RS Model as a Framework for Flavor Physics

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Free Parameters and problems in SM

- There are 27(+2) free parameters in SM

$$
\begin{aligned}
4 &\quad : \quad \alpha_1\,, \alpha_2,\ \alpha_3,\ G \\
+2 &\quad : \quad M_W\,,\ m_H \\
+6+6 &\quad : \quad m_e,\ m_\mu,\ m_\tau\,, m_{\nu_1}\,,\ m_{\nu_2}\,,\ m_{\nu_3}\,,\ m_u,\ m_c,\ m_t,\ m_d,\ m_s,\ m_b \\
+1+4+4 &\quad : \quad \theta_{QCD},\ U_{CKM}\,,\ U_{PMNS} \\
(+2) &\quad : \quad \text{Majorana phases}
\end{aligned}
$$

  Roughly speaking, the first class involves gauge interaction and how the symmetries are broken. The second classes will be referred as general flavor physics.

- The ultima goal of HEP theorist is to reduce the number of free parameters. For example,
  - GUT makes three couplings to one
  - flavor symmetry to reduce the $21(+2)$ flavor parameters to only few
  - ... etc
- There are also tension or hierarchy among these fundamental constants in each group:
  - $1/\sqrt{G} \gg M_W\,,\ m_H$
  - $m_t \gg m_q, m_l \gg m_\nu,\ \theta^{12}_{CKM} \gg \theta^{23}_{CKM} \gg \theta^{13}_{CKM}$

Resolutions proposed to understand the flavor physics

In the past few decades, a few ways have been proposed to either reduce the number of free parameters or to explain the hierarchy.

- three copies of fermion family with exactly the same quantum numbers
  $\Rightarrow$ PREON model
- Larger symmetry group $G \supset SU(3)_c \times SU(2)_L \times U(1)_Y \times H_{flavor}$
  e.g. $SU(8)$, $SO(10+4k)$, $E8$ etc.
- Pattern in the fermion mass matrices
  By guessing/ phenomenological fit: Structure zeros
  By higher scale symmetry: Froggatt-Nielsen
- Statistics
- Giving up the explanation: Landscape

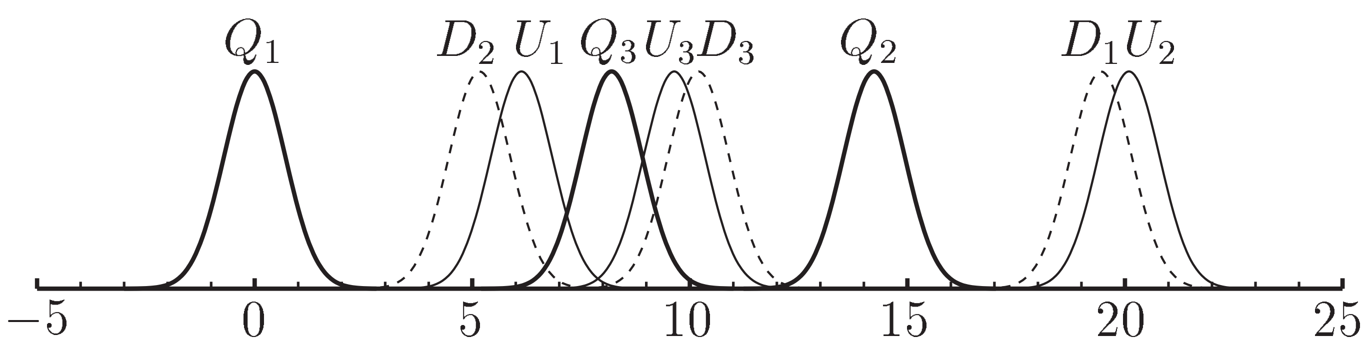
A New Paradigm for studying the flavor physics

Recent development of extra-dimensional model provides a new way to look at the flavor problem.

Flavor Problem $\Leftrightarrow$ Geometry in extra dimension

Split fermion model as an example:

- Linear displacement between left-handed and right-handed fermions in the fifth dimension becomes exponentially suppressed 4D Yukawa.
- A realistic configuration to fit quark masses and mixings



RS Model is one of the promising candidates

- Randall-Sundrum can explain the hierarchy in the first class.

$$EW \sim ke^{-kr_c\pi}$$

  with $kr_c \sim 11.7$, where k is the 5D curvature $\sim M_{planck}$ and r_c is the radius of the compactified fifth dimension.
- Due to the special profile of bulk fermion in RS, the second hierarchy can be achieved without fine tuning in Yukawa couplings.
- The number of free parameters (in flavor sector) is smaller than in SM

Introduction to the Randall-Sundrum Model

- There are more than 4 dim. Indeed RS assumes a 1+4 dim with a warp or conformal metric, AdS
- 5D interval (S_1/Z_2)is given by

$$ds^2 = G_{AB}dx^A dx^B = e^{-2kr_c|\phi|}\eta_{\mu\nu}dx^\mu dx^\nu - r_c^2 d\phi^2$$

- Two branes are localizes at $\phi = 0$(UV) and $\phi = \pi$(IR)
- The metric is, $-\pi \leq \phi \leq \pi,\ \sigma \equiv kr_c|\phi|$

$$G_{AB} = \begin{pmatrix} e^{-2\sigma}\eta_{\mu\nu} & 0 \\ 0 & -r_c^2 \end{pmatrix},\ G^{AB} = \begin{pmatrix} e^{+2\sigma}\eta_{\mu\nu} & 0 \\ 0 & -\frac{1}{r_c^2} \end{pmatrix}$$

Fermions in 5D Bulk

- 5D fermions are 4-component spinors, i.e. vector-like fermions

$$\Psi(x^\mu, y) = \begin{pmatrix} \psi_R(x^\mu, y) \\ \psi_L(x^\mu, y) \end{pmatrix}$$

- the Dirac matrices in 5D are $\gamma^M = (\gamma^\mu, i\gamma^5)$
- Project out the L(R) chiral state by boundary conditions or orbifold parities, i.e. how the field transforms under $Z_2 : y \to -y$

$$\begin{pmatrix} \psi_R(x^\mu, y) \\ \psi_L(x^\mu, y) \end{pmatrix} \to \pm \begin{pmatrix} \psi_R(x^\mu, -y) \\ -\psi_L(x^\mu, -y) \end{pmatrix}$$

- 5D action for fermions is

$$\int d^4x d\phi \sqrt{G} E_a^A \bar{\Psi} \gamma^a D_A \Psi - m \, \mathrm{sgn}(\phi) \bar{\Psi} \Psi$$

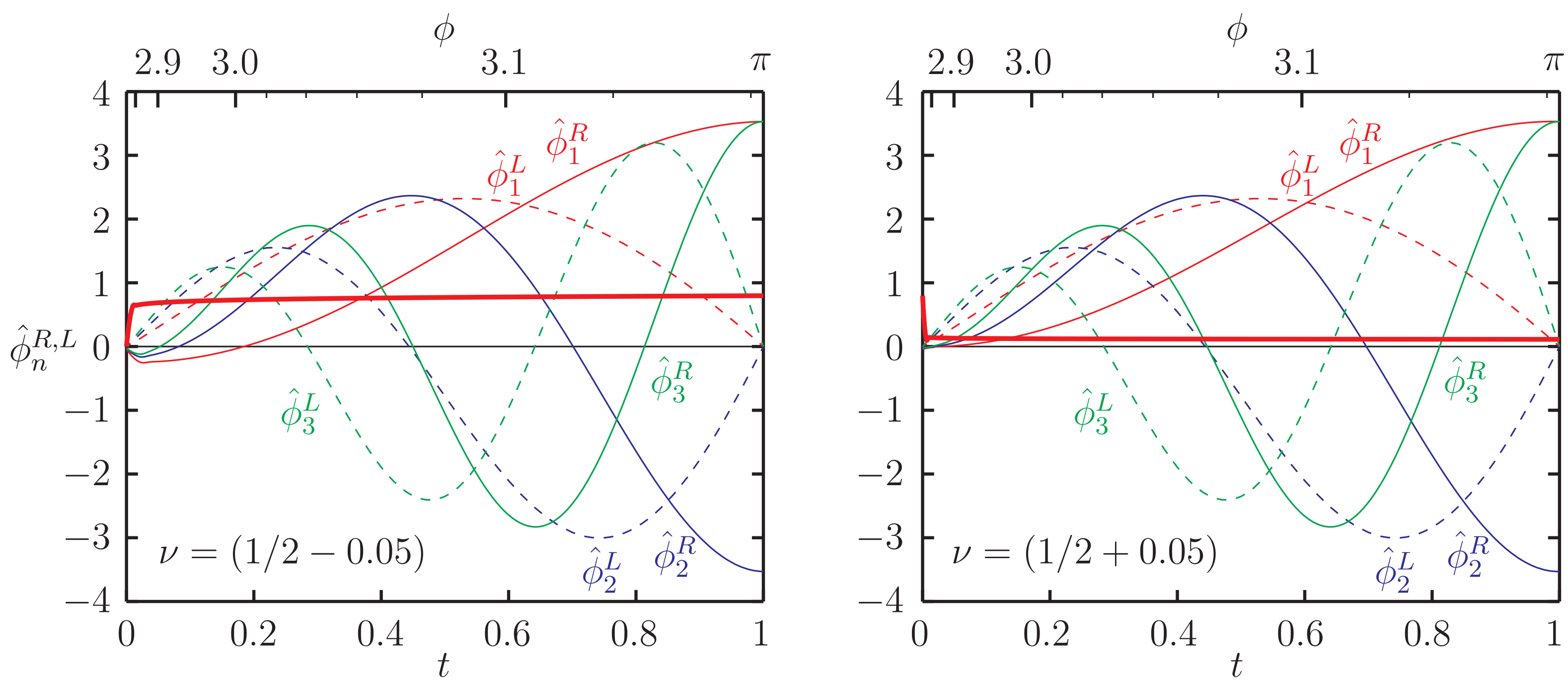
where $E_a^A = diag(e^\sigma, e^\sigma, e^\sigma, e^\sigma, 1/r_c)$, and we define a dimensionless $\nu \equiv m/k$

Profiles of bulk fermions

- Do the usual KK decomposition:

$$\Psi_{L,R}(x,\phi) = \frac{e^{\frac{3}{2}\sigma}}{\sqrt{r_c}} \sum_n \Psi_n^{L,R}(x)\hat{\phi}_n^{L,R}(\phi)\,,\ \langle\hat{\phi}_n|\hat{\phi}_m\rangle = \delta_{m,n}$$

- The RH chiral zero mode lives near the UV(IR) brane if $\nu > 1/2(\nu < 1/2)$



The thick red lines are the zero mode wave function of RH chiral fermion and t is the comformal variable

$$t \equiv e^{-kr_c(\pi-\phi)}$$

Fermion Masses in RS

- The coefficients $\nu_{L,R}$ control the zero modes peak at either UV or IR
- Localize the Higgs at the IR brane
- Have the zero modes of the SM chiral fermions localized near UV brane then the overlap after SSB will be very small
- No need to fine tune Yukawa's
- Quark masses are naturally small
- If all the fermions both LH doublets and RH singlets are localized near UV then top quark comes out too light
- RH top quark or (t_L, b_L) must not too far from IR brane

Quark Masses in RS

- The quark masses are given by

$$\left\langle M_{ij}^{f}\right\rangle = \frac{\lambda_{5,ij}^{f} v_W}{k r_c \pi} f_L^0(\pi, \nu_{f_i}^L) f_R^0(\pi, \nu_{f_j}^R)$$

  where the label f denotes up-type or down-type quark species, $v_W = 174$ GeV, and

$$f_{L,R}^0(\phi, \nu_{L,R}) = \sqrt{\frac{k r_c \pi (1 \mp 2\nu_{L,R})}{e^{k r_c \pi (1 \mp 2\nu_{L,R})} - 1}} \exp\left[k r_c \phi (1/2 \mp \nu_{L,R})\right]$$

  where the upper(lower) sign applies to the LH(RH) zero mode

- The Yukawa couplings λ_{ij} are not necessarily symmetric in i, j
- $f_{L,R}$ shows that the masses are controlled by values of $\nu_{L,R}$
- The task is find configurations that fit the CKM matrix
- Bonus: both LH and RH rotations are given for each solution
- In SM, only the LH rotations are detectable. $V_{CKM} = V_L^{u\dagger} V_L^d$

General Configurations

- In general quark mass matrices are not symmetrical in RS. Several configurations found. For example,

$$\begin{aligned} \nu_Q &= \{0.634, 0.556, 0.256\} \\ \nu_U &= \{-0.664, -0.536, 0.185\} \\ \nu_D &= \{-0.641, -0.572, -0.616\} \end{aligned}$$

- The u and d quark mass matrices (at TeV scale)

$$\langle|M_u|\rangle = \begin{pmatrix} 0.000897 & 0.049 & 0.767 \\ 0.010 & 0.554 & 8.69 \\ 0.166 & 9.06 & 142.19 \end{pmatrix}, \langle|M_d|\rangle = \begin{pmatrix} 0.0019 & 0.017 & 0.0044 \\ 0.022 & 0.196 & 0.050 \\ 0.352 & 3.209 & 0.813 \end{pmatrix}$$

(in GeV), where we have used $ke^{kr_c\pi} = 1.5TeV$

RS Quark Masses

- The CKM matrix elements for the above

$$|V^L_{us}| = 0.16(14)\,,\; |V^L_{ub}| = 0.009(11)\,,\; |V^L_{cb}| = 0.079(74)$$
$$|V^R_{us}| = 0.42(24)\,,\; |V^R_{ub}| = 0.12(10)\,,\; |V^R_{cb}| = 0.89(13)$$

- Note the RH rotations are larger than the LH ones
- Appears to be true from the numerical searches we found
- How to test it?

Symmetrical Mass Matrices in RS

- Most of the “constructions” starts from conjecture assuming that they are symmetrical
- Put zeros (1 to 3) in appropriate places to fit CKM and the observed mass hierarchies
- Can RS accomodates these without fine tuning the Yukawa couplings?
- One ONE texture zero patterns are allowed.
- By construction $U_L = U_R$

All is not well

- The main problem is that the new KK modes will modify EWPT
- Ths S, T parameters will receive tree level corrections
- It's known that $\rho = 1$ is protected by a custodial $SU(2)$ symmetry
- Promote that to a bulk gauge symmetry
- Tree level KK gauge effects are suppressed
- The gauge symmetry is now $SU(2)_L \times SU(2)_R \times U(1)_X$
- Take $X = B - L$

Custodial RS model

- Break $SU(2)_R \to U(1)_R$ by orbifold B.C.

	UV	IR
$\tilde{W}_\mu^{1,2}$	$-$	$+$
other Gauge Fields	$+$	$+$

- $U(1)_R \times U(1)_X \to U(1)_Y$ by VEV on UV brane. We have a Z' and B_μ

$$Z'_\mu = \frac{g_5 \tilde{W}_\mu^3 - g'_5 \tilde{B}_\mu}{\sqrt{g_5^2 + g_5'^2}}$$

and

$$B_\mu = \frac{g'_5 \tilde{W}_\mu^3 + g_5 \tilde{B}_\mu}{\sqrt{g_5^2 + g_5'^2}}$$

- B_μ is the SM hypercharge gauge boson and broken with $SU(2)_L$ on the IR brane by Higgs (a bi-doublet)

Quark Representation

- Zero modes have parity $(++)$
- Usual assignment doesn't work

$$\begin{array}{cc} SU(2)_L & SU(2)_R \\ \begin{pmatrix} t_L \\ b_L \end{pmatrix} & \begin{pmatrix} t_R \\ b_R \end{pmatrix} \end{array}$$

because t_R is a zero mode and $SU(2)_R$ is broken on UV

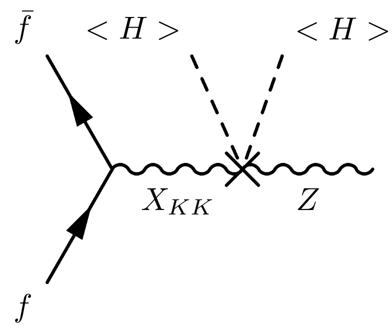
- d_R and t_R must have their own $(-+)$ partners

$$\begin{array}{ccc} SU(2)_L & SU(2)_R & SU(2)_R \\ \begin{pmatrix} t_L \\ b_L \end{pmatrix} & \begin{pmatrix} T_R \\ b_R \end{pmatrix} & \begin{pmatrix} t_R \\ B_R \end{pmatrix} \end{array}$$

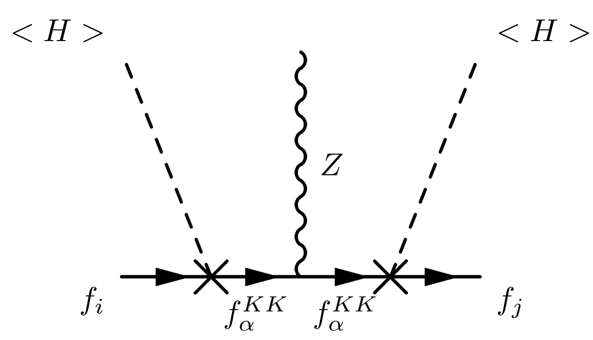
- They don't affect the quark mass matrix

FCNC in the minimal Constrained RS model

- Besides the direct production of the KK Z (≥ 2.5 TeV) is tree level FCNC
- FCNC $Z - Z_{KK}$ and $Z - Z'_{KK}$ mixing



- KK- fermion mixings



- Going to the mass basis the unitarity is broken $\rightarrow$ FCNC

$t \to Z + jets$

- The BR is:

$$
\begin{aligned}
& Br(t \to Zc(u)) \\
&= \frac{2}{\cos^2\theta_W}\left(|Q_Z(t_L)\hat{\kappa}^L_{tc(u)}|^2 + |Q_Z(t_R)\hat{\kappa}^R_{tc(u)}|^2\right)\left(\frac{1-x_t}{1-y_t}\right)^2\left(\frac{1+2x_t}{1+2y_t}\right)\frac{y_t}{x_t} \\
&= 1.8677 \times \left(|Q_Z(t_L)\hat{\kappa}^L_{tc(u)}|^2 + |Q_Z(t_R)\hat{\kappa}^R_{tc(u)}|^2\right)
\end{aligned}
$$

- LH and RH decays are different because $\kappa^R > \kappa^L$ in the configs we found
- $Br(t_R \to Z + c(u)_R) < Br(t_L \to Z + c(u)_L)$ by $\sim 2 - 10$
- The BR is $\sim 10^{-5}$ c.f. SM $\sim 10^{-13}$
- Compare the decays in $t\bar{t}$ vs single tW channels.

Conclusions

- We have found that the RS model can accommodate good quark mass matrices without fine tuning Yukawa
- It is possible to accommodate symmetrical mass matrices if there is one and no more texture zero
- For asymmetrical configurations $U_R > U_L$
- Tree level FCNC best probed in $t \to Z + jets$
- BR is $\sim 10^{-5}$ makes it very interesting at the LHC
- Predicts that LH decays are dominant.