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PHOTONS AS A GRAVITY PROBE

MACQUARIE UNIVERSITY
SYDNEY ~ AUSTRALIA

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Lecture Hall I, Science and Engineering Building, NDHU

OUTLINE

Qubits & gates & protocols
Communications
Metrology
• trajectories
• waves & rays
• PPN
• Schwarzschild & Kerr
Phases & gauges
Outlook

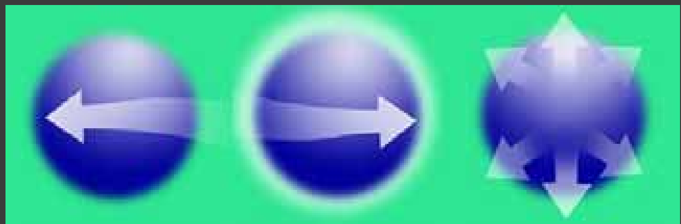
Units

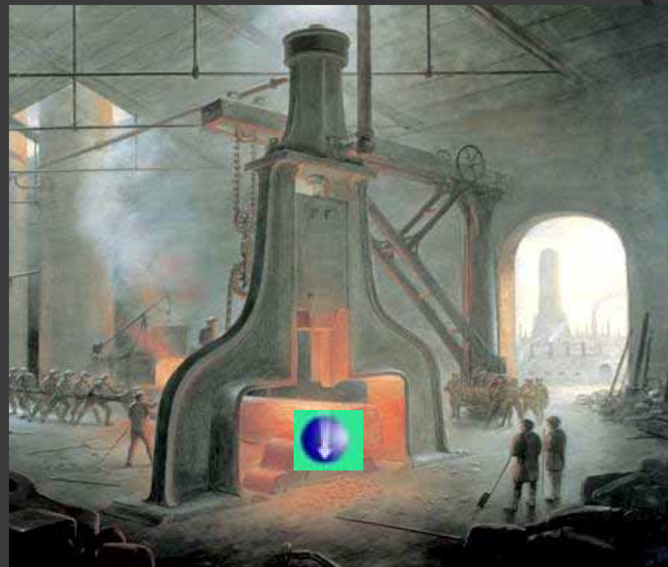
$$G=c=\hbar=1$$

signature: $-+++$

SETTING

Quantum info abstractions:
qubits & gates





Qubits:
photon polarization states

Gates:
what happens to polarization after propagation
in a gravitational field

Protocols

Communication: gravity induces noise
Parameter estimation: a gravity probe

QUBITS AND GATES[1]

Qubits

Classical photons

- well-defined position and momentum
- move on null geodesics $p = (p^0, \mathbf{p})$

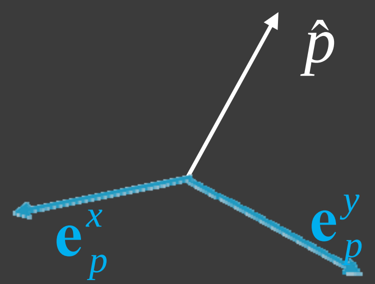
$$E = \hbar\omega = -p_0 c$$

$$p^2 = p_\mu p^\mu = 0$$

Qubit

- polarization / helicity $p_\mu e^\mu = 0, \ \ \mathbf{p}\cdot\mathbf{e} = 0 \quad \mathbf{e}^2 = 1$

$$f_p = f_+ e_p^+ + f_- e_p^-$$



$$e_p^\pm = \tfrac{1}{\sqrt{2}}\left(e_p^x \pm i e_p^y\right)$$

Classical to quantum

$$X \mapsto \hat{X} \qquad |p, \pm\rangle \Leftrightarrow (p, e_p^\pm)$$

Gates$_1$

Photons

- Parallel transport $\nabla_p p = 0, \quad \nabla_p f = 0$

More precisely

$$k^\mu = \dot{x}^\mu \equiv \frac{dx^\mu}{d\lambda} \qquad p \leftrightarrow k$$

$$k^\mu \nabla_\mu k^\nu = 0$$

$$k_\mu f^\mu = 0 \qquad \begin{cases} \mathbf{k} \cdot \mathbf{f} = 0 \\ f^0 = 0 \end{cases}$$

$$k^\mu \nabla_\mu f^\nu = 0$$

Schwarzschild & Kerr: use conserved quantities

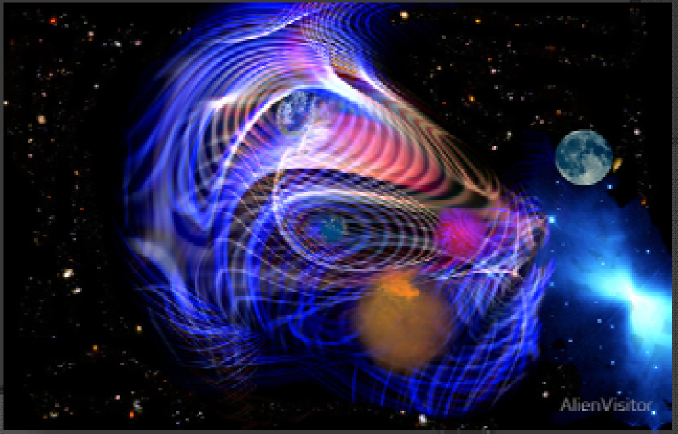
Gates$_2$

Eikonal equation [0th order in $1/|\mathbf{p}|$]: rays

1st order: polarization (normalized electric field)

Gate summary [1]

- no birefringence
- act as polarization rotation
- act individually



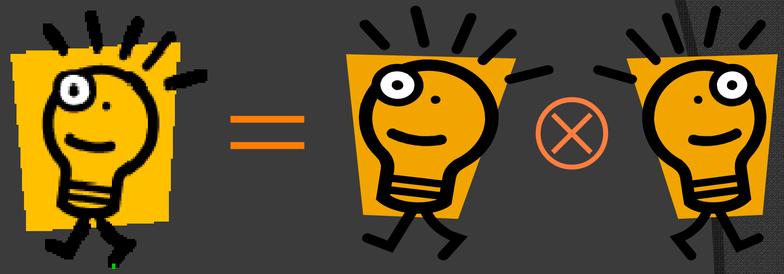
IMPLICATIONS Communication

- Single-qubit action

  decoherence-free subsystems/subspaces take care of the noise.

- Polarization rotation

  $U|p,\pm\rangle = e^{\pm i\psi}|p',\pm'\rangle$



- Source of noise:

  if the rotation is unknown, how to put the polarizer?

$$|0_L\rangle = \tfrac{1}{\sqrt{2}}\left(|p,+\rangle|p,-\rangle + |p,-\rangle|p,+\rangle\right)$$

$$|1_L\rangle = \tfrac{1}{\sqrt{2}}\left(|p,+\rangle|p,-\rangle - |p,-\rangle|p,+\rangle\right)$$

Bartlett and DRT, Phys Rev A **71** 012302 (2005)

IMPLICATIONS Gravity probe

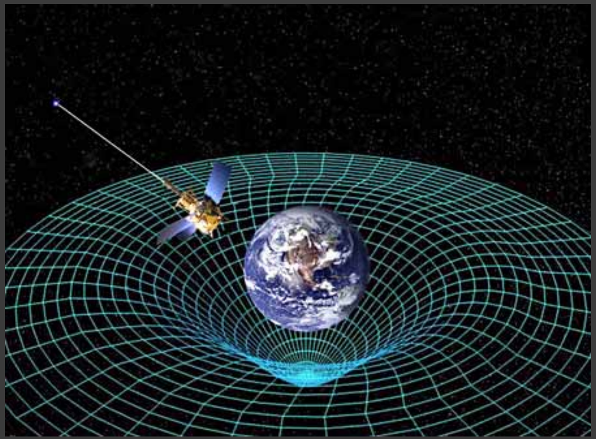
- Schwarzschild:

rigid rotation of $(\mathbf{e}_k^x, \mathbf{e}_k^y, \mathbf{k})$

- Kerr:

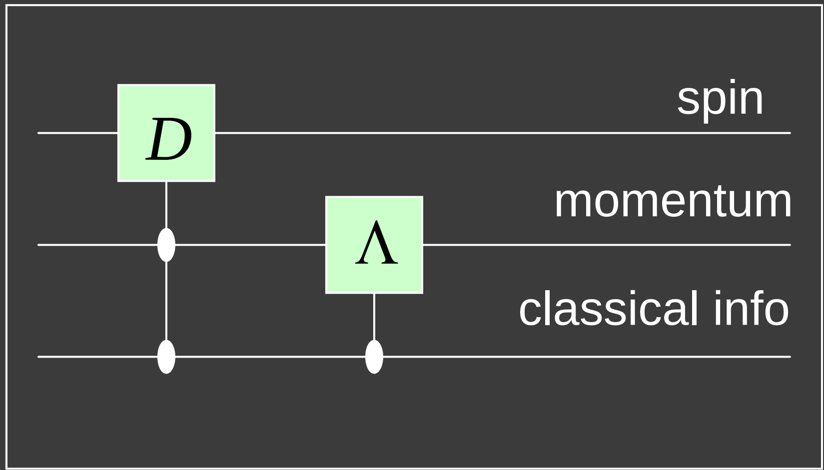
Faraday/gravimagnetic polarization rotation

Challenges



Theory: comparison & local frames	Sci-fi: Gravity probe L/Q

PHASES & FRAMES [1]



$$U(\Lambda)\left|p,\sigma\right\rangle = \sum_{\xi} D_{\xi\sigma}[W(\Lambda,p)]\left|\Lambda p,\xi\right\rangle$$

Step 1: standard states & standard Lorentz transformations

massive particles: $k_S = (m,0,0,0)$

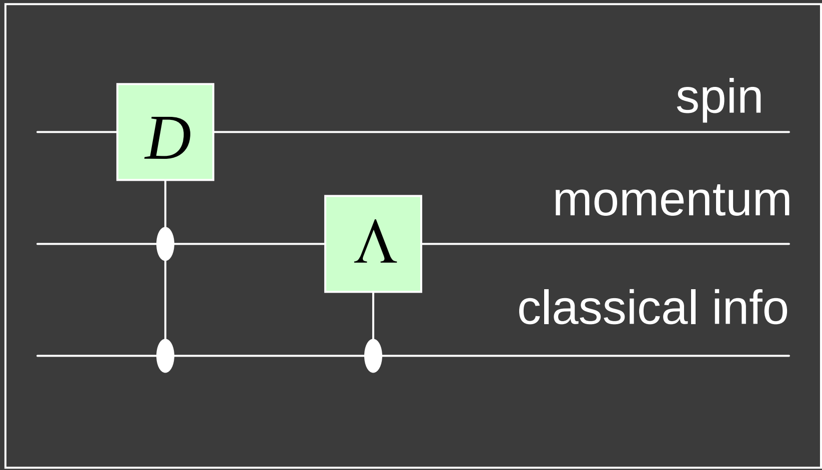
massless particles: $k_S = (1,0,0,1)$

$$p = L_p k_S$$

Step 2: state conventions $\left|p,\sigma\right\rangle \equiv U(L_p)\left|k_S,\sigma\right\rangle$

Quantum Lorentz transformations

Quantum Lorentz transformations



$$U(\Lambda)\left|p,\sigma\right\rangle = \sum_{\xi} D_{\xi\sigma}[W(\Lambda,p)]\left|\Lambda p,\xi\right\rangle$$

Step 3: transformation $\quad U(\Lambda)\left|p,\sigma\right\rangle = U(L_{\Lambda p})U(L_{\Lambda p}^{-1}\Lambda L_p)\left|k_S,\sigma\right\rangle$

Step 4: little group $\quad W(\Lambda,p) \equiv L_{\Lambda p}^{-1}\Lambda L_p$

$Wk_S = k_S$

$$k_S \xrightarrow{L_p} p \xrightarrow{\Lambda} \Lambda p \xrightarrow{L_{\Lambda p}^{-1}} k_S$$

Step 5: $\quad U(\Lambda)\left|p,\sigma\right\rangle = U(L_{\Lambda p}^{-1}\Lambda L_p)\left|\Lambda p,\sigma\right\rangle$

Massless particles

$$k_S = (1,0,0,1)$$

$$\sigma = \text{helicity} = \pm 1$$

$$W = S(\alpha,\beta)R_z(\psi) \in E(2)$$

$$D_{\xi\sigma} = e^{i\sigma\psi}\delta_{\xi\sigma}$$

Standard transform

$$L_p = R(\hat{\mathbf{p}})\, B_z(|\mathbf{p}|)$$

$$B_z(|\mathbf{p}|)k_S = (|\mathbf{p}|,0,0,|\mathbf{p}|)$$

$$R(\hat{\mathbf{p}})\,\hat{\mathbf{z}} \equiv \hat{\mathbf{p}}$$

$$R(\hat{\mathbf{p}}) = R_z(\phi)R_y(\theta)$$

Rotations & rotations

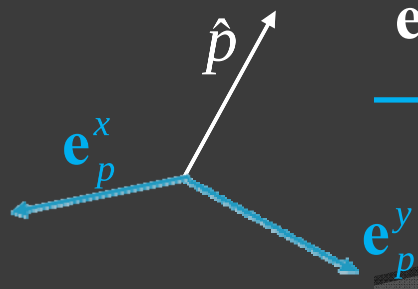
$$|k_S,\pm\rangle \rightarrow |p,\pm\rangle \qquad p = L_p k_S$$

$$\mathbf{e}^{\pm}_{k_S} \rightarrow \mathbf{e}^{\pm}_{p} = R(\hat{\mathbf{p}})\,\mathbf{e}^{\pm}_{k_S}$$

$$p' = R\,p \qquad |p,\pm\rangle \rightarrow e^{\pm i\psi(R,\hat{\mathbf{p}})}|R\,p,\pm\rangle$$

$$e^{\pm}_{p} \rightarrow R\,e^{\pm}_{p} = e^{\pm i\psi(R,\hat{\mathbf{p}})}e^{\pm}_{R\,p}$$

$$\mathbf{e}^{\pm}_{p} \rightarrow R\,\mathbf{e}^{\pm}_{p} \neq \mathbf{e}^{\pm}_{p'}$$



Photons: how to

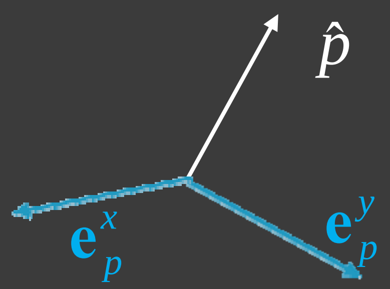
Wigner phase and rotation class

$$R = R(\hat{\mathbf{p}})R_z(\psi)R^{-1}(\hat{\mathbf{p}}) \qquad W(R, p) = R^{-1}(R\,p)R\,R(p) = R_z(\psi)$$

Zero phase

$$p' = R_{e_p^y}(\omega)\,p \qquad R_{\mathbf{e}_p^y}(\omega) = R(\hat{\mathbf{p}})R_y(\omega)R^{-1}(\hat{\mathbf{p}})$$

$$W(R_{e_p^y}(\omega), p) = R^{-1}(\hat{\mathbf{p}}\,)R_{e_p^2}(\omega)R(\hat{\mathbf{p}}) = R_y^{-1}(\theta+\omega)R_y(\theta)R_y(\omega) = I$$



Lindner et al., J. Phys. A **36**, L449 (2003)
Bergou et al., Phys. Rev. A **68**, 042102 (2003)
Alsing and Milburn, Quant. Info. Comp. **2**, 487 (2002)

Photons: more details

PPN$_1$

What's in:

- Far field
- Slow motion of the gravitating bodies
- Leading post-Newtonian correction in metric form
- Harmonic gauge $g^{\mu\nu}\Gamma^{\lambda}_{\mu\nu}=0 \Leftrightarrow \Box^{2} x^{\lambda} = 0$
- Some redefinitions
- Flat space-time & modified equations of motion

$$g_{00} \approx -1 - 2\phi$$

$$g_{\mu\nu} = \begin{pmatrix} -1 + \frac{2GM}{c^2 r} - \left(\frac{\sqrt{2}GM}{c^2 r}\right)^2 & \frac{2GJy}{c^3 r^3} & -\frac{2GJx}{c^3 r^3} & 0 \\ \frac{2GJy}{c^3 r^3} & 1 + \frac{2GM}{c^2 r} & 0 & 0 \\ -\frac{2GJx}{c^3 r^3} & 0 & 1 + \frac{2GM}{c^2 r} & 0 \\ 0 & 0 & 0 & 1 + \frac{2GM}{c^2 r} \end{pmatrix}$$

PPN gauge

GRAVITY AS A DIELECTRIC

J. Tamm, J. Russ. Phys.-Chem. Soc. Phys. Div. **56**, 248 (1924)
G. V. Skrotskii, Soviet Phys. Doklady **2**, 226 (1957)
J. Plebanski, Phys. Rev. **115**, 1396 (1960)
* S. Kopeikin and B. Mashhoon, Phys. Rev. D **65**, 064025 (2002)

$$\varsigma = \sqrt{-\det g}$$

$$E_a = F_{0a} \qquad B_a = \tfrac{1}{2}\epsilon_{abc}F_{cb}$$

$$D_a = \varsigma F^{a0} \qquad H_a = \tfrac{1}{2}\epsilon_{abc}\varsigma F^{cb}$$

$$D_a = \varepsilon_{ab}E_b + \epsilon_{abc}g_b H_c$$

$$B_a = \varepsilon_{ab}H_b - \epsilon_{abc}g_b E_c$$

$$\varepsilon_{ab} = -\frac{\varsigma}{g_{00}}g^{ab} \qquad g_a = \frac{g_{a0}}{g_{00}}$$

PPN dielectric

$$\mathbf{D} = \varepsilon\mathbf{E} + \mathbf{g}\times\mathbf{H}$$

$$\mathbf{H} = \varepsilon\mathbf{B} - \mathbf{g}\times\mathbf{E}$$

Schwarzschild PPN as an example

$\mathbf{g} = 0$ The shortest derivation: *à la* Born and Wolf, *Optics*

$$\frac{d\hat{\mathbf{k}}}{ds} = \boldsymbol{\Omega} \times \hat{\mathbf{k}} \qquad \frac{\mathbf{e}^{(i)}}{ds} = \boldsymbol{\Omega} \times \mathbf{e}^{(i)}$$

$$\boldsymbol{\Omega} = -(\mathbf{e}^2 ?\nabla n\ \mathbf{e}^1 + (\mathbf{e}^1 ?\nabla n\ \mathbf{e}^2 \qquad n := \sqrt{\varepsilon\mu} \equiv \varepsilon$$

$$\nabla n = -\frac{2}{c^2}\left(1 - \frac{\phi}{c^2}\right)\nabla\phi$$

SCHWARZSCHILD PHOTONS

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu = -(1-2M/r)dt^2 + (1-2M/r)^{-1} dr^2 + r^2 d\Omega^2$$

Coordinate transformation to PPN:
harmonic coordinates

$$r = r_h \left(1 - \frac{M}{2r_h}\right)^2$$

Conserved quantities: $E(\to 1)$, $\mathbf{L}$

Choice of the frame: $\theta = \pi/2$, $k^\theta \equiv 0$

Trajectory:
a plane curve

Polarization:
a rigid rotation

$f^{\hat\theta} = \mathrm{const}$

$f^{\hat r}, f^{\hat\varphi}$ are rotated $\Rightarrow \Omega$

PHASES & FRAMES [2]

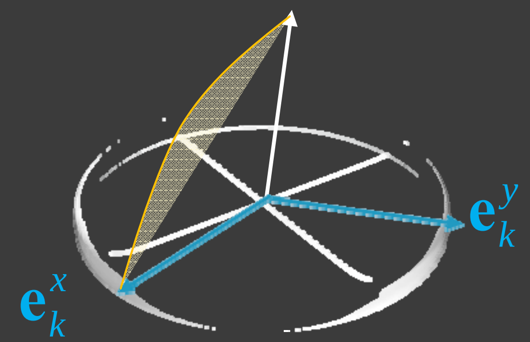
To get zero Wigner phase

$$\mathbf{e}^i_{k'} \equiv \mathbf{e}^{i\,\prime} \approx \mathbf{e}^i_k + \boldsymbol{\Omega} \times \mathbf{e}^i_k ds$$

was
is
the standard polarization direction

A necessary & sufficient condition:

$$\boldsymbol{\Omega} \parallel \mathbf{e}^y_k \Leftrightarrow \boldsymbol{\Omega} ? \mathbf{e}^x_k =$$



How to rotate a polarizer?

A physical direction: set z

$$\hat{\mathbf{z}} \propto \nabla n \propto \nabla \phi$$

Generalization:
proper frame acceleration
of an observer at rest

$$w^{\hat{r}} = -\frac{M}{r^2} \frac{1}{\sqrt{1 - 2M/r}}$$

$$\hat{\mathbf{z}} \propto \mathbf{w} \quad \mathbf{e}^y_k \propto \mathbf{k} \times \hat{\mathbf{z}}$$

KERR

$$ds^2 = -\left(1 - \frac{2Mr}{\rho^2}\right)dt^2 + \frac{\rho^2}{\Delta}dr^2 + \rho^2 d\theta^2$$
$$+\left(r^2 + a^2 + \frac{2Mra^2}{\rho^2}\sin^2\theta\right)\sin^2\theta d\varphi^2 - \frac{4Mra}{\rho^2}\sin^2\theta d\varphi dt$$

$$\rho^2 = r^2 + a^2\cos^2\theta \qquad \Delta = r^2 - 2Mr - a^2 \qquad a = J/M$$

Coordinate transformation to PPN:
harmonic coordinates (still)

Conserved quantities: $E, L_z, K_1 + iK_2$

Equatorial motion:
like Schwarzschild

Tecniques

PPN waves:

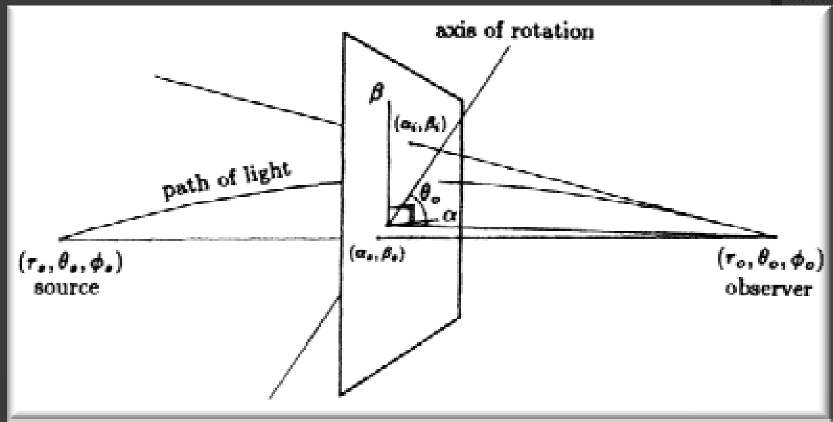
Skrotskii equation

$$\mathbf{f} = f^{\,n}\mathbf{n} + f^{\,b}\mathbf{b} \qquad \psi = \arctan\frac{f^{\,n}}{f^{\,b}}$$

$$\frac{d\psi}{ds} = \tau + \tfrac{1}{2}\mathbf{k}\cdot(\nabla\times\mathbf{k})$$

$$\frac{d\mathbf{k}}{ds} = \kappa\mathbf{n} \qquad \mathbf{b} = \mathbf{k}\times\mathbf{n}$$

$$\frac{d\mathbf{b}}{ds} = -\tau\mathbf{n}$$



Photons

[and the gravitational lens]

$$r_{\rm s} \to r_{\rm min} \to r_{\rm o}$$

Expansions in powers of $1/r_{\rm min}$

I. Bray, Phys. Rev. D **34**, 367 (1986)

H. Ishihara et al, Phys Rev D **38**, 482 (1988)

Frame dragging

Inertial frames are dragged in the direction of motion of the sources of the gravitational field.

The differential rotation between adjacent frames (far field)

$$\mathbf{\Omega}_D = -\tfrac{1}{2}\sqrt{g_{00}}\,\nabla\times(g_{00}\mathbf{g})$$

Assumption: Coriolis force acts on polarization as on gyros

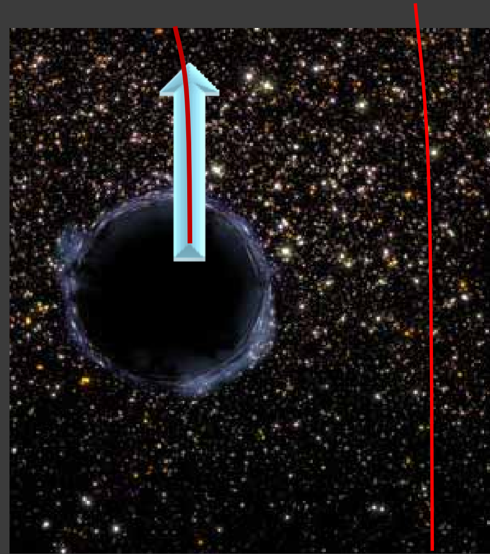
$$\Delta\psi_D = -\tfrac{1}{2}\int_{\mathrm{s}}^{\mathrm{o}}\sqrt{g_{00}}\,\nabla\times(g_{00}\mathbf{g})\cdot\mathbf{k}\,ds$$

B. B. Godfrey, Phys. Rev. D **1**, 2721 (1970)

M. Sereno, Phys. Rev D **69**, 087501 (2004)

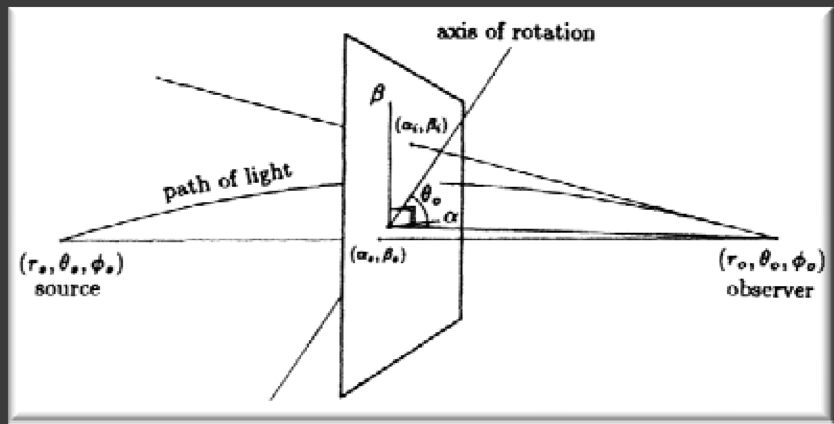
Results

Order of magnitude agreement (only)



$$\Delta\psi \sim \frac{J}{r_{\min}^2}$$

$$\Delta\psi \sim \frac{MJ}{r_{\min}^3}$$



PHASES & FRAMES [3]

$$w_{\hat{r}} = \frac{M^2(\rho^2 - 2r^2)\sqrt{\Delta}}{\rho^3(\rho^2 - 2Mr)} = -\frac{M}{r^2} - \frac{M^2}{r^3} + \dots$$

$$w_{\hat{\theta}} = \frac{Mra^2 \sin 2\theta}{\rho^3(\rho^2 - 2Mr)} = a^2 M \frac{\sin 2\theta}{r^4} + \dots$$

Local reference frame

$$\hat{\mathbf{z}} \propto \mathbf{w} \quad \mathbf{e}_k^y \propto \mathbf{k} \times \hat{\mathbf{z}}$$

OUTLOOK

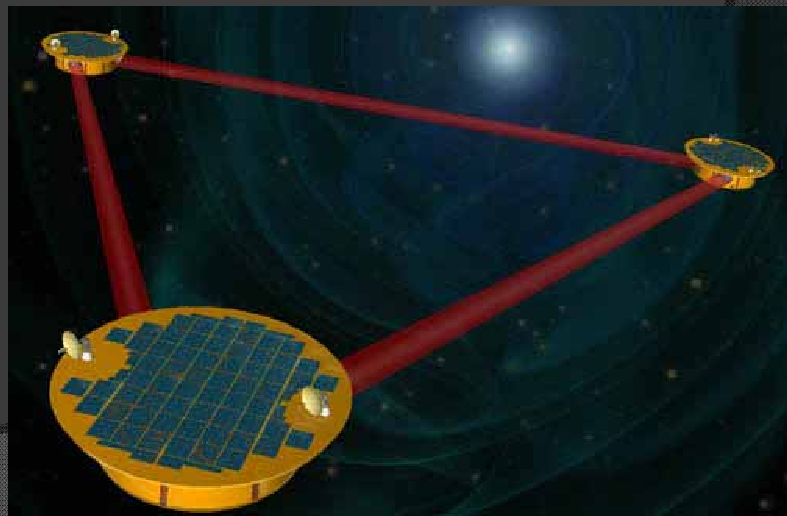
Question 1:
true physical rotations in different regimes

Question 2:
space & mirrors

$$\Delta\psi = \int_{s}^{o} \mathbf{\Omega}_{\text{phys}} \cdot \mathbf{k}\,ds \qquad \mathbf{\Omega}_{\text{phys}} \neq \nabla\Phi$$

$$\Delta\psi_{\circ} = \oint_{\circ} \mathbf{\Omega}_{\text{phys}} \cdot \mathbf{k}\,ds \neq 0$$

+ quantum phase estimation?



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