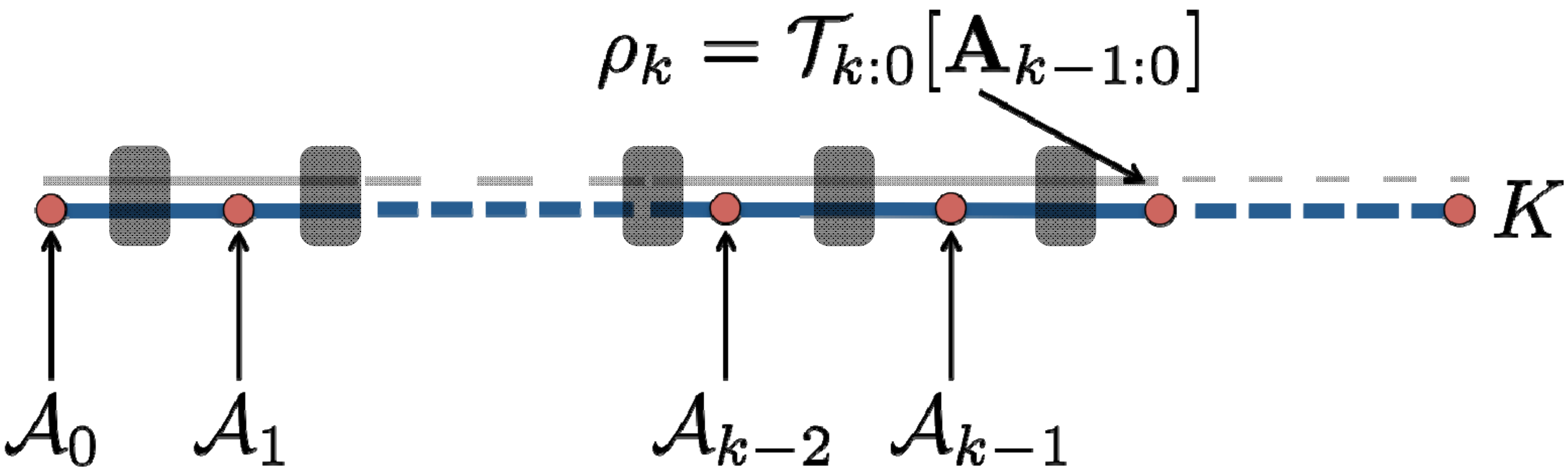
- Operational definition of non-Markovianity

http://monqis.physics.monash.edu

1. Introduction
2. Group (co-authors)
3. Dynamical maps (Complete Positivity)
4. Stinespring
5. Initial correlations/non-Markovianity
6. Pechucas theorem
7. Resolution (give up states)
8. Superchannel
9. Experiment
10. What is a process?
11. What is characterisation?
12. Process tensor
13. Properties of PT
14. Proof of Universality OQE to PT
15. Proof of Universality PT to OQE
16. Direct tomography (scaling)
17. CJI states
18. Substates of CJI state
19. Causal break
20. Non-Markov condition
21. Measure for non-Markovianity
22. Applications
23. Holevo, transport
24. Quantum information theory

Can we generalise this?

Quantum process



$$\mathbf{A}_{k-1:0} = [\mathcal{A}_{k-1}; \mathcal{A}_{k-2}; \ldots; \mathcal{A}_1; \mathcal{A}_0]$$

These actions are the history of buttons grad student pressed