PPP8

RARE ELECTROWEAK DECAYS

OF *K* AND *B* MESONS

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Introduction

- Standard Model (SM) successful.
- Calculation for strong QCD processes not well established.
- Electroweak processes simple at the quark level.
- Calculations complicated for hadronic states.
- Quarks are tightly bound inside hadrons.
- Non-perturbative nature of QCD forces.
- Interactions of bound quarks in hadrons not well understood.

Electroweak rare decay processes

$$M_1 \to M_2 \ell \bar{\ell} \quad \ell = \nu, e^-$$

Quark Level

$$q_1 \to q_2 \ell \bar{\ell} \quad \ell = \nu, e^-$$

Neutrino Pair Emission $q_1 \to q_2 \nu\bar{\nu}$

$$\Gamma\left(q_1 \to q_2 \nu\bar{\nu}\right) = \frac{3\alpha^2 G_F^2}{12288\pi^3 \sin^4\theta_W} m_1^5 R(y) \left| \sum_j \lambda_j f(x_j) \right|^2$$

$$x_j = m_j^2 / M_W^2 \,,\; y = m_2^2 / m_1^2 \,,\; \lambda_j = V_{j2}^* V_{j1}$$

$$R(y) = 1 - 8y + 8y^3 - y^4 - 12y^2 \ln y$$

$$f(x) = f_B(x) + f_Z(x)$$

$$f_B(x) = \frac{4x}{(x-1)^2}(x - 1 - \ln x)$$

$$f_Z(x) = \frac{x}{2(x-1)^2}\left[(-6+x)(x-1) + (2+3x)\ln x\right]$$

Charged Lepton Pair Emission $q_1 \to q_2 \ell^+ \ell^-$

$$\Gamma(q_1 \to q_2 \ell^+ \ell^-) = \frac{\alpha^2 G_F^2}{192\pi^3} m_1^5 \Bigg\{ (H_1 + H_2) \left| \sum_j \lambda_j f^V(x_j) \right|^2 + (H_1 - H_2) \left| \sum_j \lambda_j f^A(x_j) \right|^2 + H_3 \left| \sum_j \lambda_j f^T(x_j) \right|^2 + H_4 \,\mathrm{Re} \left(\sum_j \lambda_j f^V(x_j) \right) \left(\sum_j \lambda_j f^T(x_j) \right)^* \Bigg\}$$

Dynamical form factors:

$$f^V(x) = -\frac{1}{32\pi \sin^2\theta_W} f_B(x) + \frac{4\sin^2\theta_W - 1}{8\pi \sin^2\theta_W} f_Z(x) + \frac{1}{\pi} f_{\gamma 1}(x)$$

$$f^A(x) = \frac{1}{32\pi \sin^2\theta_W} f_B(x) + \frac{1}{8\pi \sin^2\theta_W} f_Z(x)$$

$$f^T(x) = \frac{1}{\pi} f_{\gamma 2}(x)$$

$$f_{\gamma 1}(x) = \frac{1}{72(1-x)^4} \left[x^2(-25 + 19x)(1-x) - 2(8 - 32x + 54x^2 - 30x^3 + 3x^4) \ln x \right]$$

$$f_{\gamma 2}(x) = \frac{x}{24(1-x)^4} \left[(-7 + 5x + 8x^2)(x-1) + 6x(2-3x) \ln x \right]$$

Kinematic Factors:

$$H_1 = m_1^{-8}\{-2I(2) + (\Sigma + 2m_\ell^2)I(1) + (D^2 - 4m_\ell^2\Sigma)I(0) + 2m_\ell^2 D^2 I(-1)\}$$

$$H_2 = 6m_1^{-8} m_\ell^2[\Sigma I(0) - I(1)\}$$

$$H_3 = m_1^{-8}\{-\Sigma I(1) - [\Sigma(\Sigma + 2m_\ell^2) + 12m_1^2 m_2^2]I(0)$$
$$+[2\Sigma^2(\Sigma - m_\ell^2) - 24m_\ell^2 m_1^2 m_2^2]I(-1) + 4m_\ell^2\Sigma D^2 I(-2)\}$$

$$H_4 = 6m_1^{-8}\{\Sigma I(1) + [-D^2 + 2m_\ell^2\Sigma]I(0) - 2m_\ell^2 D^2 I(-1)\}$$

$$\Sigma = m_1^2 + m_2^2$$
$$D = m_1^2 - m_2^2$$

$$I(n) = \int_{s_-}^{s_+} ds\, s^n \sqrt{(1 - 4m_\ell^2/s)(s^2 - 2\Sigma s + D^2)}$$
$$s_+ = (m_1 - m_2)^2$$
$$s_- = 4m_\ell^2$$

CKM mixing matrix

$V_{ud} = 0.97418 \pm 0.00027$

$V_{us} = 0.22552 \pm 0.0019$

$V_{ub} = 0.00393 \pm 0.00036$

$V_{cd} = 0.230 \pm 0.011$

$V_{cs} = 0.9723 \pm 0.0026$

$V_{cb} = 0.0412 \pm 0.0011$

$V_{td} = 0.0081 \pm 0.0006$

$V_{ts} = 0.0387 \pm 0.0023$

$V_{tb} = 0.999134 \pm 0.000045$

Dynamical Form Factors

Current quark masses

$m_u = 2.4 \pm 0.9$ MeV

$m_c = 1.27^{+0.07}_{-0.11}$ GeV

$m_t = 171.2 \pm 2.1$ GeV

Neutrino Pair Production

m_j	x_j	$f(x_j)$
0.2400E-02	0.89110480E-09	0.54816915E-07
0.1270E+01	0.24952480E-03	0.59628445E-02
0.1712E+03	0.45343361E+01	0.62775693E+01

Lepton Pair Production

m_j	$f^V(x_j)$	$f^A(x_j)$	$f^T(x_j)$
0.2400E-02	0.14740260E+01	0.31787867E-09	-0.82731128E-10
0.1270E+01	0.58651859E+00	0.88889967E-04	-0.23267408E-04
0.1712E+03	-0.27343687E+00	0.72681391E+00	-0.11448136E+00

Bound Quarks in Hadronic States

- Consider M_1 changes into meson M_2 electroweak process
- Bound q_1 in M_1 changes into bound q_2 in M_2
- Bound quarks in hadrons behave like free particles
- Both q_1 and q_2 treated as free quarks
- All QCD effects adequately taken care of by assigning effective masses to the quarks
- Empirical determination of effective quark masses
- Effective quark masses determined from weak decay processes

Processes considered

$$K^- \to \pi^- \nu\bar{\nu}$$
$$B^- \to \pi^- \nu\bar{\nu}$$
$$B^- \to K^- \nu\bar{\nu}$$

$$K^- \to \pi^- e^+ e^-$$
$$B^- \to \pi^- e^+ e^-$$
$$B^- \to K^- e^+ e^-$$

Effective quark masses

- d-quark mass in π^- is taken to be negligible.
- s-quark mass in K^- = 339.2 MeV
- b-quark mass in B^- = 2779 MeV

Kinematic Factors

- R Factor

Transition	m_1 (MeV)	m_2 (MeV)	R
$K^- \to \pi^-$	339.2	0	1.0
$B^- \to \pi^-$	2779	0	1.0
$B^- \to K^-$	2779	339.2	0.892045

***** H FACTORS *****

M1= 339.2 M2= 1.0E-10

H(1)	H(2)	H(3)	H(4)
0.500000D+00	0.000000D+00	-0.144006D+02	-0.200000D+01

M1= 2229.0 M2= 1.0E-10

H(1)	H(2)	H(3)	H(4)
0.500000D+00	0.000000D+00	-0.144006D+02	-0.200000D+01

M1= 2229.0 M2= 339.2

H(1)	H(2)	H(3)	H(4)
0.419536D+00	0.000000D+00	-0.155275D+02	-0.167814D+01

Magnitudes of $\lambda_j f(x_j)$ for the decay $q_1 \to q_2 \nu \bar{\nu}$

Transition	$\lvert\lambda_u f(x_u)\rvert$	$\lvert\lambda_c f(x_c)\rvert$	$\lvert\lambda_t f(x_t)\rvert$
$s \to d$	1.2043E-8	1.3333E-3	1.9680E-3
$b \to d$	1.5979E-10	5.6504E-5	5.2098E-2
$b \to s$	4.8584E-11	2.3887E-4	2.4275E-1

Magnitudes of $\lambda_j f^{\alpha}(x_j)$, $\alpha = V, A, T$ for the decay $q_1 \to q_2 \ell^+ \ell^-$

Transition	j	$\left|\lambda_j f^V(x_j)\right|$	$\left|\lambda_j f^A(x_j)\right|$	$\left|\lambda_j f^T(x_j)\right|$
$s \to d$	u	3.2384E-1	6.9838E-11	1.8176E-11
	c	1.3115E-1	1.9876E-5	5.2026E-6
	t	8.57225E-5	2.2786E-4	3.5890E-5
$b \to d$	u	4.2968E-3	9.2662E-13	2.4116E-13
	c	5.5579E-3	8.4232E-7	2.2048E-7
	t	2.2693E-3	6.0318E-3	9.5008E-4
$b \to s$	u	1.3064E-3	2.8174E-13	7.3325E-14
	c	2.3496E-2	3.5609E-6	9.3209E-7
	t	1.0574E-2	2.8106E-2	4.4270E-3

Results

Neutrino Pair Emission

$K^- \to \pi^- \nu \bar{\nu}$

- Dominant contributions: internal *c*- and *t*-quarks.
- Because relative phases of CKM matrix elements are not known, only a range for the estimated branching ratio is obtained:

$$BR_{(th)}(K^- \to \pi^- \nu \bar{\nu}) = (0.39 - 10.55) \times 10^{-10}$$

- Compared to experimental value of:

$$BR_{(\exp)}(K^- \to \pi^- \nu \bar{\nu}) = (1.5^{+1.3}_{-0.9}) \times 10^{-10}$$

$B^- \to \pi^- \nu\bar{\nu}, \quad B^- \to K^- \nu\bar{\nu}$

- Internal *t*-quark contribution dominates.

$$BR_{(th)}(B^- \to \pi^- \nu\bar{\nu}) = 1.283 \times 10^{-6}$$

$$BR_{(th)}(B^- \to K^- \nu\bar{\nu}) = 2.485 \times 10^{-5}$$

- Experimental upper bounds:

$$BR_{(\exp)}(B^- \to \pi^- \nu\bar{\nu}) < 1.0 \times 10^{-4}$$

$$BR_{(\exp)}(B^- \to K^- \nu\bar{\nu}) < 1.4 \times 10^{-5}$$

Charged Lepton Pair Emission

Analysis considerably more involved.

$\underline{K^- \to \pi^- e^+ e^-}$

Dominant contributions:

$$\lambda_u f^V(x_u) \text{ and } \lambda_c f^V(x_c)$$

$\underline{B^- \to \pi^- e^+ e^-}$, $\underline{B^- \to K^- e^+ e^-}$

Dominant contributions:

$$\lambda_u f^V(x_u)\ ,\ \lambda_c f^V(x_c),\ \lambda_t f^V(x_t),$$

$$\lambda_t f^A(x_t) \text{ and } \lambda_t f^T(x_t)$$

Charged lepton pair emissions

Decay Mode	$BR_{(th)}$	$BR_{(exp)}$
$K^- \to \pi^- e^+ e^-$	1.954E-6	(2.88 ± 0.13)E-7
$B^- \to \pi^- e^+ e^-$	(0.690 – 1.362)E-8	$<$ 1.8E-7
$B^- \to K^- e^+ e^-$	(1.186 – 1.266)E-7	(4.9 ± 1.0)E-7

Conclusion

- A simple model to describe electroweak interactions of quarks inside hadrons.
- A quark tightly bound inside a hadron behaves as free particle with an effective mass.
- This simple model allows calculation of electroweak interaction processes at the quark level directly to hadronic processes.

- Experimental data on the charged weak decays of mesons are used to obtain estimates of effective masses of quarks in the hadrons.
- The model is applied to the rare electroweak decays of mesons.
- Reasonable agreement is obtained when comparing to the experimental values.