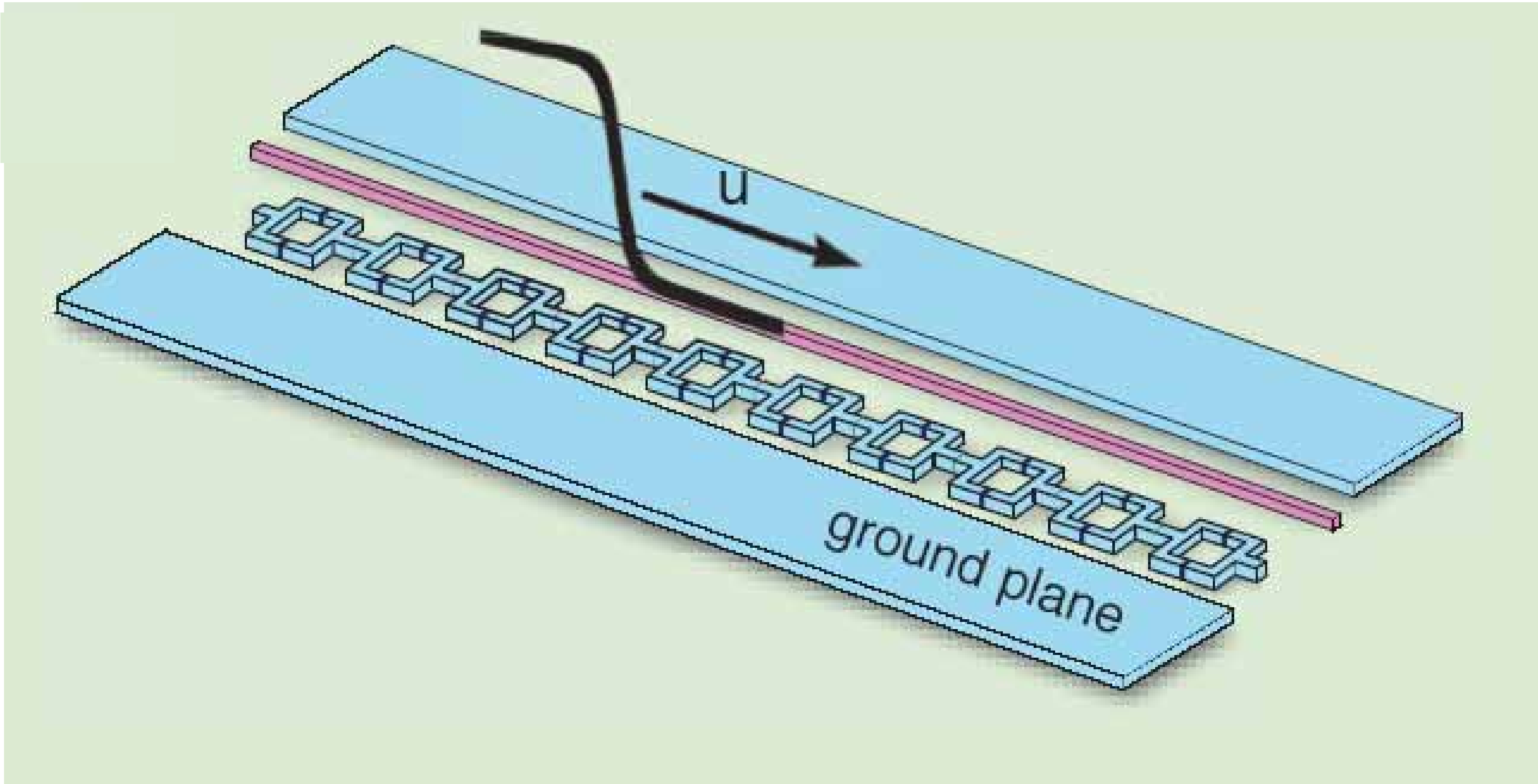
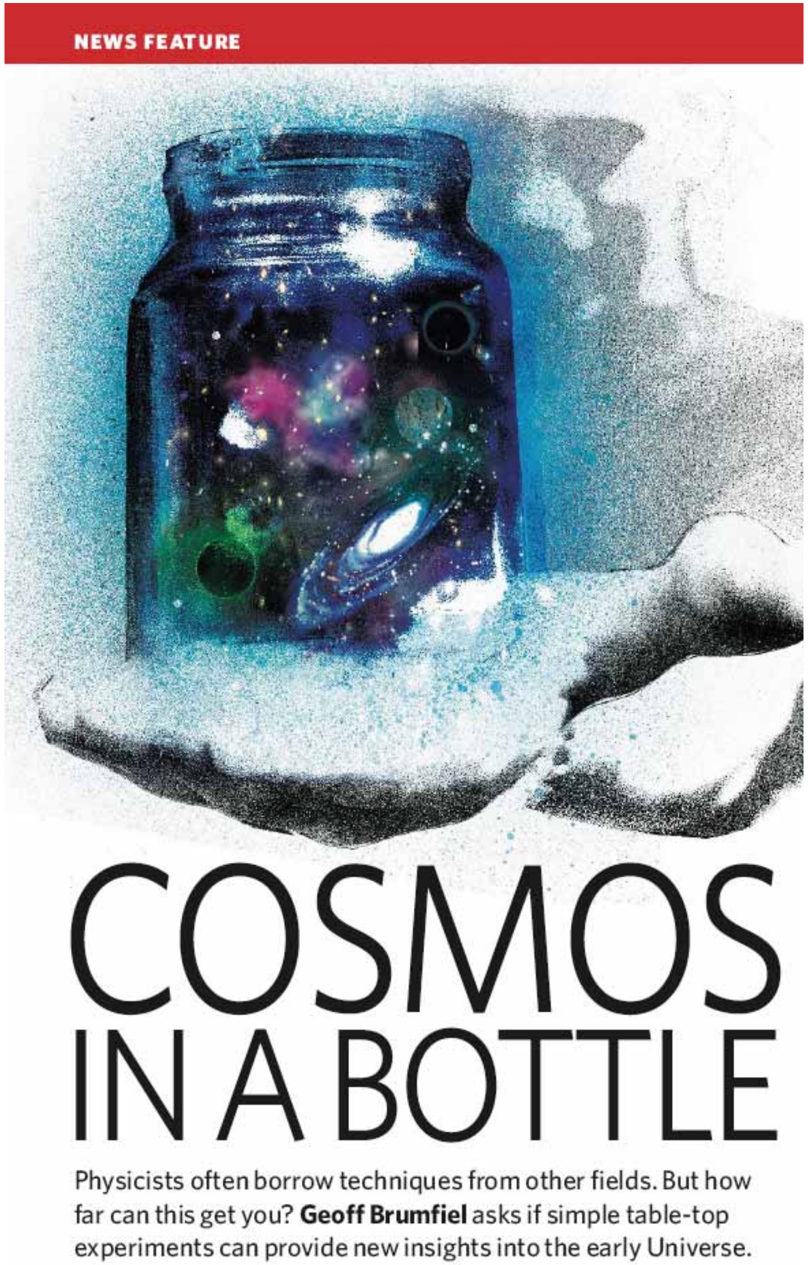
# III. cQED and Analogue Gravity




[P. Nation *et al*., *Phys. Rev. Lett.* **103**, 087004 (2009)]

Quantum field + gravity analogues in the lab

“Physicists often borrow techniques from other fields. But how far can this get you? Can simple table-top experiments provide new insights into the early universe?”


NEWS FEATURE

# COSMOS IN A BOTTLE

Physicists often borrow techniques from other fields. But how far can this get you? **Geoff Brumfiel** asks if simple table-top experiments can provide new insights into the early Universe.

[Nature **451**, 236 (17 January 2008)]

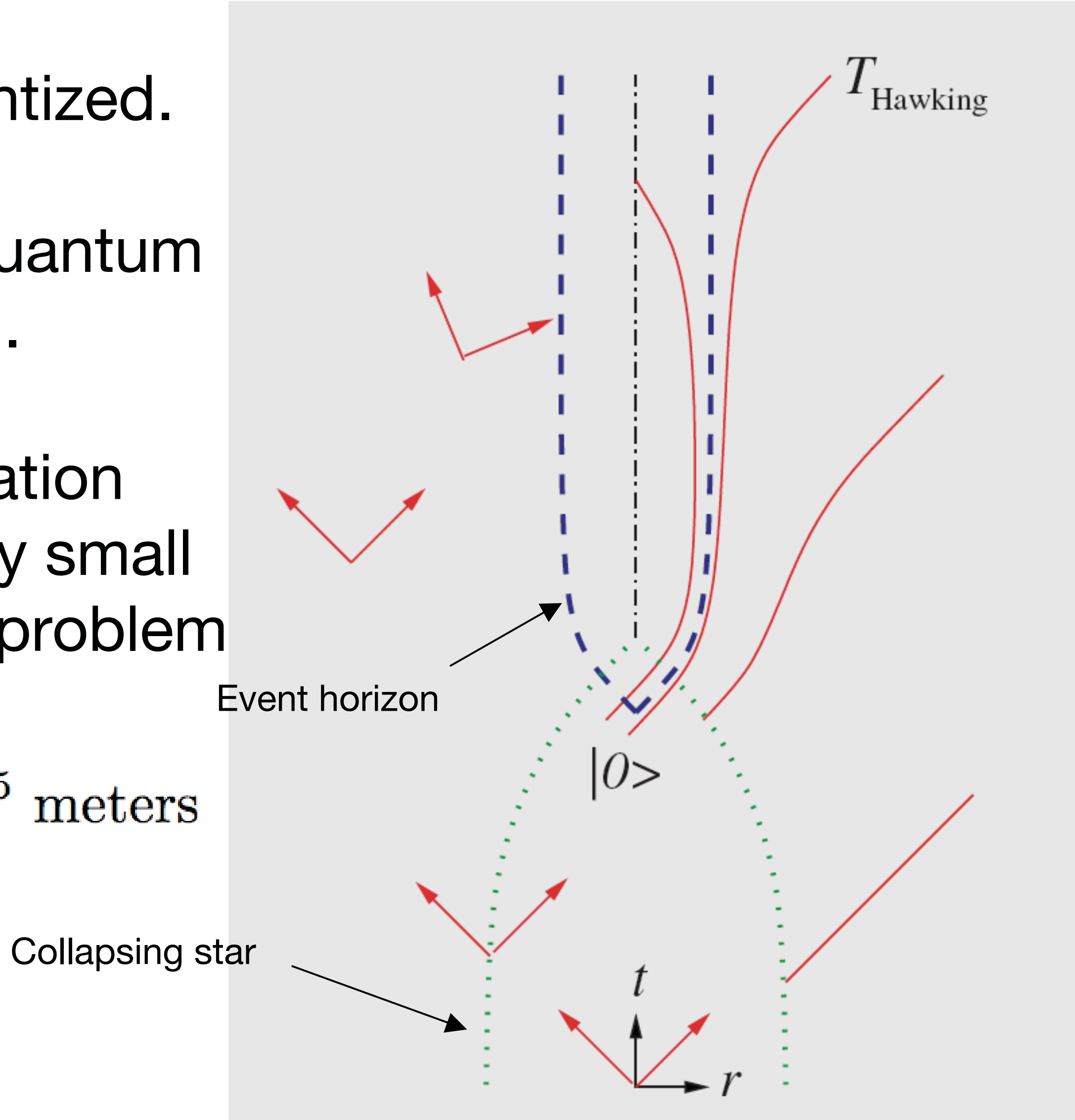
Hawking ('74): Black holes radiate through photon pair production at the event horizon. To distant observer, appears as a black body with temperature:

$$T_{BH} = \frac{\hbar c^3}{8\pi k_B G M}$$

But original semiclassical derivation has unjustified assumptions/approximations:

- Gravitational field unquantized.
- Neglect back action of quantum fields on gravitational field.
- Quantum field wave equation assumed valid to arbitrarily small scales: “trans-Planckian” problem

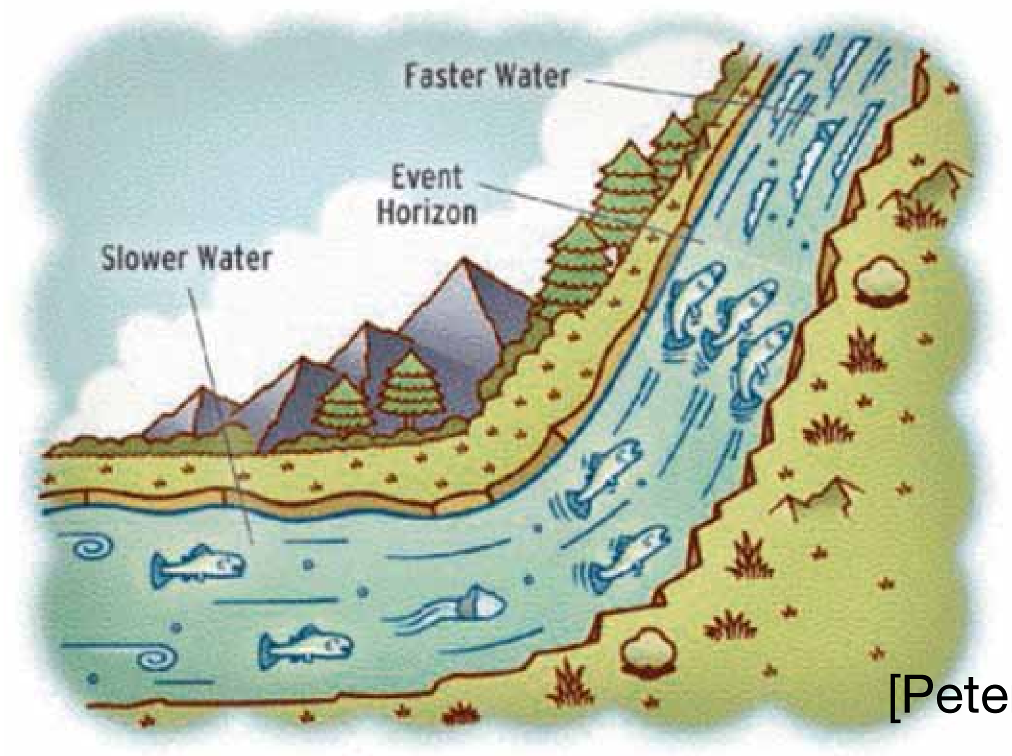
$$L_{\text{planck}} = \sqrt{\frac{\hbar G}{c^3}} \approx 1.6 \times 10^{-35} \text{ meters}$$




[From Schützhold, Lect. Notes Phys. ‘07]

# Black Hole Analogues

Address validity of Hawking's derivation using low energy, laboratory analogues of a black hole [Unruh, PRL '81]: steady state fluid flow with fluid velocity exceeding sound velocity at some closed surface $\Rightarrow$ sonic horizon.




[Peter Hoey, Science '08]

Semiclassical approach (à la Hawking): Quantize fluid fluctuations (phonons) about background classical fluid flow in vicinity of sonic horizon ⇒ thermal phonon radiation

Radial fluid flow velocity

$$T = \frac{\hbar}{2\pi k_B} \frac{\partial v^r}{\partial r}\bigg|_{\text{horizon}}$$

“Trans-Planckian” problem: Breakdown of continuum fluid approximation on intermolecular distance scales.

“Quantum Gravity” approach: solve (in principle) Schrödinger’s equation for the known microscopic Hamiltonian of the fluid system. Hawking radiation?

Alternatively, perform experiment.

Assume $\partial v^r/\partial r \sim c/R$

Local sound velocity (→ $c$) Horizon radius (→ $R$)

Then $$T \sim 10^{-7}\ \mathrm{K} \left(\frac{c}{300\ \mathrm{m\ sec^{-1}}}\right)\left(\frac{1\ \mathrm{mm}}{R}\right)$$

Nontrivial to maintain transonic flows at such small *R*'s without turbulence.

- BEC's: $T \sim 10\ \mathrm{nK}$ [Garay et al., PRL '00; Giovanazzi, PRA '04]

Thus, difficult to observe Hawking radiation in these analogue systems, assuming it exists.

But note:

PRL **106,** 021302 (2011) PHYSICAL REVIEW LETTERS week ending 14 JANUARY 2011

## Measurement of Stimulated Hawking Emission in an Analogue System

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(Received 30 August 2010; published 10 January 2011)

Hawking argued that black holes emit thermal radiation via a quantum spontaneous emission. To address this issue experimentally, we utilize the analogy between the propagation of fields around black holes and surface waves on moving water. By placing a streamlined obstacle into an open channel flow we create a region of high velocity over the obstacle that can include surface wave horizons. Long waves propagating upstream towards this region are blocked and converted into short (deep-water) waves. This is the analogue of the stimulated emission by a white hole (the time inverse of a black hole), and our measurements of the amplitudes of the converted waves demonstrate the thermal nature of the conversion process for this system. Given the close relationship between stimulated and spontaneous emission, our findings attest to the generality of the Hawking process.

See also:

[“Hawking Radiation from Ultrashort Laser Pulse Filaments,” F. Belgiorno *et al*., *Phys. Rev. Lett.* **105**, 203901 (2010); Comment: R. Schützhold *et al*., arXiv:1012.2686]

Alternative approach to moving medium:

What matters are the effective properties of the medium; not necessary to physically move a medium to form an event horizon.

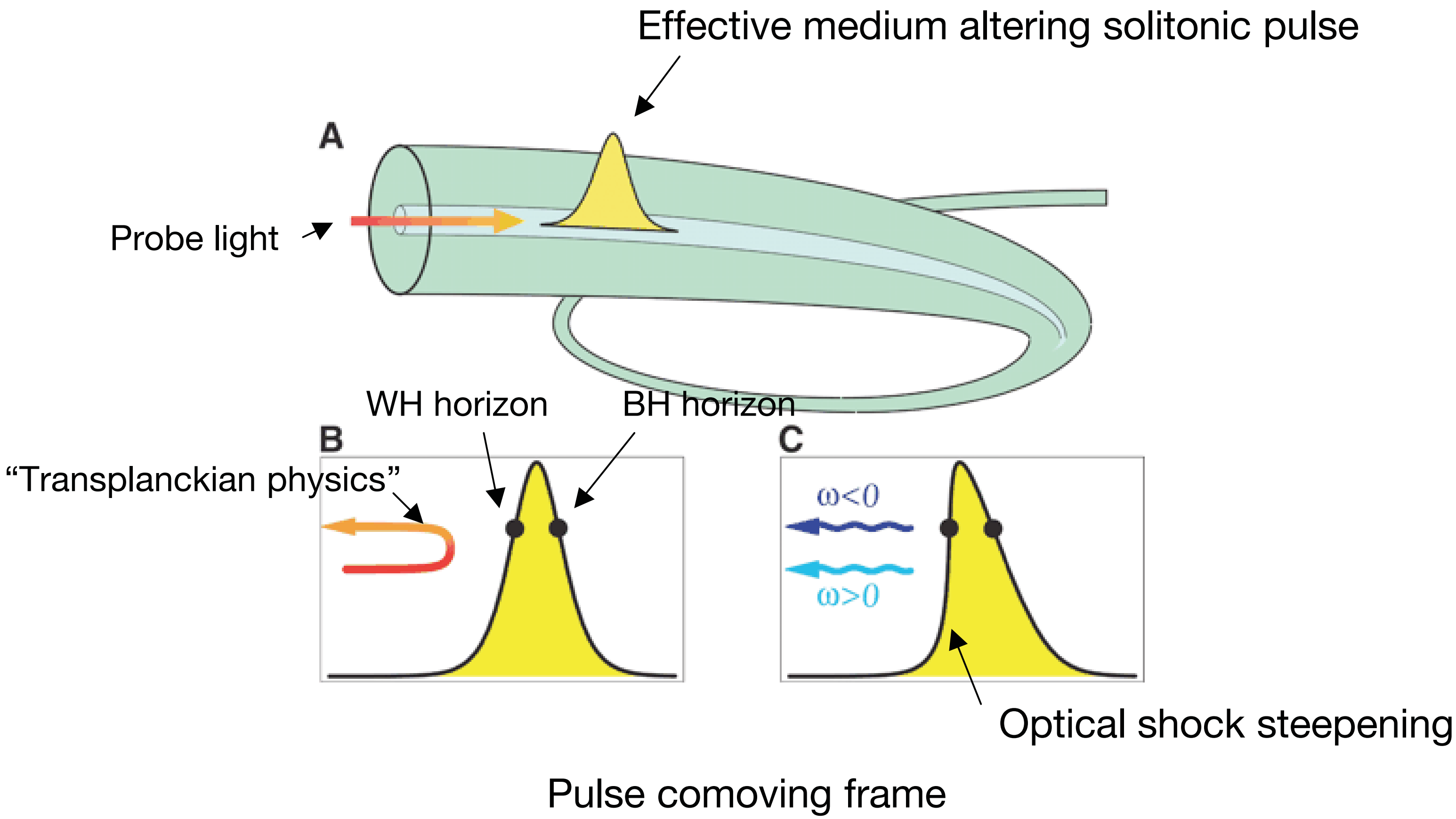
Instead, change effective properties of a medium at rest, e.g., as a propagating wave front. Essentially indistinguishable from moving medium.

E.g., use nonlinear optical fibre [Philbin et al., Science ‘08] where effective index of refraction satisfies

$$n = n_0 + \delta n, \ \ \delta n \propto I(z,t)$$

Instantaneous light pulse intensity at ($z$,$t$)

[Recall $n = c/v_p$ , $c$ is velocity of light in vacuum, $v_p$ is (phase) velocity of light in medium]




[Philbin et al., Science '08]

# Another effective medium proposal using a microwave cavity:

PRL **95,** 031301 (2005) PHYSICAL REVIEW LETTERS week ending 15 JULY 2005

**Hawking Radiation in an Electromagnetic Waveguide?**

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(Received 23 August 2004; published 15 July 2005)

It is demonstrated that the propagation of electromagnetic waves in an appropriately designed waveguide is (for large wavelengths) analogous to that within a curved space-time—such as around a black hole. As electromagnetic radiation (e.g., microwaves) can be controlled, amplified, and detected (with present-day technology) much easier than sound, for example, we propose a setup for the experimental verification of the Hawking effect. Apart from experimentally testing this striking prediction, this would facilitate the investigation of the trans-Planckian problem.

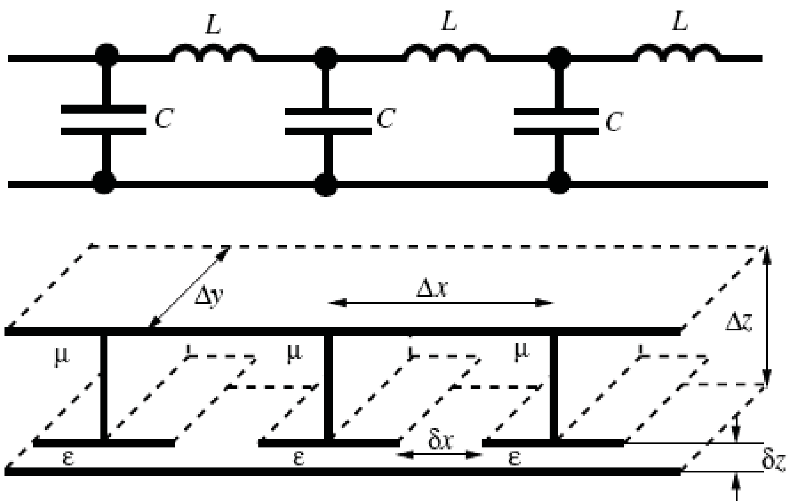



FIG. 1. Circuit diagram and sketch of the waveguide.

Dielectric between capacitor plates controlled by means of an external laser beam.

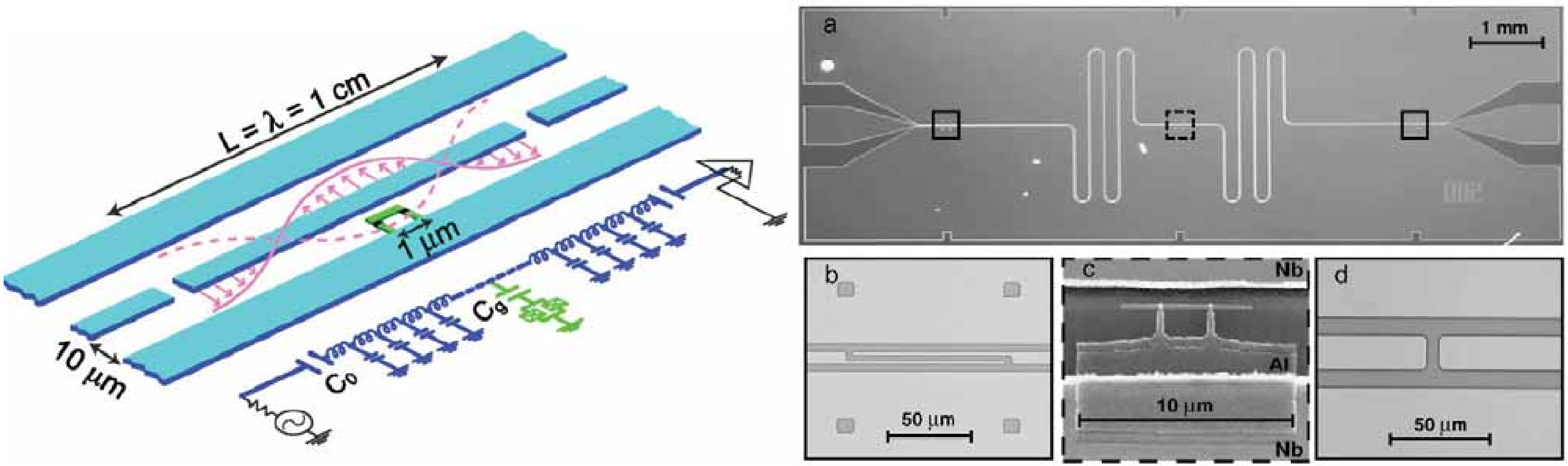
$T$~10-100 mK

But, must avoid heating due to laser beam

# A SQUID Array Microwave Resonator Proposal

Start with a low loss, superconducting coplanar microwave guide

C.f., circuit-QED scheme [Wallraff et al., Nature '04]:

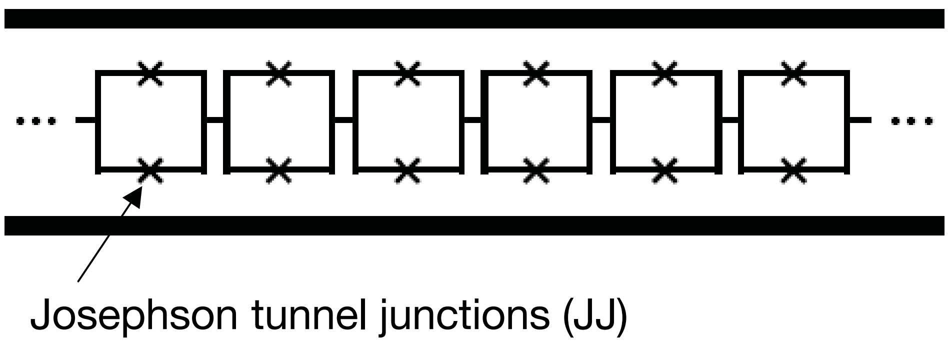


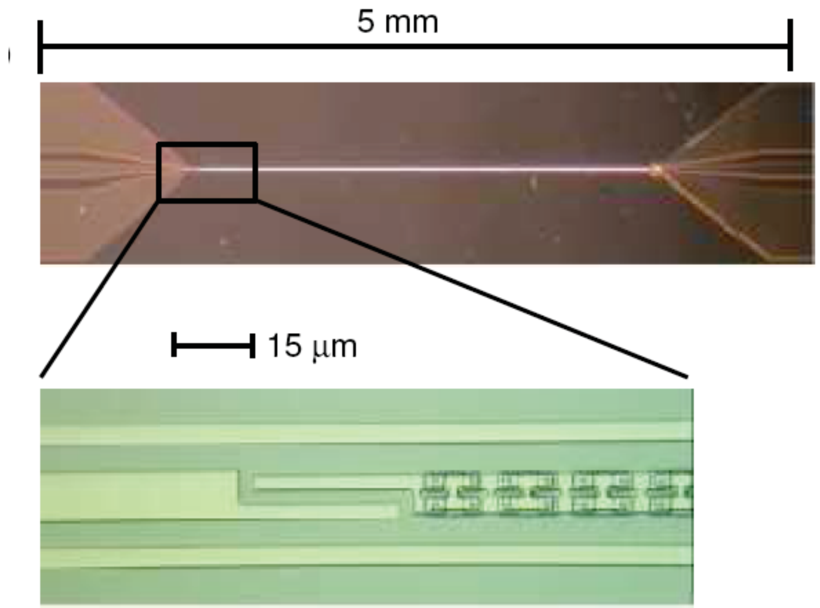

Effective 1D, planar cavity better than, e.g., 3D coaxial cavity because vacuum zero-point energy distributed over much smaller effective volume ⇒much larger zero-point $V_{rms}$ between center conductor and ground plane.

But, require nonlinear, effective microwave medium (recall fibre optic scheme).

One way: drive cavity close to it's superconducting critical current value for onset of normal (electron) current flow (kinetic inductance).

Another, less dissipative way: replace center conductor stripline with a DC Superconducting quantum interference device (SQUID) array.

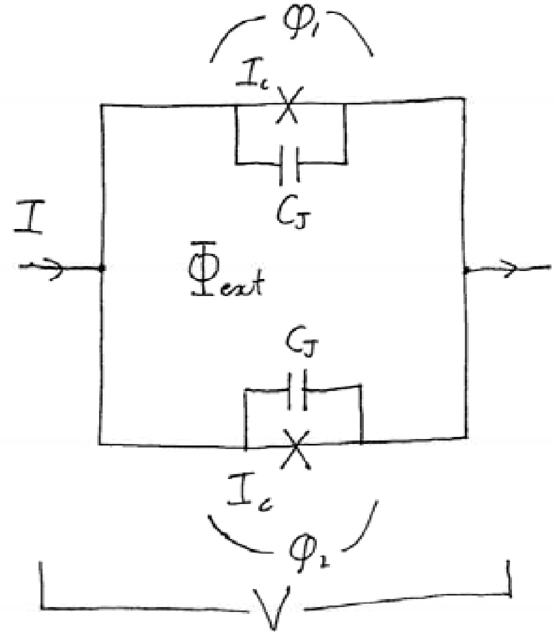


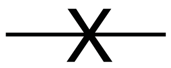




[Castellanos-Beltran et al.,Nature Phys '08]

Josephson junction (JJ) voltage/current relations in terms of phase coordinate $\phi$:

DC SQUID



$$
\begin{aligned}
V_{JJ} &= \frac{\Phi_0}{2\pi}\frac{d\phi}{dt} \\
I_{JJ} &= I_c \sin(\phi(t))
\end{aligned}
$$

$$\Phi_0 = \frac{h}{2e}$$ flux quantum

DC SQUID equations of motion in terms of phase coords. $\gamma_\pm = (\phi_1 \pm \phi_2)/2$

$$
\begin{aligned}
\frac{1}{\omega_p^2}\frac{d^2\gamma_+}{dt^2} + \sin(\gamma_+)\cos(\gamma_-) &= \frac{I}{2I_c} \\
\frac{1}{\omega_p^2}\frac{d^2\gamma_-}{dt^2} + \sin(\gamma_-)\cos(\gamma_+) &= \frac{2}{\beta_L}\left(\frac{\pi\Phi_{\mathrm{ext}}}{\Phi_0} - \gamma_-\right)
\end{aligned}
$$

Plasma frequency $\omega_p = \sqrt{\dfrac{2\pi I_c}{C_J\Phi_0}}$

Loop self inductance $\beta_L = \dfrac{2\pi L I_c}{\Phi_0}$

Voltage across SQUID:

$$V = \frac{d}{dt}\left[L_{\text{eff}}(\Phi_{\text{ext}}, I)I)\right]$$

where $$L_{\text{eff}} = \frac{\Phi_0 \gamma_+}{2\pi I} + \frac{L}{4}$$

Provided microwave frequency << JJ plasma frequency $\omega_p$, can neglect inertial properties of the SQUID.

Also assume SQUID loop inductance is small: $\beta_L \ll 1$.

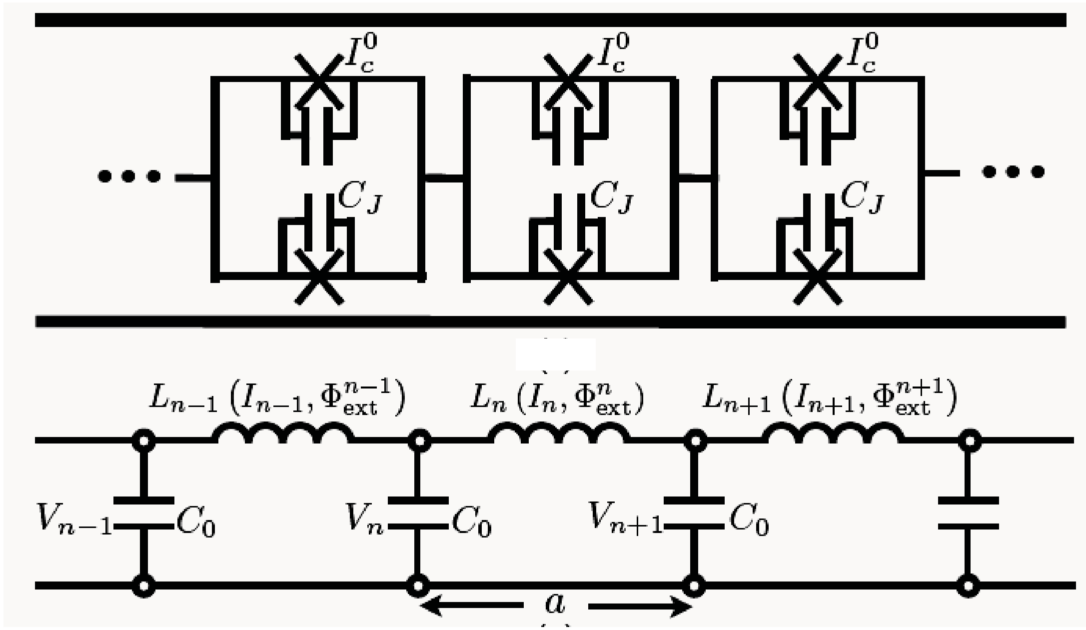
$$\Rightarrow \quad L_{\text{eff}}(\Phi_{\text{ext}}, I) = \frac{\Phi_0}{2\pi I}\arcsin\left(\frac{I\sec(\pi\Phi_{\text{ext}}/\Phi_0)}{2I_c}\right)$$

Thus, SQUID behaves simply as a *nonlinear, current and external flux-dependent inductor*.

# Nonlinear Wave (Field) Equation

DC SQUID transmission line

Effective circuit model valid for frequencies << plasma frequency




$a$ is length of single SQUID element
$C_0$ is capacitance to ground of single SQUID

Applying Kirchhoff's laws assuming space and time varying flux/current (and hence $L_{\text{eff}}$) :

$$V_{n+1} - V_n = -\frac{d(L_n I_n)}{dt}$$

$$I_{n+1} - I_n = -C_0 \frac{dV_{n+1}}{dt}$$

Introduce potential $A_n$:

$$\begin{aligned} I_n &= -C_0 \frac{dA_n}{dt} \\ V_n &= A_n - A_{n-1} \end{aligned}$$

⇒ equations of motion

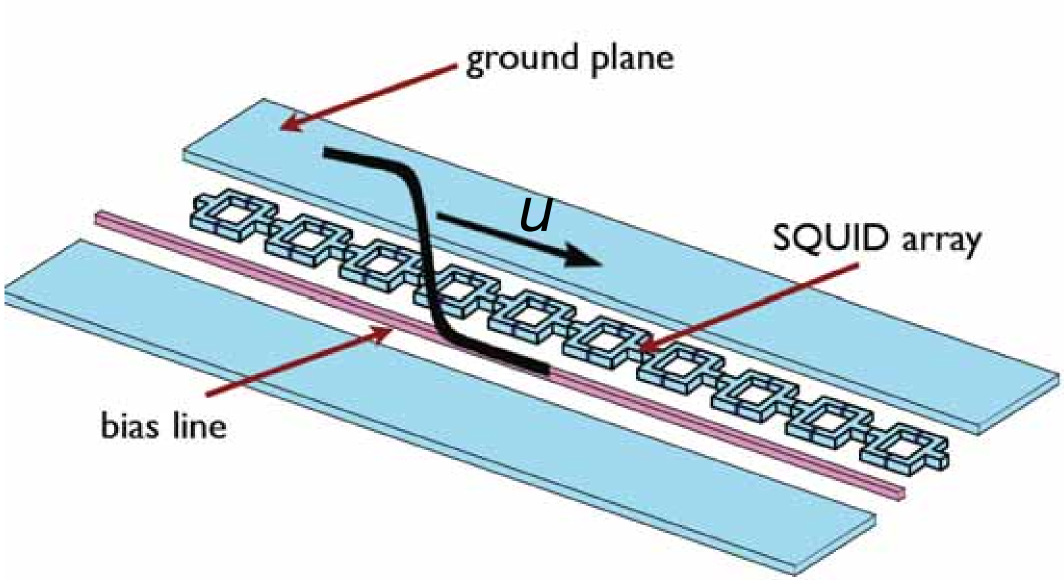
$$\frac{A_{n+1} - 2A_n + A_{n-1}}{a^2} = \frac{d}{dt}\left(\frac{L_n C_0}{a^2}\frac{dA_n}{dt}\right)$$

Continuum limit: $\lambda >> a$ (SQUID size) ⇒

$$\left(\frac{\partial}{\partial t}\frac{1}{c^2}\frac{\partial}{\partial t} - \frac{\partial^2}{\partial x^2}\right) A = 0$$

where microwave (light) velocity is $c(x,t) = \dfrac{a}{\sqrt{L(x,t)C_0}}$

Consider external flux pulse propagating with some velocity *u*; assume *I*=0.




$$L(x - ut)|_{I=0} = \frac{\Phi_0}{4\pi I_c} \sec\left(\frac{\pi \Phi_{\text{ext}}(x - ut)}{\Phi_0}\right)$$

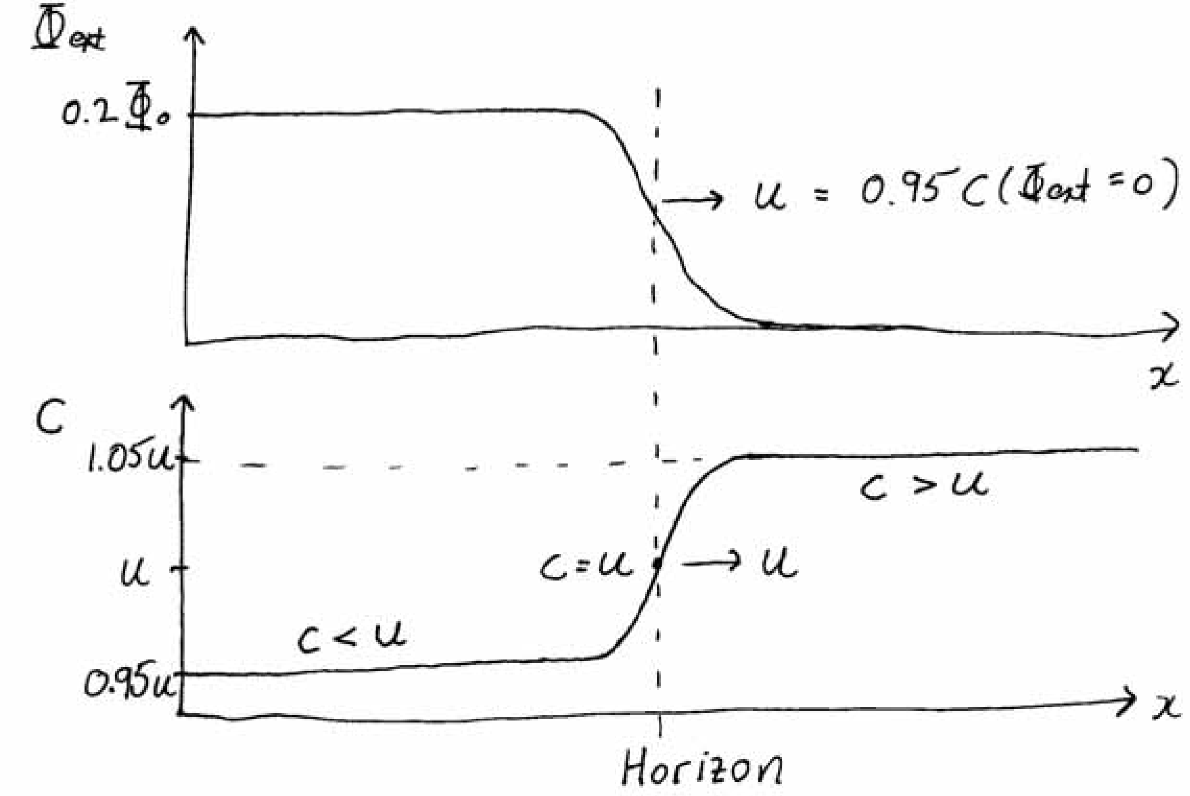
$$c^2(x - ut) = \frac{4\pi I_c a^2}{\Phi_0 C_0} \cos\left(\frac{\pi \Phi_{\text{ext}}(x - ut)}{\Phi_0}\right)$$

Velocity of light *decreases* as enter region of increasing flux.

Example: consider flux pulse with front velocity $u = 0.95c(\Phi_{\text{ext}} = 0)$ and flux amplitude

$$\Phi_{\text{ext}} = 0.2\Phi_0$$

$\Rightarrow$ light ahead of the front moves faster than front, while light behind front moves slower than front.



# Effective Geometry

Transform to frame comoving with flux pulse front:

$$x' = x - ut,\ t' = t,\ \frac{\partial}{\partial t} = \frac{\partial}{\partial t'} - u\frac{\partial}{\partial x'},\ \frac{\partial}{\partial x} = \frac{\partial}{\partial x'}$$

Wave equation becomes (dropping primes):

$$\left[\left(\frac{\partial}{\partial t} - u\frac{\partial}{\partial x}\right)\frac{1}{c^2}\left(\frac{\partial}{\partial t} - u\frac{\partial}{\partial x}\right) - \frac{\partial^2}{\partial x^2}\right]A = 0$$

Can be formally written as $\partial_\mu(g^{\mu\nu}_{\text{eff}}\partial_\nu A) = 0$
where effective metric is

$$g^{\mu\nu}_{\text{eff}} = \frac{1}{c^2}\begin{pmatrix} 1 & -u \\ -u & u^2 - c^2 \end{pmatrix}$$

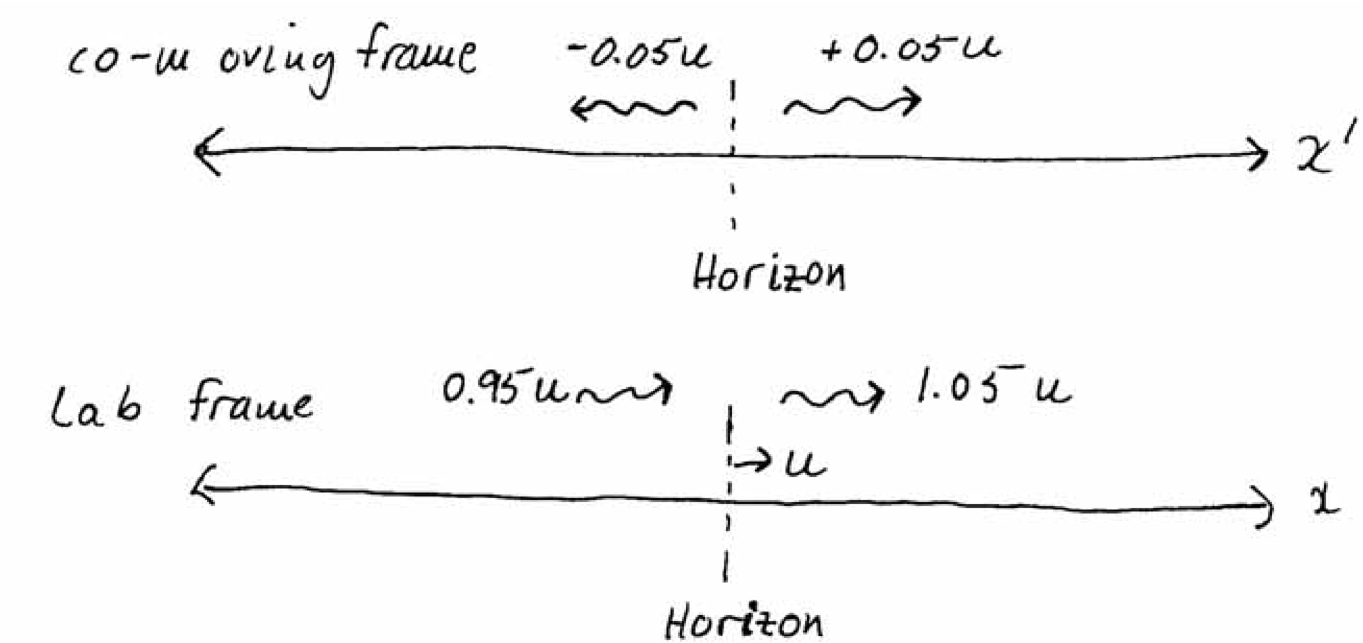
Horizon where $c(x)=u$

Similar to metric of black hole expressed in Painlevé-Gullstrand coordinates (free fall frame time coordinate).

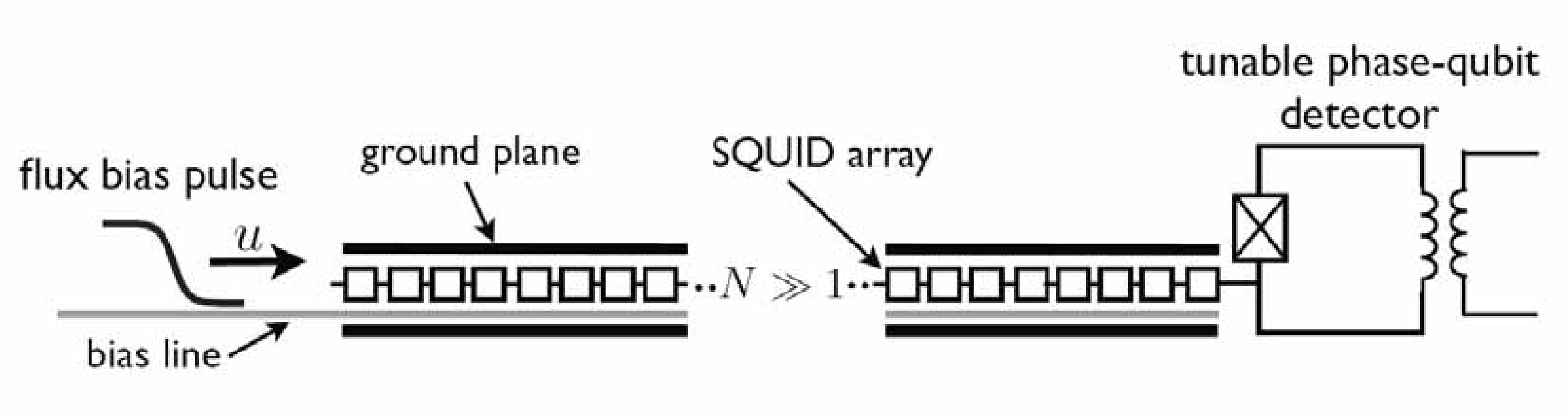
Hawking: existence of horizon⇒vacuum fluctuations converted into photons:

In co-moving frame, $$T_H = \frac{\hbar}{2\pi k_B}\left|\frac{\partial c(x)}{\partial x}\right|_{c(x)=u}$$

Radiated power $$P = \frac{\pi}{12\hbar}(k_B T)^2$$



# Possible Implementation




Example achievable parameters [Castellanos-Beltran *et al*., APL '07]:

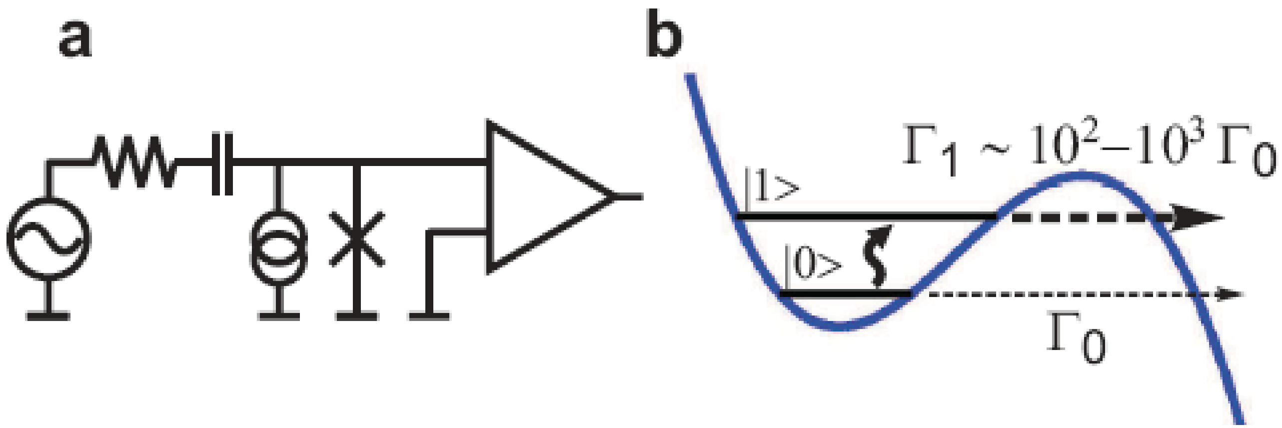
$$I_c = 2\ \mu\text{A},\ \omega_p = 2\pi \times 10^{12}\ \text{s}^{-1},\ C_0 = 5 \times 10^{-17}\ \text{F},\ a = 0.25\ \mu\text{m}$$

$$\Rightarrow\ c(\Phi_{\text{ext}} = 0) \sim c_{\text{vac}}/100$$

Assume $\left|\frac{\partial c(x)}{\partial x}\right|_{c(x)=u} = \frac{1}{10}\omega_p/2\pi$ . Then $T_H = 120\ \text{mK}$ .

With pulse dispersion, radiated power formula ⇒one photon pair/pulse for ~4800 SQUID transmission line.

# Single shot microwave photon detection [Y. Chen *et al*., *ArXiv*:1011.4329 (2010)]




(a) Schematic diagram of Josephson-junction microwave detector. The junction is biased with a dc current, and microwaves are coupled to the junction via an on-chip capacitor. The voltage across the junction is read out by a room temperature preamplifier and comparator. (b) Junction potential energy landscape. The junction is initialized in the $|0\rangle$ state. An incident photon induces a transition to the $|1\rangle$ state, which rapidly tunnels to the continuum.

# Analogue Planck-Scale Physics

For considered parameters, relevant short distance scales are $0.1\ \mu\text{m} < c/\omega_p \sim a < 1\ \mu\text{m}$.

'Solve' for full quantum dynamics of nonlinear SQUID transmission line equations with horizon forming flux pulse bias: does one get thermal Hawking radiation?

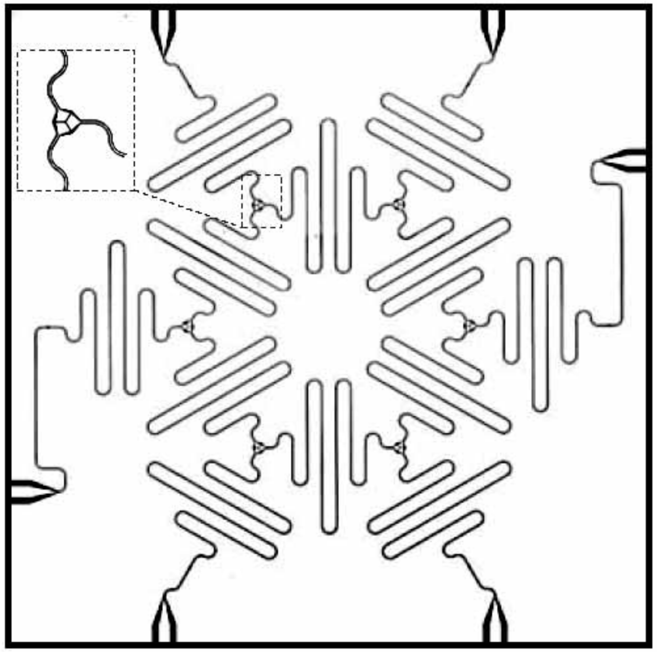
Squids are quantum coherent devices. Quantum fluctuations in phase $\gamma_+$ become more significant as $\Phi_{\text{ext}} \to \Phi_0/2 \Rightarrow$ fluctuations in *c* (i.e., fluctuations in effective spacetime geometry). Thus can tune magnitude of quantum fluctuations by applying uniform $\Phi_{\text{ext}}$: analogue 'quantum gravity' physics.

Simpler “warm-up” problem (in progress):

Microwave photon propagation down SQUID array transmission line: effective medium with quantum fluctuating index of refraction (≡ fluctuating light speed ≡fluctuating metric)

# Other (fundamental physics) applications of cQED

- Interacting photon lattices [J. Koch and A. Houck]



- What possible applications can *you* dream up?