Bell correlations in a Bose-Einstein Condensate

Roman Schmied, Jean-Daniel Bancal,
Baptiste Allard, Matteo Fadel, Valerio Scarani,
Philipp Treutlein, Nicolas Sangouard

Tainan, December 12, 2015

University of Basel

Centre for Quantum Technologies
National University of Singapore

= nonlocality

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What? Bell correlations witnessed by collective observables

Why? quantum state characterization in many-body systems

How? projective measurements of spin-squeezed states of ^{87}Rb BECs

plus connection to entanglement

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Local causality

conditional probabilities can be expressed as

$$P(a, b, c, \dots | x, y, z, \dots) = \int \mathrm{d}\lambda \, P(\lambda) \, P(a|x,\lambda) P(b|y,\lambda) P(c|z,\lambda) \cdots$$

only (shared) local knowledge

Bi-partite or multi-partite Bell correlations cannot be expressed in this form.

(compare to separable states (↔ no entanglement):

$$\hat{\rho} = \sum_k p_k \, \hat{\rho}_k^{(1)} \otimes \hat{\rho}_k^{(2)} \otimes \hat{\rho}_k^{(3)} \otimes \cdots$$

)

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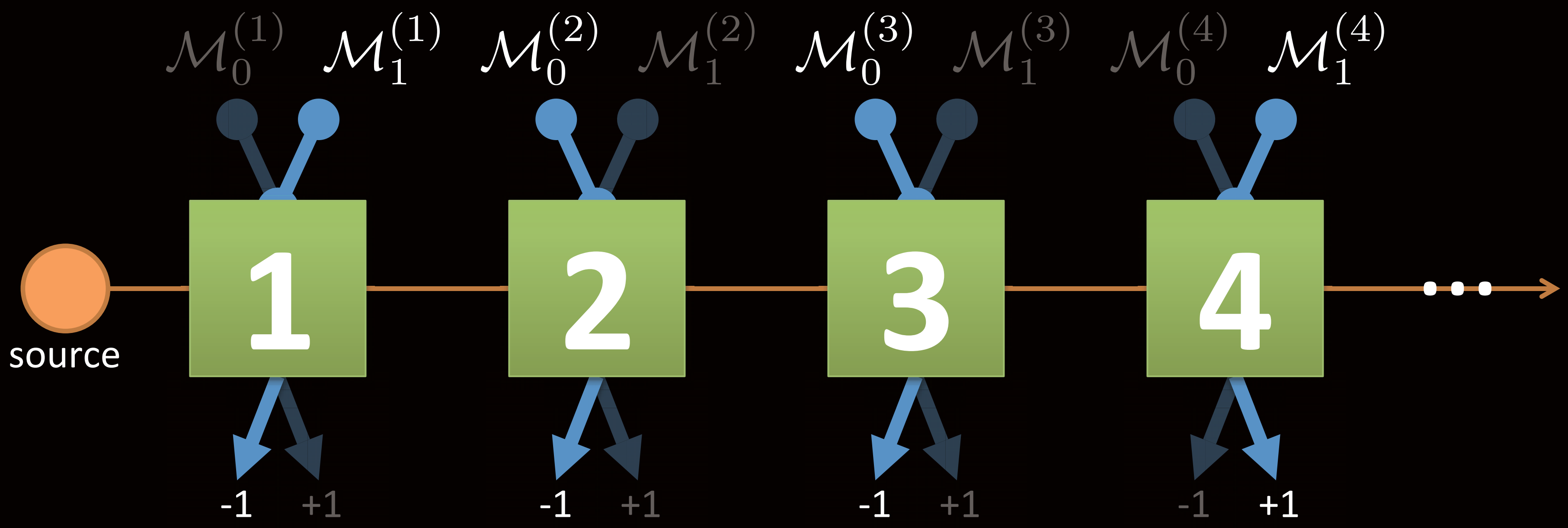
NEW: useable many-body Bell inequalities



$$S_k = \sum_i \langle \mathcal{M}_k^{(i)} \rangle = \sum_i \sum_{a=\pm 1} a\, P_i(a|k)$$

S_0: all switches to the left

$$S_{k\ell} = \sum_{i \neq j} \langle \mathcal{M}_k^{(i)} \mathcal{M}_\ell^{(j)} \rangle = \sum_{i \neq j} \sum_{a,b=\pm 1} ab\, P_{ij}(ab|k\ell)$$

$$2S_0 + \frac{1}{2} S_{00} + S_{01} + \frac{1}{2} S_{11} + 2N \geq 0$$

J. Tura, R. Augusiak, A. B. Sainz, T. Vértesi, M. Lewenstein, A. Acín Science **344**, 1256 (2014)

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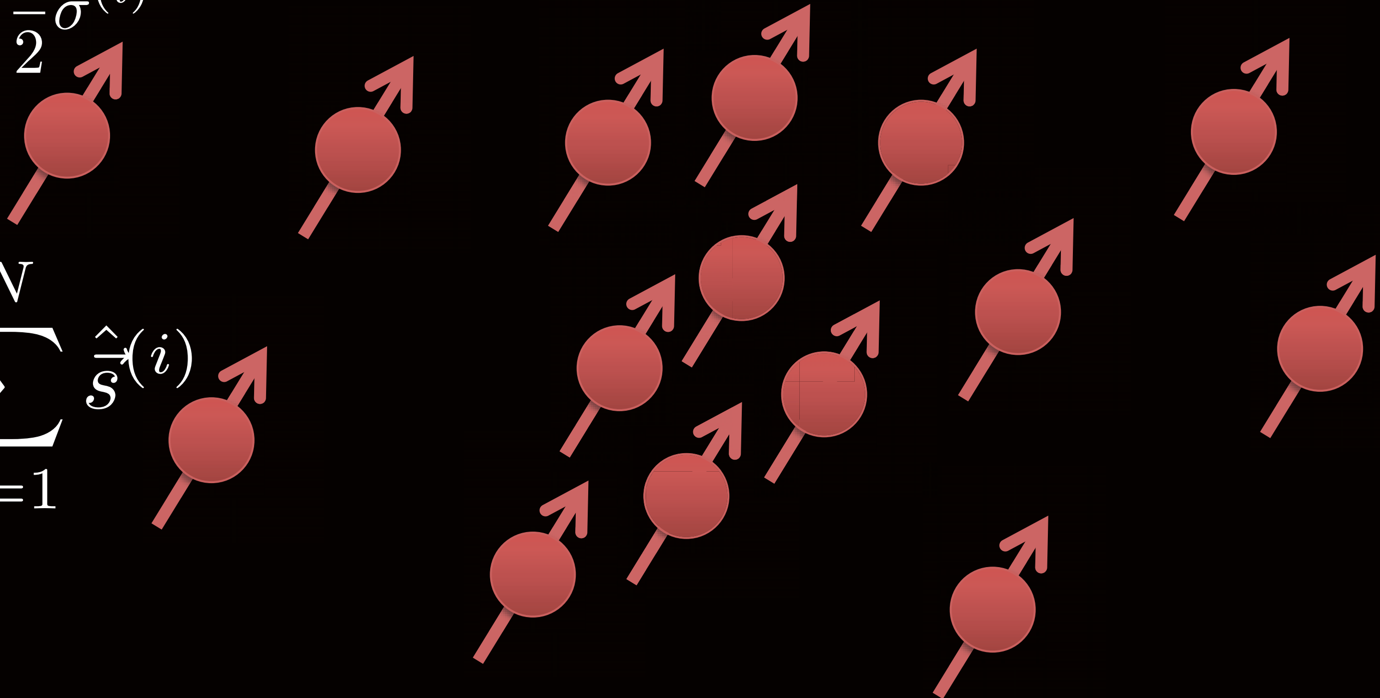
many-body system from qubits

$$\hat{\vec{s}}^{(i)} = \frac{1}{2}\hat{\vec{\sigma}}^{(i)}$$

$$\hat{\vec{S}} = \sum_{i=1}^{N} \hat{\vec{s}}^{(i)}$$



- Collective measurements: total spin along chosen quantization axis $\vec{a}$ or $\vec{n}$: $\hat{S}_{\vec{a}} = \vec{a} \cdot \hat{\vec{S}}$

$$\hat{S}_{\vec{n}} = \vec{n} \cdot \hat{\vec{S}}$$

many-body system from qubits: assumptions on the measurements

measure along axis $\vec{a}$ $\vec{n}$

(pseudo-)spin ½

spin projection along chosen axis -½ +½

$$\vec{m} = 2(\vec{a}\cdot\vec{n})\vec{a} - \vec{n}$$

$$S_0 = 2\langle \hat{S}_{\vec{n}} \rangle$$

$$S_{00} = 4\langle \hat{S}^2_{\vec{n}} \rangle - N$$

$$S_{11} = 4\langle \hat{S}^2_{\vec{m}} \rangle - N$$

$$S_{01} = \langle (\hat{S}_{\vec{n}} + \hat{S}_{\vec{m}})^2 \rangle - \langle (\hat{S}_{\vec{n}} - \hat{S}_{\vec{m}})^2 \rangle - N(\vec{n}\cdot\vec{m})$$

(no assumptions on the state)

J. Tura, R. Augusiak, A. B. Sainz, T. Vértesi, M. Lewenstein, A. Acín
Science **344**, 1256 (2014)

many-body system: **Bell operator**

$\hat{\vec{s}}^{(i)} = \frac{1}{\ldots}$

- assuming quantum-mechanical projective spin measurements
- all local states satisfy this inequality
- Bell correlations are necessary to violate this inequality

$$2S_0 + \frac{1}{2}S_{00} + S_{01} + \frac{1}{2}S_{11} + 2N \geq 0$$

$$2\langle \hat{S}_{\vec{n}} \rangle + 4(\vec{a}\cdot\vec{n})^2 \langle \hat{S}^2_{\vec{a}} \rangle + N[1-(\vec{a}\cdot\vec{n})^2] \geq 0$$

(no Bell inequality: assumptions on measurements, but no assumptions on the state)

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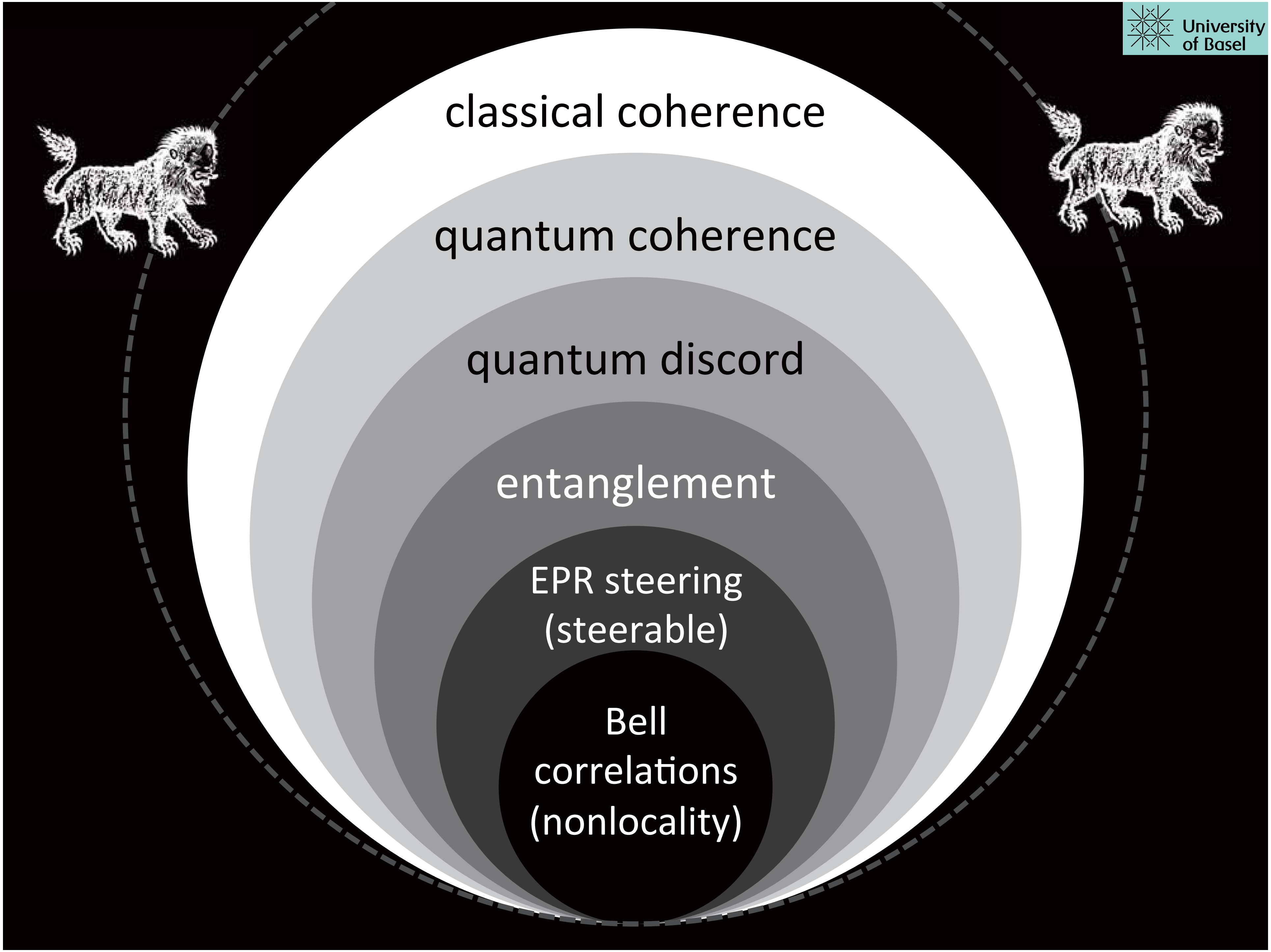
plus connection to entanglement

Motivation: many-body systems

- Understand quantum correlations in many-body systems
- Generate and characterize interesting many-body states
- links between **global dynamics** and **internal correlations**



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What lies beyond local causality?

Bell correlations
violate Bell inequality

≠

local hidden variable models
satisfy Bell inequality

- entanglement is necessary but insufficient
- resource for device-independent QIP:
  - provable randomness generation
  - reliable QKD

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many-body Bell test

test QM

(state and measurement independent)

characterize state and/or measurement

many-body Bell test

test QM
(state and measurement independent)

characterize state and/or measurement

“witnessing Bell correlations”

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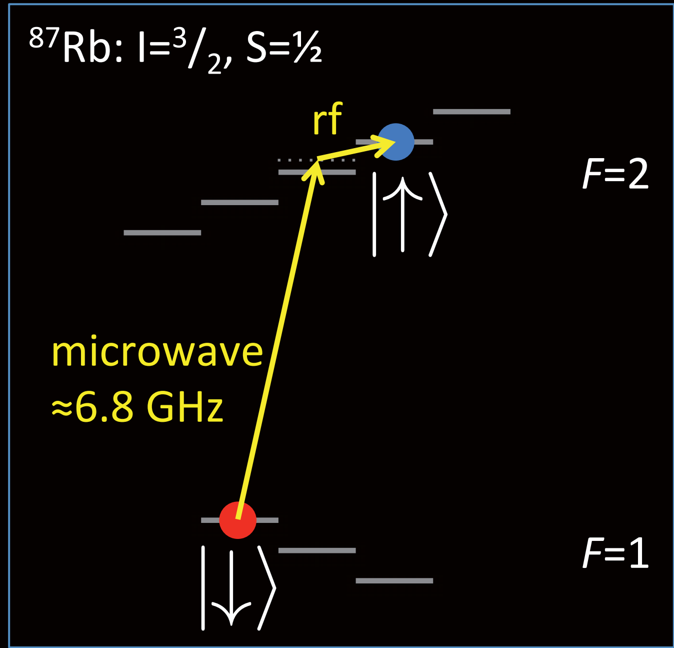
our system

- ^{87}Rb BEC with $10^2 \ldots 10^3$ atoms
- internal (hyperfine) states: "pseudospin"

$$|\downarrow\rangle = |F = 1, M_F = -1\rangle$$

$$|\uparrow\rangle = |F = 2, M_F = +1\rangle$$

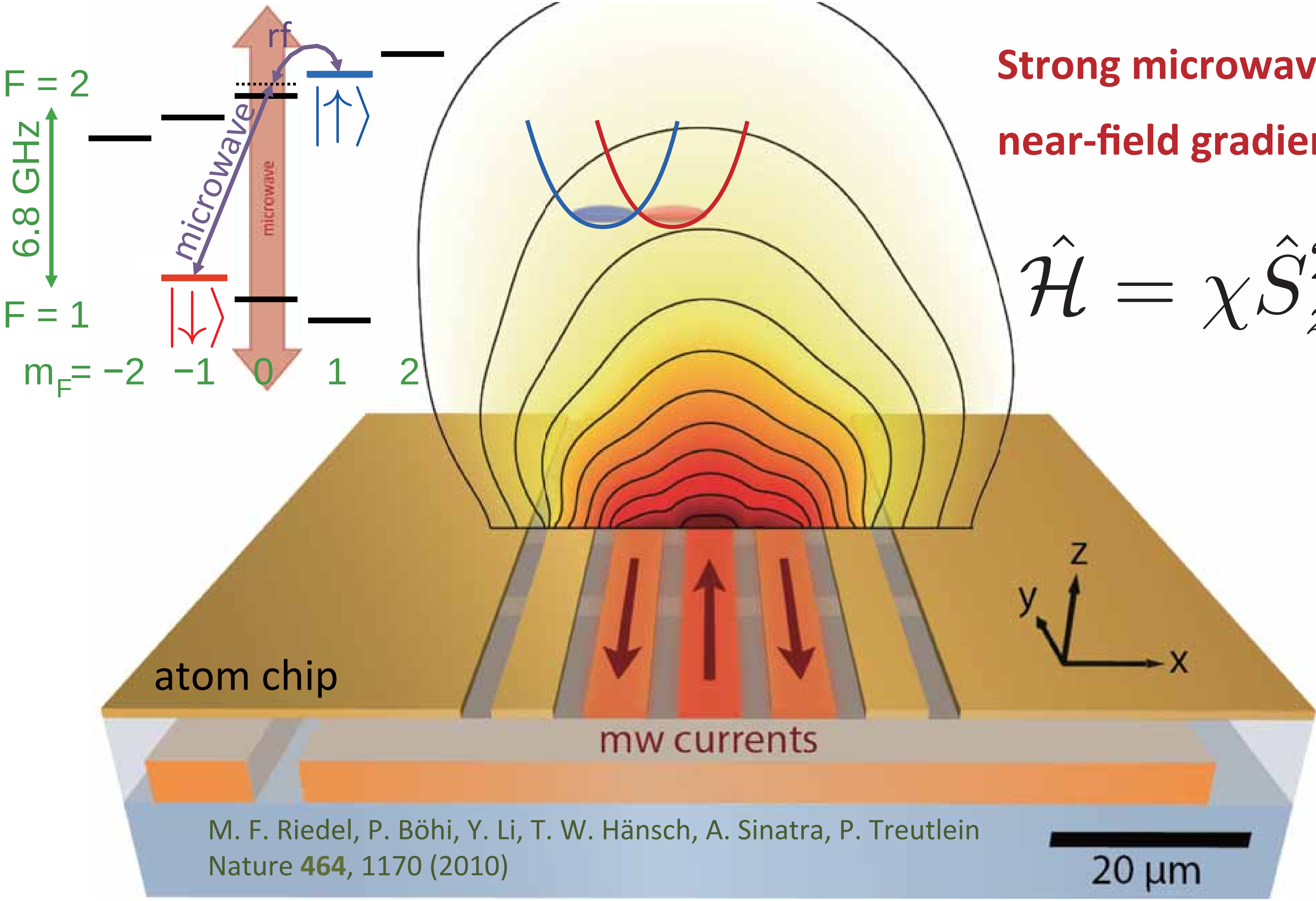


- total pseudospin:

$$\hat{\vec{S}} = \sum_{i=1}^{N} \frac{1}{2} \hat{\vec{\sigma}}^{(i)}$$

- total symmetry: $S = N/2$
- Readout by absorption imaging: $M = (N_2 - N_1)/2$ ("Stern-Gerlach")

spin squeezing: state-selective potential

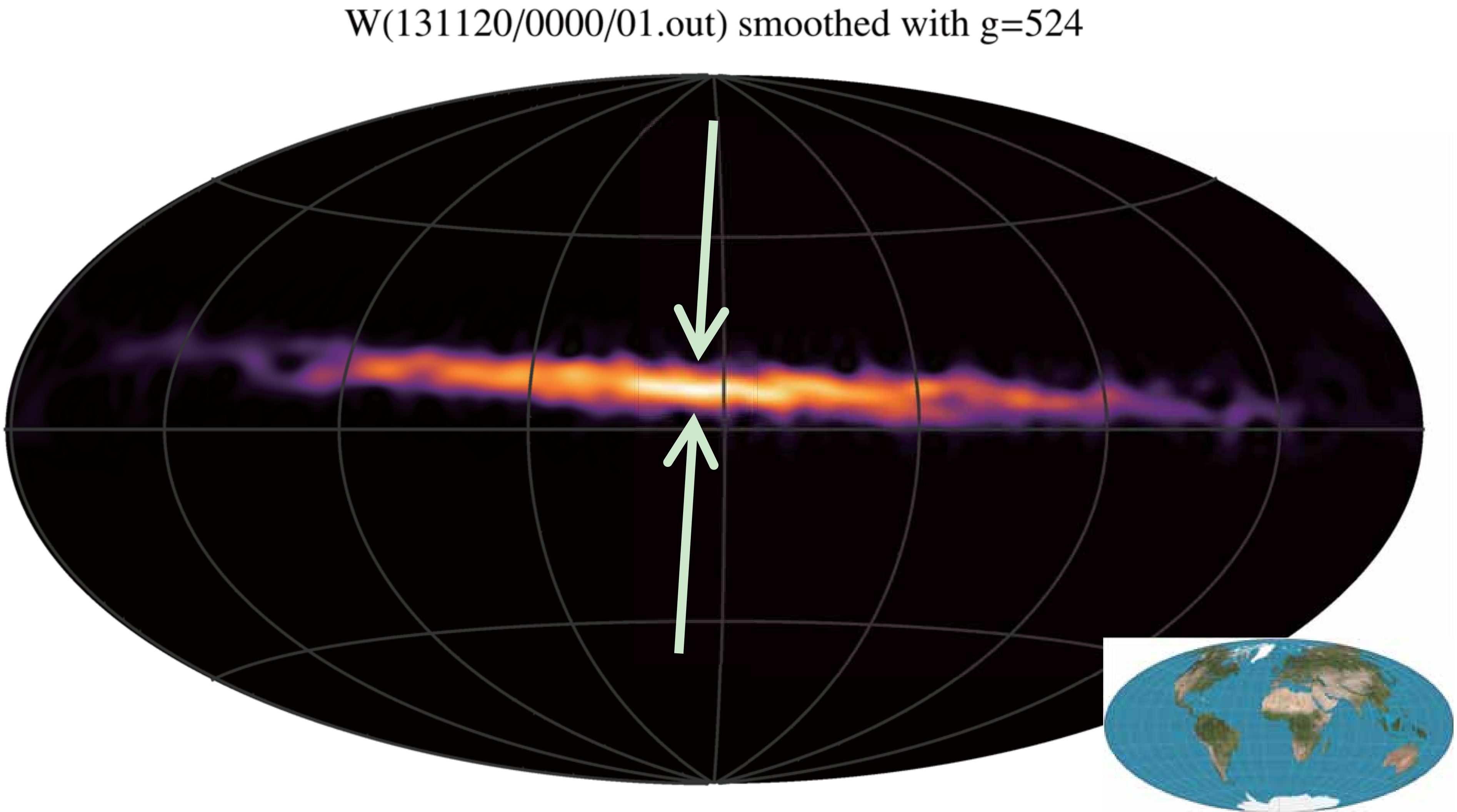


Strong microwave near-field gradient

$$\hat{\mathcal{H}} = \chi \hat{S}_z^2$$

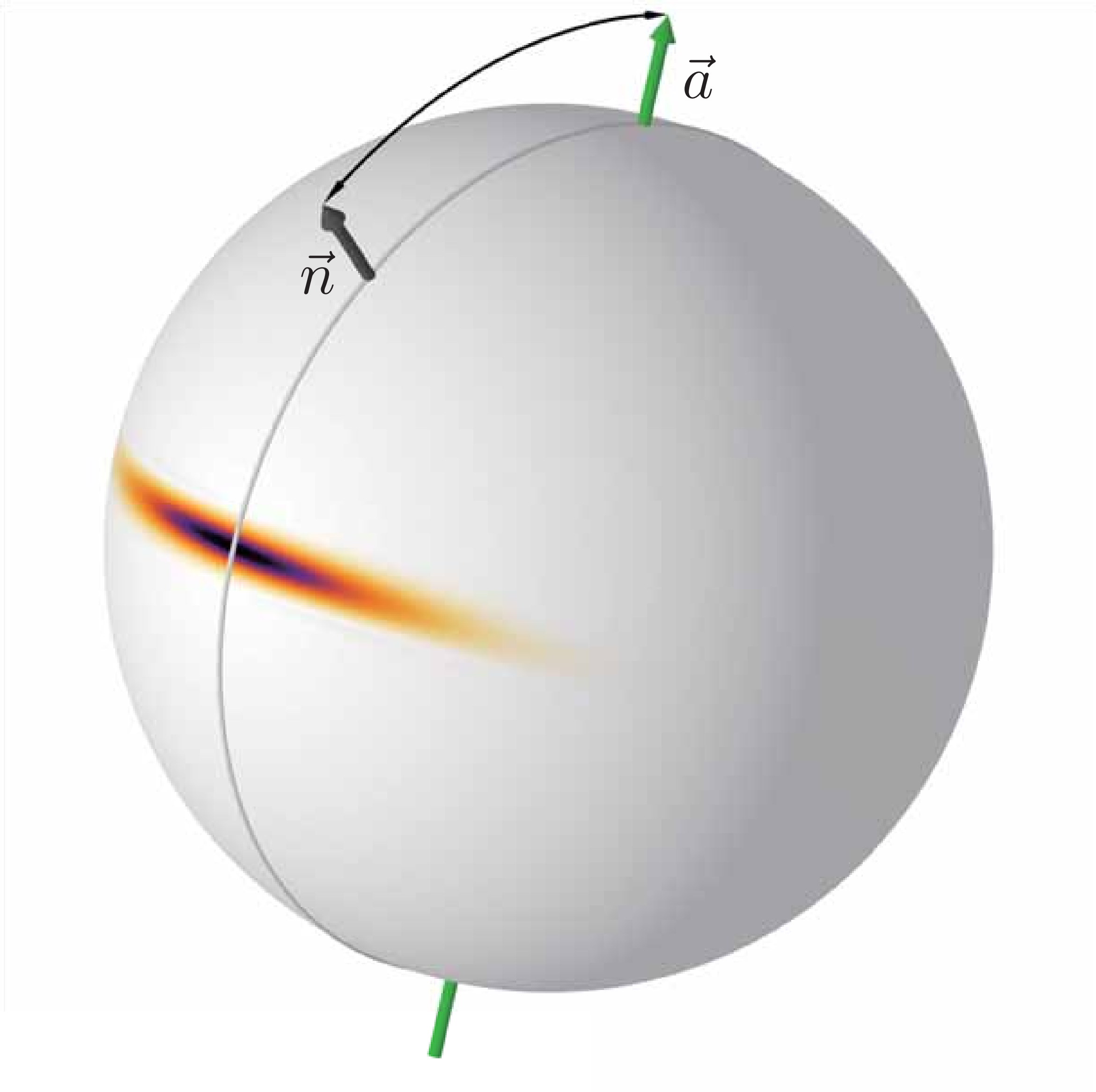
M. F. Riedel, P. Böhi, Y. Li, T. W. Hänsch, A. Sinatra, P. Treutlein
Nature **464**, 1170 (2010)

Spin-squeezed state tomography



Hammer projection

RS, P. Treutlein, NJP **13**, 065019 (2011)



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squeezed states

- one-axis twisting

M. Kitagawa, M. Ueda
PRA **47**, 5138 (1993)

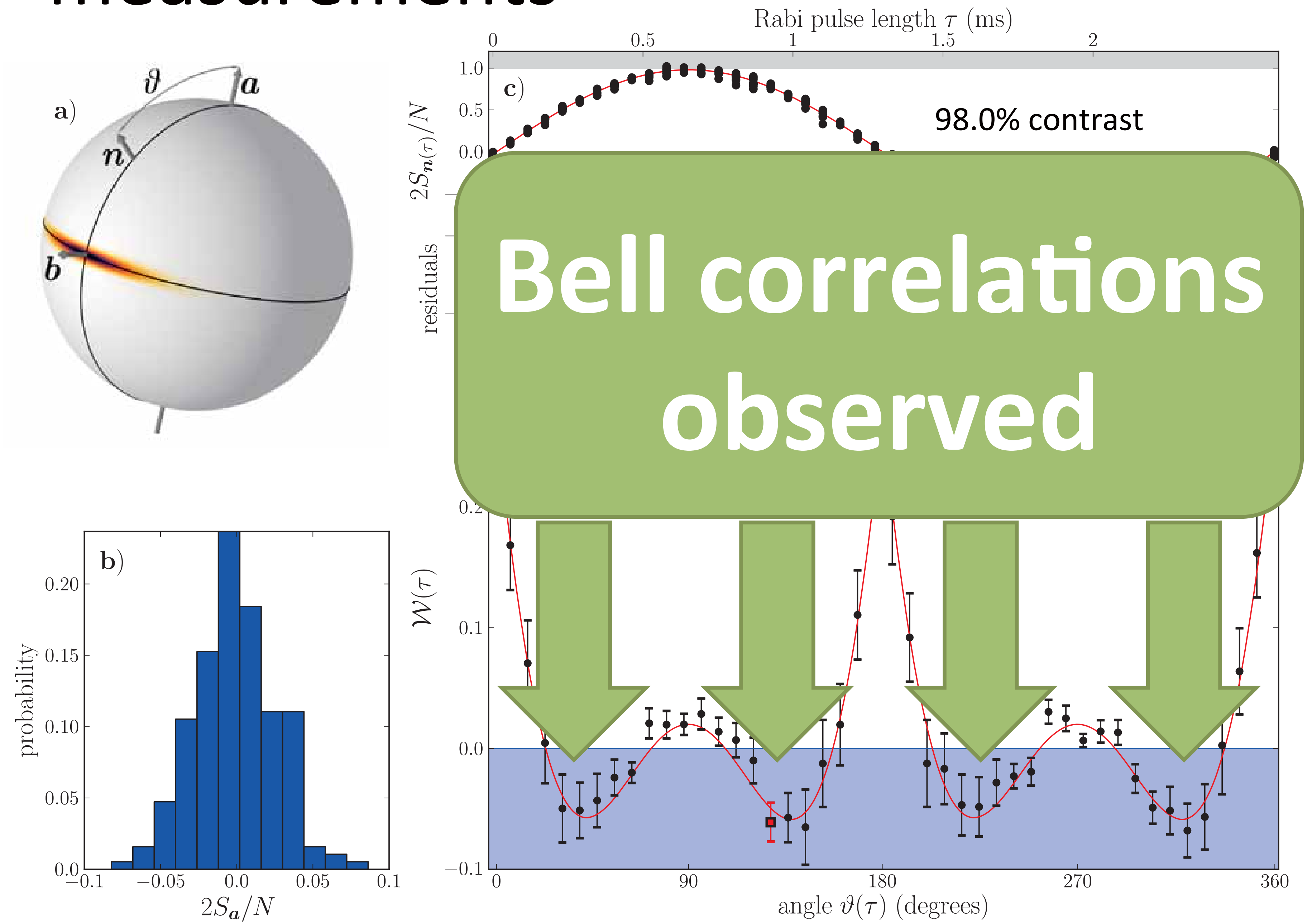
M. F. Riedel, P. Böhi, Y. Li, T. W. Hänsch, A. Sinatra, P. Treutlein
Nature **464**, 1170 (2010)

C. F. Ockeloen, RS, M. F. Riedel, P. Treutlein
PRL **111**, 143001 (2013)

- scan $\vec{n}$ in the squeezing plane
- Measure $\langle \hat{S}_{\vec{a}}^2 \rangle$, not the variance!

$$2\langle \hat{S}_{\vec{n}} \rangle + 4(\vec{a}\cdot\vec{n})^2 \langle \hat{S}_{\vec{a}}^2 \rangle + N[1-(\vec{a}\cdot\vec{n})^2] \geq 0$$

measurements



98.0% contrast

Bell correlations observed

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plus connection to entanglement

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Witnessing **entanglement**

- Spin squeezing witnesses entanglement between atoms:

$$\xi^2 = \frac{N\left[\langle \hat{S}_{\vec{a}}^2\rangle - \langle \hat{S}_{\vec{a}}\rangle^2\right]}{\langle \hat{S}_{\vec{b}}\rangle^2} < 1$$

assuming $\vec{a} \perp \vec{b}$

D. J. Wineland, J. J. Bollinger, W. M. Itano
PRA **50**, 67 (1994)

A. Sørensen, L.-M. Duan, J.I. Cirac, P. Zoller
Nature **409**, 63 (2001)

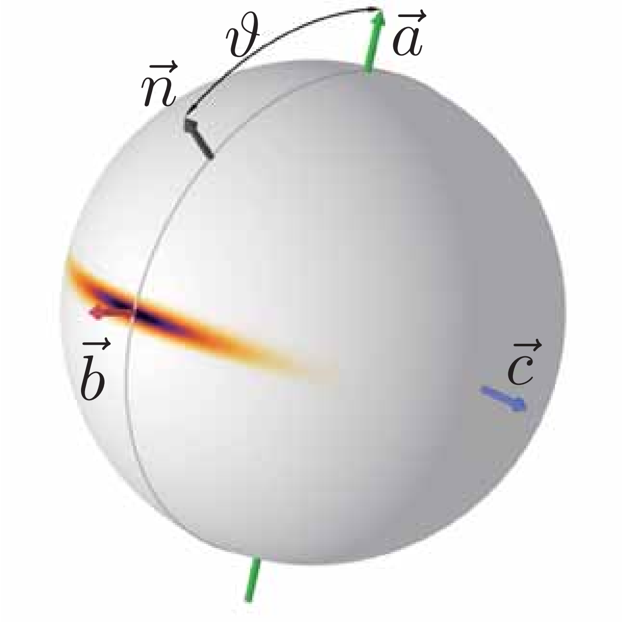
- We can reliably create and use squeezed states: $\xi^2 \lesssim 0.3 \quad (-5\,\mathrm{dB})$

J. Appel, P. J. Windpassinger, D. Oblak, U. B. Hoff, N. Kjærgaard, E. S. Polzik
PNAS **106**, 10960 (2009)

M. H. Schleier-Smith, I. D. Leroux, V. Vuletić
PRL **104**, 073604 (2010)

C. Gross, T. Zibold, E. Nicklas, J. Estève, M. K. Oberthaler
Nature **464**, 1165 (2010)

M.F. Riedel, P. Böhi, Y. Li, T.W. Hänsch, A. Sinatra, P. Treutlein
Nature **464**, 1170 (2010)



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comparing our Bell correlation witness to entanglement witnesses

$$\langle \hat{S}_{\vec{a}} \rangle = 0$$

$$\langle \hat{S}_{\vec{n}} \rangle = \langle \hat{S}_{\vec{b}} \rangle \sin(\vartheta)$$

$$\vec{a} \cdot \vec{n} = \cos(\vartheta)$$

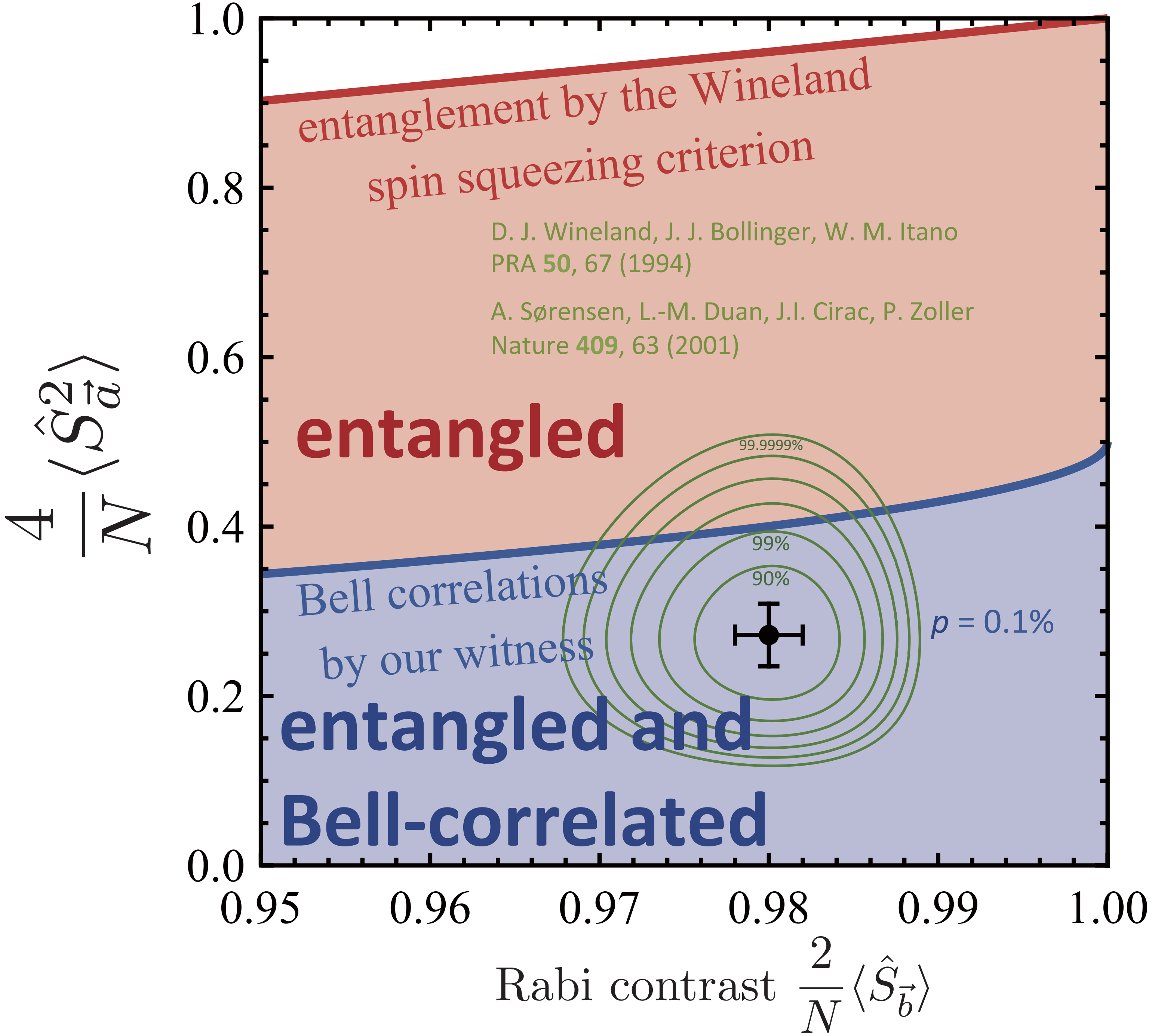
$$2\langle \hat{S}_{\vec{n}} \rangle + 4(\vec{a} \cdot \vec{n})^2 \langle \hat{S}^2_{\vec{a}} \rangle + N[1 - (\vec{a} \cdot \vec{n})^2] \geq 0$$

becomes

$$\frac{\langle \hat{S}^2_{\vec{a}} \rangle}{N/4} \geq \frac{1 - \sqrt{1 - \left(\frac{\langle \hat{S}_{\vec{b}} \rangle}{N/2} \right)^2}}{2}$$

Bell correlation witness with assumptions on state and measurements

All locally causal states satisfy this.



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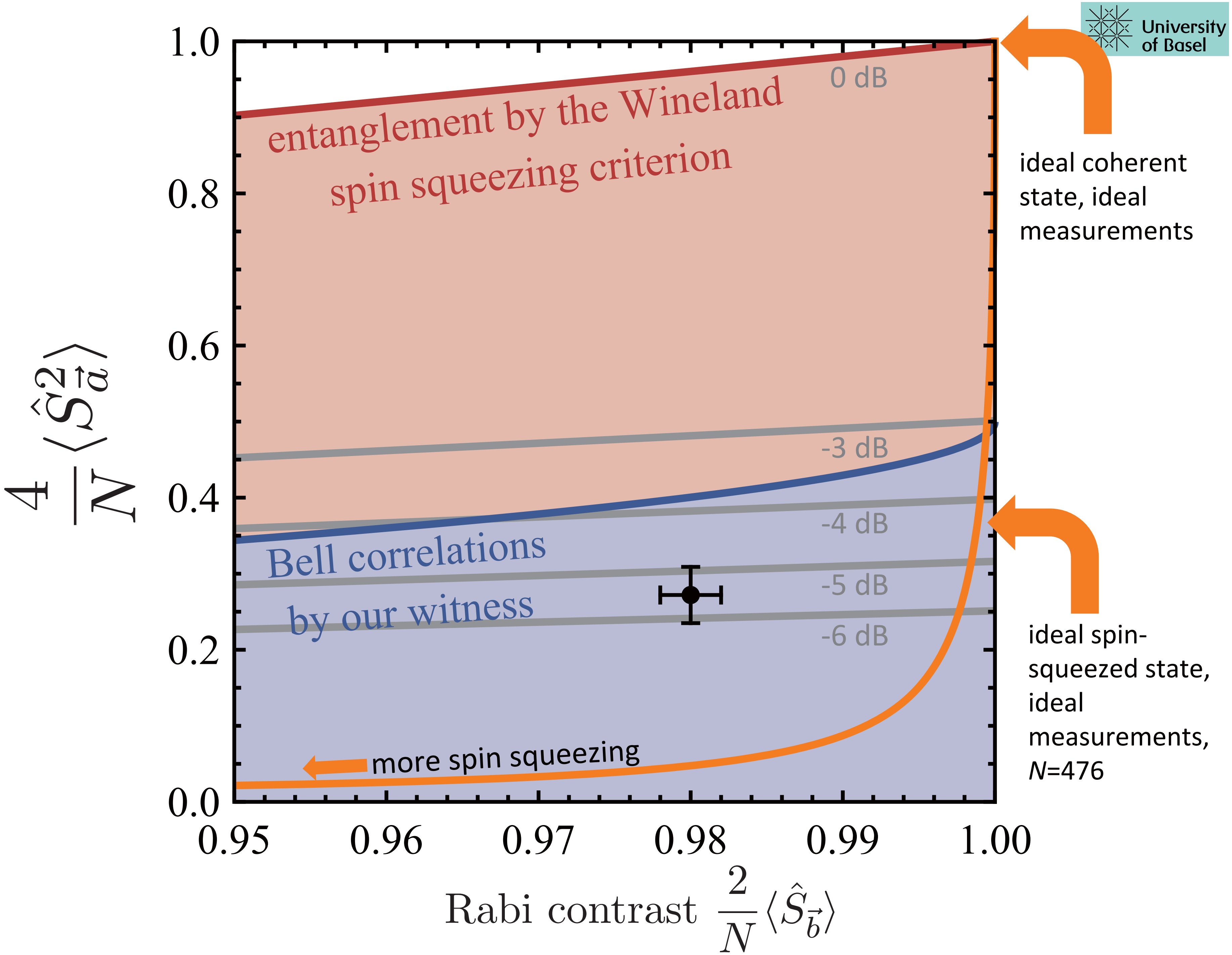
entanglement by the Wineland
spin squeezing criterion
D. J. Wineland, J. J. Bollinger, W. M. Itano
PRA 50, 67 (1994)
A. Sørensen, L.-M. Duan, J.I. Cirac, P. Zoller
Nature 409, 63 (2001)
entangled
99.9999%
99%
90%
Bell correlations
by our witness
p = 0.1%
entangled and
Bell-correlated
1.0
0.8
0.6
0.4
0.2
0.0
0.95
0.96
0.97
0.98
0.99
1.00
$\frac{4}{N}\langle \hat{S}_{\vec{a}}^2 \rangle$
Rabi contrast $\frac{2}{N}\langle \hat{S}_{\vec{b}} \rangle$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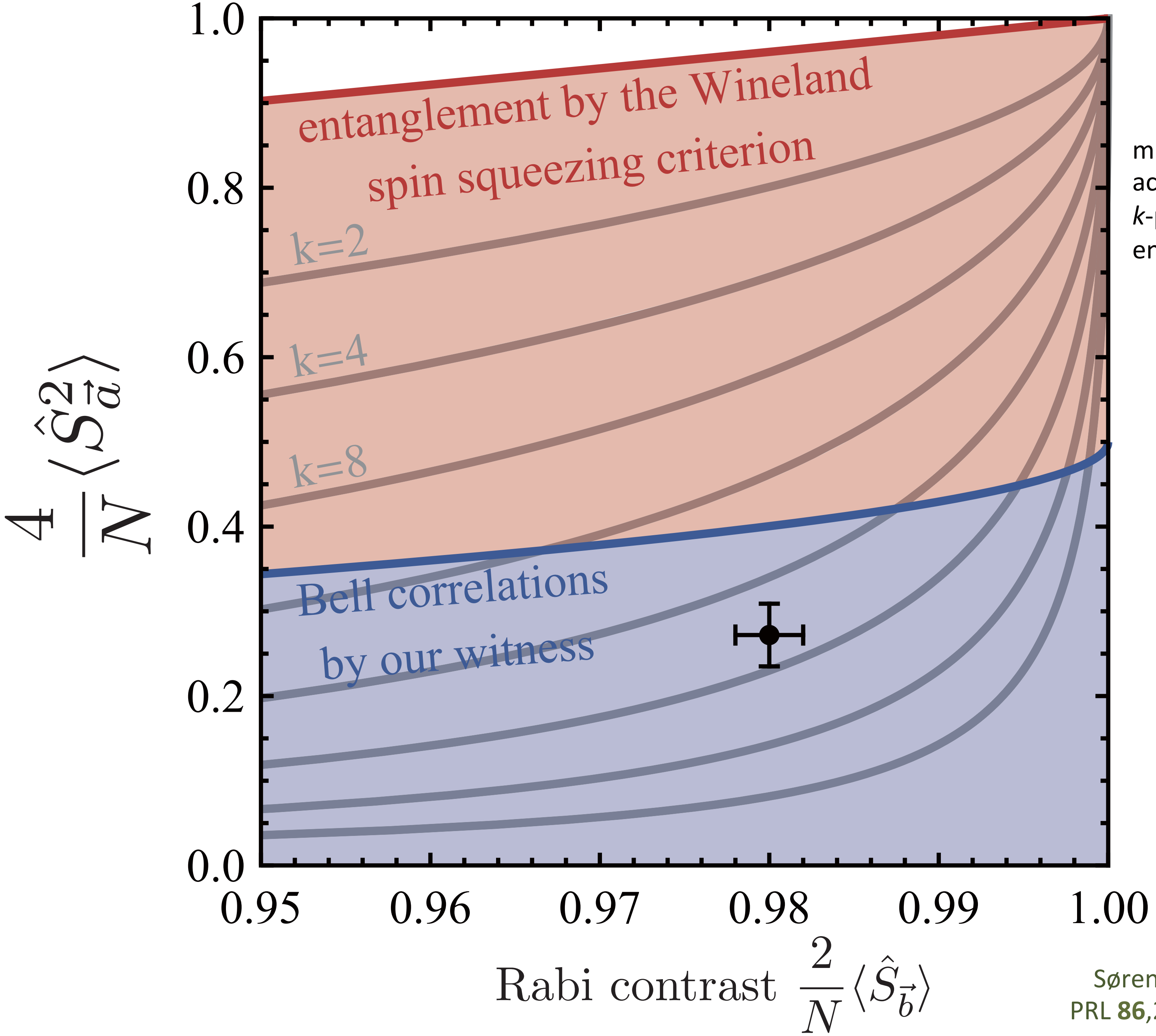
entanglement by the Wineland
spin squeezing criterion
0 dB
-3 dB
-4 dB
-5 dB
-6 dB
Bell correlations
by our witness
more spin squeezing
ideal coherent state, ideal measurements
ideal spin-squeezed state, ideal measurements, N=476
1.0
0.8
0.6
0.4
0.2
0.0
0.95
0.96
0.97
0.98
0.99
1.00
$\frac{4}{N}\langle \hat{S}^2_{\vec{a}} \rangle$
Rabi contrast $\frac{2}{N}\langle \hat{S}_{\vec{b}} \rangle$



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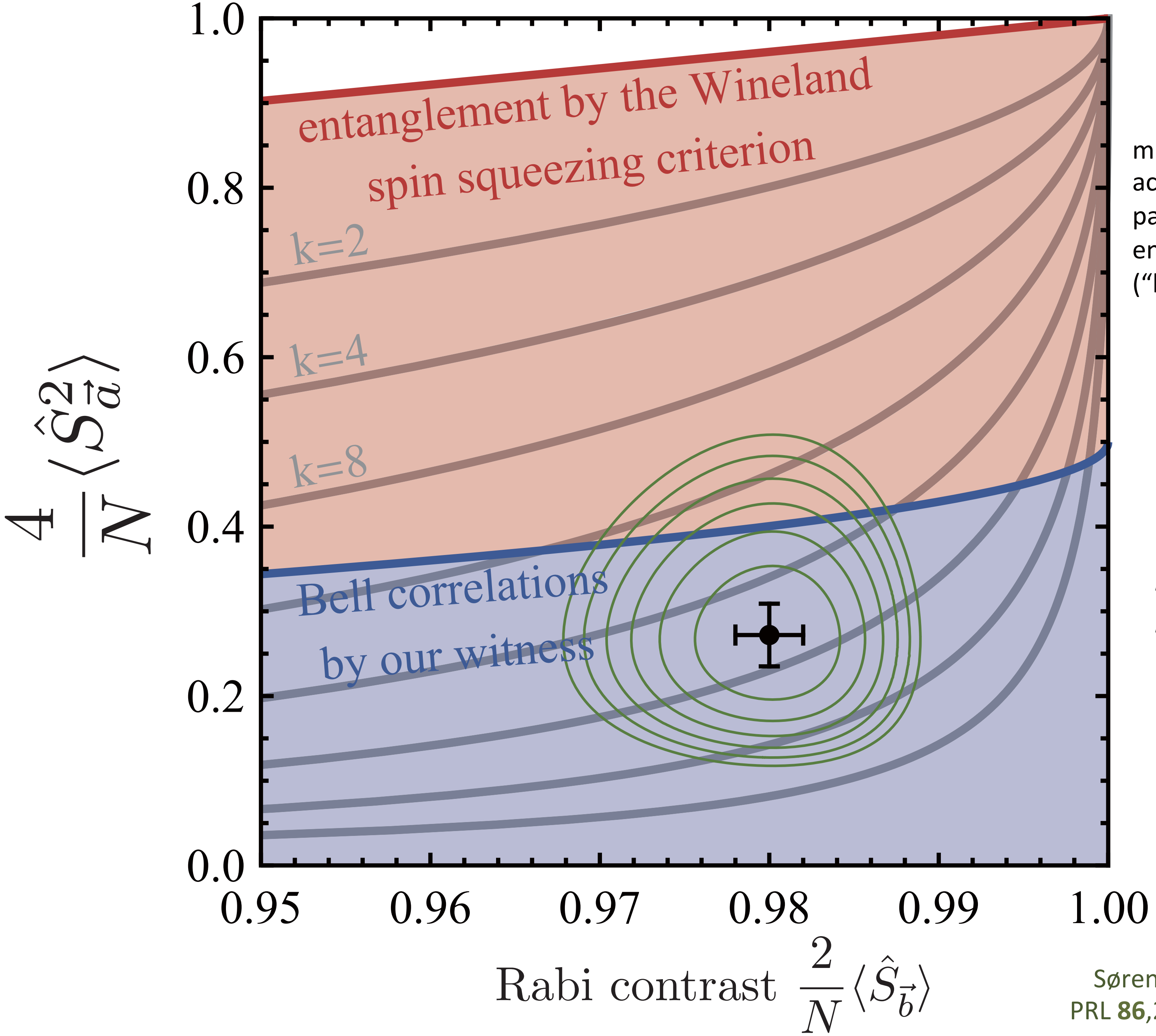
minimum values achievable with *k*-particle entanglement

Sørensen and Mølmer
PRL **86**,20 p.4431 (2001)



minimum values achievable with *k*-particle entanglement ("k-producibility")

k>24 (*p*<1%)
k>29 (*p*<5%)

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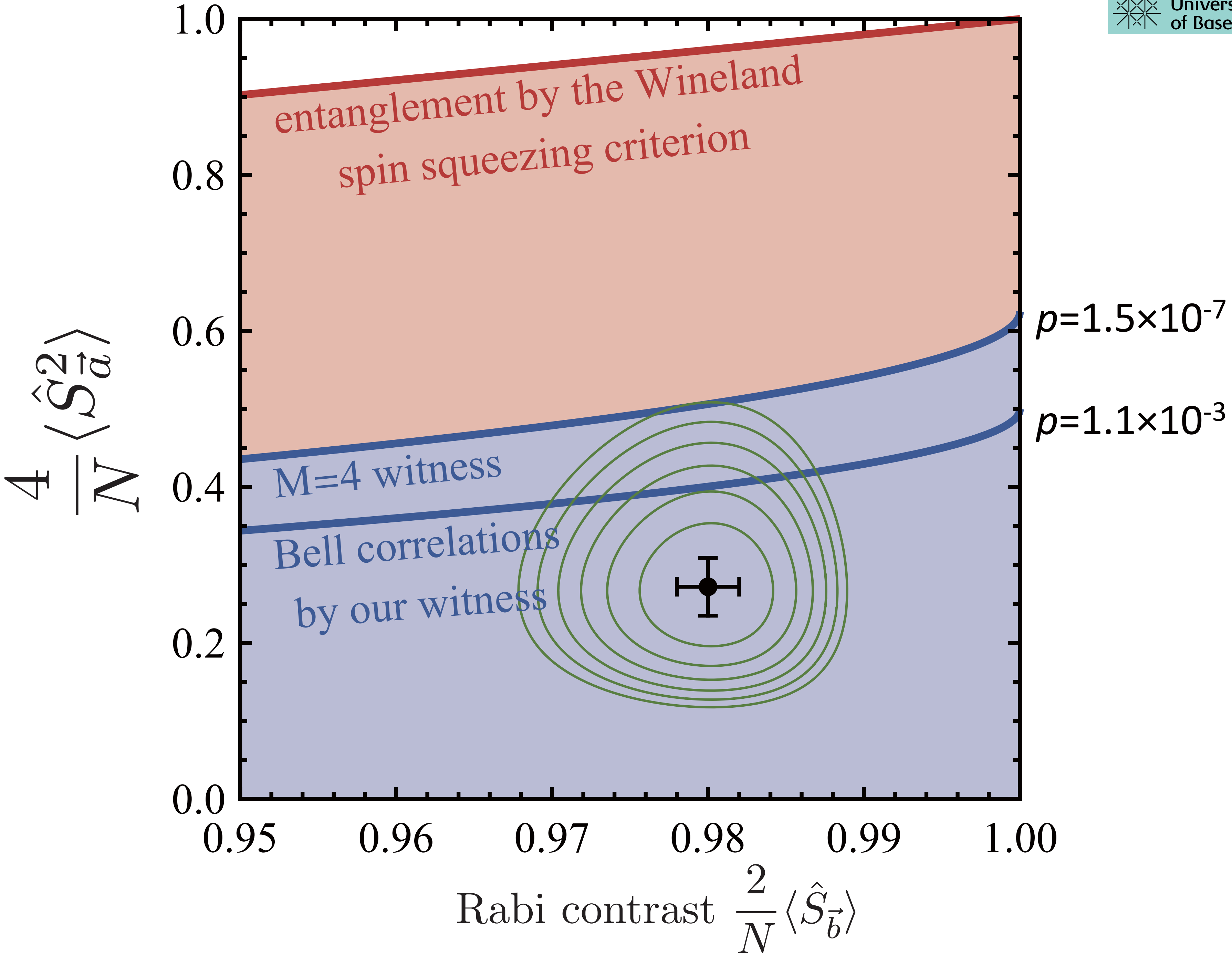
Sørensen and Mølmer
PRL **86**,20 p.4431 (2001)



entanglement by the Wineland spin squeezing criterion
M=4 witness
Bell correlations by our witness
p=1.5×10^-7
p=1.1×10^-3
1.0
0.8
0.6
0.4
0.2
0.0
0.95
0.96
0.97
0.98
0.99
1.00
Rabi contrast $\frac{2}{N}\langle\hat{S}_{\vec{b}}\rangle$
$\frac{4}{N}\langle\hat{S}^2_{\vec{a}}\rangle$

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conclusions

- We have derived a useable witness from Tura et al.’s multi-partite Bell inequality.
- We have detected Bell correlations with this witness in spin-squeezed states of a ^{87}Rb BEC.
- At least −3 dB of spin squeezing are necessary for observing Bell correlations with our witness. Probably much less with better witnesses.
- We do not address the locality loophole.

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Roman Schmied

Baptiste Allard

Matteo Fadel

Philipp Treutlein

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tomorrow afternoon

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What do these correlations mean?

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III.5 ON THE EINSTEIN PODOLSKY ROSEN PARADOX*

JOHN S. BELL†

I. Introduction

THE paradox of Einstein, Podolsky and Rosen [1] was advanced as an argument that quantum mechanics could not be a complete theory but should be supplemented by additional variables. These additional variables were to restore to the theory causality and locality [2]. In this note that idea will be formulated mathematically and shown to be incompatible with the statistical predictions of quantum mechanics. It is the requirement of locality, or more precisely that the result of a measurement on one system be unaffected by operations on a distant system with which it has interacted in the past, that creates the essential difficulty. There have been attempts [3] to show that even without such a separability or locality requirement no "hidden variable" interpretation of quantum mechanics is possible. These attempts have been examined elsewhere [4] and found wanting. Moreover, a hidden variable interpretation of elementary quantum theory [5] has been explicitly constructed. That particular interpretation has indeed a grossly non-local structure. This is characteristic, according to the result to be proved here, of any such theory which reproduces exactly the quantum mechanical predictions.

"absence of communication between **measurements**"

Physics **1**, 195 (1964)

Gühne and Tóth, Physics Reports 474, 1 (2009)

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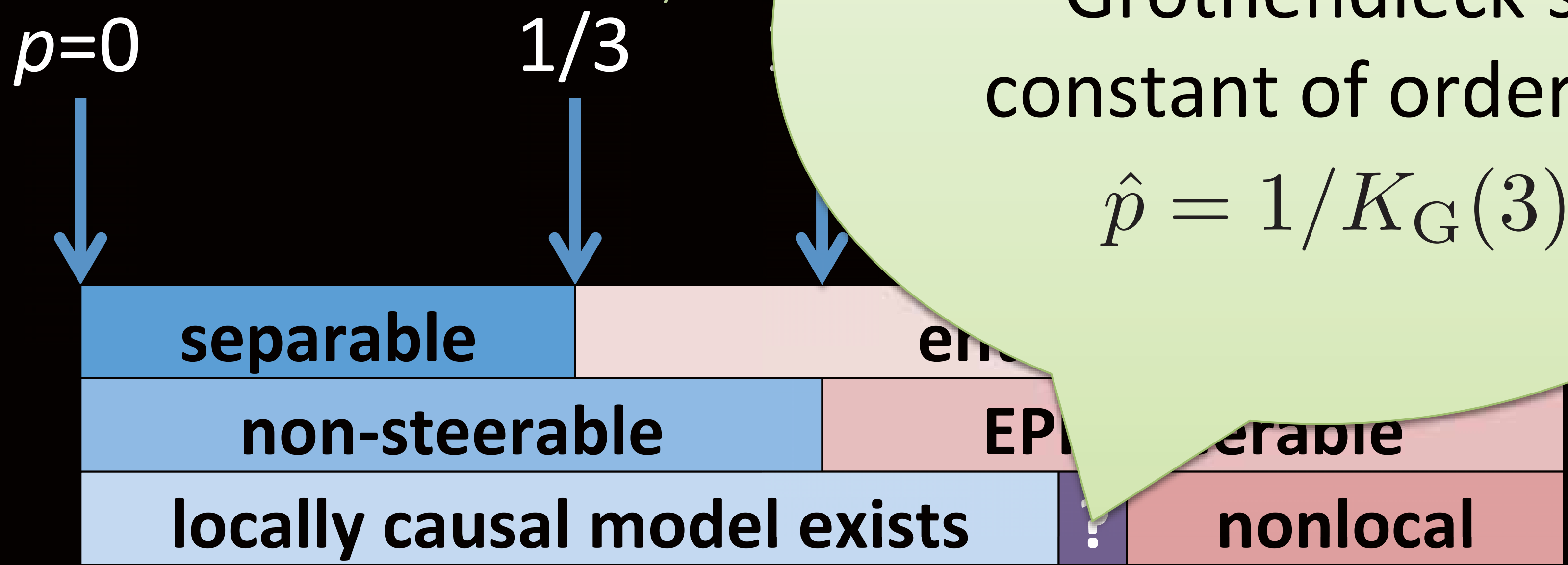
toy model: Werner state

$$|\Psi\rangle = \frac{|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle}{\sqrt{2}}$$

$$\hat{\rho}(p) = p|\Psi\rangle\langle\Psi|$$

Phys. Rev.



related to
Grothendieck's
constant of order 3:
$\hat{p} = 1/K_{\mathrm{G}}(3)$