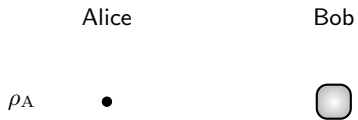


Postquantum steering

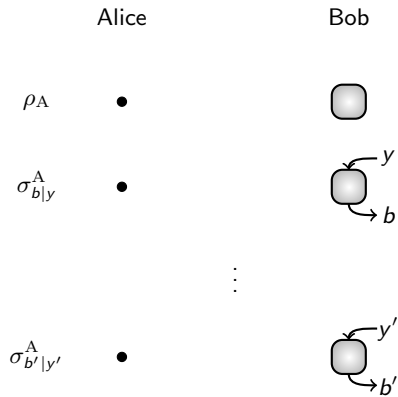
Ana Belén Sainz, Nicolas Brunner, Daniel Cavalcanti
Paul Skrzypczyk and Tamás Vértesi

Phys. Rev. Lett. 115, 190403 (2015)

Steering



Steering



Steering

Fix y \longrightarrow ensemble: $\{\sigma_{b|y}^A\}_b$, \longrightarrow $\rho_A = \sum_b \sigma_{b|y}^A$

Assemblage: $\{\sigma_{b|y}^A\}_{b,y}$. $p(b|y) = \text{tr}(\sigma_{b|y}^A)$.

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Given an assemblage, **could it have a classical explanation?**

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Set of “classical” assemblages

Steering Inequality:
 $\text{tr} \sum_{by} F_{by} \sigma_{b|y}^A \leq \beta_{\text{US}}$

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Bipartite steering

$$\text{Given } \{\sigma_{b|y}^A\}_{b,y}, \quad \rho_A = \sum_b \sigma_{b|y}^A, \quad \text{tr}(\rho_A) = 1$$

$$\exists \rho_{AB}, \quad \{M_{b|y}\}_{b,y} \quad \text{st} \quad \sigma_{b|y}^A = \text{tr}_B(\mathbb{1}_A \otimes M_{b|y} \rho_{AB})$$

¹N. Gisin, Helvetica Physica Acta 62, 363 (1989).

L. P. Hughston, R. Jozsa and W. K. Wootters, Phys. Lett. A 183, 14 (1993).

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- Alice and Bob: **Yes !** GHJW theorem¹
- **Multipartite scenarios?**

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Steering: multipartite scenarios

Alice

$$\bullet$$
$$\sigma_{bc|yz}^A$$

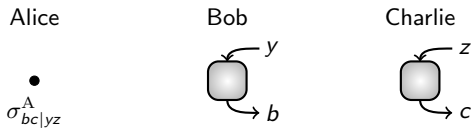
Bob



Charlie



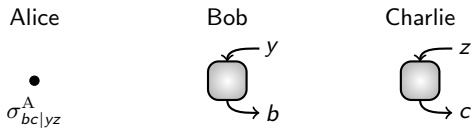
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Assemblage: $\{\sigma_{bc|yz}^A\}_{b,y,c,z}. \quad p(bc|yz) = \text{tr}(\sigma_{bc|yz}^A).$

Steering: multipartite scenarios

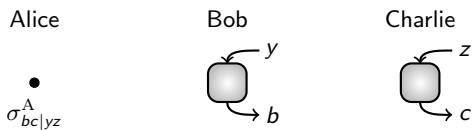


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No Signalling: $\sum_b \sigma_{bc|yz}^A = \sum_b \sigma_{bc|y'z}, \quad \sum_c \sigma_{bc|yz}^A = \sum_c \sigma_{bc|yz'}$

Steering: multipartite scenarios



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$\exists \rho_{ABC}, \{M_{b|y}\}_{b,y}, \{M_{c|z}\}_{c,z} \quad \text{st} \quad \sigma_{bc|yz}^A = \text{tr}_B(\mathbb{1}_A \otimes M_{b|y} \otimes M_{c|z} \rho_{ABC})$

Postquantum steering: example

Alice

$$\sigma_{bc|yz}^A$$

Bob



Charlie



$$b, c, y, z \in \{0, 1\}$$

$$\rho_A = \frac{1}{2}.$$

Postquantum steering: example

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$$\sigma_{bc|yz}^A = \begin{cases} \frac{1}{4}, & \text{if } b = c, \\ 0, & \text{if } b \neq c, \end{cases}$$

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$$\sum_b \sigma_{bc|yz}^A = \frac{1}{4}, \quad \sum_c \sigma_{bc|yz}^A = \frac{1}{4}$$

No quantum realisation for the assemblage

Postquantum steering: example without postquantum nonlocality

(1) Postquantum assemblage $\{\sigma_{bc|yz}^A\}_{b,y,c,z}$

(2) Quantum correlations for every measurement by Alice:

$$p(abc|xyz) = \text{tr} (M_{a|x} \otimes \sigma_{bc|yz}^A)$$

(1) Postquantum assemblage $\sigma_{bc|yz}^A$

Steering inequality: F_{bcyz}

$$\text{tr} \sum_{bcyz} F_{bcyz} \sigma_{bc|yz}^A$$

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Almost quantum assemblages: $\tilde{Q} \supset Q$

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$$\text{tr} \sum_{bcyz} F_{bcyz} \sigma_{bc|yz}^A > \beta_{\tilde{Q}} \Rightarrow \sigma_{bc|yz}^A \text{ is postquantum}$$

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(2) Quantum correlations $p(abc|xyz)$

- (i) $p(abc|xyz)$ is local
- (ii) Real qubit assemblage, local for all projective measurements
- (iii) Qutrit assemblage, local for all POVMs².

²F. Hirsch, M. T. Quintino, J. Bowles and N. Brunner, Phys. Rev. Lett, 111, 160402 (2013).

(ii) Qubit assemblage, local for all PVM

$$\Pi_{a|x}(\mu) = \mu \Pi_{a|x} + (1 - \mu) \mathbb{1}/2, \quad \sigma_{bc|yz}^A(\mu) = \mu \sigma_{bc|yz}^A + (1 - \mu) \text{tr}(\sigma_{bc|yz}^A) \mathbb{1}/2$$

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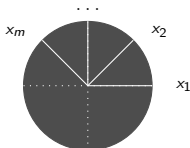
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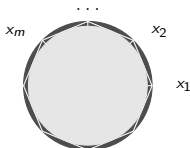


$\sigma_{bc|yz}^A$ **local for** $\{x_1, \dots, x_m\}$

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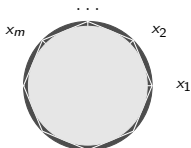


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Postquantum steering: example without postquantum nonlocality

- Four dichotomic measurements (X, Z)
- Search:

Fix $F_{bc|yz}$:

- Compute $\beta_{\tilde{Q}}$ (SDP).
- Find max violation of the inequality by the 'local' assemblages (SDP).
→ $\sigma_{bc|yz}^A$
- Compute $\beta^* = \text{tr} \sum_{bcyz} F_{bcyz} \sigma_{bc|yz}^*$, $\sigma_{bc|yz}^* := \sigma_{bc|yz}^A(\mu = \cos(\frac{\pi}{8}))$
If $\beta^* > \beta_{\tilde{Q}}$: done! , otherwise, change $F_{bc|yz}$, start over.

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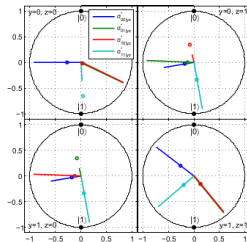
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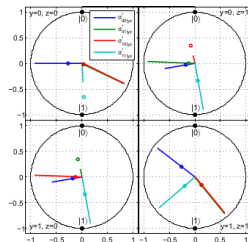
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Summary and open questions

- Steering beyond quantum theory \rightarrow multipartite scenarios
- Genuinely new effect
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- Fundamental difference between bipartite and multipartite scenarios

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- Genuinely new effect
 \rightarrow postquantum steering $\not\Rightarrow$ postquantum nonlocality
- Fundamental difference between bipartite and multipartite scenarios
- Insight on the characterisation of quantum phenomena
- General framework for non-signalling assemblages
 \rightarrow quantify postquantumness
- Information-theoretic applications of postquantum steering

Thanks !!!

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