Wavefunction uncollapse and related topics

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Outline:
- Uncollapse (measurement reversal): theory
- Experiments on partial collapse and uncollapse
- Decoherence (T1) suppression by uncollapse
- Some related topics

Acknowledgements

Theory: A. Jordan, K. Keane

Experiment: N. Katz, J. Martinis, et al.

Undoing a weak measurement of a qubit ("uncollapse")

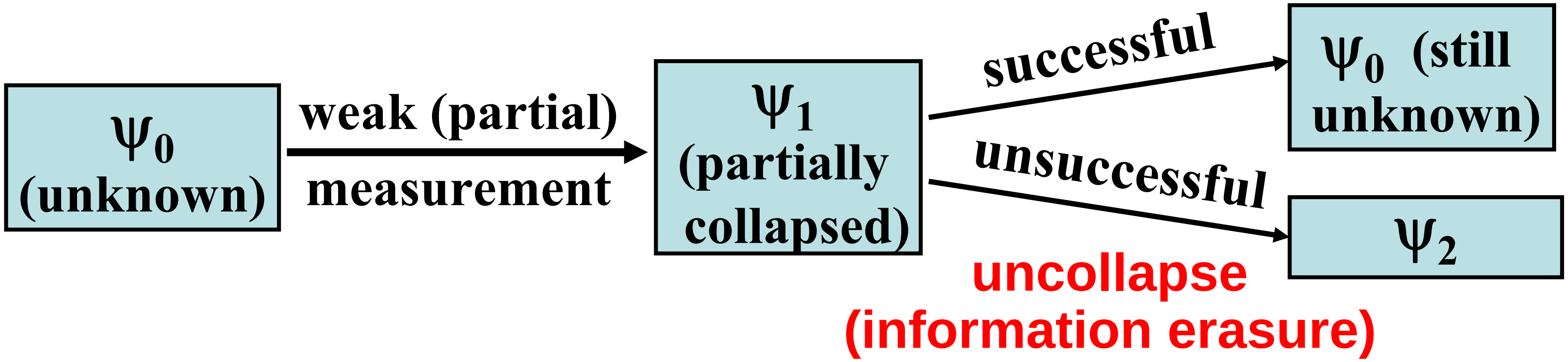
A.K. & Jordan, PRL-2006

It is impossible to undo "orthodox" quantum measurement (for an unknown initial state)

Is it possible to undo partial quantum measurement?
(To restore a "precious" qubit accidentally measured)

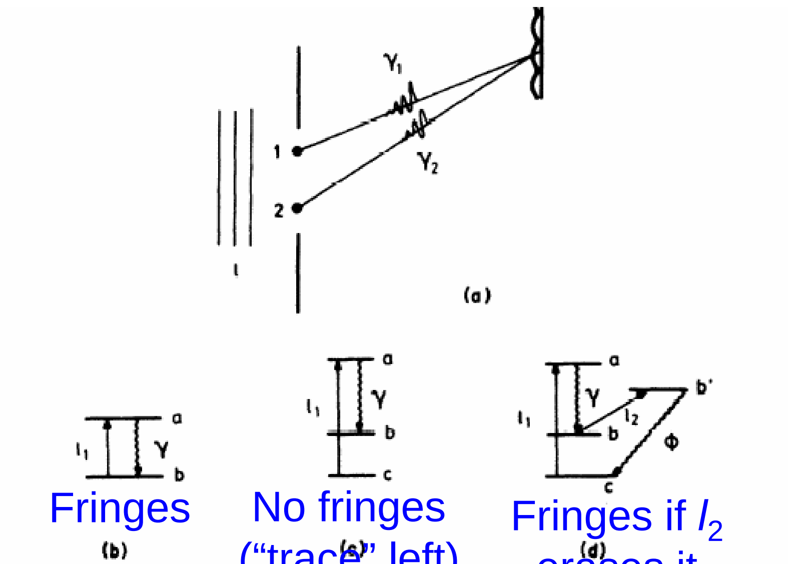
Yes! (but with a finite probability)

If undoing is successful, an unknown state is **fully** restored



Quantum erasers in optics

Quantum eraser proposal by Scully and Drühl, PRA (1982)

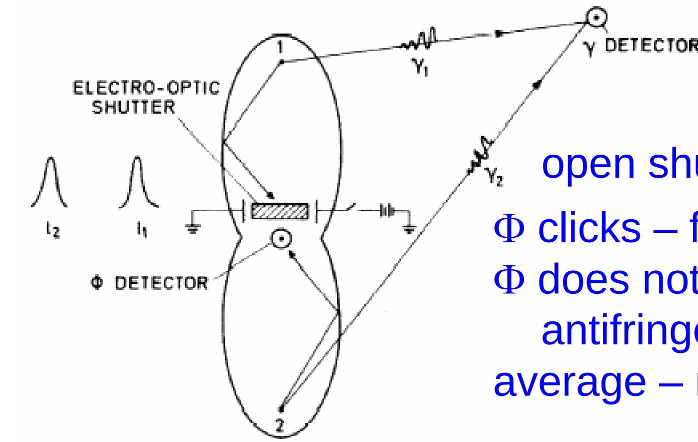


Fringes

No fringes (“trace” left)

Fringes if l_2 erases it

FIG. 1. (a) Figure depicting light impinging from left on atoms at sites 1 and 2. Scattered photons γ_1 and γ_2 produce interference pattern on screen. (b) Two-level atoms excited by laser pulse l_1, and emit γ photons in $a \to b$ transition. (c) Three-level atoms excited by pulse l_1 from $c \to a$ and emit photons in $a \to b$ transition. (d) Four-level system excited by pulse l_1 from $c \to a$ followed by emission of γ photons in $a \to b$ transition. Second pulse l_2 takes atoms from $b \to b'$. Decay from $b' \to c$ results in emission of ϕ photons.



open shutter:

Φ clicks – fringes,

Φ does not click – antifringes,

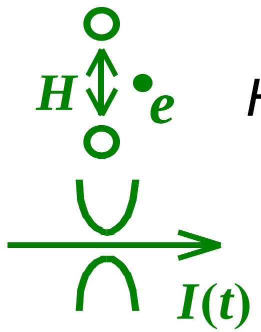
average – no fringes

FIG. 2. Laser pulses l_1 and l_2 incident on atoms at sites 1 and 2. Scattered photons γ_1 and γ_2 result from $a \to b$ transition. Decay of atoms from $b' \to c$ results in ϕ photon emission. Elliptical cavities reflect ϕ photons onto common photodetector. Electro-optic shutter transmits ϕ photons only when switch is open. Choice of switch position determines whether we emphasize particle or wave nature of γ photons.

Interference fringes restored for two-detector correlations (since “which-path” information is erased)

Our idea of uncollapsing is quite different:
we really extract quantum information and then erase it

Evolution of a charge qubit



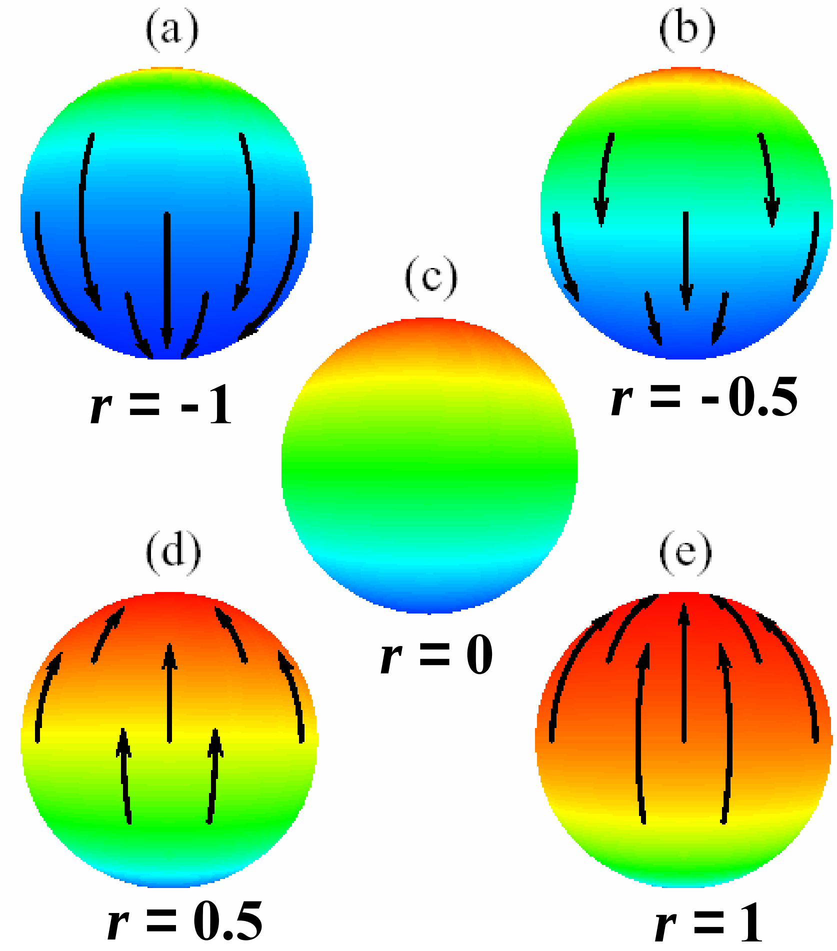
H=0

$$\frac{\rho_{11}(t)}{\rho_{22}(t)} = \frac{\rho_{11}(0)}{\rho_{22}(0)} \exp[2r(t)]$$

$$\frac{\rho_{12}(t)}{\sqrt{\rho_{11}(t)\rho_{22}(t)}} = \text{const}$$

where measurement result $r(t)$ is

$$r(t) = \frac{\Delta I}{S_I}\left[\int_0^t I(t')\,dt' - I_0 t\right]$$



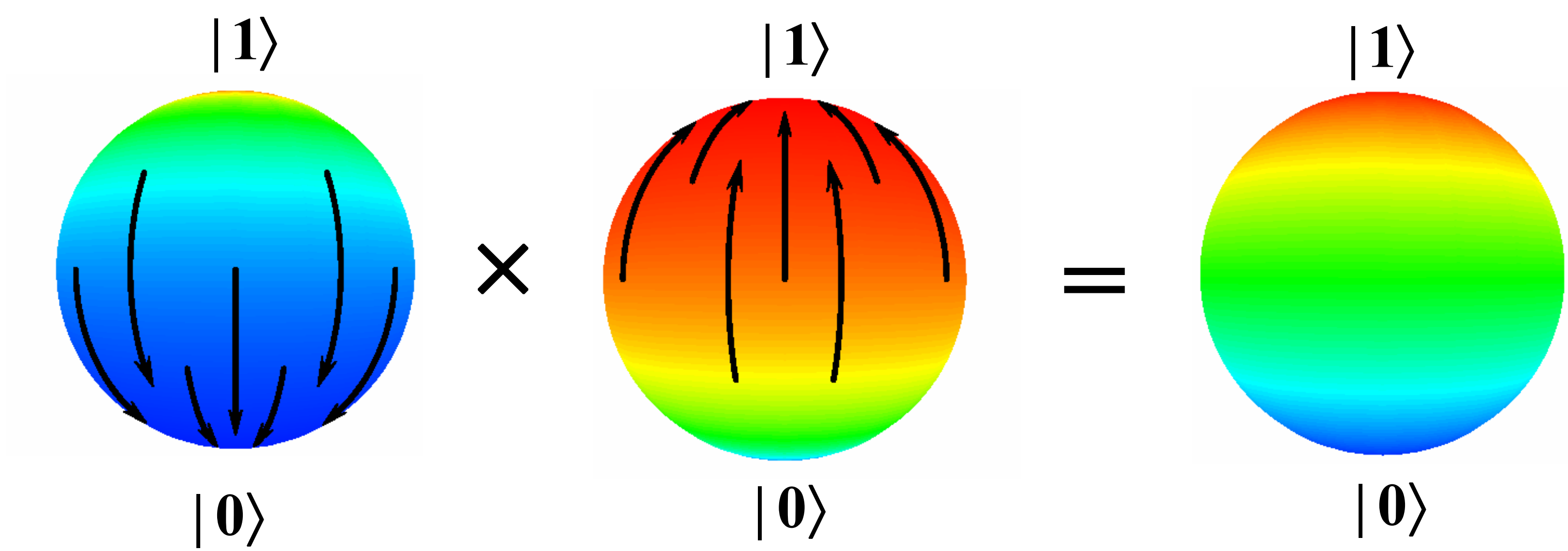
Jordan-A.K.-Büttiker, PRL-06

If $r = 0$, then no information and no evolution!

Uncollapse of a qubit state

Evolution due to partial (weak, continuous, etc.) measurement is **non-unitary**, so impossible to undo it by Hamiltonian dynamics.

How to undo? One more measurement!



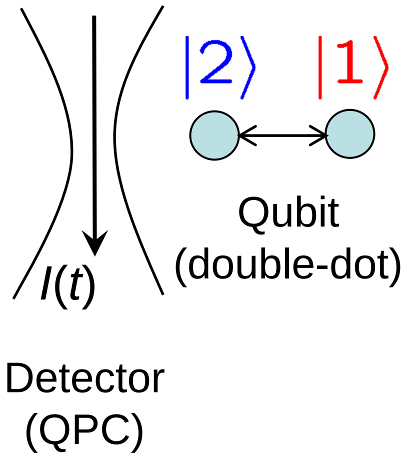
need ideal (quantum-limited) detector

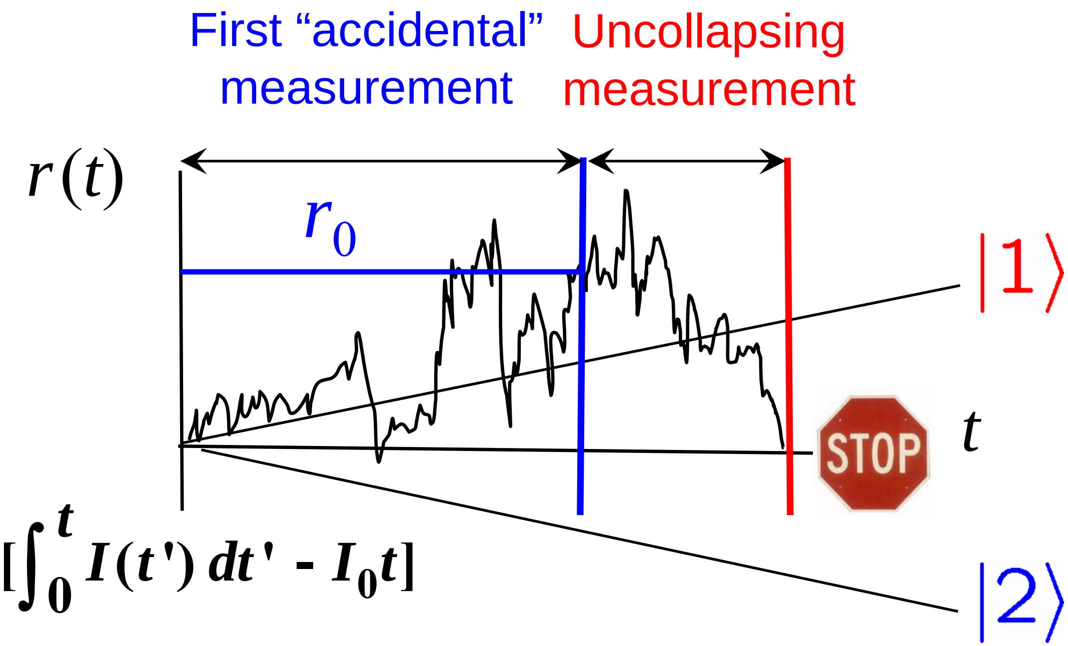
(similar to Koashi-Ueda, PRL-1999)

(Figure partially adopted from Jordan-A.K.-Büttiker, PRL-06)

Uncollapsing for qubit-QPC system

A.K. & Jordan, 2006





$$r(t) = \frac{\Delta I}{S_I}\left[\int_0^t I(t')\,dt' - I_0 t\right]$$

Simple strategy: continue measuring until *r*(*t*) becomes zero!

Then any unknown initial state is fully restored.

(same for an entangled qubit)

It may happen though that $r = 0$ never happens;
then undoing procedure is unsuccessful.

Probability of success

Trick: since non-diagonal matrix elements are not directly involved, we can analyze classical probabilities (as if qubit is in some certain, but unknown state); then simple diffusion with drift

Results:

Probability of successful uncollapsing

$$P_S = \frac{e^{-|r_0|}}{e^{|r_0|}\rho_{11}(0) + e^{-|r_0|}\rho_{22}(0)}$$

where r_0 is the result of the measurement to be undone, and $\rho(0)$ is initial state (traced over entangled qubits)

Larger $|r_0|$ ⇒ more information ⇒ less likely to uncollapse

Averaged probability of success (over result r_0)

$$P_{av} = 1 - \mathrm{erf}[\sqrt{t/2T_m}\,]$$

(does not depend on initial state; cannot!)

where $T_m = 2S_I/(\Delta I)^2$ (“measurement time”)

General theory of uncollapsing

POVM formalism
(Nielsen-Chuang, p.100)

Measurement operator M_r: $\rho \to \dfrac{M_r \rho M_r^\dagger}{\mathrm{Tr}(M_r \rho M_r^\dagger)}$

Probability: $P_r = \mathrm{Tr}(M_r \rho M_r^\dagger)$ Completeness: $\sum_r M_r^\dagger M_r = 1$

Uncollapsing operator: $C \times M_r^{-1}$ (to satisfy completeness, eigenvalues cannot be >1)

$$\max(C) = \min_i \sqrt{p_i}\,, \quad p_i \text{ – eigenvalues of } M_r^\dagger M_r$$

Probability of success: $$P_S \le \frac{\min_i p_i}{P_r(\rho_{\text{in}})} = \frac{\min P_r}{P_r(\rho_{\text{in}})}$$ A.K. & Jordan, 2006

$P_r(\rho_{in})$ – probability of result r for initial state ρ_{in},

min P_r – probability of result r minimized over all possible initial states

(similar to Koashi-Ueda, 1999)

General theory of uncollapsing (cont.)

Overall probability: result *r* and successful uncollapsing

$$\tilde{P}_S = P_r[\rho_{in}] \times P_S$$

It cannot depend on initial state

(otherwise we learn something after uncollapsing)

Exact upper bound: $\tilde{P}_S \leq \min P_r$

(probability of result *r* minimized over initial states)

Averaged (over *r*) overall probability of uncollapsing:

$$P_{S,av} \leq \sum_r \min P_r$$

(independent of initial state as well)

Characterization of (irrecoverable) collapse strength:

$$1 - P_{S,av} = 1 - \sum_r \min P_r$$

Comparison of the general bound for DQD-QPC uncollapsing success

General bound: $$P_S \le \frac{\min P_r}{P_r[\rho(0)]}$$

⇒ **for DQD+QPC** $$P_S \le \frac{\min(p_1, p_2)}{p_1\rho_{11}(0) + p_2\rho_{22}(0)}$$

where $$p_i = (\pi S_I / t)^{-1/2} \exp[-(\bar{I} - I_i)^2 t / S_I]\, d\bar{I}$$

Actual result: $$P_S = \frac{e^{-|r_0|}}{e^{|r_0|}\rho_{11}(0) + e^{-|r_0|}\rho_{22}(0)} \qquad r_0 = \frac{\Delta I}{S_I}\left[\int_0^t I(t')\,dt' - I_0 t\right]$$

The two results coincide, so the upper bound is reached,

therefore uncollapsing strategy is optimal

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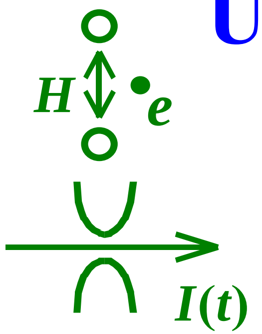


Uncollapsing of evolving charge qubit

$$\hat{H}_{QB} = (\varepsilon/2)(c_1^\dagger c_1 - c_2^\dagger c_2) + H(c_1^\dagger c_2 + c_2^\dagger c_1)$$

(now non-zero *H* and ε, qubit evolves during measurement)

1) Bayesian equations to calculate measurement operator
2) unitary operation, measurement by QPC, unitary operation

More general: uncollapsing for *N* entangled charge qubits

1) unitary transformation of *N* qubits
2) null-result measurement of a certain strength by a strongly nonlinear QPC (tunneling only for state |11..1⟩)
3) repeat 2^N times, sequentially transforming the basis vectors of the diagonalized measurement operator into |11..1⟩

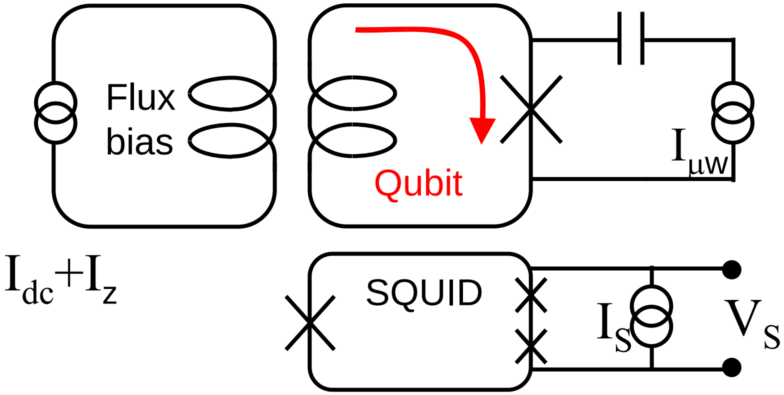
(also reaches the upper bound for success probability)

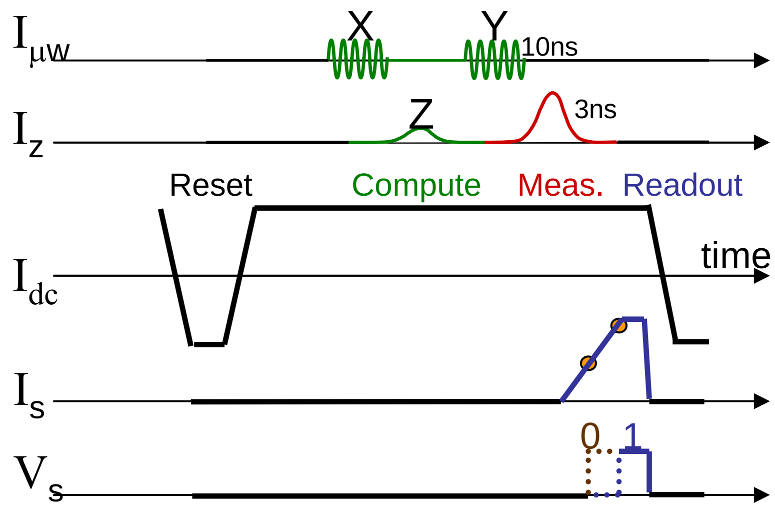
Jordan & A.K., Contemp. Phys., 2010

No experiment yet for DQD-QPC system,
but uncollapsing has been demonstrated
for a superconducting phase qubit

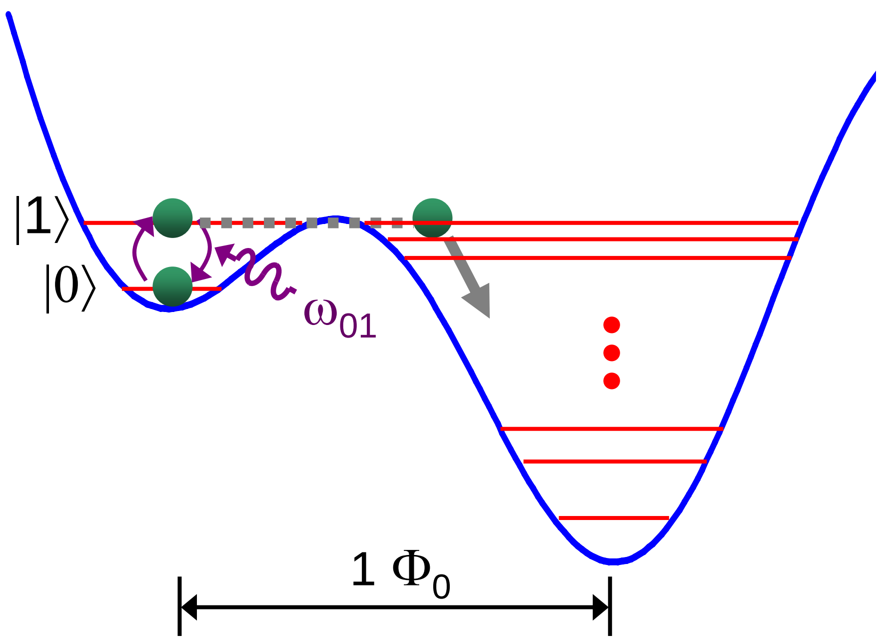
Superconducting phase qubit at UCSB

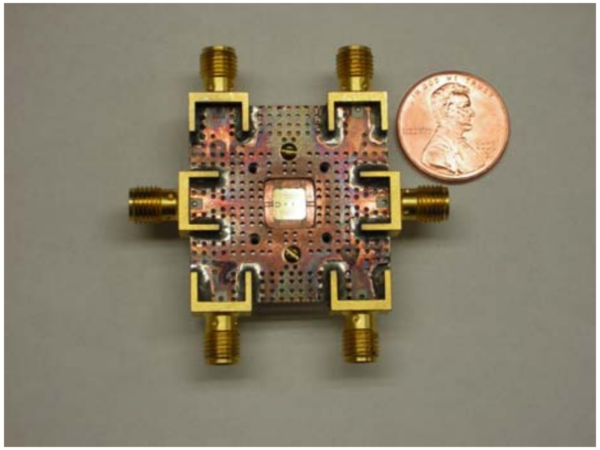
Courtesy of Nadav Katz (UCSB, now at Hebrew University)



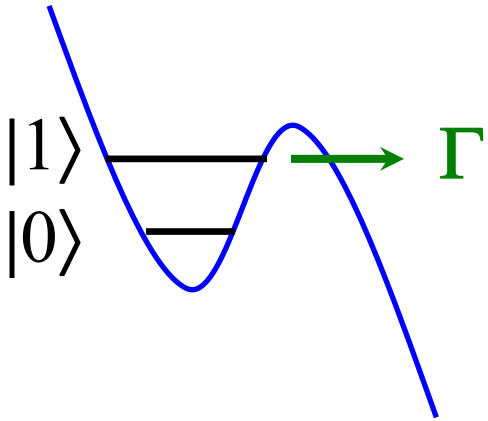


Repeat 1000x
prob. 0,1





Partial collapse of a Josephson phase qubit



N. Katz, M. Ansmann, R. Bialczak, E. Lucero, R. McDermott, M. Neeley, M. Steffen, E. Weig, A. Cleland, J. Martinis, A. Korotkov, Science-06

How does a qubit state evolve in time before tunneling event?

(What happens when nothing happens?)

Qubit "ages", in contrast to a radioactive atom

Main idea:

$$\psi = \alpha\,|0\rangle + \beta\,|1\rangle \;\rightarrow\; \psi(t) = \begin{cases} |out\rangle, \text{ if tunneled} \\ \dfrac{\alpha\,|0\rangle + \beta\, e^{-\Gamma t/2} e^{i\varphi}\,|1\rangle}{\sqrt{|\alpha|^2 + |\beta|^2\, e^{-\Gamma t}}}, \text{ if not tunneled} \end{cases}$$

(better theory: Pryadko & A.K., 2007)

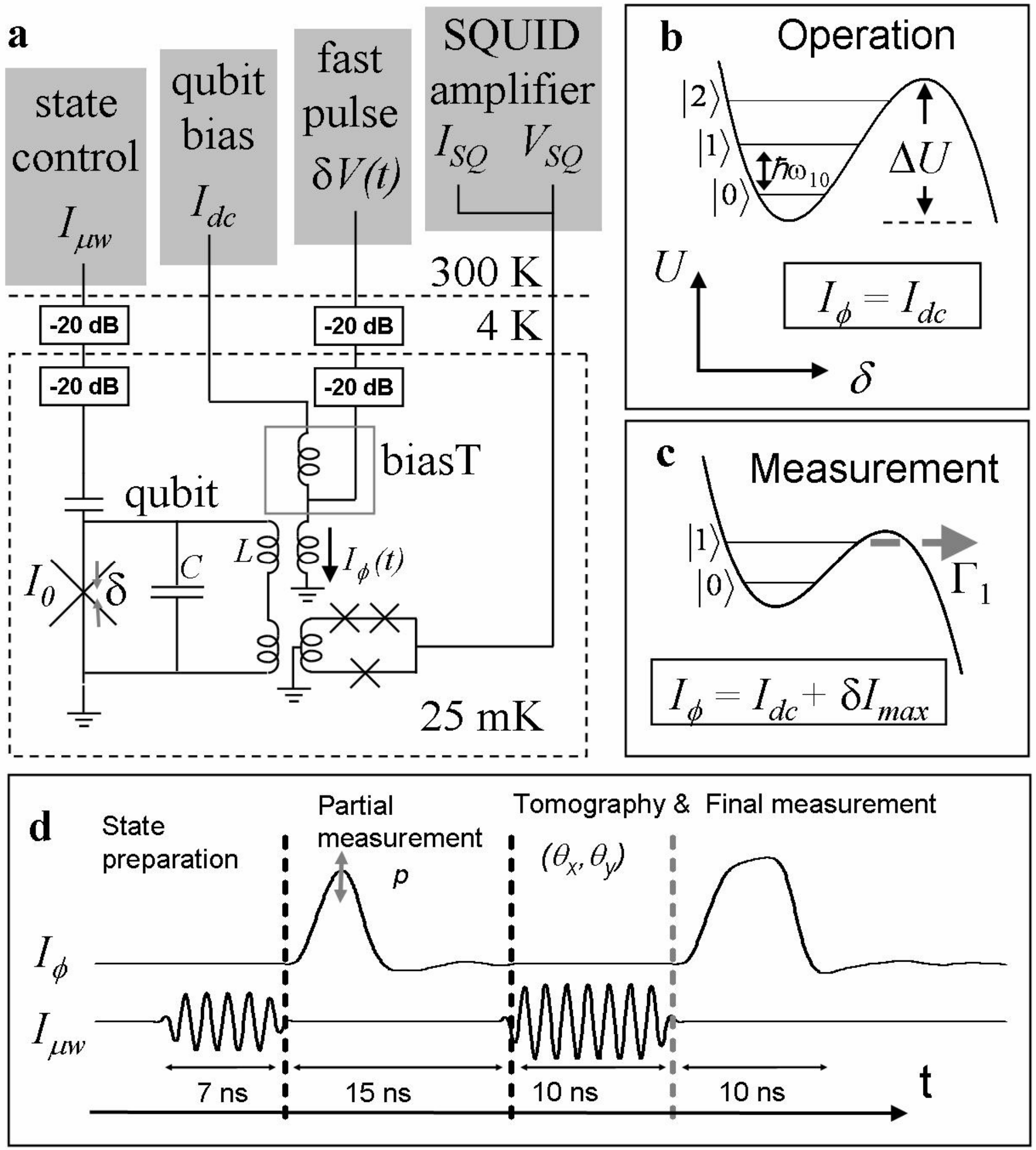
amplitude of state |0⟩ grows without physical interaction

finite linewidth only after tunneling

continuous null-result collapse

(similar to optics, Dalibard-Castin-Molmer, PRL-1992)

Experimental technique for partial collapse



Nadav Katz *et al.*
(John Martinis group)

Protocol:

1) State preparation (via Rabi oscillations)

2) Partial measurement by lowering barrier for time *t*

3) State tomography (micro-wave + full measurement) trick: subtract probability

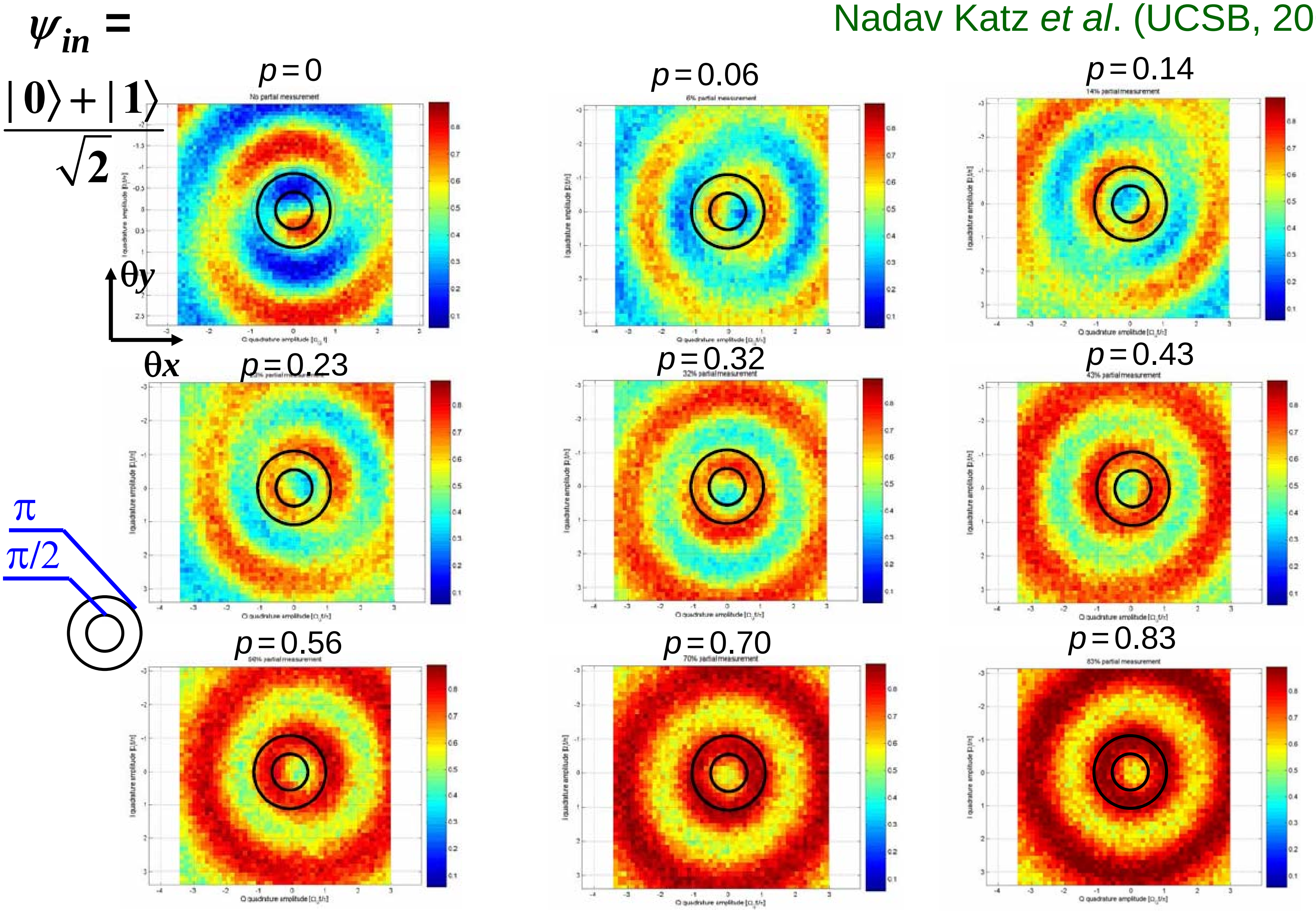
Measurement strength
$p = 1 - \exp(-\Gamma t)$
is actually controlled by Γ, not by t

p=0: no measurement

p=1: orthodox collapse

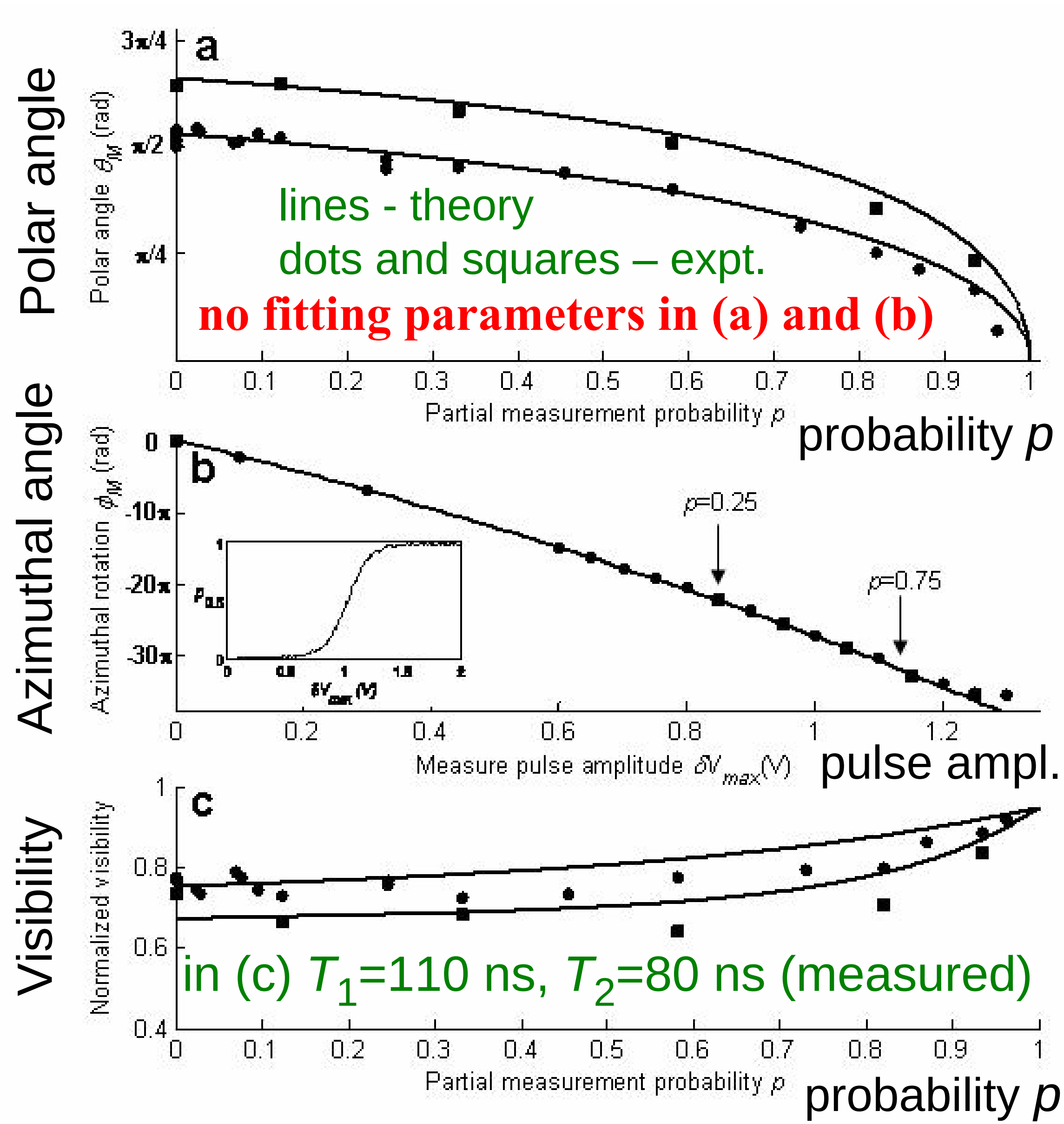
Experimental tomography data

Nadav Katz *et al*. (UCSB, 2005)



Partial collapse: experimental results

N. Katz *et al.*, Science-06



- In case of no tunneling phase qubit evolves
- Evolution is described by the Bayesian theory without fitting parameters
- Phase qubit remains coherent in the process of continuous collapse (expt. ~80% raw data, ~96% corrected for T_1,T_2)

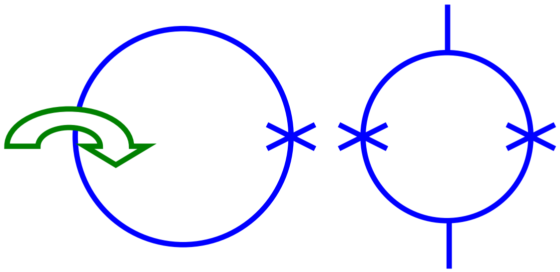
quantum efficiency

$\eta_0 > 0.8$

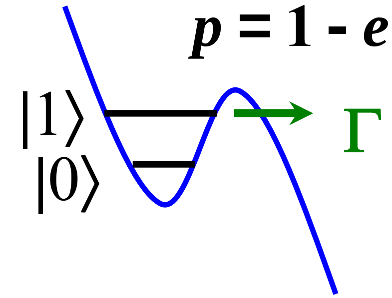
Uncollapse of a phase qubit state

A.K. & Jordan, 2006

1) Start with an unknown state
2) Partial measurement of strength p
3) π-pulse (exchange $|0\rangle \leftrightarrow |1\rangle$)
4) One more measurement with the **same strength** p
5) π-pulse



$p = 1 - e^{-\Gamma t}$



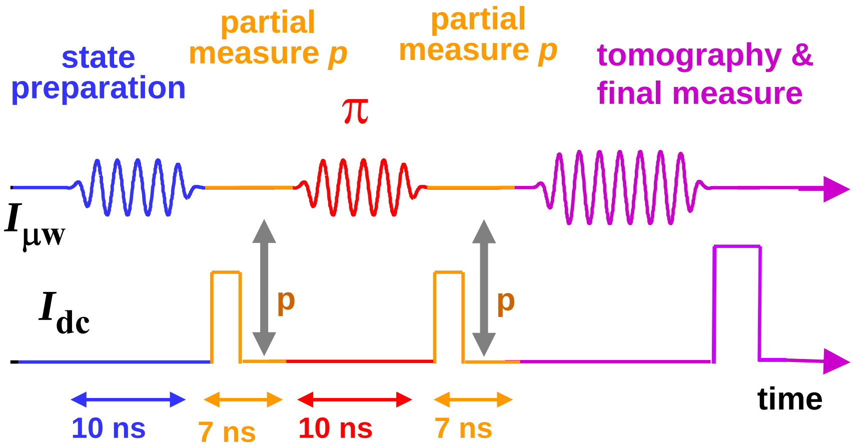
If no tunneling for both measurements, then initial state is fully restored!

$$\alpha|0\rangle + \beta|1\rangle \rightarrow \frac{\alpha|0\rangle + e^{i\phi}\beta e^{-\Gamma t/2}|1\rangle}{\text{Norm}} \rightarrow$$

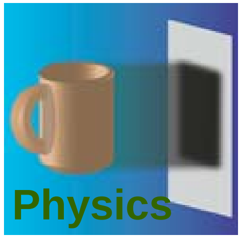
$$\frac{e^{i\phi}\alpha e^{-\Gamma t/2}|0\rangle + e^{i\phi}\beta e^{-\Gamma t/2}|1\rangle}{\text{Norm}} = e^{i\phi}(\alpha|0\rangle + \beta|1\rangle)$$

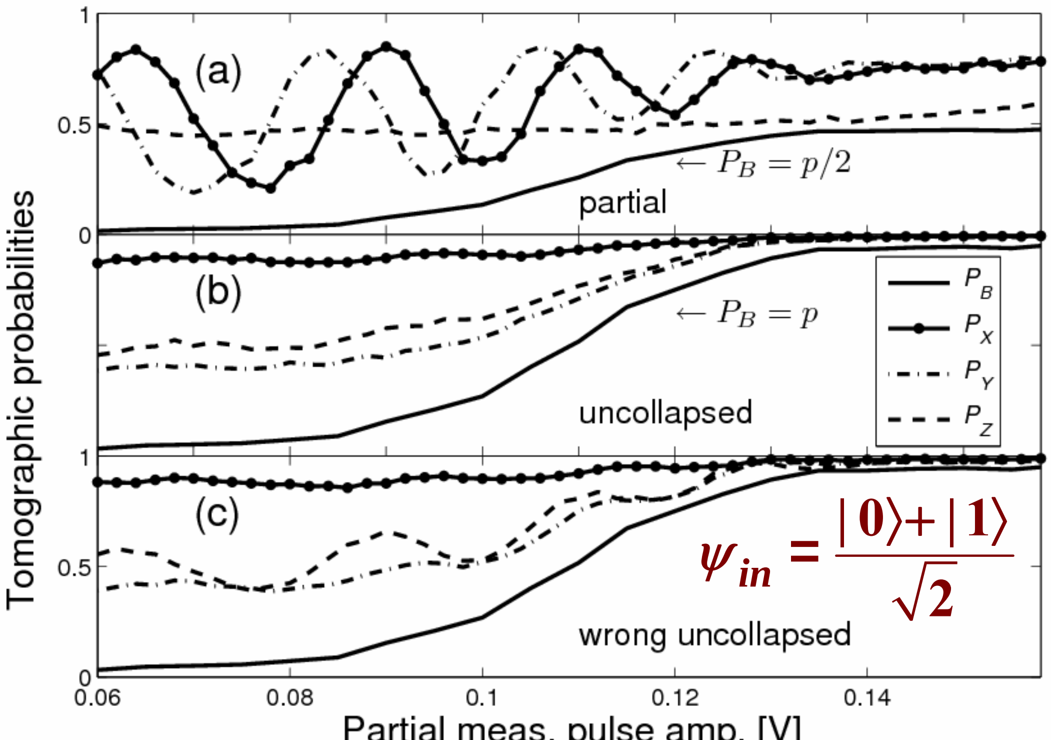
phase is also restored (spin echo)

Experiment on wavefunction uncollapse



N. Katz, M. Neeley, M. Ansmann, R. Bialzak, E. Lucero, A. O'Connell, H. Wang, A. Cleland, J. Martinis, and A. Korotkov, PRL-2008





Uncollapse protocol:

- partial collapse
- π-pulse
- partial collapse
  (same strength)

State tomography with *X*, *Y*, and no pulses

Background P_B should be subtracted to find qubit density matrix

Experimental results on the Bloch sphere

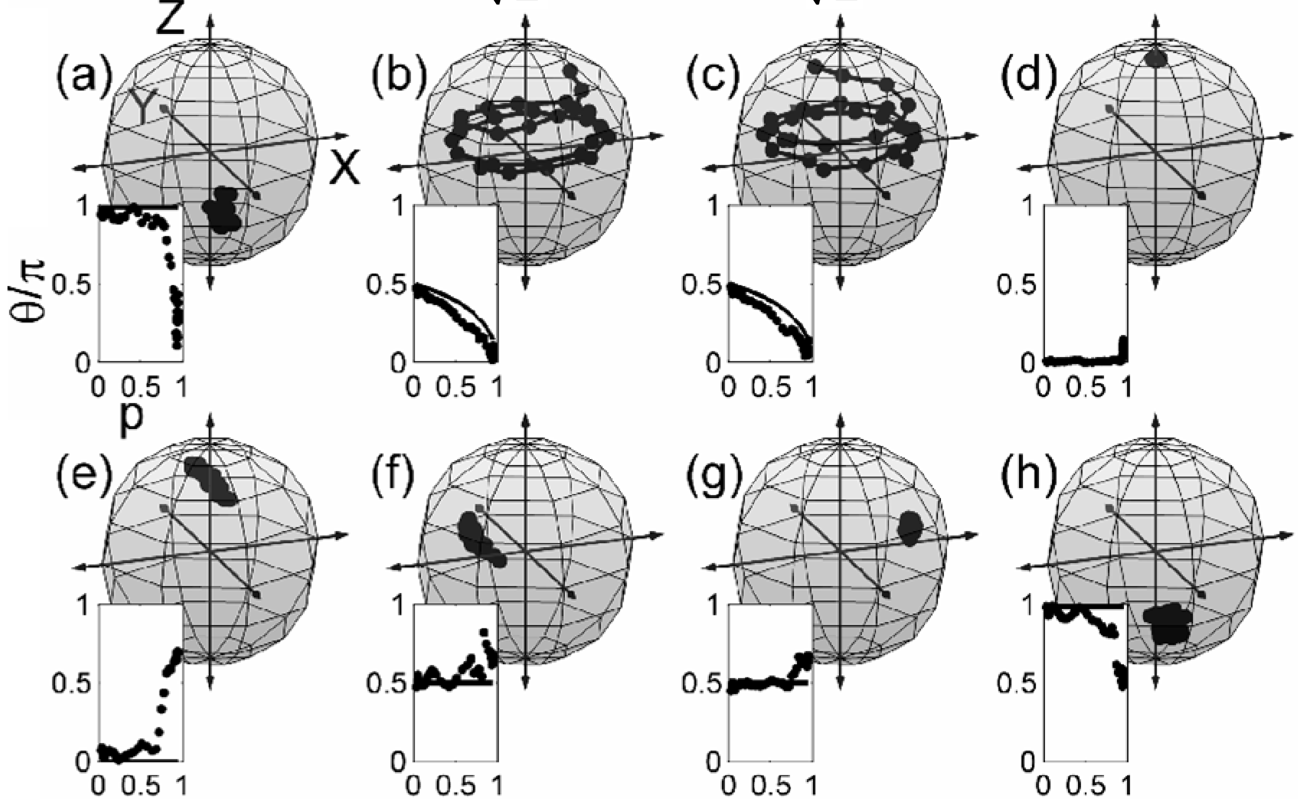
N. Katz et al.

Initial state: $|1\rangle$, $\frac{|0\rangle - i|1\rangle}{\sqrt{2}}$, $\frac{|0\rangle + |1\rangle}{\sqrt{2}}$, $|0\rangle$

Partially collapsed

Uncollapsed

uncollapsing works well!

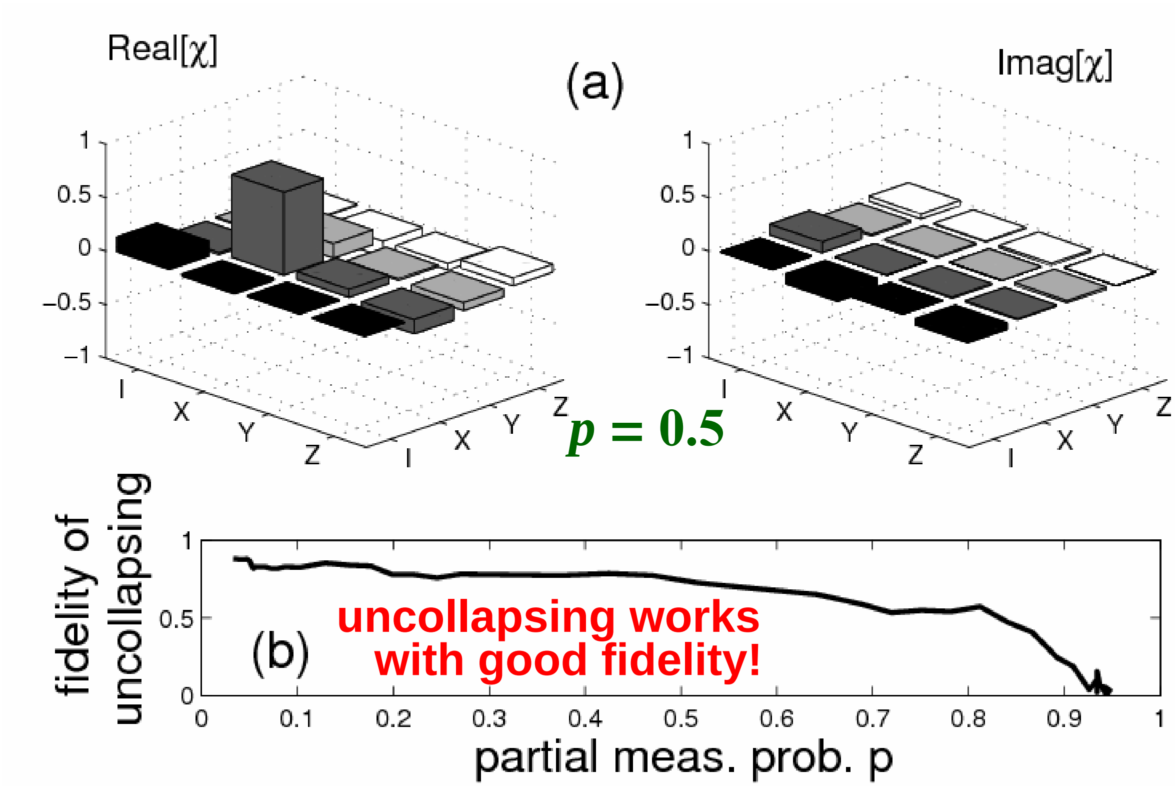


Both spin echo (azimuth) and uncollapsing (polar angle)

Difference: spin echo – undoing of an <u>unknown unitary</u> evolution, uncollapsing – undoing of a <u>known, but non-unitary</u> evolution

Quantum process tomography

N. Katz et al.
(Martinis group)



Why getting worse at p>0.6?

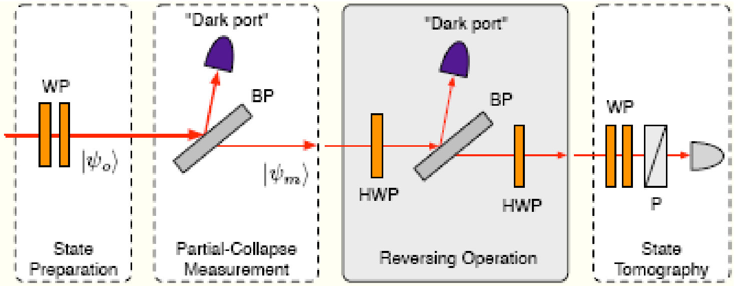
Energy relaxation $\mathbf{p_r = t/T_1 = 45ns/450ns = 0.1}$

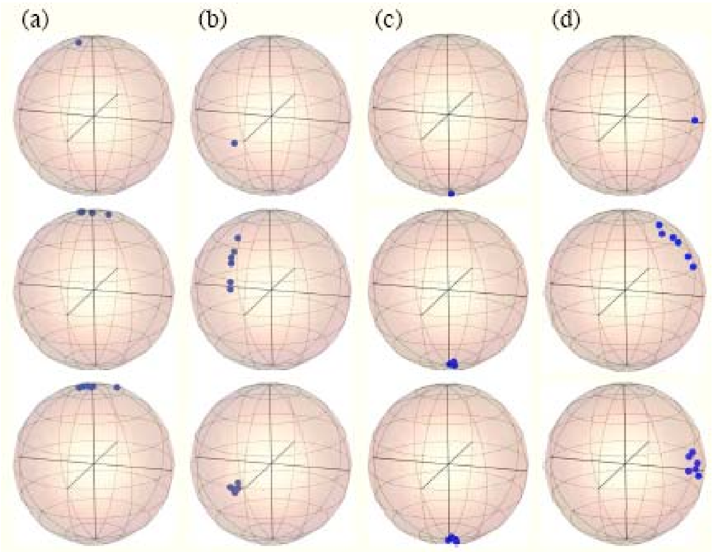
Selection affected when $\mathbf{1-p \sim p_r}$

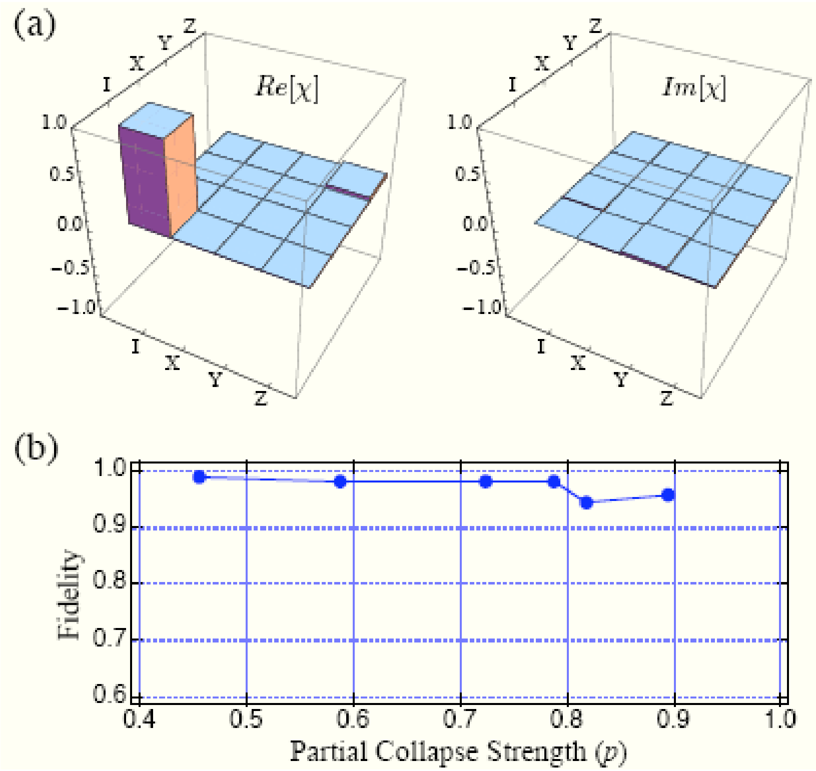
Overall: uncollapsing is well-confirmed experimentally

Experiment on uncollapsing using single photons

Kim et al., Opt. Expr.-2009



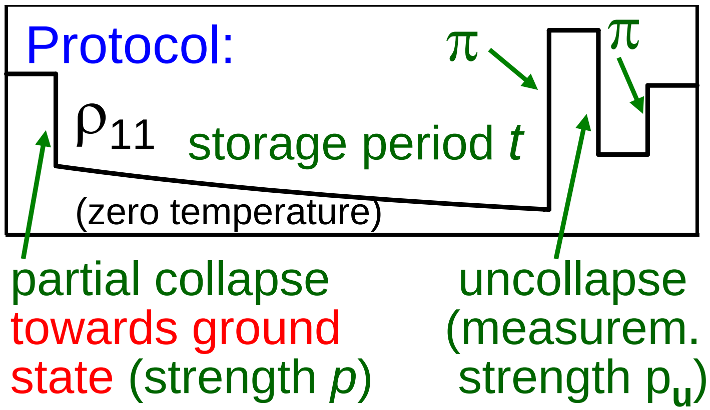




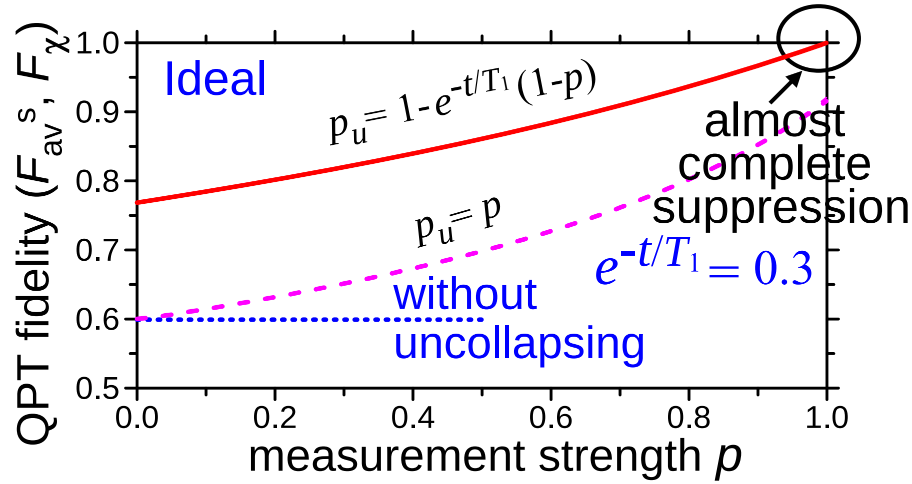
- very good fidelity of uncollapsing (>94%)
- measurement fidelity is probably not good (normalization by coincidence counts)

Suppression of T_1-decoherence by uncollapsing

A.K. & Keane, PRA-2010



(almost same as existing experiment!)



Ideal case (T_1 during storage only, T=0)

for initial state $|\psi_{in}\rangle = \alpha |0\rangle + \beta |1\rangle$

$|\psi_f\rangle = |\psi_{in}\rangle$ with probability $(1-p)\, e^{-t/T_1}$

$|\psi_f\rangle = |0\rangle$ with $(1-p)^2 |\beta|^2 e^{-t/T_1}(1-e^{-t/T_1})$

procedure preferentially selects events without energy decay

Trade-off: fidelity vs. selection probability

Unraveling of energy relaxation

$$\begin{pmatrix} |\beta|^2 e^{-t/T_1} & \alpha\beta^* e^{-t/2T_1} \\ \alpha^*\beta e^{-t/2T_1} & 1-|\beta|^2 e^{-t/T_1} \end{pmatrix} =$$

$$= p_t |0\rangle\langle 0| + (1-p_t)|\tilde{\psi}\rangle\langle\tilde{\psi}|$$

where $p_t = |\beta|^2 (1-e^{-t/T_1})$

$$|\tilde{\psi}\rangle = (\alpha|0\rangle + \beta e^{-t/2T_1}|1\rangle)/Norm$$

⇒ optimum: $1 - p_u = e^{-t/T_1}(1-p)$

An issue with quantum process tomography (QPT)

QPT fidelity is usually $F_\chi = \mathrm{Tr}(\chi_{desired}\chi)$
where χ is the QPT matrix.

However, QPT is developed for a linear quantum process, while uncollapsing (after renormalization) is non-linear.

A better way: average state fidelity

$$F_{av} = \mathrm{Tr}(\rho_f U_0 |\psi_{in}\rangle\langle\psi_{in}|)\, d|\psi_{in}\rangle$$

Without selection

$$F_\chi = F_{av}^s = \frac{(d+1)F_{av} - 1}{d}, \; d = 2$$

Another way: "naïve" QPT fidelity
(via 4 standard initial states)

The two ways practically coincide
(within line thickness)

Analytics for the ideal case

Average state fidelity

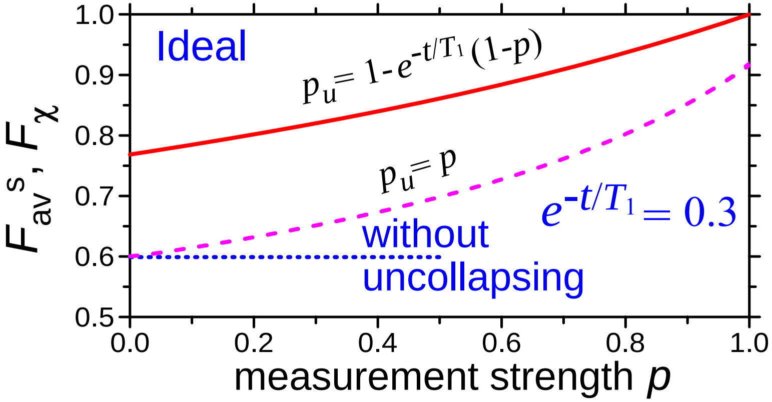
$$F_{av} = \frac{1}{2} + \frac{1}{C} + \frac{\ln(1+C)}{C^2}$$

"Naïve" QPT fidelity

$$F_\chi = -\frac{1}{4} + \frac{1}{4(1+C)} + \frac{4+C}{2(2+C)}$$

where $C = (1-p)(1-e^{-\Gamma t})$

$$p_u = 1 - e^{-\Gamma t}(1-p)$$



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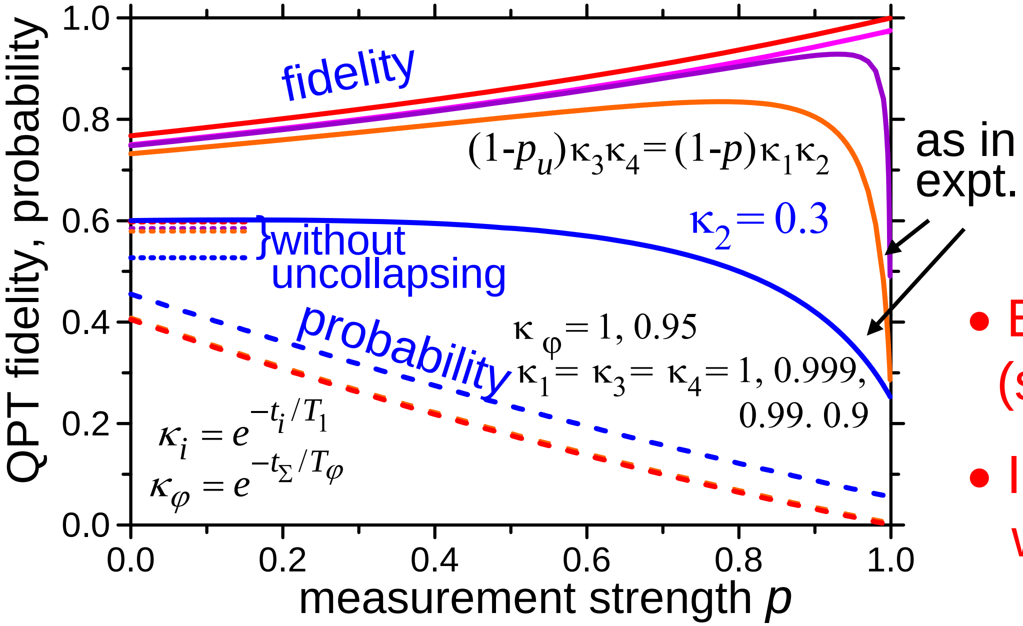
Realistic case (T_1 and T_φ at all stages)



- Easy to realize experimentally (similar to existing experiment)
- Improved fidelity can be observed with just one partial measurement

- Tφ-decoherence is not affected
- fidelity decreases at $p\to1$ due to T_1 between 1st π-pulse and 2nd meas.

Uncollapse seems **the only way** to protect against T_1-decoherence without quantum error correction

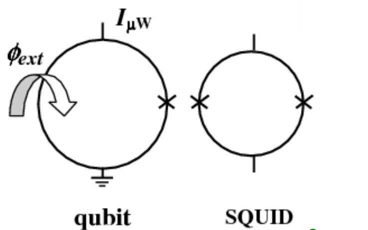
A.K. & Keane, 2010

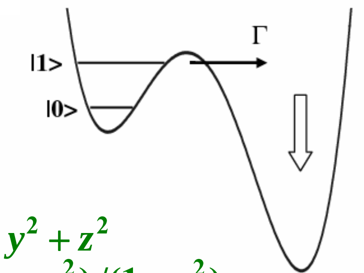
Trade-off: fidelity vs. selection probability

Some other related effects, proposals, and theories

Crossover of phase qubit dynamics in presence of weak collapse and μwaves

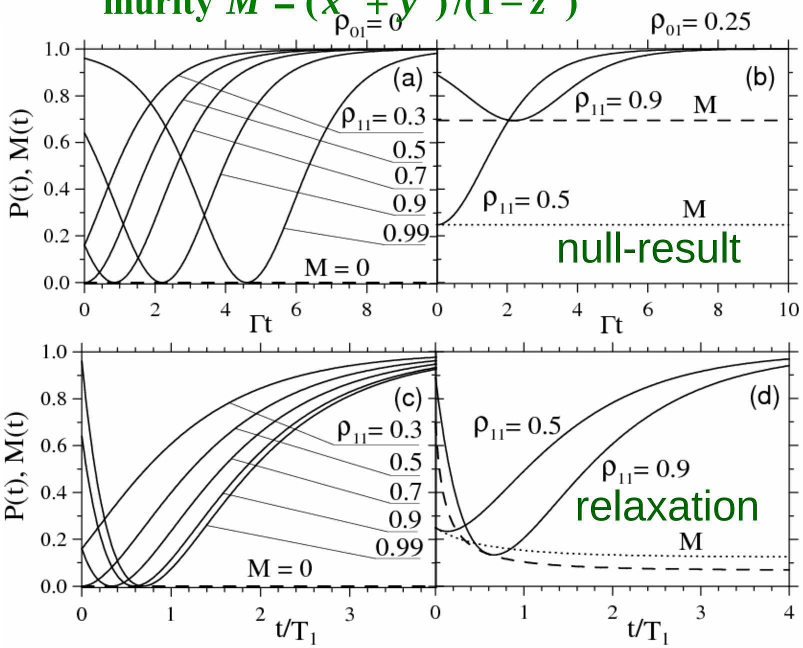
R. Ruskov, A. Mizel, and A.K., 2007





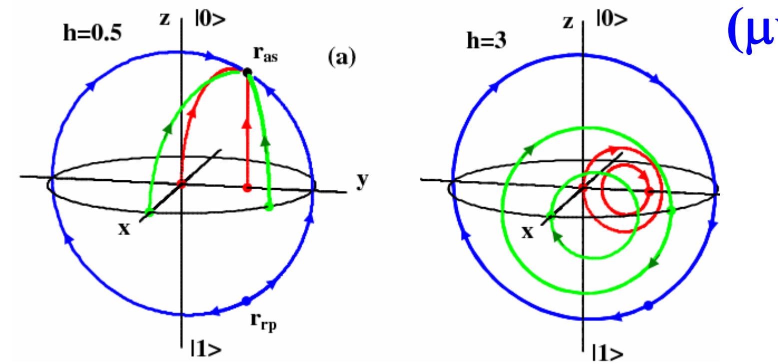
purity $P = x^2 + y^2 + z^2$

murity $M = (x^2 + y^2)/(1 - z^2)$



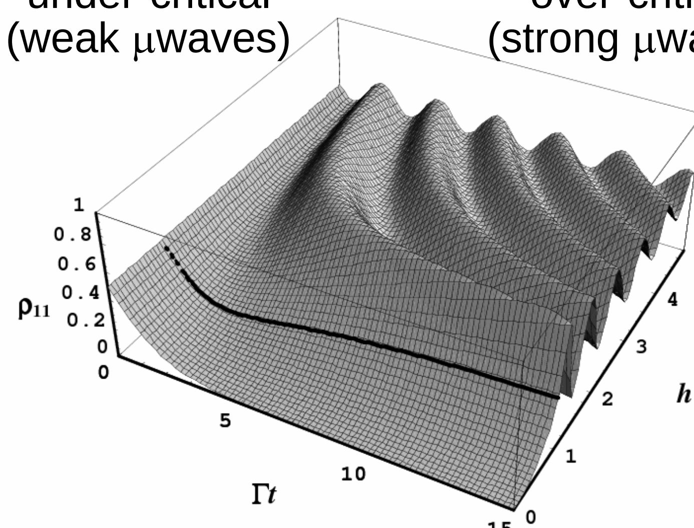
Evolutions due to null-result measurement and relaxation are clearly distinguishable

Null-result measurement + Rabi oscillations (μwaves)



under-critical (weak μwaves)

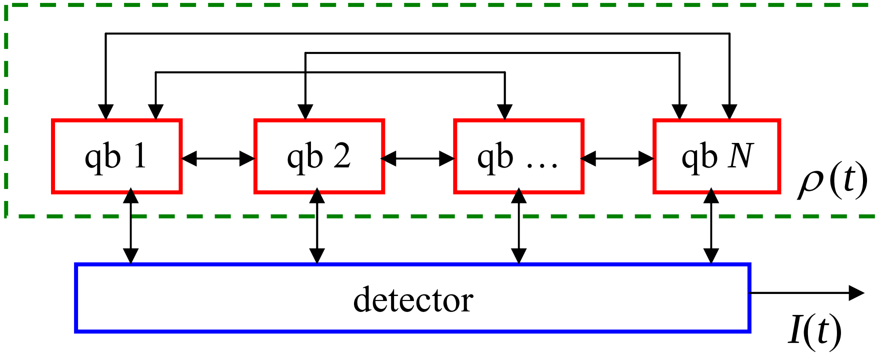
over-critical (strong μwaves)



$$h = \frac{2\Omega_R}{\Gamma}$$

Crossover between asymptotic stability and non-decaying oscillations

Bayesian formalism for *N* entangled qubits measured by one detector



Up to 2^N levels of current

$$\frac{d}{dt}\rho_{ij} = \frac{-i}{\hbar}[\hat{H}_{qb},\rho]_{ij} + \rho_{ij}\frac{1}{S}\sum_k \rho_{kk}\Big[\Big(I(t)-\frac{I_k+I_i}{2}\Big)(I_i-I_k) +$$

$$+\Big(I(t)-\frac{I_k+I_j}{2}\Big)(I_j-I_k)\Big] - \gamma_{ij}\rho_{ij}$$

(Stratonovich form)

$$\gamma_{ij} = (\eta^{-1}-1)(I_i-I_j)^2/4S_I \qquad I(t) = \sum_i \rho_{ii}(t) I_i + \xi(t)$$

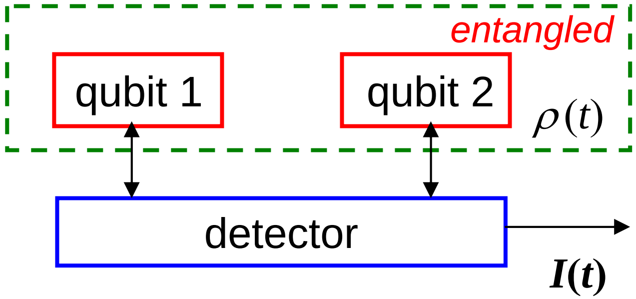
Averaging over ξ(t) ⟹ master equation

No measurement-induced dephasing between states |*i*⟩ and |*j*⟩ if $I_i = I_j$!

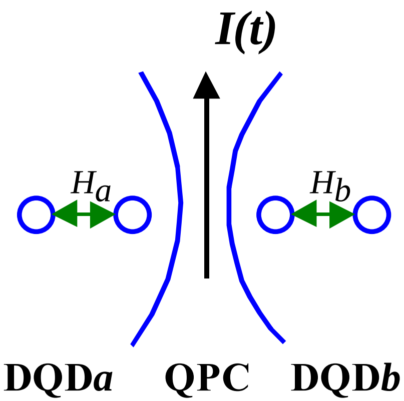
A.K., PRA 65 (2002), PRB 67 (2003)

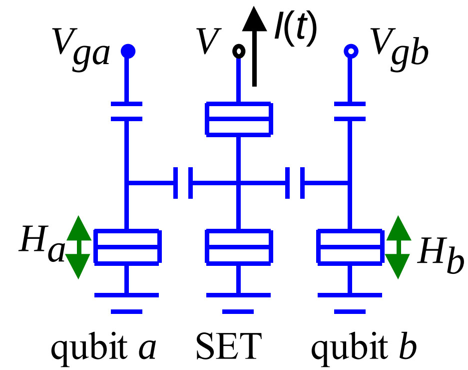
Two-qubit entanglement by measurement

Ruskov & A.K., 2002

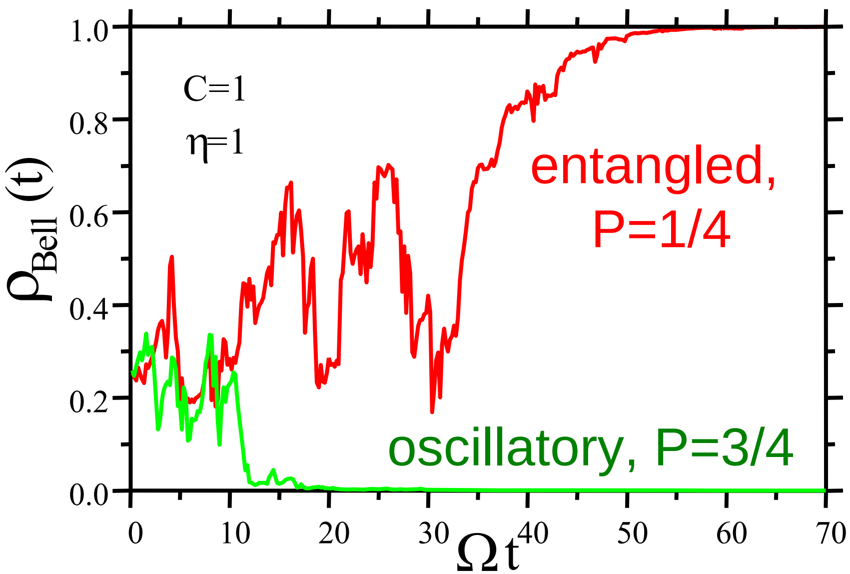


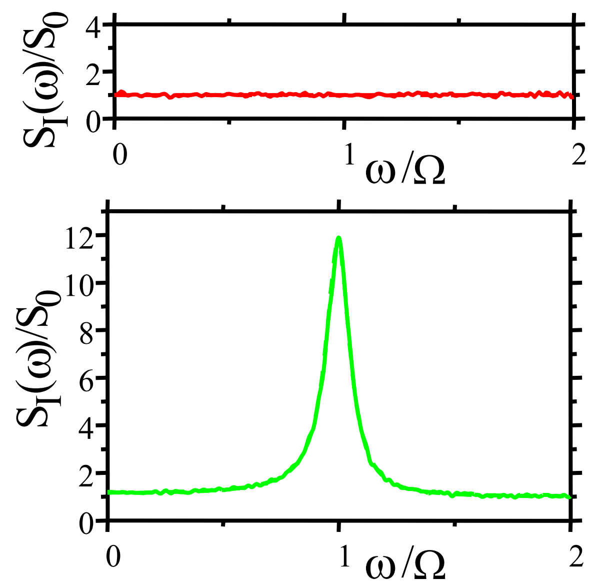
Symmetric setup, no qubit interaction





Two evolution scenarios:



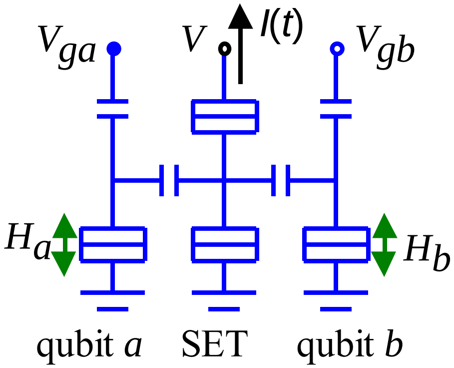


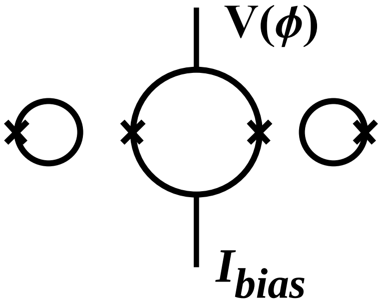
Peak/noise = (32/3) η

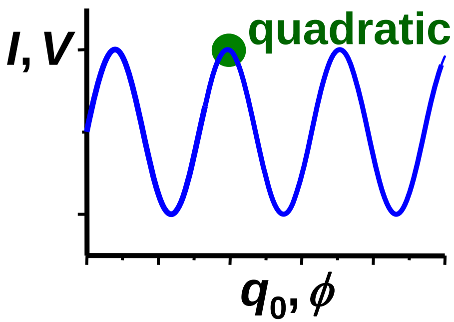
Collapse into |Bell⟩ state (spontaneous entanglement) with probability 1/4 starting from fully mixed state

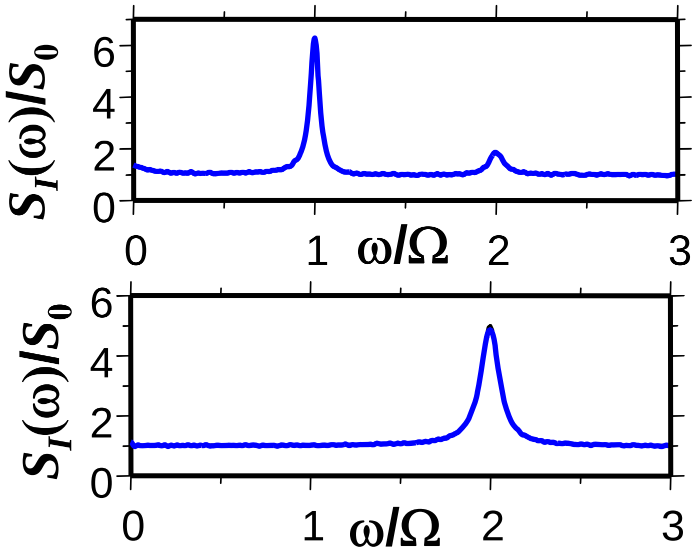
Quadratic quantum detection

Mao, Averin, Ruskov, A.K., PRL-2004









Nonlinear detector:

spectral peaks at Ω, 2Ω and 0

Quadratic detector:

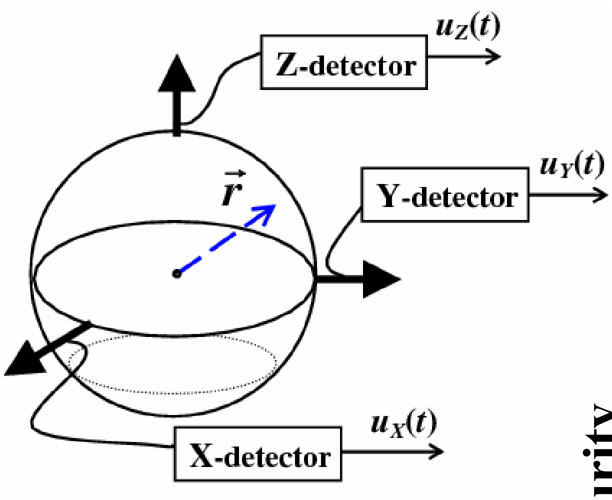
Peak only at 2Ω, peak/noise = 4η

$$S_I(\omega) = S_0 + \frac{4\Omega^2 (\Delta I)^2 \Gamma}{(\omega^2 - 4\Omega^2)^2 + \Gamma^2 \omega^2}$$

Three evolution scenarios: 1) collapse into $|\uparrow\downarrow - \downarrow\uparrow\rangle$, current $I_{\uparrow\downarrow}$, flat spectrum 2) collapse into $|\uparrow\uparrow - \downarrow\downarrow\rangle$, current $I_{\uparrow\uparrow}$, flat spectrum; 3) collapse into remaining subspace, current $(I_{\uparrow\downarrow} + I_{\uparrow\uparrow})/2$, spectral peak at 2Ω

Entangled states distinguished by average detector current

Qubit monitoring via 3 complementary observables

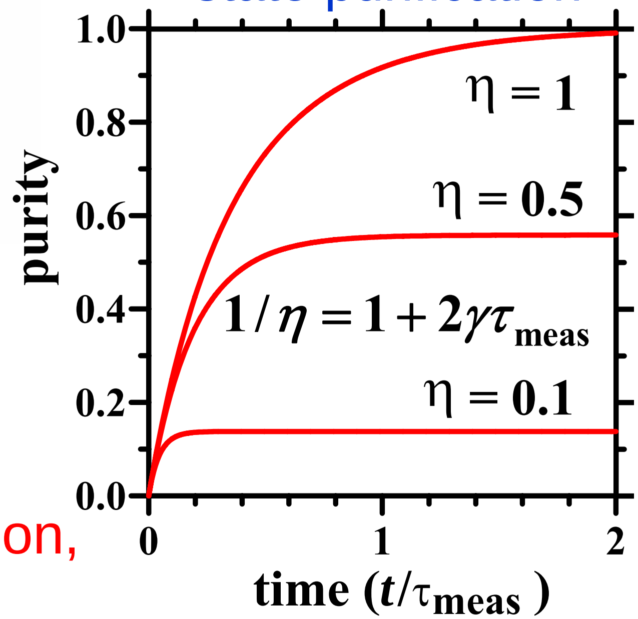


evolution

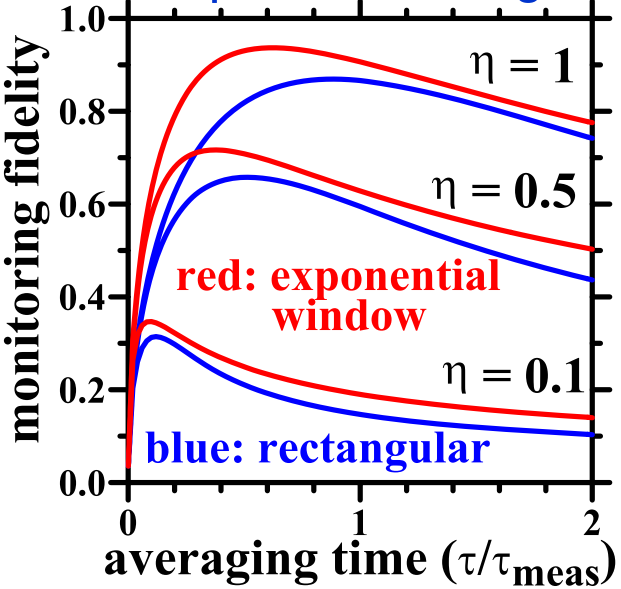
$$\frac{d\vec{r}}{dt} = -2\gamma\,\vec{r} + a\{\vec{u}(t)(1-r^2) - [\vec{r}\times[\vec{r}\times\vec{u}(t)]]\}$$

a – coupling, γ - extra dephasing

state purification



simple monitoring



Isotropic evolution,
3 times faster purification,
good fidelity of simple
monitoring (up to 0.94)

Ruskov, Korotkov, Molmer, PRL-2010

Binary-output qubit detector (non-destructive, single-shot)

General characterization

general POVM (superoperator) for each result:

16 +16 – 4 = **28** real parameters to describe (too many!)

28 = 2 (meas. axis) + 2 (fidelity) + 2×3 (unitary) + 2×9 (decoherence)

↖ F_0 – prob. to get 0 if |0>
F_1 – prob. to get 1 if |1>

Simplifications:

1) Textbook projective only 2 parameters (meas. axis)

2) Perfect fidelity F_0=F_1=1; then only meas. axis is interesting
(6 more parameters affect only reinitialization)

3) QND $|0\rangle \to |0\rangle$, $|1\rangle \to |1\rangle$; then 6 parameters

QND binary-output detector

A.K., 2008

6 parameters: fidelity (F_0, F_1), decoherence (D_0, D_1), and phases (ϕ_0, ϕ_1)

result 0: $$\begin{pmatrix} \rho_{00} & \rho_{01} \\ \rho_{10} & \rho_{11} \end{pmatrix} \to \frac{1}{P_0} \begin{pmatrix} F_0\rho_{00} & \sqrt{F_0(1-F_1)}\, e^{-D_0} e^{i\phi_0} \rho_{01} \\ c.c. & (1-F_1)\rho_{11} \end{pmatrix}$$

result 1: $$\begin{pmatrix} \rho_{00} & \rho_{01} \\ \rho_{10} & \rho_{11} \end{pmatrix} \to \frac{1}{P_1} \begin{pmatrix} (1-F_0)\rho_{00} & \sqrt{(1-F_0)F_1}\, e^{-D_1} e^{i\phi_1} \rho_{01} \\ c.c. & F_1\rho_{11} \end{pmatrix}$$

(simple Bayes)

$$P_0 = F_0\rho_{00} + (1-F_1)\rho_{11}, \;\; P_1 = (1-F_0)\rho_{00} + F_1\rho_{11}$$

Corresponding quantum limits

result 0: $$\frac{|\rho_{01}^{\text{after}}|}{|\rho_{01}|} \le \frac{1}{P_0}\sqrt{F_0(1-F_1)}$$ **result 1:** $$\frac{|\rho_{01}^{\text{after}}|}{|\rho_{01}|} \le \frac{1}{P_1}\sqrt{F_1(1-F_0)}$$

ensemble decoherence: $$\frac{|\rho_{01}^{\text{after}}|}{|\rho_{01}|} \le \sqrt{F_0(1-F_1)} + \sqrt{(1-F_0)F_1}$$

natural to introduce quantum efficiencies by comparing with quantum limits

(easy to realize η_0=1, but difficult η_0=η_1=1)

Natural definitions of quantum efficiency (actual decoherence vs. informational bound)

Ensemble decoherence (averaged over result, similar to the definition for linear detectors)

$$\eta = D_{\min} / D_{av}$$

Also for each result of measurement

$$1-\eta_0 = \frac{D_0}{D_0 - \ln\sqrt{F_0(1-F_1)}}$$

$$1-\eta_1 = \frac{D_1}{D_1 - \ln\sqrt{(1-F_0)F_1}}$$

(useful for “asymmetric” and “half-destructive” detectors, as for phase qubits)

Niels Bohr:

"If you are not confused by quantum physics then you haven't really understood it"

Richard Feynman:

"I think I can safely say that nobody understands quantum mechanics"

Quantum measurement is the most confusing and also fascinating part of QM

Two main puzzles:

- Non-locality of collapse

  Now well-studied (understood?), in many QM textbooks, being used (quant. cryptography, CHSH as calibration, etc.)

- What is "inside" collapse

  We know basic answer (many equivalent approaches), still to be included into QM textbooks, may lead to important practical applications (q. feedback, etc.)

Conclusions (to 3 lectures)

- It is easy to see what is "inside" collapse: simple Bayesian formalism works for many solid-state setups
- Rabi oscillations are persistent if weakly measured
- Quantum feedback can synchronize persistent Rabi oscillations
- Collapse can sometimes be undone if we manage to erase extracted information
- Continuous/partial measurements, quantum feedback, and uncollapsing may have useful applications
- Three direct solid-state experiments have been realized, many interesting experimental proposals are still waiting

Thank you!