University of Waterloo

PERIMETER INSTITUTE FOR THEORETICAL PHYSICS

Acceleration and Entanglement: a Deteriorating Relationship

R.B. Mann

D. Ahn
P. Alsing
I. Fuentes-Schuller
F. Haque
B.L. Hu
S.Y. Lin

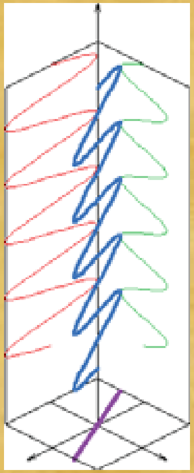
N. Menicucci
D. Ostapchuk
G. Salton
T. Tessier
V. Villalba

Phys. Rev. Lett. 95 120404 (2005)
Phys. Rev. A74 032326 (2006)
Phys. Rev. A79 042333 (2009)
Phys. Rev. A80 02230 (2009)

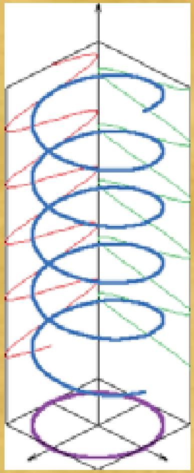
Qubits
The Simplest Quantum Systems

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle \qquad |\alpha|^2 + |\beta|^2 = 1$$

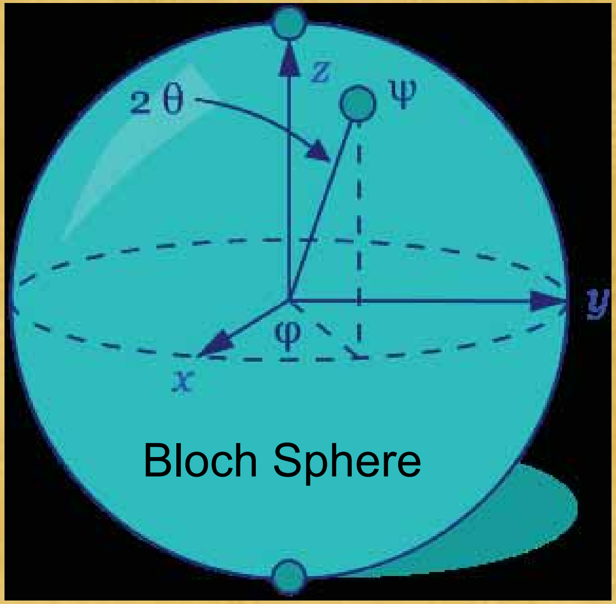
Photon Polarization



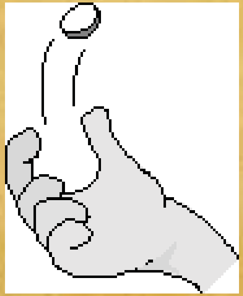
Linear



Circular

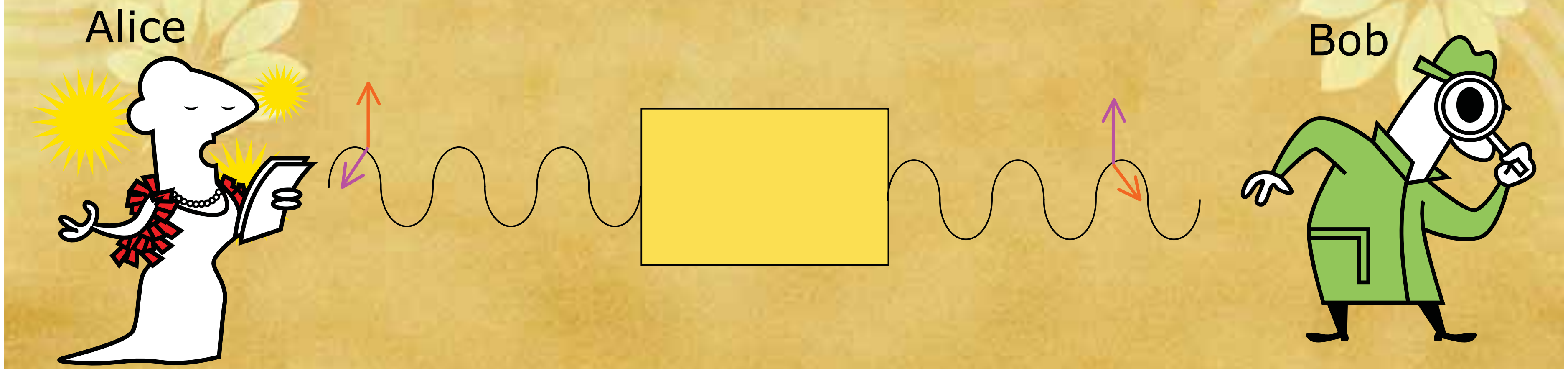


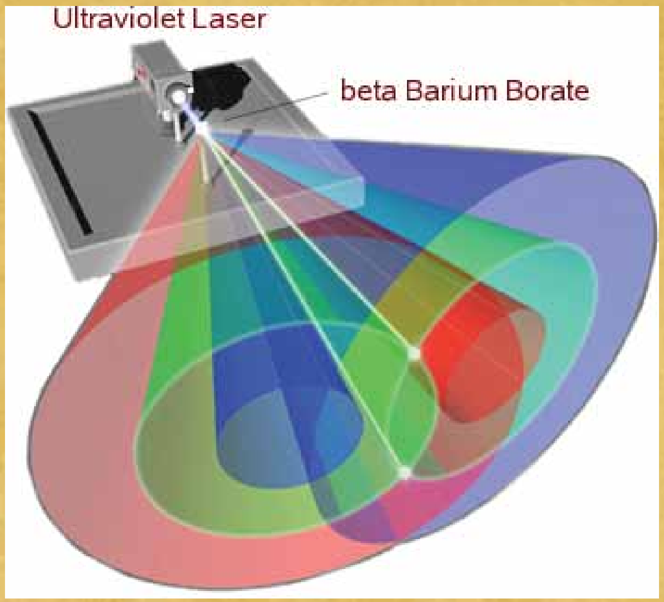
Bloch Sphere

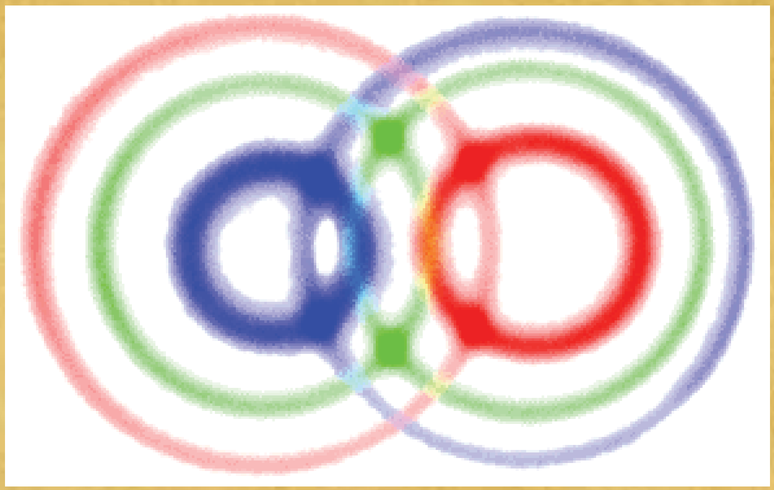


$= \alpha|$  $\rangle + \beta|$  $\rangle$

Entanglement







Resource for information tasks:
teleportation,cyptography, etc.

Image copyright © Anton Zeilinger, Institute of Experimental Physics, University of Vienna.

Describing Entanglement

Separable state:

$$|\phi\rangle_{AB} = |\phi\rangle_1^A \otimes |\psi\rangle_2^B \Rightarrow$$

Local Operations and
Classical Communication

Entangled state:
Defined as non-separable

Cannot be prepared by LOCC

$$|\phi\rangle_{AB} = \frac{1}{\sqrt{2}}\left(|0\rangle_1^A |0\rangle_2^B + |1\rangle_1^A |1\rangle_2^B\right) \Rightarrow$$

Maximal correlations when
measured in any basis

Density Matrix

Quantum State $|\psi\rangle \Rightarrow |\psi\rangle\langle\psi| \equiv \rho$ ⇐ Density Matrix

Expectation Value: $\langle\psi|A|\psi\rangle = \mathrm{Tr}\left[\rho A\right]$

Pure State: $\alpha|0\rangle + \beta|1\rangle \Rightarrow \rho = \begin{pmatrix} |\alpha|^2 & \alpha\beta^* \\ \alpha^*\beta & |\beta|^2 \end{pmatrix}$ $\mathrm{Tr}\left[\rho^2\right] = 1$

Entangled state:

$$\alpha|0\rangle_1^A|0\rangle_2^B + \beta|1\rangle_1^A|1\rangle_2^B \Rightarrow \rho = \begin{pmatrix} |\alpha|^2 & 0 & 0 & \alpha\beta^* \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \alpha^*\beta & 0 & 0 & |\beta|^2 \end{pmatrix}$$

Mixed State: $\mathrm{Tr}\left[\rho^2\right] < 1$

Quantifying Entanglement

For pure states:

$$|\phi\rangle_{AB} = \sum_{ij}\left(\omega_{ij}|i\rangle_1^A|j\rangle_2^B\right) \Longrightarrow |\phi\rangle_{AB} = \sum_{n}\left(\omega_n|n\rangle_1^A|n\rangle_2^B\right)$$

Schmidt decomposition

Measure of entanglement: von-Neumann entropy

$$\rho_{AB} = |\phi\rangle\langle\phi| \Longrightarrow S(\rho) = -\mathrm{Tr}\left[\rho\log_2\rho\right] \Longrightarrow S(\rho_{AB}) = 0$$

Reduce

$$\rho_A = -\mathrm{Tr}_\mathrm{B}\left[\rho_{AB}\right] \Longrightarrow S(\rho_A) = S(\rho_B)$$

Problem: mixed states

$$\rho_{AB} = \sum_{ijkl}\left(\omega_{ijkl}|i\rangle|j\rangle\langle k|\langle l|\right) \Longrightarrow$$

No analogous Schmidt decomposition

Entropy no longer quantifies entanglement….

Mixed State Entanglement

Separable state: $\rho_{AB} = \sum_i \alpha_{ij} \rho_A^i \otimes \rho_B^j$

Necessary separability condition:

The partial transpose of the density matrix has positive eigenvalues if the state is separable.

To determine separability is easy

To measure entanglement is not

Mutual information

$$I(\rho_{AB}) = S(\rho_A) + S(\rho_B) - S(\rho_{AB})$$

Total correlations quantum + classical

Measuring Entanglement

Logarithmic negativity $N\left(\rho\right)=\log_2\left\|\rho^T\right\|$

Entanglement of Formation

$$E_F = \lim_{\text{copies}\to\infty} \frac{\text{\# of maximally entangled pairs}}{\text{\# of pure state copies created}}$$

$$E_D \le E_F \le N$$

Upper bound on all entanglement!

Entanglement of Distillation

$$E_D = \lim_{\text{\#states}\to\infty} \frac{\text{\# of non-maximally entangled states}}{\text{\# of maximally entangled states}}$$

Relativistic Entanglement

What happens if Bob moves
with respect to Alice?

Inertial Frames:

- Entanglement constant between inertially moving parties
- Some degrees of freedom can be transferred to others

Peres,Scudo and Terno
Alsing and Milburn
Gingrich and Adami
Pachos and Solano

Non-inertial frames:

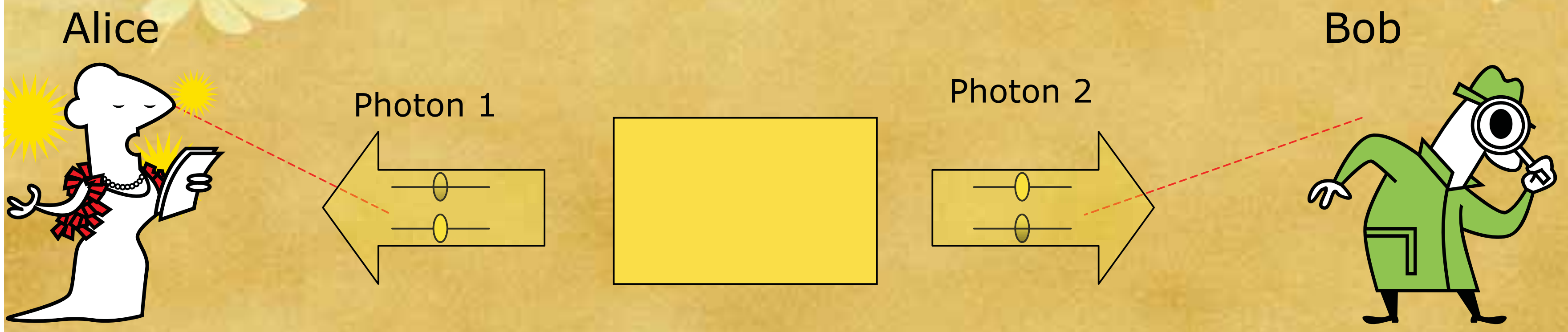
Alsing and Milburn, PRL 91 180404, (2003)

Teleportation with a uniformly accelerated partner

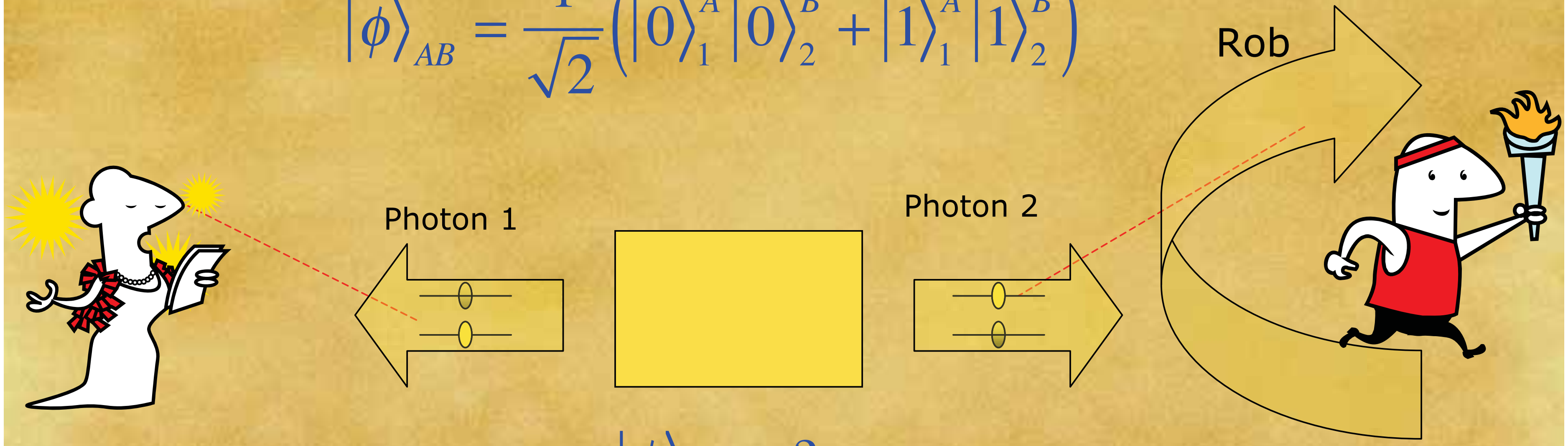
- teleportation fidelity decreases as the acceleration grows
- Indication of entanglement degradation.

Our motivation: Study entanglement in this setting

Bosonic Entanglement for Uniformly Accelerated Observers



$$|\phi\rangle_{AB} = \frac{1}{\sqrt{2}}\left(|0\rangle_1^A|0\rangle_2^B + |1\rangle_1^A|1\rangle_2^B\right)$$



$$|\phi\rangle_{AB} = ?$$

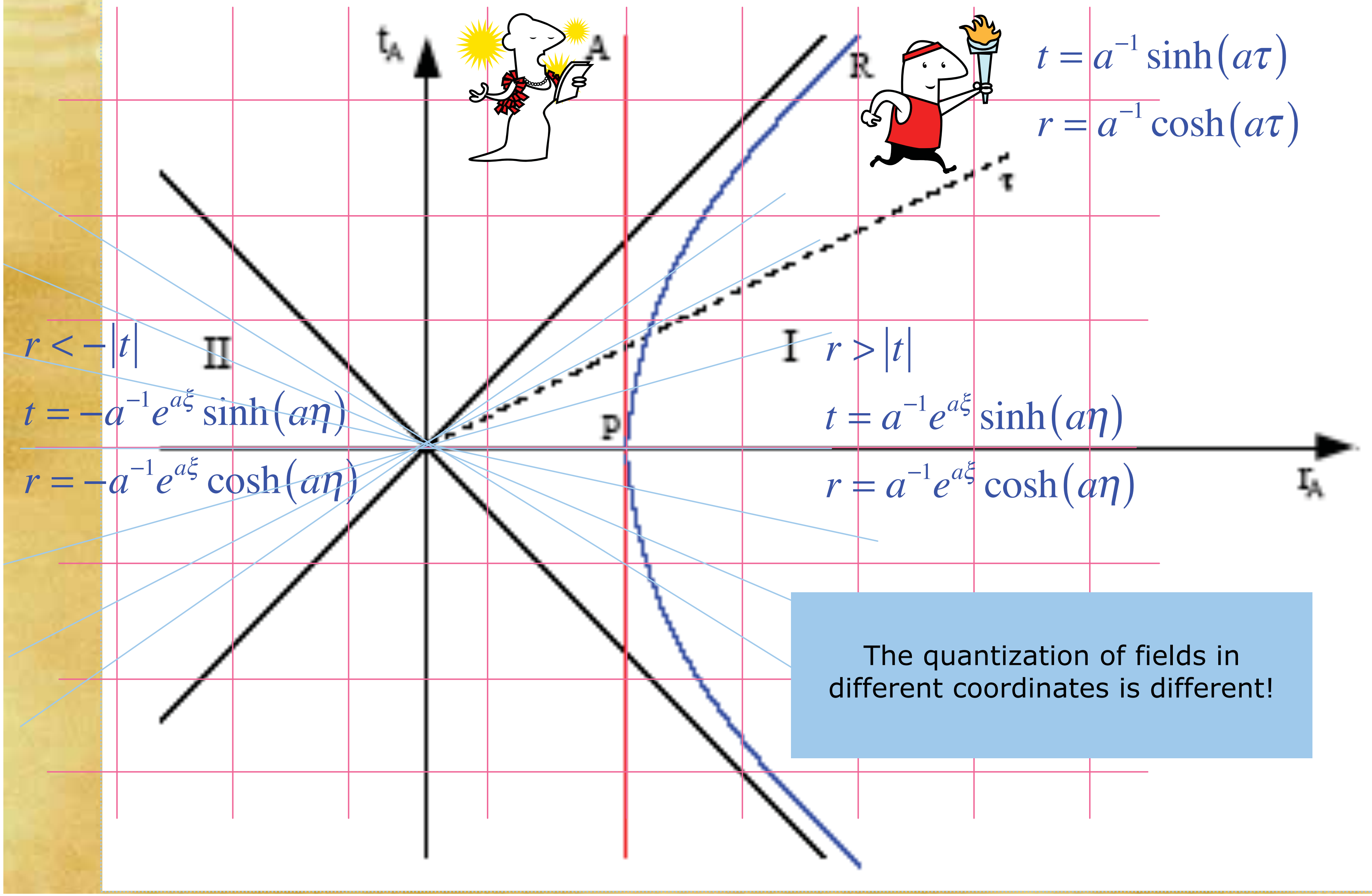
Fuentes-Schuller/Mann
Phys. Rev. Lett. 95:120404 (2005)

Minkowski space

$$ds^2 = dt^2 - dr^2$$

Rindler space

$$ds^2 = e^{2a\xi}\left(d\eta^2 - d\xi^2\right)$$



$$t = a^{-1}\sinh(a\tau)$$

$$r = a^{-1}\cosh(a\tau)$$

$$r < -|t|$$

$$t = -a^{-1}e^{a\xi}\sinh(a\eta)$$

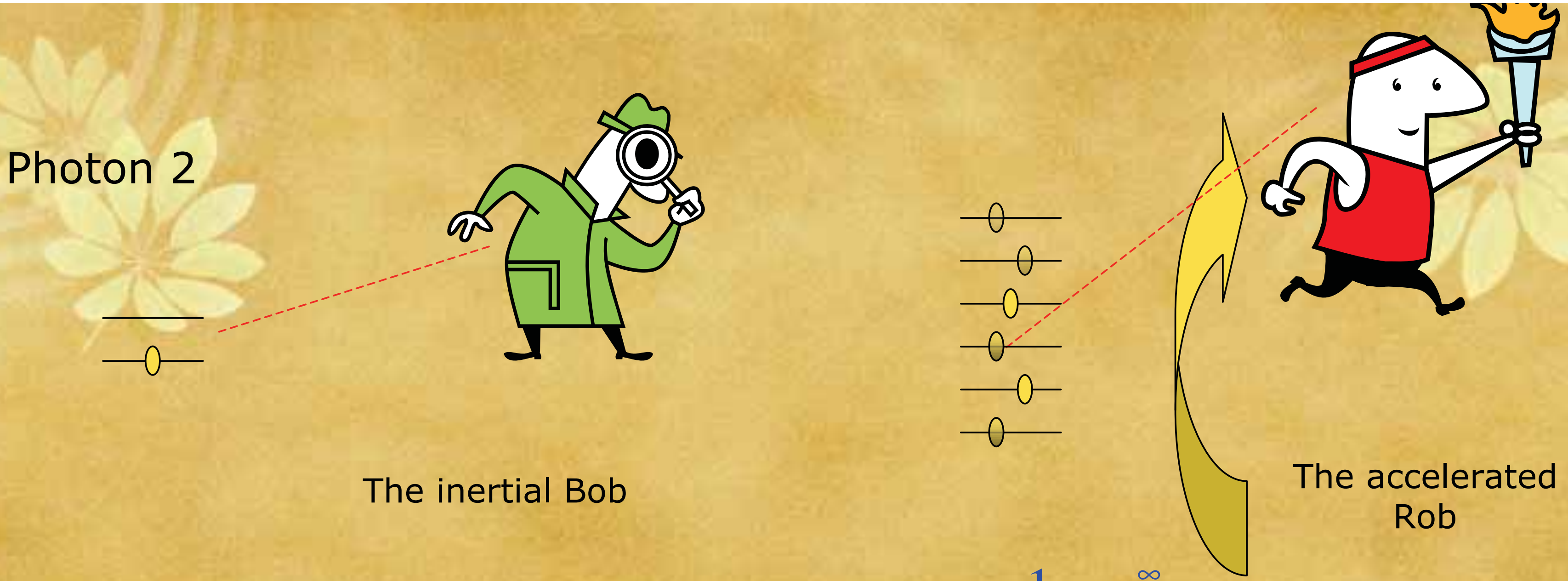
$$r = -a^{-1}e^{a\xi}\cosh(a\eta)$$

$$r > |t|$$

$$t = a^{-1}e^{a\xi}\sinh(a\eta)$$

$$r = a^{-1}e^{a\xi}\cosh(a\eta)$$

The quantization of fields in different coordinates is different!



Photon 2

The inertial Bob

The accelerated Rob

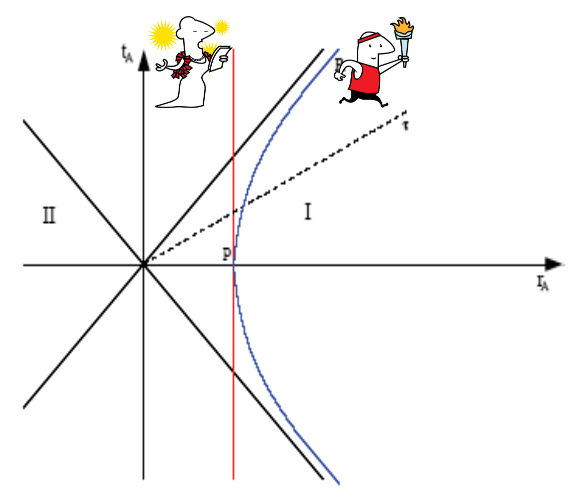
$$|0\rangle_2^B \Rightarrow |0\rangle_2^B = \frac{1}{\cosh r}\sum_{n=0}^{\infty}\tanh^n r|n\rangle_I|n\rangle_{II}$$

$$\cosh r = \frac{1}{\sqrt{1-\exp(-2\pi\Omega)}}$$

$$\Omega = \frac{\omega c}{a}$$

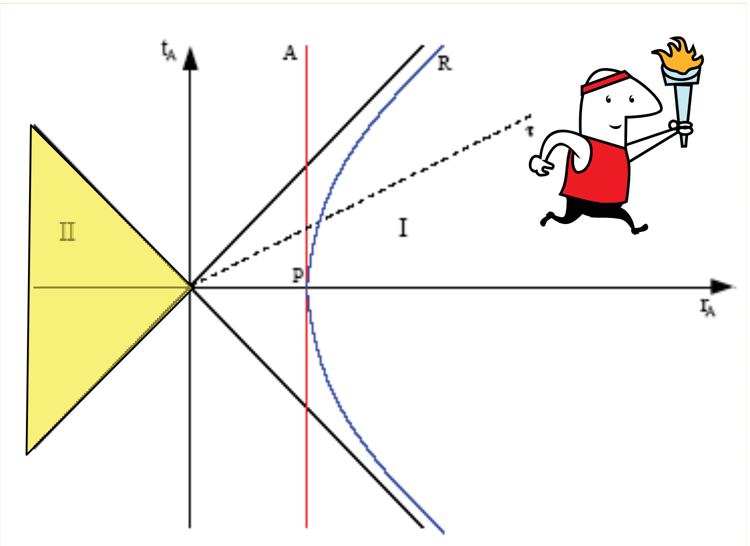
Rob is causally disconnected from region II

Unruh effect!



$$|0\rangle_2^B = \frac{1}{\cosh r}\sum_{n=0}^{\infty}\tanh^n r|n\rangle_I|n\rangle_{II}$$

$$|1\rangle_2^B = \frac{1}{\cosh r}\sum_{n=0}^{\infty}\tanh^n r\sqrt{n+1}|n+1\rangle_I|n\rangle_{II}$$



Trace over region II

$$\begin{aligned}\rho_{AR} &= \frac{1}{2\cosh^2 r}\sum_n (\tanh r)^{2n}\rho_n \\ \rho_n &= |0\,n\rangle\langle 0\,n| + \frac{\sqrt{n+1}}{\cosh r}|0\,n\rangle\langle 1\,(n+1)| \\ &+ \frac{\sqrt{n+1}}{\cosh r}|1\,(n+1)\rangle\langle 0\,n| + \frac{(n+1)}{\cosh^2 r}|1\,(n+1)\rangle\langle 1\,(n+1)|\end{aligned}$$

$$|nm\rangle = |n\rangle^A|m\rangle_I$$

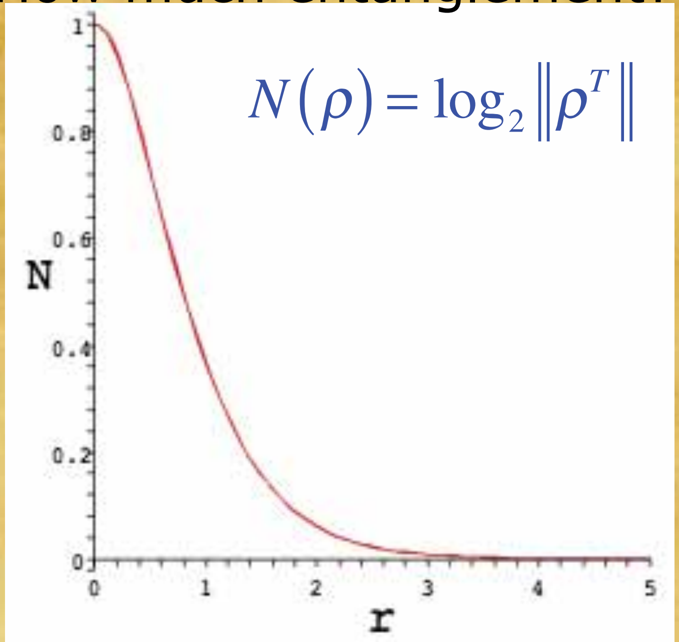
Entanglement?

Check eigenvalues of positive partial transpose

$$\left.\begin{aligned} \lambda_{\pm}^{n} &= \frac{\tanh^{2n} r}{4\cosh^2 r}\left[\left(\frac{n}{\sinh^2 r}+\tanh^2 r\right)\pm\sqrt{Z_n}\right] \\ Z_n &= \left(\frac{n}{\sinh^2 r}+\tanh^2 r\right)^2+\frac{4}{\cosh^2 r} \end{aligned}\right\} \quad \lambda_{-}^{n}<0$$

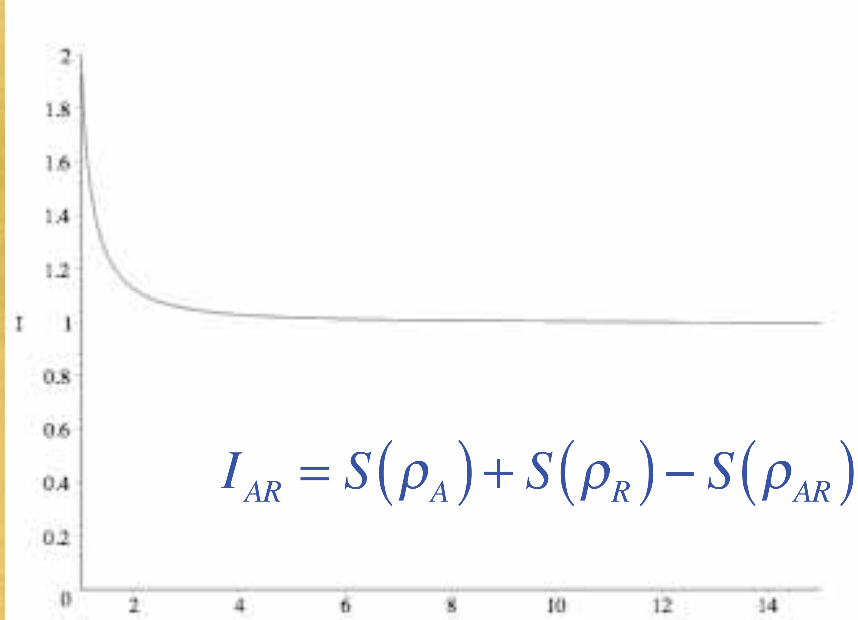
Always entangled for finite acceleration!

How much entanglement?



$$N(\rho)=\log_2\left\|\rho^T\right\|$$

Logarithmic negativity vs. acceleration parameter r



$$I_{AR}=S(\rho_A)+S(\rho_R)-S(\rho_{AR})$$

Mutual information vs cosh(r)

Distillable entanglement decreases as acceleration increases

Where did the entanglement go?

Pure state: $S\left(\rho_{ARI,II}\right)=0 \Longrightarrow S\left(\rho_{ARI}\right)=S\left(\rho_{RII}\right)$

Entanglement between Alice+Rob in region I
With modes in region II

Zero acceleration

$S\left(\rho_{ARI}\right)=0$

Alice+Bob are maximally entangled and there is no entanglement with region II

Finite acceleration

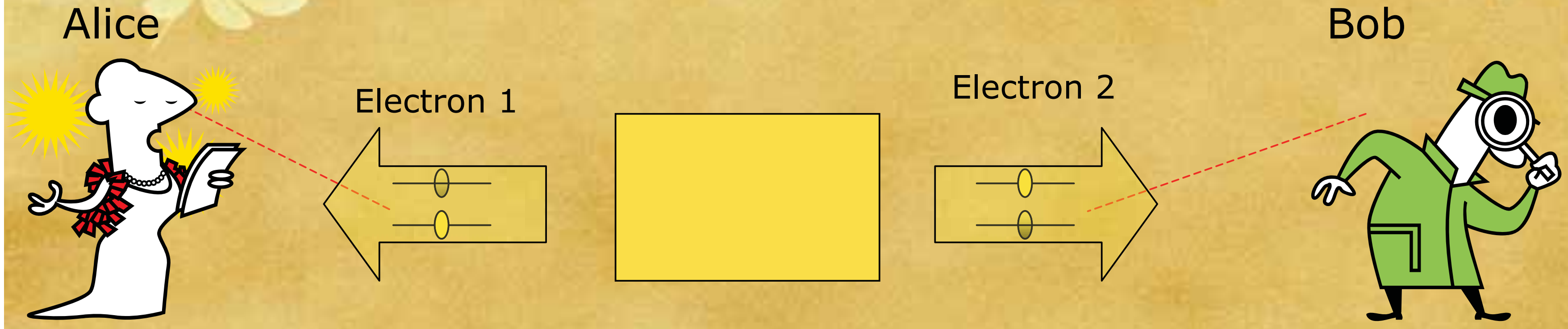
the entanglement between Alice+Rob is degraded and entanglement with region II grows

Infinite acceleration

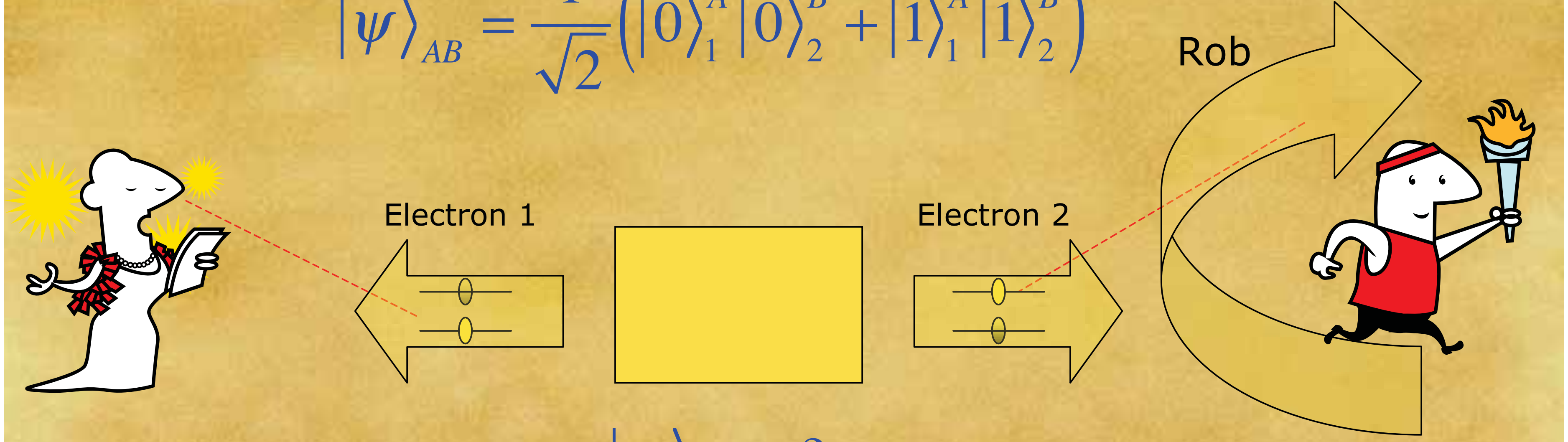
$S\left(\rho_{ARI}\right)=1$

Alice+Rob are disentangled and entanglement with region II is maximal

Fermionic Entanglement for Uniformly Accelerated Observers



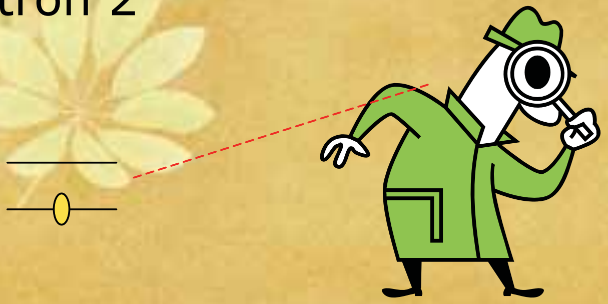
$$|\psi\rangle_{AB} = \frac{1}{\sqrt{2}}\left(|0\rangle_1^A|0\rangle_2^B + |1\rangle_1^A|1\rangle_2^B\right)$$



$$|\psi\rangle_{AB} = ?$$

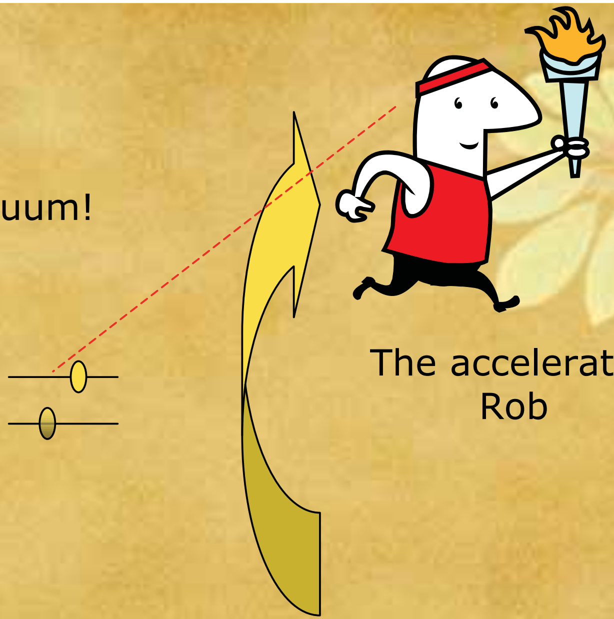
Alsing/Mann/Fuentes-Schuller/Tessier
Phys. Rev. A74 032326 (2006)

Electron 2



The inertial Bob

Fermionic vacuum!



The accelerated Rob

$$|0\rangle_2^B \Rightarrow |0\rangle_2^B = \cos r|0\rangle_I|0\rangle_{II} + \sin r|1\rangle_I|1\rangle_{II}$$

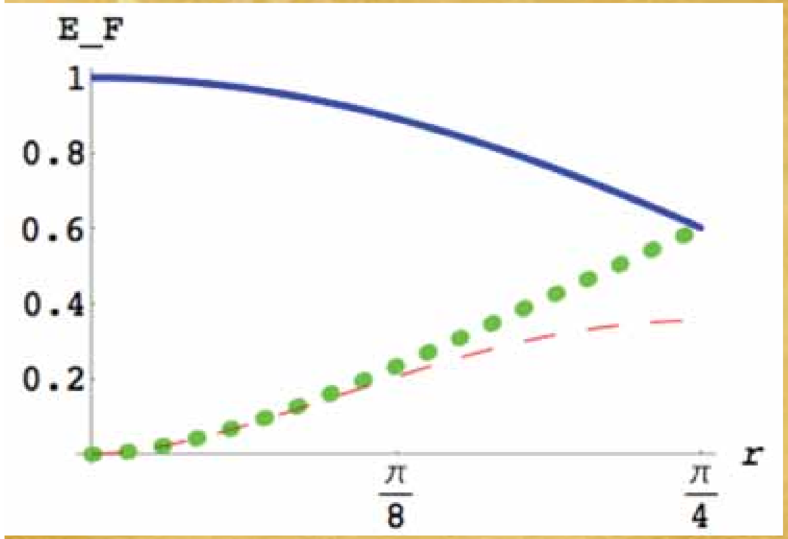
$$\cos r = \frac{1}{\sqrt{1+\exp(-2\pi\Omega)}} \qquad \Omega = \frac{\omega c}{a}$$

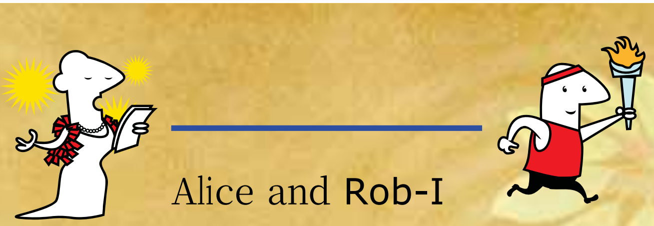
Entangement of Formation

$$E_F = -\frac{1+\sin r}{2}\log_2\left[\frac{1+\sin r}{2}\right] - \frac{1-\sin r}{2}\log_2\left[\frac{1-\sin r}{2}\right]$$

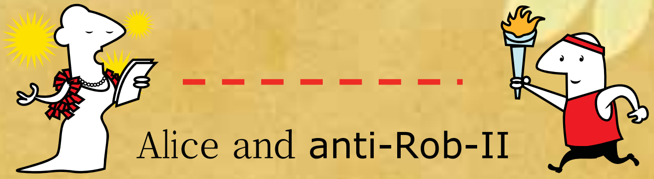
Negativity

$$N = \log_2\left[1+\cos^2 r\right]$$

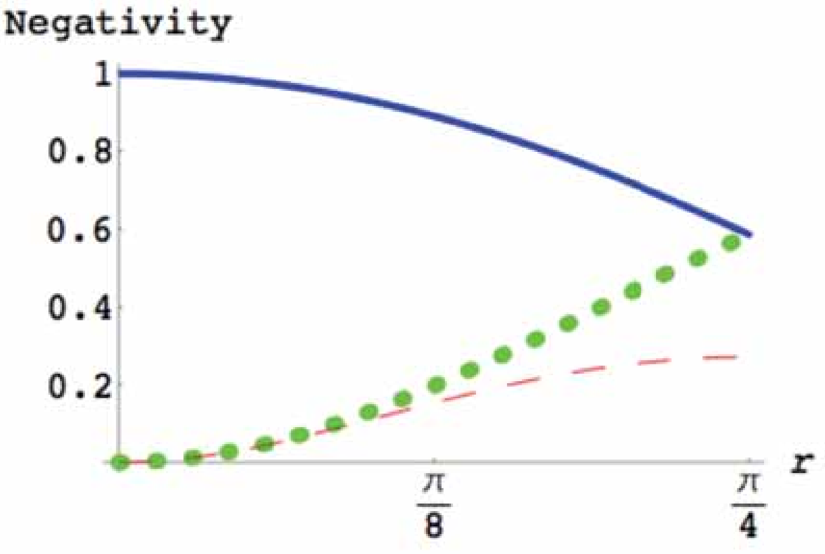


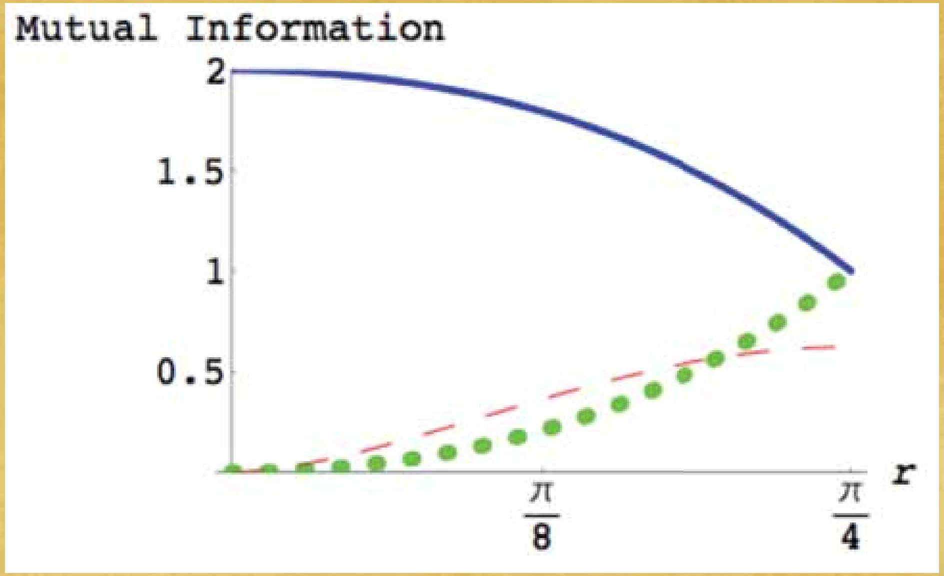


Alice and Rob-I



Alice and anti-Rob-II





Rob-I and anti-Rob-II

Infinite Acceleration Limit

Bosons

- Entanglement
  - goes to zero
- Mutual information
  - goes to 1
- State
  - maximally entangled to region II

The state is only classically correlated

Fermions

- Entanglement
  - goes to $1/\sqrt{2}$
- Mutual information
  - goes to 1
- State
  - maximally entangled to region II

The state always has some quantum correlation

How Robust is this Difference?

- Does the number of excited modes matter?
  - No for fermions -- entanglement degradation is still limited for fermions

Leon/Martin-Martinez
Phys.Rev. A80 012314

  - Yes for bosons -- entanglement loss depends on the number of modes occupied

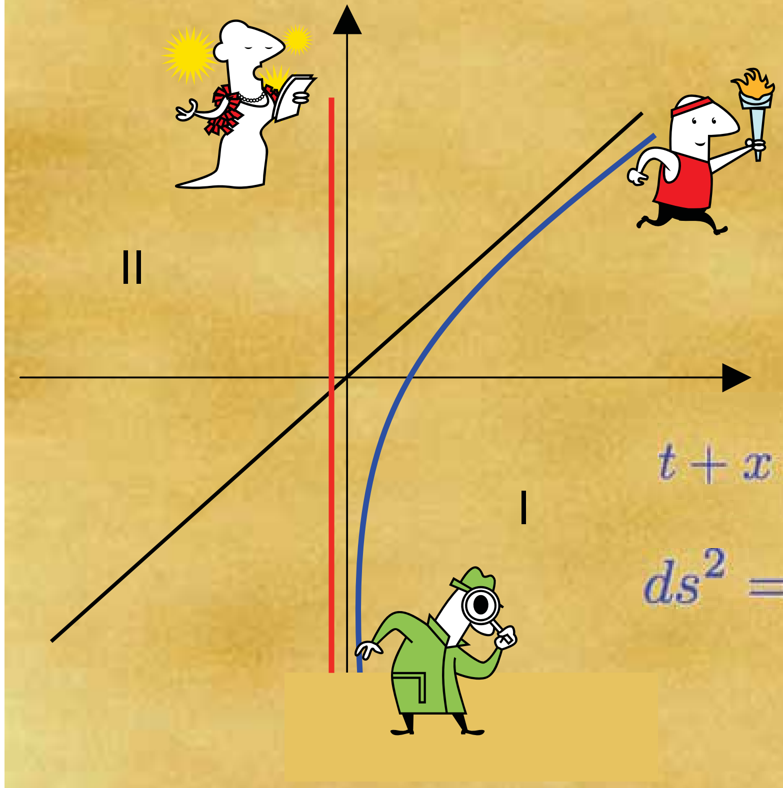
Leon/Martin-Martinez
Phys.Rev. A81 032320

- Does the surviving entanglement depend on the choice of maximally entangled state?
  - No

Leon/Martin-Martinez
1003.3550

- Does spin matter?
  - Yes, but only quantitatively -- entanglement degradation is greater but still does not vanish in the large acceleration limit

Non-Uniform Acceleration



No Timelike Killing Vector!

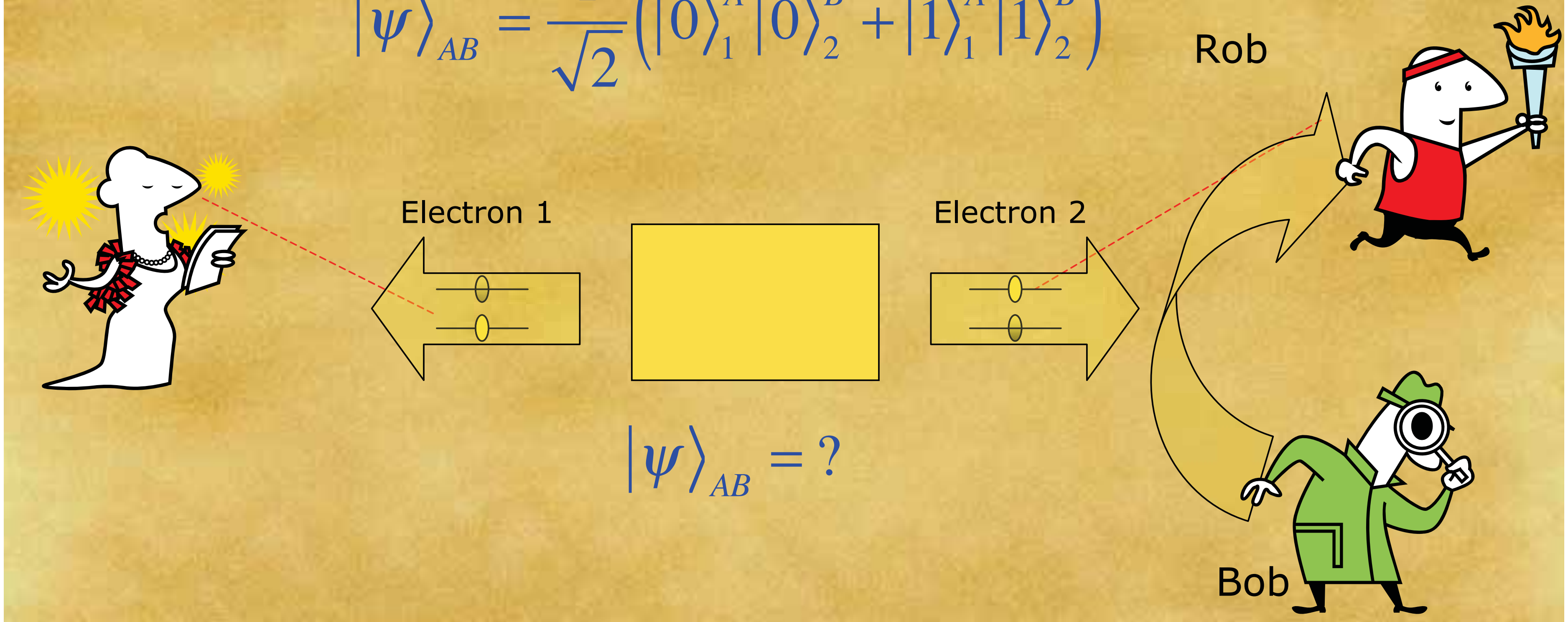
$$t + x = \frac{2\sinh(a(T+X))}{a} \qquad t - x = -\frac{e^{-a(T-X)}}{a}$$

$$ds^2 = [\exp(-2aT) + \exp(2aX)]\,[dT^2 - dX^2]$$

$$a(T, X_0) = \frac{a e^{2aX_0}}{\left[e^{-2aT} + e^{2aX_0}\right]^{3/2}}$$

Costa Rev.Bras.Fis. 17 (1987) 585
Percocca/Villalba CQG 9 (1992) 307

$$|\psi\rangle_{AB} = \frac{1}{\sqrt{2}}\left(|0\rangle_1^A |0\rangle_2^B + |1\rangle_1^A |1\rangle_2^B\right)$$

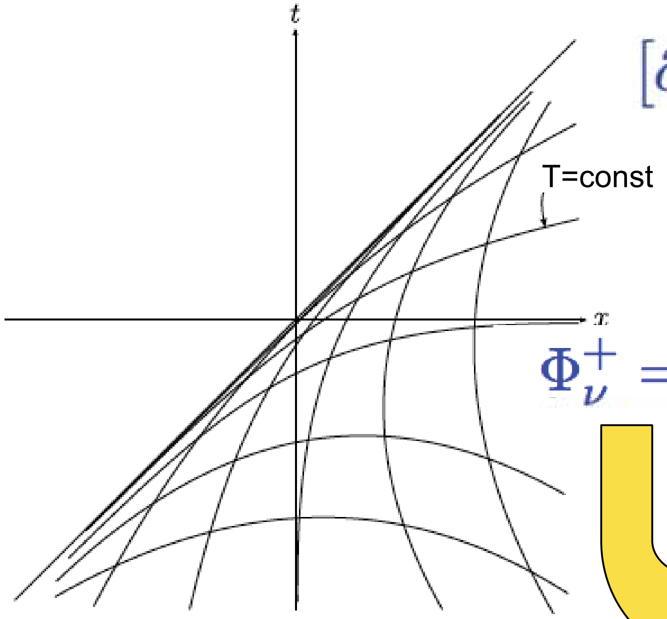


$$|\psi\rangle_{AB} = ?$$

Possible Scenario

Mann/Villalba
Phys. Rev. A80 02230 (2009)

Let Vic define +ve/-ve frequency on a constant time-slice



KG eqn

$$\left[\partial_T^2 - \partial_X^2 + m^2\left(e^{-2wT} + e^{2wX}\right)\right] \Phi(T,X) = 0$$

Separate variables and solve

$$\Phi_\nu^+ = \left(c_\mu\,(T_0)\, J_{i\nu}(\tilde{T}) + d_\mu\,(T_0)\, J_{-i\nu}(\tilde{T})\right) K_{i\nu}(\tilde{X})$$

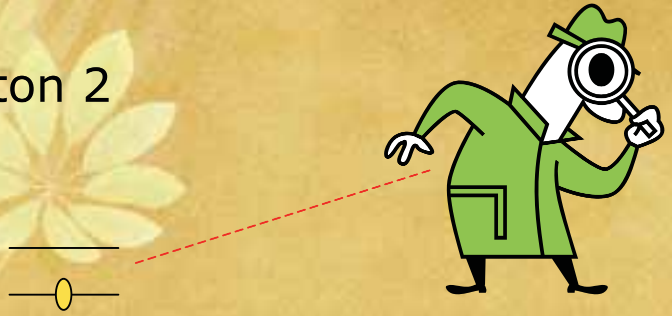
$$\begin{cases} \longrightarrow e^{\mp iKT_0} K_{i\nu}(\tilde{X}) & T_0 \to +\infty \\ \longrightarrow e^{\mp iKt} K_{i\nu}(\tilde{X}) & T_0 \to -\infty \end{cases}$$

$$c_\nu\,(T) = \frac{-i\nu\pi W^{1/2}}{2K\sinh(\pi\nu)}\left(\dot{J}_{-i\nu}(\tilde{T})W^{-1} + iJ_{-i\nu}(\tilde{T})\right)$$

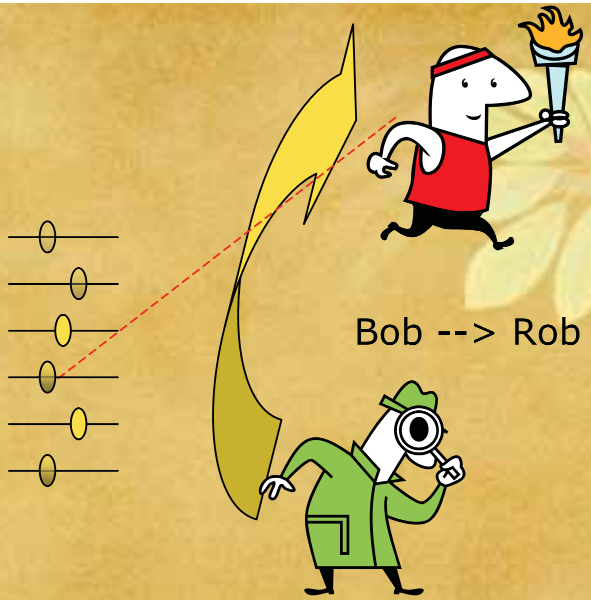
$$d_\nu\,(T) = \frac{i\nu\pi W^{1/2}}{2K\sinh(\pi\nu)}\left(\dot{J}_{i\nu}(\tilde{T})W^{-1} + iJ_{i\nu}(\tilde{T})\right)$$

$$\nu = K/w$$

Photon 2



The inertial Bob



Bob --> Rob

$$|0\rangle_2^B \Longrightarrow |0_k\rangle_2^B = \sqrt{1-|q|^2}\sum_{n=0}^{\infty} q^n |n_k\rangle_I |n_k\rangle_{II}$$

$$q = \frac{e^{-\pi\nu}c^*_{\nu+}(T_0) - c^*_{\nu-}(T_0)}{c_{\nu+}(T_0)+e^{-\pi\nu}c_{\nu-}(T_0)}$$

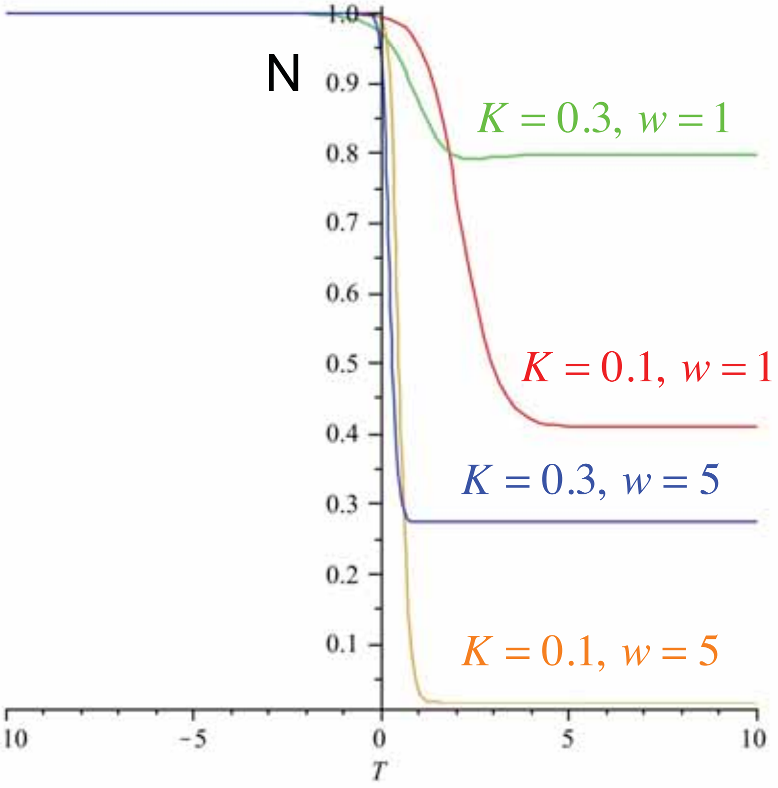
$$\rho = \frac{1-|q|^2}{2}\sum_n q^{2n}\rho_n$$

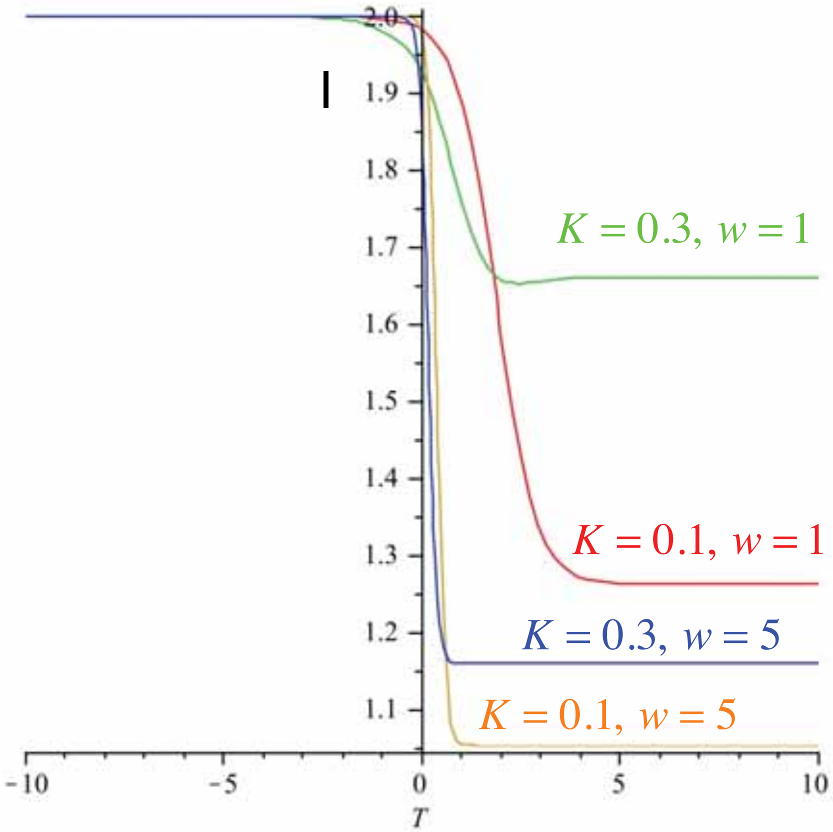
$$\rho_n = |0n\rangle\langle 0n| + \sqrt{1-|q|^2}\sqrt{n+1}\,|0n\rangle\langle 1\; n+1|$$
$$+ \sqrt{1-|q|^2}\sqrt{n+1}\,|1\; n+1\rangle\langle 0n| + \left(1-|q|^2\right)(n+1)|1\; n+1\rangle\langle 1\; n+1|$$

$$I = 1 - \frac{1}{2}\log_2\left(|q|^2\right) + \frac{1-|q|^2}{2}\sum_{n=0}^{\infty}|q|^{2n} D_n$$

$$D_n = \left(1 + \frac{n\left(1-|q|^2\right)}{|q|^2}\right)\log_2\left(1 + \frac{n\left(1-|q|^2\right)}{|q|^2}\right)$$

$$-\left(1+(n+1)\left(1-|q|^2\right)\right)\log_2\left(1+(n+1)\left(1-|q|^2\right)\right)$$





$$v = K/w$$

$$N = \log_2\left(\frac{1-|q|^2}{2} + \sum_{n=0}^{\infty}\frac{|q|^{2n}}{4}\left(1-|q|^2\right)\sqrt{Z_n}\right)$$

$$Z_n = \left(\frac{n|q|^2}{\sqrt{1-|q|^2}} + |q|^2\right)^2 + 4\left(1-|q|^2\right)$$

Generating Entanglement

Measurements of an accelerated observer generate entanglement!

Han/Olson/Dowling
Phys. Rev. A78 022302 (2008)
Mann/Ostapchuk
Phys. Rev. A79 042333 (2009)

Fermions

$$\left|0\right\rangle^{M+} = N\prod_k \exp\left(\left(\tan r\right) c_k^{I\,\dagger} d_k^{II\,\dagger}\right)\left|0_k\right\rangle_I^+ \left|0_k\right\rangle_{II}^-$$

Minkowski particle vacuum

$$\left|0\right\rangle^{M-} = N\prod_k \exp\left(\left(-\tan r\right) d_k^{I\,\dagger} c_k^{II\,\dagger}\right)\left|0_k\right\rangle_I^- \left|0_k\right\rangle_{II}^+$$

Minkowski antiparticle vacuum

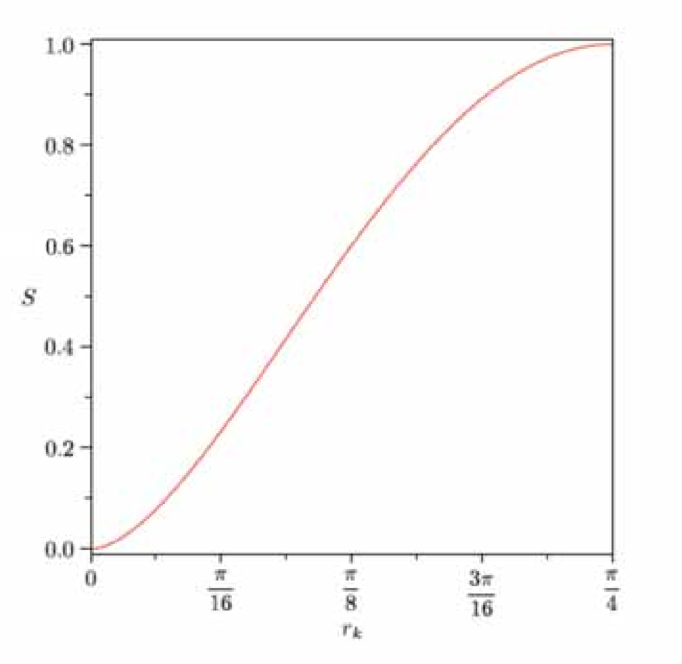
Project onto a single particle mode *k* in region I

$$\left|\psi_+(k)\right\rangle = P_k^I \left|0\right\rangle^{M+} \underset{\uparrow}{=} \left[\sin r + \cos r \; a_k^\dagger b_k^\dagger\right]\left|0\right\rangle$$

Bogoliubov tmf

Trace over antiparticle states:

$$S = \log_2\left(\csc^2 r\right) + \cos^2 r \log_2\left(\tan^2 r\right)$$



Detector Responses

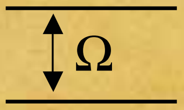
Can entanglement be used to distinguish curvature effects from Minkowski-space heating?

$$H_I(\tau)=\eta(\tau)\phi(x(\tau))\left(e^{i\Omega\tau}\sigma^{+}+e^{-i\Omega\tau}\sigma^{-}\right)$$

coupling

detector worldline

$$\Box\phi+\frac{1}{6}R\phi=0$$



Compare:

- Minkowski space at finite temperature T

$$ds^2=-dt^2+d\vec{x}\bullet d\vec{x}$$

- de Sitter space with expansion rate κ=2πT

$$ds^2=-dt^2+e^{-2\kappa t}d\vec{x}\bullet d\vec{x}$$

A single detector can’t distinguish these 2 possibilities

Gibbons/Hawking
Phys. Rev. D15 2738 (1977)

$$\rho=A|1\rangle\langle 1|+(1-A)|0\rangle\langle 0|$$

$$A=\int_{-\infty}^{\infty}d\tau\int_{-\infty}^{\infty}d\tau'\eta(\tau)\eta(\tau')e^{-i\Omega(\tau-\tau')}D^{+}(x,x')$$

$$D^{+}(x,x')=\langle\phi(\tau)\phi(\tau')\rangle=-\frac{T^2}{4\sinh^2(\pi T(t-t'-i\varepsilon))}$$

What about 2 detectors?

Negativity

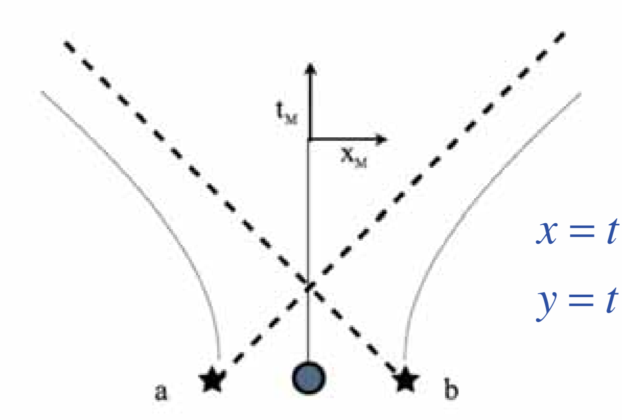
Menicucci/Ver Steeg
Phys. Rev. D79 042007 (2009)

amplitude detector clicks

$$N = \max\left(\left|X\right| - A, 0\right)$$

amplitude for virtual particle exchange

$$X = -2\int_{\tau' < \tau}^{\infty} d\tau d\tau' \eta(\tau)\eta(\tau') e^{-i\Omega(\tau+\tau')} D^{+}(x,x')$$

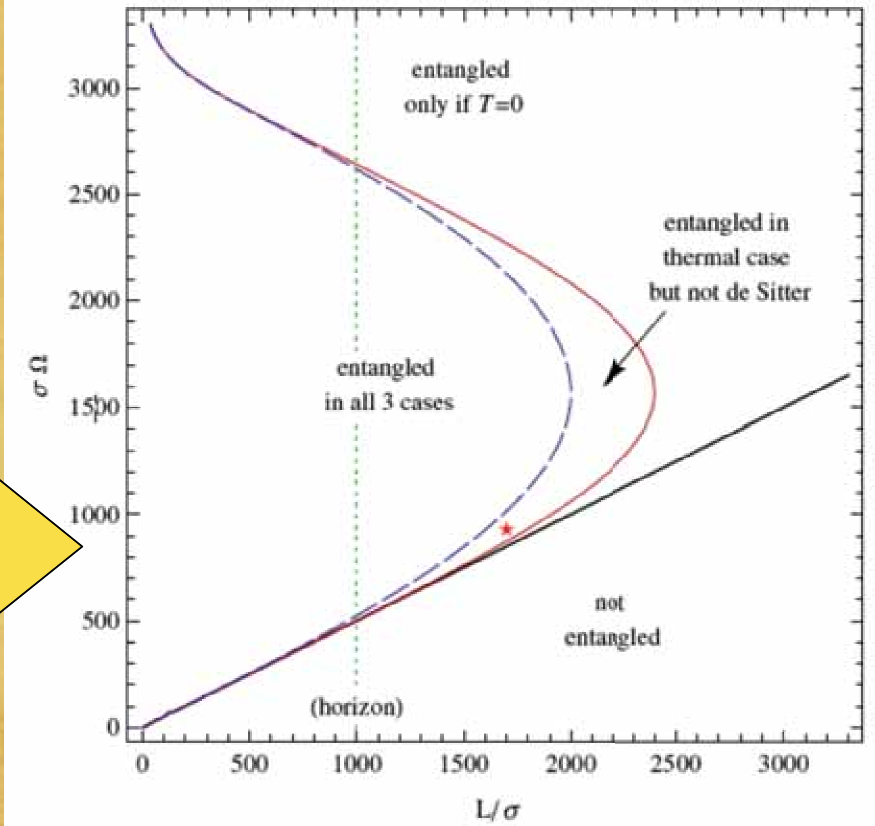


$x = t + t'$

$y = t - t' - i\varepsilon$

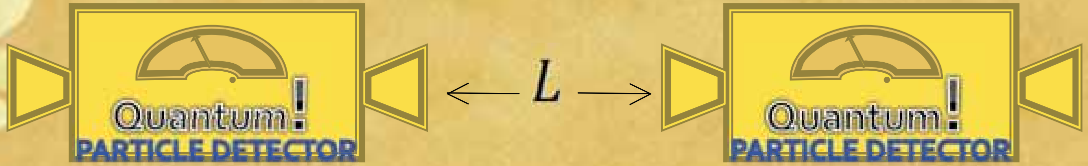
$$D^{+}_{\text{Mink}}(x,x') = -\frac{T}{8\pi L}\left[\coth\left(\pi T(L-y)\right) + \coth\left(\pi T(L-y)\right)\right]$$

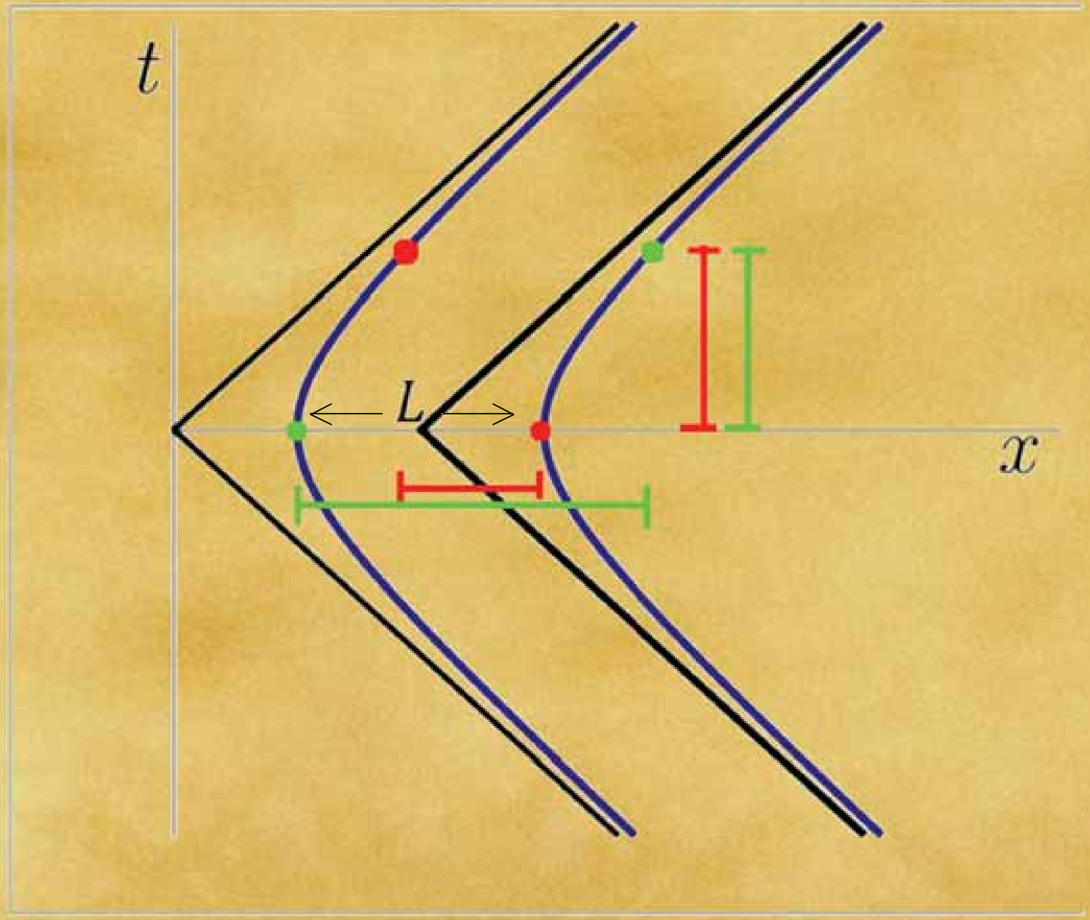
$$D^{+}_{\text{dS}}(x,x') = -\frac{T}{4\pi^2}\left(\frac{\sinh^2(\pi T y)}{\pi^2 T^2} - e^{2\pi T x} L^2\right)^{-1}$$



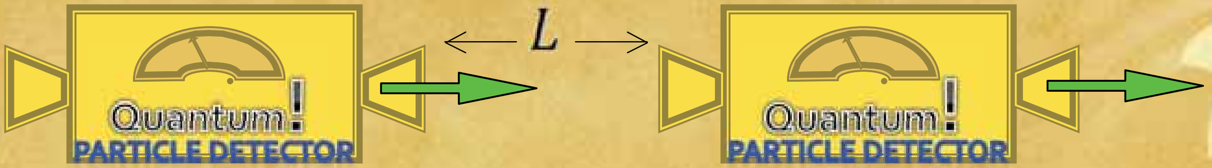
Mann/Menicucci/Salton

Now compare 2 uniformly accelerated detectors

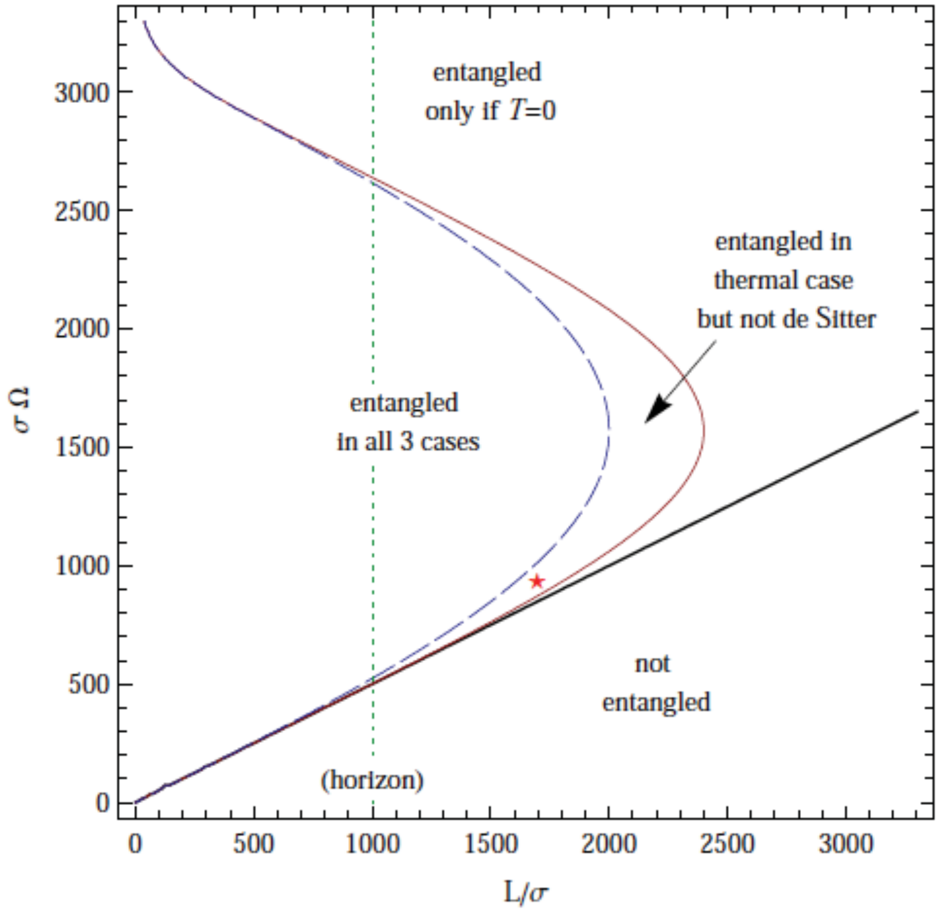




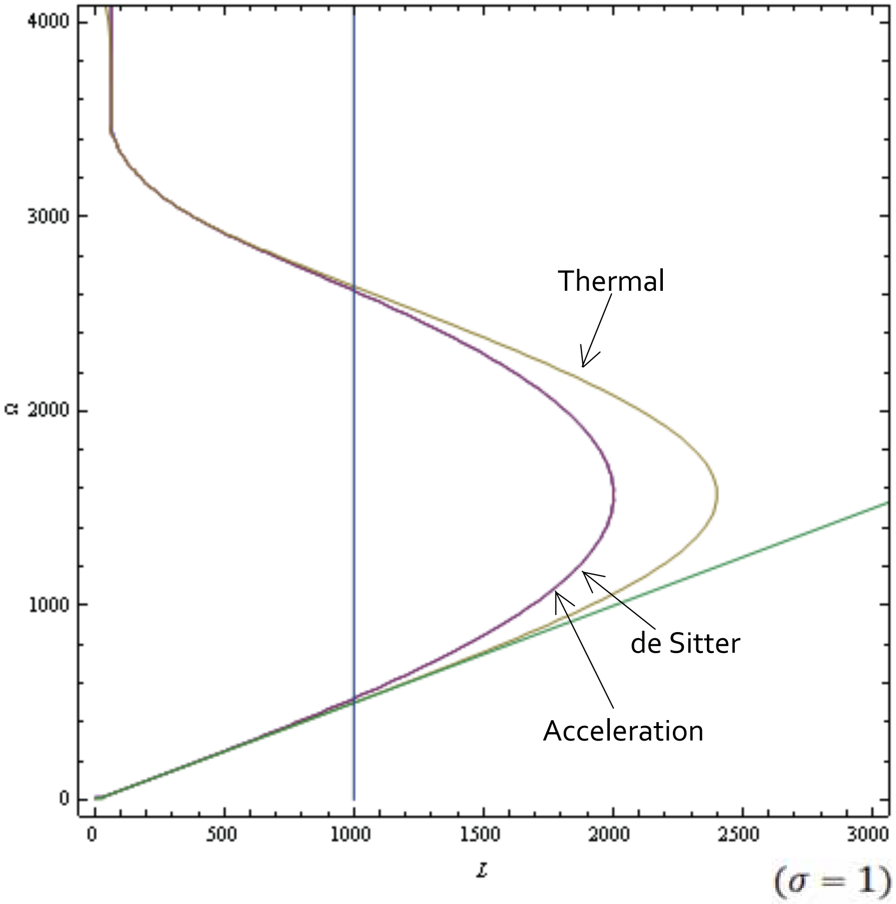
Same acceleration



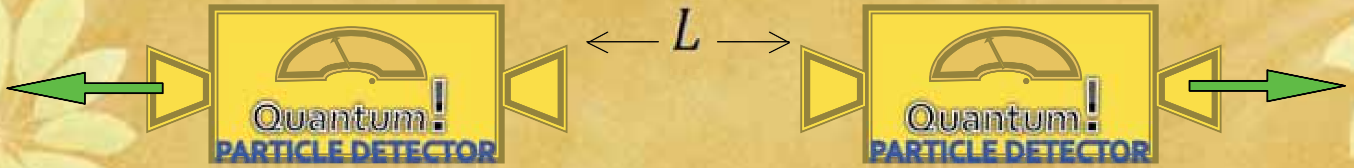
Thermal and de Sitter (Ver Steeg and Menicucci, 2009)

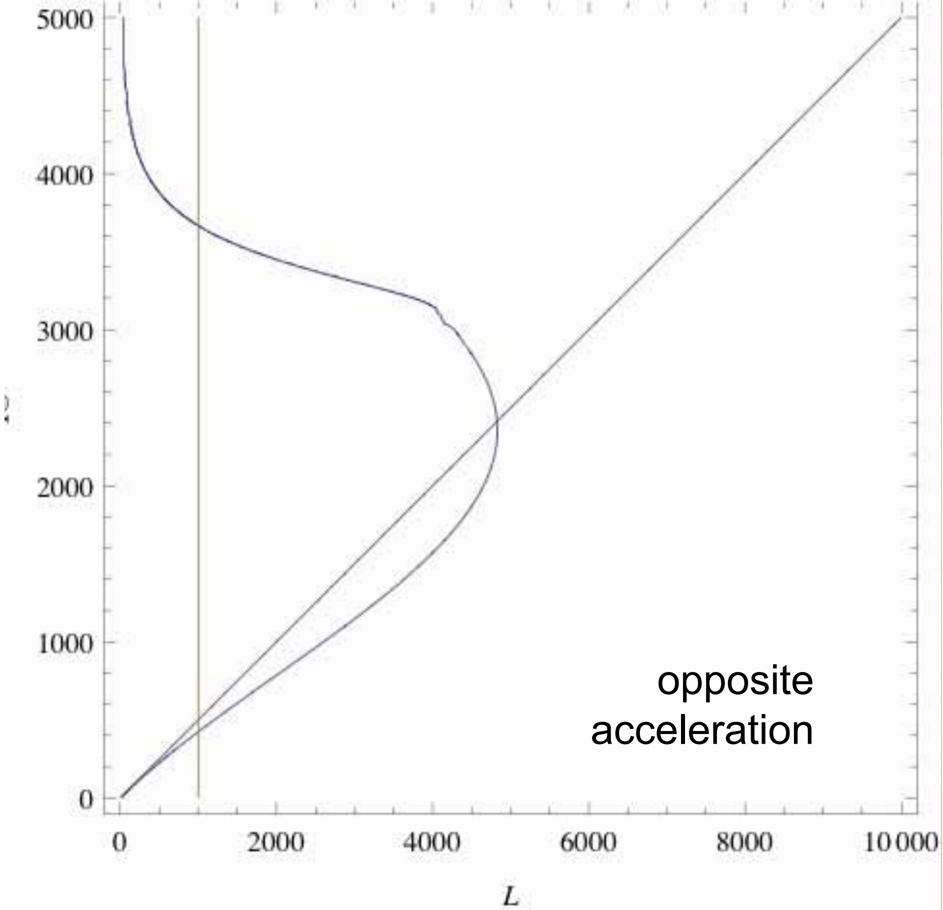


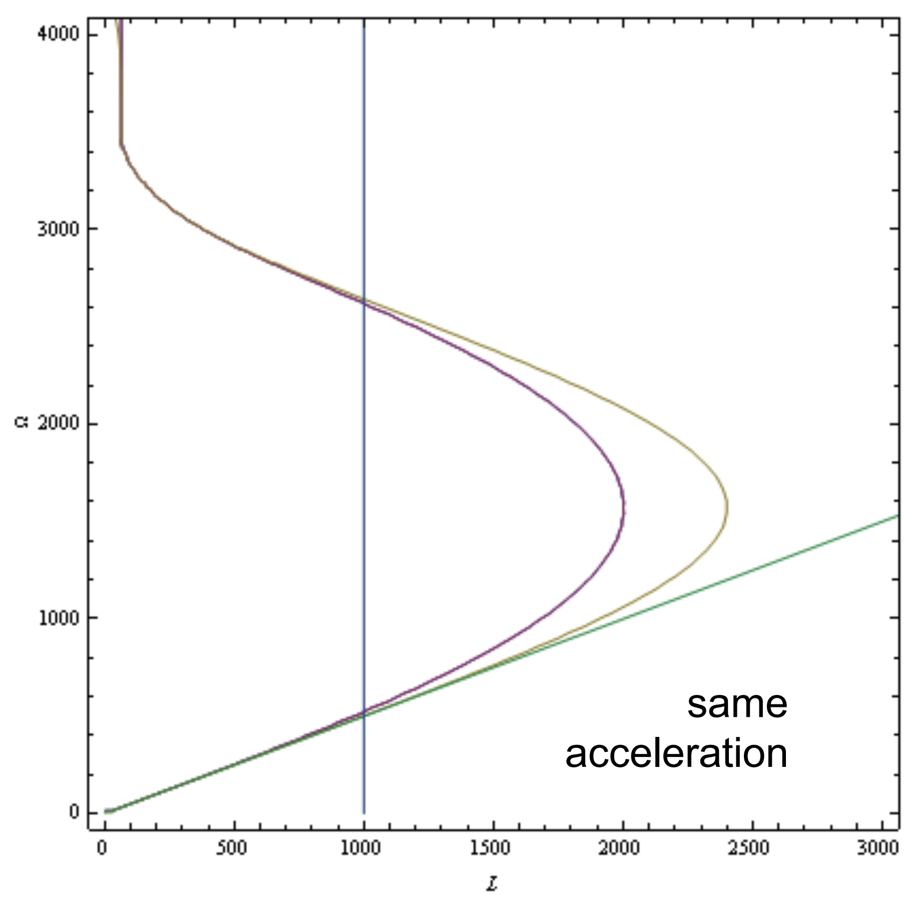
Thermal, de Sitter and Acceleration



Opposite acceleration

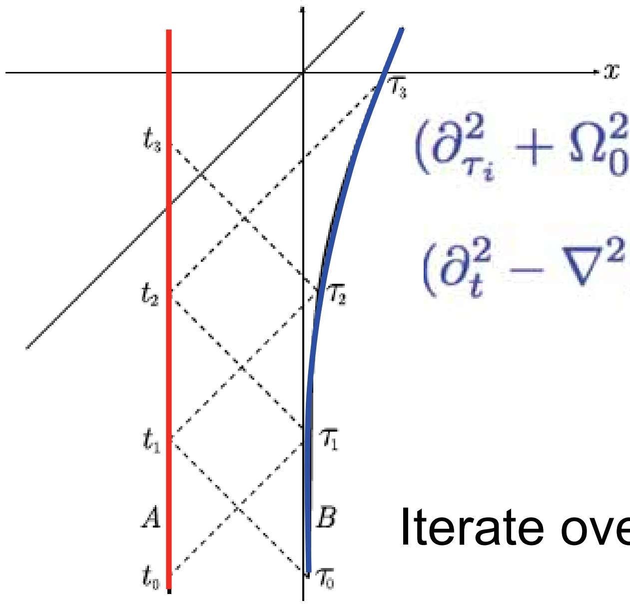






Hu/Haque/Lin/Mann/Ostapchuk

Alice and Rob each have a detector that couples to a scalar field



$$(\partial_{\tau_i}^2 + \Omega_0^2)\hat{Q}_i(\tau_i) = \frac{\lambda_0}{m_0}$$

Driven Damped Harmonic Oscillator

Frequency Ω

Damping Coefficient ~ λ^2_0

$$(\partial_t^2 - \nabla^2)\hat{\Phi}(x) = \lambda_0 \left\{ \int_{\tau_A(t_0)}^{\infty} d\tau_A\, \hat{Q}_A(\tau_A)\delta^4(x - z_A(\tau_A)) + \int_{\tau_\psi(t_0)}^{\infty} d\tau_B\, \hat{Q}_B(\tau_B)\delta^4(x - z_B(\tau_\psi)) \right\}$$

Iterate over retarded mutual influences in powers of λ_0

Compute Correlation Matrix $\mathrm{V} = \langle R_\mu R_\nu \rangle = \langle AB | R_\mu R_\nu | AB \rangle + \langle 0 | R_\mu R_\nu | 0 \rangle$

detectors vacuum

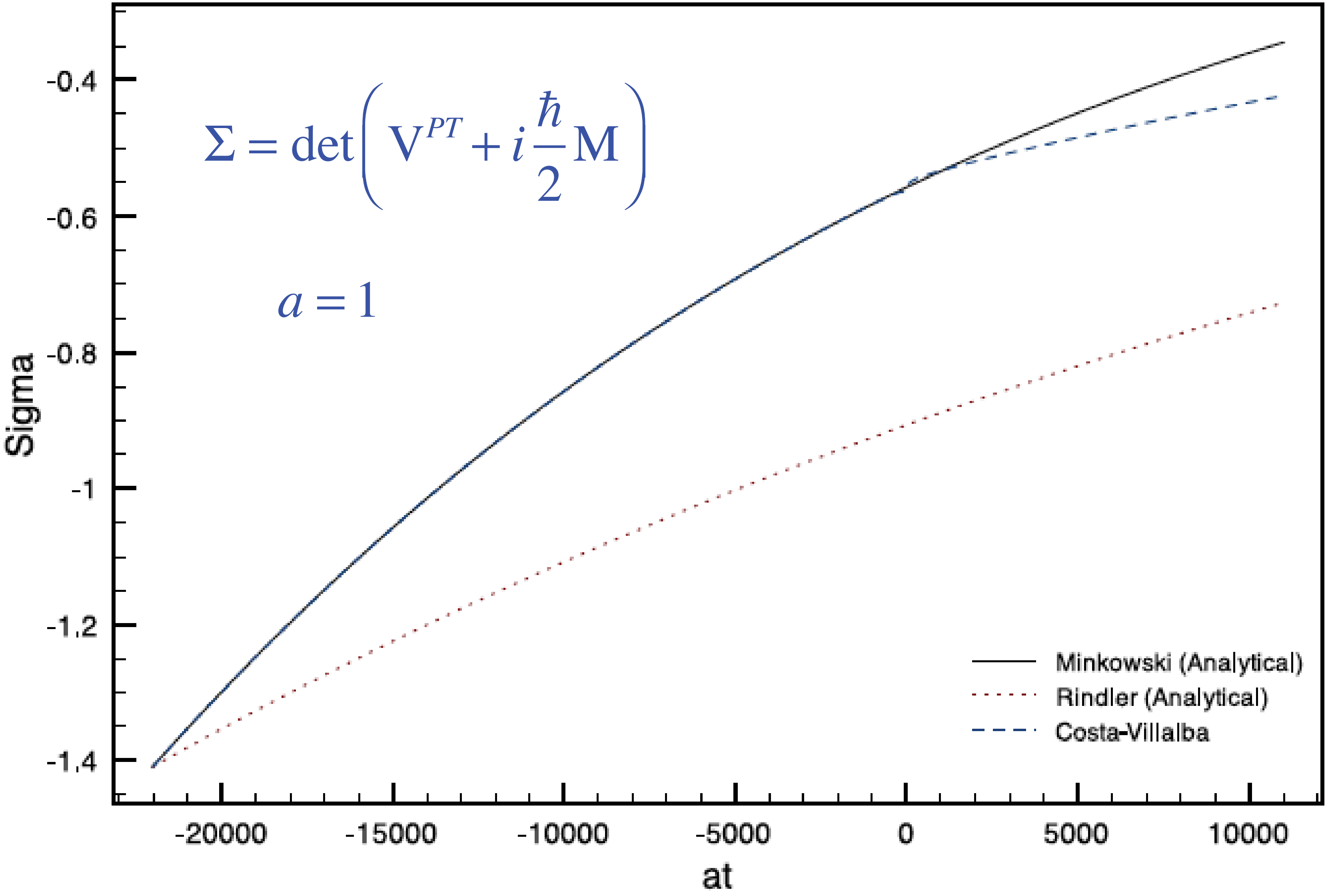
$$R_\mu = \{Q_A, P_A, Q_B, P_B\}$$

$$V^{PT} = \Lambda V \Lambda$$

Entanglement

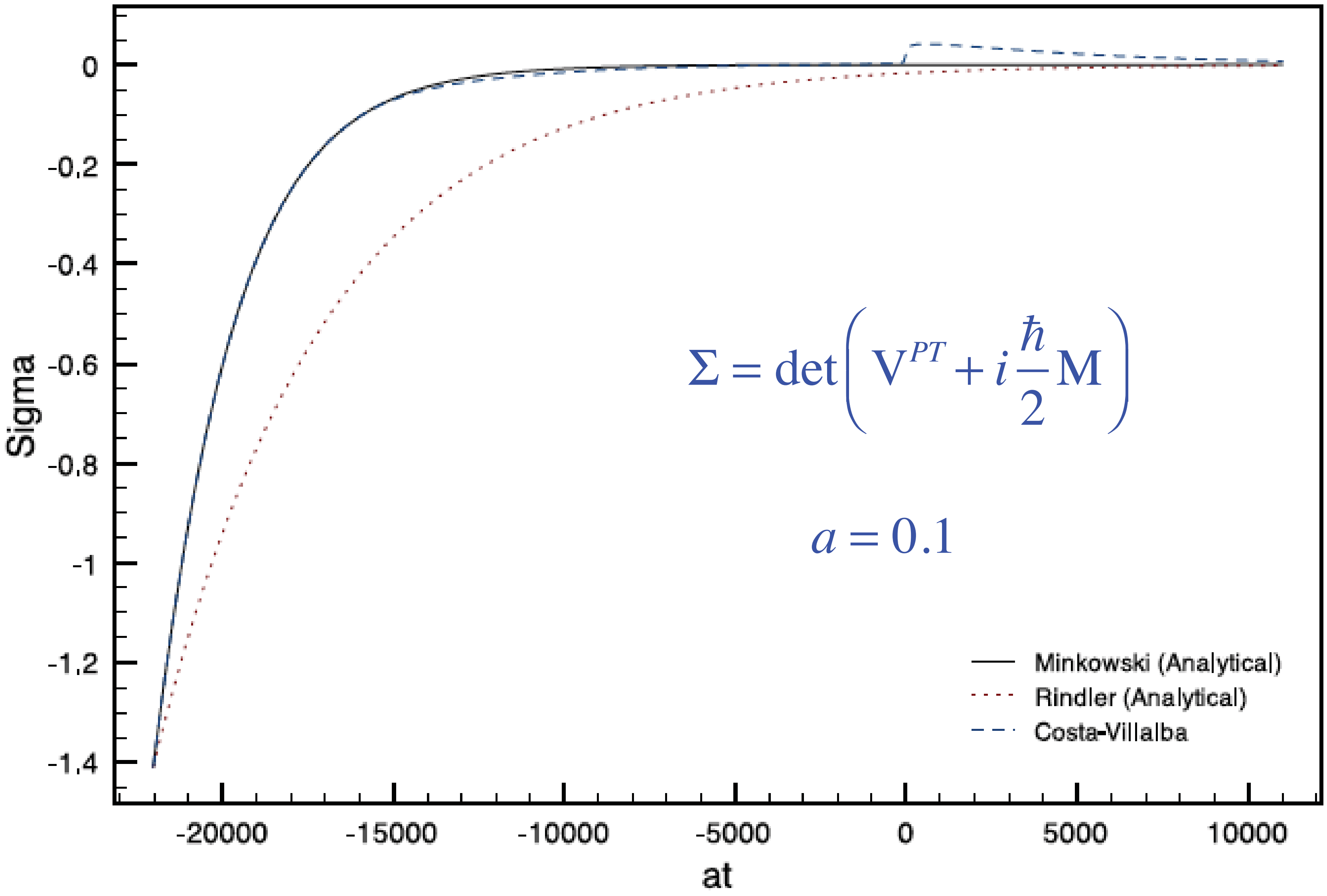
$$\Sigma = \det\left(\mathrm{V}^{PT} + i\frac{\hbar}{2}\mathrm{M}\right) < 0$$

$$\Lambda = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \qquad M = \begin{pmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \end{pmatrix}$$



$$\Sigma = \det\left(V^{PT} + i\frac{\hbar}{2}M \right)$$

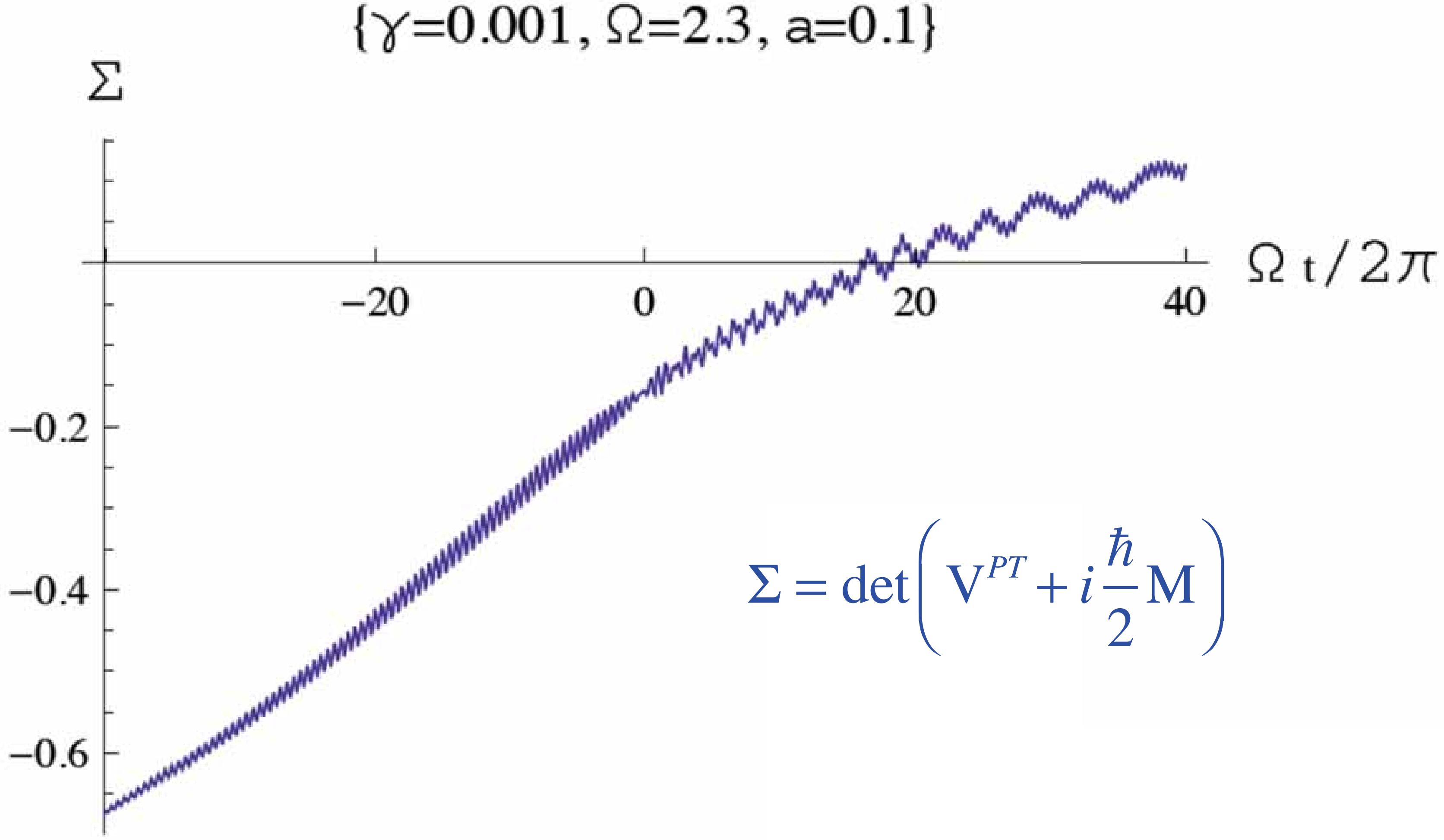
$$a = 1$$



$$\Sigma = \det\left(\mathrm{V}^{PT} + i\frac{\hbar}{2}\mathrm{M} \right)$$

$$a = 0.1$$

Better Regulator



$$\Sigma = \det\left(V^{PT} + i\frac{\hbar}{2} M \right)$$

Summary

- Entanglement is an observer/detector dependent phenomenon
- As acceleration increases
  - Entanglement goes to
    - zero (bosons)
    - finite (fermions)
  - Mutual information goes to unity
  - Entanglement with region II gets larger
- As observer increases acceleration
  - Entanglement decreases with time
  - Maximum change takes place when change in acceleration is maximal