Nucleon form factors and the lattice

施羅斯 / Wolfram Schroers

中央研究院

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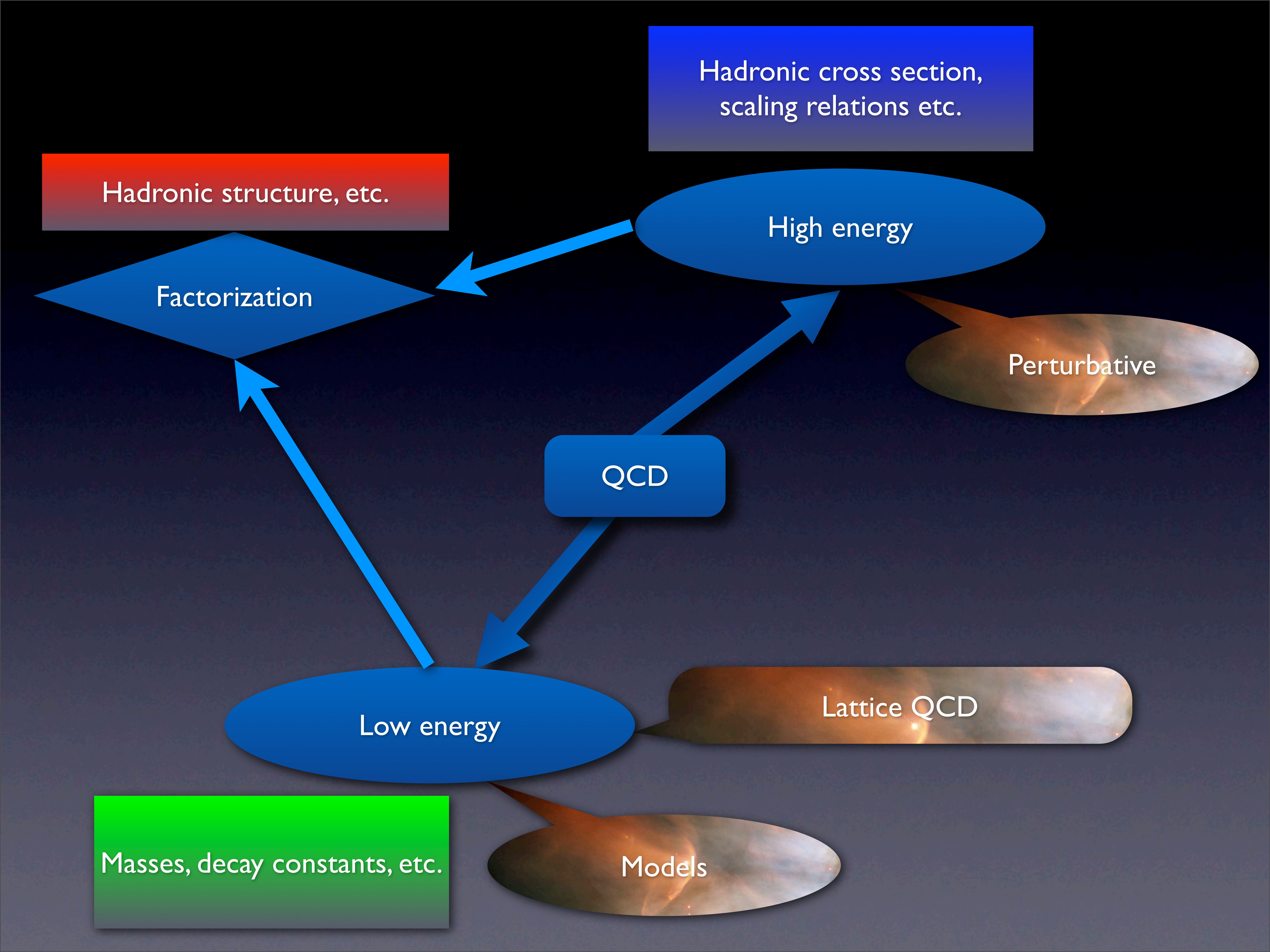
D.B. Renner, and A. Walker-Loud


Hadronic cross section, scaling relations etc.
Hadronic structure, etc.
High energy
Factorization
Perturbative
QCD
Lattice QCD
Low energy
Masses, decay constants, etc.
Models

- Vary parameters, e.g. m_q, N_c, N_f
- Test models of QCD
- Qualitative & quantitative post- and predictions from first principles

Quark masses!

Fermion discretizations

- (Improved) Wilson fermions
- (Improved) staggered fermions
- Twisted mass Wilson fermions
- Ginsparg-Wilson fermions
  - Domain-wall
  - Overlap

QCDSF

Fermion discretizations

Simple, well understood

- (Improved) Wilson fermions
- (Improved) staggered fermions
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  - Overlap

LHPC

Fermion discretizations

- (Improved) Wilson fermions
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  - Domain-wall
  - Overlap

Cheap

Chiral, $O(a^2)$, **but** expensive

Hybrid action

Hybrid action

- **LHPC**: Asqtad sea quarks (MILC), N_f=2+1, a=0.124 fm
- DWF valence quarks, L_5=16
- Lightest m_π=290 MeV, V=(2.5 fm)^3
- Conceptual question: 1/sqrt vs. cont. limit
- Excellent price/performance ratio!

LHPC publications

Nucleon Structure with Domain Wall Fermions at a = 0.084 fm.	PoS LATTICE2008:169,2008.
Aspects of Precision Calculations of Nucleon Generalized ...	PoS LATTICE2008:141,2008.
Light hadron spectroscopy using domain wall valence quarks ...	arXiv:0806.4549 [hep-lat]
Ab initio hadron structure from lattice QCD.	J.Phys.Conf.Ser.78:012019,2007.
Generalized parton distributions from domain wall valence ...	PoS LAT2007:160,2007.
Hadron spectrum with domain-wall valence quarks on an ...	PoS LAT2006:195,2006.
Nucleon Generalized Parton Distributions from Full Lattice ...	Phys.Rev.D77:094502,2008.
Nucleon structure in the chiral regime with domain wall ...	PoS LAT2006:121,2006.
Calculation of the nucleon axial charge in lattice QCD.	J.Phys.Conf.Ser.46:152-156,2006.
The Nucleon axial charge in full lattice QCD.	Phys.Rev.Lett.96:052001,2006.
First principles calculations of nucleon and pion form factors: ...	J.Phys.Conf.Ser.16:174-178,2005.
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Moments of nucleon spin dependent generalized parton ...	Nucl.Phys.Proc.Suppl.129:907-909,2004.
Insight into nucleon structure from generalized parton ...	Nucl.Phys.Proc.Suppl.129:910-912,2004.

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  - Domain-wall
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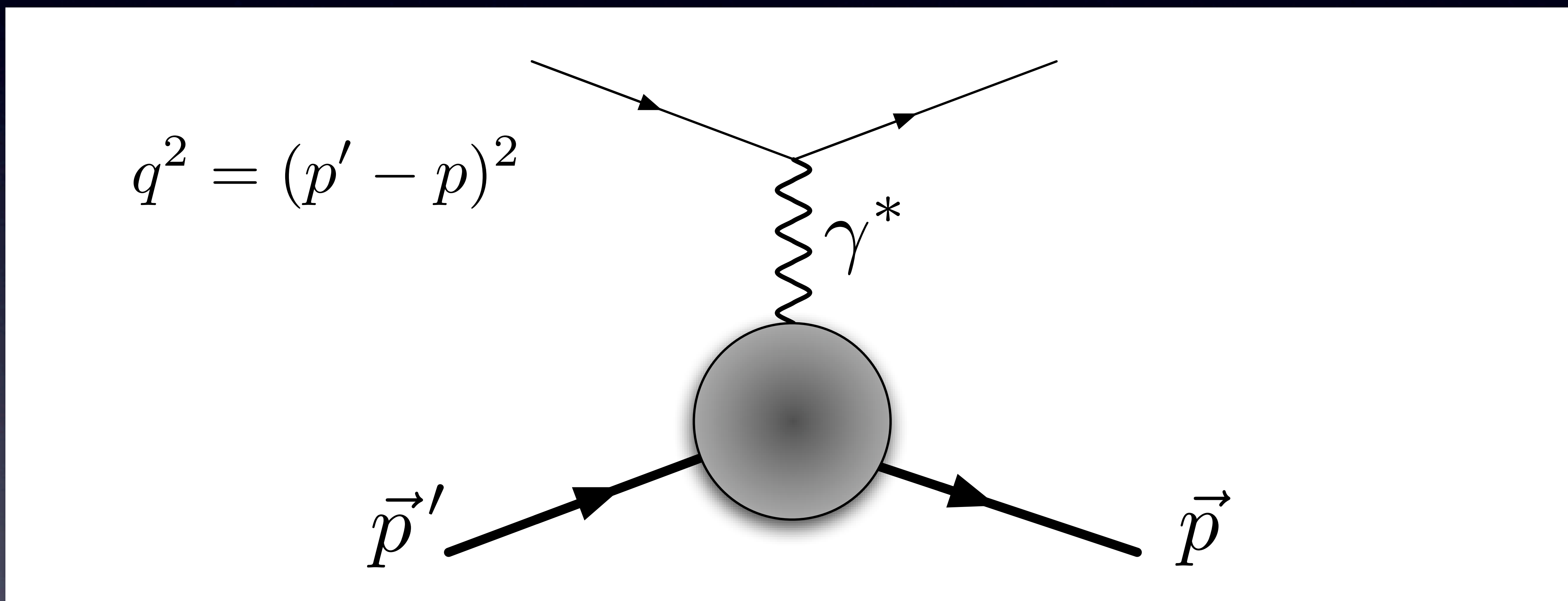
← Chiral, $O(a^2)$, **but** expensive

RBC/UKQCD

- LHPC uses: Full DWF with N_f=2+1, a=0.116 fm (24^3x64) and a=0.083 fm (32^3x64)
- DWF quarks all use L_5=32
- Lightest m_π=290 MeV, V=(2.7 fm)^3
- Similar technology as the hybrid calculation
- Serves both as extension of older work and as cross-check

The nucleon

Form factors



Form factors

$$\langle p'|J_\mu|p\rangle = \bar{u}(p')\Big[\gamma_\mu F_1(q^2) + \frac{\mathrm{i}\, q^\alpha}{2m_N}\sigma_{\alpha\mu}F_2(q^2)\Big]u(p)$$

Experiment

- Reviews: Phys.Rev. C71:055202 (2005) J.Phys. G34:S23-S25(2007)
- Two techniques:
  - Rosenbluth: elastic scattering $\sigma(e + p \to e + p)$
  - Spin-transfer: $\vec{e} + \vec{p} \to e + p$

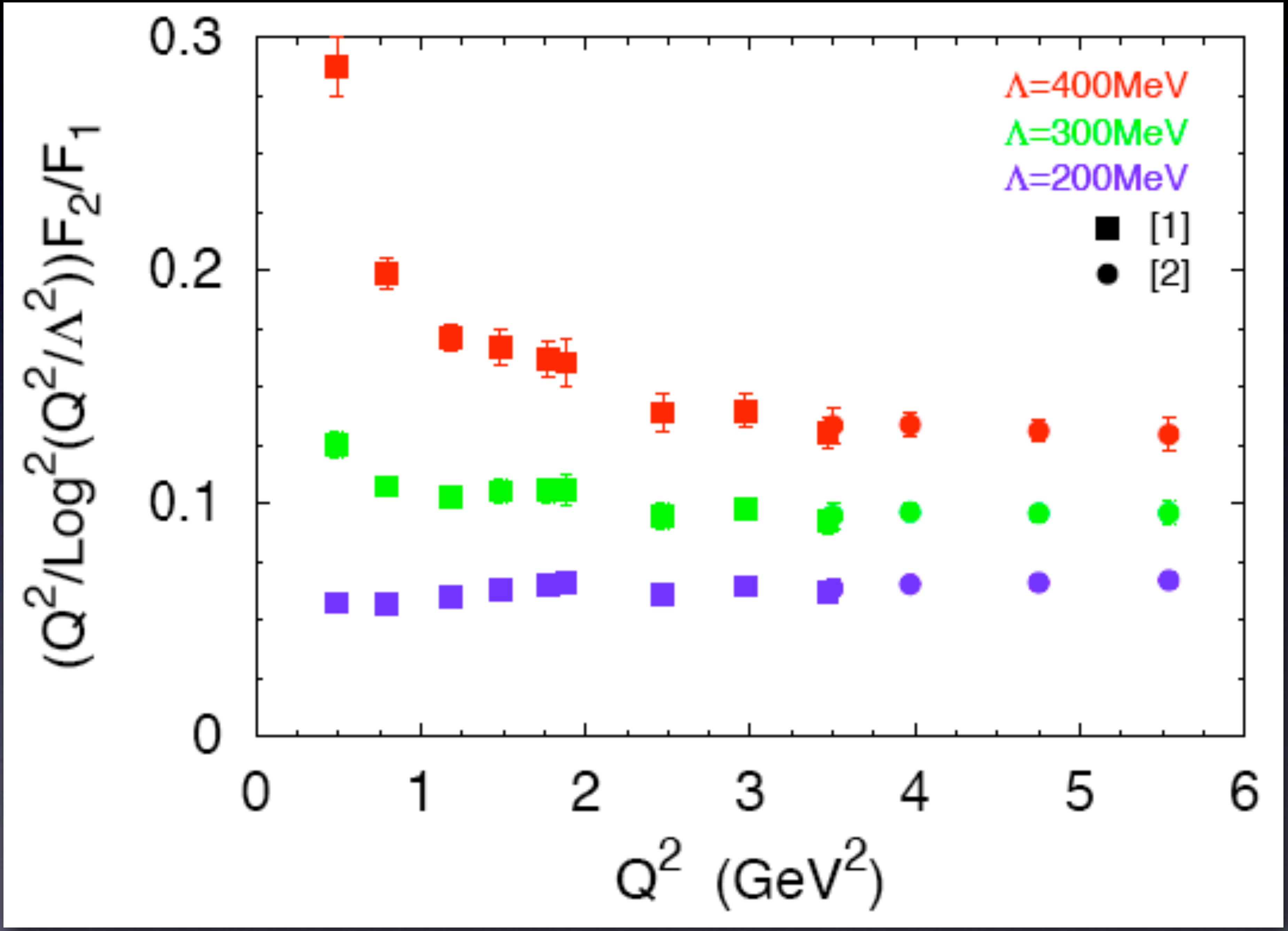
    $\vec{e} + p \to e + \bar{p}$

Asymptotic scaling

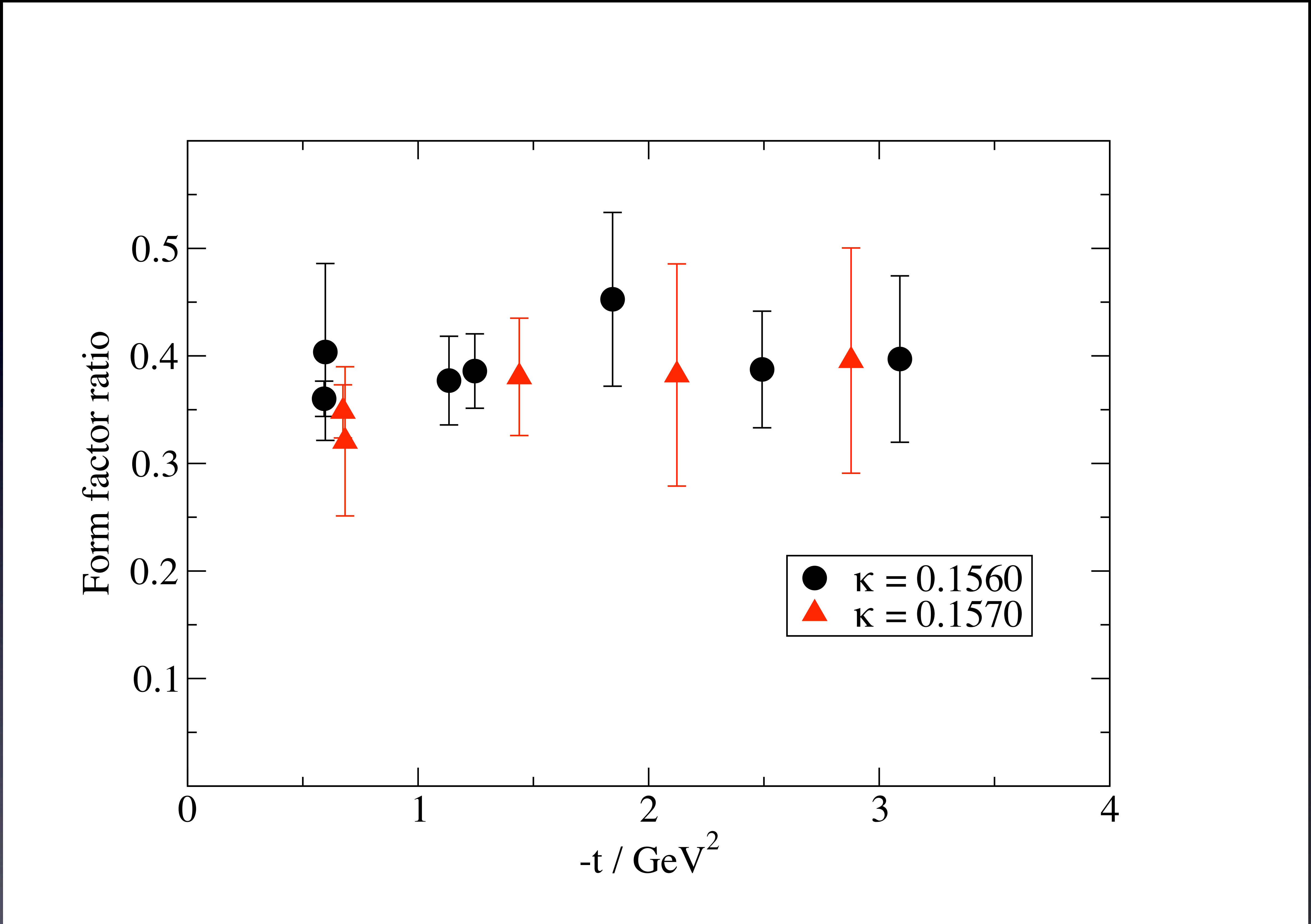
$t = Q^2$ dependence

$\frac{tF_2(t)}{F_1(t)}$	$\propto \mathrm{const}$	Naive quark counting rules
$\frac{\sqrt{t}F_2(t)}{F_1(t)}$	$\propto \mathrm{const}$	JLab spin transfer experiment
$\frac{tF_2(t)}{\log^2(t)F_1(t)}$	$\propto \mathrm{const}$	PRL91:092003 (2003)

Form factors in Nature

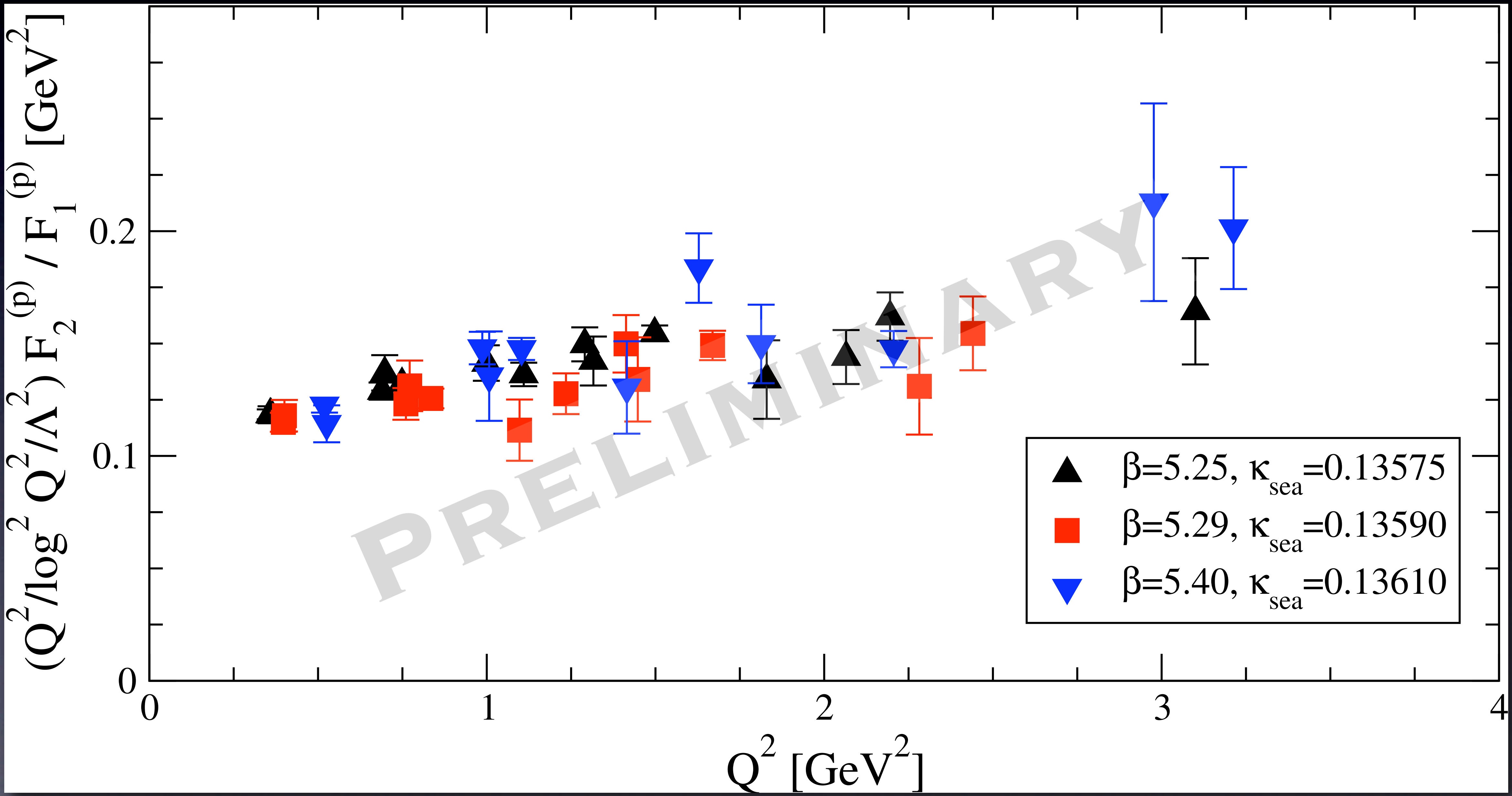


Phys.Rev.Lett. 91:092003 (2003)

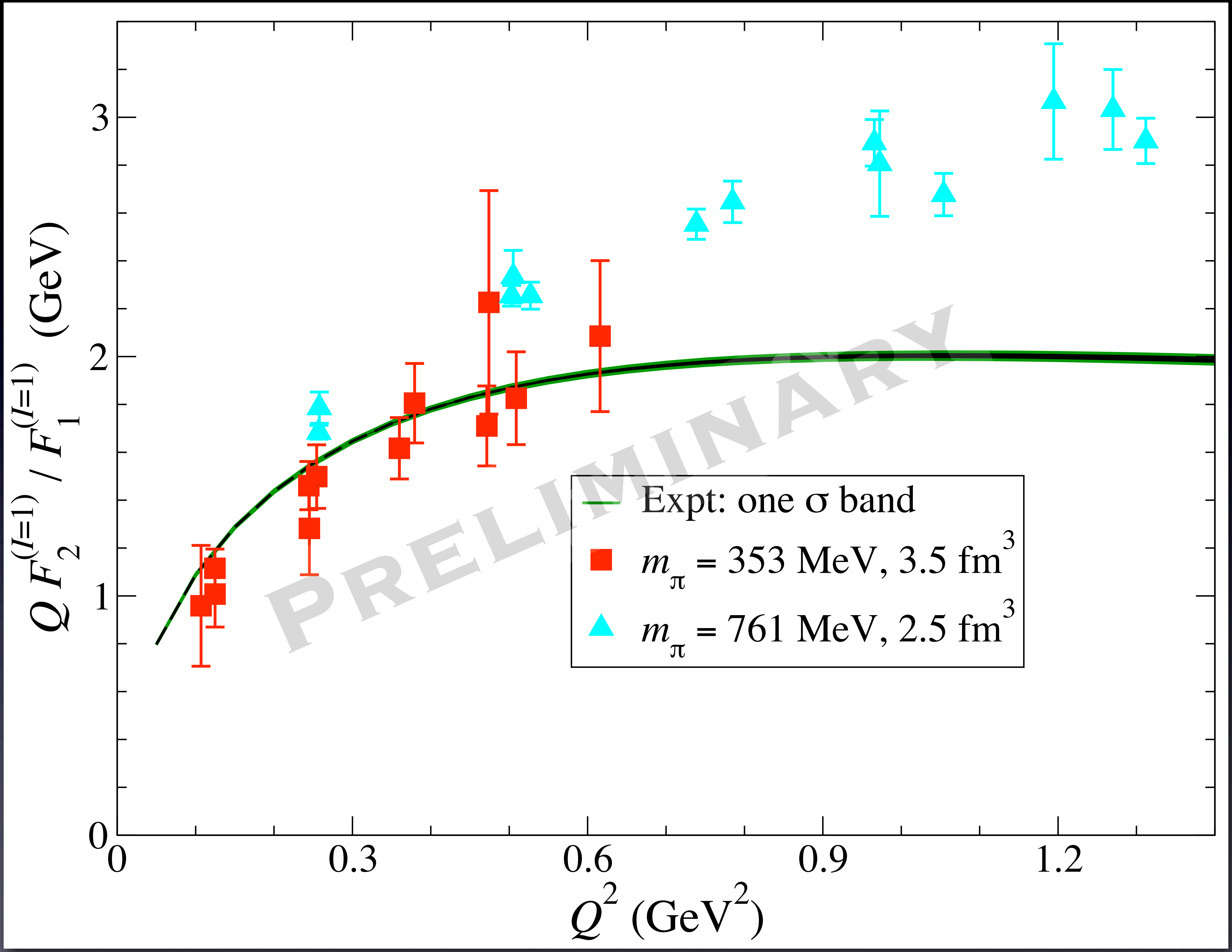


Nucl.Phys.Proc.Suppl. 128:170-178 (2004) (Negele et al)

Ratio scaling



QCDSF collab., $m_\pi \approx 600$ MeV, a=0.070...0.084 fm



LHPC collaboration, in preparation

- Charge radii:

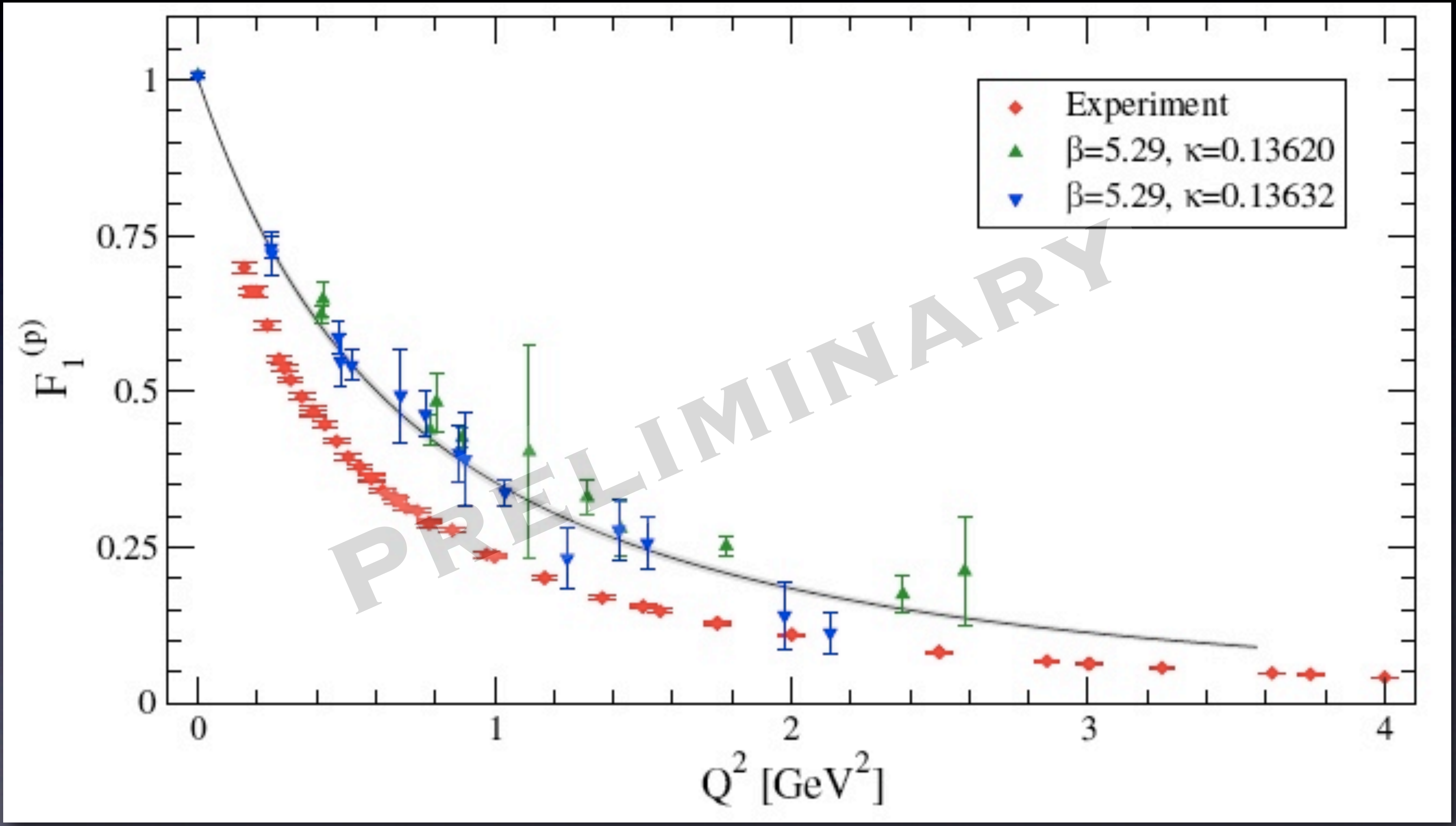
$$F_i(q^2) = F_i(0)\left(1 + \frac{1}{6} r_i^2 q^2 + \mathcal{O}(q^4)\right)$$

- Magnetic moment:

$$\mu \equiv F_1(0) + F_2(0) \equiv 1 + \kappa$$

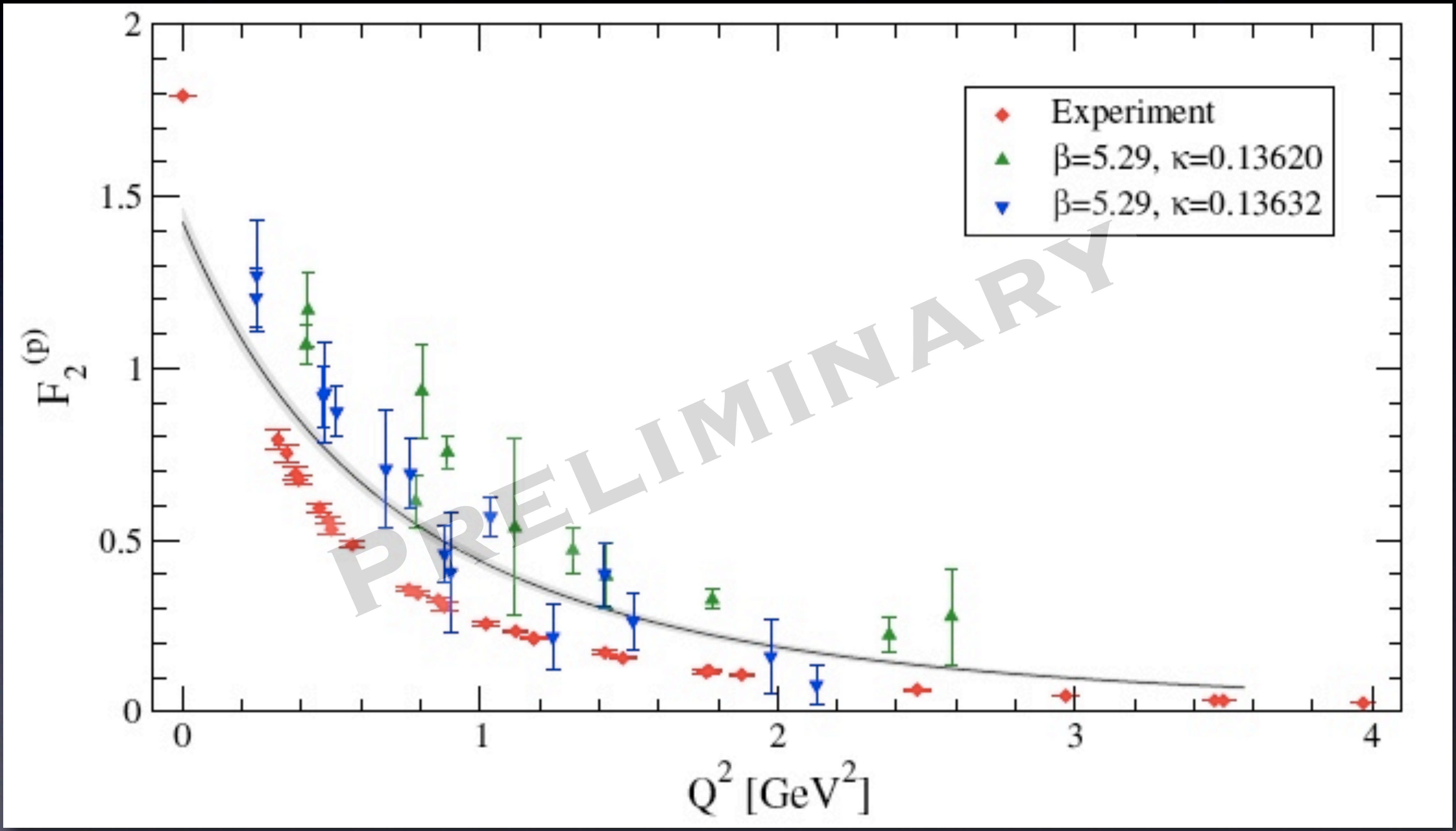
Related to small-Q^2,
playing to the lattice method's strength

Lattice ↔ Experiment



QCDSF collaboration, selected data

Lattice ↔ Experiment



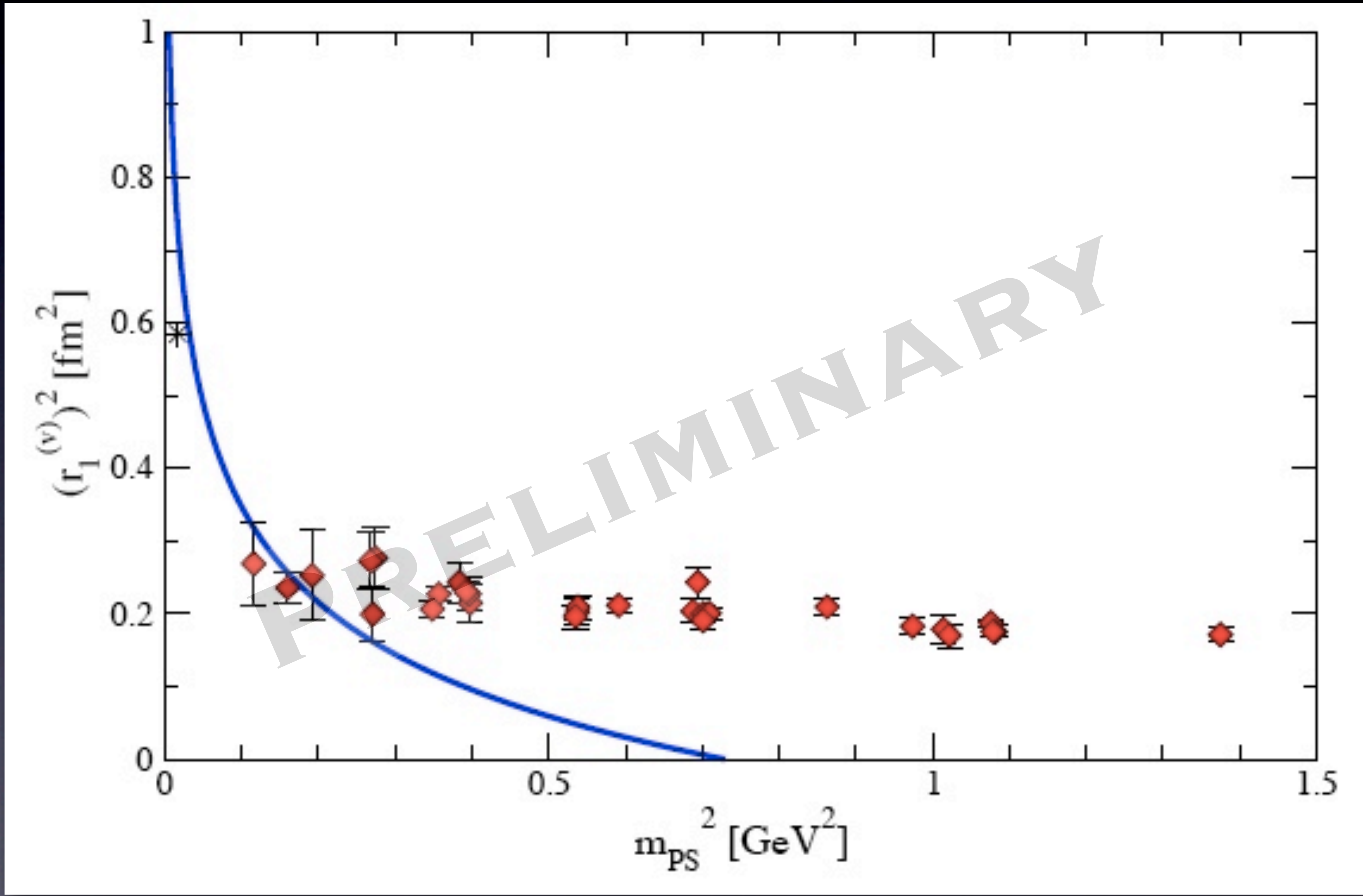
QCDSF collaboration, selected data

Chiral effective FT

- Expressions for $(r_1^v)^2, (r_2^v)^2, \kappa^v$ (Phys.Rev. D71:034508 (2005))

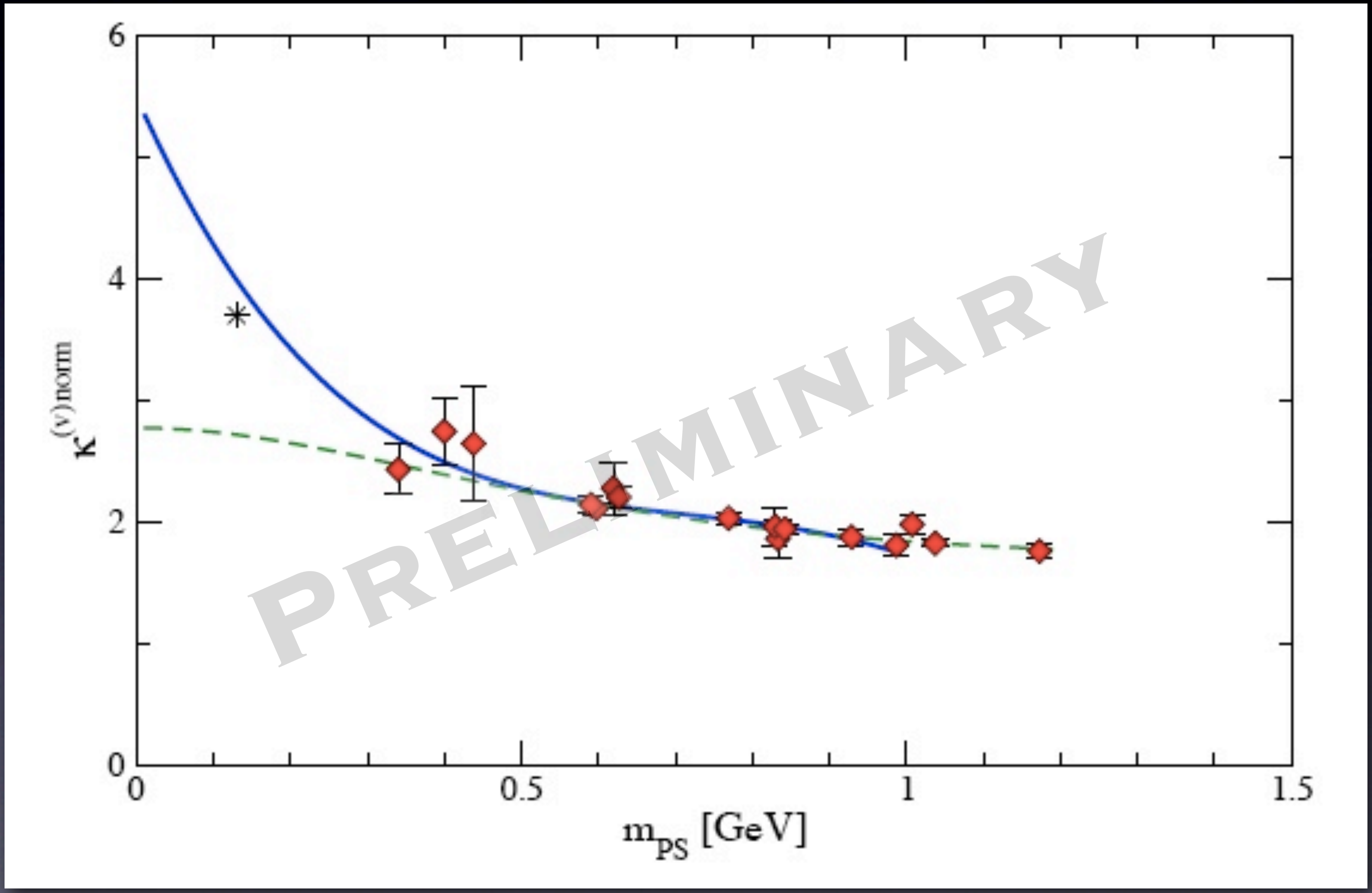
$$
\begin{aligned}
(r_1^v)^2 \quad = \quad & -\frac{1}{(4\pi f_\pi)^2}\left(1 + 7g_A^2 + (10g_A^2 + 2)\ln\left[\frac{m_{\mathrm{PS}}}{\lambda}\right]\right) \\
& + \frac{c_A^2}{54\pi^2 f_\pi^2}\left(26 + 30\ln\left[\frac{m_{\mathrm{PS}}}{\lambda}\right]\right. \\
& \left. +30\frac{\Delta}{\sqrt{\Delta^2 - m_{\mathrm{PS}}^2}}\ln\left[\frac{\Delta}{m_{\mathrm{PS}}} + \sqrt{\frac{\Delta^2}{m_{\mathrm{PS}}^2} - 1}\right]\right)
\end{aligned}
$$

Result for $r_1^{(v)}$



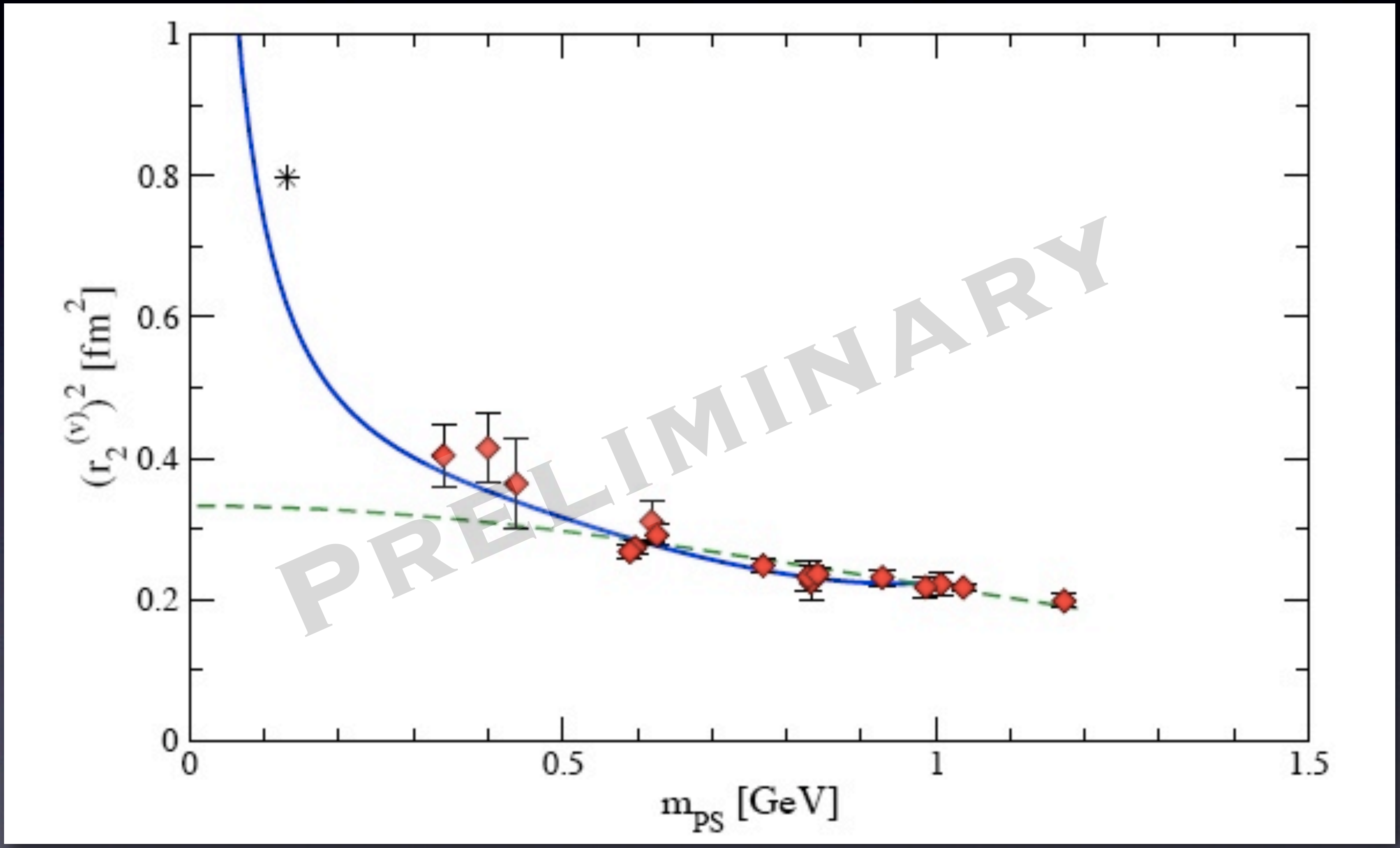
Review, Baryons '07

Result for $\kappa^{(v)}$



Review, Baryons '07

Result for $r_2^{(v)}$

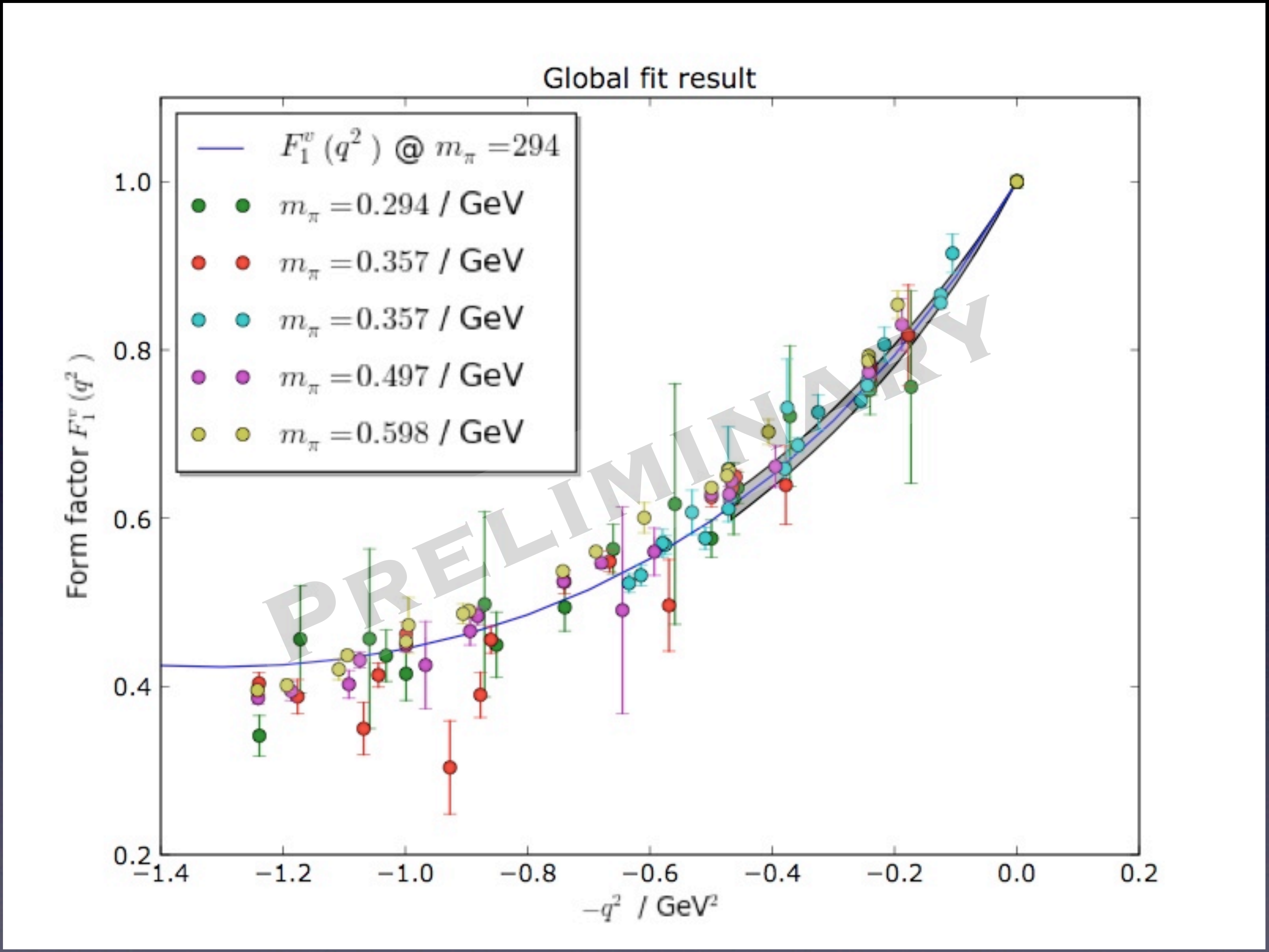


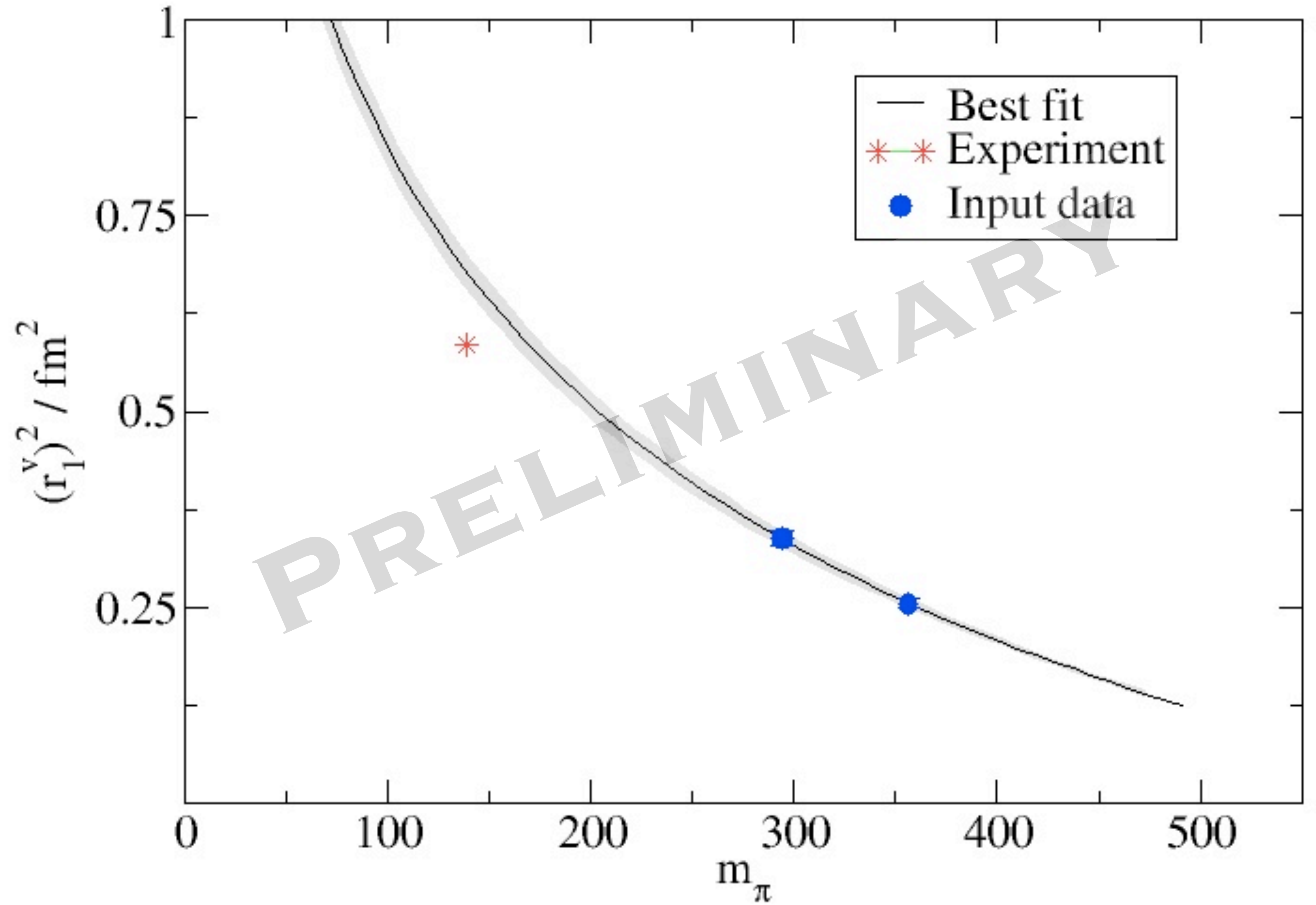
Review, Baryons '07

$$F_1^v(q^2) = 1 + \frac{1}{(4\pi F_\pi)^2}\left\{q^2\left(\frac{68}{81}c_A^2 - \frac{2}{3}g_A^2 - 2B_{10}^{(r)}(\lambda)\right) + q^2\left(\frac{40}{27}c_A^2 - \frac{5}{3}g_A^2 - \frac{1}{3}\right)\log\left[\frac{m_\pi}{\lambda}\right]\right.$$
$$+ \int_0^1 dx \left[\frac{16}{3}\Delta^2 c_A^2 + m_\pi^2\left(3g_A^2 + 1 - \frac{8}{3}c_A^2\right) - q^2 x(1-x)\left(5g_A^2 + 1 - \frac{40}{9}c_A^2\right)\right]\log\left[\frac{\tilde{m}^2}{m_\pi^2}\right]$$
$$+ \int_0^1 dx \left[\frac{32}{9}c_A^2 q^2 x(1-x)\frac{\Delta \log R(\tilde{m})}{\sqrt{\Delta^2 - \tilde{m}^2}}\right]$$
$$\left. - \int_0^1 dx\, \frac{32}{3}c_A^2\Delta\left[\sqrt{\Delta^2 - m_\pi^2}\log R(m_\pi) - \sqrt{\Delta^2 - \tilde{m}^2}\log R(\tilde{m})\right]\right\} + \mathcal{O}(\epsilon^4)\,. \qquad (38)$$

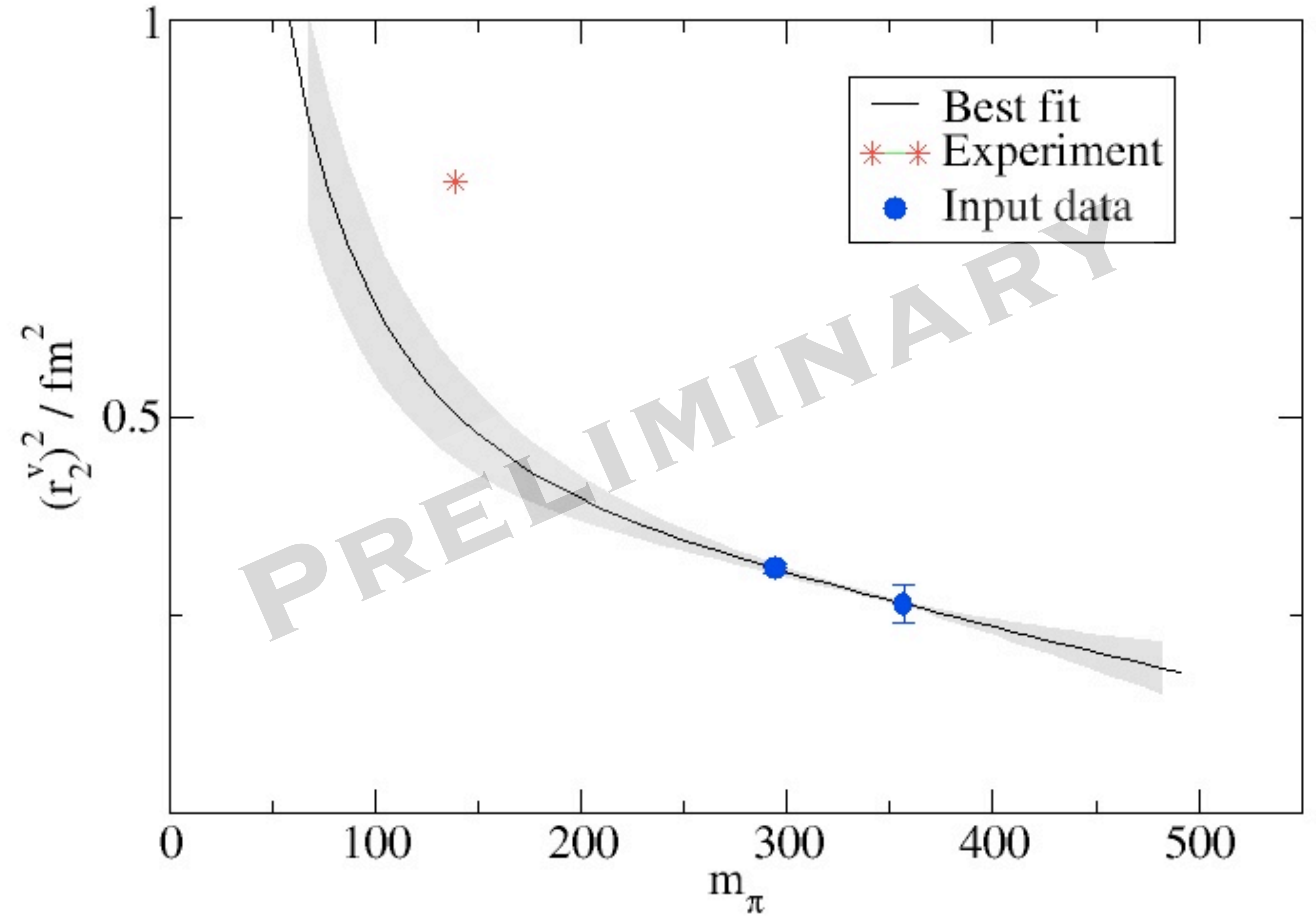
$$F_2^v(q^2) = \kappa_v(m_\pi) - g_A^2\frac{4\pi M_N}{(4\pi F_\pi)^2}\int_0^1 dx\left[\sqrt{\tilde{m}^2} - m_\pi\right]$$
$$+ \frac{32c_A^2 M_N\Delta}{9(4\pi F_\pi)^2}\int_0^1 dx\left[\frac{1}{2}\log\left[\frac{\tilde{m}^2}{4\Delta^2}\right] - \log\left[\frac{m_\pi}{2\Delta}\right]\right.$$
$$\left. + \frac{\sqrt{\Delta^2 - \tilde{m}^2}}{\Delta}\log R(\tilde{m}) - \frac{\sqrt{\Delta^2 - m_\pi^2}}{\Delta}\log R(m_\pi)\right] \qquad (39)$$

$$\kappa_v(m_\pi) = \kappa_v^0 - \frac{g_A^2\, m_\pi M_N}{4\pi F_\pi^2} + \frac{2c_A^2\Delta M_N}{9\pi^2 F_\pi^2}\left\{\sqrt{1 - \frac{m_\pi^2}{\Delta^2}}\log R(m_\pi) + \log\left[\frac{m_\pi}{2\Delta}\right]\right\}$$
$$- 8E_1^{(r)}(\lambda)M_N m_\pi^2 + \frac{4c_A c_V g_A M_N m_\pi^2}{9\pi^2 F_\pi^2}\log\left[\frac{2\Delta}{\lambda}\right] + \frac{4c_A c_V g_A M_N m_\pi^3}{27\pi F_\pi^2\Delta}$$
$$- \frac{8c_A c_V g_A \Delta^2 M_N}{27\pi^2 F_\pi^2}\left\{\left(1 - \frac{m_\pi^2}{\Delta^2}\right)^{3/2}\log R(m_\pi) + \left(1 - \frac{3m_\pi^2}{2\Delta^2}\right)\log\left[\frac{m_\pi}{2\Delta}\right]\right\} \qquad (41)$$

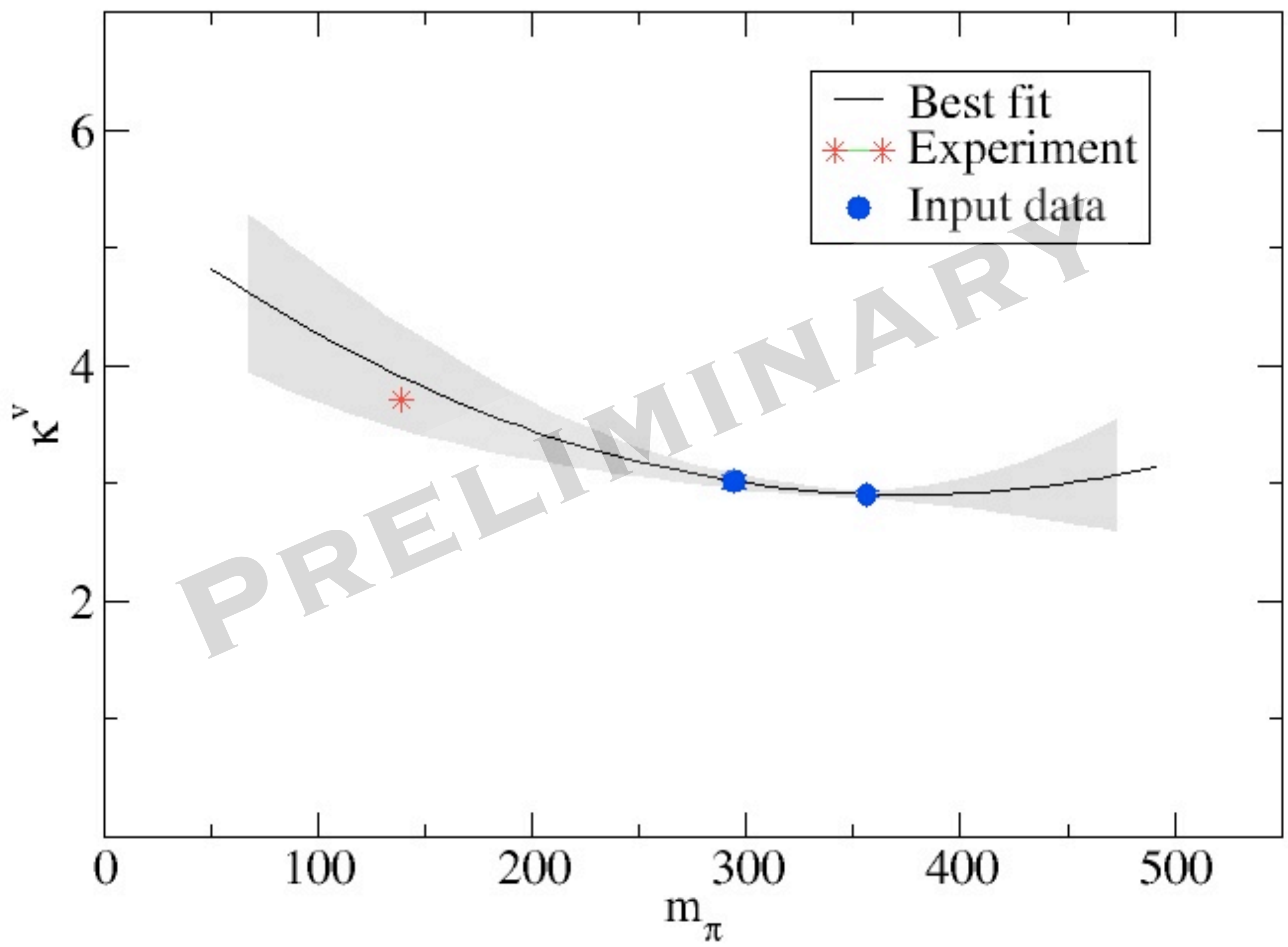

Global fit result
$F_1^v(q^2)$ @ m_π =294
m_π =0.294 / GeV
m_π =0.357 / GeV
m_π =0.357 / GeV
m_π =0.497 / GeV
m_π =0.598 / GeV
PRELIMINARY
Form factor $F_1^v(q^2)$
$-q^2$ / GeV²
1.0
0.8
0.6
0.4
0.2
−1.4
−1.2
−1.0
−0.8
−0.6
−0.4
−0.2
0.0
0.2



Best fit
Experiment
Input data
PRELIMINARY
$(r_1^v)^2$ / fm^2
1
0.75
0.5
0.25
0
100
200
300
400
500
m_π



Best fit
Experiment
Input data
PRELIMINARY
$(r_2^v)^2$ / fm^2
1
0.5
0
100
200
300
400
500
m_π



Best fit
Experiment
Input data
PRELIMINARY
6
4
2
κ^v
0
100
200
300
400
500
m_π

Summary

- Form factors remain an important subject
- Lattice simulations have provided insight into their scaling and nucleon size
- Previously: qualitative insight
- Upcoming generation of results will provide quantitative insights
- Interplay of models, experiments and lattice results becoming crucial