Entanglement and Random Measurements

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MESSAGE

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Pure entangled states are more correlated in random local measurements than product states.

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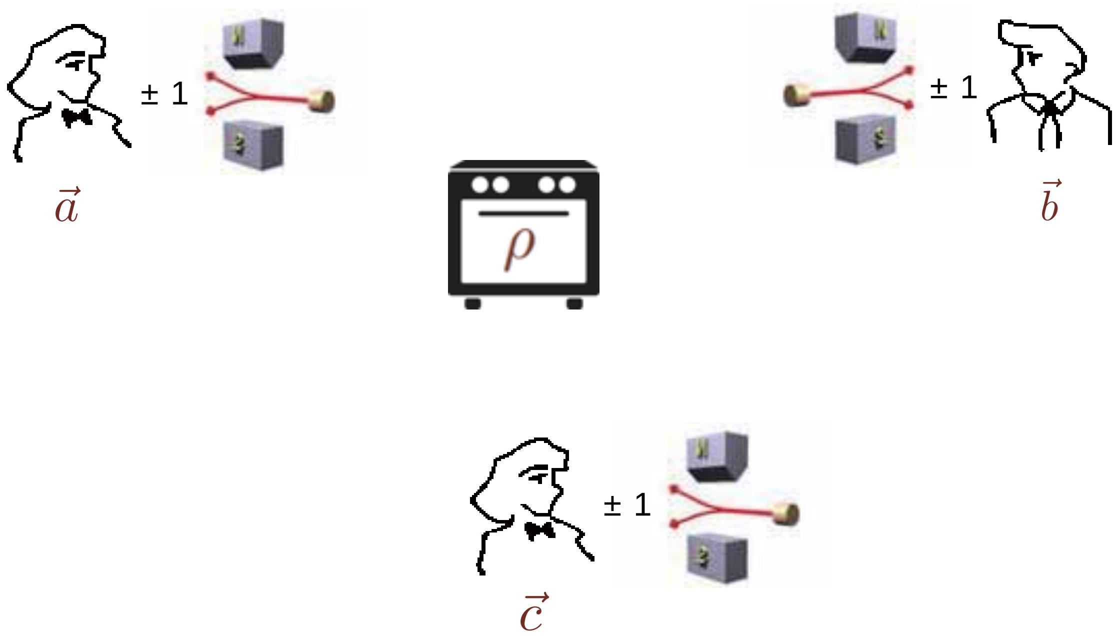
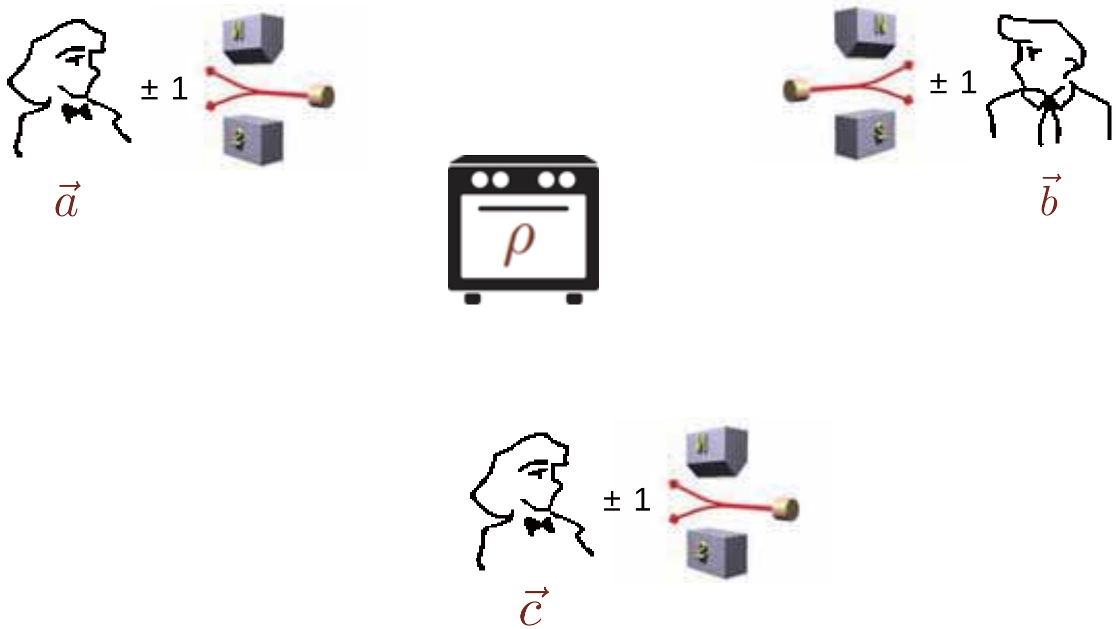
… and consequences


± 1
$\vec{a}$
± 1
$\vec{b}$
ρ
± 1
$\vec{c}$

QUANTUM CORRELATION FUNCTIONS



$$\rho = \frac{1}{2^3} \sum_{\mu,\nu,\eta=0}^{3} T_{\mu\nu\eta}\, \sigma_\mu \otimes \sigma_\nu \otimes \sigma_\eta$$

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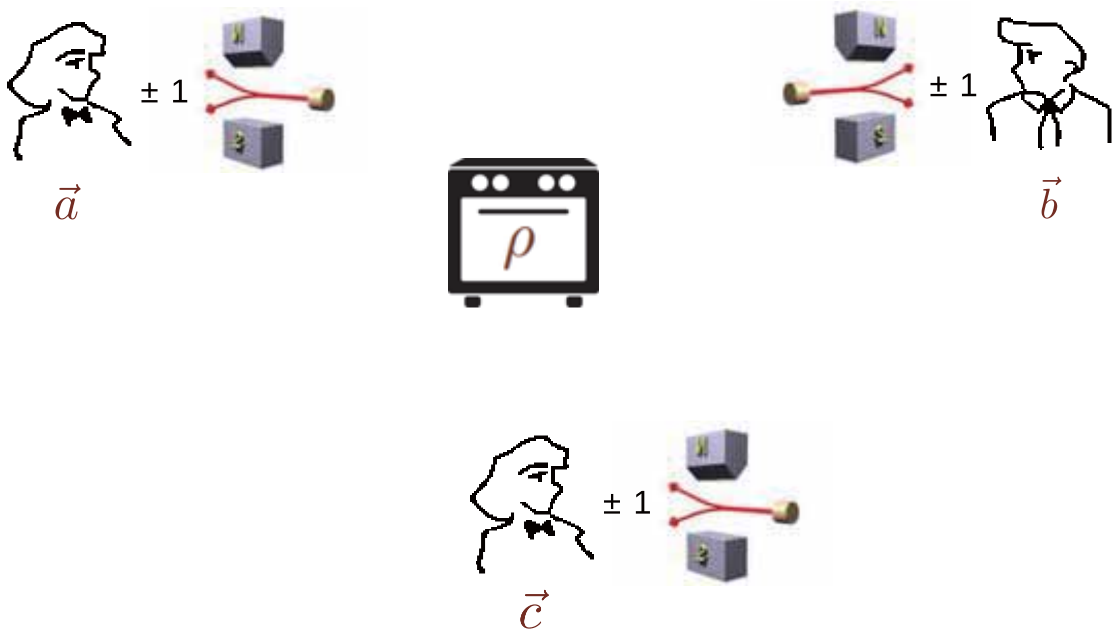

$$\rho = \frac{1}{2^3} \sum_{\mu,\nu,\eta=0}^{3} T_{\mu\nu\eta}\, \sigma_\mu \otimes \sigma_\nu \otimes \sigma_\eta$$

$$E(\vec{a},\vec{b},\vec{c}) = \sum_{j,k,l=1}^{3} T_{jkl}\, a_j b_k c_l$$

$$\mathcal{R} \equiv \frac{1}{(4\pi)^N} \int d\vec{u}_1 \ldots \int d\vec{u}_N \; E^2(\vec{u}_1, \ldots, \vec{u}_N)$$

E: *expectation value of the product of local results*

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E: *expectation value of the product of local results*

Universal lower bound:
For all pure states of N qubits

$$\mathcal{R} \geq 1/3^N$$

Length of correlations

$$\mathcal{R} = \frac{1}{3^N} \sum_{j_1,\ldots,j_N=1}^{3} T^2_{j_1\ldots j_N}$$

Linearisation

$$T^2_{j_1\ldots j_N} = \langle\psi|\sigma_{j_1} \otimes \cdots \otimes \sigma_{j_N}|\psi\rangle\langle\psi|\sigma_{j_1} \otimes \cdots \otimes \sigma_{j_N}|\psi\rangle$$

$$T^2_{j_1\ldots j_N} = \langle\psi|\langle\psi|\sigma_{j_1} \otimes \cdots \otimes \sigma_{j_N} \otimes \sigma_{j_1} \otimes \cdots \otimes \sigma_{j_N}|\psi\rangle|\psi\rangle$$

$$\mathcal{R} = \frac{1}{3^N}\langle\psi|\langle\psi| \sum_{j_1,\ldots,j_N=1}^{3} \sigma_{j_1} \otimes \cdots \otimes \sigma_{j_N} \otimes \sigma_{j_1} \otimes \cdots \otimes \sigma_{j_N}|\psi\rangle|\psi\rangle$$

$$|\psi_{1 \ldots N}\rangle \neq |\phi_1\rangle \otimes |\phi_2\rangle \otimes \cdots \otimes |\phi_N\rangle$$

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A pure state is entangled if and only if $\mathcal{R} > 1/3^N$

A. S. M. Hassan, P. S. Joag QIC **8**, 773 (2007); PRA **77**, 062334 (2008); PRA **80**, 042302 (2009)

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- *Operational significance of entanglement*
- *Entanglement is solely characterised by correlations between all parties*
- *Entangled states are more correlated in random measurements*

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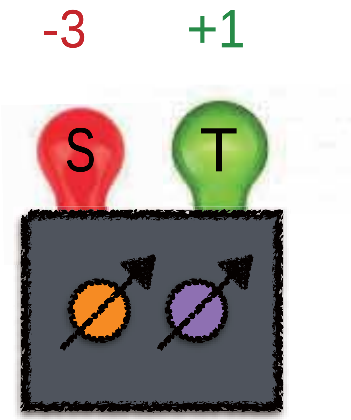
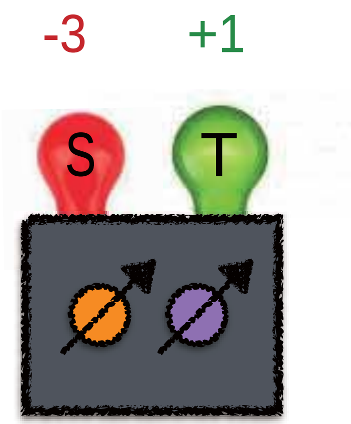
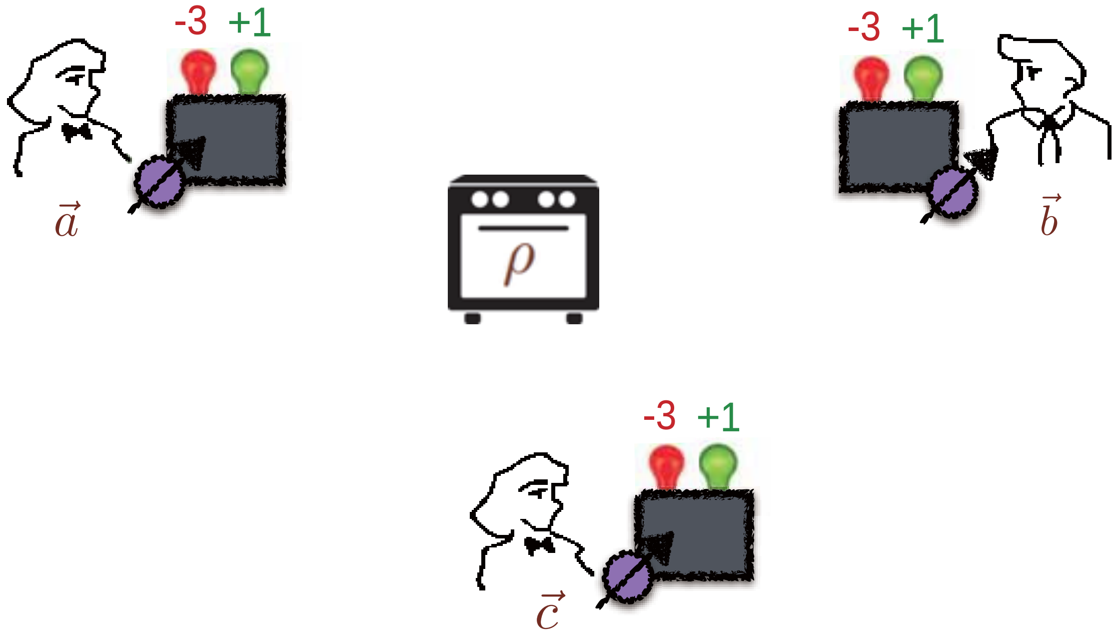

-3
+1
S
T

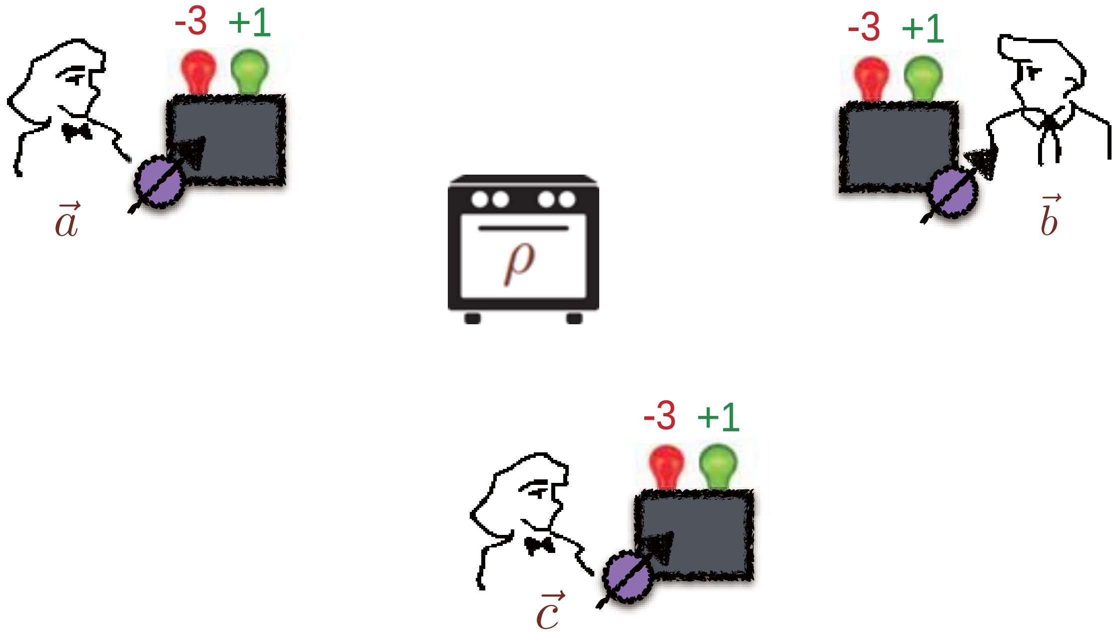


Observable of the nth party

$$\mathcal{M} = +1(\mathbb{1} - |\psi^-\rangle\langle\psi^-|) - 3|\psi^-\rangle\langle\psi^-| = \sum_{j=1}^{3} \sigma_j \otimes \sigma_j$$

CORRELATIONS WITH MICROSCOPIC REFERENCES





$$E(\vec{a}, \vec{b}, \vec{c})$$

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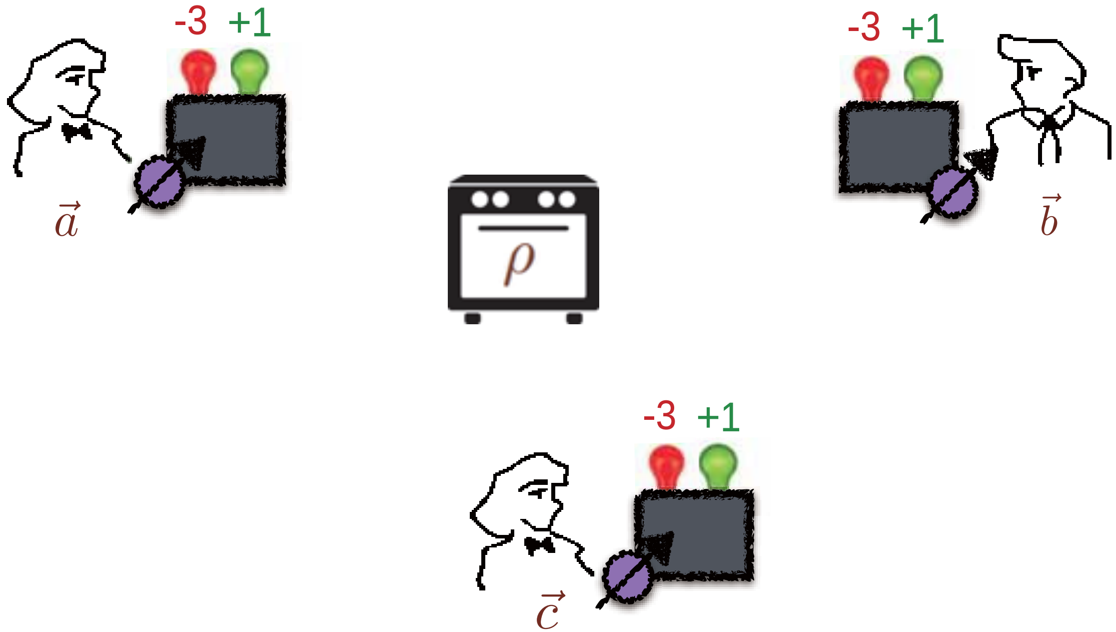
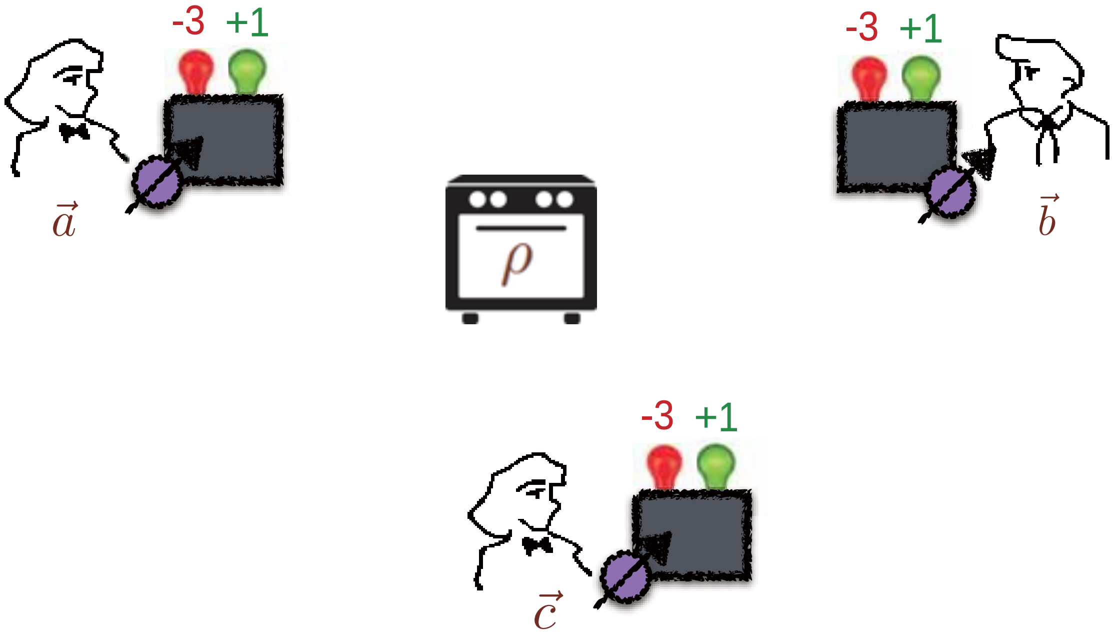


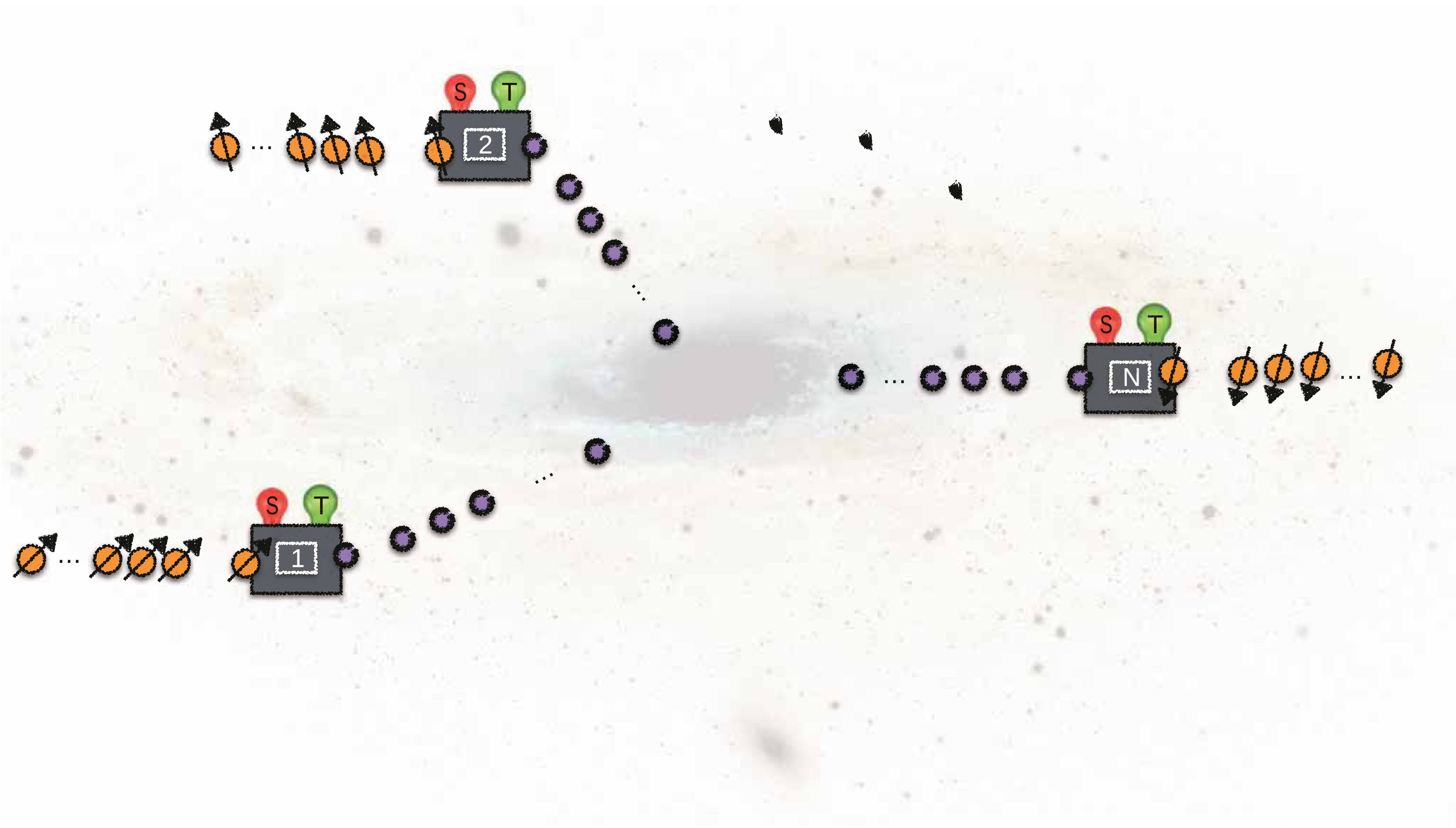
$$E(\vec{a},\vec{b},\vec{c}) \longrightarrow \rho_{123} \otimes a \otimes b \otimes c$$



$$E(\vec{a}, \vec{b}, \vec{c}) \longrightarrow \rho_{123} \otimes a \otimes b \otimes c \longrightarrow \sum_{j,k,l=1}^{3} T_{jkl}\, a_j b_k c_l$$

$\mathcal{M}_A$ $\mathcal{M}_C$

$\mathcal{M}_B$


S
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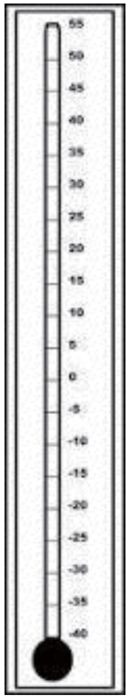
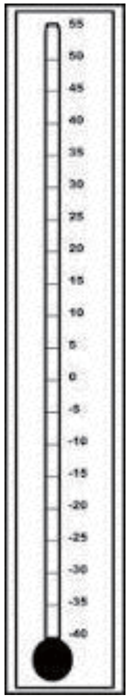
N

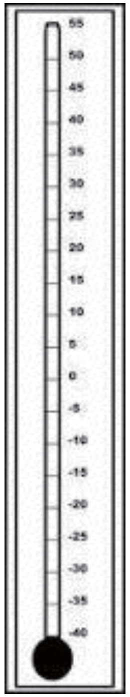
SPONTANEOUS MAGNETISATION

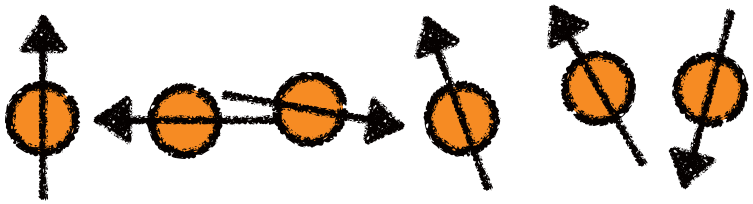


SPONTANEOUS MAGNETISATION

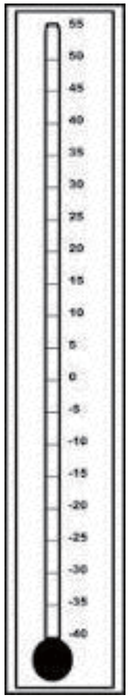


SPONTANEOUS MAGNETISATION





SPONTANEOUS MAGNETISATION



- *Random sampling reveals properties of the whole population with remarkable accuracy*
- *Random strategies give raise to Nash equilibrium in every game*
- *Cryptography is based on randomness*
- *Gambling…*

AVI WIGDERSON

What if the setting is not random?

There is always not entangled state mimicking entangled one

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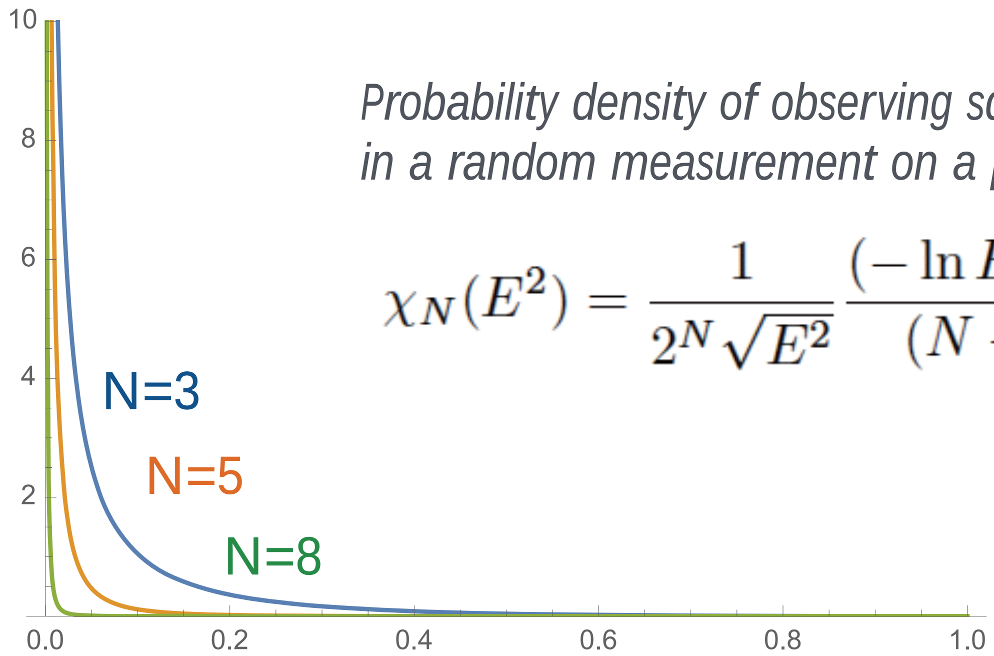
There is always not entangled state mimicking entangled one

And if it is random…?



Probability density of observing squared correlations in a random measurement on a product state

$$\chi_N(E^2) = \frac{1}{2^N\sqrt{E^2}}\frac{(-\ln E^2)^{N-1}}{(N-1)!}$$

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$$\frac{1}{\sqrt{2}}(|0\ldots0\rangle + |1\ldots1\rangle)$$

Probability to detect GHZ entanglement with confidence 95.4%

N	3	4	5	6	7	8	9	10
	26%	44%	48%	63%	67%	77%	80%	86%

M: *number of random measurement settings*
K: *number of experimental runs to estimate correlations*

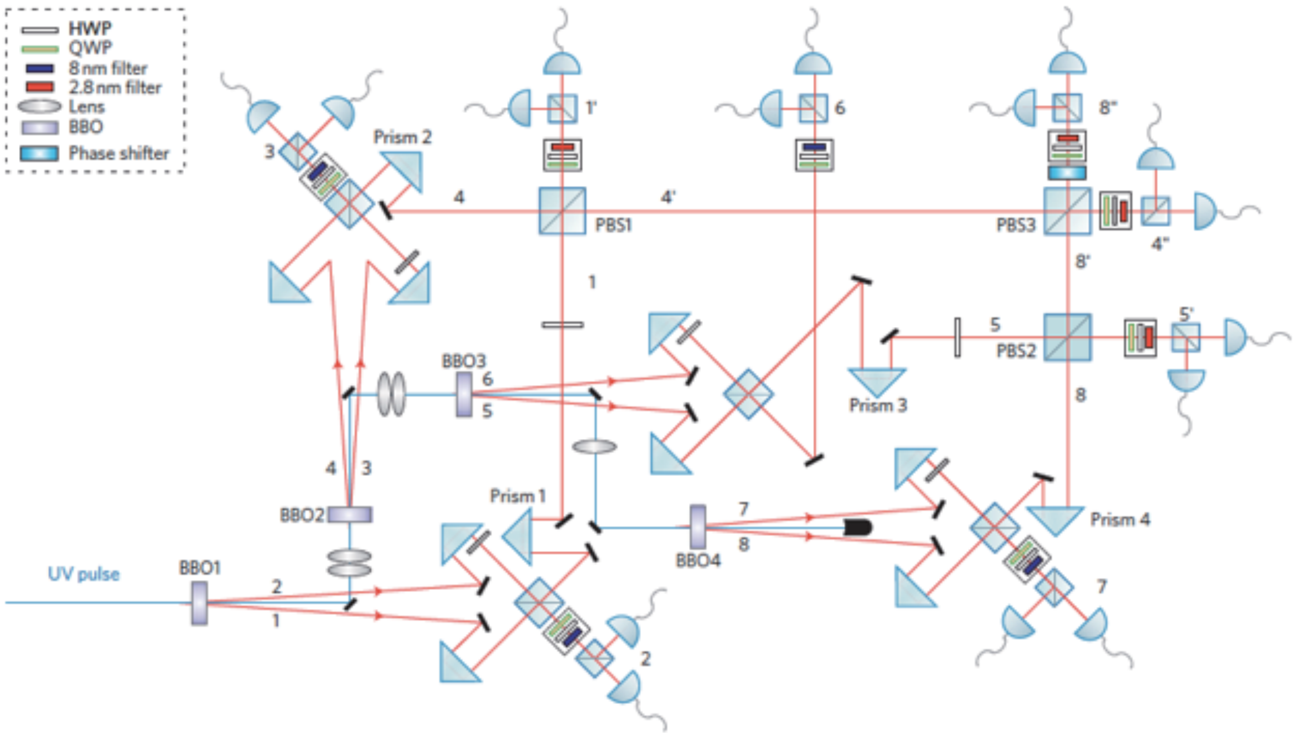
$$\mathcal{R}_{M,K} > 1/3^N + \gamma\,\Delta_{M,K}$$

then likely state is entangled

Probability to detect GHZ entanglement in 1000 trials with confidence 95.4%

N	3	4	5	6	7	8	9	10
	26%	44%	47%	57%	52%	48%	41%	34%
	26%	44%	48%	63%	67%	77%	80%	86%

PRACTICAL IMPLICATION



X.-C. Yao, T.-X. Wang, …, J.-W. Pan, Nature Phot. **6**, 225 (2012)

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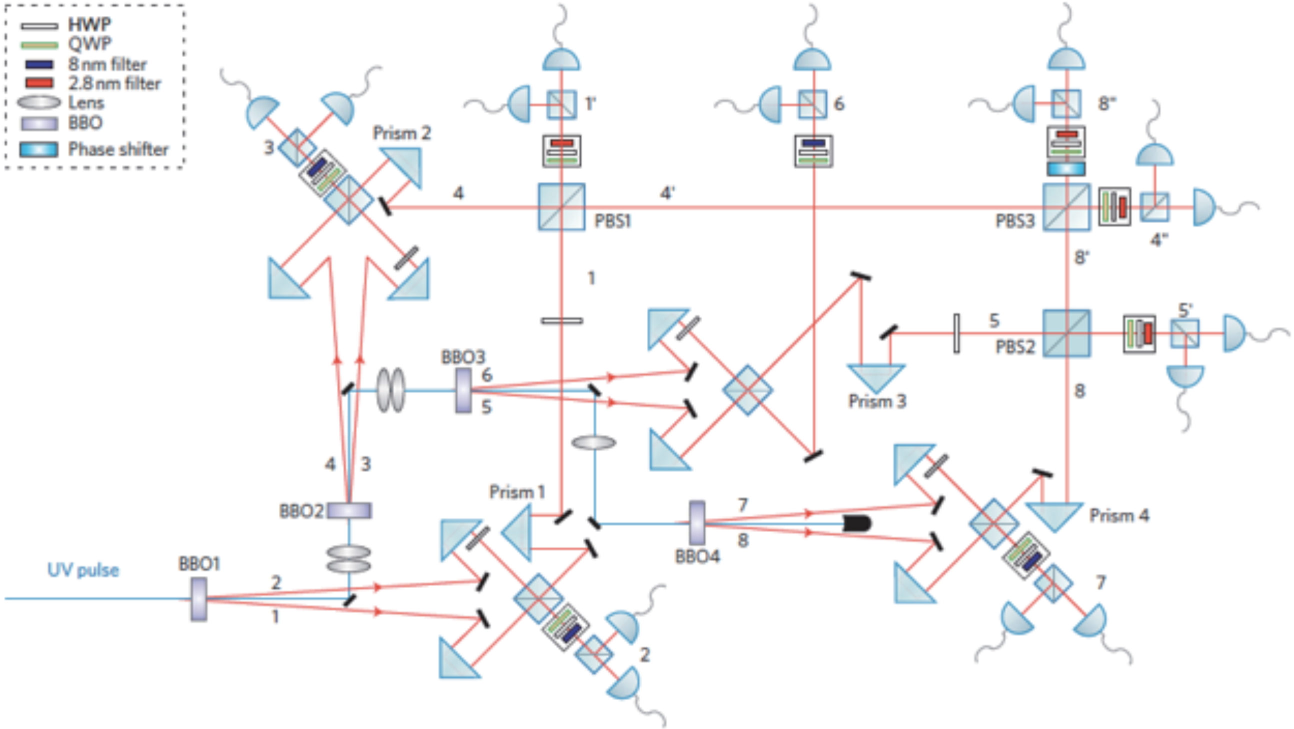

Coincidence click only about every 6 minutes!

X.-C. Yao, T.-X. Wang, …, J.-W. Pan, Nature Phot. **6**, 225 (2012)

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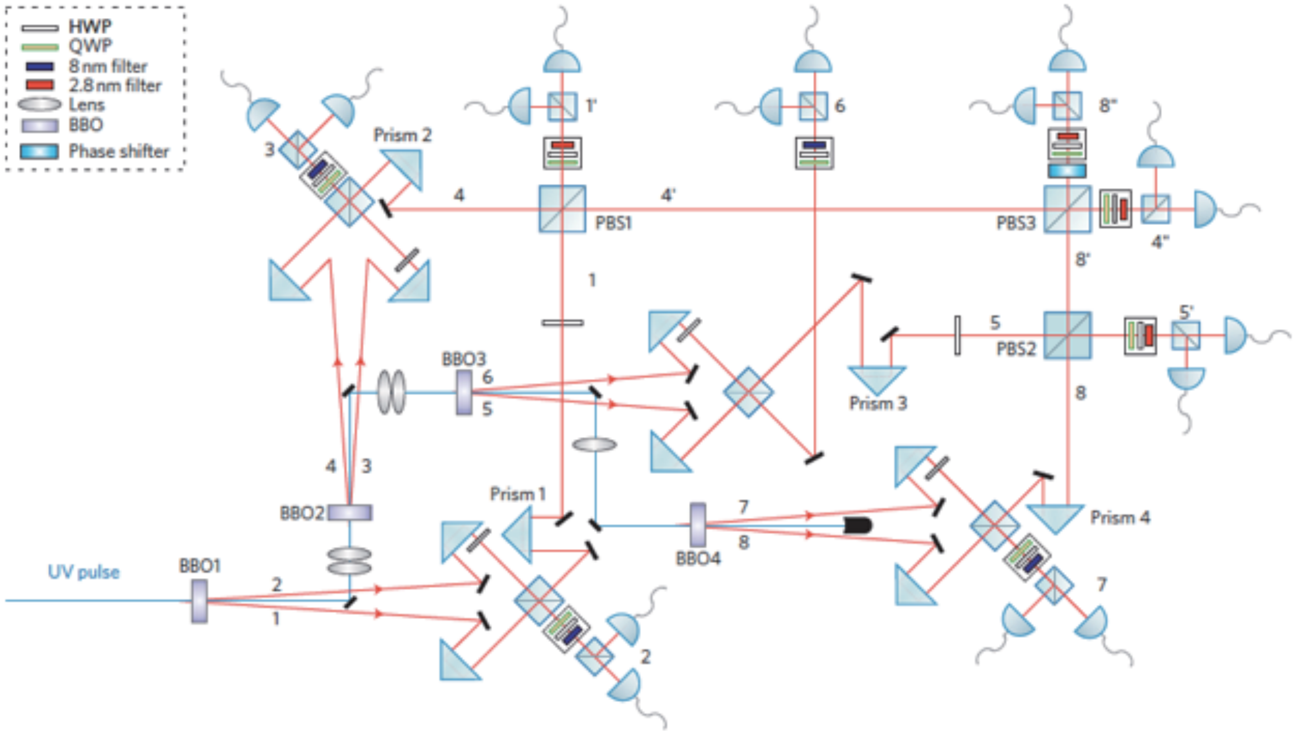

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Density matrix reconstruction would take 75 years!

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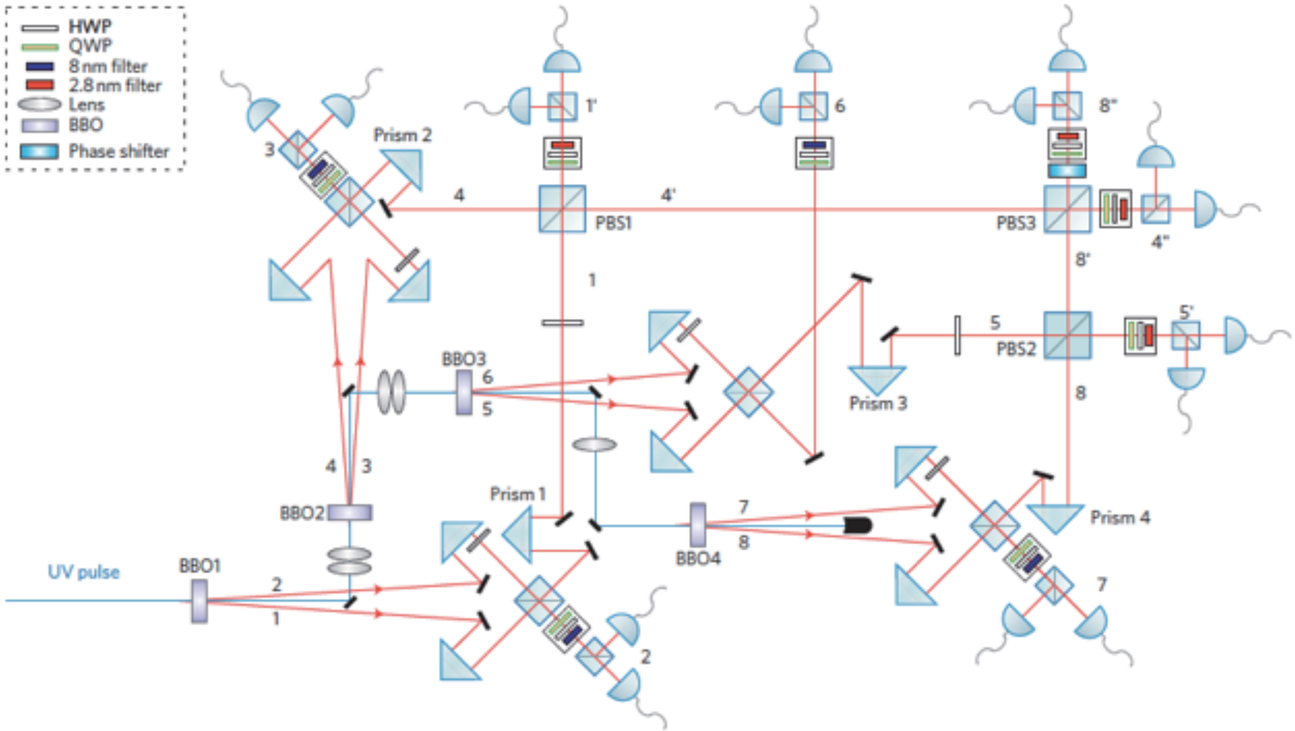

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With the random setting you likely detect entanglement in 4.5 days

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SUMMARY

Pure state entanglement is solely characterised by N-party correlations

Entangled states are more correlated in random measurements

Randomness empowers entanglement detection with one setting

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