ENTANGLEMENT OF BOSONIC AND FERMIONIC FIELDS IN AN EXPANDING UNIVERSE

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OUTLINE

- Motivation
- PART ONE: theoretical considerations
    - entanglement basics (very short)
    - quantum field theory in curved spacetime basics
- PART TWO: entanglement in an expanding Universe
    - bosonic fields
    - fermionic fields

MOTIVATION

- Entanglement is a key resource in quantum information theory which has mainly been studied in flat spacetime
- However spacetime is generally curved
- Questions:

  Does the underlying curvature has effects on entanglement?

  Can the dynamics of spacetime create or destroy entanglement?

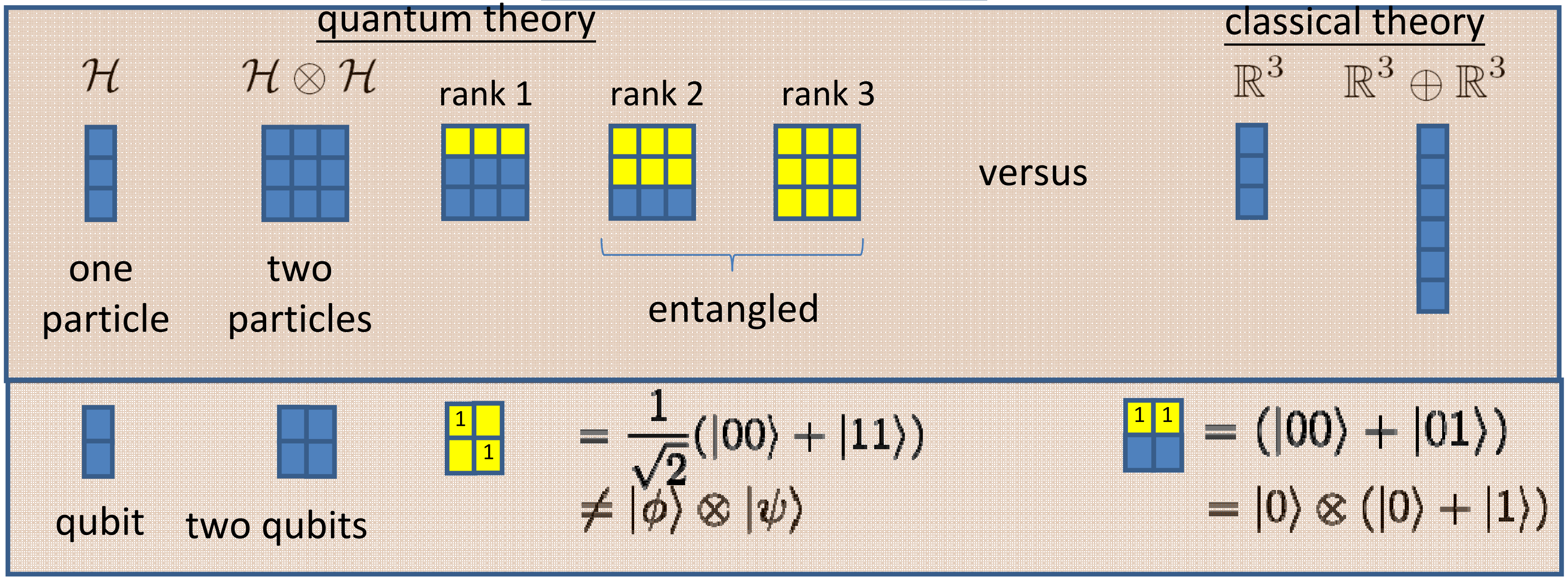
  Entanglement is hard to defined in curved spacetime but what can we learn about it?

PART ONE

THEORETICAL CONSIDERATIONS

ENTANGLEMENT AND ITS USES

COMPOSITE SYSTEMS



HOW DOES THIS WORK IN A RELATIVISTIC QUANTUM THEORY?

QUANTIFYING ENTANGLEMENT

BIPARTITE PURE STATES

Schmidt basis

$$|\Phi\rangle_{AB} = \sum_{ij} \omega_{ij} |i\rangle_A \otimes |j\rangle_B \quad \Rightarrow \quad |\Phi\rangle_{AB} = \sum_n \omega_n |n\rangle_A \otimes |n\rangle_B$$

density matrix

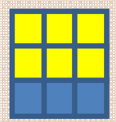
$$\rho_{AB} = |\Phi\rangle\langle\Phi|_{AB}$$

reduced density matrix

$$\rho_A = \mathbf{Tr}_B(\rho_{AB})$$

ENTANGELMENT

between A and B



$$S(\rho_A) = S(\rho_B)$$

von Neumann entropy

$$S(\rho) = -\mathbf{Tr}(\rho \log_2(\rho))$$

PARTICLES FROM FIELDS

Quantum field theory on curved spacetime

↓

Quantum field fundamental • Particles derived notion (if at all)

QUANTUM FIELD THEORY

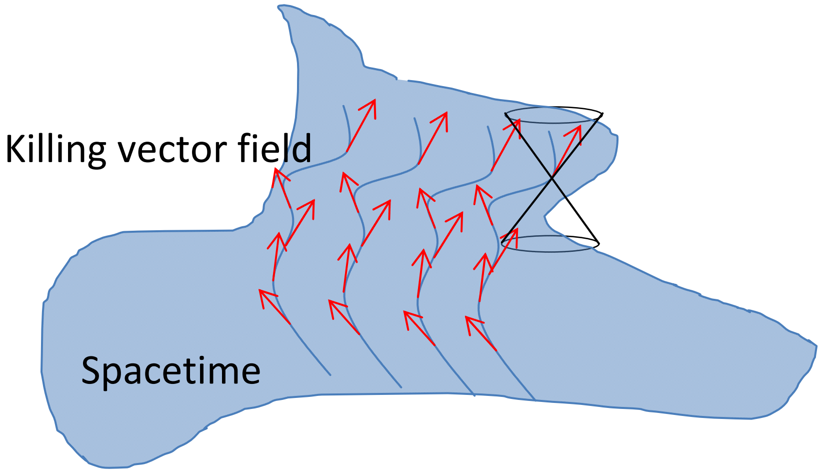
linear field equation $(\Box + m^2)\Phi = 0$

↓

vectorspace of solutions $\mathrm{span}\{\tilde{\Phi}_k\}$

$\tilde{\Phi} \in \mathcal{H} \quad (\phi, \psi) = i \int_\Sigma d\Sigma^\mu [\psi^* \partial_\mu \phi - \phi \partial_\mu \psi^*]$

Inner product not positive definite!



PARTICLE INTERPRETATION

requires classification of modes into positive ↔ negative frequency ω

$$i\partial_T \tilde{\Phi} = \omega \tilde{\Phi} \quad \begin{cases} \tilde{\Phi}^* & \text{pos.} \\ \Updownarrow & \\ \tilde{\Phi} & \text{neg.} \end{cases}$$

↑ timelike Killing vector field

$$\mathcal{F} = \mathbb{C} \oplus \mathcal{H} \oplus \mathcal{H} \circ \mathcal{H} \oplus \ldots \oplus \mathcal{H}^n$$

boson Fock space

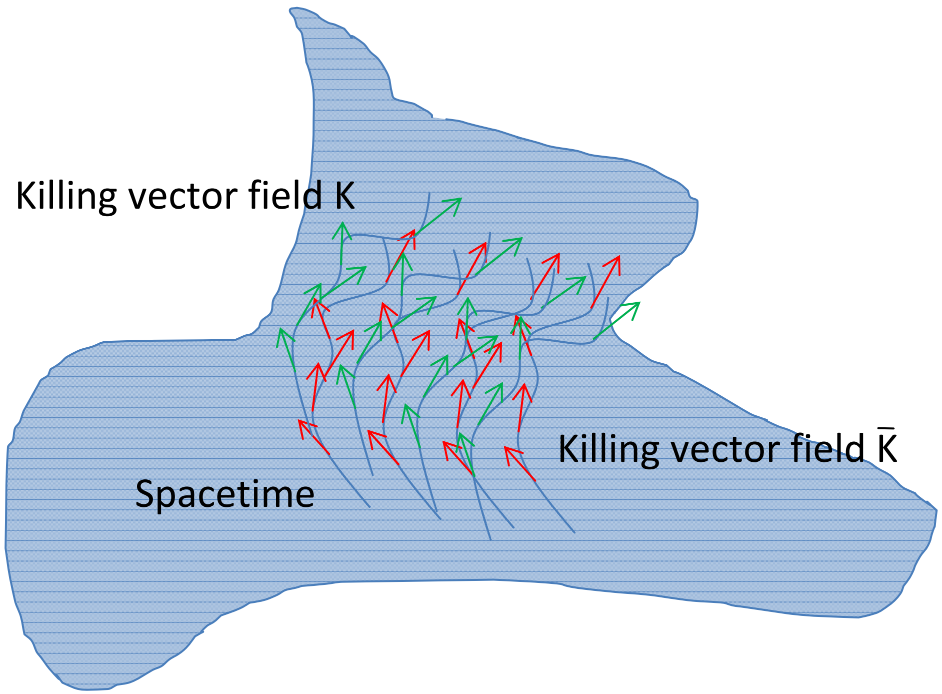
$$\phi(x) = \int dk \left[a_k^+ \tilde{\Phi}_k + a_k^- \tilde{\Phi}_k^* \right]$$

↑ creation ↑ annihilation

KILLING OBSERVERS

INSIGHTS

- particles present ill-defined subsystems!
- particles well-defined only for killing observers
- particle interpretation may change with change of Killing vector field



KILLING OBSERVERS

different timelike Killing vectors K and $\bar{K}$

$\Rightarrow$ different splits of basis in pos/neg

$$\{u_p, u_p^*\} \longrightarrow \{\bar{u}_p, \bar{u}_p^*\}$$

Bogoliubov transformation

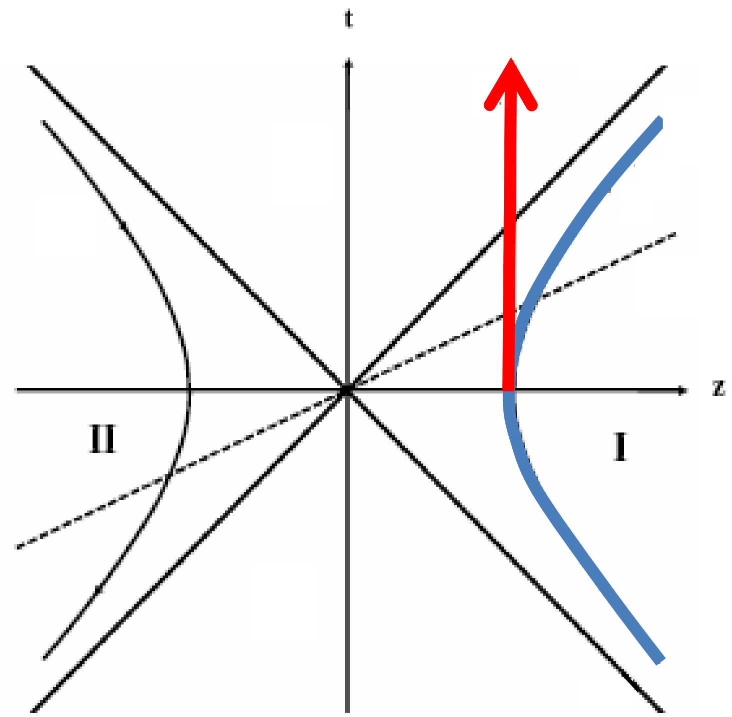
$$\bar{a}_p = \int_{q \in \mathcal{P}} [\alpha_{pq}^* a_q - \beta_{pq}^* a_q^\dagger],$$

Squeezed states

$$|0\rangle = e^{\sum_{i \neq j} \gamma_{ij} a_i^\dagger a_j^\dagger - \gamma_{ij}^* a_i a_j} |\bar{0}\rangle$$

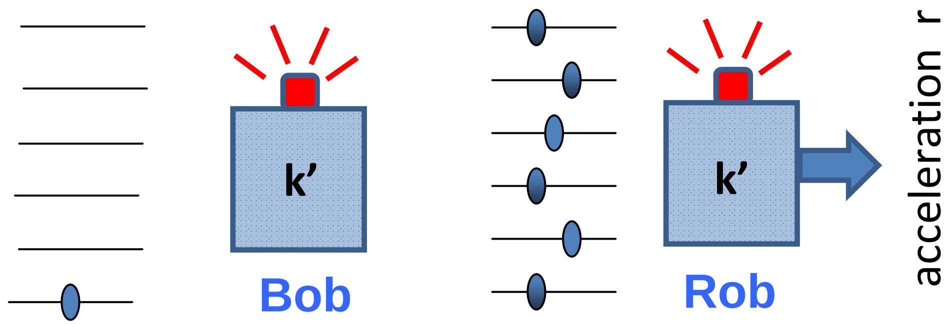
EXAMPLE: UNRUH EFFECT

Minkowski spacetime in 1+1 dimensions (flat spacetime = no gravity!)



$|0_k\rangle^{\mathcal{M}}$

Rob is causally disconnected from region II



<u>Timelike killing observers</u>

(a) inertial observer

(b) uniformly accelerated observers

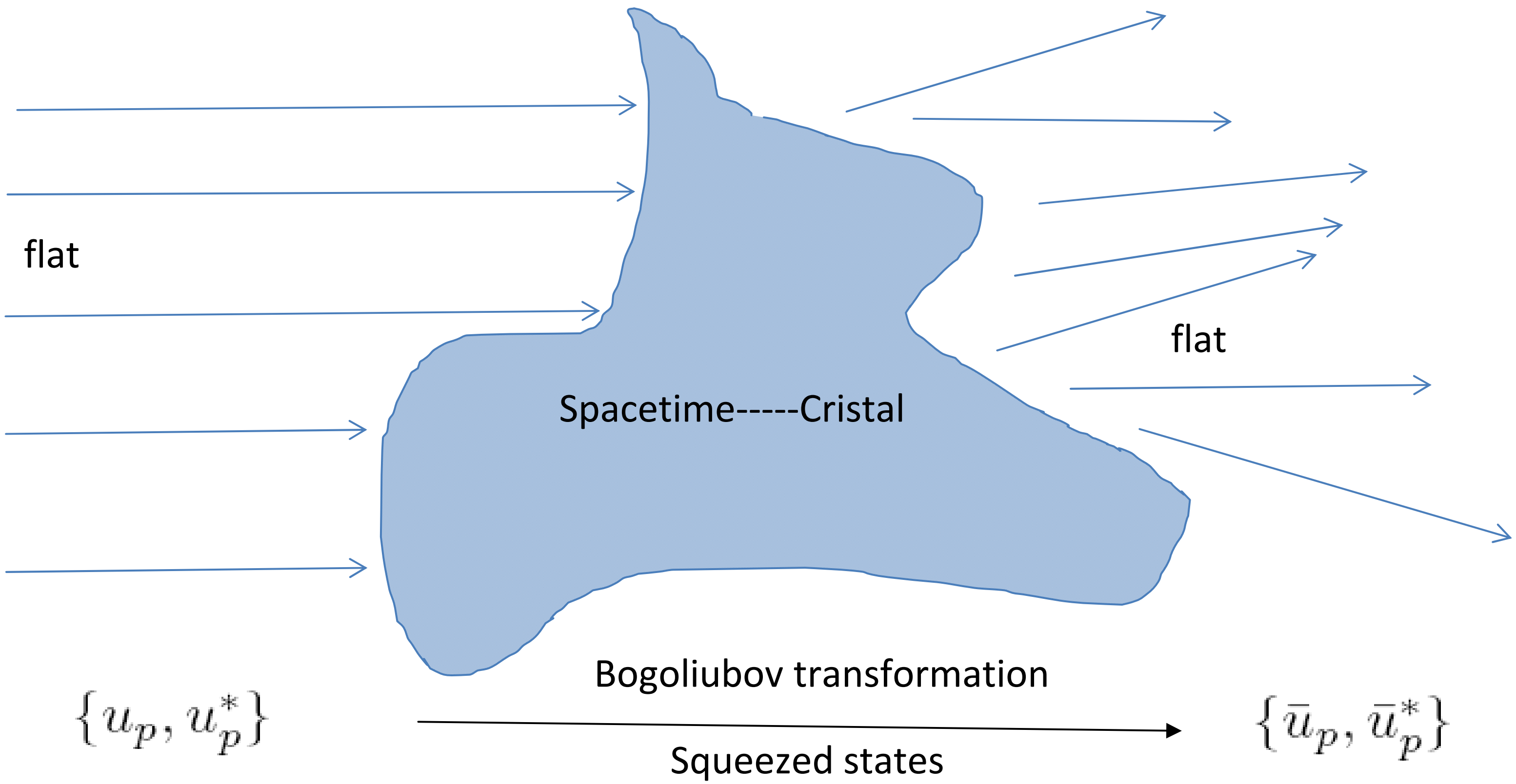
trace → thermal state

Similar effect in black holes: Hawking radiation

SPACETIME AS A CRISTAL

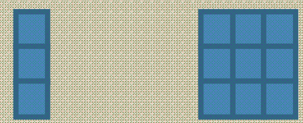
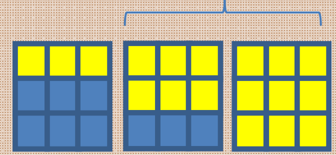
Curve spacetimes generally do not admit timelike killing vector fields…

particular spacetimes with asymptotically flat regions



Just like in quantum optics!

THE CHALLENGE

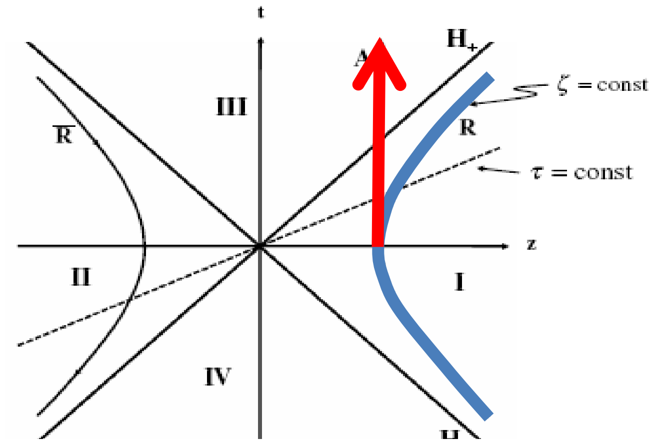
Theory	Particles	Particle number	Mathematics	Effects
non-relativistic	massive	obs-independent conserved	$\mathcal{H}$ $\quad$ $\mathcal{H} \otimes \mathcal{H}$ 	entangled 

ENTANGLEMENT IN QUANTUM FIELD THEORY

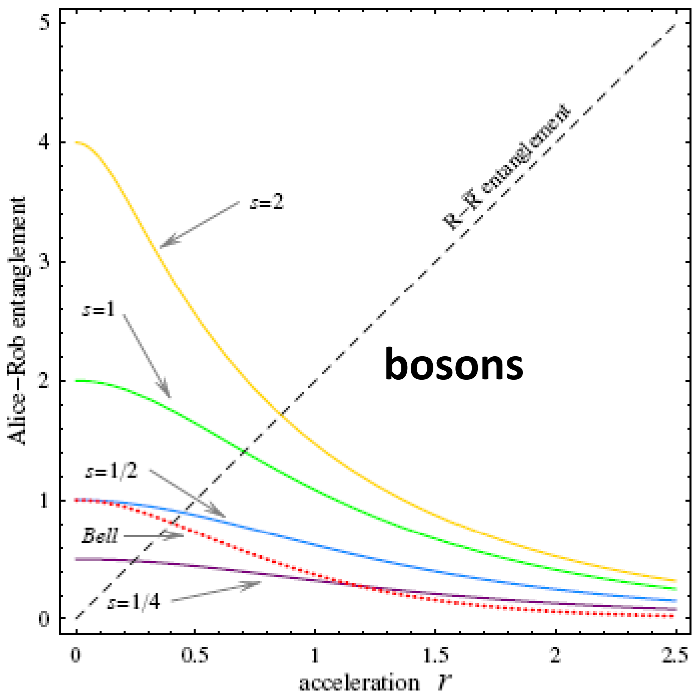
IFS, Mann PRL (2005)
Adesso, IFS , Ericsson PRA (2007)
Alsing, IFS, Mann, Tessier PRA (2006)

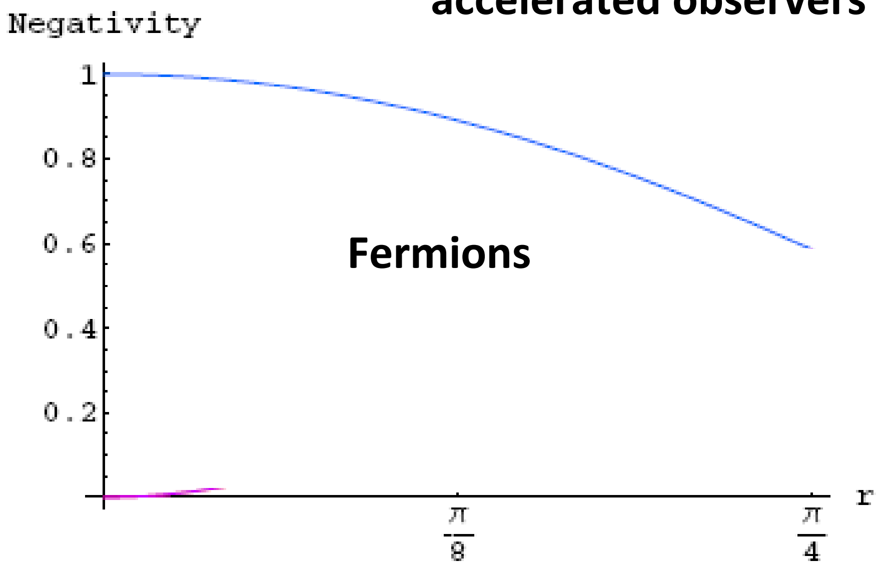
RESULTS ON ENTANGLEMENT IN FLAT SPACETIME

- Entanglement is observer dependent
- It can be degraded due to horizons
- Fermionic and Bosonic entanglement is very different



accelerated observers





PART TWO

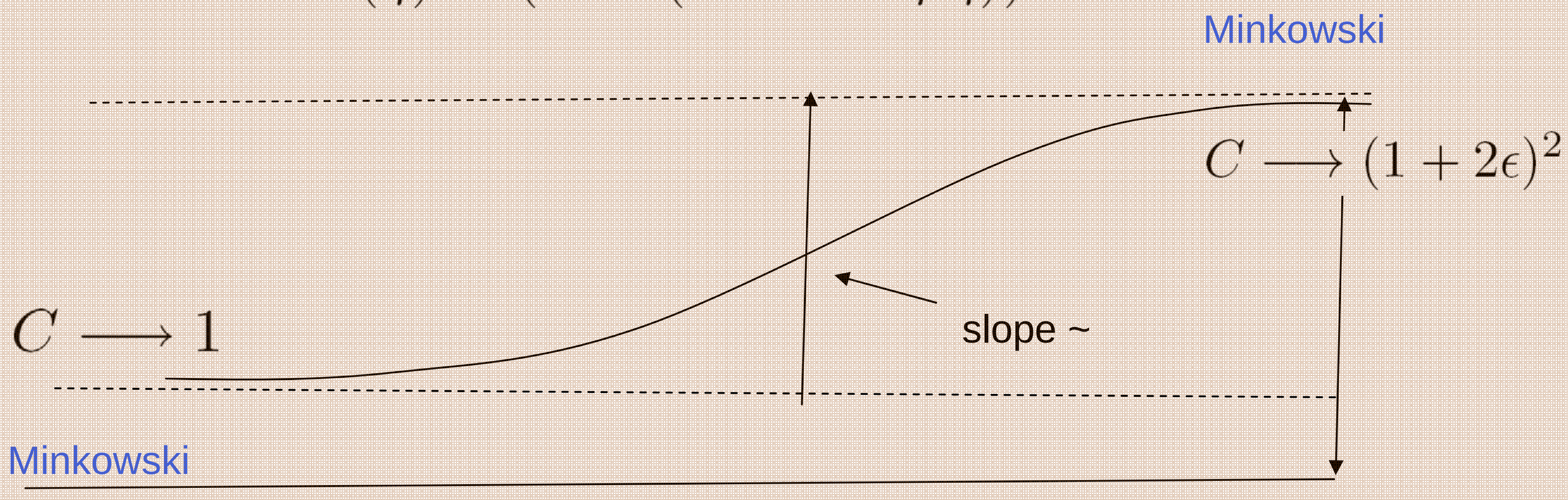
ENTANGLEMENT IN AN EXPANDING SPACETIME

ROBERTSON-WALKER UNIVERSE

spacetime with metric $ds^2 = C(\eta)(-d\eta^2 + dx_i dx^i)$

EXPANDING UNIVERSE

$$C(\eta) = (1 + \epsilon(1 + \tanh \rho\eta))^2$$



COSMOLOGYCAL PARAMETERS

expansion rate ρ

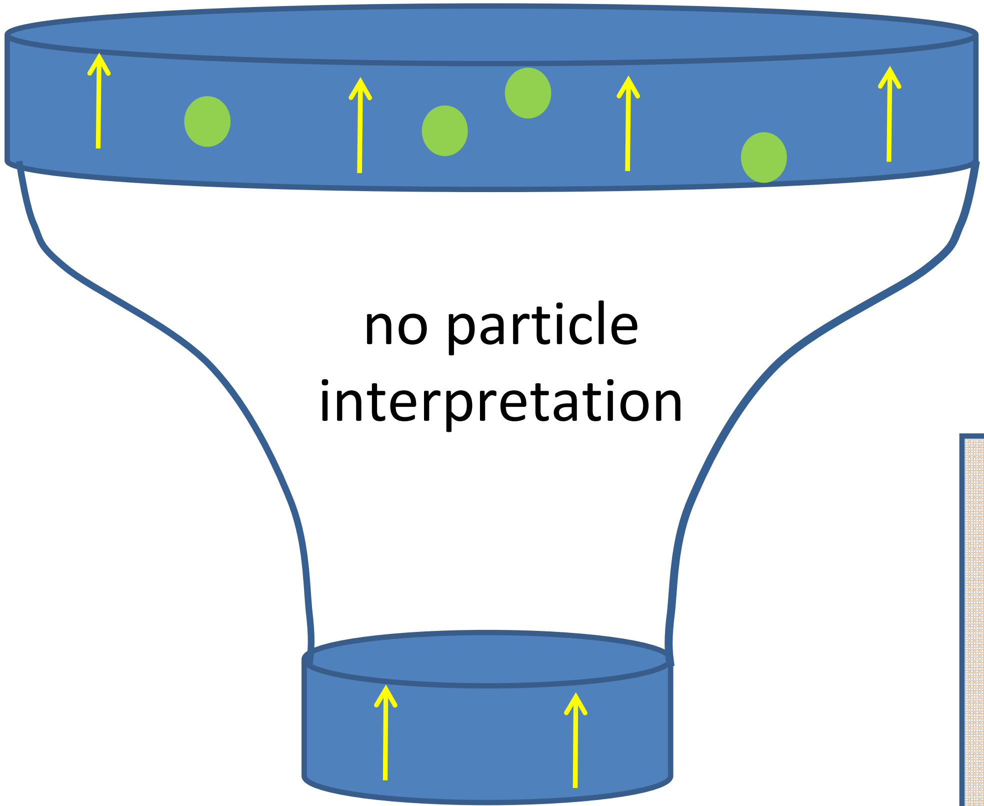
expansion factor ε

TIMELIKE KILLING VECTOR FIELD

$$\pm\partial\eta$$

PARTICLE CREATION AND ENTANGLEMENT

particle interpretation in the asymptotic past and future



QUANTIFYING ENTANGLEMENT FOR BOSONS AND FERMIONS

- asymptotic past $S = 0$

  asymptotic future $S = S(\sigma, \epsilon)$

- excitingly, can solve for

$$\epsilon = \epsilon(S)$$

FERMIONIC IN AND OUT STATES

Duncan PRD 1978

DIRAC EQUATION

$$\{i\gamma^\mu(\partial_\mu - \Gamma_\mu) + m\}\psi = 0.$$

1. SOLVE THE EQUATION IN THE GIVEN SPACE-TIME

Exploiting spatial translational invariance

$$\psi_k(\eta, x) = e^{i\mathbf{k}\cdot\mathbf{x}} C^{(1-d)/4} \left(\bar{\gamma}^0 \partial_\eta + i\bar{\gamma} \cdot \mathbf{k} - mC^{1/2}\right) \phi_k(\eta)$$

therefore

Flat spacetime matrices

$$\left(\partial_\eta^2 + m^2 C \pm im\dot{C}C^{-1/2} + |\mathbf{k}|^2\right) \phi_k^{(\pm)} = 0$$

where

$$k = |\mathbf{k}| = \left(\sum_{i=1}^{d-1} k_i^2\right)$$

2. IN THE ASYMPTOTIC LIMIT IDENTIFY POSITIVE AND NEGATIVE SOLUTIONS

$$\eta \to -\infty \qquad \phi_{in}^{(\pm)} \sim e^{\pm i\omega_{in}\eta}$$

$$\eta \to +\infty \qquad \phi_{out}^{(\pm)} \sim e^{\pm i\omega_{out}\eta}$$

$$\begin{aligned} \omega_{in} &= (|\mathbf{k}|^2 + \mu_{in}^2)^{1/2} \\ \omega_{out} &= (|\mathbf{k}|^2 + \mu_{out}^2)^{1/} \\ \mu_{in} &= mC^{1/2}(-\infty), \\ \mu_{out} &= mC^{1/2}(+\infty). \end{aligned}$$

therefore the in and out solution are

$$\phi_{in}^{(\pm)} = \exp\left(-i\omega_+\eta - \frac{i\omega_-}{\rho}\ln[2\cosh\rho\eta]\right) F_1\left(1 + \frac{i\omega_-}{\rho} \pm \frac{im\epsilon}{\rho}, \frac{i\omega_-}{\rho} \mp \frac{im\epsilon}{\rho}, 1 - \frac{i\omega_{in}}{\rho}, \frac{1+\tanh(\rho\eta)}{2}\right)$$

$$\phi_{out}^{(\pm)} = \exp\left(-i\omega_+\eta - \frac{i\omega_-}{\rho}\ln[2\cosh\rho\eta]\right) F_1\left(1 + \frac{i\omega_-}{\rho} \pm \frac{im\epsilon}{\rho}, \frac{i\omega_-}{\rho} \mp \frac{im\epsilon}{\rho}, 1 + \frac{i\omega_{out}}{\rho}, \frac{1-\tanh(\rho\eta)}{2}\right)$$

where $\omega_\pm = (\omega_{out} \pm \omega_{in})/2$

and F1 are hypergeometric functions

3. EXPRESS THE FIELD IN TERMS OF THE IN AND OUT SOLUTIONS WITH THE CORRESPONDING CREATION AND ANNIHILATION OPERATORS

$$\psi(x) = (2\pi)^{-(1-d)/2} \int d^{d-1}k \left[\frac{\mu_{in}}{\omega_{in}}\right] \sum_{\lambda=1}^{d/2-1} \{a_{in}(\mathbf{k},\lambda) U_{in}(\mathbf{k},\lambda;\mathbf{x},\eta) + b_{in}^{\dagger}(\mathbf{k},\lambda) V_{in}(\mathbf{k},\lambda;\mathbf{x},\eta)\}$$

where the curved spacetime spinor solutions are

$$\begin{aligned} U_{in}(\mathbf{k},\lambda;x,t) &\equiv K_{in}(k) C^{(1-d)/4}(\eta)(-i\partial_t + i\mathbf{k}\cdot\bar{\gamma} - mC^{1/2})\phi_k^{in(-)(\eta)} e^{i\mathbf{k}\cdot\mathbf{x}} u(0,\lambda) \\ V_{in}(\mathbf{k},\lambda;x,t) &\equiv K_{in}(k) C^{(1-d)/4}(\eta)(i\partial_t - i\mathbf{k}\cdot\bar{\gamma} - mC^{1/2})\phi_k^{in(+)*(\eta)} e^{-i\mathbf{k}\cdot\mathbf{x}} v(0,\lambda) \end{aligned}$$

flat spacetime spinors

with

$$K_{in/out} = \frac{1}{|k|}\left(\frac{\omega_{in/out}(k) - \mu_{in/out}}{\mu_{in/out}}\right)^{1/2}.$$

4. USE INNER PRODUCT TO FIND BOGOLIUBOV TRANSFORMATIONS BETWEEN THE IN AND OUT SOLUTIONS

$$\phi_{in}^{(\pm)} = \alpha_k^{(\pm)} \phi_{out}^{(\pm)} + \beta_k^{(\pm)} \phi_{out}^{(\mp)*}$$

5. FIND TRANSFORMATION BETWEEN THE OPERATORS

$$b_{in}(\bar{k}) = A(\alpha^-(k)^* b_{out}(\bar{k}) \; + \; \beta^-(k)^* \chi(\bar{k}) a_{out}^\dagger(-\bar{k})).$$

particles

$$a_{out} = \left(\frac{\mu_{in}\omega_{out}}{\omega_{in}\mu_{out}}\right)^{\frac{1}{2}} \frac{K_{in}}{K_{out}} \left(\alpha_k^{(-)} a_{in} + \beta_k^{(-)*} \sum_{\lambda'} X_{\lambda\lambda'}(-\mathbf{k}) b_{in}^\dagger(-\mathbf{k}, \lambda')\right)$$

antiparticles

$$b_{out} = \left(\frac{\mu_{in}\omega_{out}}{\omega_{in}\mu_{out}}\right)^{\frac{1}{2}} \frac{K_{in}}{K_{out}} \left(\alpha_k^{(-)} b_{in} + \beta_k^{(-)*} \sum_{\lambda'} X_{\lambda\lambda'}(-\mathbf{k}) a_{in}^\dagger(-\mathbf{k}, \lambda')\right)$$

where

$$X_{\lambda\lambda'}(-\mathbf{k}) = -2\mu_{out}^2 K_{out}^2 \bar{u}_{out}(-\mathbf{k}, \lambda') v(0, \lambda).$$

6. FIND TRANSFORMATION BETWEEN THE STATES

$$0 = b_{in}(\bar{k})|0\rangle_{in} = \alpha^-(k)^* A_1|1_{-k}\rangle + \beta^-(k)^* \chi(\bar{k}) A_0 |1_{-k}\rangle$$

$$A_1 = -\frac{\beta^-(k)^*}{\alpha^-(k)^*}\chi(\bar{k})A_0 = -\gamma^{-*}\chi(\bar{k})A_0$$

where

$$|\gamma^-|^2 \equiv |\frac{\beta_k^{(-)}}{\alpha_k^{(-)}}|^2 = \frac{(\omega_- + m\epsilon)(\omega_+ + m\epsilon)\sinh\frac{\pi}{\rho}(\omega_- - m\epsilon)\sinh\frac{\pi}{\rho}(\omega_- + m\epsilon)}{(\omega_- - m\epsilon)(\omega_+ - m\epsilon)\sinh\frac{\pi}{\rho}(\omega_+ + m\epsilon)\sinh\frac{\pi}{\rho}(\omega_+ - m\epsilon)}$$

therefore,

$$|0\rangle_{in} = \Pi_k \frac{1}{(1 + |\gamma^-(k)\chi(\bar{k})|^2)^{1/2}}(|0\rangle_{out} - \gamma^{-*}(k)\chi(\bar{k})|1_k 1_{-k}\rangle_{out})$$

FERMIONIC ENTANGLEMENT

Fuentes, Mann, Moradi, Martin-Martinez (in preparation)

7. COMPUTE THE ENTANGLEMENT BETWEEN K AND –K MODES

$$S_F = \log\left(\frac{1+|\gamma_F^-|^2}{|\gamma_F^-|^{\frac{2|\gamma_F^-|^2}{|\gamma_F^-|^2+1}}}\right)$$

$$|\gamma_F| = |\gamma_k^- \chi(\bar{k})|$$

in the limit $\epsilon \rightarrow \infty$

$$k = m = \rho = 1$$

$$S_F(\epsilon \rightarrow \infty) \approx 0.0048$$

for light particles

$$\frac{\mu_{out}^2}{|k|^2}\left(1 - \frac{\omega_{out}}{\mu_{out}}\right)^2 \approx \frac{\omega_{out}^2}{|k|^2}$$

$$\epsilon \approx \frac{\gamma_F^- \varepsilon}{\gamma_F^- + 1}$$

BOSONIC IN AND OUT STATES

KLEIN-GORDON EQUATION

$$\left(\partial_\eta^2 + k^2 + C(\eta)m^2\right)\chi_k(\eta) = 0$$

massive conformally coupled case

POSITIVE AND NEGATIVE SOLUTIONS

$$\chi_{in}(\eta) = \exp\left(-i\omega_+\eta - \frac{i\omega_-}{\rho}\ln[2\cosh\rho\eta]\right) F\left(\frac{1}{2} - \frac{i\bar{\omega}}{2\rho} + \frac{i\omega_-}{\rho}, \frac{1}{2} + \frac{i\bar{\omega}}{2\rho} + \frac{i\omega_-}{\rho}, 1 - i\frac{\omega_{in}}{\rho}, \frac{1+\tanh(\rho\eta)}{2}\right)$$

$$\chi_{out}(\eta) = \exp\left(-i\omega_+\eta - \frac{i\omega_-}{\rho}\ln[2\cosh\rho\eta]\right) F\left(\frac{1}{2} - \frac{i\bar{\omega}}{2\rho} + \frac{i\omega_-}{\rho}, \frac{1}{2} + \frac{i\bar{\omega}}{2\rho} + \frac{i\omega_-}{\rho}, 1 - i\frac{\omega_{out}}{\rho}, \frac{1-\tanh(\rho\eta)}{2}\right)$$

where $\omega_\pm = (\omega_{out} \pm \omega_{in})/2$ $\qquad \bar{\omega} = (m^2(2\epsilon+1)^2 - \rho^2)^{1/2}$

BOSONIC GAMMA

$$|\gamma_B^-|^2 = \frac{\cosh\frac{\pi}{\rho}\left(\frac{\bar{\omega}}{2} - \omega_-\right)\cosh\frac{\pi}{\rho}\left(\frac{\bar{\omega}}{2} + \omega_-\right)}{\cosh\frac{\pi}{\rho}\left(\frac{\bar{\omega}}{2} - \omega_+\right)\cosh\frac{\pi}{\rho}\left(\frac{\bar{\omega}}{2} + \omega_+\right)} = \frac{\cosh\frac{\pi}{\rho}\bar{\omega} + \cosh\frac{2\pi}{\rho}\omega_-}{\cosh\frac{\pi}{\rho}\bar{\omega} + \cosh\frac{2\pi}{\rho}\omega_+}$$

BOSONIC ENTANGLEMENT

Ball, IFS, Schuller PLA (2006)

HISTORY OF THE UNIVERSE ENCODED IN ENTANGLEMENT

$$S_B = \log\left(\frac{|\gamma_B^-|^{\frac{2|\gamma_B^-|^2}{|\gamma_B^-|^2 - 1}}}{1 - |\gamma_B|^2} \right)$$

in the limit $\epsilon \to \infty$

$k = m = \rho = 1$

$$S_B(\epsilon \to \infty) \approx 0.0913$$

$$S_F(\epsilon \to \infty) \approx 0.0048$$

for light particles $m\sqrt{\epsilon(1+\epsilon)} \ll E \ll \rho$

$E = \sqrt{k^2 + m^2}$

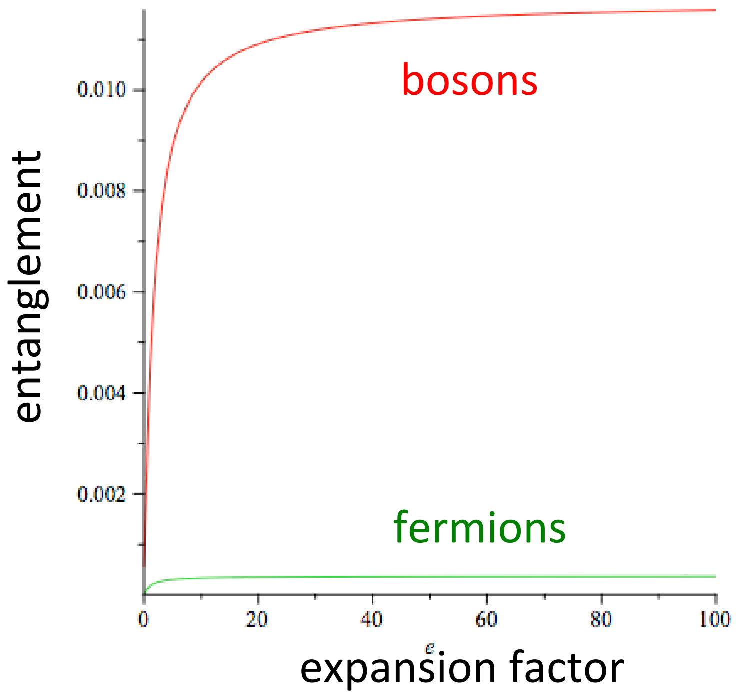
$$\epsilon \approx \varepsilon \sqrt{\gamma_B}$$

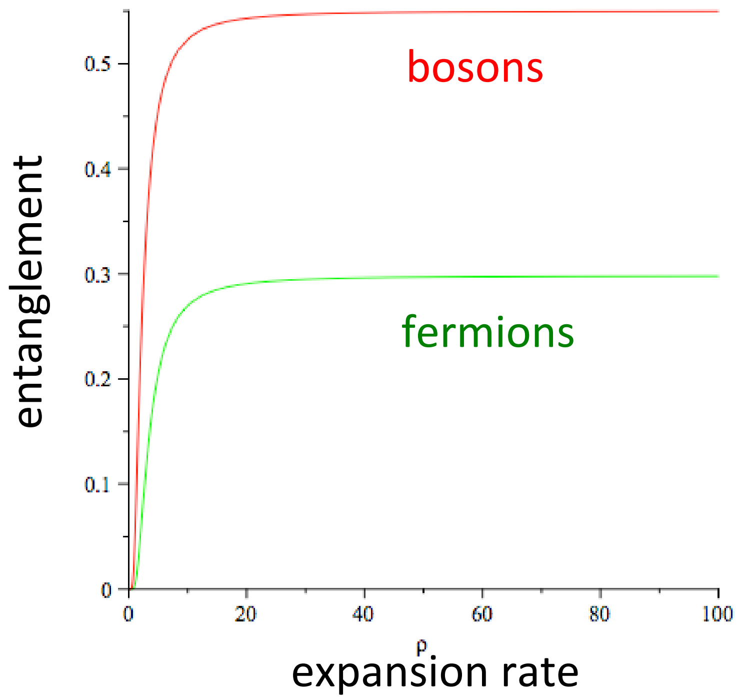
$\varepsilon = E/m \gg 1$

Entanglement contains information about the underlying spacetime

ENTANGLEMENT COSMOLOGY

Fuentes, Mann, Moradi, Martin-Martinez (in preparation)

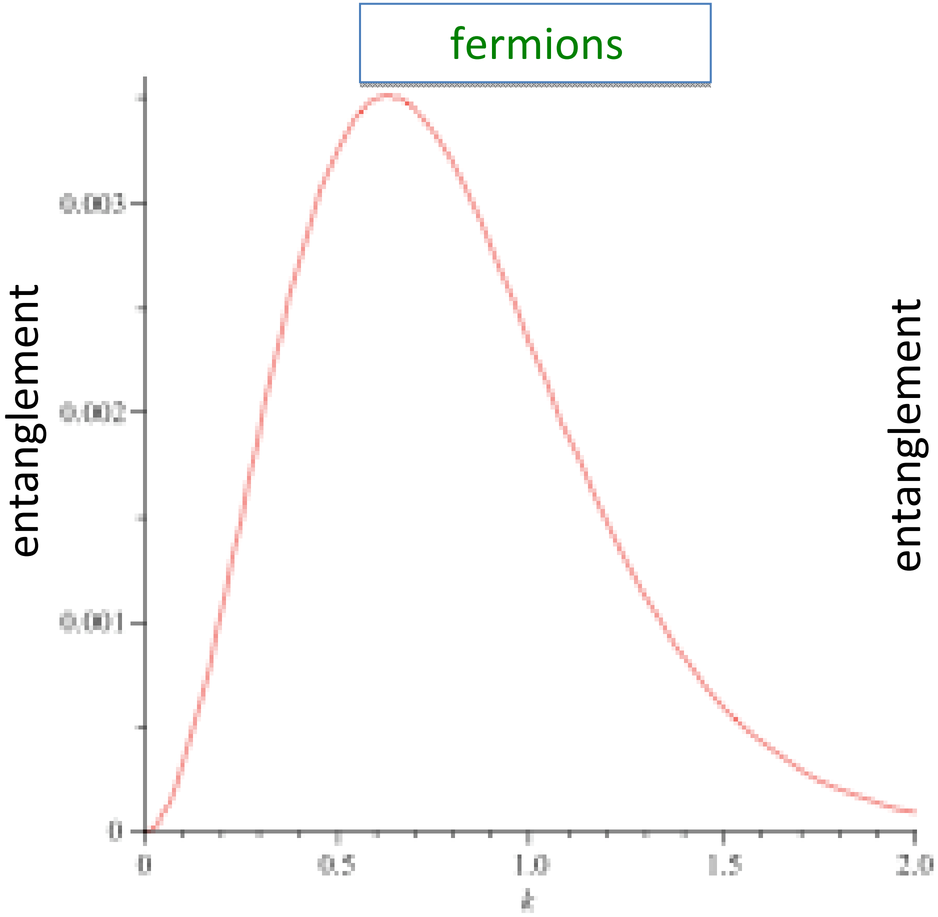


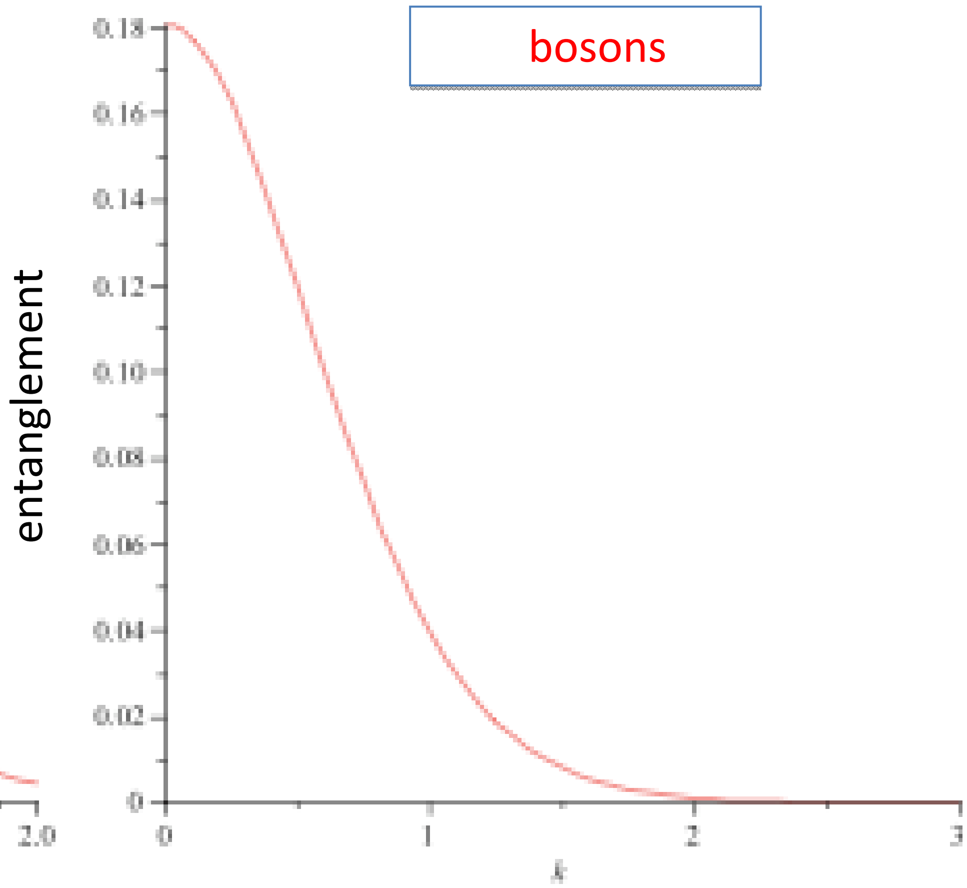


"The Universe entangles less fermionic fields"

Entanglement vs. frequency

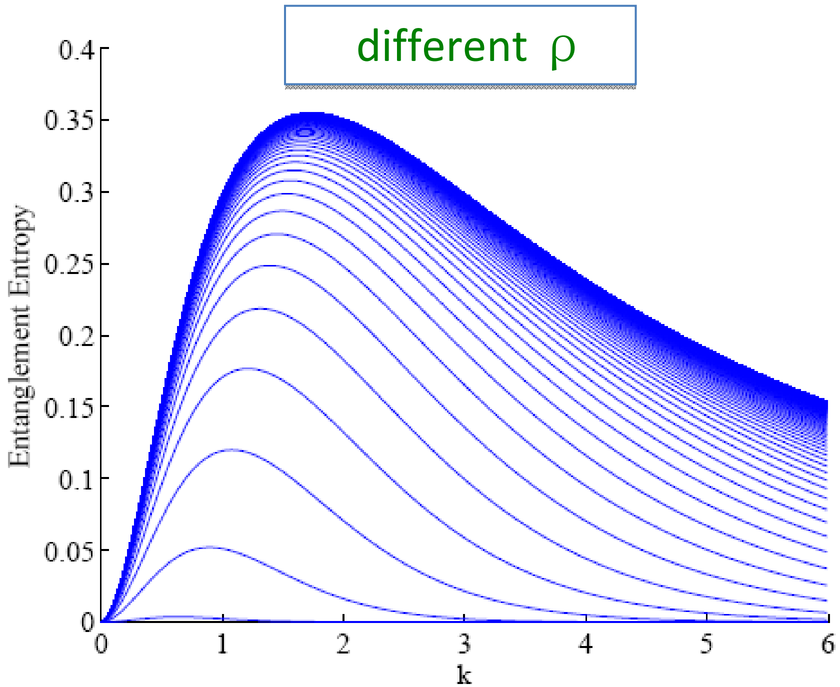
Fuentes, Mann, Moradi, Martin-Martinez (in preparation)

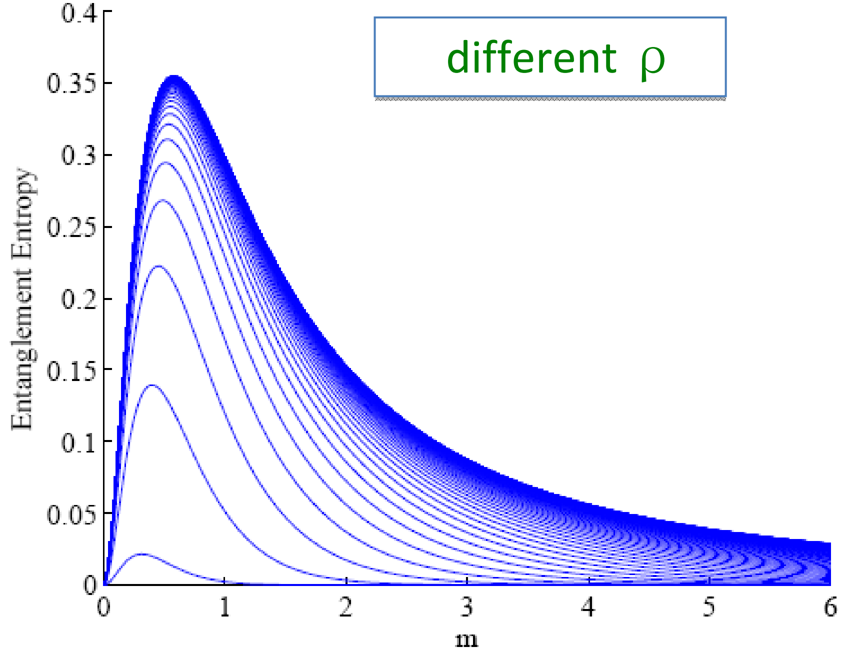




Entropy for bosons and fermions as a function of k

EXPANSION PARAMETERS FROM ENTANGLEMENT

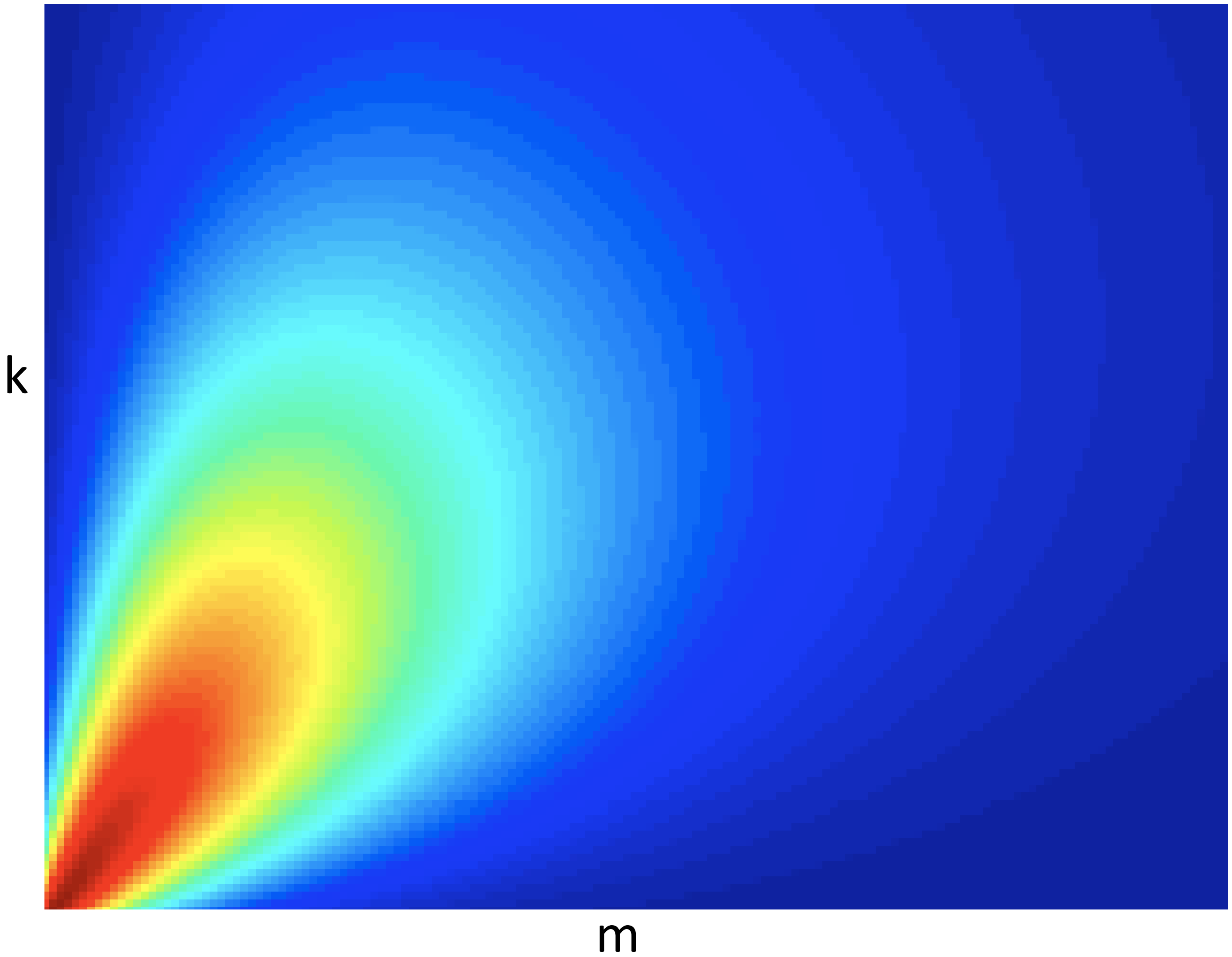




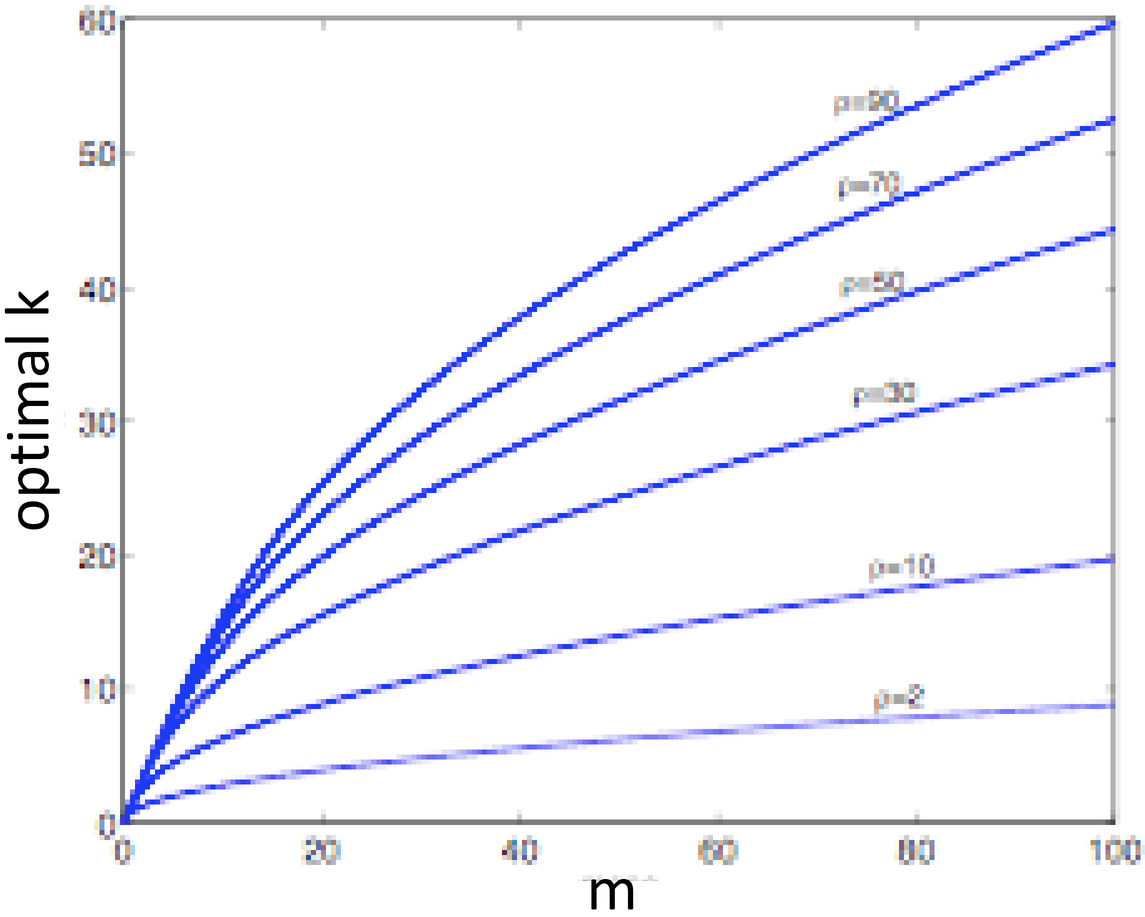
Entropy for fermions as a function of k and m

EXPANSION PARAMETERS FROM ENTANGLEMENT

Entropy for fermions as a function of k and m

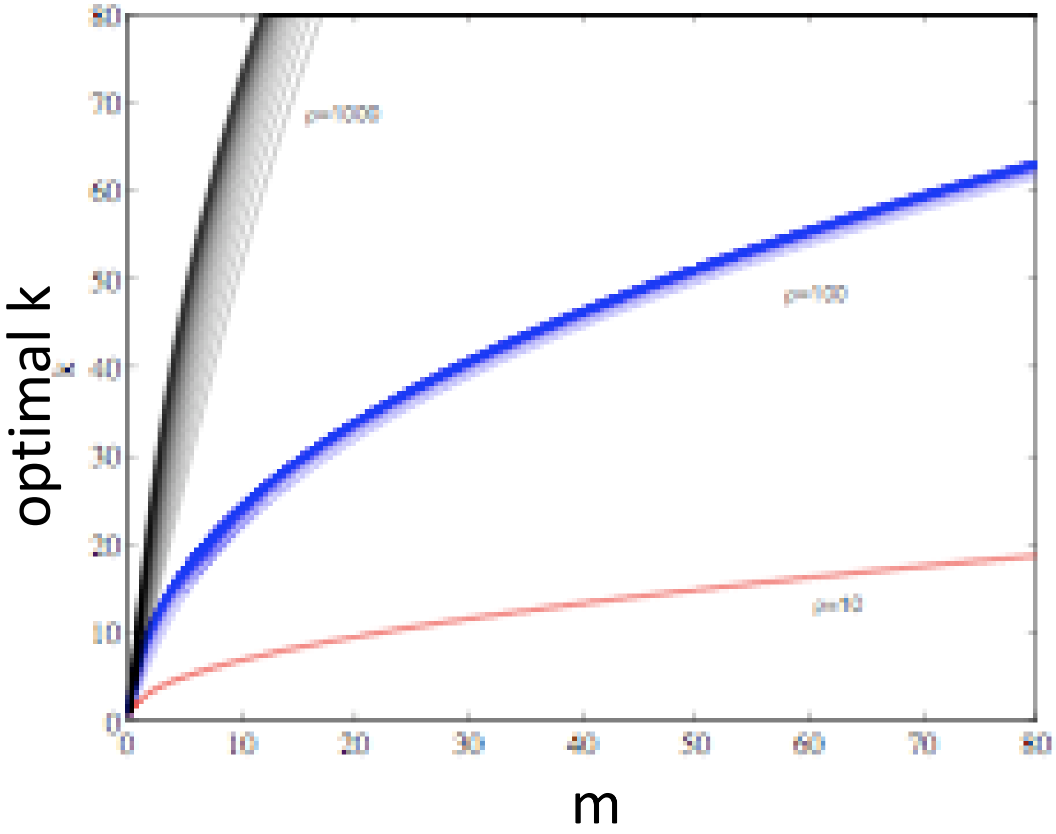


EXPANSION PARAMETERS FROM ENTANGLEMENT



Entropy for fermions as a function of k and m

EXPANSION PARAMETERS FROM ENTANGLEMENT



Entropy for fermions as a function of k and m

CONCLUSIONS

- Entanglement is created by the expansion of the Universe
- Fermionic and bosonic entanglement is very different
- Fermionic entangelment is less sensitive to the underlying spacetime
- In the fermionic case, we find that entanglement is maximum for a given frequency
- The entanglement encodes information about the past history of the Universe
    - This information can be better extracted using fermions