New Physics in $B \to K\pi$ decays?

1) Is there any puzzle of new physics in $B \to K\pi$ decays?
2) Possible new physics from $B \to K\pi$ decays : Unparticle, Leptophobic Z'

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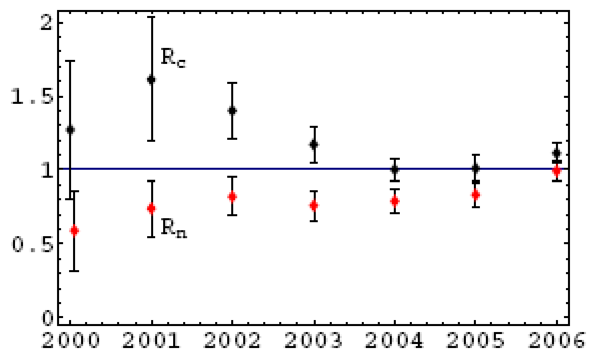
B → K π Puzzle

- Branching Ratios - HFAG March 2007

Measurement	BABAR	Belle	CLEO	Average
$\mathcal{B}(K^0\pi^+)$	$23.9 \pm 1.1 \pm 1.0$	$22.8^{+0.8}_{-0.7} \pm 1.3$	$18.8^{+3.7+2.1}_{-3.3-1.8}$	23.1 ± 1.0
$\mathcal{B}(K^+\pi^0)$	$13.3 \pm 0.6 \pm 0.6$	$12.4 \pm 0.5^{+0.7}_{-0.6}$	$12.9^{+2.4+1.2}_{-2.2-1.1}$	12.8 ± 0.6
$\mathcal{B}(K^+\pi^-)$	$19.1 \pm 0.6 \pm 0.6$	$20.0 \pm 0.4 \pm 0.8$	$18.0^{+2.3+1.2}_{-2.1-0.9}$	19.4 ± 0.6
$\mathcal{B}(K^0\pi^0)$	$10.5 \pm 0.7 \pm 0.5$	$9.2^{+0.7+0.6}_{-0.8-0.7}$	$12.8^{+4.0+1.7}_{-3.3-1.4}$	10.0 ± 0.6



Fleischer Hep-ph/0701217

$$R_c \equiv 2\frac{\mathcal{B}(B^+ \to K^+\pi^0)}{\mathcal{B}(B^+ \to K^0\pi^+)}, \quad R_n \equiv \frac{1}{2}\frac{\mathcal{B}(B^0 \to K^+\pi^-)}{\mathcal{B}(B^0 \to K^0\pi^0)}$$

At March 2007

Rc = 1.11 ± 0.07

Rn = 0.97 ± 0.07

B → K π Puzzle

- CP Asymmetries - HFAG March 2007

Measurement	BABAR	Belle	CLEO	Average
$\mathcal{A}_{CP}(K^0\pi^+)$	$-0.029 \pm 0.039 \pm 0.010$	$0.03 \pm 0.03 \pm 0.01$	$0.18 \pm 0.24 \pm 0.02$	0.009 ± 0.025
$\mathcal{A}_{CP}(K^+\pi^0)$	$0.016 \pm 0.041 \pm 0.012$	$0.07 \pm 0.03 \pm 0.01$	$-0.29 \pm 0.23 \pm 0.02$	0.047 ± 0.026
$\mathcal{A}_{CP}(K^+\pi^-)$	$-0.108 \pm 0.024 \pm 0.008$	$-0.093 \pm 0.018 \pm 0.008$	$-0.04 \pm 0.16 \pm 0.02$	-0.095 ± 0.013
$\mathcal{A}_{CP}(K^0\pi^0)$	$-0.20 \pm 0.16 \pm 0.03$	$-0.05 \pm 0.14 \pm 0.05$		-0.12 ± 0.11
$S_{K_S\pi^0}$	$0.33 \pm 0.26 \pm 0.04$	$0.33 \pm 0.35 \pm 0.08$		0.33 ± 0.21

with CDF measurement $\mathcal{A}_{CP}(K^+\pi^-) = -0.086 \pm 0.023 \pm 0.009$

$$\mathcal{A}_{CP}(\boldsymbol{B}^+ \to \boldsymbol{K}^+\boldsymbol{\pi}^0) - \mathcal{A}_{CP}(\boldsymbol{B}^0 \to \boldsymbol{K}^+\boldsymbol{\pi}^-) = 0.14 \pm 0.03$$

$$(\sin 2\boldsymbol{\beta})_{K_S\pi^0} - (\sin 2\boldsymbol{\beta})_{c\bar{c}s} = -0.35 \pm 0.21$$

Quark Diagram Approach in B → K π

- Amplitude parameterization

$$\boldsymbol{A}\left(\boldsymbol{B}^+ \to \boldsymbol{K}^0 \boldsymbol{\pi}^+\right) = \mathcal{P}' + \mathcal{A}'$$

$$\boldsymbol{A}(\boldsymbol{B}^0 \to \boldsymbol{K}^+ \boldsymbol{\pi}^-) = -\mathcal{P}' - \mathcal{P}'^{C}_{\boldsymbol{EW}} - \mathcal{T}'$$

$$\sqrt{2}\boldsymbol{A}(\boldsymbol{B}^+ \to \boldsymbol{K}^+ \boldsymbol{\pi}^0) = -\mathcal{P}' - \mathcal{P}'_{\boldsymbol{EW}} - \mathcal{P}'^{C}_{\boldsymbol{EW}} - \mathcal{T}' - \mathcal{C}' - \mathcal{A}'$$

$$\sqrt{2}\boldsymbol{A}\left(\boldsymbol{B}^0 \to \boldsymbol{K}^0 \boldsymbol{\pi}^0\right) = \mathcal{P}' - \mathcal{P}'_{\boldsymbol{EW}} - \mathcal{C}'$$

with re-definition of

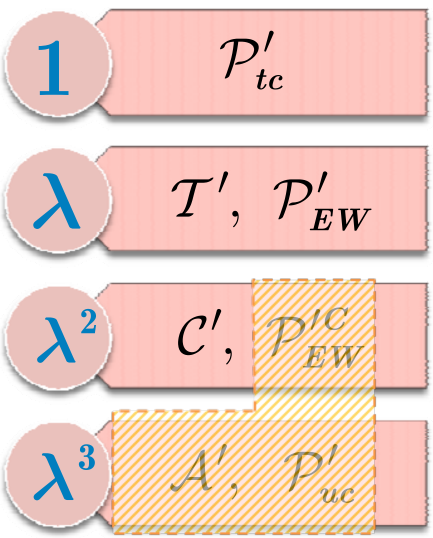
$$\mathcal{P}' + \mathcal{E}\mathcal{P}' - \frac{1}{3}\mathcal{P}'^{C}_{\boldsymbol{EW}} - \frac{1}{3}\mathcal{E}\mathcal{P}'^{C}_{\boldsymbol{EW}} \to \mathcal{P}'$$

$$\mathcal{A}' + \mathcal{E}\mathcal{P}'^{C}_{\boldsymbol{EW}} \to \mathcal{A}'$$

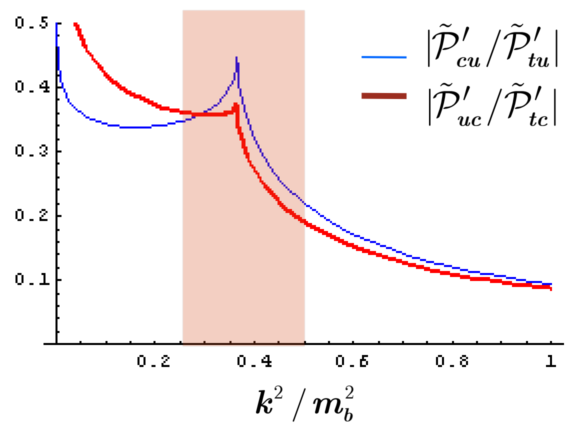
Quark Diagram Approach in B → K π

- Hierarchy between the parameters

$$\mathcal{P}' = \underbrace{V_{tb}^* V_{ts}}_{\lambda^2} \tilde{\mathcal{P}}'_{tc} + \underbrace{V_{ub}^* V_{us}}_{\lambda^4} \tilde{\mathcal{P}}'_{uc} \equiv \mathcal{P}'_{tc} + \mathcal{P}'_{uc}$$



$$\frac{\mathcal{P}'_{uc}}{\mathcal{P}'_{tc}} \approx \frac{G(m_u,k,\mu) - G(m_c,k,\mu)}{E(x_t) + \frac{2}{3}\ln\left(\frac{\mu^2}{M_W^2}\right) - G(m_c,k,\mu)}$$



$$\frac{1}{4} \le \frac{k^2}{m_b^2} \le \frac{1}{2}$$

Buras, Fleischer

PLB. 341. 379 (1995)

$$0.2 \le \left|\frac{\tilde{\mathcal{P}}'_{uc}}{\tilde{\mathcal{P}}'_{tc}}\right| \le 0.4$$

Mishima, Yoshikawa

PRD. 70. 094024 (2004)

$$\mathcal{C}' > |\mathcal{P}'^{C}_{EW}|$$

Quark Diagram Approach in B → K π

- Final form

$$A\left(B^{+} \to K^{0}\pi^{+}\right) \equiv A^{0+} = -P'$$

$$A\left(B^{0} \to K^{+}\pi^{-}\right) \equiv A^{+-}e^{i\alpha^{+-}} = P'(1 - r_T e^{i\gamma} e^{i\delta'_T})$$

$$A\left(B^{+} \to K^{+}\pi^{0}\right) \equiv A^{+0}e^{i\alpha^{+0}} = \frac{1}{\sqrt{2}} P'\left(1 - r_T e^{i\gamma} e^{i\delta'_T} - r_C e^{i\gamma} e^{i\delta'_C} + r_{EW} e^{i\delta'_{EW}}\right)$$

$$A\left(B^{0} \to K^{0}\pi^{0}\right) \equiv A^{00}e^{i\alpha^{00}} = \frac{1}{\sqrt{2}} P'\left(-1 - r_C e^{i\gamma} e^{i\delta'_C} + r_{EW} e^{i\delta'_{EW}}\right)$$

$$P' = \left|\mathcal{P}'_{tc}\right|, \quad r_T = \left|\frac{\mathcal{T}'}{\mathcal{P}'_{tc}}\right|, \quad r_C = \left|\frac{\mathcal{C}'}{\mathcal{P}'_{tc}}\right|, \quad r_{EW} = \left|\frac{\mathcal{P}'_{EW}}{\mathcal{P}'_{tc}}\right|$$

- We Neglect P'_{uc}, P'^{C}_{EW}, A'
- We set the strong phase of P to be zero → all phase is relative to it
- We hold 7 unknown parameters $P', r_T, r_C, r_{EW}, \delta'_T, \delta'_C, \delta'_{EW}$
- We use γ value given by other analysis
- A^{ij} are real and positive, α^{ij} are phases of their amplitude

Re-Parameterization Invariance

- Re-parameterization Invariance

  Botella, Silva 2005

  - For any phase φ

$$e^{i\phi} = \frac{\sin(\phi-\eta)}{\sin(\theta-\eta)} e^{i\theta} - \frac{\sin(\phi-\theta)}{\sin(\theta-\eta)} e^{i\eta}$$

  - We can choose arbitrary θ, η at will, for any given φ

- We assume NP comes into P_{EW} part (or C part)

$$\frac{r^N}{\sqrt{2}} e^{i\phi^N} e^{i\delta^N} = \frac{r^N}{\sqrt{2}} \frac{\sin\phi^N}{\sin\gamma} e^{i\delta^N} e^{i\gamma} - \frac{r^N}{\sqrt{2}} \frac{\sin(\phi^N-\gamma)}{\sin\gamma} e^{i\delta^N}$$

$$\begin{pmatrix} \theta = \gamma \\ \eta = 0 \end{pmatrix}$$

Absorbed into C

Absorbed into EW

Re-Parameterization Invariance

NP term is absorbed into SM term

$$\boldsymbol{A}^{+0}, \boldsymbol{A}^{00} \supset \frac{1}{\sqrt{2}} \boldsymbol{P}' \left(-\boldsymbol{r}_C \boldsymbol{e}^{i\gamma} \boldsymbol{e}^{i\delta'_C} + \boldsymbol{r}_{EW} \boldsymbol{e}^{i\delta_{EW}} + \boldsymbol{r}^N \boldsymbol{e}^{i\phi^N} \boldsymbol{e}^{i\delta^N} \right)$$

$$= \frac{1}{\sqrt{2}} \boldsymbol{P}' \left(-\boldsymbol{r}_C \boldsymbol{e}^{i\gamma} \boldsymbol{e}^{i\delta_C} + \boldsymbol{r}_{EW} \boldsymbol{e}^{i\delta'_{EW}} + \boldsymbol{r}^N \frac{\sin \phi^N}{\sin \gamma} \boldsymbol{e}^{i\delta^N} \boldsymbol{e}^{i\gamma} - \boldsymbol{r}^N \frac{\sin(\phi^N - \gamma)}{\sin \gamma} \boldsymbol{e}^{i\delta^N} \right)$$

$$= \frac{1}{\sqrt{2}} \boldsymbol{P}' \left(-\boldsymbol{r}_C^M \boldsymbol{e}^{i\gamma} \boldsymbol{e}^{i\delta_C^M} + \boldsymbol{r}_{EW}^M \boldsymbol{e}^{i\delta_{EW}^M} \right)$$

$$\boldsymbol{r}_C^M \boldsymbol{e}^{i\delta_C^M} = \boldsymbol{r}_C \boldsymbol{e}^{i\delta'_C} - \boldsymbol{r}^N \frac{\sin \phi^N}{\sin \gamma} \boldsymbol{e}^{i\delta^N}$$

$$\boldsymbol{r}_{EW}^M \boldsymbol{e}^{i\delta_{EW}^M} = \boldsymbol{r}_{EW} \boldsymbol{e}^{i\delta'_{EW}} - \boldsymbol{r}^N \frac{\sin(\phi^N - \gamma)}{\sin \gamma} \boldsymbol{e}^{i\delta^N}$$

Analytic re-Solution (CSK, S Oh, Y Yoon, PLB665(2008)231)

- Original Form does not change

$$A\left(B^+ \to K^0\pi^+\right) \equiv A^{0+} = -P'$$

$$A\left(B^0 \to K^+\pi^-\right) \equiv A^{+-}e^{i\alpha^{+-}} = P'(1 - r_T e^{i\gamma} e^{i\delta'_T})$$

$$A\left(B^+ \to K^+\pi^0\right) \equiv A^{+0}e^{i\alpha^{+0}} = \frac{1}{\sqrt{2}}P'\left(1 - r_T e^{i\gamma} e^{i\delta'_T} - r_C^M e^{i\gamma} e^{i\delta_C^M} + r_{EW}^M e^{i\delta_{EW}^M}\right)$$

$$A\left(B^0 \to K^0\pi^0\right) \equiv A^{00}e^{i\alpha^{00}} = \frac{1}{\sqrt{2}}P'\left(-1 - r_C^M e^{i\gamma} e^{i\delta_C^M} + r_{EW}^M e^{i\delta_{EW}^M}\right)$$

- If there is NP $r_C^M \neq (r_C)_{SM}$, $\delta_C^M \neq (\delta_C)_{SM}$

  $r_{EW}^M \neq (r_{EW})_{SM}$, $\delta_{EW}^M \neq (\delta_{EW})_{SM}$

Analytic Solution

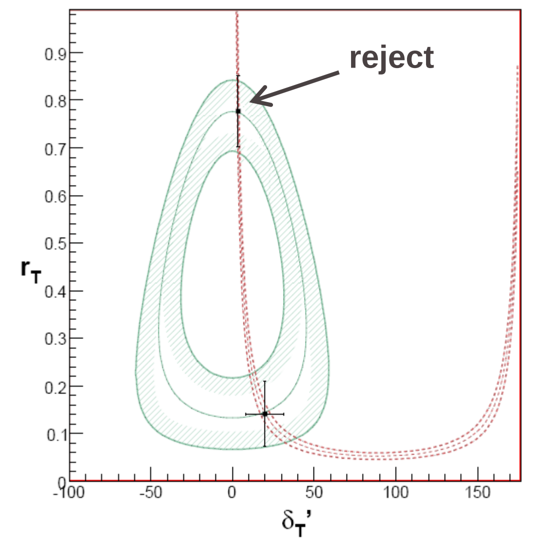
- Step 1 - P', r_T, δ'_T

$$A^{0+} = -P'$$
$$A^{+-} e^{i\alpha^{+-}} = P'(1 - r_T e^{i\gamma} e^{i\delta'_T})$$

$$R \equiv \frac{\mathcal{B}^{+-}}{\mathcal{B}^{0+}} \frac{\tau_{B^+}}{\tau_{B^0}} = 0.90 \pm 0.05$$

$$R = 1 + r_T^2 - 2 r_T \cos\delta'_T \cos\gamma$$
$$-\mathcal{A}_{CP}^{+-} R = 2 r_T \sin\delta'_T \sin\gamma$$

$$P' \propto \sqrt{Br^{0+}}$$
$$\cot\delta'_T = \frac{\sin 2\gamma}{(-A_{CP}^{+-})R}\left[1 \pm \sqrt{1 + \frac{1}{\cos^2\gamma}\left(R - 1 - \left(\frac{-\mathcal{A}_{CP}^{+-} R}{2\sin\gamma}\right)^2\right)}\right]$$
$$r_T = \sqrt{R(1 - \mathcal{A}_{CP}^{+-} \cot\gamma \cot\delta'_T) - 1}$$



$$P' = (49.9 \pm 1.1)\,\text{eV}$$
$$r_T = 0.14 \pm 0.07$$
$$\delta'_T = 20^\circ \pm 11^\circ$$

Analytic Solution

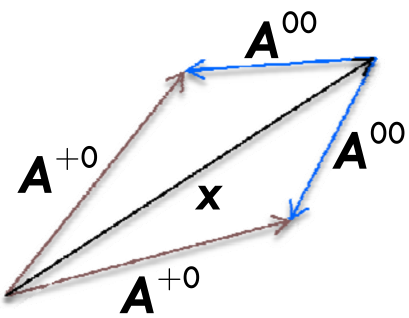
- Step 2 - $\alpha^{00}, \bar{\alpha}^{00}$

$$\sqrt{2}\left(A^{+0}e^{i\alpha^{+0}} - A^{00}e^{i\alpha^{00}}\right) = P'(2 - r_T e^{i\gamma}e^{i\delta_T}) \equiv x e^{i\zeta}$$

$$\sqrt{2}\left(\bar{A}^{+0}e^{i\bar{\alpha}^{+0}} - \bar{A}^{00}e^{i\bar{\alpha}^{00}}\right) = P'(2 - r_T e^{-i\gamma}e^{i\delta_T}) \equiv \bar{x} e^{i\bar{\zeta}}$$

$$\alpha^{00} = \zeta \pm \mathrm{ArcCos}\left(\frac{2A^{+0\,2} - 2A^{00\,2} - P'^2 x^2}{2\sqrt{2}A^{00}P'x}\right)$$

$$\bar{\alpha}^{00} = \bar{\zeta} \pm \mathrm{ArcCos}\left(\frac{2\bar{A}^{+0\,2} - 2\bar{A}^{00\,2} - P'^2 \bar{x}^2}{2\sqrt{2}\bar{A}^{00}P'\bar{x}}\right)$$



Two-fold ambiguity occurs.

2 X 2 = 4 fold ambiguities in tatal.

Analytic Solution

- Step 3 - $r_C^M, r_{EW}^M, \delta_C^M, \delta_{EW}^M$

$$-r_C^M e^{i\gamma} e^{i\delta_C^M} + r_{EW}^M e^{i\delta_{EW}^M} = \sqrt{2}\,\frac{A^{00}}{P'} e^{i\alpha^{00}} + 1 \equiv \; y e^{i\eta}$$

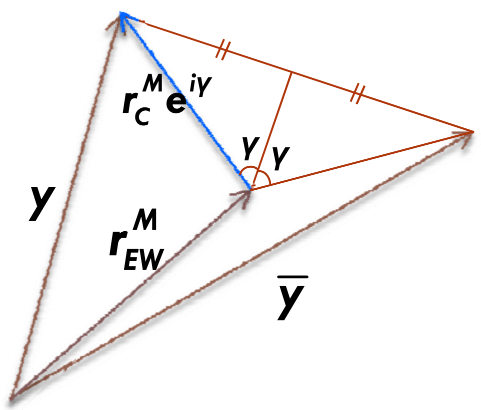
$$-r_C^M e^{-i\gamma} e^{i\delta_C^M} + r_{EW}^M e^{i\delta_{EW}^M} = \sqrt{2}\,\frac{\bar{A}^{00}}{P'} e^{i\bar{\alpha}^{00}} + 1 \equiv \; \bar{y} e^{i\bar{\eta}}$$

$$r_C^M = \frac{1}{\sin\gamma}\sqrt{\frac{|y|^2 + |\bar{y}|^2}{2} - y\bar{y}\cos(\bar{\eta} - \eta)}$$

$$r_{EW}^M = \frac{1}{\sin\gamma}\sqrt{\frac{|y|^2 + |\bar{y}|^2}{2} - y\bar{y}\cos(2\gamma + \bar{\eta} - \eta)}$$

$$\delta_C^M = \mathrm{ArcTan}\left(-\frac{y\cos\eta - \bar{y}\cos\bar{\eta}}{y\sin\eta - \bar{y}\sin\bar{\eta}}\right)$$

$$\delta_{EW}^M = \mathrm{ArcTan}\left(-\frac{y\cos(\eta - \gamma) - \bar{y}\cos(\bar{\eta} + \gamma)}{y\sin(\eta - \gamma) - \bar{y}\sin(\bar{\eta} + \gamma)}\right)$$



No discrete ambiguity

Analytic Solution

- 4 different solutions for $r_C^M, r_{EW}^M, \delta_C^M, \delta_{EW}^M$

	$\bar{\alpha}^{00} - \alpha^{00}$	r_C^M	r_{EW}^M	δ_C^M	δ_{EW}^M	$S_{K_S\pi^0}$
Case 1	$3^\circ \pm 14^\circ$	0.070 ± 0.075	0.26 ± 0.11	$231^\circ \pm 101^\circ$	$77^\circ \pm 16^\circ$	0.65 ± 0.17
Case 2	$39^\circ \pm 14^\circ$	0.36 ± 0.13	0.078 ± 0.069	$192^\circ \pm 11^\circ$	$208^\circ \pm 70^\circ$	0.08 ± 0.26
Case 3	$-25^\circ \pm 14^\circ$	0.24 ± 0.13	0.17 ± 0.09	$-18^\circ \pm 15^\circ$	$-1.1^\circ \pm 29^\circ$	0.92 ± 0.07
Case 4	$11^\circ \pm 14^\circ$	0.12 ± 0.11	0.29 ± 0.13	$227^\circ \pm 38^\circ$	$-82^\circ \pm 14^\circ$	0.53 ± 0.17

$S_{K_S\pi^0} = 0.33 \pm 0.21\,(\text{data})$

- We reject "Case 3" due to large $S_{K_s\pi^0}$ prediction
- The SM estimate $r_{EW} = 0.12 > r_C = 0.039, \quad \delta_C \approx -61^\circ, \quad \delta_{EW} = 22^\circ$
- Case 2: Large C
- Case 4: Large EW

Analytic Solution

- Step 4 - solutions for NP term

$$r_C^M e^{i\delta_C^M} = r_C e^{i\delta'_C} - r^N \frac{\sin\phi^N}{\sin\gamma} e^{i\delta^N}$$

$$r_{EW}^M e^{i\delta_{EW}^M} = r_{EW} e^{i\delta'_{EW}} - r^N \frac{\sin(\phi^N - \gamma)}{\sin\gamma} e^{i\delta^N}$$

4 Equations VS 7 unknowns - $r_C, r_{EW}, \delta'_C, \delta'_{EW}, r^N, \varphi^N, \delta^N$

Need at least 3 additional inputs to fix NP terms

Additional Inputs

- a) Additional inputs from Flavor SU(3) Sym.

From B → ππ decays

Measurement	BABAR	Belle	CLEO	Average
$\mathcal{B}(B^+ \to \pi^+\pi^0)$	$5.1 \pm 0.5 \pm 0.3$	$6.6 \pm 0.4^{+0.4}_{-0.5}$	$4.6^{+1.8+0.6}_{-1.6-0.7}$	5.7 ± 0.4
$\mathcal{B}(B^0 \to \pi^+\pi^-)$	$5.5 \pm 0.4 \pm 0.3$	$5.1 \pm 0.2 \pm 0.2$	$4.5^{+1.4+0.5}_{-1.2-0.4}$	5.16 ± 0.22
$\mathcal{B}(B^0 \to \pi^0\pi^0)$	$1.48 \pm 0.26 \pm 0.12$	$1.1 \pm 0.3 \pm 0.1$	< 4.4	1.31 ± 0.21
$\mathcal{A}_{CP}(B^+ \to \pi^+\pi^0)$	$-0.02 \pm 0.09 \pm 0.01$	$0.07 \pm 0.06 \pm 0.01$		0.04 ± 0.05
$\mathcal{A}_{CP}(B^0 \to \pi^+\pi^-)$	$0.21 \pm 0.09 \pm 0.02$	$0.55 \pm 0.08 \pm 0.05$		0.38 ± 0.07
$\mathcal{A}_{CP}(B^0 \to \pi^0\pi^0)$	$0.33 \pm 0.36 \pm 0.08$	$0.44^{+0.73+0.04}_{-0.62-0.06}$		$0.36^{+0.33}_{-0.31}$
$S_{\pi^+\pi^-}$	$-0.60 \pm 0.11 \pm 0.03$	$-0.61 \pm 0.10 \pm 0.04$		-0.61 ± 0.08

with CDF measurement $\mathcal{B}(B^0 \to \pi^+\pi^-) = 5.10 \pm 0.33 \pm 0.36$

HFAG March 2007

Assuming no NP in B → ππ

Additional Inputs

- Additional inputs from Flavor SU(3) Sym.

B → ππ parameterization

$$\sqrt{2}\,A\left(B^{+}\to\pi^{+}\pi^{0}\right) = -\left(Te^{i\gamma}e^{i\delta_T} + Ce^{i\gamma}e^{i\delta_C}\right)$$

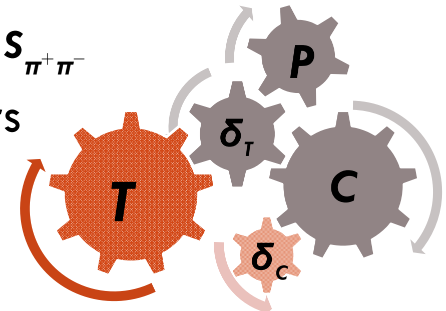
$$A(B^{0}\to\pi^{+}\pi^{-}) = -(Te^{i\gamma}e^{i\delta_T} + Pe^{-i\beta})$$

$$\sqrt{2}A(B^{+}\to\pi^{0}\pi^{0}) = -(Ce^{i\gamma}e^{i\delta_C} - Pe^{-i\beta})$$

Chisq-fitting with 5 measurements

3 – Br, $A_{CP}(\pi^{+}\pi^{-})$, $S_{\pi^{+}\pi^{-}}$

with 5 parameters



	T(eV)	C(eV)	P(eV)	δ_T	δ_C	$\mathcal{A}_{CP}(\pi^0\pi^0)$	$S_{\pi^0\pi^0}$
Sol. 1	22.5 ± 0.7	16.2 ± 1.6	7.8 ± 1.3	$40^\circ\pm7^\circ$	$-12^\circ\pm15^\circ$	0.17 ± 0.22	0.66 ± 0.12
Sol. 2	22.5 ± 0.7	15.5 ± 1.5	7.8 ± 1.3	$40^\circ\pm7^\circ$	$87^\circ\pm18^\circ$	-0.80 ± 0.09	-0.11 ± 0.25

$$\mathcal{A}_{CP}(\pi^{0}\pi^{0}) = 0.36^{+0.33}_{-0.31}\ \text{(data)}$$

$$C' = \frac{V_{us}}{V_{ud}}C = (3.8\pm0.4)\,\text{eV}$$

$$r_{EW}e^{i\delta'_{EW}} = -\frac{3}{2}\frac{c_9+c_{10}}{c_1+c_2}\frac{1}{\lambda^2 R_b}\left(r_T e^{i\delta'_T} + r_C e^{i\delta'_C}\right)$$

Gronau, Pirjol, Yan (1999)

→

$$(r_C,\ \delta'_C) = (0.076\pm0.008,\ \ -12^\circ\pm15^\circ)$$

$$(r_{EW},\delta'_{EW}) = (\ 0.14\pm0.04,\ \ \ 9^\circ\pm10^\circ)$$

Additional Inputs

- b) Additional inputs from PQCD result

Li, Mishima, Sanda, PRD72, 114005 (2005)

TABLE V. Topological amplitudes in units of 10^{-5} GeV for the $B \to \pi K$, $\pi\pi$ decays in the NDR scheme.

Topology	LO	$\mathrm{LO_{NLOWC}}$	+VC	+QL	+MP	+NLO
P'	$36.6e^{i2.9}$	$50.6e^{i2.9}$	$49.6e^{i2.9}$	$52.1e^{i2.9}$	$43.7e^{i2.8}$	$44.1e^{i2.9}$
T'	$6.9e^{i0.0}$	$6.6e^{i0.0}$	$6.6e^{i0.1}$	$6.6e^{i0.0}$	$6.6e^{i0.0}$	$6.6e^{i0.1}$
C'	$0.5e^{-i2.5}$	$0.6e^{-i0.4}$	$1.9e^{-i1.3}$	$0.6e^{-i0.2}$	$0.6e^{-i0.4}$	$1.7e^{-i1.3}$
P'_{ew}	$5.8e^{i3.1}$	$5.8e^{-i3.1}$	$5.4e^{-i3.0}$	$5.8e^{-i3.1}$	$5.8e^{-i3.1}$	$5.4e^{-i3.0}$
T	$24.3e^{i0.0}$	$23.5e^{i0.0}$	$23.1e^{i0.0}$	$23.6e^{-i0.1}$	$23.5e^{i0.0}$	$23.2e^{i0.0}$
P	$4.7e^{-i0.4}$	$6.5e^{-i0.4}$	$6.3e^{-i0.3}$	$6.7e^{-i0.3}$	$5.7e^{-i0.4}$	$5.6e^{-i0.4}$
C	$0.8e^{i2.6}$	$2.2e^{i0.2}$	$4.8e^{-i1.1}$	$2.3e^{i0.4}$	$2.2e^{i0.2}$	$4.3e^{-i1.1}$
P_{ew}	$0.7e^{i0.0}$	$0.7e^{i0.0}$	$0.7e^{-i0.1}$	$0.7e^{i0.0}$	$0.7e^{i0.0}$	$0.7e^{-i0.1}$

$$(r_C,\ \delta'_C) = (0.039,\ -61^\circ)$$
$$(r_{EW},\delta'_{EW}) = (0.12,\quad 22^\circ\ )$$

Determining NP parameters

- Solution for NP term with additional inputs

Defining

$$\Delta r_C e^{i\Delta\delta_C} \equiv r_C^M e^{i\delta_C^M} - r_C e^{i\delta_C'}$$

$$\Delta r_{EW} e^{i\Delta\delta_{EW}} \equiv r_{EW}^M e^{i\delta_{EW}^M} - r_{EW} e^{i\delta_{EW}'}$$

$$\Delta r_C e^{i\Delta\delta_C} = -r^N \frac{\sin\phi^N}{\sin\gamma} e^{i\delta^N}$$

$$\Delta r_{EW} e^{i\Delta\delta_{EW}} = -r^N \frac{\sin(\phi^N - \gamma)}{\sin\gamma} e^{i\delta^N}$$

$$\delta^N = \Delta\delta_C \ \text{or} \ \Delta\delta_C - \pi$$

$$\frac{\Delta r_C}{\Delta r_{EW}} = \frac{\sin\phi^N}{\sin(\phi^N - \gamma)}$$

$$r^N = \frac{\sin\gamma}{\sin\phi^N} \Delta r_C$$

With inputs from SU(3) sym.

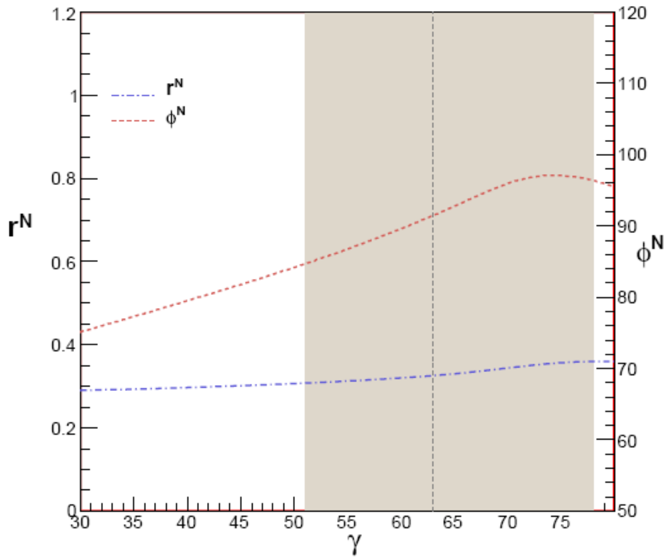
	r^N	ϕ^N	δ^N
Case 2	0.39 ± 0.13	$92^\circ \pm 15^\circ$	$7^\circ \pm 26^\circ$
Case 4	0.29 ± 0.19	$148^\circ \pm 25^\circ$	$25^\circ \pm 17^\circ$

With inputs from PQCD results

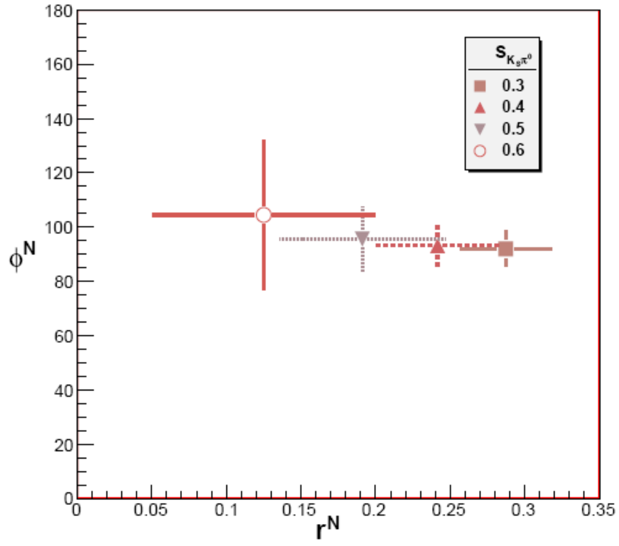
	r^N	ϕ^N	δ^N
Case 2	0.34 ± 0.13	$94^\circ \pm 16^\circ$	$6^\circ \pm 27^\circ$
Case 4	0.31 ± 0.29	$160^\circ \pm 21^\circ$	$29^\circ \pm 14^\circ$

Cases 2&4 are suitable and consistent each other between two methods.

Discussions



Dependance on γ



Dependance on $S_{K_S\pi^0}$

Summary

- Due to the Re-parameterization Invariance(RI) the NP terms absorbed into the SM terms C and P_{EW} in pair.
- In order to extract NP parameters we need at least 3 additional inputs.
- We could pin down each hadronic parameter under four-fold discrete ambiguity using analytic method. And also NP parameter for given additional inputs
- Results shows that there should be quite large NP contribution with maximal weak phase

Unparticle Physics

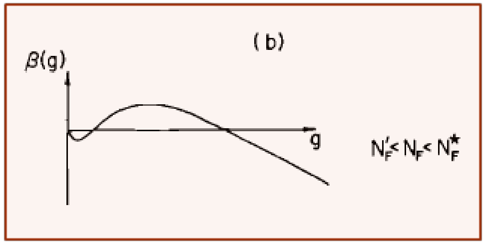
in $B\bar{B}$ mixing and $B \to (\pi, K)\pi$ decays

Collaboration with Chuan-Hung Chen and Yeo-Woong Yoon

PLB671(2009)250

Unparticle Physics

Georgi, PRL. 98. 221601 (2007)



Banks, Zaks, NPB.196.189(1982)

scale Invarant

$\mathcal{BZ}$ field with IR fixed point

at $M_{\mathcal{U}}(>1\,\text{TeV})$ scale physics

can not be described by ordinary particle

☞ ***Unparticle*** stuff

Interaction with the SM particle:

$$\frac{1}{M_{\mathcal{U}}^{k}} O_{sm} O_{\mathcal{BZ}}$$

dimensional transmulation

at scale of $\Lambda_{\mathcal{U}}$

Matching onto Unparticle operator

$$\frac{C_{\mathcal{U}} \Lambda_{\mathcal{U}}^{d_{\mathcal{BZ}}-d_{\mathcal{U}}}}{M_{\mathcal{U}}^{k}} O_{sm} O_{\mathcal{U}}$$

$d_{\mathcal{U}}$: scaling dimension of Unparticle Op.

Unparticle Physics

The vacuum matrix element

$$\langle 0|O_{\mathcal{U}}(0)O_{\mathcal{U}}^{\dagger}|0\rangle = \int \frac{d^4p}{(2\pi)^4} e^{-ip\cdot x} |\langle 0|O_{\mathcal{U}}(0)|P\rangle|^2 \rho(P^2)$$

should scale with dimension $2d_{\mathcal{U}}$, by virtue of scale invariance. Therefore,

$$|\langle 0|O_{\mathcal{U}}(0)|P\rangle|^2 \rho(P^2) = A_{d_{\mathcal{U}}}\theta(P^0)\theta(P^2)(P^2)^{d_{\mathcal{U}}-2},$$

This characterizes the unparticle phase space.
And, it resembles the phase space for n massless particles

$$(2\pi)^4\delta^4\left(P-\sum_{j=1}^{n}p_j\right)\prod_{j=1}^{n}\delta(p_j^2)\theta(p_j^0)\frac{d^4p_j}{(2\pi)^3} = A_n\theta(P^0)\theta(P^2)(P^2)^{n-2}$$

Where, $A_n = \frac{16\pi^{5/2}}{(2\pi)^{2n}}\frac{\Gamma(n+1/2)}{\Gamma(n-1)\Gamma(2n)}$

Unparticle Physics

Georgi's proporsal:

Identifying $A_n \Rightarrow A_{d_{\mathcal{U}}}$, $n \Rightarrow d_{\mathcal{U}}$

$$A_{d_{\mathcal{U}}} = \frac{16\pi^{5/2}}{(2\pi)^{2d_{\mathcal{U}}}} \frac{\Gamma(d_{\mathcal{U}} + 1/2)}{\Gamma(d_{\mathcal{U}} - 1)\Gamma(2d_{\mathcal{U}})}$$

" Unparticle with scaling dimension $d_{\mathcal{U}}$ "

" Fractional number $d_{\mathcal{U}}$ of invisible particles "

Unparticle Physics

Because of scale invariance, The Unparticle propagator is

$$\int d^4x e^{ip\cdot x} \langle 0 | T(O_{\mathcal{U}}^{\mu}(x) O_{\mathcal{U}}^{\nu}(0) | 0 \rangle = i \frac{A_{d_{\mathcal{U}}}}{2\sin(d_{\mathcal{U}}\pi)} \frac{1}{(p^2)^{2-d_{\mathcal{U}}}} \left(-g^{\mu\nu} + \frac{p^{\mu}p^{\nu}}{p^2} \right) e^{-i\phi_{\mathcal{U}}}$$

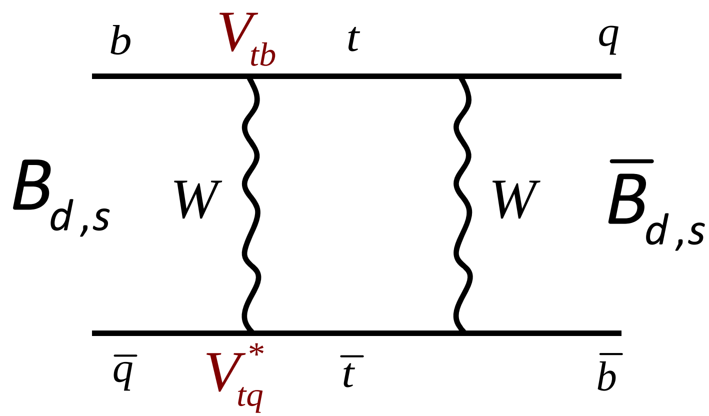
Georgi, PLB 650:275(2007), Cheung et al, PRL.99:051803(2007)

It carries CP conserving phase $\phi_{\mathcal{U}} = (d_{\mathcal{U}} - 2)\pi$

The Effective Lagrangian for the interaction with vector Unparticle is

$$\frac{C_L^{q'q}}{\Lambda_{\mathcal{U}}^{d_{\mathcal{U}}-1}} \bar{q}' \gamma_{\mu} (1-\gamma_5) q O_{\mathcal{U}}^{\mu} + \frac{C_R^{q'q}}{\Lambda_{\mathcal{U}}^{d_{\mathcal{U}}-1}} \bar{q}' \gamma_{\mu} (1+\gamma_5) q O_{\mathcal{U}}^{\mu}$$

$B\overline{B}$ Mixing Constraints

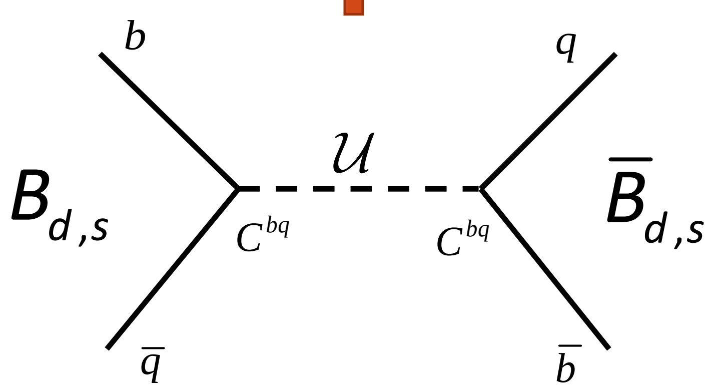


$= \mathcal{M}_{12}^{q,\,SM}$

Containing Weak Phase

From CKM Factor

+



$= \mathcal{M}_{12}^{q,\,\mathcal{U}}$

Containing Strong Phase

From unparticle propagator

$$\mathcal{M}_{12}^{q} = \mathcal{M}_{12}^{q,SM} + \mathcal{M}_{12}^{q,\mathcal{U}}$$

$B\bar{B}$ Mixing Constraints

$$\mathcal{M}_{12}^{q,\,SM} = \frac{G_F^2 m_W^2}{12\pi^2} m_{B_q} f_{B_q}^2 \hat{B}_{B_q} (V_{tq}^* V_{tb})^2 \hat{\eta}^B S_0(x_t) = |\mathcal{M}_{12}^{q,\,SM}|\, e^{i\phi_q^{SM}}$$

Lattice QCD results for the Non-perturbative parameters.

	$f_{B_d}\sqrt{\hat{B}_{B_d}}$	$f_{B_s}\sqrt{\hat{B}_{B_s}}$
$JLQCD:$	$0.215 \pm 0.019^{+0}_{-0.023}$	$0.245 \pm 0.021^{+0.003}_{-0.002}$
$(HP+JL)QCD:$	0.244 ± 0.026	0.295 ± 0.036

The values for the SM mixing amplitudes

	$2\|\mathcal{M}_{12}^{d,\,SM}\|$	$2\|\mathcal{M}_{12}^{s,\,SM}\|$
$JLQCD:$	$0.75^{+0.20}_{-0.26}\ \mathrm{ps}^{-1}$	$16.4 \pm 2.8\ \mathrm{ps}^{-1}$
$(HP+JL)QCD:$	$0.97 \pm 0.29\ \mathrm{ps}^{-1}$	$23.8 \pm 5.9\ \mathrm{ps}^{-1}$

$$\phi_d^{SM} = 2\beta = 45.2^\circ \pm 5.7^\circ, \qquad \phi_s^{SM} = -2\lambda^2\eta = -2.3^\circ \pm 0.2^\circ$$

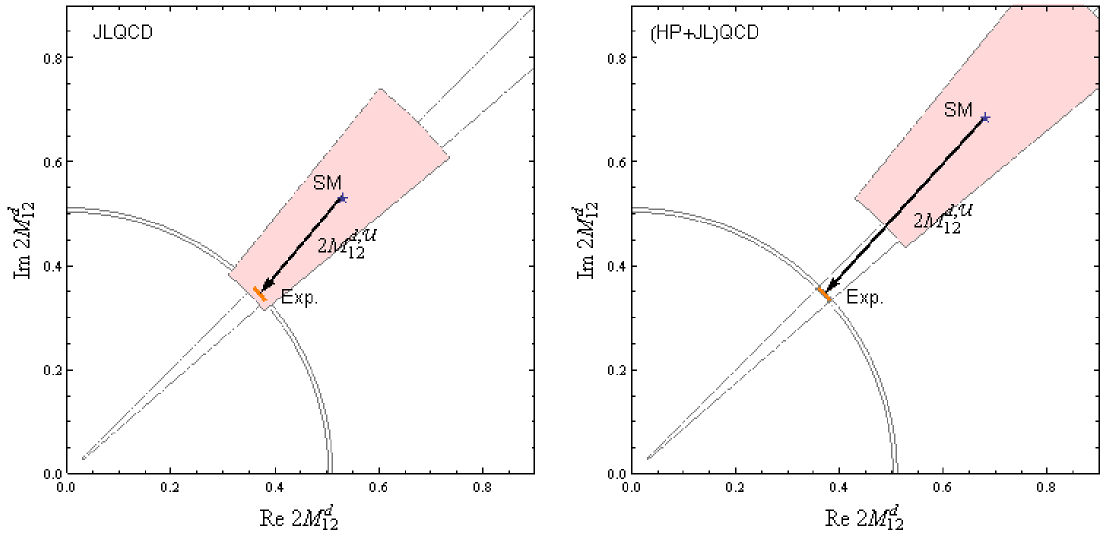
$B\bar{B}$ Mixing Constraints

Current Experimental data (HFAG)

$$\Delta M_d = (0.507 \pm 0.004)\ \text{ps}^{-1} \qquad \phi_d = 43^\circ \pm 2^\circ$$

$$\Delta M_s = (17.77 \pm 0.12)\ \text{ps}^{-1}$$ *CDF, PRL. 97. 242003 (2006)*

Strongly constraining on the $B_d\bar{B}_d$ mixing



$$\Delta M_q = 2\,|\mathcal{M}_{12}^q|$$

Especially on phases of $\mathcal{M}_{12}^{d,\mathcal{U}}$

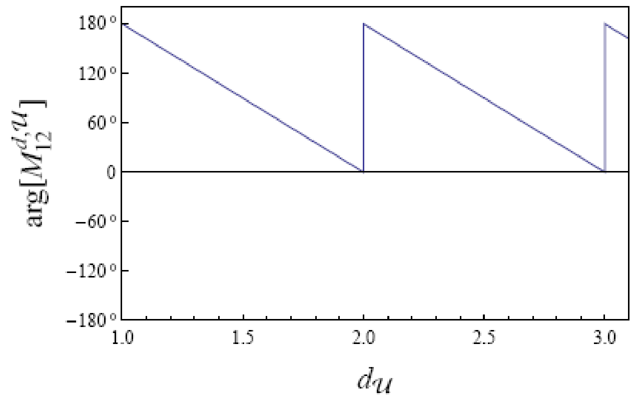
$$JLQCD:\ \phi_d^{\mathcal{U}} = -130^\circ \pm 180^\circ, \quad (HP+JL)QCD:\ \phi_d^{\mathcal{U}} = -132^\circ \pm 12^\circ$$

$B\bar{B}$ Mixing Constraints

$$\mathcal{M}_{12}^{q,\mathcal{U}} = c_{\mathcal{U}}(p^2) m_{B_q} f_{B_q}^2 \hat{B}_{B_q} \times \left[\left((C_L^{qb})^2 + (C_R^{qb})^2 \right) \left(\frac{4}{3} - \frac{5}{6} \frac{m_{B_q}^2}{p^2} \right) + C_L^{qb} C_R^{qb} \left(-\frac{10}{3} + \frac{14}{6} \frac{m_{B_q}^2}{p^2} \right) \right]$$

$$c_{\mathcal{U}}(q^2) = \frac{A_{d_{\mathcal{U}}} e^{-i\phi_{\mathcal{U}}}}{2 \sin d_{\mathcal{U}} \pi \Lambda_{\mathcal{U}}^{2d_{\mathcal{U}}-2} (p^2)^{2-d_{\mathcal{U}}}}, \qquad \phi_{\mathcal{U}} = (d_{\mathcal{U}} - 2)\pi$$

$c_{\mathcal{U}}(q^2)$ determine the phase of $\mathcal{M}_{12}^{q,\mathcal{U}}$



Therefore (HP+JL)QCD can not give the right value of scaling dimension $d_{\mathcal{U}}$

JLQCD allows all value of $d_{\mathcal{U}}$

We choose JLQCD case and set $d_{\mathcal{U}} = 1.5$

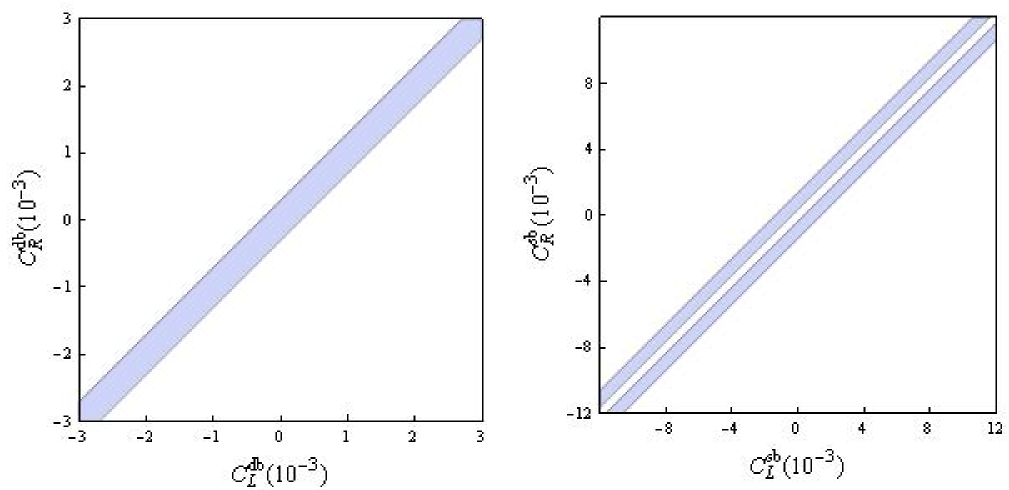
$B\bar{B}$ Mixing Constraints

Constraints on unparticle mixing amplitudes from experimental data

$$2|\mathcal{M}_{12}^{d,\mathcal{U}}| = 0.25 \pm 0.26 \text{ ps}^{-1} \qquad 2|\mathcal{M}_{12}^{s,\mathcal{U}}| = 7.6 \pm 6.6 \text{ ps}^{-1}$$

C_L^{db}, C_R^{db}, $C_L^{sb} C_R^{sb}$ are strongly constrained as

$$|C_L^{db} - C_R^{db}| < 3.1 \times 10^{-4}, \qquad 3.5 \times 10^{-4} < |C_L^{sb} - C_R^{sb}| < 1.4 \times 10^{-3},$$



Unparticle contribution in $B \to (\pi, K)\pi$ decays

The SM decay amplitudes

$$\begin{aligned}
\sqrt{2}A^{SM}(B^+ \to \pi^+\pi^0) &= -Te^{i\gamma} - Ce^{i\gamma} - P_{EW}e^{-i\beta}, \\
A^{SM}(B_d \to \pi^+\pi^-) &= -Te^{i\gamma} - Pe^{-i\beta}, \\
\sqrt{2}A^{SM}(B_d \to \pi^0\pi^0) &= -Ce^{i\gamma} + Pe^{-i\beta} - P_{EW}e^{-i\beta}, \\
A^{SM}(B^+ \to K^0\pi^+) &= P', \\
A^{SM}(B_d \to K^+\pi^-) &= -P' - T'e^{i\gamma}, \\
\sqrt{2}A^{SM}(B^+ \to K^+\pi^0) &= -P' - T'e^{i\gamma} - C'e^{i\gamma} - P'_{EW}, \\
\sqrt{2}A^{SM}(B_d \to K^0\pi^0) &= P' - C'e^{i\gamma} - P'_{EW} ,
\end{aligned}$$

The recent PQCD result for the SM parameters

Topology	Abs	Arg	Topology	Abs	Arg
P'	$43.6^{+10.8}_{-8.0}$	$2.9^{+0.1}_{-0.2}$	T	$23.2^{+8.0}_{-6.1}$	0.0 ± 0.0
T'	$6.5^{+2.4}_{-1.8}$	0.1 ± 0.0	P	$5.6^{+1.2}_{-0.8}$	$-0.4^{+0.2}_{-0.1}$
P'_{EW}	$5.4^{+1.4}_{-1.0}$	-1.3 ± 0.1	C	$4.3^{+2.1}_{-1.5}$	-1.1 ± 0.0
C'	$1.7^{+0.9}_{-0.6}$	-3.0 ± 0.0	P_{EW}	$0.7^{+0.1}_{-0.1}$	-0.1 ± 0.0

Unparticle contribution in $B \to (\pi, K)\pi$ decays

Effective Hamiltonian for $b \to q\bar{q}'q'$

$$\mathcal{H}_{\mathcal{U}} = -C_{\mathcal{U}}(q^2)\left(C_L^{qb}(\bar{q}b)_{V-A} + C_R^{qb}(\bar{q}b)_{V+A}\right)\left(C_L^{q'q'}(\bar{q}'q')_{V-A} + C_R^{q'q'}(\bar{q}'q')_{V+A}\right)$$

Unparticle contribution in decays

$$A^{\mathcal{U}}(B \to \pi^i\pi^j) = C_{\mathcal{U}}(q_1^2) f_\pi m_B^2 F_0^{B\pi}(m_\pi^2) a_{dec}^{\mathcal{U},\pi^i\pi^j}$$

$$A^{\mathcal{U}}(B \to K^i\pi^j) = C_{\mathcal{U}}(q_1^2) f_K m_B^2 F_0^{B\pi}(m_K^2) a_{dec}^{\mathcal{U},K^i\pi^j}$$

$$q_1^2 \approx m_B(m_B - m_b), \quad q_2^2 = m_\pi^2$$

$$r_1^\pi = \frac{m_\pi^2}{m_b(m_u + m_d)}, \quad r_2^\pi = \frac{m_\pi^2}{m_b(m_d + m_d)}$$

$$r_1^K = \frac{m_K^2}{m_b(m_u + m_s)}, \quad r_2^K = \frac{m_K^2}{m_b(m_d + m_s)}$$

decay mode	$a_{\rm dec}^{\mathcal{U}}$
$\pi^+\pi^-$	$-\frac{1}{N_c}\Big((C_L^{db}C_L^{uu} - C_R^{db}C_R^{uu}) + 2r_1^\pi (C_L^{db}C_R^{uu} - C_R^{db}C_L^{uu}) \Big)$
$\pi^+\pi^0$	$-\frac{1}{\sqrt{2}N_c}\Big((C_L^{db}(C_L^{uu} - C_L^{dd}) - C_R^{db}(C_R^{uu} - C_R^{dd}))$ $+2r_2^\pi (C_L^{db}(C_R^{uu} - C_R^{dd}) - C_R^{db}(C_L^{uu} - C_L^{dd})) \Big)$ $-\frac{C_{\mathcal{U}}(q_2^2)}{\sqrt{2}C_{\mathcal{U}}(q_1^2)} (C_L^{db} + C_R^{db}) (C_L^{uu} - C_L^{dd} - C_R^{uu} + C_R^{dd})$
$\pi^0\pi^0$	$\frac{1}{\sqrt{2}N_c}\Big((C_L^{db}C_L^{dd} - C_R^{db}C_R^{dd}) + 2r_2^\pi (C_L^{db}C_R^{dd} - C_R^{db}C_L^{dd}) \Big)$ $-\frac{C_{\mathcal{U}}(q_2^2)}{\sqrt{2}C_{\mathcal{U}}(q_1^2)} (C_L^{db} + C_R^{db}) (C_L^{uu} - C_L^{dd} - C_R^{uu} + C_R^{dd})$
$K^0\pi^-$	$\frac{1}{N_c}\Big((C_L^{sb}C_L^{dd} - C_R^{sb}C_R^{dd}) + 2r_1^K (C_L^{sb}C_R^{dd} - C_R^{sb}C_L^{dd}) \Big)$
$K^+\pi^-$	$-\frac{1}{N_c}\Big((C_L^{sb}C_L^{uu} - C_R^{sb}C_R^{uu}) + 2r_1^K (C_L^{sb}C_R^{uu} - C_R^{sb}C_L^{uu}) \Big)$
$K^+\pi^0$	$-\frac{1}{\sqrt{2}N_c}\Big((C_L^{sb}C_L^{uu} - C_R^{sb}C_R^{uu}) + 2r_1^K (C_L^{sb}C_R^{uu} - C_R^{sb}C_L^{uu}) \Big)$ $-\frac{C_{\mathcal{U}}(q_2^2)}{\sqrt{2}C_{\mathcal{U}}(q_1^2)} \frac{f_\pi}{f_K} \frac{F_0^{BK}(m_\pi^2)}{F_0^{B\pi}(m_K^2)} (C_L^{sb} + C_R^{sb}) (C_L^{uu} - C_L^{dd} - C_R^{uu} + C_R^{dd})$
$K^0\pi^0$	$\frac{1}{\sqrt{2}N_c}\Big((C_L^{sb}C_L^{dd} - C_R^{sb}C_R^{dd}) + 2r_2^K (C_L^{sb}C_R^{dd} - C_R^{sb}C_L^{dd}) \Big)$ $-\frac{C_{\mathcal{U}}(q_2^2)}{\sqrt{2}C_{\mathcal{U}}(q_1^2)} \frac{f_\pi}{f_K} \frac{F_0^{BK}(m_\pi^2)}{F_0^{B\pi}(m_K^2)} (C_L^{sb} + C_R^{sb}) (C_L^{uu} - C_L^{dd} - C_R^{uu} + C_R^{dd})$

Unparticle contribution in $B \to (\pi, K)\pi$ decays

$$A(B \to f) = A^{SM}(B \to f) + A^{\mathcal{U}}(B \to f)$$

Unparticle Parameters:

$\Lambda_{\mathcal{U}},\ d_{\mathcal{U}}$ → We set $\Lambda_{\mathcal{U}} = 1\text{TeV}, \quad d_{\mathcal{U}} = 1.5$

$C_L^{db}, C_R^{db}, C_L^{sb}, C_R^{sb}$ (*Strong constraints from mixing*)

$C_L^{uu}, C_R^{uu}, C_L^{dd}, C_R^{dd}$

Free parameters

Perform minimum χ^2 analysis for 8 free parameters with 16 experimental data of $B \to (\pi, K)\pi$ decays.

The fitted values are

$$C_L^{db} = 1.5\times10^{-4},\ C_R^{db} = 2.3\times10^{-4},\quad C_L^{sb} = 5.8\times10^{-4},\ C_R^{sb} = 9.3\times10^{-5}$$

$$C_L^{uu} = 3.9,\quad C_R^{uu} = 12.4,\quad C_L^{uu} = 3.7,\quad C_R^{uu} = 12.2$$

$$\chi^2 = 4.6,\quad d.o.f = 8$$

Unparticle contribution in $B \to (\pi, K)\pi$ decays

The unparticle contribution with fitted values of the parameters

(w/o : without Unparticle contribution)

observables	data	theory (w/o)	theory	χ^2 (w/o)	χ^2
$\mathcal{B}(K^0\pi^+)$	23.1 ± 1.0	23.5 ± 12	22.2 ± 11	0.001	0.006
$\mathcal{B}(K^+\pi^0)$	12.9 ± 0.6	13.0 ± 6.2	12.6 ± 6.0	0.001	0.003
$\mathcal{B}(K^+\pi^-)$	19.4 ± 0.6	19.7 ± 10	19.2 ± 10	0.001	0.0
$\mathcal{B}(K^0\pi^0)$	9.9 ± 0.6	8.8 ± 4.9	10.4 ± 5.3	0.046	0.009
$\mathcal{A}_{CP}(K^0\pi^+)$	0.009 ± 0.025	0.0 ± 0.0	0.0 ± 0.0	0.13	0.13
$\mathcal{A}_{CP}(K^+\pi^0)$	0.050 ± 0.025	-0.017 ± 0.068	0.056 ± 0.067	0.84	0.008
$\mathcal{A}_{CP}(K^+\pi^-)$	-0.097 ± 0.012	-0.099 ± 0.073	-0.088 ± 0.071	0.001	0.017
$\mathcal{A}_{CP}(K^0\pi^0)$	-0.14 ± 0.11	-0.065 ± 0.040	-0.051 ± 0.032	0.41	0.60
$S_{K_S\pi^0}$	0.38 ± 0.19	0.74 ± 0.08	0.74 ± 0.07	3.1	3.2
$\mathcal{B}(\pi^+\pi^0)$	$5.59^{+0.41}_{-0.40}$	4.03 ± 2.53	4.05 ± 2.53	0.37	0.36
$\mathcal{B}(\pi^+\pi^-)$	5.16 ± 0.22	6.80 ± 4.43	7.10 ± 4.43	0.14	0.19
$\mathcal{B}(\pi^0\pi^0)$	1.31 ± 0.21	0.23 ± 0.13	1.33 ± 0.30	19	0.002
$\mathcal{A}_{CP}(\pi^+\pi^0)$	0.06 ± 0.05	0.00 ± 0.01	0.055 ± 0.017	1.6	0.01
$\mathcal{A}_{CP}(\pi^+\pi^-)$	0.38 ± 0.07	0.17 ± 0.10	0.38 ± 0.14	2.8	0.0
$\mathcal{A}_{CP}(\pi^0\pi^0)$	$0.48^{+0.32}_{-0.31}$	0.64 ± 0.23	0.53 ± 0.13	0.17	0.024
$S_{\pi^+\pi^-}$	-0.61 ± 0.08	-0.55 ± 0.44	-0.55 ± 0.42	0.021	0.023

Chisq is Much Reduced

Summary

- We searched Unparticle contribution in $B_{d,s}\overline{B}_{d,s}$ mixing and $B \longrightarrow (\pi, K)\pi$ decays
- $B_{d,s}\overline{B}_{d,s}$ mixing could give strong constraints on unparticle parameters. The scaling dimension $d_{\mathcal{U}}$ also can be constrained when more precise estimation of the SM mixing amplitude is provided.
- The Unparticle contribution could successfully resolve the discrepancy between theory and data for the $B \longrightarrow (\pi, K)\pi$ decays, such as $Br(B_d \longrightarrow \pi^0\pi^0)$, $A_{CP}(B_d \longrightarrow \pi^+\pi^-)$ and $A_{CP}(B_d \longrightarrow K^+\pi^0)$

Leptophobic Z' model

in $B\bar{B}$ mixing and $B \to (\pi, K)\pi$ decays

Collaboration with S. W. Baek and J. H. Jeon

PLB664(2008)84

LEPTOPHOBIC Z' MODEL

- ✓ Extra neutral U(1) gauge boson, Z'
  - has been considered one of the extensions of the SM
  - motivated by
    - String-inspired GUTs (J.L.Hewett, T.G.Rizzo, M.Cvetic, P.Langacker, etc)
    - Dynamical symmetry breaking models (G.Buchalla, G.Burdman, etc)
    - Extra dimension models (M.Masip, A.Pomarol)
    - Little higgs models (N.Arkani-Hamed, A.G.Cohen, T.Han, etc)

- ✓ Leptophobic Z'
  - does not couple to SM leptons
  - introduced to explain the R_b-R_c puzzle at LEP and anomalous high-E_T jet cross section at CDF
  - by introducing the superstring inspired models, i.e., E_6 or Flipped SU(5)

E_6 GUTs

- ✓ comes from heterotic superstring ($E_8 \to SU(3) \times E_6$)
- ✓ was the natural anomaly free choice for a GUT group after SO(10)
- ✓ could have several intermediate mass breaking scales
- ✓ Maximal breakings of E_6 : $\begin{cases} 1.\ \ SO(10)\times U(1) \\ 2.\ \ [SU(3)]^3 \\ 3.\ \ SU(2)\times SU(6) \end{cases}$

If we consider the following breaking chain

$$\begin{aligned} E_6 &\to SO(10) \times U(1)_\psi \\ &\to SU(5) \times U(1)_\chi \times U(1)_\psi \\ &\to SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)' \end{aligned}$$

U(1)' can be a linear combination of

1. two U(1)s $[U(1)' = U(1)_\psi \sin\theta - U(1)_\chi \cos\theta,\ Flipped\ SU(5)]$
2. three U(1)s [ambiguity of embeddings, $Flipped\ SU(5) + Ma$]

$$E_6 \to SO(10) \times U(1)_\psi$$
$$\to SU(5) \times U(1)_\chi \times U(1)_\psi$$
$$\to SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)'$$

✓ directly SU(5) → SM : SU(5) (Geogri-Glashow)

- $SU(5)_{GG}$: $F = (10, \frac{1}{2}) = \{Q, u^c, e^c\}$ $\bar{f} = (\bar{5}, -\frac{3}{2}) = \{L, d^c\}$ $l^c = (1, \frac{5}{2}) = \{\nu^c\}$
- $U(1)' = U(1)_\psi \sin\theta - U(1)_\chi \cos\theta$

✓ SU(5)XU(1)$_\chi$ → SM : Flipped SU(5) (S.M.Barr,1982)

Flipped SU(5) is a different breaking pattern of SO(10)

- Flipped $SU(5)$: $F = (10, \frac{1}{2}) = \{Q, d^c, \nu^c\}$ $\bar{f} = (\bar{5}, -\frac{3}{2}) = \{L, u^c\}$ $l^c = (1, \frac{5}{2}) = \{e^c\}$
- $Y/2 = \alpha\, U(1)_{SU(5)} + \beta\, U(1)_\chi$ $(\alpha = \beta = -1/5)$

Leptophobic Z' does not couple to multiplet(f) and singlet(ℓ^C)

Leptophobic Z' *in stringy flipped* $SU(5)$

(J.L Lopez, D.V. Nanopoulos, and K.J.Yuan (NPB399,654(1993))

- *Gauge group* : $\underbrace{SU(5) \times U(1)}_{observable} \times \underbrace{SO(10) \times SU(4)}_{hidden} \times \underbrace{U(1)^5}_{U(1)'}$

In addition to its own beauty this scenario has the following phenomenologically interesting features:

- The new Z' coupling is generation dependent and can generate FCNC processes.
- The FCNC couplings allow large CP violation.
- It violates the isospin symmetry in the right-handed up- and down-quarks.
- The new gauge boson interaction maximally violates the parity in the up-quark sector.

In the mass eigenstates the interactions of Z' gauge boson with the quarks can be written as

$$\mathcal{L} = -\frac{g_2}{\cos\theta_W}\delta Z'_\mu \left(\overline{u}\gamma^\mu P_L\left[V_L^u \hat{c} V_L^{u\dagger}\right]u + \overline{d}\gamma^\mu P_L\left[V_L^d \hat{c} V_L^{d\dagger}\right]d + \overline{d}\gamma^\mu P_R\left[V_R^d \hat{c} V_R^{d\dagger}\right]d\right),$$

We introduce complex parameters, L and R,

$$\left[V_L^d \hat{c} V_L^{d\dagger}\right]_{23} \equiv \frac{1}{2}L_{sb}^{Z'}, \quad \left[V_R^d \hat{c} V_R^{d\dagger}\right]_{23} \equiv \frac{1}{2}R_{sb}^{Z'}.$$

$$c_L^u \equiv \left[V_L^u \hat{c} V_L^{u\dagger}\right]_{11}, \quad c_L^d \equiv \left[V_L^d \hat{c} V_L^{d\dagger}\right]_{11}, \quad c_R^d \equiv \left[V_R^d \hat{c} V_R^{d\dagger}\right]_{11}.$$

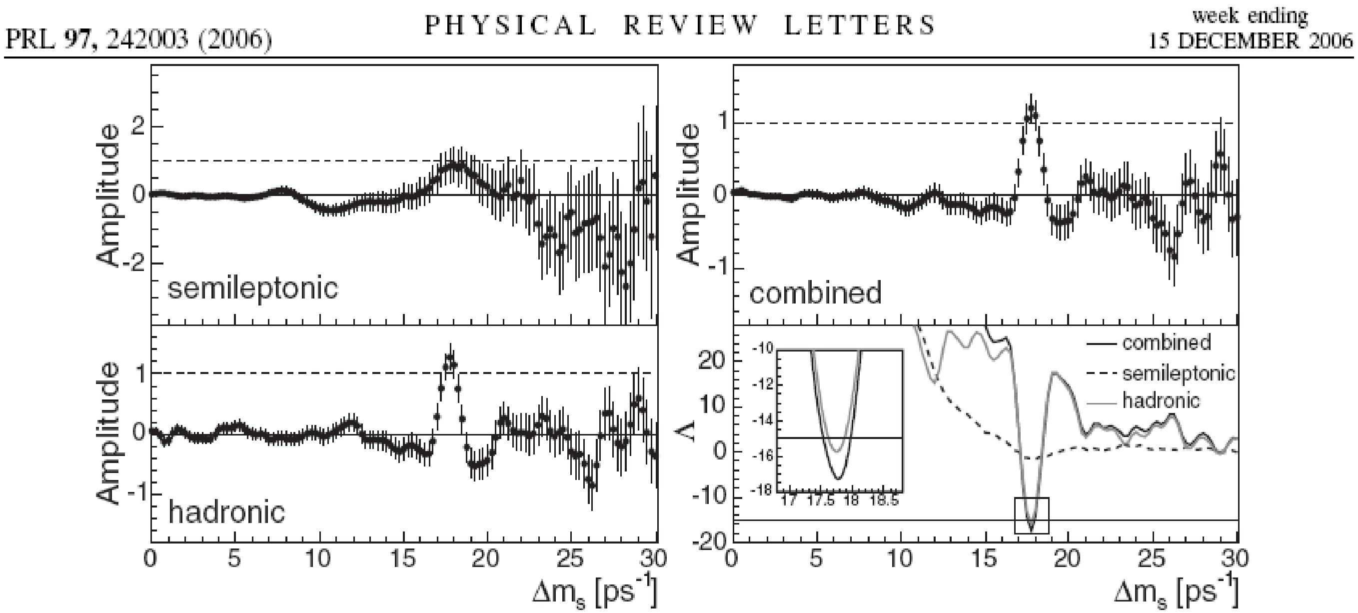
Neutral – Current Interaction

$$\mathcal{L}_{\text{FCNC}}^{Z'} = -\frac{g_2}{2\cos\theta_W}\left[L_{sb}^{Z'}\overline{s}_L\gamma_\mu b_L Z'^\mu + R_{sb}^{Z'}\overline{s}_R\gamma_\mu b_R Z'^\mu\right] + h.c,$$

$$\mathcal{L}(Z'\overline{q}q) = -\frac{g_2}{\cos\theta_W}\delta\, Z'^\mu\left[\overline{u}\gamma_\mu c_L^u P_L u + \overline{d}\gamma_\mu(c_L^d P_L + c_R^d P_R)d\right],$$

B_s Mixing

Experimental result



$$\Delta m_s^{\mathrm{exp}} = 17.77 \pm 0.10(\mathrm{stat}) \pm 0.07(\mathrm{syst})\ \mathrm{ps}^{-1}$$

less than 1%

Lattice QCD result

$$\Delta m_s^{\mathrm{SM}}\Big|_{(\mathrm{HP+JL})\mathrm{QCD}} = 22.57^{+5.88}_{-5.22}\ \mathrm{ps}^{-1}.$$

about 27%

BRs

Experimental results

Mode	$BR[10^{-6}]$	A_{CP}	S_{CP}	
$B^+ \to \pi^+ K^0$	23.1 ± 1.0	0.009 ± 0.025		$\sim 4.3\%$
$B^+ \to \pi^0 K^+$	12.9 ± 0.6	0.050 ± 0.025		$\sim 4.7\%$
$B^0 \to \pi^- K^+$	19.4 ± 0.6	-0.097 ± 0.012		$\sim 3.1\%$
$B^0 \to \pi^0 K^0$	9.9 ± 0.6	-0.14 ± 0.11	0.38 ± 0.19	$\sim 6.1\%$

PQCD results $\square 1\sigma$

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TABLE III. Branching ratios in the NDR scheme ($\times 10^{-6}$). The label $\mathrm{LO}_{\mathrm{NLOWC}}$ means the LO results with the NLO Wilson coefficients, and +VC, +QL, +MP, and +NLO mean the inclusions of the vertex corrections, the quark loops, the magnetic penguin, and all the above NLO corrections, respectively. The errors in the parentheses were defined in the context.

Mode	Data [1]	LO	$\mathrm{LO}_{\mathrm{NLOWC}}$	+VC	+QL	+MP	+NLO
$B^\pm \to \pi^\pm K^0$	24.1 ± 1.3	17.0	32.3	31.0	34.2	24.1	$24.5^{+13.6(+12.9)}_{-8.1(-7.8)}$
$B^\pm \to \pi^0 K^\pm$	12.1 ± 0.8	10.2	18.4	17.4	19.4	14.0	$13.9^{+10.0(+7.0)}_{-5.6(-4.2)}$
$B^0 \to \pi^\mp K^\pm$	18.9 ± 0.7	14.2	27.7	26.7	29.4	20.5	$20.9^{+15.6(+11.0)}_{-8.3(-6.5)}$
$B^0 \to \pi^0 K^0$	11.5 ± 1.0	5.7	12.1	11.8	12.8	8.7	$9.1^{+5.6(+5.1)}_{-3.3(-2.9)}$

Direct CP Asymmetries

Experimental results

Mode	$BR[10^{-6}]$	A_{CP}	S_{CP}
$B^+ \to \pi^+ K^0$	23.1 ± 1.0	0.009 ± 0.025	
$B^+ \to \pi^0 K^+$	12.9 ± 0.6	0.050 ± 0.025	
$B^0 \to \pi^- K^+$	19.4 ± 0.6	-0.097 ± 0.012	
$B^0 \to \pi^0 K^0$	9.9 ± 0.6	-0.14 ± 0.11	0.38 ± 0.19

$2\ \sigma$

PQCD results

TABLE IV. Direct *CP* asymmetries in the NDR scheme.

Mode	Data [1]	LO	LO_{NLOWC}	+VC	+QL	+MP	+NLO
$B^\pm \to \pi^\pm K^0$	-0.02 ± 0.04	-0.01	-0.01	-0.01	0.00	-0.01	$0.00 \pm 0.00(\pm 0.00)$
$B^\pm \to \pi^0 K^\pm$	0.04 ± 0.04	-0.08	-0.06	-0.01	-0.05	-0.08	$-0.01^{+0.03(+0.03)}_{-0.05(-0.05)}$
$B^0 \to \pi^\mp K^\pm$	-0.115 ± 0.018	-0.12	-0.08	-0.09	-0.06	-0.10	$-0.09^{+0.06(+0.04)}_{-0.08(-0.06)}$
$B^0 \to \pi^0 K^0$	0.02 ± 0.13	-0.02	0.00	-0.07	0.00	0.00	$-0.07^{+0.03(+0.01)}_{-0.03(-0.01)}$

Mixing-induced CP Asymmetries

Experimental results

Mode	$BR[10^{-6}]$	A_{CP}	S_{CP}
$B^+ \to \pi^+ K^0$	23.1 ± 1.0	0.009 ± 0.025	
$B^+ \to \pi^0 K^+$	12.9 ± 0.6	0.050 ± 0.025	
$B^0 \to \pi^- K^+$	19.4 ± 0.6	-0.097 ± 0.012	
$B^0 \to \pi^0 K^0$	9.9 ± 0.6	-0.14 ± 0.11	0.38 ± 0.19

$2\,\sigma$

PQCD results

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TABLE VI. Mixing-induced *CP* asymmetries in the NDR scheme.

	Data	LO	LO_{NLOWC}	+VC	+QL	+MP	+NLO
$S_{\pi^0 K_S}$	0.31 ± 0.26	0.70	0.73	0.74	0.73	0.73	$0.74^{+0.02(+0.01)}_{-0.03(-0.01)}$

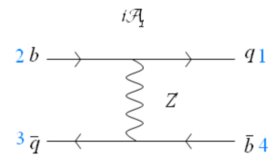
$B_s^0 - \bar{B}_s^0$ Mixing

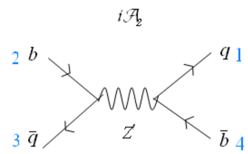
$$\Delta m_s = 2|M_{12}^s| = \Delta m_s^{SM}\,|1+R|\,.$$

$$R = M_{12}^{s,SM} / M_{12}^{s,Z'}$$

RG

$$M_{12}^{s,Z'} = \frac{1}{3}M_{B_s} f_{B_s}^2 \Bigg[\Big(C_1(\mu_b) + \widetilde{C}_1(\mu_b)\Big)B_1(\mu_b) + \frac{1}{4}\left(\frac{M_{B_s}}{m_b(\mu_b)+m_s(\mu_s)}\right)^2\Big(3C_4(\mu_b)B_4(\mu_b) + C_5(\mu_b)B_5(\mu_b)\Big)\Bigg]$$





$$C_1(M_{Z'}) = \frac{g_Z^2}{8M_{Z'}^2}\left(L_{sb}^{Z'}\right)^2,$$

$$\widetilde{C}_1(M_{Z'}) = \frac{g_Z^2}{8M_{Z'}^2}\left(R_{sb}^{Z'}\right)^2,$$

$$C_5(M_{Z'}) = \frac{g_Z^2}{8M_{Z'}^2}\left(-2L_{sb}^{Z'}R_{sb}^{Z'}\right).$$

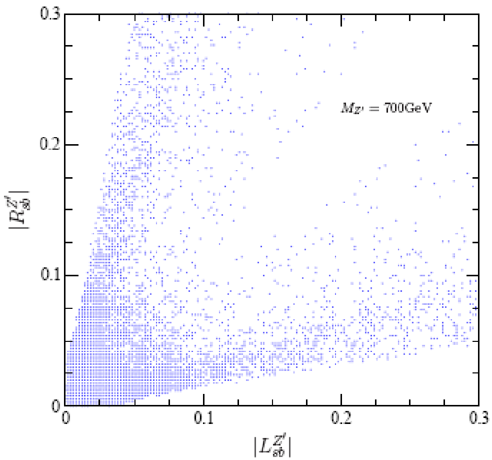


FIG. 1: The allowed region in $(|L_{sb}^{Z'}|, |R_{sb}^{Z'}|)$ plane by Δm_s alone.

$B \to \pi K$ decays

$$\begin{aligned}
A(B^+ \to \pi^+ K^0) &= -P'_{tc} - \frac{1}{3}P'^C_{\rm EW} + P'_{uc}e^{i\gamma}, \\
\sqrt{2}A(B^+ \to \pi^0 K^+) &= P'_{tc} - P'_{\rm EW} - \frac{2}{3}P'^C_{\rm EW} - \left(T' + C' + P'_{uc}\right) e^{i\gamma}, \\
A(B^0 \to \pi^- K^+) &= P'_{tc} - \frac{2}{3}P'^C_{\rm EW} - \left(T' + P'_{uc}\right) e^{i\gamma}, \\
\sqrt{2}A(B^0 \to \pi^0 K^0) &= -P'_{tc} - P'_{\rm EW} - \frac{1}{3}P'^C_{\rm EW} - \left(C' - P'_{uc}\right) e^{i\gamma},
\end{aligned}$$

$$\begin{array}{cc}
O(1) & |P'_{tc}|, \\
O(\bar\lambda) & |T'|, |P'_{\rm EW}|, \\
O(\bar\lambda^2) & |C'|, |P'_{uc}|, |P'^C_{\rm EW}|, \\
O(\bar\lambda^3) & |A'|.
\end{array}$$

Leptophobic Z' contributions

$$\begin{aligned}
P'_{EW} &= \delta \frac{m_Z^2}{m_{Z'}^2}\Big[\Big(-1.03\ c_R^d\ - 1.026\ (c_L^u - c_L^d)\ \Big)\, L_{sb}^{Z'} \\
&\qquad - \Big(\ 1.03\ (c_L^u - c_L^d) + 1.026\ c_R^d\ \Big)\, R_{sb}^{Z'}\Big]\, A_{K\pi} \\
P'^{C}_{EW} &= \delta \frac{m_Z^2}{m_{Z'}^2}\Big[\Big(-0.113\ (c_L^u - c_L^d) + 0.626\ c_R^d\ r_\chi^K\ \Big)\ L_{sb}^{Z'} \\
&\qquad + \Big(-0.113\ c_R^d\ + 0.626(c_L^u - c_L^d) r_\chi^K\Big)\ R_{sb}^{Z'}\Big]\, A_{\pi K} \\
P'_{tc} &= \delta \frac{m_Z^2}{m_{Z'}^2}\Big[\Big(\ 0.081\ \frac{1}{3}\ (c_L^u + 2c_L^d) + 0.607\ \frac{2}{3}\ c_R^d\ r_\chi^K\Big)\, L_{sb}^{Z'} \\
&\qquad - \Big(\ 0.081\ \frac{2}{3}\ c_R^d\ + 0.607\ \frac{1}{3}\ (c_L^u + 2c_L^d) r_\chi^K\Big)\, R_{sb}^{Z'}\Big]\, A_{\pi K}
\end{aligned}$$

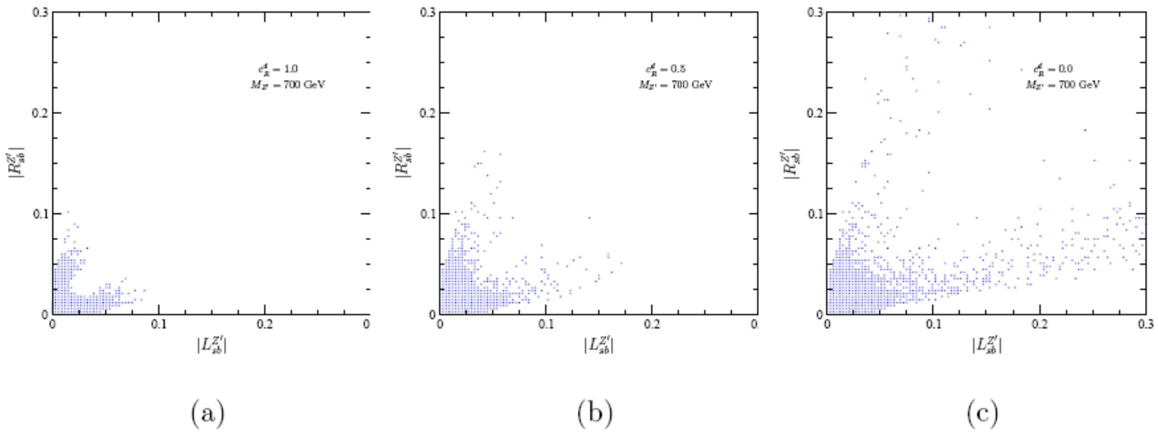


FIG. 3: The allowed region in $(\left|L_{sb}^{Z'}\right|, \left|R_{sb}^{Z'}\right|)$ plane by Δm_s and the four $BR(B \to \pi K)$'s. We fixed $c_R^d = 1.0, 0.5, 0.0$ from the left.

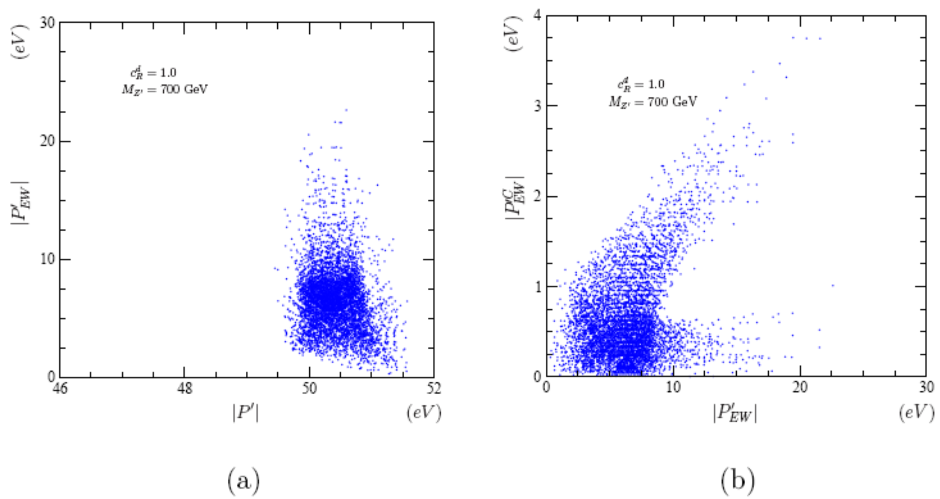
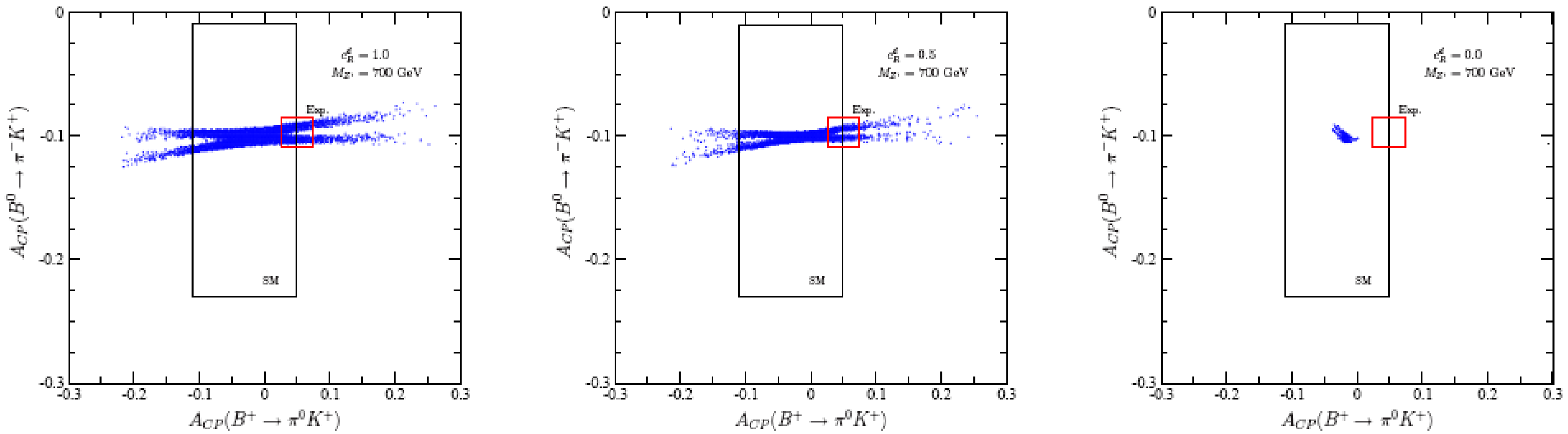
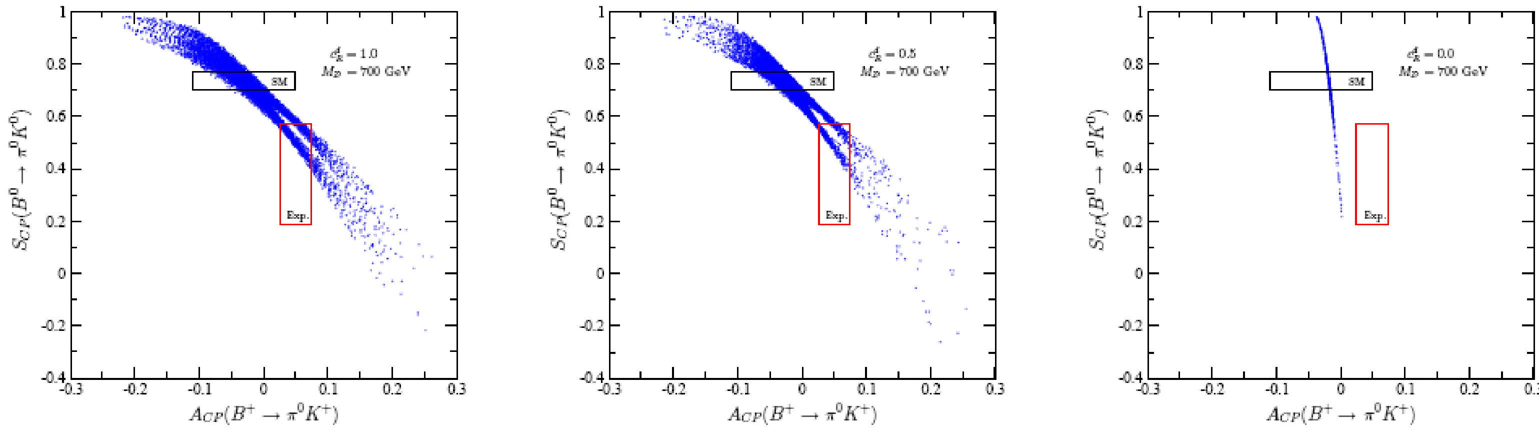


FIG. 2: The correlations between P'_{tc} and $P'_{\rm EW}$ (a) and between P'_{EW} and $P'^C_{\rm EW}$ (b) for $M_{Z'} = 700$ GeV and $c_R^d = 1$.



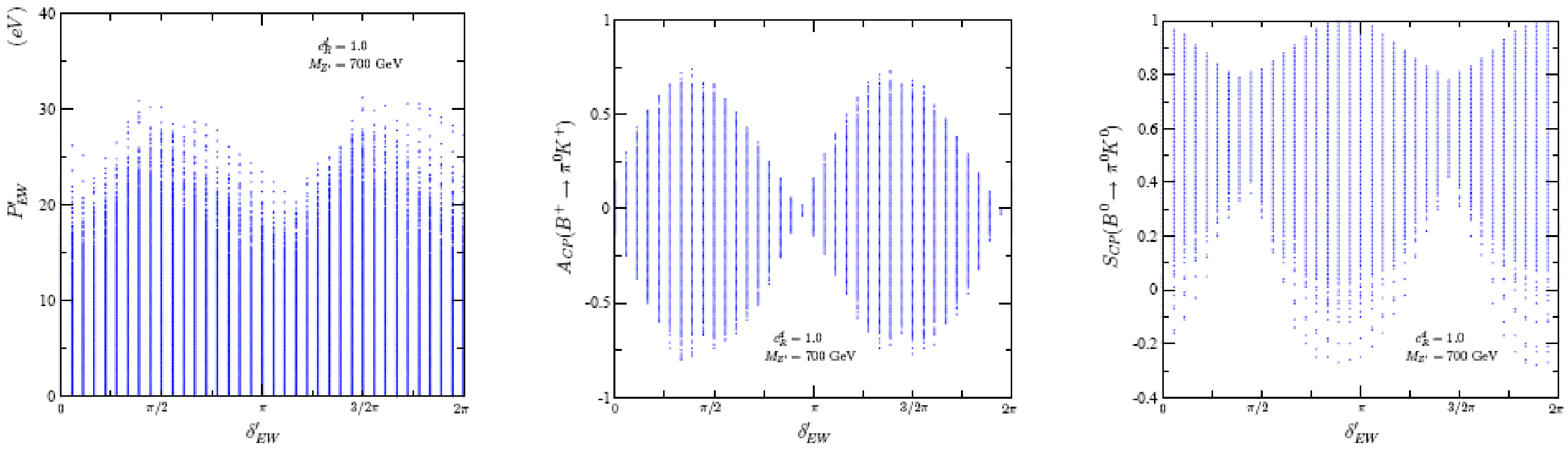
(a) (b) (c)

FIG. 4: The predictions for $A_{\rm CP}(B^+ \to \pi^0 K^+)$ and $A_{\rm CP}(B^0 \to \pi^- K^+)$ for $M_{Z'} = 700$ GeV and (a) $c_R^d = 1.0$, (b) $c_R^d = 0.5$, (c) $c_R^d = 0.0$.



(a) (b) (c)

FIG. 5: The correlation between $A_{\rm CP}(B^+ \to \pi^0 K^+)$ and $S_{\rm CP}(B^0 \to \pi^0 K^0)$ for $M_{Z'} = 700$ GeV and (a) $c_R^d = 1.0$, (b) $c_R^d = 0.5$, (c) $c_R^d = 0.0$.



(a) (b) (c)

FIG. 6: The $|P'_{\rm EW}|$ (a), $A_{\rm CP}(B^+ \to \pi^0 K^+)$ (b), and $S_{\rm CP}(B^0 \to \pi^0 K^0)$ (c) as a function of strong phase, δ'_{EW}, of the electroweak penguin. We fixed $c_R^d = 1, M_{Z'} = 700$ GeV.

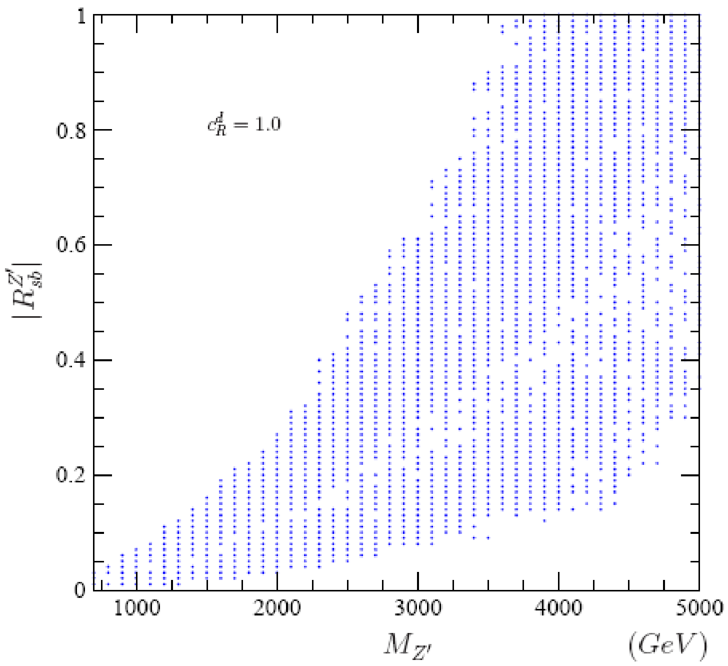


Figure 7. A scattered plot in $(M_{Z'}, |R_{sb}^{Z'}|)$ plane. For this plot we imposed $A_{\rm CP}(B^+ \to \pi^0 K^+)$, $A_{\rm CP}(B^0 \to \pi^- K^+)$, and $S_{\rm CP}(B^0 \to \pi^0 K^0)$ constraints as well as the Δm_s and $BR(B \to \pi K)$'s.

Conclusion

- ✓ Stringy leptophobic Z' can possibly explain the apparent deviations from the SM predictions in the $B \to \pi K$ decays

- ✓ This is phenomenologically interesting because

  - The new Z' coupling is generation dependent and can generate FC
  - The FCNC couplings allow large CP violation
  - The couplings also violate the isospin symmetry and can give large contributions to the EW penguins (P_{EW} and P^C_{EW})

Back up

There is an ambiguity in the assignment of the various fields

Table 2.1. Charges of fermions contained in the **27** representation of E_6 within the conventional particle embedding [1].

$SO(10)$	$SU(5)$	Particles	$SU(3)_c$	$Y/2$	$2\sqrt{10}Q_\chi$	$2\sqrt{6}Q_\psi$
16	**10**	$Q=(u,d)^T$	$\mathbf{3}$	$1/6$	-1	1
		u^c	$\bar{\mathbf{3}}$	$-2/3$	-1	1
		e^c	$\mathbf{1}$	1	-1	1
	$\bar{\mathbf{5}}$	$L=(\nu,e)^T$	$\mathbf{1}$	$-1/2$	3	1
		d^c	$\bar{\mathbf{3}}$	$1/3$	3	1
	1	ν^c	$\mathbf{1}$	0	-5	1
10	$\bar{\mathbf{5}}$	$H=(N,E)^T$	$\mathbf{1}$	$-1/2$	-2	-2
		h^c	$\bar{\mathbf{3}}$	$1/3$	-2	-2
	5	$H^c=(N^c,E^c)^T$	$\mathbf{1}$	$1/2$	2	-2
		h	$\mathbf{3}$	$-1/3$	2	-2
1	**1**	S^c	$\mathbf{1}$	0	0	4

1) $Q_{\varphi,\chi}$ of the fields (L,d^c,ν^c) can be interchanged with those of (H,h^c,S^c)

2) The pairs (u^c,e^c) and (d^c,ν^c) are interchanged : Flpped SU(5)

3) We can consider the interchange of both (1) and (2) simutaneously