Hilltop Inflation and Supersymmetry

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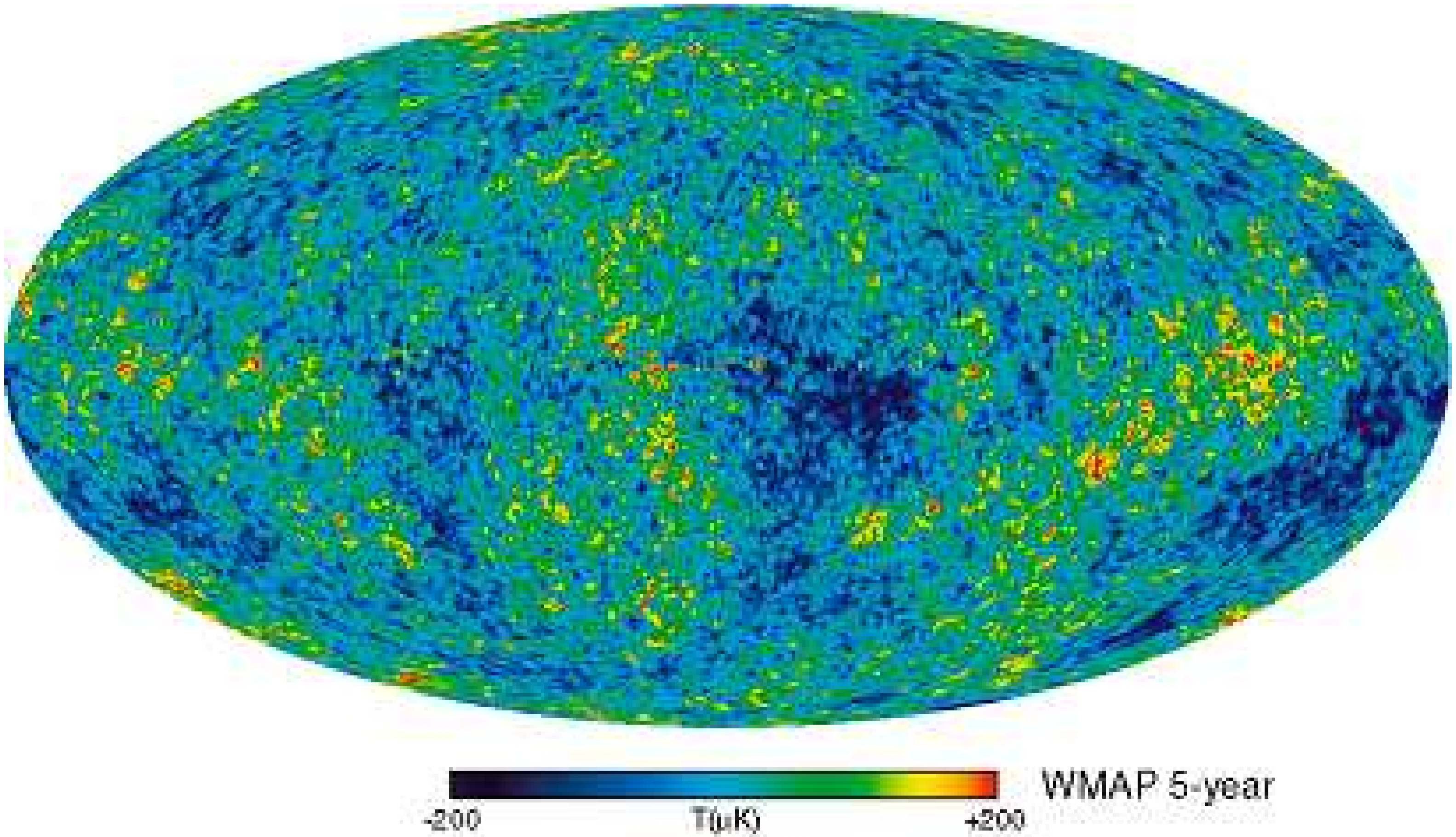
This talk is based on following works with my collaborators

Kazunori Kohri, Chia-Min Lin, David H. Lyth
JCAP 0712: 004, 2007 0707.3826 [hep-ph]

Chia-Min Lin, John McDondald
Phys. Rev. D74:063510, 2006 hep-ph/0604245

Chia-Min Lin, Kingman Cheung
JCAP 0903: 012, 2009 0812.2731 [hep-ph]

Chia-Min Lin, Kingman Cheung
Phys. Rev. D79:083509, 2009 0901.3280 [hep-ph]



-200
T(μK)
+200
WMAP 5-year

Basic Equations (I)

Friedmann Equation

$$H^2 = \frac{\rho}{3M_P^2} - \frac{k}{a^2}$$

$$H \equiv \frac{\dot{a}}{a} \quad \text{(Hubble parameter)}$$

The fluid Equation

$$\dot{\rho} + 3H(\rho + P) = 0$$

Matter Domination:

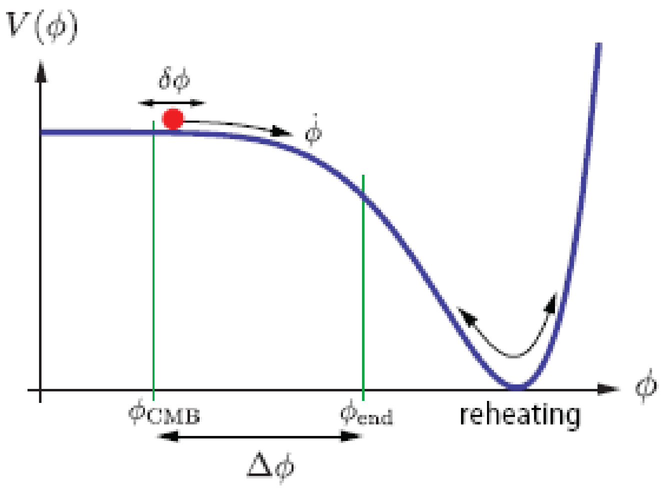
$$P = 0 \Rightarrow \ a \propto t^{2/3}, \quad \rho \propto a^{-3}$$

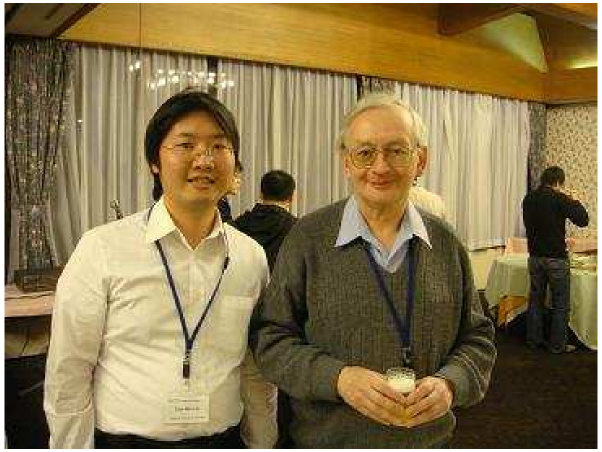
Radiation Domination:

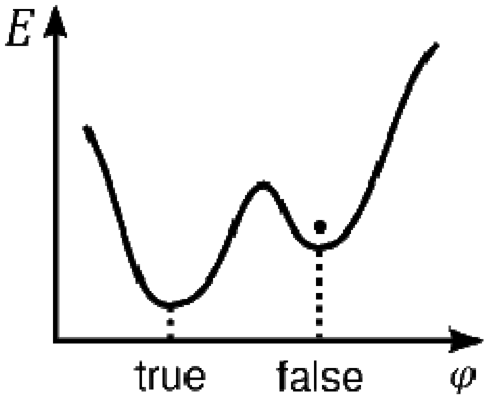
$$P = \frac{\rho}{3} \Rightarrow \ a \propto t^{1/2}, \quad \rho \propto a^{-4}$$

Inflation

Alan Guth (1981)







CML and A. Starobinsky

CML and K. Sato

Vacuum Energy Dominated

$$\frac{\dot{a}}{a} = H = const.$$

$$\frac{da}{adt} = H$$

$$\frac{da}{a} = Hdt$$

$$a \propto e^{H\Delta t} \sim e^{-N}$$

De Sitter phase

N: number of e-folds

Basic Equations (II)

Scalar field in cosmological background

$$\rho = \frac{1}{2}\dot{\phi}^2 + V$$

$$P = \frac{1}{2}\dot{\phi}^2 - V$$

$$\ddot{\phi} + 3H\dot{\phi} + V'(\phi) = 0$$

$$H^2 = \frac{1}{3M_P^2}[V(\phi) + \frac{1}{2}\dot{\phi}^2]$$

Basic Equations (III)

Slow roll parameters: $\varepsilon << 1$

$$\varepsilon \equiv \frac{1}{2} M_P^2 \left(\frac{V'}{V}\right)^2 \qquad \eta \equiv M_P^2 \frac{V''}{V} \qquad |\eta| << 1$$

Number of e-folds

$$N(\phi) = \int_{\phi_{end}}^{\phi} M_P^{-2} \frac{V}{V'} d\phi = \int_{\phi_{end}}^{\phi} \frac{1}{\sqrt{2\varepsilon}} d\phi$$

Spectrum of primordial curvature perturbation

$$P_R = \left[\left(\frac{H}{\dot{\phi}}\right)\left(\frac{H}{2\pi}\right)\right]^2 = \frac{1}{12\pi^2 M_P^6} \frac{V^3}{V'^2} = \frac{1}{24\pi^2 M_P^4} \frac{V}{\varepsilon}$$

$P_R^{1/2} \approx 5 \times 10^{-5}$ We called this CMB normalization.

The spectral index

$$n_s - 1 \equiv \frac{d \ln P_R}{d \ln k}$$

$$n_s = 1 + 2\eta - 6\varepsilon$$

FIVE-YEAR WILKINSON MICROWAVE ANISOTROPY PROBE (WMAP[1]) OBSERVATIONS: COSMOLOGICAL INTERPRETATION

E. KOMATSU [1], J. DUNKLEY [2,3,4], M. R. NOLTA [5], C. L. BENNETT [6], B. GOLD [6], G. HINSHAW [7], N. JAROSIK [2], D. LARSON [6], M. LIMON [8] L. PAGE [2], D. N. SPERGEL [3,9], M. HALPERN [10], R. S. HILL [11], A. KOGUT [7], S. S. MEYER [12], G. S. TUCKER [13], J. L. WEILAND [11], E. WOLLACK [7], AND E. L. WRIGHT [14]

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ABSTRACT

The *WMAP* 5-year data provide stringent limits on deviations from the minimal, 6-parameter ΛCDM model. We report these limits and use them to constrain the physics of cosmic inflation via Gaussianity, adiabaticity, the power spectrum of primordial fluctuations, gravitational waves, and spatial curvature. We also constrain models of dark energy via its equation of state, parity-violating interaction, and neutrino properties such as mass and the number of species. We detect no convincing deviations from the minimal model. The 6 parameters and the corresponding 68% uncertainties, derived from the *WMAP* data combined with the distance measurements from the Type Ia supernovae (SN) and the Baryon Acoustic Oscillations (BAO) in the distribution of galaxies, are: $\Omega_b h^2 = 0.02267^{+0.00058}_{-0.00059}$, $\Omega_c h^2 = 0.1131 \pm 0.0034$, $\Omega_\Lambda = 0.726 \pm 0.015$, $n_s = 0.960 \pm 0.013$, $\tau = 0.084 \pm 0.016$, and $\Delta^2_{\mathcal{R}} = (2.445 \pm 0.096) \times 10^{-9}$ at $k = 0.002$ Mpc^{-1}. From these we derive $\sigma_8 = 0.812 \pm 0.026$, $H_0 = 70.5 \pm 1.3$ km s^{-1} Mpc^{-1}, $\Omega_b = 0.0456 \pm 0.0015$, $\Omega_c = 0.228 \pm 0.013$, $\Omega_m h^2 = 0.1358^{+0.0037}_{-0.0036}$, $z_{\rm reion} = 10.9 \pm 1.4$, and $t_0 = 13.72 \pm 0.12$ Gyr. With the *WMAP* data combined with BAO and SN, we find the limit on the tensor-to-scalar ratio of $r < 0.22$ (95% CL),

Hybrid Inflation

$$V(\phi,\psi) = \kappa^2 \left(M^2 - \frac{\psi^2}{4} \right)^2 + \frac{\lambda^2 \psi^2 \phi^2}{4} + \frac{m^2 \phi^2}{2}$$

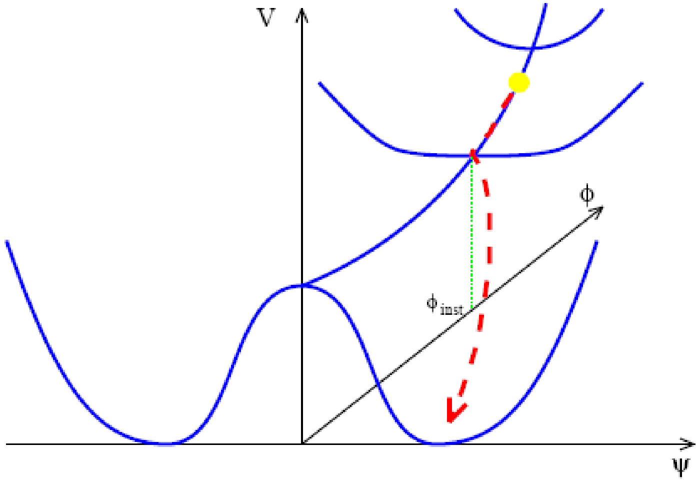
The vacua lie at

$$\langle \psi \rangle = \pm 2M \qquad \langle \phi \rangle = 0$$

Linde astro-ph/9307002

There is a valley of minima for

$$|\phi| > \phi_c = \frac{\sqrt{2}\kappa M}{\lambda} \qquad \text{where} \qquad \psi = 0$$

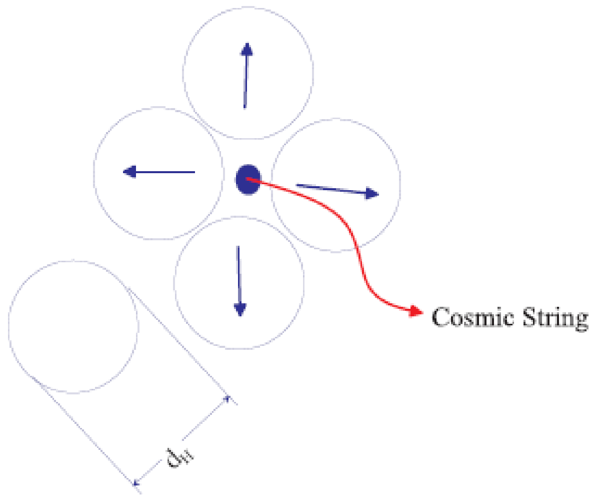


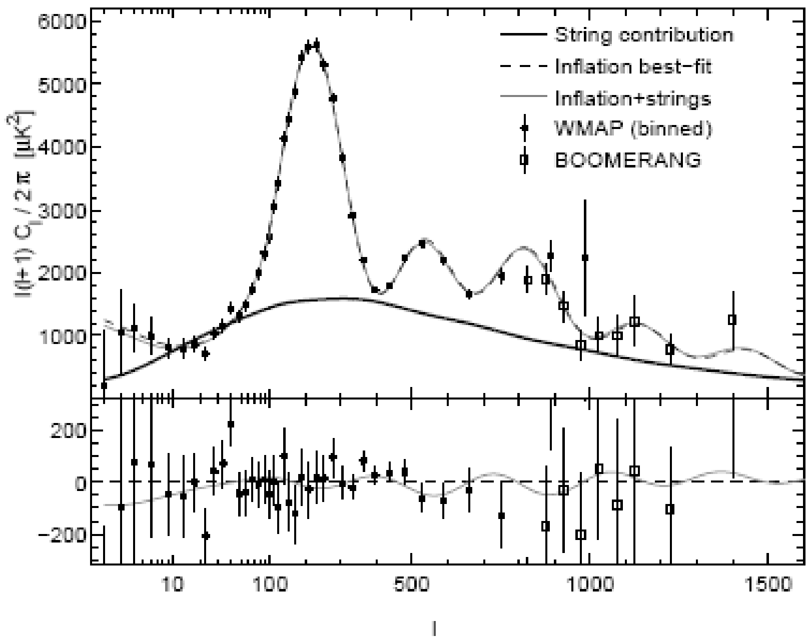
In this regime

$$V(\phi) = V_0 + \frac{m^2}{2}\phi^2 \qquad V_0 = \kappa^2 M^4$$

Some other issues

- Non-Gaussianity f_{NL}
- Gravity waves $r \equiv P_T / P_S$
- Cosmic Strings





Hilltop Inflation

Hill Top - the home of Beatrix Potter

L. Boubekeur and D. H. Lyth   hep-ph/0502047

$$V(\phi) = V_0 - \frac{m^2}{2}\phi^2 + \cdots = V_0\left(1 - \frac{1}{2}|\eta_0|\left(\frac{\phi}{M_P}\right)^2 + \cdots\right)$$

More hilltop Inflation models   0707.3826 [hep-ph]

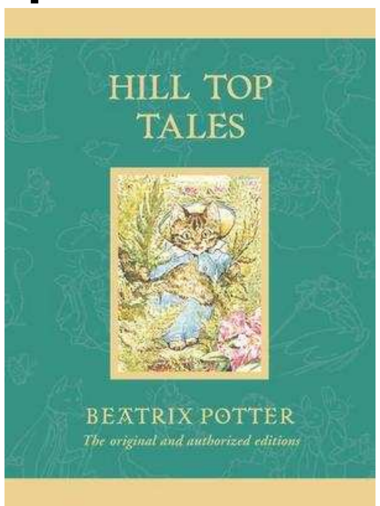
$$V(\phi) = V_0 \pm \frac{1}{2}m^2\phi^2 - \lambda\frac{\phi^p}{M_P^{p-4}} + \cdots$$

$$\equiv V_0\left(1 + \frac{1}{2}\eta_0\frac{\phi^2}{M_P^2}\right) - \lambda\frac{\phi^p}{M_P^{p-4}} + \cdots$$

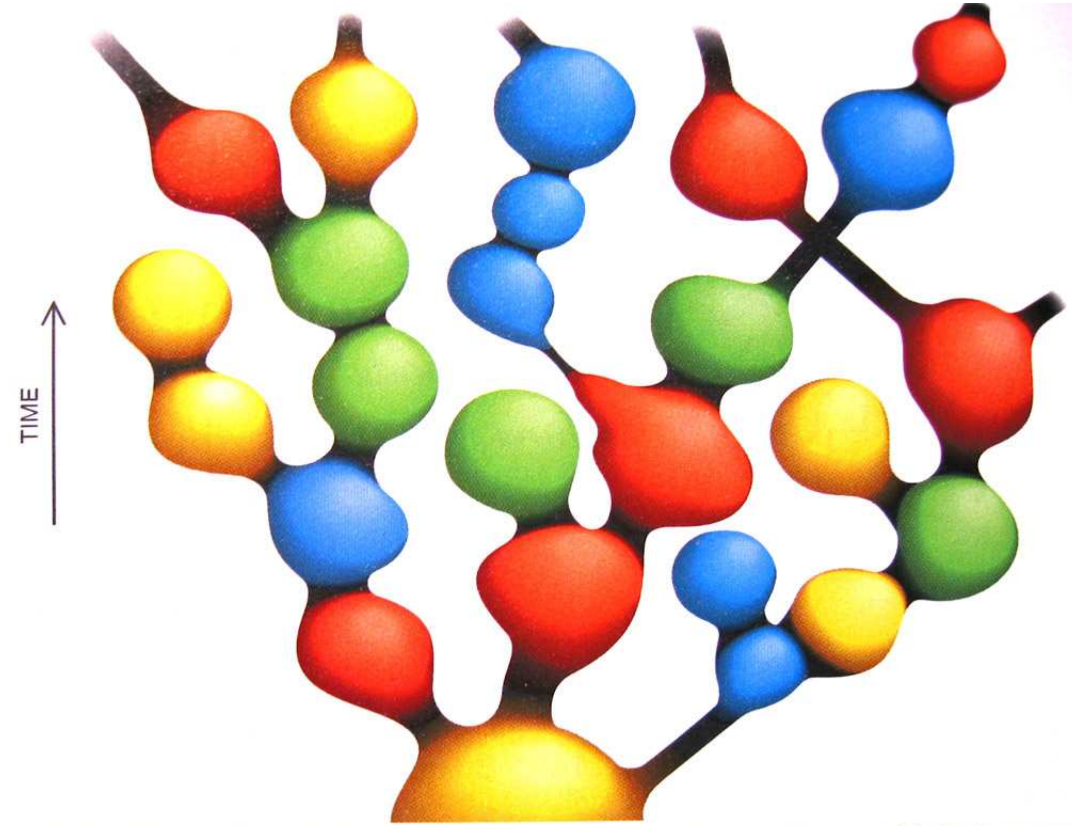
CML, D. Lyth, K. Kohri

Benefits of Hilltop Inflation

- Produce “Eternal Inflation”
- Produce the spectral index required from (WMAP) observation.
- Reduce the inflation scale. (Solve the cosmic string problem of hybrid inflation.)

Eternal Inflation



SELF-REPRODUCING COSMOS appears as an extended branching of inflationary bubbles. Changes in color represent "mutations" in the laws of physics from parent universes. The properties of space in each bubble do not depend on the time when the bubble formed. In this sense, the universe as a whole may be stationary, even though the interior of each bubble is described by the big bang theory.

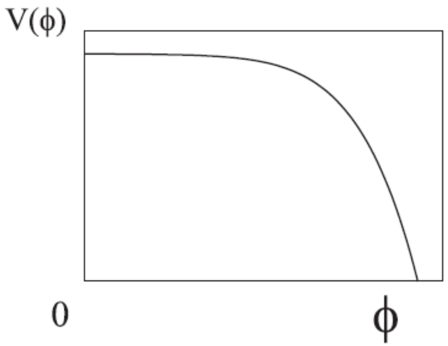
Analytical Solutions

$$P_{\varsigma} = \frac{1}{12\pi^2}\left(\frac{V_0}{M_P^4}\right)^{\frac{p-4}{p-2}} e^{-2\eta_0 N}\, \frac{\left[p\lambda\left(e^{(p-2)\eta_0 N}-1\right)+\eta_0 x\right]^{\frac{2p-2}{p-2}}}{\eta_0^{\frac{2p-2}{p-2}}\left(\eta_0 x - p\lambda\right)^2}$$

$$n_s = 1 + 2\eta_0\left[1 - \frac{\lambda p(p-1)e^{(p-2)\eta_0 N}}{\eta_0 x + p\lambda\left(e^{(p-2)\eta_0 N}-1\right)}\right]$$

$$x = \frac{p(p-1)\lambda}{1+\eta_0}$$

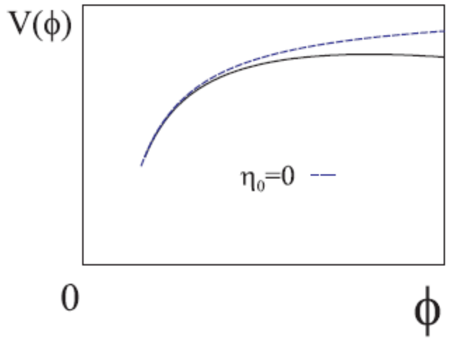
Three classes of Hilltop Inflation



$\eta_0 \leq 0$

$p > 2$

R invariance New Inflation

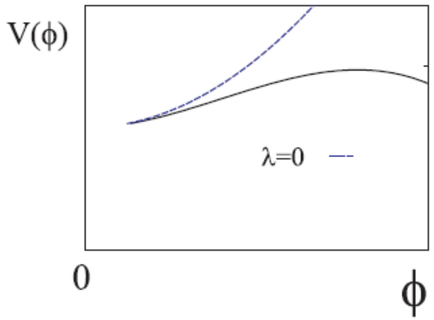


$\eta_0 < 0$

$p < 0$

(Hilltop) D-term Inflation

(Hilltop)F-term Inflation



$\eta_0 > 0$

$p > 2$

(Hilltop) Supernatural Inflation

F-term Hybrid Inflation

$$W = \kappa S(\phi_+ \phi_- - M^2)$$

$$V = V_0 \left(1 + \frac{\kappa^2}{8\pi^2} \ln\left(\frac{\phi}{\Lambda}\right) \right) \qquad V_0 = \kappa^2 M^4 \qquad \phi = \sqrt{2}\,\mathrm{Re}(S)$$

D-term Hybrid Inflation

$$W = \lambda S \Phi_+ \Phi_-$$

$$V = V_0 \left(1 + \frac{g^2}{4\pi^2} \ln\left(\frac{\phi}{\Lambda}\right) \right) \qquad V_0 = \frac{g^2 \xi^2}{2}$$

For both F- and D-term inflation:

Those are NOT hilltop inflation

$$n_s \simeq 1 - \frac{1}{N} = 0.983$$

Hilltop F-term Inflation from quadratic correction

From higher order term of the Kahler potential

$$K = |S|^2 + \frac{c_1 |S|^4}{M_P^2}$$

$$V = e^{\frac{K}{M_P^2}} \left[\left(W_m + \frac{W K_m}{M_P^2} \right) K^{m^\dagger n} \left(W_n + \frac{W K_n}{M_P^2} \right) - \frac{3|W|^2}{M_P^2} \right]$$

$$V = V_0 \left(1 + \frac{\kappa^2}{8\pi^2} \ln\left(\frac{\phi}{\Lambda} \right) \right) - c_2 H^2 \phi^2$$

This is regarded as “eta-problem”, but it can also be used as “eta-correction”

M. Bastero-Gil, S. F. King and Q. Shafi, hep-ph/0614198

Hilltop F-term Inflation from quartic correction

$$K = |S|^2 + \kappa_S \frac{|S|^4}{M_P^2} + \kappa_{SS} \frac{|S|^6}{6M_P^4}$$

$$V \simeq \kappa^2 M^4 \left(1 - \kappa_S \frac{\phi^2}{2M_P^2} + \gamma_S \frac{\phi^4}{8M_P^4} + \frac{\kappa^2}{8\pi^2} \ln\left(\frac{\phi}{\Lambda}\right) \right)$$

$$\gamma_S = \left(1 - \frac{7\kappa_S}{2} + 2\kappa_S^2 - 3\kappa_{SS} \right)$$

CML and Kingman Cheung 0812.2731[hep-ph]

Hilltop D-term Inflation from quadratic correction

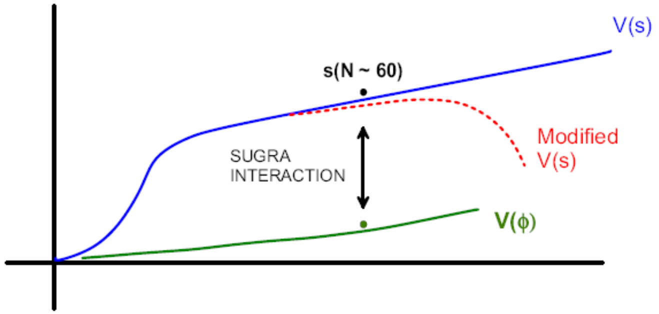
From Right-Handed sneutrino

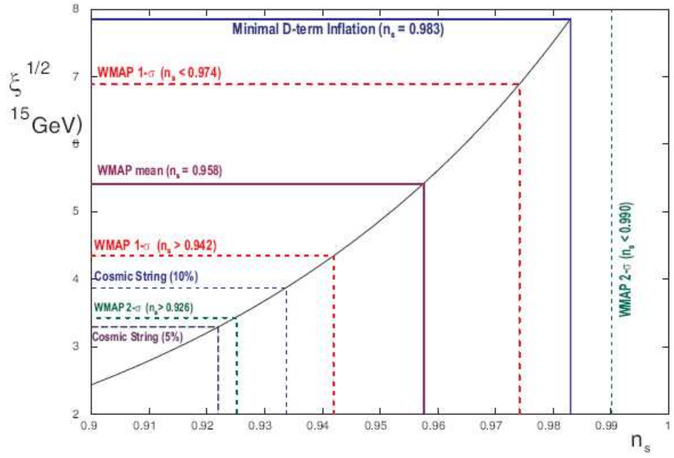
$$W_\nu = \lambda_\nu \Phi H_u L + \frac{m_\Phi}{2}\Phi^2$$

$$K = |S|^2 + |\Phi|^2 + \frac{c\,|S|^2|\Phi|^2}{M_P^2}$$

$$V = V_0\left(1 + \frac{\kappa^2}{8\pi^2}\ln\left(\frac{\phi}{\Lambda}\right)\right) - \frac{(c-1)m_\Phi^2\,|\Phi|^2|\phi|^2}{M_P^2}$$

CML and John McDonald hep-ph/0604245





(a) WMAP data only.

Hilltop D-term Inflation from quartic correction

$$V_D = \frac{1}{2}\left(\mathrm{Re}\, f\right)^{-1} g^2 \left(q_n K_n \Phi^n + \xi\right)^2$$

$$f^{-1} = 1 + \alpha \frac{\phi^2}{M_P^2} - \beta \frac{\phi^4}{M_P^4}$$

$$V = V_0 \left(1 + \alpha \frac{\phi^2}{M_P^2} - \beta \frac{\phi^4}{M_P^4} + \frac{g^2}{4\pi^2} \ln\left(\frac{\phi}{\Lambda}\right)\right)$$

CML and Kingman Cheung 0812.2731[hep-ph]

Supernatural Inflation

$$V(\phi) = V_0 + \frac{m^2}{2}\phi^2 \qquad n_s \geq 1$$

L. Randall, M. Soljacic and A. H. Guth   hep-ph/9512439

Hilltop Supernatural Inflation

$$W = \lambda_p \frac{\phi^p}{M_P^{p-3}}$$

$$V(\phi) = V_0 + \frac{m^2}{2}\phi^2 - \frac{\lambda_p A \phi^p}{p M_p} \left(+ \lambda_p^2 \frac{\phi^{2(p-1)}}{M_p^{2(p-3)}} \right)$$

$$n_s = 0.96$$

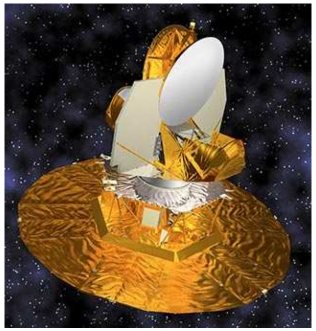
CML and Kingman Cheung 0901.3280 [hep-ph]

Planck Satellite:

$$\Delta n_s \leq 0.01$$

$$f_{NL} < 5$$

$$r < 0.01$$



WMAP

PLANCK May 14, 2009

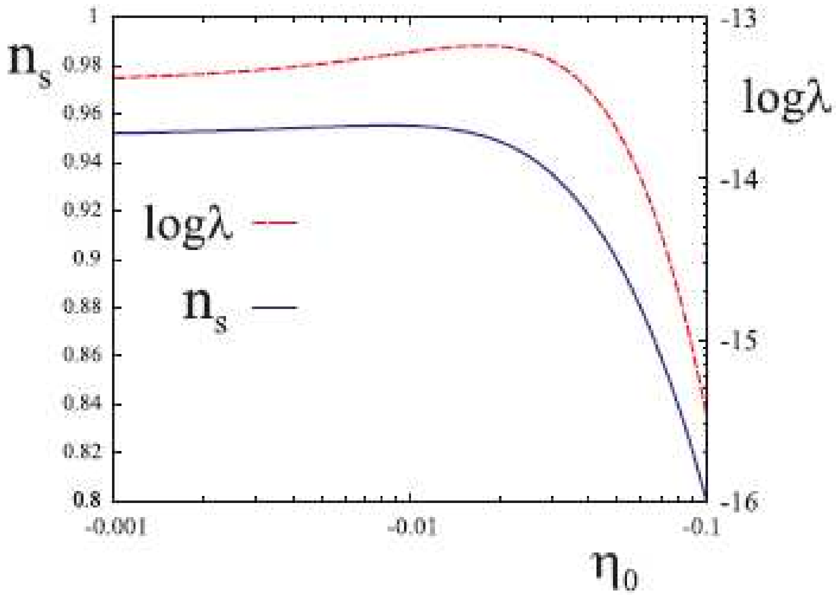
Finale

- “Now is the time to be a cosmologist.”--- Mark Kamionkowsky 0706.2986[astro-ph]
- “Our universe is an ultimate test of fundamental physics.”---

  Renata Kallosh hep-th/0702059

Model One, p=4

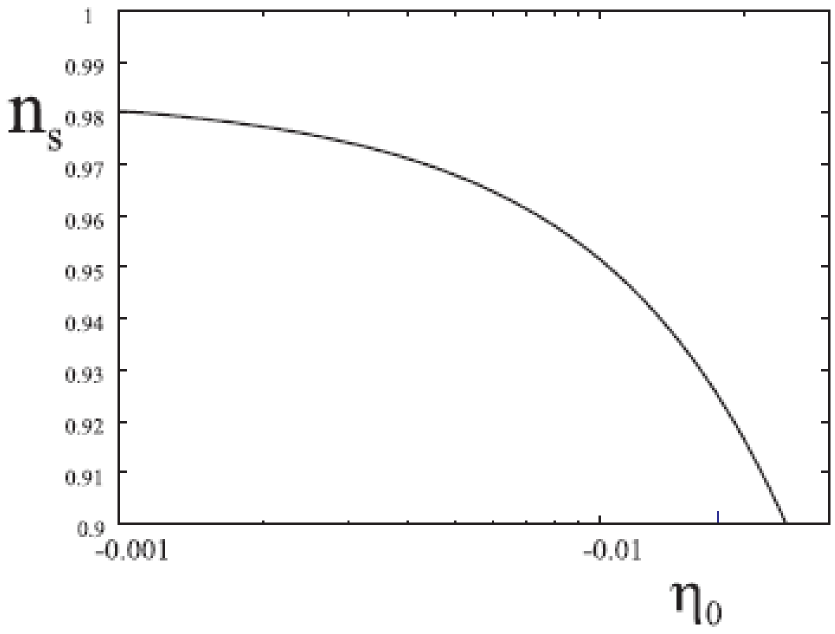
New Inflation



Model 2, p=0

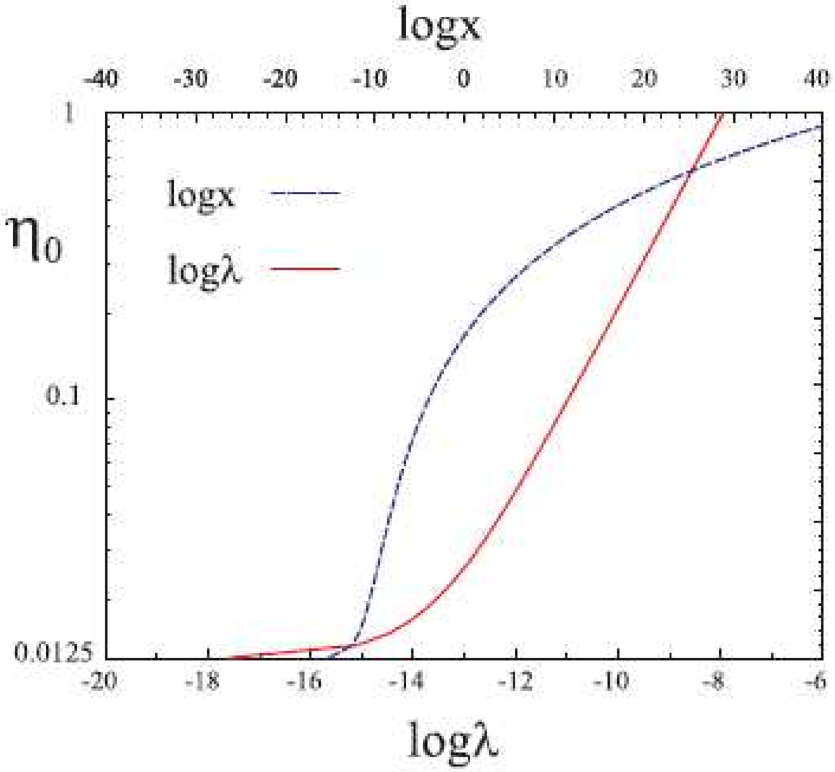
Take $p \rightarrow 0$ **with** $\lambda p = -\frac{V_0}{M_P^4}\frac{g^2}{4\pi^2}$

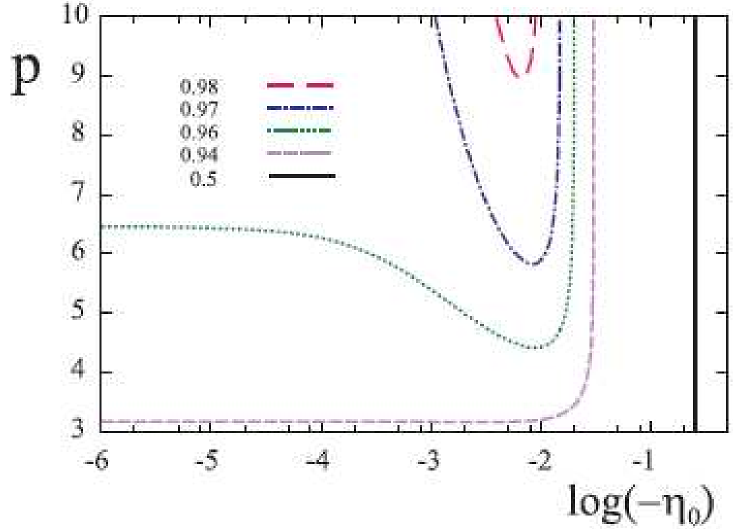
Modified F- and D-term Inflation



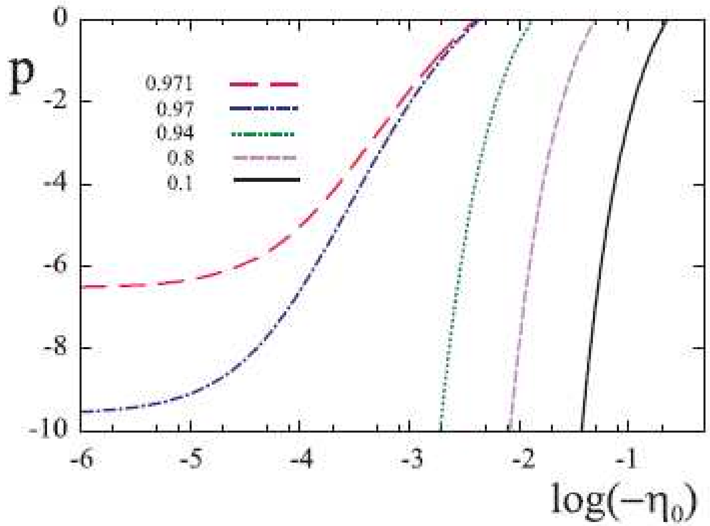
Model 3, p=4

We fix n=0.95 in this case.





Model 1



Model 2