Circuit Quantum Electrodynamics
or
Fundamental Physics with Tiny Pieces of Cold Metal

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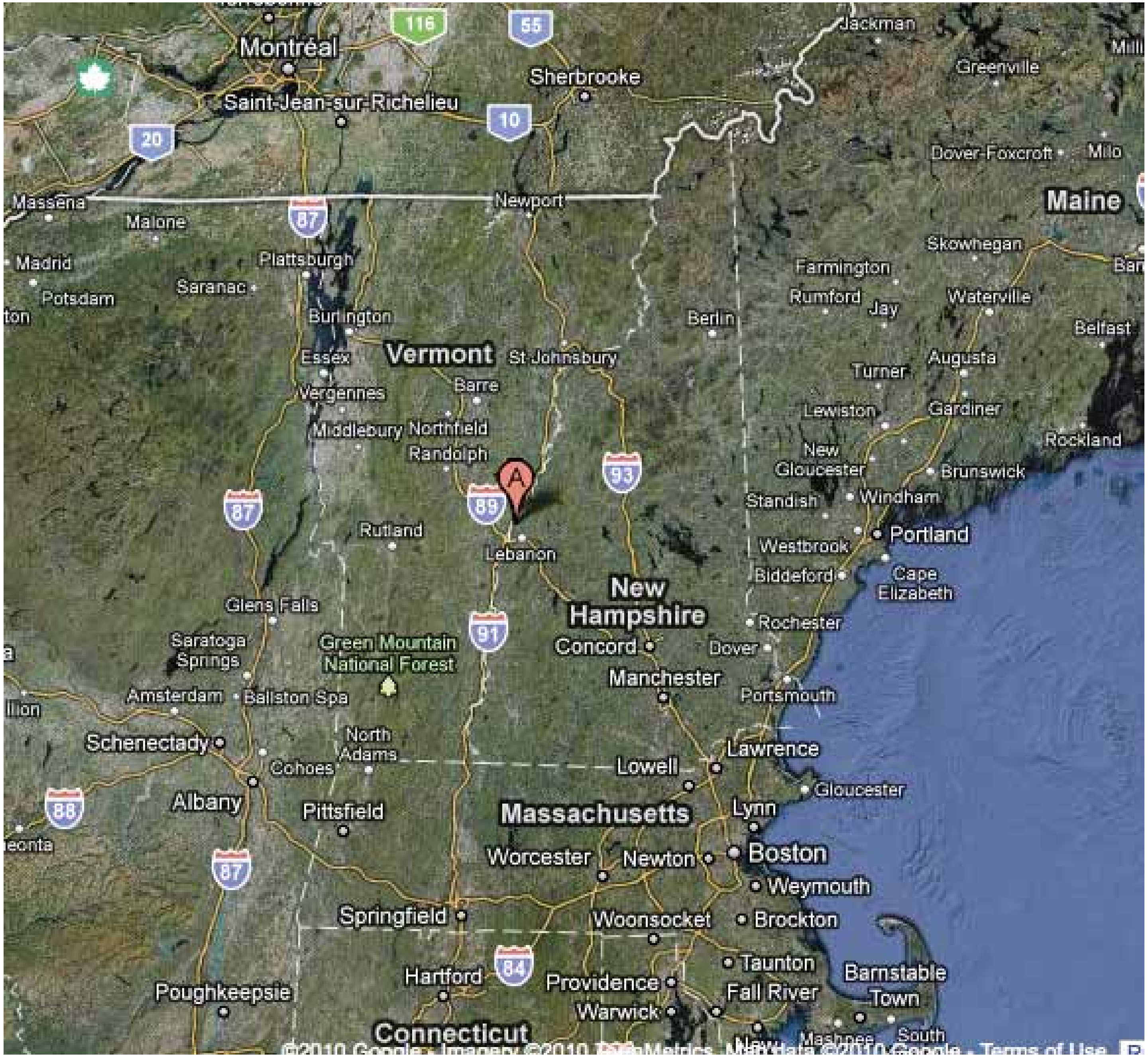
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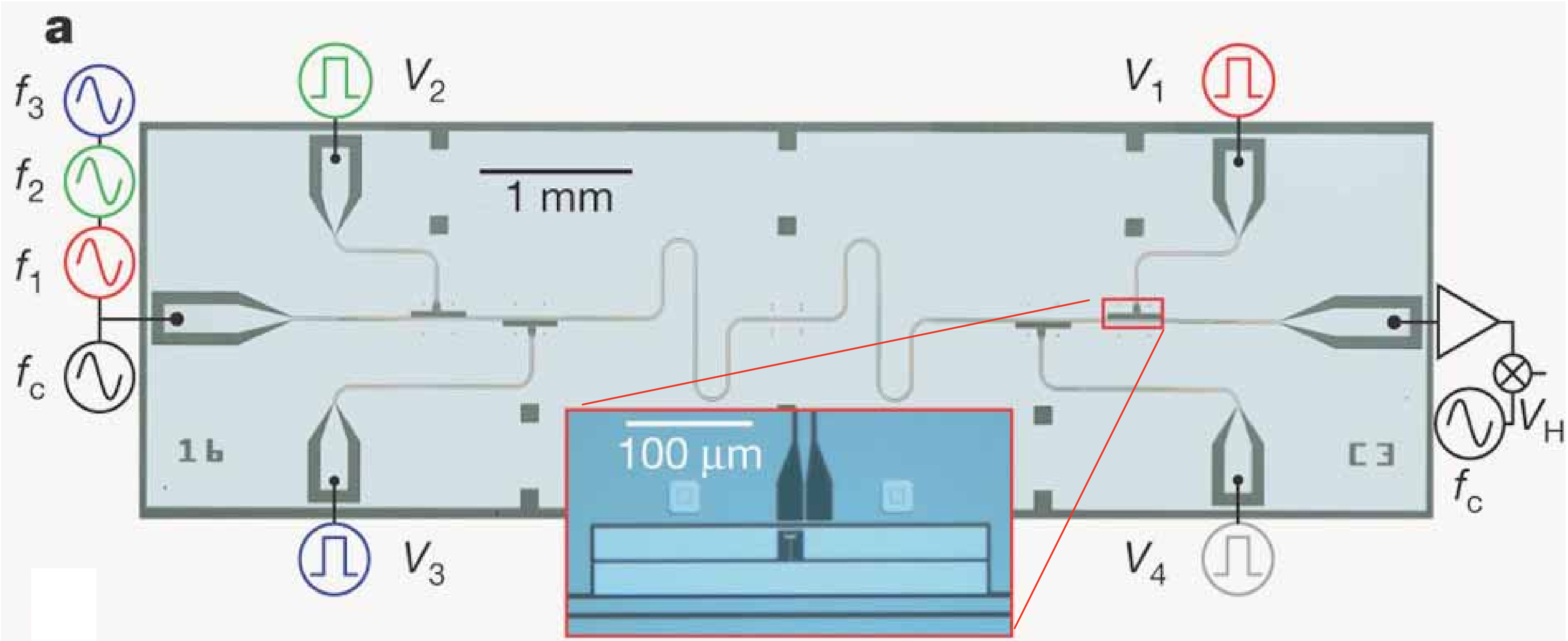
Outline

I. Circuit Quantum Electrodynamics (cQED) for Quantum Information Processing

II. cQED and the Quantum-Classical Correspondence

III. cQED and Analogue Gravity

I. Circuit Quantum Electrodynamics (cQED) for Quantum Information Processing

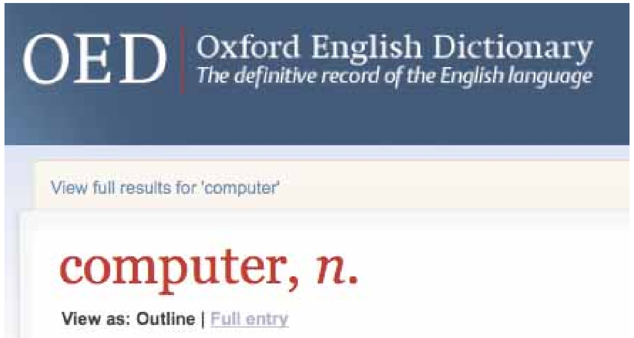


Four-qubit cQED processor

[L DiCarlo *et al. Nature* **467**, 574-578 (2010)]

Goal of this first lecture is to understand the physics behind the cQED scheme for quantum computing.

A classical computer is…



a. An electronic device (or system of devices) which is used to store, manipulate, and communicate information, perform complex calculations, or control or regulate other devices or machines, and is capable of receiving information (data) and of processing it in accordance with variable procedural instructions (programs or software); *esp.* a small, self-contained one for individual use in the home or workplace, used esp. for handling text, images, music, and video, accessing and using the Internet, communicating with other people (e.g. by means of email), and playing games.

Binary data (…10110111…) encoded in state of a physical system, for example, as presence (“1”) or absence (“0”) of electrical charge in a capacitor.

A quantum computer is…

quantum computer *n.* a computer in which quantum effects are significant; *spec.* a (hypothetical) computer in which information is represented in terms of the possible quantum states of a set of physical systems (such as spinning particles), and in which the existence of superposed states would allow operations to be performed on all possible values of a variable simultaneously.

Superposition principle ⇒ quantum bit, or "qubit":

$$|\Psi\rangle = \alpha|0\rangle + \beta|1\rangle \qquad |\alpha|^2 + |\beta|^2 = 1$$

Unitary operator

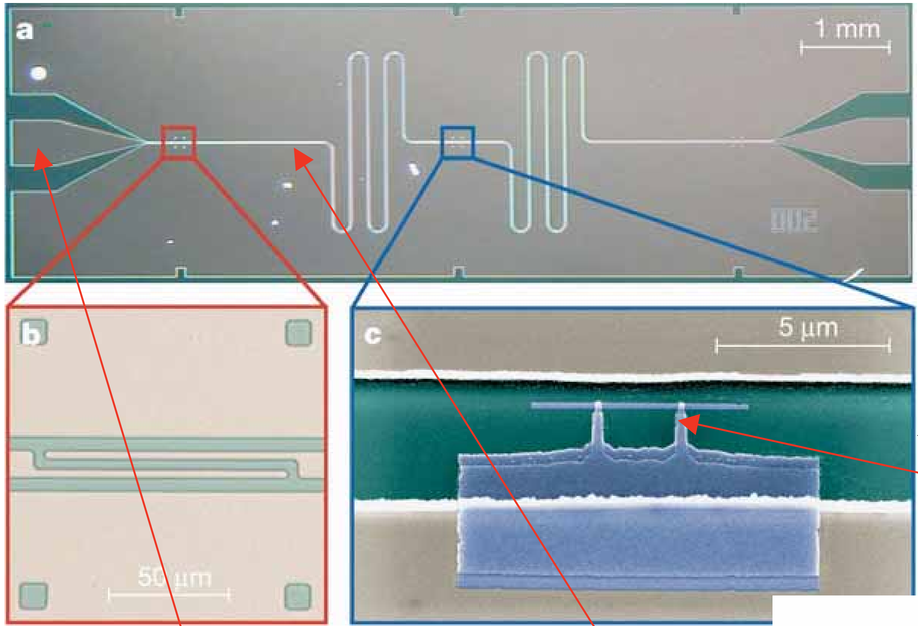
$$|\Psi\rangle = \alpha|0\rangle + \beta|1\rangle \xrightarrow{\hat{U}} \gamma|0\rangle + \delta|1\rangle$$

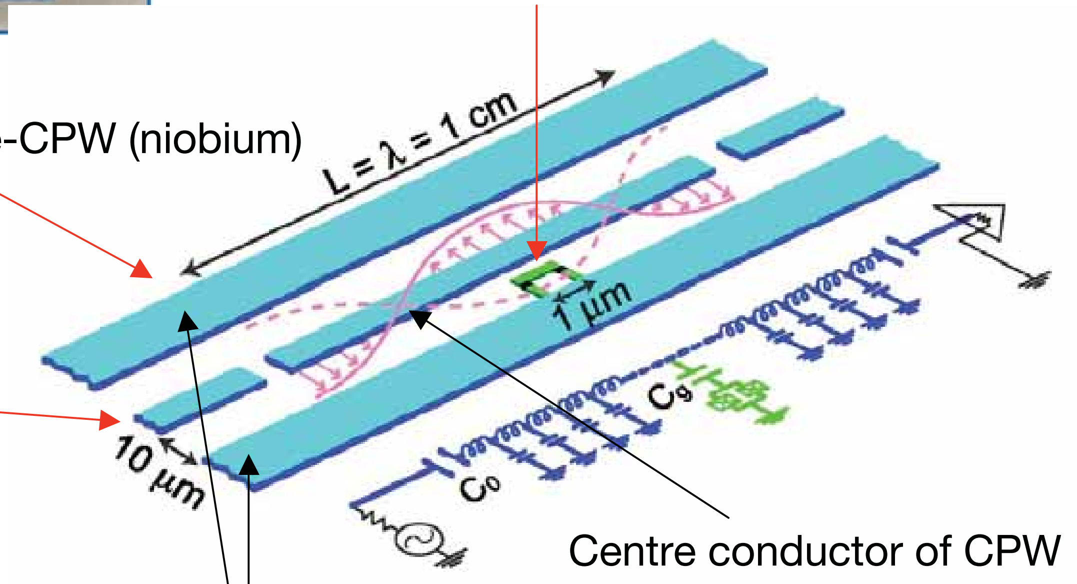
Encode qubit in two-level quantum system. Examples:

- Spin 'down' and spin 'up' states of electron
- Clockwise and anticlockwise electrical currents of superconducting ring

Will now analyze following, simpler cQED device

[A. Wallraff *et al.*, *Nature* **431**, 162 (2004);
A. Blais *et al.*, *Phys. Rev.* A **69**, 062320 (2004)]





Electron microscope image

Cooper pair box qubit (aluminium)

Coplanar waveguide-CPW (niobium)

Control/measurement line

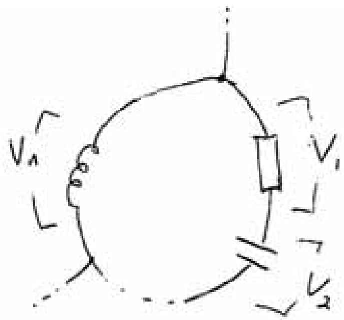
Centre conductor of CPW

Ground plane of CPW

Derive Hamiltonian for Cooper pair box (CPB) qubit - coplanar waveguide (CPW) resonator system

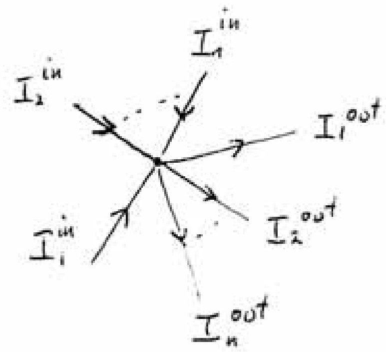
Use Kirchhoff's laws:

- Voltages add up to zero around a closed loop



$$V_1 + V_2 + \ldots + V_n = 0$$

- Total current entering a junction equals total exiting a junction

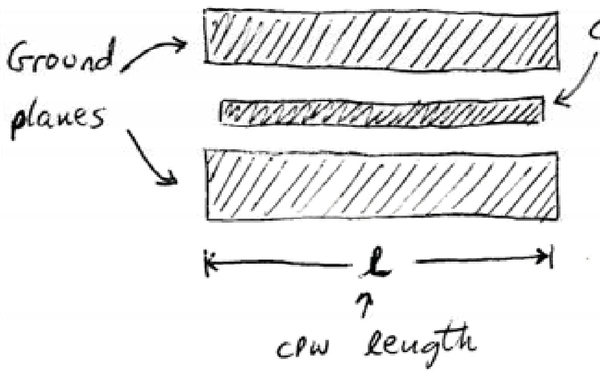


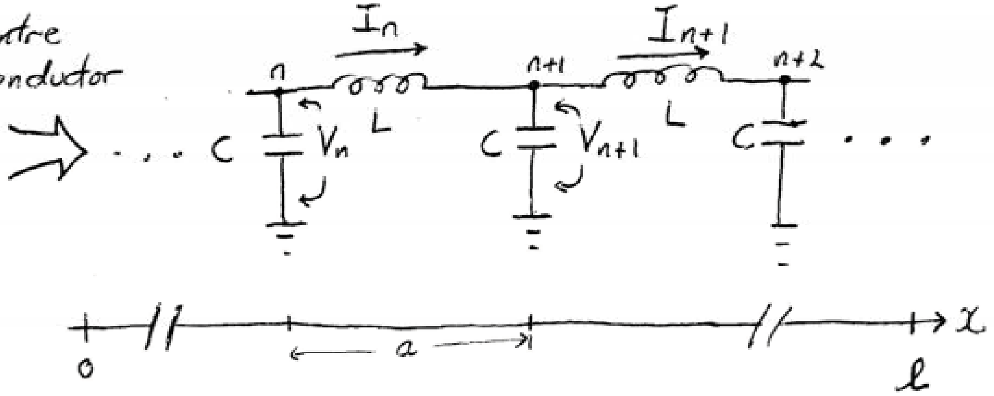
$$I_1^{in} + I_2^{in} + \ldots + I_n^{in} = I_1^{out} + I_2^{out} + \ldots + I_n^{out}.$$

Consider first, isolated CPW resonator without the CPB qubit

Isolated CPW resonator

One dimensional LC network model





Kirchhoff:

$$I_n - I_{n+1} = \dot{Q}_{n+1} = C\frac{dV_{n+1}}{dt}$$

$$V_n - V_{n+1} = L\frac{dI_n}{dt}$$

Take limit $a \to 0$ $\Rightarrow$

$$\frac{\partial I(x,t)}{\partial x} = -\mathcal{C}\frac{\partial V(x,t)}{\partial t}$$

$$\frac{\partial V(x,t)}{\partial x} = -\mathcal{L}\frac{\partial I(x,t)}{\partial t}$$

where $\mathcal{C} = \frac{C}{a}$ = capacitance/length, $\mathcal{L} = \frac{L}{a}$ = inductance/length

Introduce phase field coordinate $\varphi(x,t)$:

$$\varphi(x,t) = \frac{2\pi}{\Phi_0} \int^{t} dt' \, V(x,t') \quad , \quad \Phi_0 = \frac{h}{2e} \leftarrow \text{flux quantum}$$

$$\therefore \quad V(x,t) = \frac{\Phi_0}{2\pi} \frac{\partial \varphi(x,t)}{\partial t}$$

and $$I(x,t) = -\frac{\Phi_0}{2\pi \mathcal{L}} \frac{\partial \varphi(x,t)}{\partial x} .$$

$$\therefore \quad \frac{\partial I}{\partial x} = -\mathcal{C} \frac{\partial V}{\partial t} \quad \Rightarrow \quad \boxed{\frac{\partial^2 \varphi}{\partial t^2} = v^2 \frac{\partial^2 \varphi}{\partial x^2}} \leftarrow \text{Wave equation.}$$

where $v = \frac{1}{\sqrt{\mathcal{L}\mathcal{C}}}$ is the phase velocity of EM microwaves in the CPW.

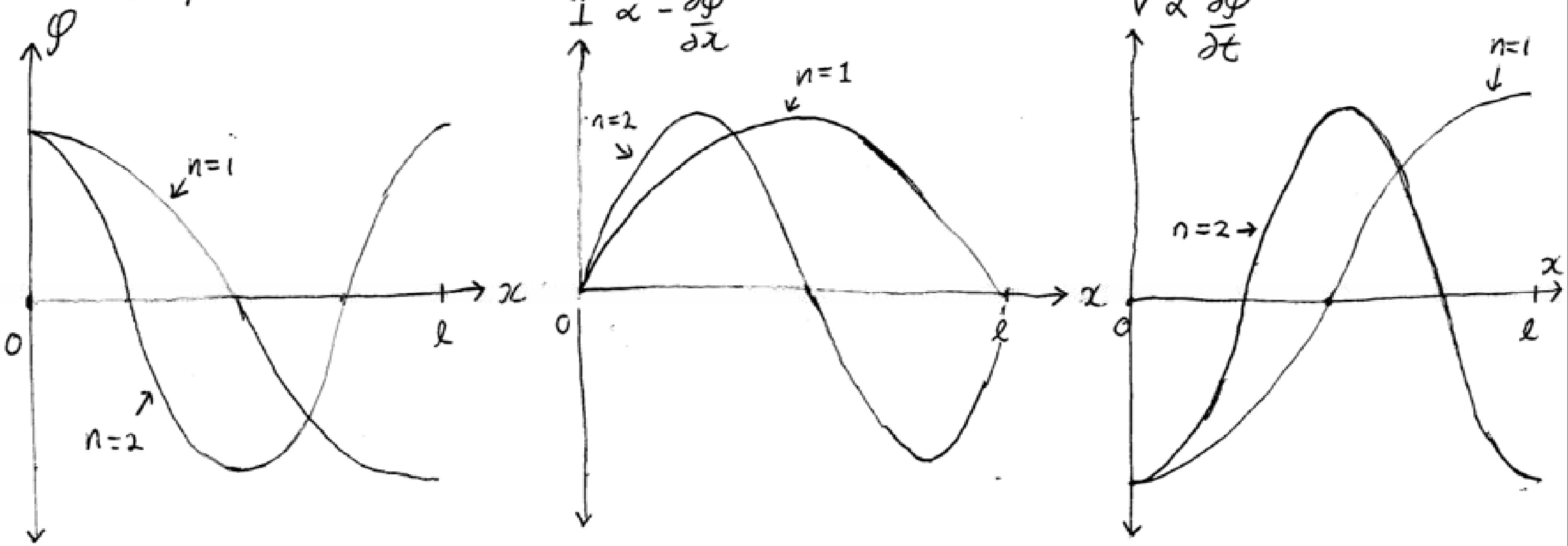
Solve wave equation subject to boundary conditions

$$I(x=0,t) = I(x=\ell,t) = 0$$ (current vanishes at ends of CPW)

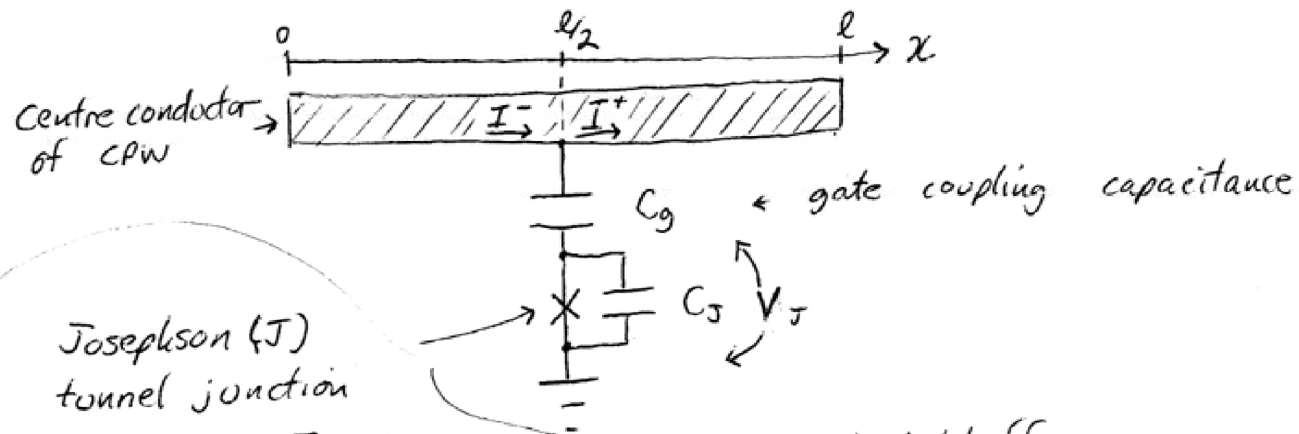
$$\Rightarrow \frac{\partial\varphi}{\partial x}(x=0,t) = \frac{\partial\varphi}{\partial x}(x=\ell,t) = 0$$

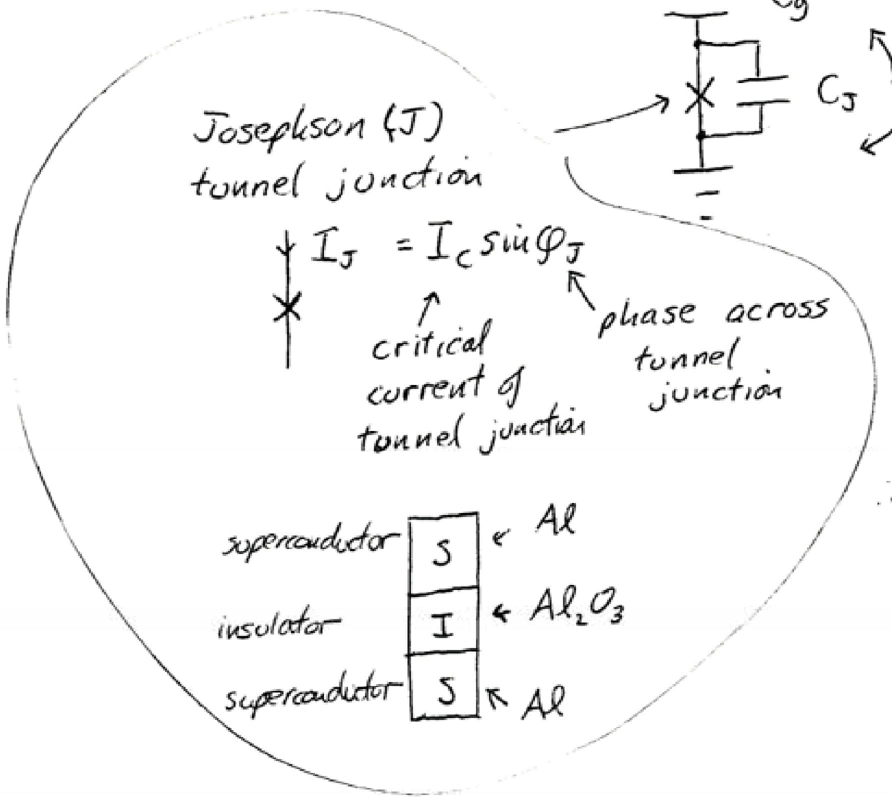
Get standing wave solutions: $\boxed{\varphi(x,t) = A\cos\left(\frac{\pi n v t}{\ell}\right)\cos\left(\frac{\pi n x}{\ell}\right)}$

Snapshots:



Now include Cooper pair box (CPB)





Kirchhoff :

$$V(x=\tfrac{\ell}{2},t) = V_g + V_J$$

$$\therefore \frac{\Phi_0}{2\pi}\frac{\partial\varphi}{\partial t}(x=\tfrac{\ell}{2},t) = \frac{Q_g(t)}{C_g} + \frac{\Phi_0}{2\pi}\frac{d\varphi_J(t)}{dt}$$

$$\therefore \frac{\Phi_0}{2\pi}\frac{\partial^2\varphi}{\partial t^2}(x=\tfrac{\ell}{2},t) = \frac{I^- - I^+}{C_g} + \frac{\Phi_0}{2\pi}\frac{d^2\varphi_J}{dt^2}$$

$$I^- - I^+ = I_c \sin\varphi_J + C_J\frac{\Phi_0}{2\pi}\frac{d^2\varphi_J}{dt^2}$$

Eliminate $I^- - I^+$ term $\Rightarrow$

$$\underbrace{(C_J + C_g)}_{C_\Sigma}\frac{\Phi_0}{2\pi}\frac{d^2\varphi_J}{dt^2} + I_c\sin\varphi_J = C_g\frac{\Phi_0}{2\pi}\frac{\partial^2\varphi}{\partial t^2}(x=\tfrac{\ell}{2},t)$$

CPB equation of motion:

$$C_\Sigma \frac{\Phi_0}{2\pi} \frac{d^2\varphi_J}{dt^2} + I_c \sin\varphi_J = C_g \frac{\Phi_0}{2\pi} \frac{\partial^2 \varphi}{\partial t^2}\left(x = \frac{\ell}{2}, t\right)$$

- Assuming $C_g \ll \mathcal{C}\ell$ (CPW capacitance), then spatial dependence of CPW mode solutions only weakly affected by coupling to CPB: $\varphi(x,t) \approx \varphi(t) \cos\left(\frac{n\pi x}{\ell}\right)$.
- To couple CPW and CPB, must have $n = 2, 4, 6, \ldots$ (current node / voltage antinode).

Restrict to $n=2$ mode: harmonic oscillator approx.

Lagrangian

$$L = \frac{1}{2} \mathcal{C}\ell \left(\frac{\Phi_0}{2\pi}\right)^2 \left[\frac{\dot\varphi^2}{2} - \left(\frac{2\pi v}{\ell}\right)^2 \frac{\varphi^2}{2}\right] + \frac{1}{2} C_\Sigma \left(\frac{\Phi_0}{2\pi}\right)^2 \dot\varphi_J^2 + I_c \frac{\Phi_0}{2\pi} \cos\varphi_J - C_g \left(\frac{\Phi_0}{2\pi}\right)^2 \dot\varphi\, \dot\varphi_J$$

Generalized momenta : $p_\varphi = \frac{\partial \mathcal{L}}{\partial \dot{\varphi}} = \frac{c\ell}{2}\left(\frac{\Phi_0}{2\pi}\right)^2 \dot{\varphi} - C_g\left(\frac{\Phi_0}{2\pi}\right)^2 \dot{\varphi}_J$

$$p_{\varphi_J} = \frac{\partial \mathcal{L}}{\partial \dot{\varphi}_J} = C_\Sigma\left(\frac{\Phi_0}{2\pi}\right)^2 \dot{\varphi}_J - C_g\left(\frac{\Phi_0}{2\pi}\right)^2 \dot{\varphi}$$

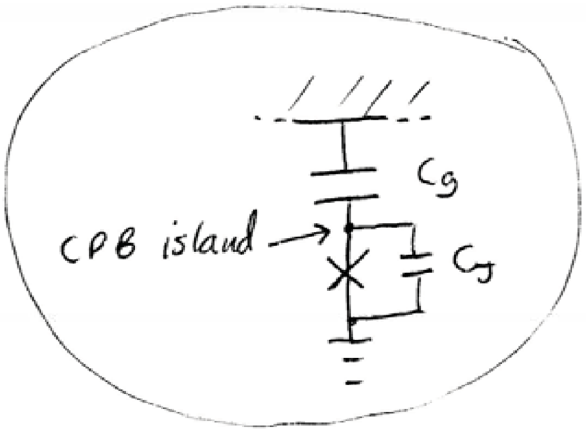
$$\left(\dot{\varphi} \equiv \frac{d\varphi}{dt} \right)$$

Define $N = -\frac{p_{\varphi_J}}{\hbar}$

$$= -\frac{1}{\hbar}\underbrace{\left(\frac{\Phi_0}{2\pi}\right)}_{=\hbar/2e}\Big(\underbrace{C_\Sigma}_{=C_J+C_g} V_J - C_g V(x=\tfrac{\ell}{2})\Big)$$

$$= -\frac{1}{2e}\left[C_J V_J - C_g\left(V(x=\tfrac{\ell}{2}) - V_J\right)\right]$$

$$= -\frac{1}{2e}\left[C_J V_J - C_g V_g\right]$$



$\therefore N \equiv$ the number of excess Cooper pairs on the CPB island

Hamiltonian

include controllable dc voltage bias on centre conductor $N_g = -\frac{C_g V_{dc}}{2e}$

$$H = \frac{(2e)^2}{2C_\Sigma}(N - N_g)^2 - \frac{I_c \Phi_0}{2\pi}\cos\varphi_J$$

$$+ \frac{2}{Cl}\left(\frac{2\pi}{\Phi_0}\right)^2 \frac{p_\varphi^2}{2} + \frac{1}{2} Cl \left(\frac{\Phi_0}{2\pi}\right)^2 \left(\frac{2\pi v}{l}\right)^2 \frac{\varphi^2}{2}$$

$$- 4e \frac{C_g}{Cl\, C_\Sigma} N \frac{2\pi}{\Phi_0} p_\varphi$$

Now quantize (we are nearly there!)

$$\hat{N} = \sum_{N=-\infty}^{+\infty} N |N\rangle\langle N|$$

$$\hat{\cos\varphi_J} = \frac{1}{2}\sum_{N=-\infty}^{+\infty} \left(|N\rangle\langle N+1| + |N+1\rangle\langle N| \right)$$

$$\hat{a}^{\pm} = -\frac{1}{\sqrt{2m_c\omega_c\hbar}} \left(\hat{p}_\varphi \pm i\, m_c\omega_c \hat{\varphi} \right)$$

$$m_c = \frac{C\ell}{2}\left(\frac{\Phi_0}{2\pi}\right)^2$$

$$\omega_c = \frac{2\pi v}{\ell}.$$

Suppose $N \leq N_g \leq N+1$.

Truncate CPB to two-dimensional subspace:

$$|N\rangle \equiv \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$|N+1\rangle \equiv \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$\hat{N} = N|N\rangle\langle N| + (N+1)|N+1\rangle\langle N+1|$$

$$= \left(N+\frac{1}{2}\right) I + \frac{1}{2}\hat{\sigma}_z$$

$$\hat{\cos\varphi_J} = \frac{1}{2}\left(|N\rangle\langle N+1| + |N+1\rangle\langle N| \right) = \frac{1}{2}\hat{\sigma}_x$$

$$\hat{\sigma}_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$\hat{\sigma}_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

Pauli matrices

$$\hat{H} = E_{C_\Sigma}\,\delta N\,\hat{\sigma}_z - E_J\,\hat{\sigma}_x + \hbar\omega_c\,\hat{a}^+\hat{a} + \hbar g(\hat{a} + \hat{a}^+)\hat{\sigma}_z$$

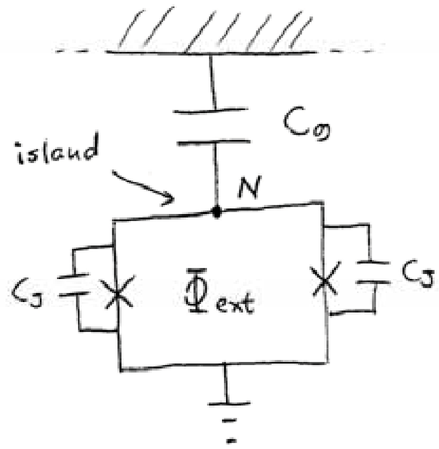
$E_{C_\Sigma} = \dfrac{(2e)^2}{2C_\Sigma}$ is the single Cooper pair charging energy

$E_J = \dfrac{I_c\Phi_0}{2\pi}$ is the Josephson energy

$\delta N = N + \frac{1}{2} - N_g$

$g = \omega_c \dfrac{C_g}{2C_\Sigma}\sqrt{\dfrac{(2e)^2/(Cl)}{\hbar\omega_c}}$ is the CPW mode - CPB qubit coupling in units of frequency

Actual CPB qubit slightly more complicated



Gives (external magnetic flux) tunable Josephson energy

$$H = E_{C_\Sigma} \delta N \sigma_z - E_J \cos(\pi \Phi_{\text{ext}}/\Phi_0)\sigma_x + \hbar\omega_c a^+ a + \hbar g(a + a^+)\sigma_z$$

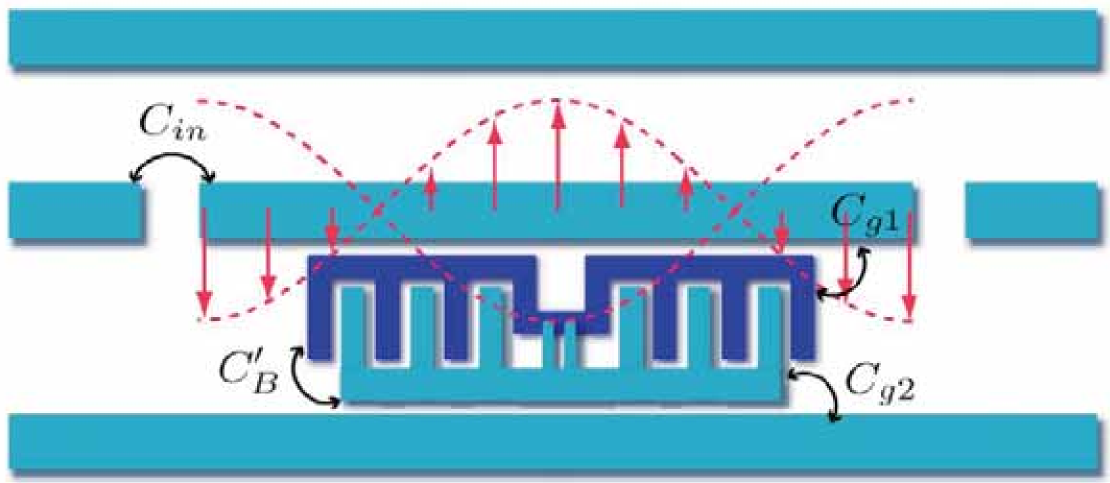
•Tuning the qubit into resonance with the cavity mode (by varying Φ_{ext}) and applying microwave pulse allows qubit state preparation.
•Tuning the qubit away from resonance with the cavity mode (dispersive limit) allows a measurement of the qubit state (shifts the cavity mode frequency).

For the details, see the original theory and experiment papers:
[A. Blais *et al.*, *Phys. Rev.* A **69**, 062320 (2004); A. Wallraff *et al.*, *Nature* **431**, 162 (2004)]

cQED milestones for quantum information processing

- The "Transmon" [J. Koch *et al*., *Phys. Rev.* A **76**, 042319 (2007)]
- Coupling superconducting qubits [J. Majer *et al*., *Nature* **449**, 443 (2007); M. Sillanpaa *et al*., *Nature* **449**, 438 (2007)]
- Synthesizing arbitrary quantum states in a superconducting resonator [M. Hofheinz *et al*., *Nature* **459**, 546 (2009)]
- Preparation and measurement of three-qubit entanglement [L. DiCarlo *et al*., *Nature* **467**, 574 (2010)]

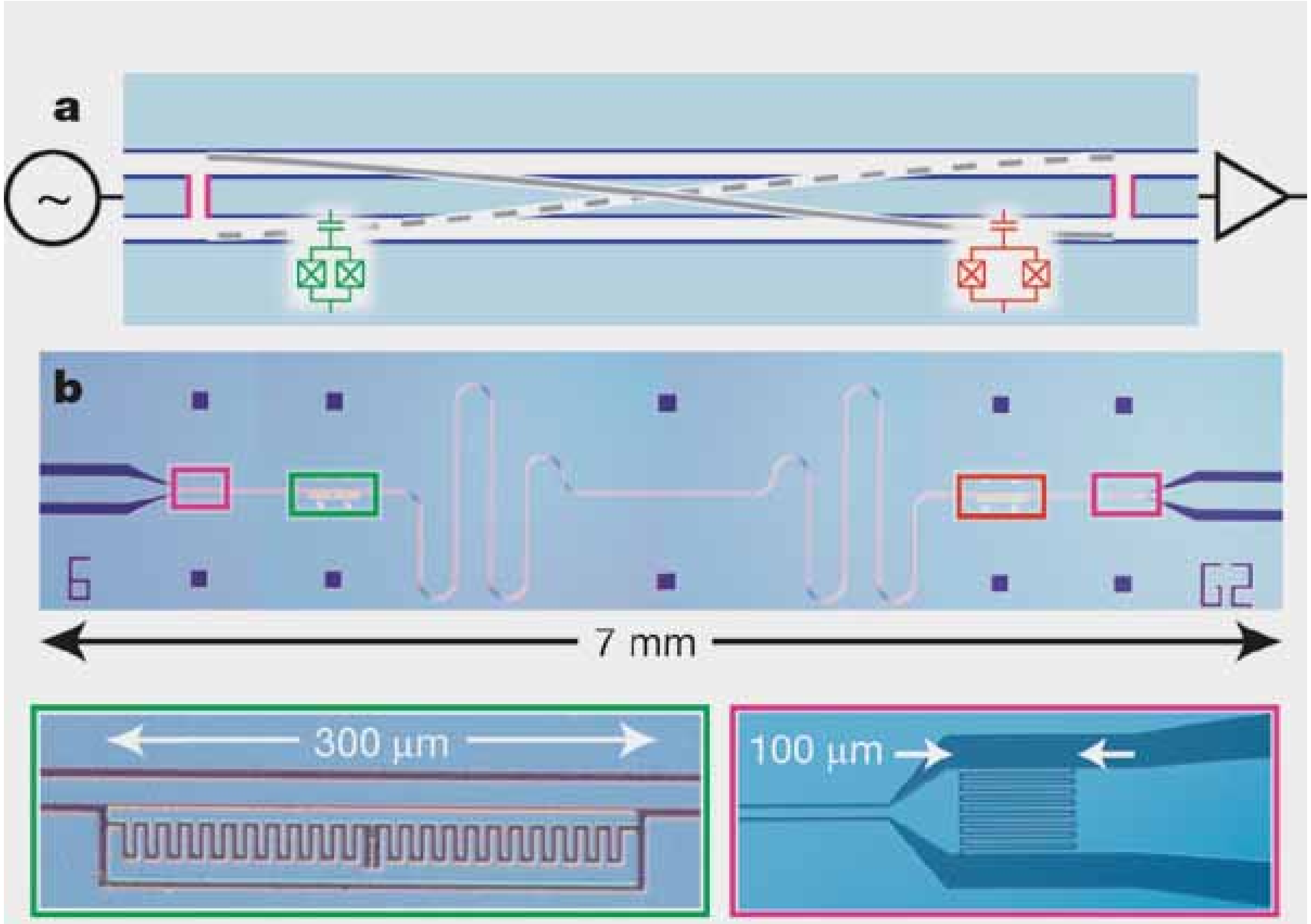
The "transmon" [J. Koch *et al*., *Phys. Rev.* A **76**, 042319 (2007); J. Schreier *et al*., *Phys. Rev.* B **77**, 180502 (2008)]



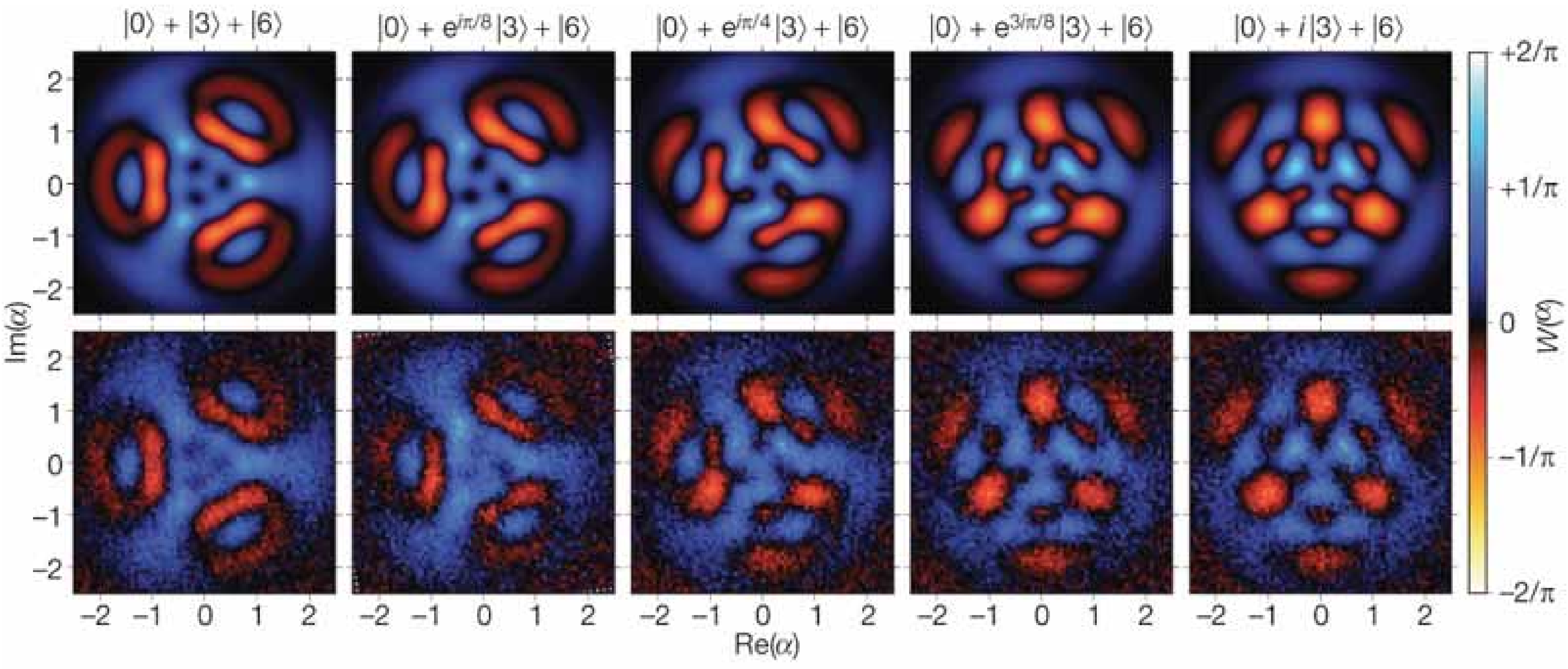
Increasing C_g reduces the CPB charging energy and hence dephasing due to 1/*f* charge fluctuations. Also, increases *g*, the CPB-CPW coupling.

$$E_{C_\Sigma} = \frac{(2e)^2}{2C_\Sigma} \qquad g = \omega_c \frac{C_g}{2C_\Sigma}\sqrt{\frac{(2e)^2/(C\ell)}{\hbar\omega_c}}$$

Coupling qubits [J. Majer *et al*., *Nature* **449**, 443 (2007); M. Sillanpää *et al*., *Nature* **449**, 438 (2007)]



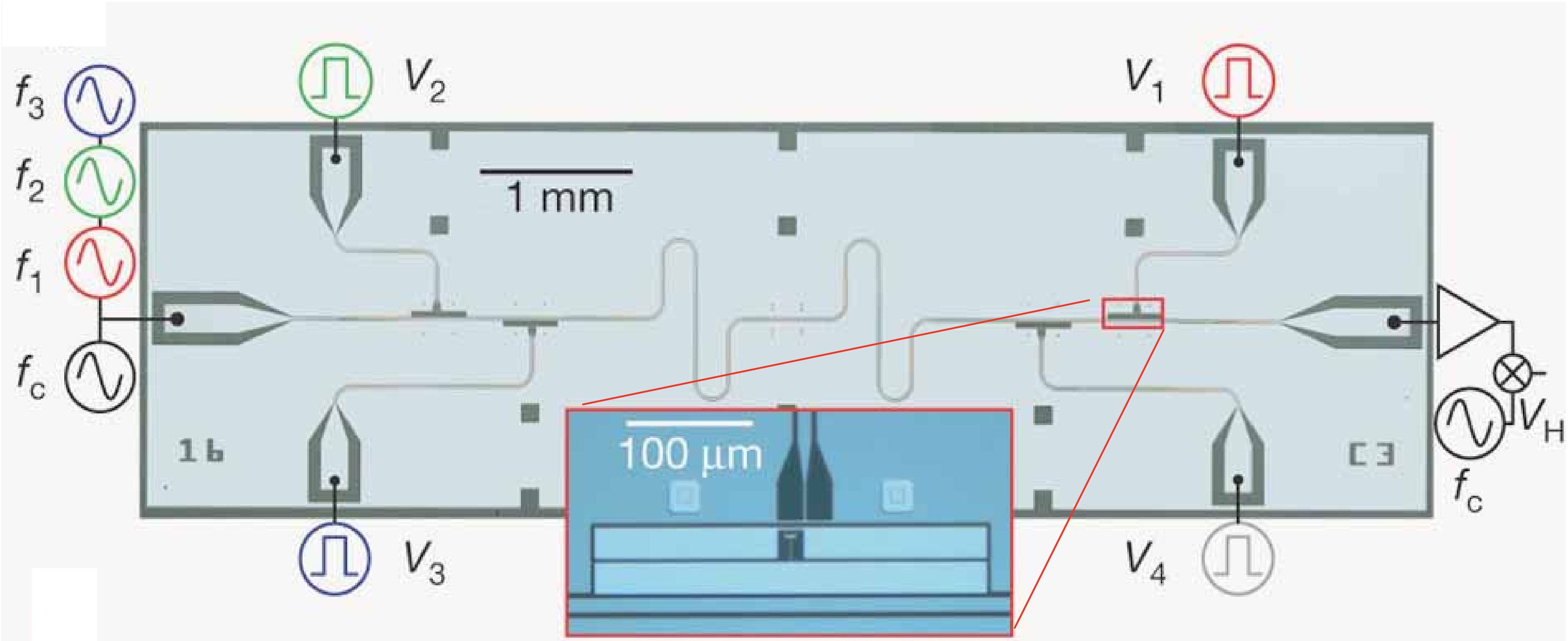
Synthesizing arbitrary microwave resonator quantum states
[M. Hofheinz *et al*., *Nature* **459**, 546 (2009)]



Wigner representation of states: theory (top) versus experiment (bottom)

Superconducting qubit used to both prepare and measure microwave resonator states

Preparation and measurement of three-qubit entanglement [L. DiCarlo *et al*., *Nature* **467**, 574 (2010)]



Transmon qubit