Quantum Interface between Light and Atoms -
Quantum Network, Quantum Communication, Quantum Repeater

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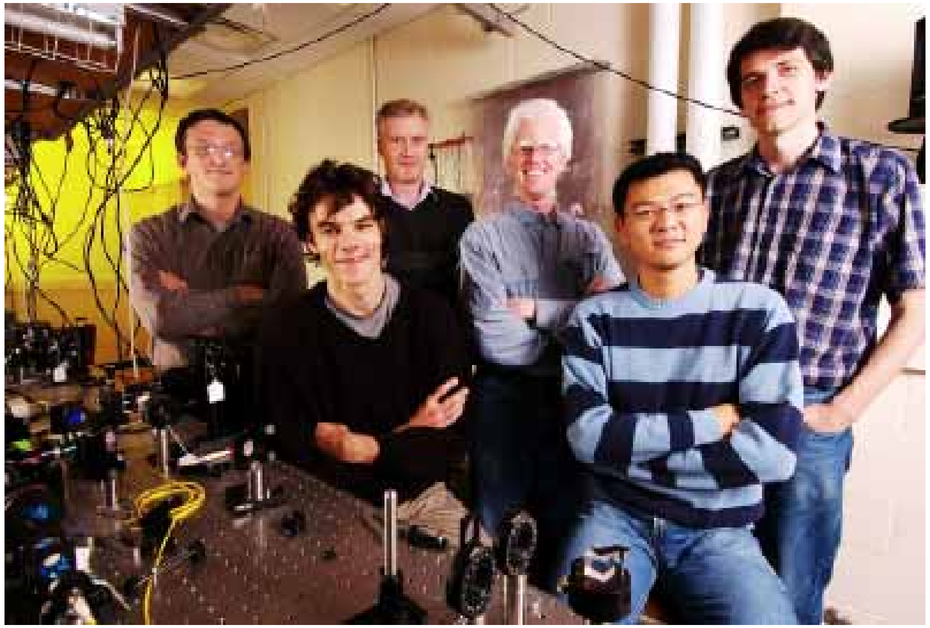

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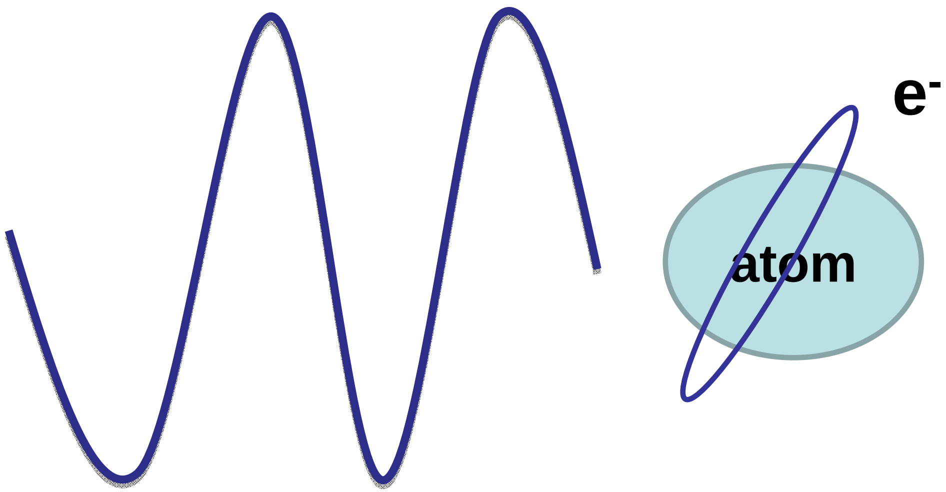
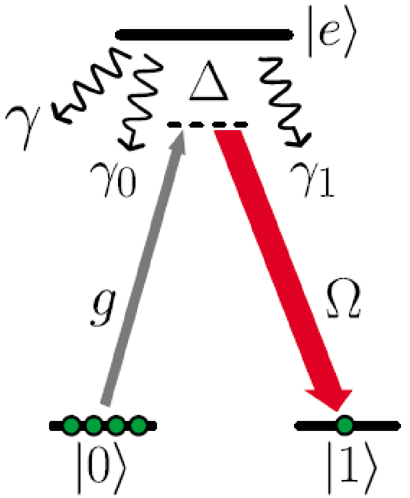
Outline

- Quantum interface through interaction of light and atoms
- Atomic media and entanglement
- Quantum computers
- Quantum memory - EIT
- Quantum repeater : DLCZ protocol
- Cascade emission
- Telecom wavelength conversion using a diamond configuration
- Outlook

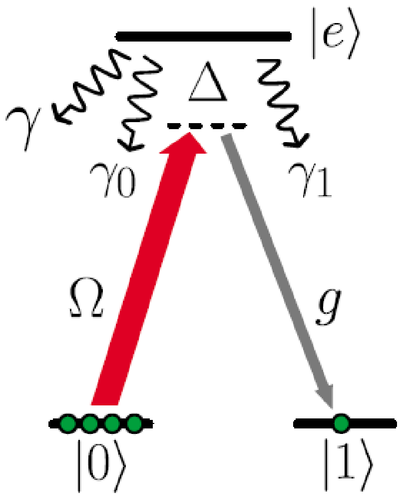
Interaction of light and atoms



Basic interaction and effective Hamiltonian



(a)



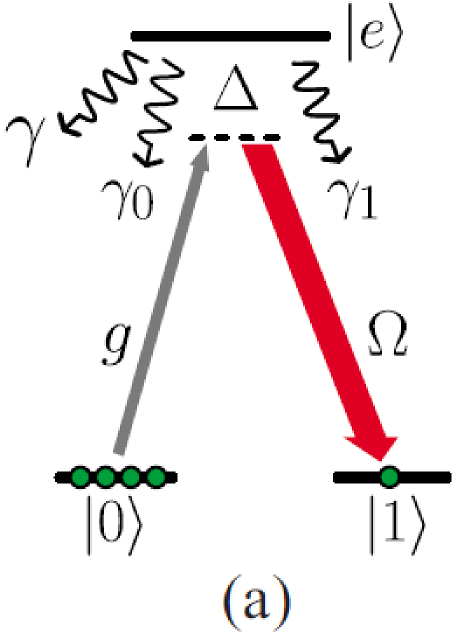
(b)

$$H_{\mathrm{BS}} = \int dz \left[\frac{-|\Omega(z,t)|^2}{4\Delta} \sum_m a^\dagger_{A,m}(z) a_{A,m}(z) - \frac{|g(z)|^2}{\Delta} \sum_m a^\dagger_{L,m}(z) a_{L,m}(z) - \left(\frac{g^*(z)\Omega(z,t)}{2\Delta} \sum_m a^\dagger_{L,m}(z) a_{A,m}(z) + \mathrm{H.c.} \right) \right]$$

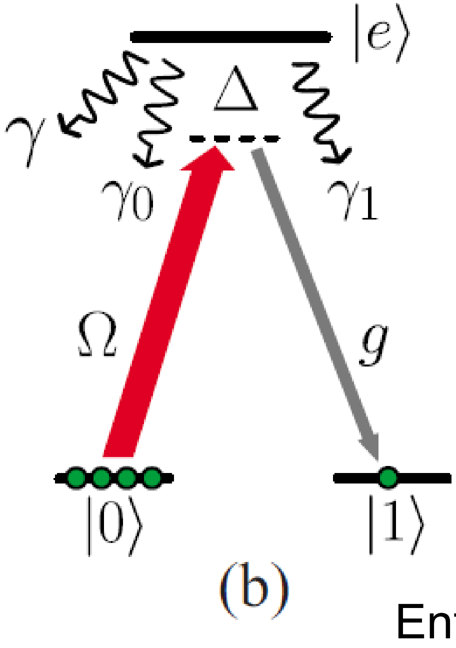
$$H_G = \int dz \left[\frac{|\Omega(z,t)|^2}{4\Delta} a^\dagger_A(z) a_A(z) - \left(\frac{g^*(z)\Omega(z,t)}{2\Delta} a^\dagger_L(z) a^\dagger_A(z) + \mathrm{H.c.} \right) \right].$$

K. Hammerer, et al., Rev. Mod. Phys. 82, 1041 (2010)

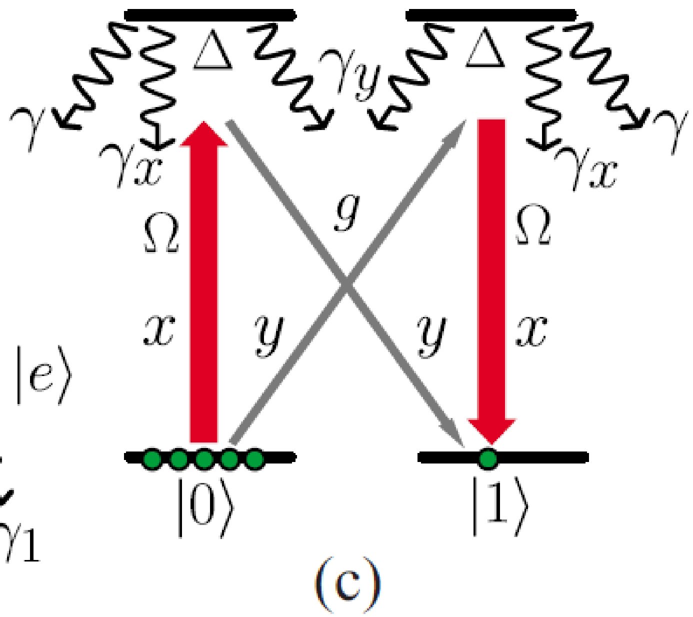
Elements for quantum interface



EIT memory scheme.



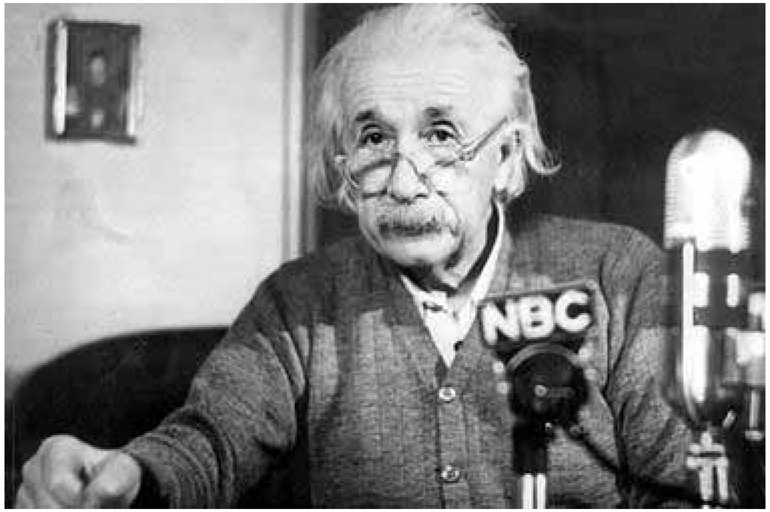
Entanglement scheme.



Memory+entanglement
+teleportation

K. Hammerer, et al., Rev. Mod. Phys. 82, 1041 (2010)

Atomic media and entanglement



Spooky!!

Features for various atomic media

(1) Room-temperature gas: Doppler broadening/atomic motion.
- (a) buffer gas – 2 ms (localization).
- (b) paraffin-coated cell – 1 s (anti-relaxation).

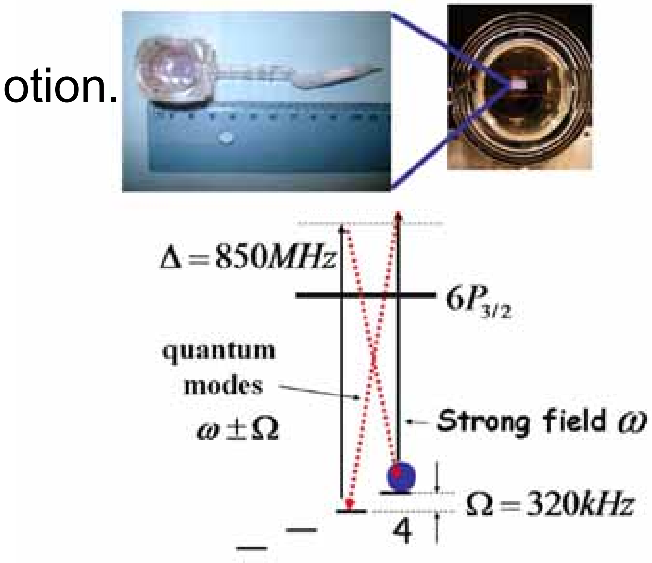
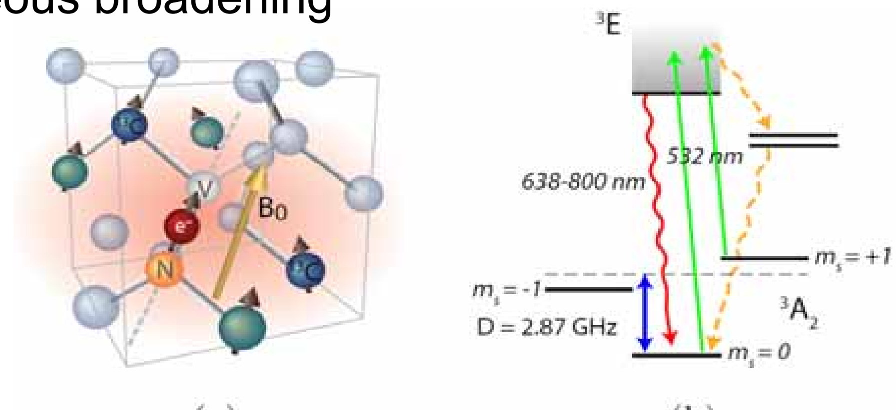


FIG. 8. (Color online) Paraffin coated cesium cell.

(2) Cold and trapped atoms: light focusing at ~10 μm, gradient magnetic fields, optical pumping difficulty.
- (a) MOT – 100 μk, 100 μs.
- (b) Dipole trap lattice + clock transition – 6 ms (limited by motional dephasing).
- (c) BEC – high OD (low generation rate). BEC on chip.

(3) Solid state: frozen motion but inhomogeneous broadening by lattice fields.
- (a) rare-earth doped crystal – 4 k, s. (Λ scheme or photo-echo rephasing).
- (b) NV-center diamond – 4k to room-temperature, 600 μs.
- (c) quantum dot.

(4) Ion crystal.



(a) (b)

K. Hammerer, et al., Rev. Mod. Phys. 82, 1041 (2010)

Separable or entangled

- ***Einstein/Podolsky/Rosen**:* An entangled wavefunction does not describe the physical reality in a complete way.
- ***J. Bell**:* . . . a correlation that is stronger than any classical correlation.
- ***A. Peres**:* " . . . a trick that quantum magicians use to produce phenomena that cannot be imitated by classical magicians."

$|\phi\rangle = |a\rangle \otimes |b\rangle = |0\rangle_a |0\rangle_b$: product state. $|\Phi^{\pm}\rangle = \frac{1}{\sqrt{2}}(|00\rangle \pm |11\rangle)$: entangled.

What about this one?

$$\begin{aligned}\rho_{CL} &= \frac{1}{2}\Big(|H_A V_B\rangle\langle H_A V_B|\Big) \\ &+ \frac{1}{2}\Big(|V_A H_B\rangle\langle V_A H_B|\Big)\end{aligned}$$

$$\rho_{CL} = \frac{1}{2}\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$\varrho = \sum_i p_i |a_i\rangle\langle a_i| \otimes |b_i\rangle\langle b_i|$ $\longrightarrow$ Separable mixed state.

J. Math. Phys. Vol. 43, 4237 (2002)

J. H. Eberly, arXiv:quant-ph/0508019v1

Entanglement measure in Schmidt basis (pure bipartite)

$$|\Psi_{AB}\rangle = \sum_i \sum_j C(i,j)|i\rangle \otimes |j\rangle, \qquad C(n,\nu) = \sum_s \sqrt{\lambda_s} f_n^s (\phi_\nu^s)^*,$$

$$\begin{aligned}
|\Psi_0\rangle &\equiv \sum_n \sum_\nu C(n,\nu)|n\rangle \otimes |\nu\rangle \\
&= \sum_n \sum_\nu \Big(\sum_s \sqrt{\lambda_s} f_n^s (\phi_\nu^s)^* \Big) |n\rangle \otimes |\nu\rangle \\
&= \sum_s \sqrt{\lambda_s} \Big(\sum_n f_n^s |n\rangle \Big) \otimes \Big(\sum_\nu (\phi_\nu^s)^* |\nu\rangle \Big) \\
&= \sum_s \sqrt{\lambda_s} |F^s\rangle \otimes |\Phi^s\rangle,
\end{aligned}$$

Von Neumann entropy: $S(\varrho_{\mathrm{red}}) = -\mathrm{Tr}(\varrho_{\mathrm{red}} \log \varrho_{\mathrm{red}})$

J. H. Eberly, arXiv:quant-ph/0508019v1

Entangled pure bipartite system

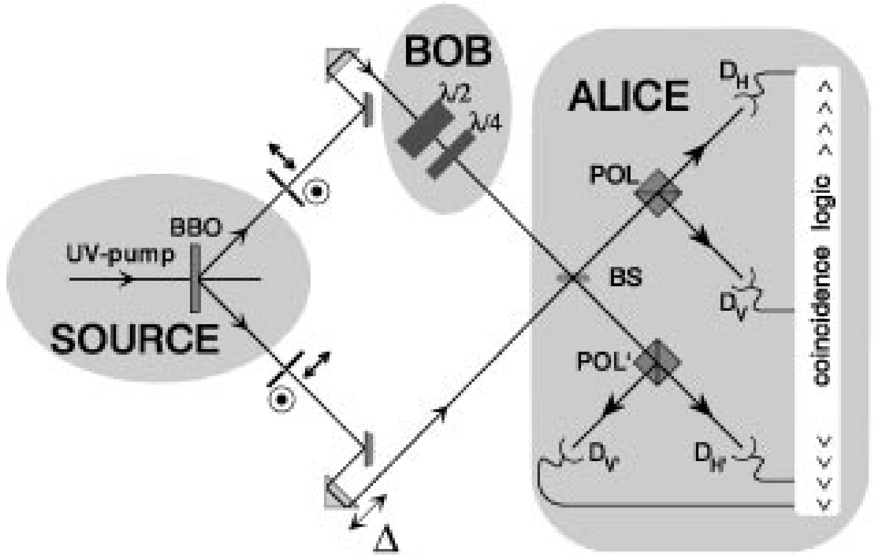
$$\rho_{QM} = |\Psi(Bell)\rangle\langle\Psi(Bell)|, \quad \text{where}$$
$$|\Psi(Bell)\rangle = \frac{1}{\sqrt{2}}\Big(|H_A V_B\rangle + |V_A H_B\rangle\Big),$$

$$\rho_{QM} = \frac{1}{2}\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

Why entangled?

Dense coding:

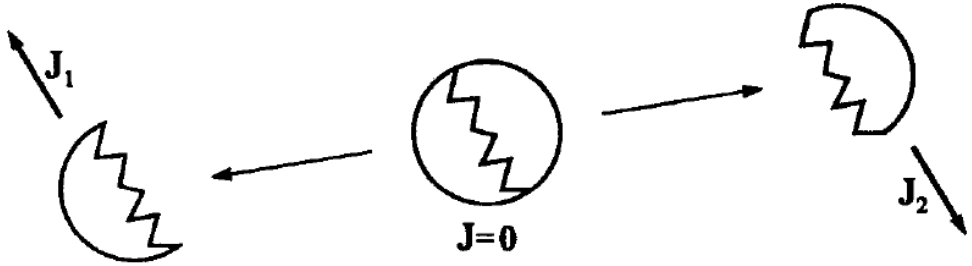
$$|\Psi^+\rangle = (|H\rangle|V\rangle + |V\rangle|H\rangle)/\sqrt{2},$$
$$|\Psi^-\rangle = (|H\rangle|V\rangle - |V\rangle|H\rangle)/\sqrt{2},$$
$$|\Phi^+\rangle = (|H\rangle|H\rangle + |V\rangle|V\rangle)/\sqrt{2},$$
$$|\Phi^-\rangle = (|H\rangle|H\rangle - |V\rangle|V\rangle)/\sqrt{2}.$$



K. Mattle, et. al., Phys. Rev. Lett. 76, 4656 (1996)

Bell inequality – classical correlation

$$a = \mathrm{sign}(\alpha \cdot J_1).$$

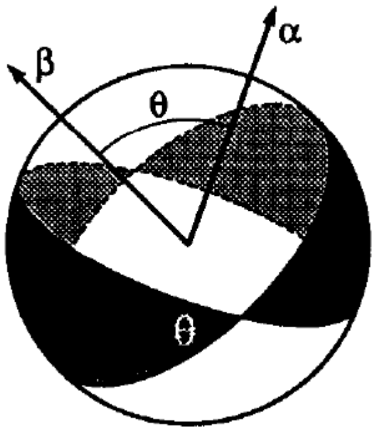


$$b = \mathrm{sign}(\beta \cdot J_2).$$

$$J_2 = -J_1$$

If directions J_1 J_2 are randomly distributed, $\langle a \rangle = \dfrac{\sum_j a_j}{N}$ and $\langle b \rangle = \dfrac{\sum_j b_j}{N} \xrightarrow[N\to\infty]{} 0.$

Correlations: $\langle ab \rangle = \sum_j a_j b_j / N \to -1$ when $\alpha = \beta.$



Classical correlations: $\langle ab \rangle = [\theta - (\pi - \theta)]/\pi = -1 + 2\theta/\pi.$

Quantum Theory: Concepts and Methods (1995), Asher Perez

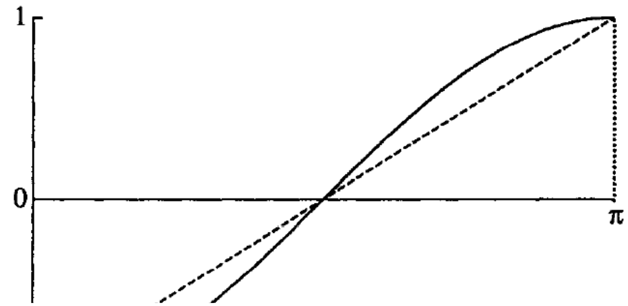
Bell inequality – quantum correlation

Consider two spin1/2 particles in a singlet state,

$$\alpha\cdot\sigma_1 \to a = \pm 1, \beta\cdot\sigma_2 \to b = \pm 1.$$

$$\langle a\rangle = 0 \text{ and } \langle b\rangle = 0. \qquad \langle ab\rangle = \psi^\dagger(\alpha\cdot\sigma_1)(\beta\cdot\sigma_2)\psi.$$

With $\sigma_2\psi = -\sigma_1\psi$ and $(\alpha\cdot\sigma)(\beta\cdot\sigma) =$
$\alpha\cdot\beta + i(\alpha\times\beta)\cdot\sigma$, we have
$\psi^\dagger(\alpha\cdot\sigma_1)(\beta\cdot\sigma_2)\psi = -\alpha\cdot\beta = -\cos\theta.$



CHSH Inequality: $|\langle ab\rangle + \langle bc\rangle + \langle cd\rangle - \langle da\rangle| \leq 2$

Three tests: $a\,(b-c) \equiv \pm(1 - b\,c),$

Statistics: $a_j\,b_j - a_j\,c_j \equiv \pm(1 - b_j\,c_j)$

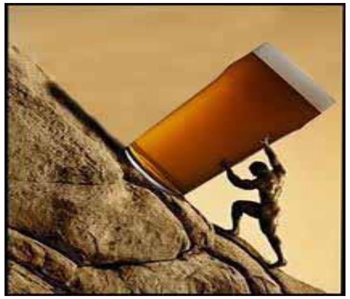
Upper bound: $|\langle ab\rangle - \langle ac\rangle| \leq 1 - \langle bc\rangle$

Inequality violation: $|\cos 2(\alpha-\beta) - \cos 2(\alpha-\gamma)| + \cos 2(\beta-\gamma) \leq 1$

Quantum Theory: Concepts and Methods (1995), Asher Perez

Quantum computation

General criteria for QC



(1) Scalability: Hilbert space can be exponential grown without an exponential cost.
(2) Universal logic gates: single-qubit and C-NOT gates.
(3) Correctability: quantum error correction.

Candidates for quantum computers:

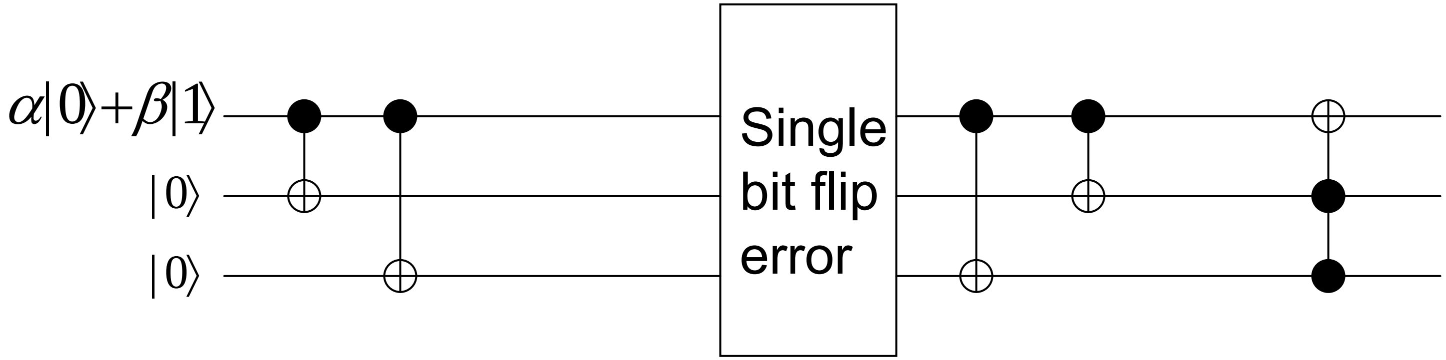
(1) Photons: KLM scheme with single photon source.
(2) Trapped atoms.
(3) Liquid-state nuclear magnetic resonance.
(4) Quantum dots and dopants in solids.
(5) Superconductors.
(6) Polar molecules. Rare-earth doped crystal. Carbon nano-materials. Graphene. Topological quantum computation.

caveat emptor -
買者自負

D. Ladd, et. al., Naure 464, 45 (2010)

QEC_3 qubit code - bit flip

$$\alpha|0\rangle|0\rangle|0\rangle+\beta|1\rangle|1\rangle|1\rangle$$



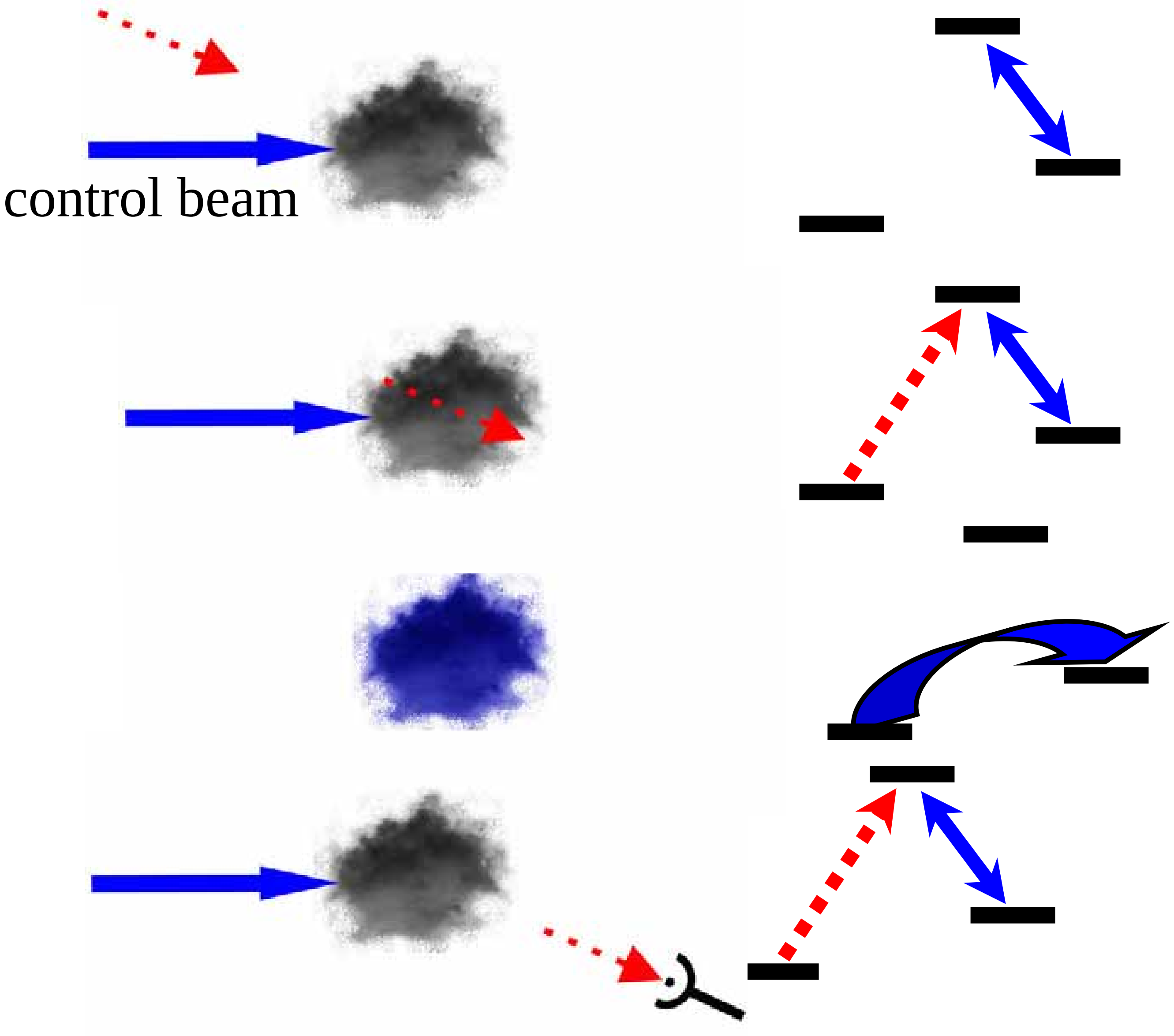
$$\begin{array}{l}
\alpha|0\rangle|0\rangle|0\rangle+\beta|1\rangle|1\rangle|1\rangle \\
\alpha|1\rangle|0\rangle|0\rangle+\beta|0\rangle|1\rangle|1\rangle \\
\alpha|0\rangle|1\rangle|0\rangle+\beta|1\rangle|0\rangle|1\rangle \\
\alpha|0\rangle|0\rangle|1\rangle+\beta|1\rangle|1\rangle|0\rangle
\end{array}$$

$$\begin{cases}
\alpha|0\rangle|0\rangle|0\rangle+\beta|1\rangle|0\rangle|0\rangle \\
\alpha|1\rangle|1\rangle|1\rangle+\beta|0\rangle|1\rangle|1\rangle \\
\alpha|0\rangle|1\rangle|0\rangle+\beta|1\rangle|1\rangle|0\rangle \\
\alpha|0\rangle|0\rangle|1\rangle+\beta|1\rangle|0\rangle|1\rangle
\end{cases}
=
\begin{cases}
(\alpha|0\rangle+\beta|1\rangle)\otimes|0\rangle|0\rangle \\
(\alpha|0\rangle+\beta|1\rangle)\otimes|1\rangle|1\rangle \\
(\alpha|0\rangle+\beta|1\rangle)\otimes|1\rangle|0\rangle \\
(\alpha|0\rangle+\beta|1\rangle)\otimes|0\rangle|1\rangle
\end{cases}$$

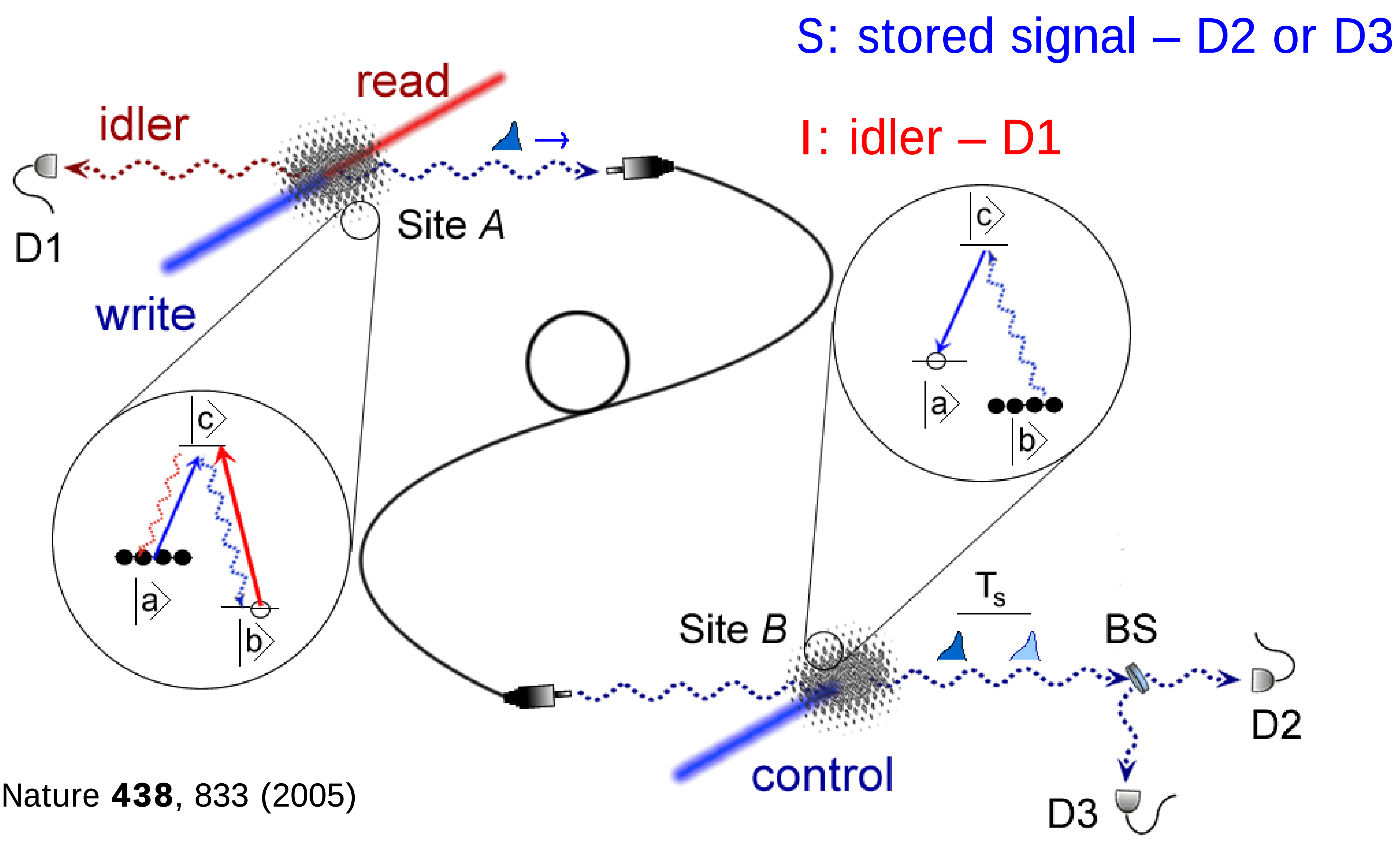
Quantum computation and quantum information (2000), Neilson&Chuang

Quantum memory - EIT

Storage of single photons



Demonstration of quantum memory for single photons



Nature **438**, 833 (2005)

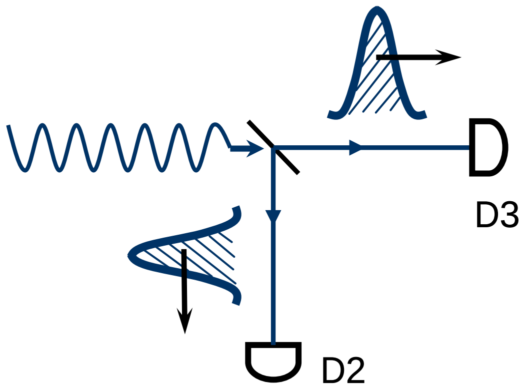
Quantum vs classical memory

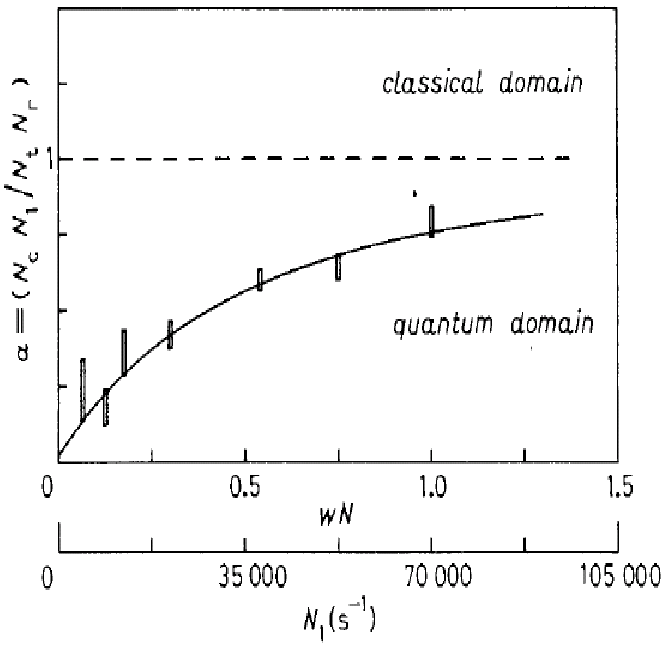
EUROPHYSICS LETTERS 15 February 1986

Europhys. Lett., **1** (4), pp. 173-179 (1986)

Experimental Evidence for a Photon Anticorrelation Effect on a Beam Splitter: A New Light on Single-Photon Interferences.

P. GRANGIER, G. ROGER and A. ASPECT (*)
Institut d'Optique Théorique et Appliquée, B.P. 43 - F 91406 *Orsay, France*





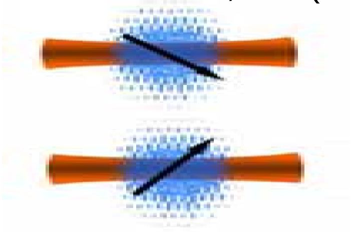
for Fock states $\alpha = 1 - \frac{1}{n}$,

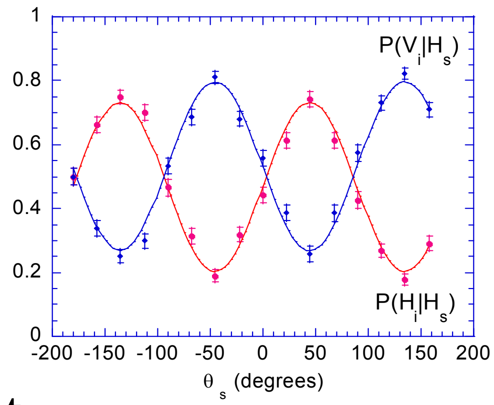
$\Rightarrow \alpha = 0$ for single photons

Matter-light entanglement generation and distribution

Two-ensemble qubit

Science **306**, 663 (2004)

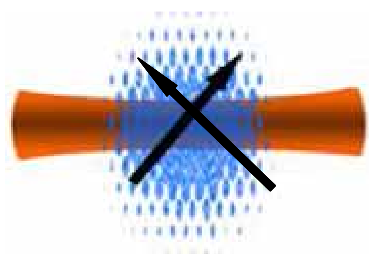




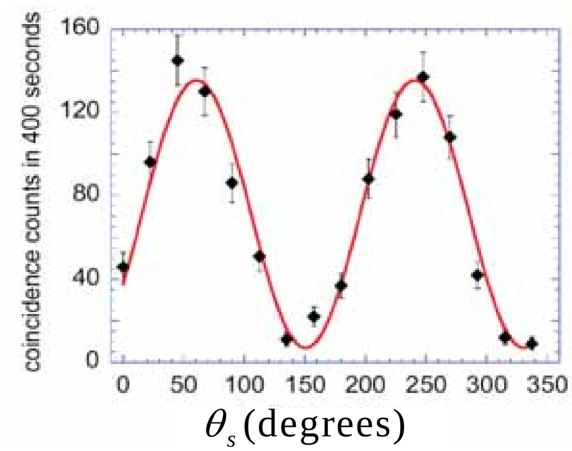
atomic qubit states |+> and |-> state can be prepared and converted into light independently

Polarization (spin) qubit

PRL **95**, 040405 (2005)

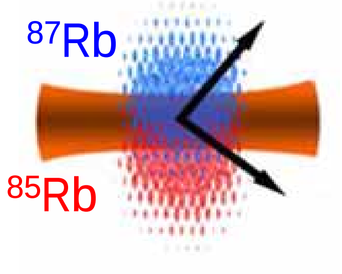


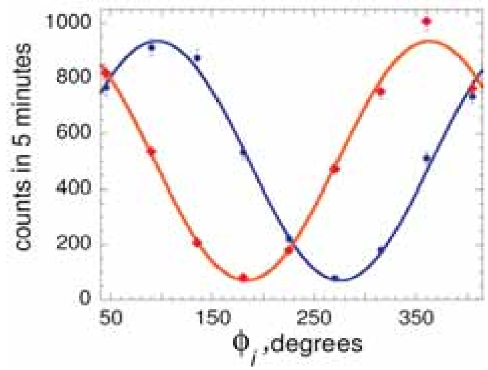
robust atom-photon entanglement



Two-species qubit

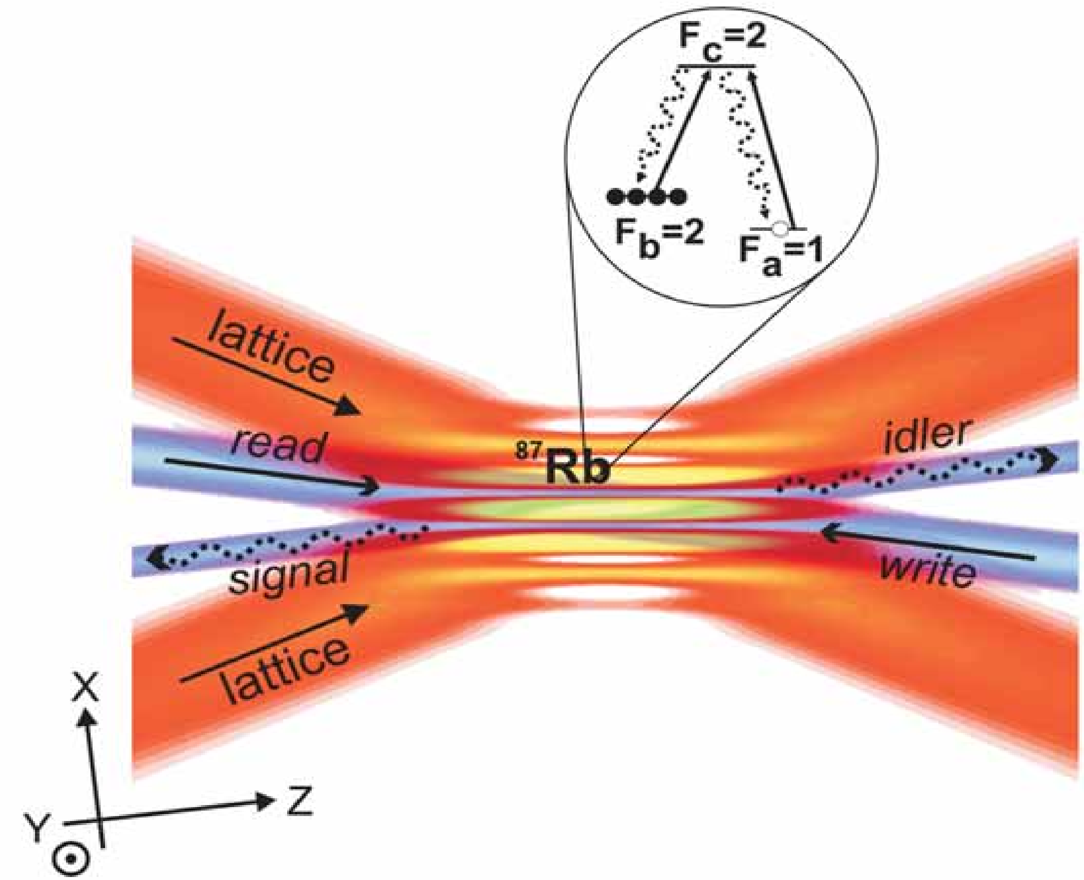
PRL **98**, 123602 (2007)

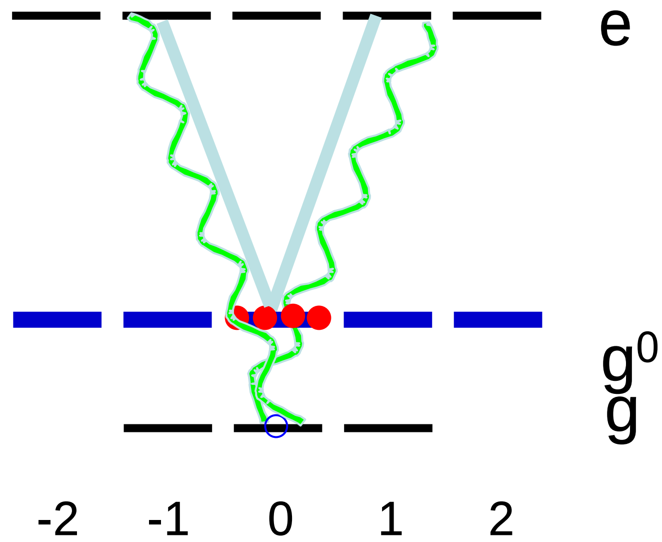


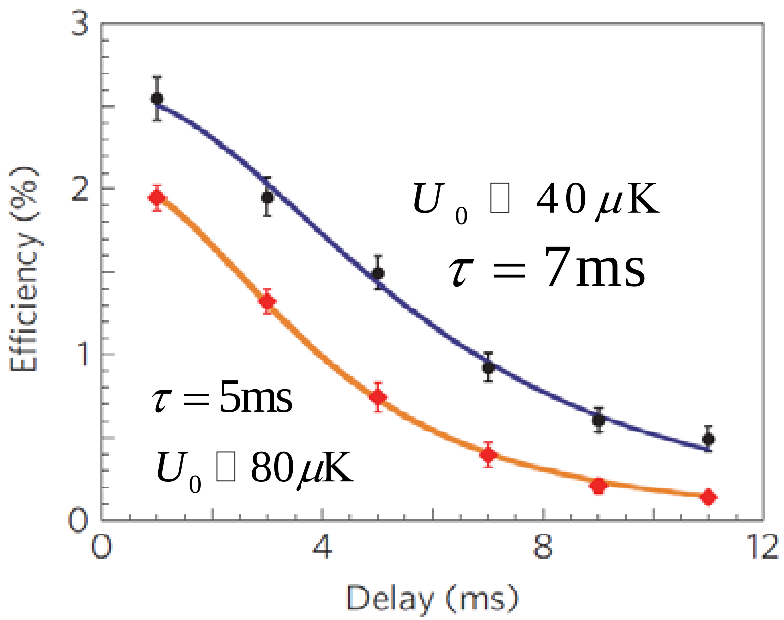


robust atom-photon entanglement;
atomic qubit states |+> and |-> states are separately addressable

Optically confined quantum memory

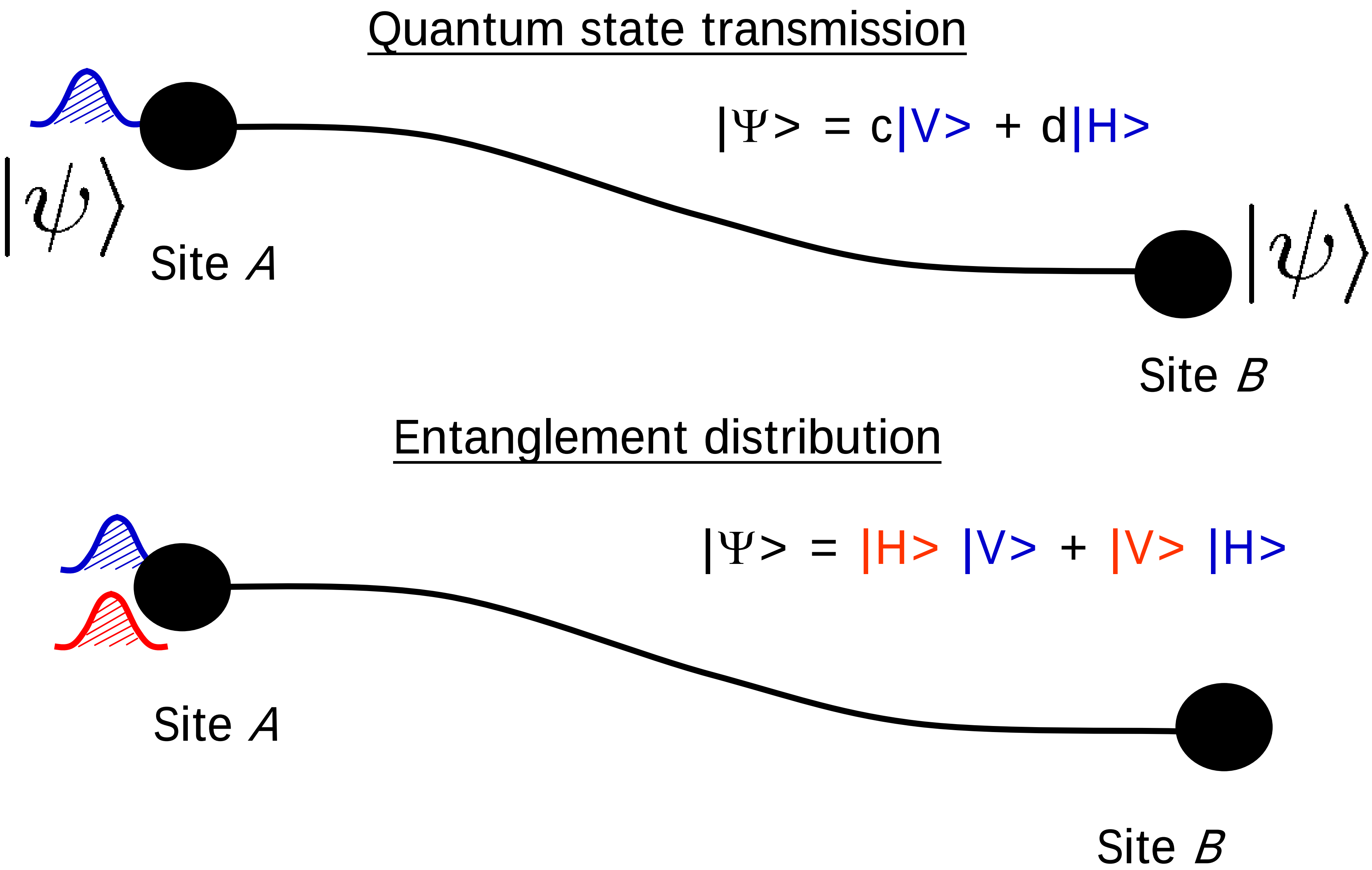




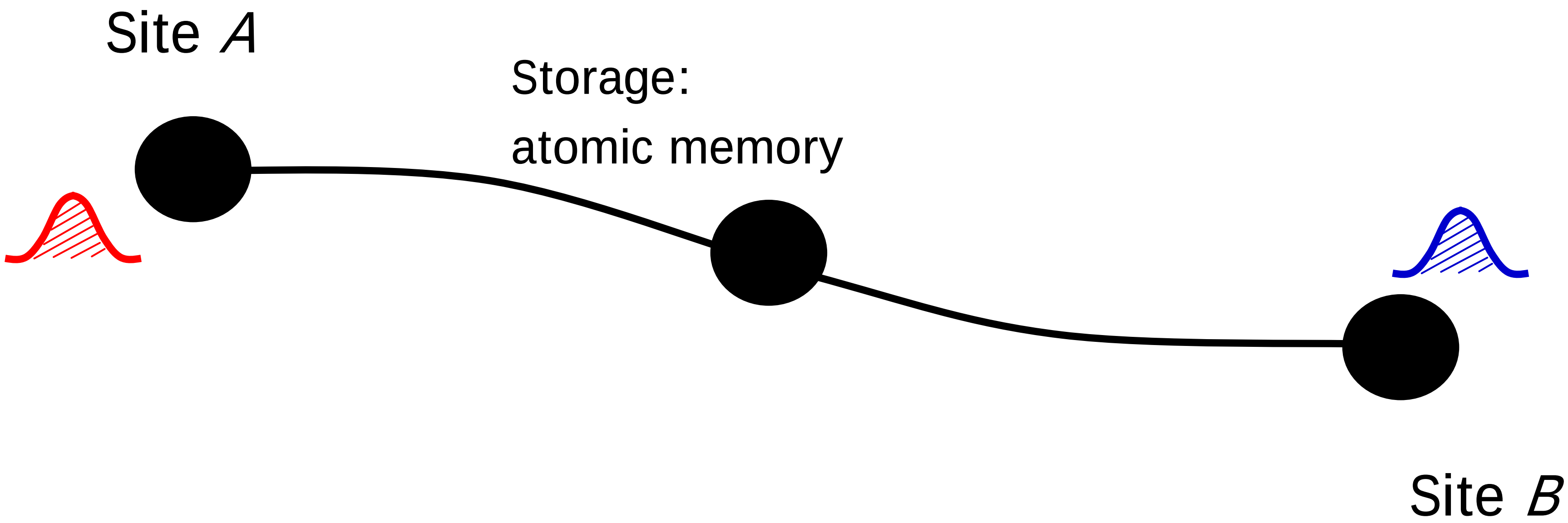


Nature Physics 5, 100 (2009)

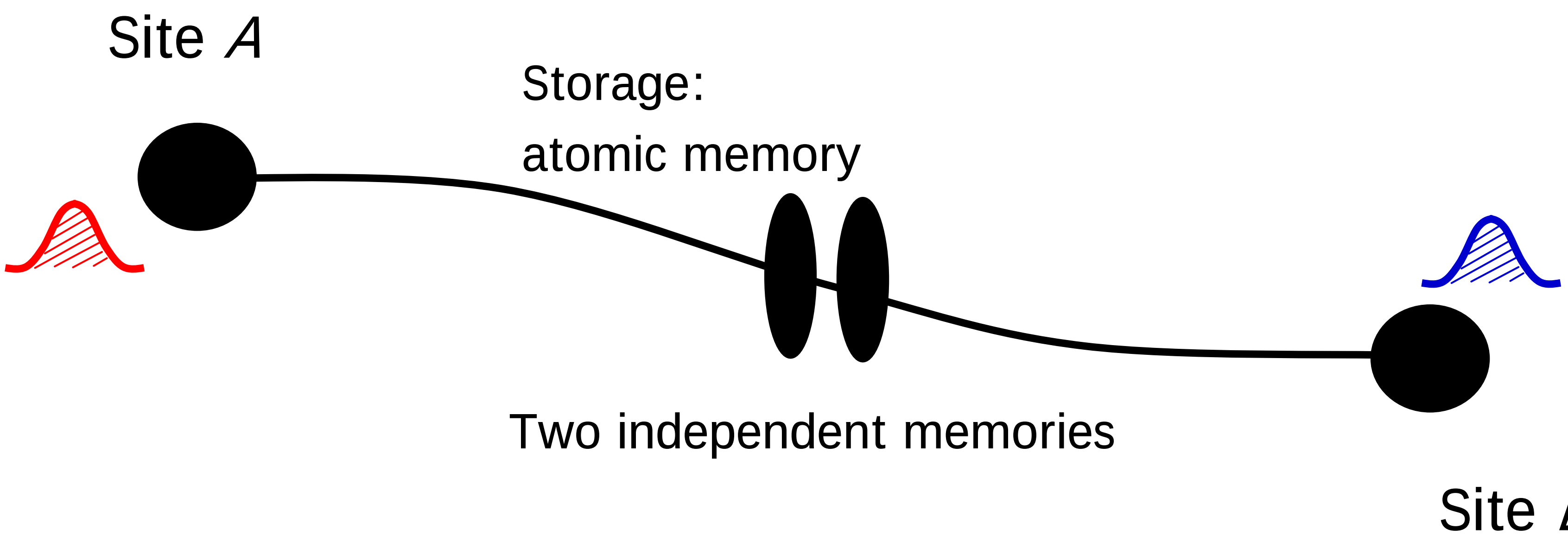
Quantum repeater


Quantum state transmission
|Ψ> = c|V> + d|H>
$|\psi\rangle$
Site A
$|\psi\rangle$
Site B
Entanglement distribution
|Ψ> = |H> |V> + |V> |H>
Site A
Site B

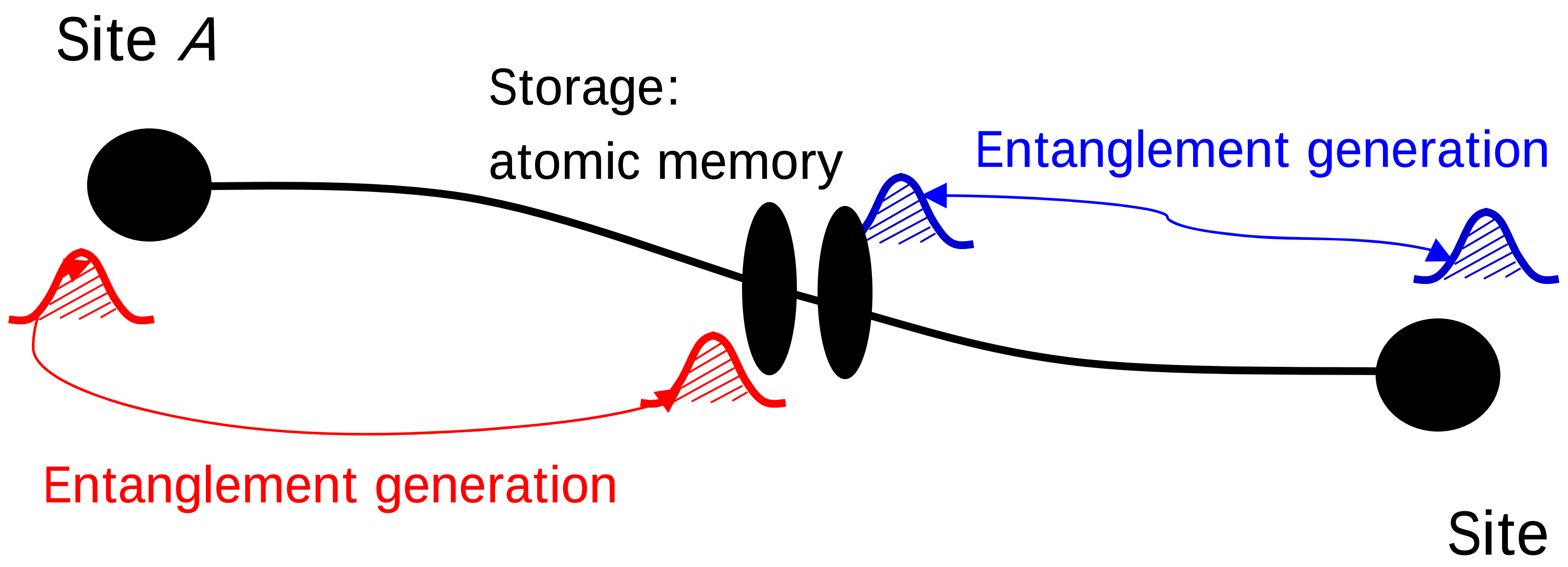
Quantum repeater



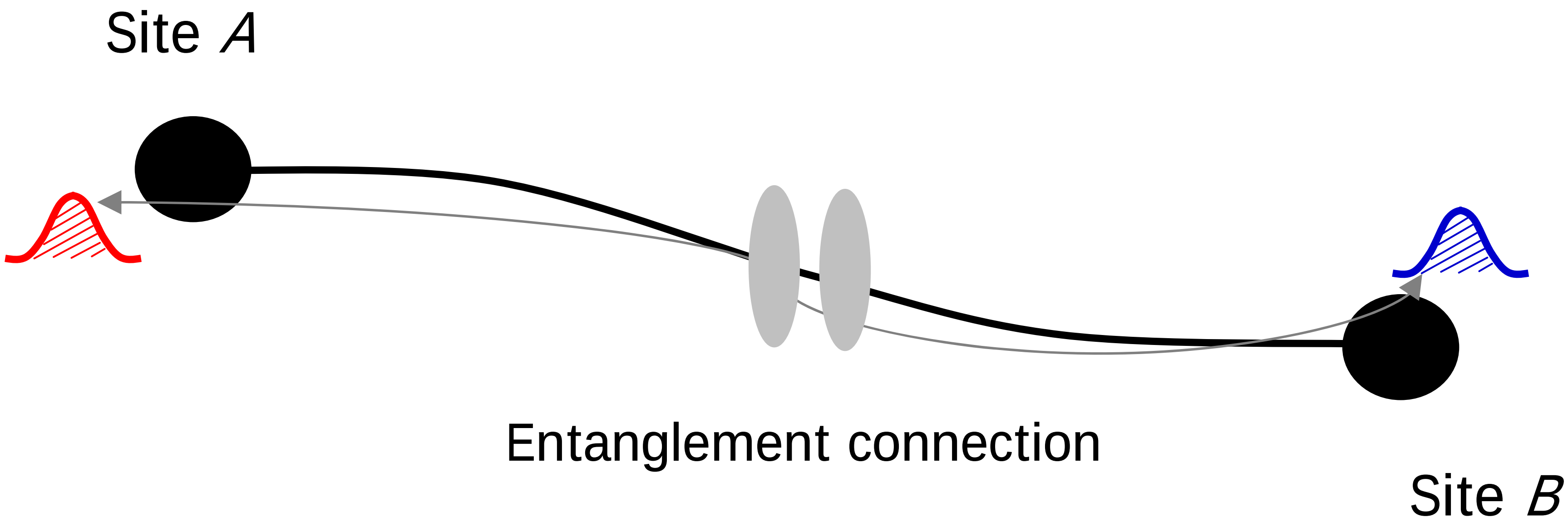
Quantum repeater



Quantum repeater

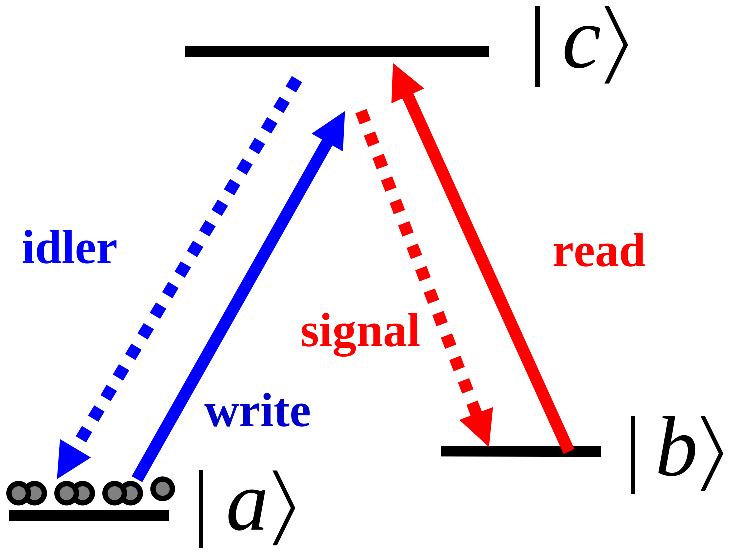


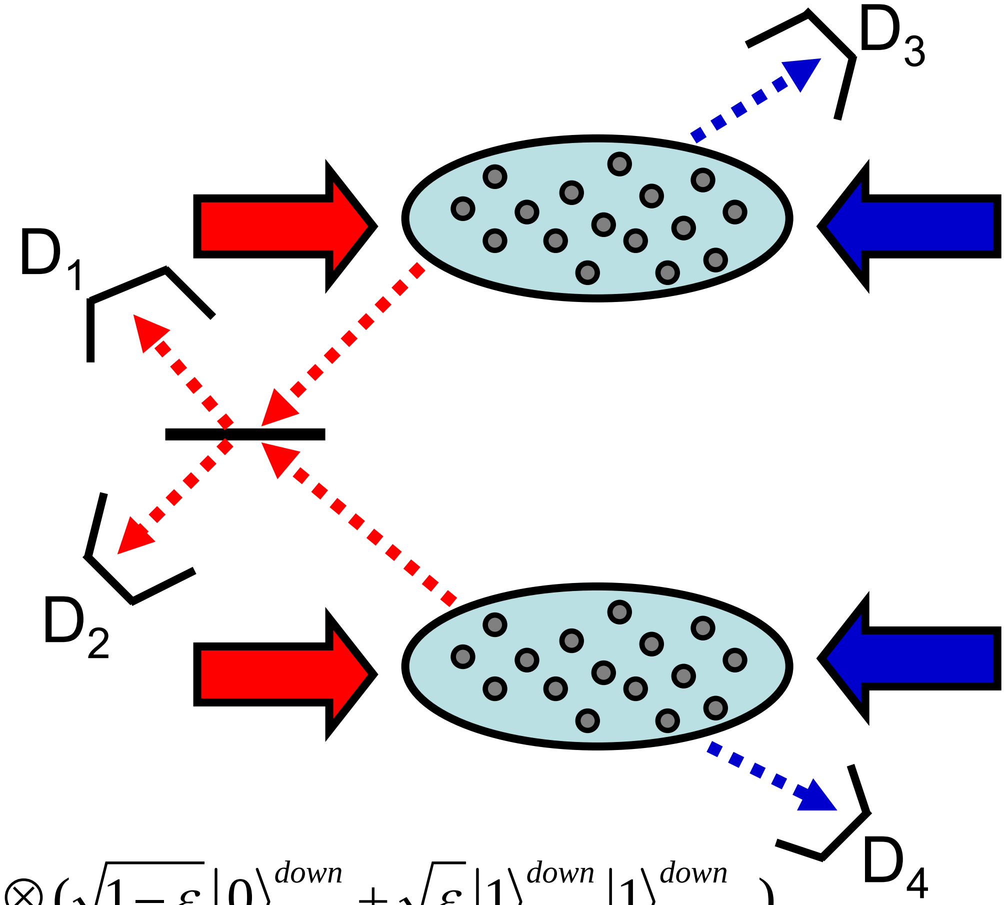
Quantum repeater



Duan-Lukin-Cirac-Zoller quantum repeater

Raman scheme



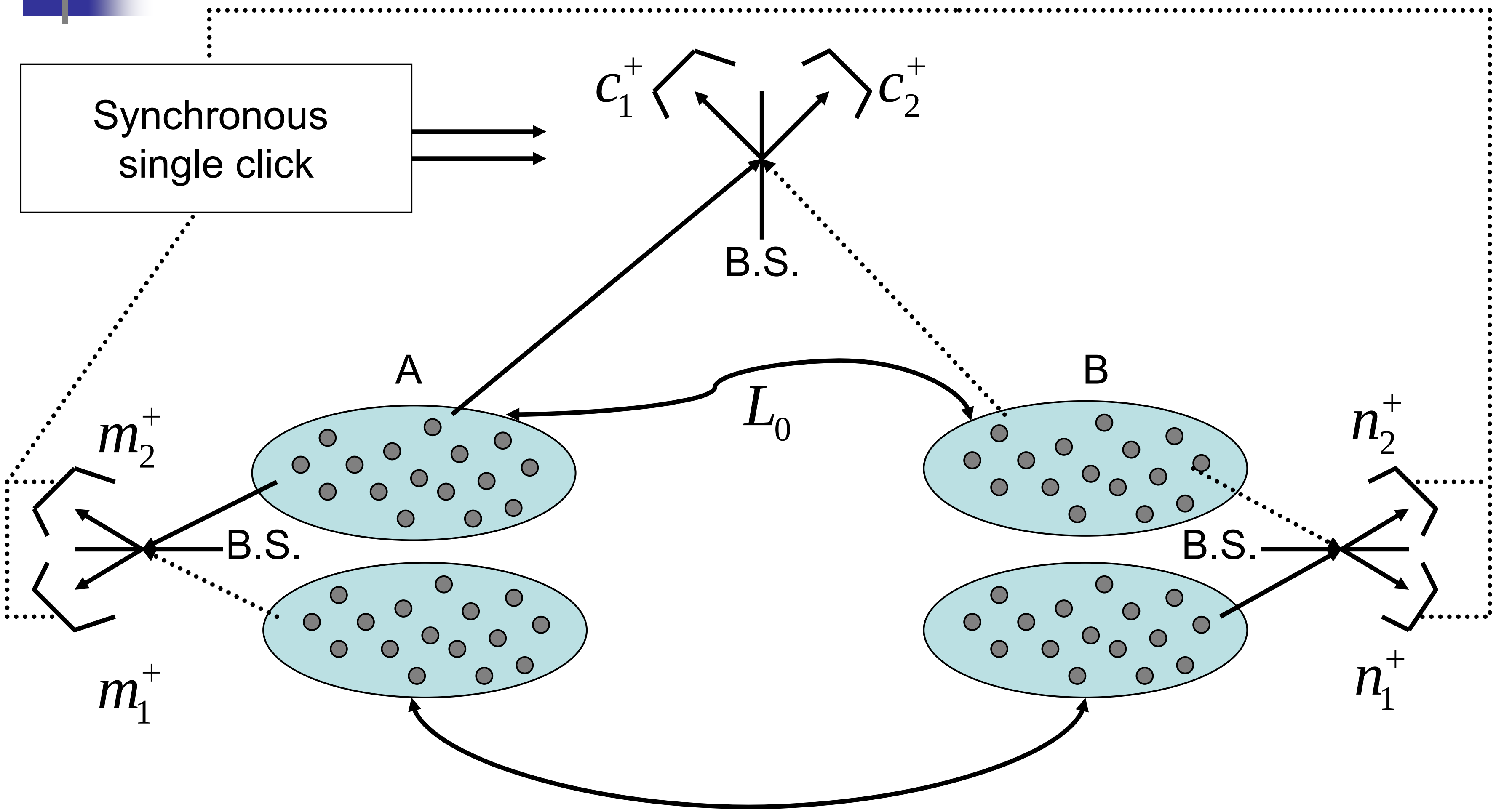


$$|\Psi\rangle = \left(\sqrt{1-\varepsilon}\,|0\rangle^{up} + \sqrt{\varepsilon}\,|1\rangle^{up}_{atom}\,|1\rangle^{up}_{photon}\right) \otimes \left(\sqrt{1-\varepsilon}\,|0\rangle^{down} + \sqrt{\varepsilon}\,|1\rangle^{down}_{atom}\,|1\rangle^{down}_{photon}\right)$$

$$|\Psi\rangle_{\text{single click}} = \sqrt{\varepsilon}\sqrt{1-\varepsilon}\left(|1\rangle^{up}_{atom}\,|0\rangle^{down}_{atom} + |0\rangle^{up}_{atom}\,|1\rangle^{down}_{atom}\right) + O(\varepsilon)$$

L.-M. Duan et al, *Nature* **414**, 413 (2001)

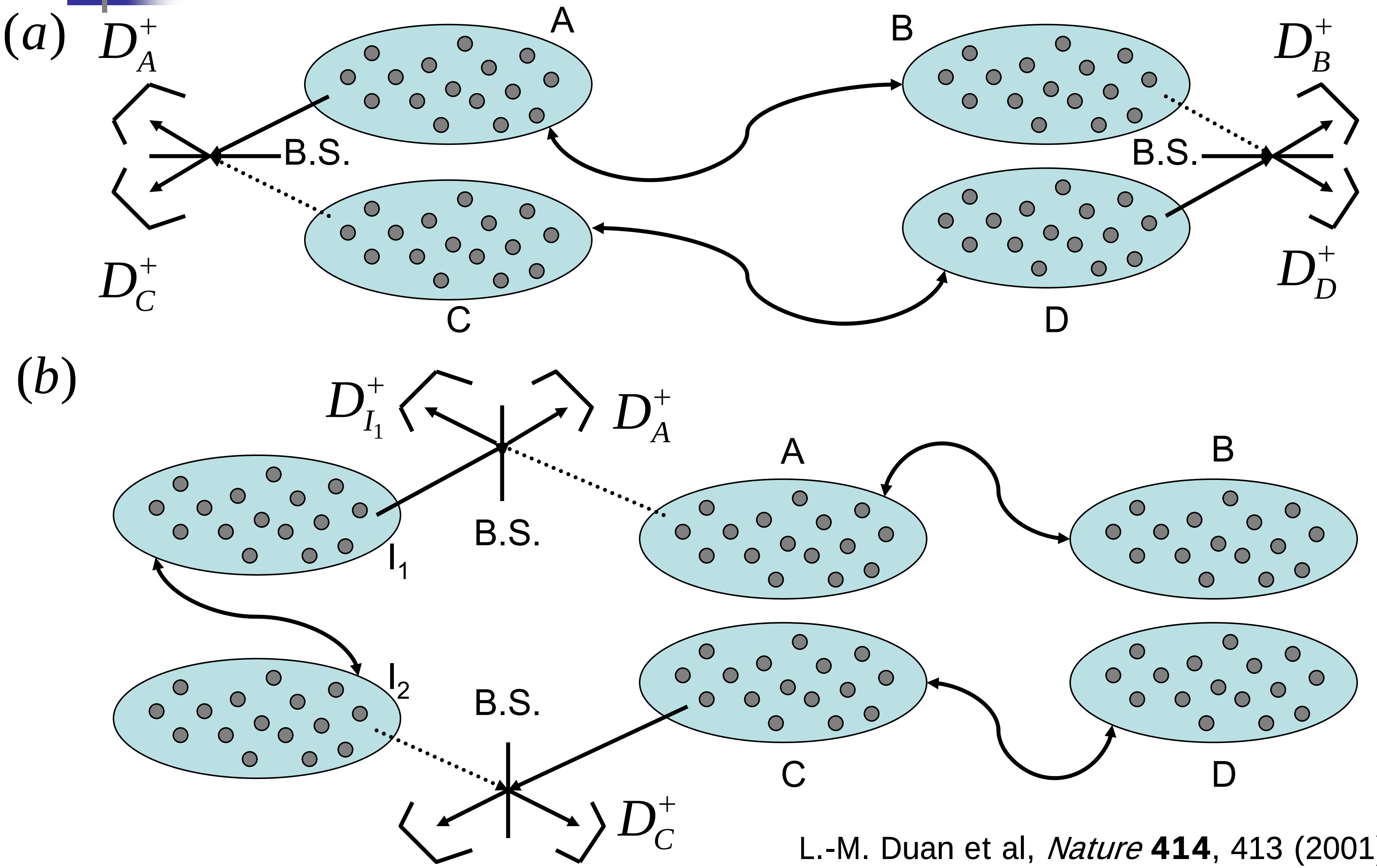
Duan-Lukin-Cirac-Zoller quantum swapping



$$|\Psi\rangle_{DLCZ} = \frac{S_A^+ + S_B^+}{\sqrt{2}} |0\rangle_{A,B}$$

L.-M. Duan et al, *Nature* **414**, 413 (2001)

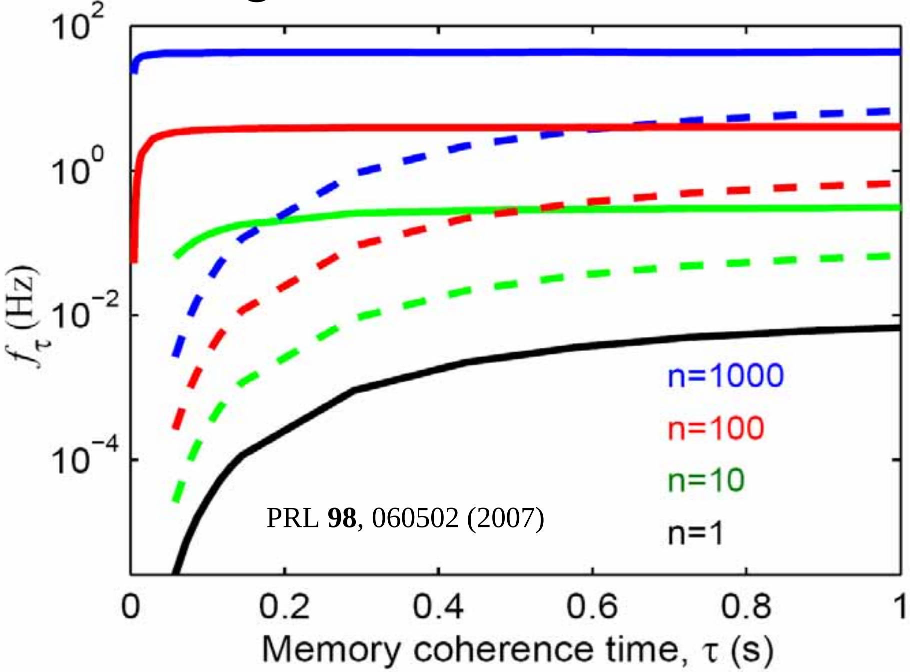
PME projection and quantum teleportation



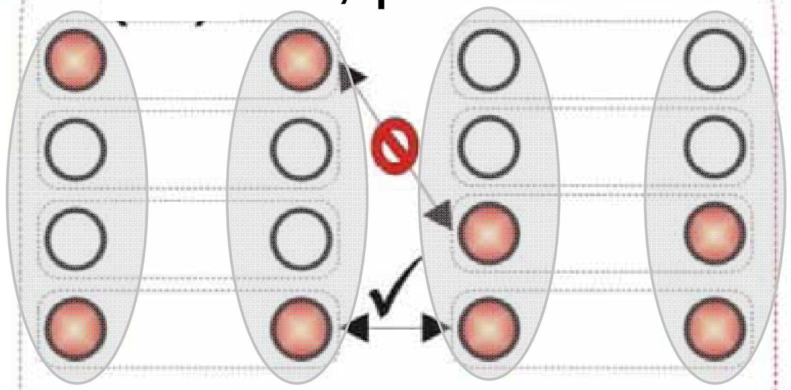
L.-M. Duan et al, *Nature* **414**, 413 (2001)

Multiplexing memory nodes

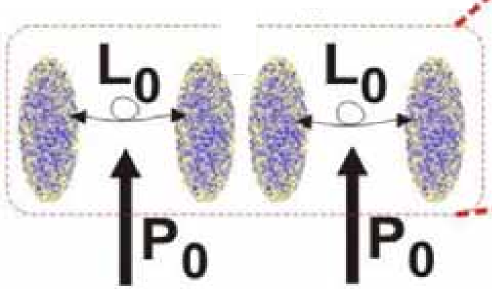
Entanglement distribution rate



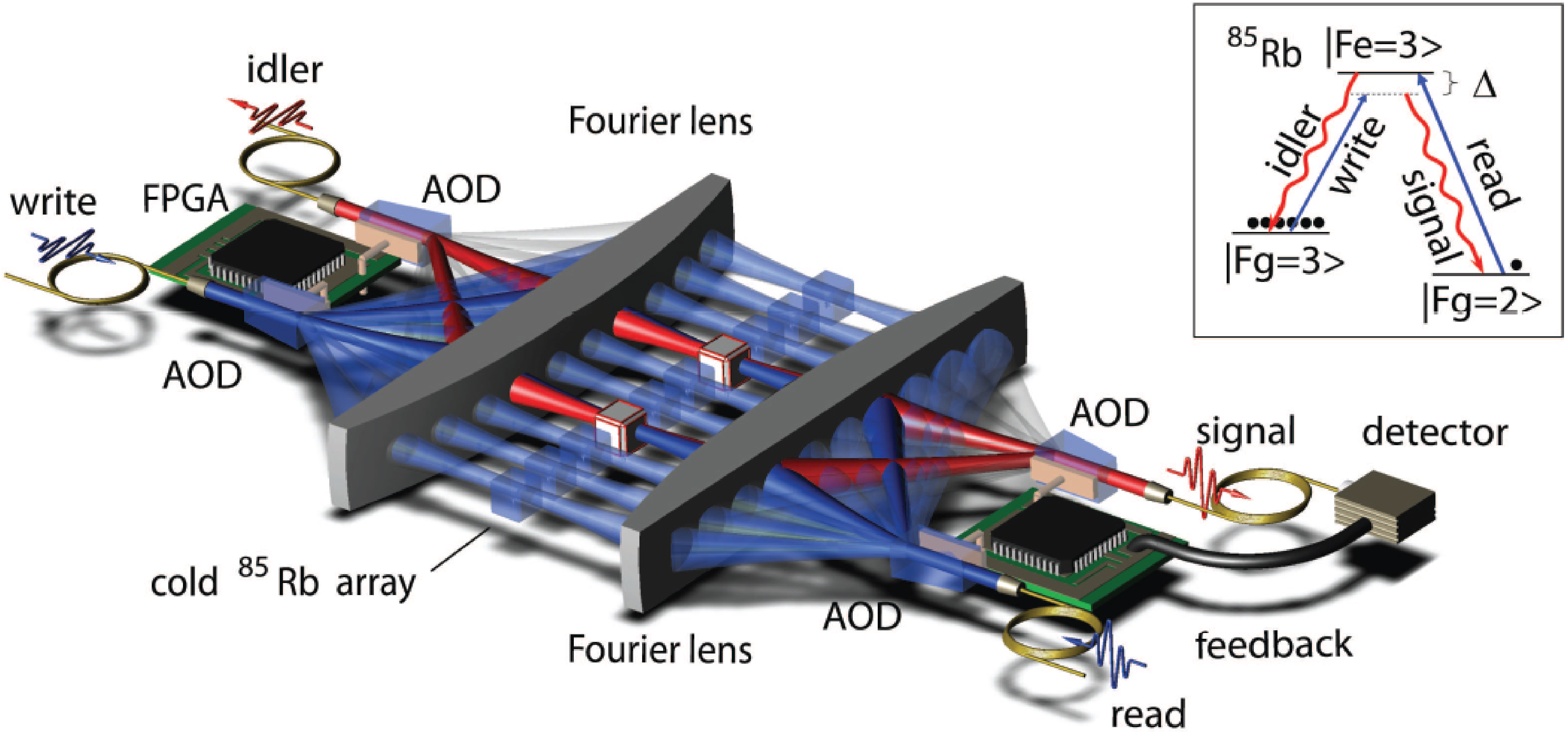
n=4, parallel



n=1

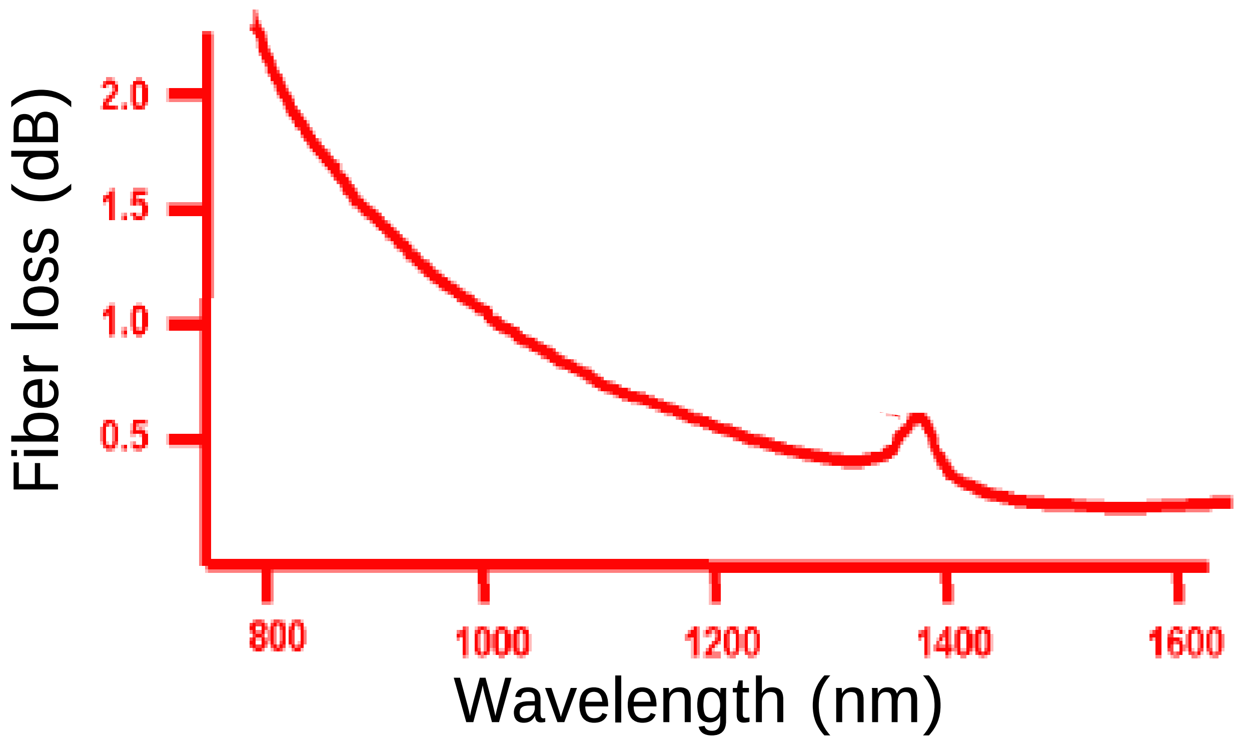


Multiplexing memory elements: experiment



S.-Y. Lan, et al, Opt. Exp., Vol. 17, No. 16, 13645 (2009)

Attenuation for fiber transmission



Atomic Cascade Transition

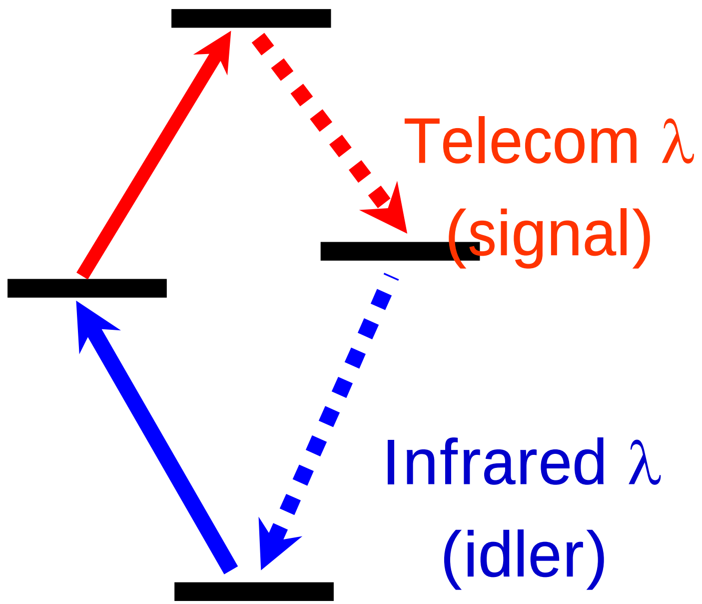
Quantum telecommunication using atomic cascade transitions

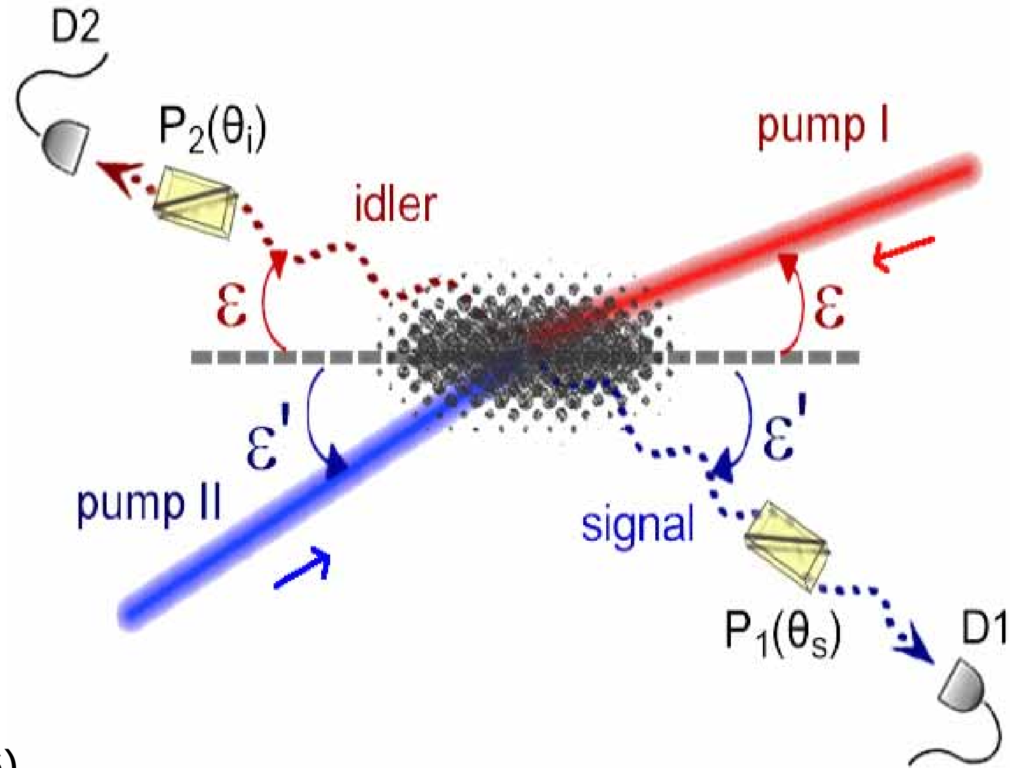
(1) Two-photon excitation of alkalis gives telecom λ,

(2) Two photons are polarization entangled,

(3) Storage of the idler then gives telecom-matter entanglement.

1.32 mm	6 $^2S_{1/2}$ to 5 $^2P_{1/2}$
1.37 mm	6 $^2S_{1/2}$ to 5 $^2P_{3/2}$
1.48 mm	4 $^2D_{3/2}$ to 5 $^2P_{1/2}$
1.53 mm	4 $^2D_{5/2}$ to 5 $^2P_{3/2}$

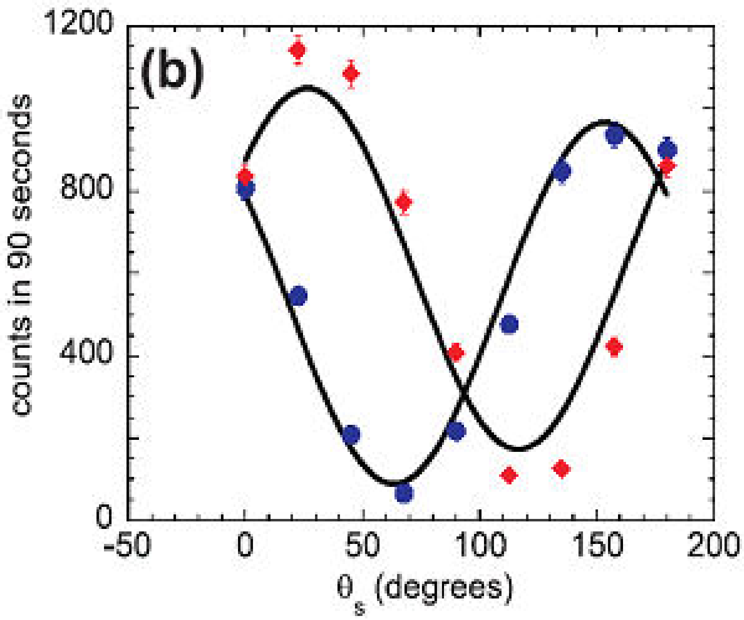




T. Chanelière, et al., *PRL* **96**, 093604 (2006)

Quantum correlations and superradiance

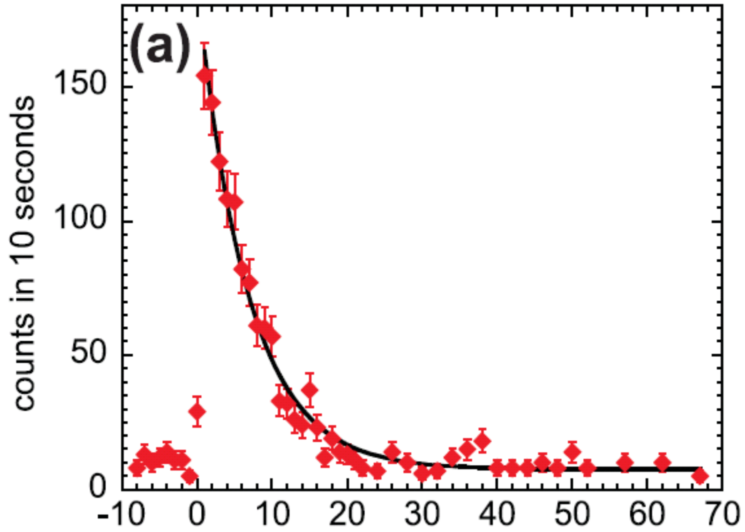
- Polarization correlations

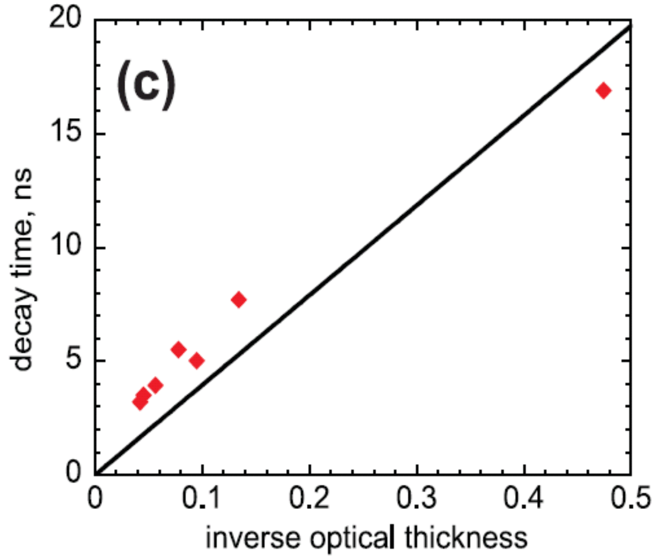


Bell inequality violation:

$$S_{\text{exp}} = 2.132 \pm 0.036 \not\leq 2$$

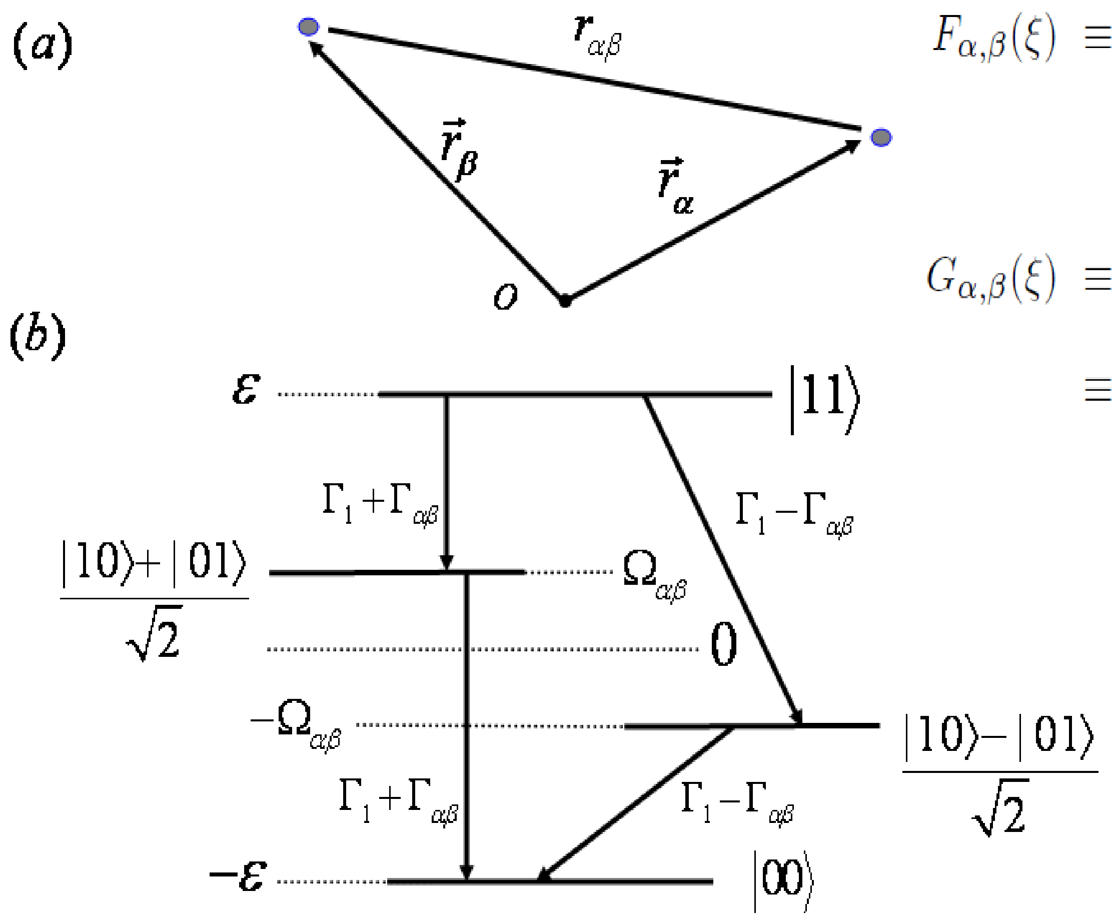
T. Chanelière, et al., *PRL* **96**, 093604 (2006)





(776 nm-780 nm cascade)

Superradiance - Dipole-dipole interaction



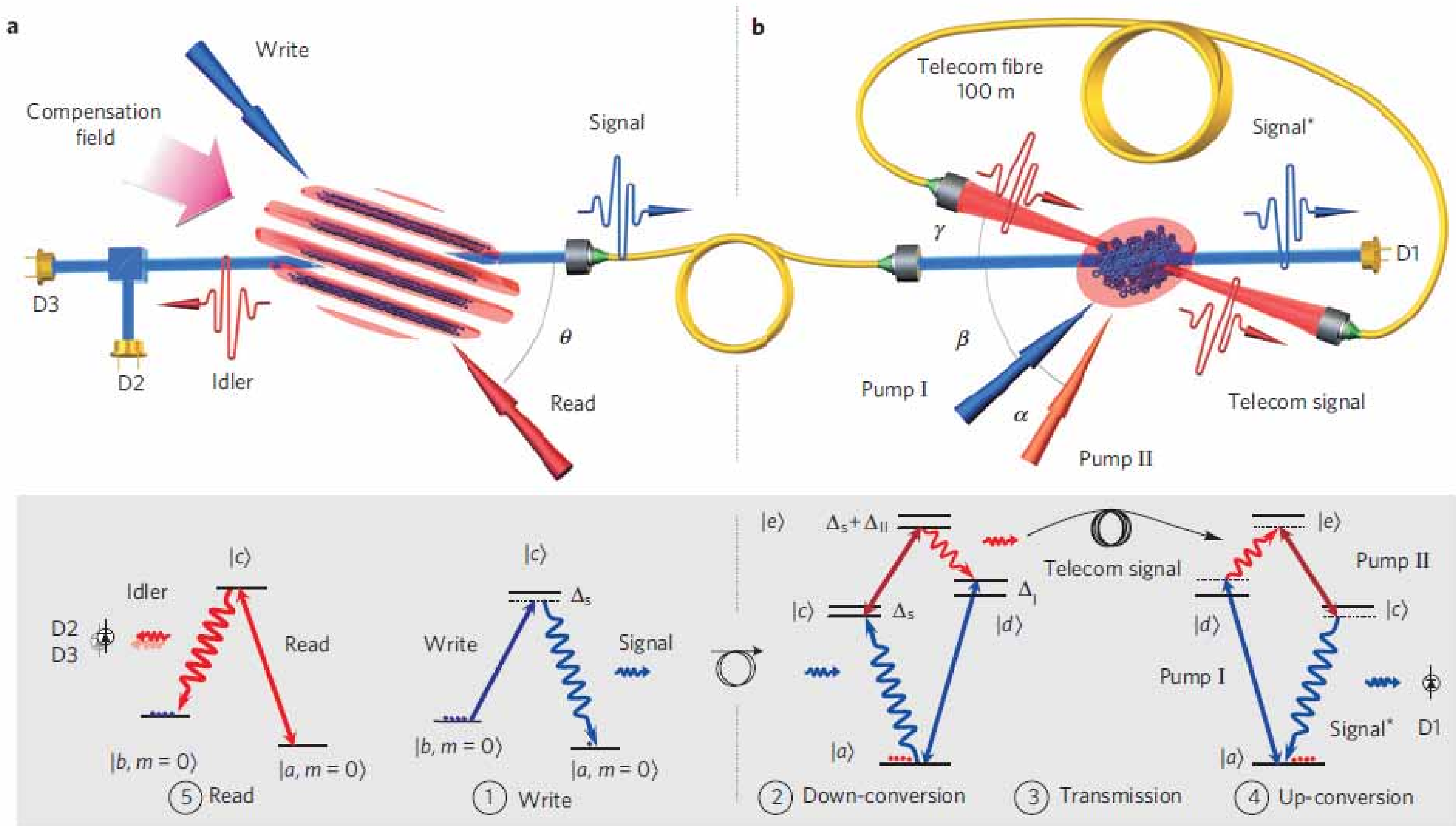
$$F_{\alpha,\beta}(\xi) \equiv \frac{3}{2}\Big\{[1-(\hat{p}\cdot\hat{r}_{\alpha\beta})^2]\frac{sin\xi}{\xi} + [1-3(\hat{p}\cdot\hat{r}_{\alpha\beta})^2] \\ (\frac{cos\xi}{\xi^2} - \frac{sin\xi}{\xi^3})\Big\} \qquad (A8)$$

$$G_{\alpha,\beta}(\xi) \equiv \Omega_{\alpha\beta}/\Gamma_3 \equiv -(\Omega^-_{\alpha\beta} + \Omega^+_{\alpha\beta})/\Gamma_3 \\ \equiv \frac{3}{4}\Big\{-[1-(\hat{p}\cdot\hat{r}_{\alpha\beta})^2]\frac{cos\xi}{\xi} + [1-3(\hat{p}\cdot\hat{r}_{\alpha\beta})^2] \\ (\frac{sin\xi}{\xi^2} + \frac{cos\xi}{\xi^3})\Big\},\ \alpha \neq \beta \qquad (A9)$$

R. H. Dicke, Phys. Rev. 93, 99 (1954); R. H. Lehmberg, PRA 2, 883 (1970)

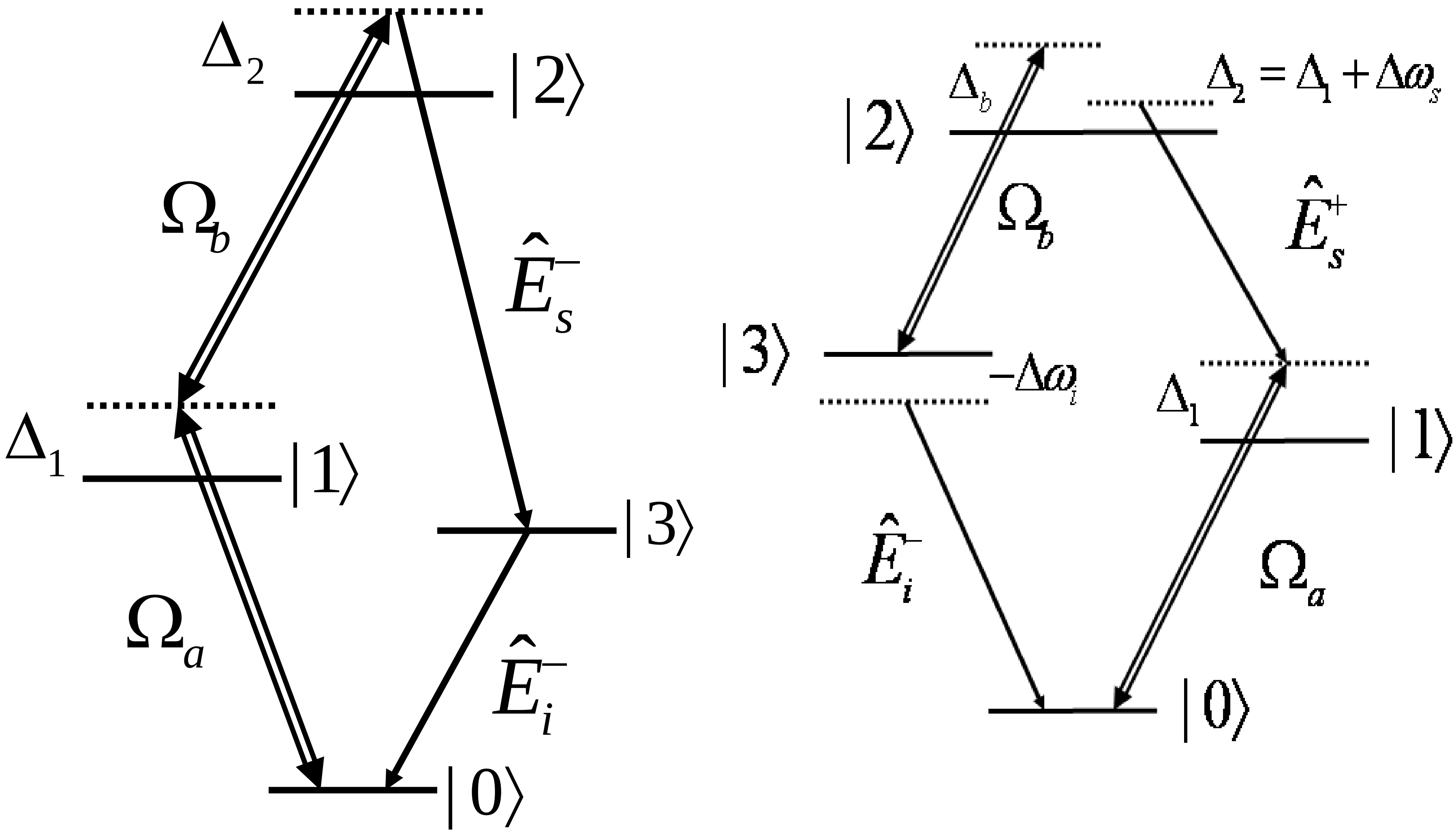
Telecom wavelength conversion

Quantum memory with telecom-wavelength conversion



A. G. Radnaev, et al., *Nature Physics* 6, 894 (2010)

Cascade: noise-initiated emission Vs Diamond: input-output



Hamiltonian and field quantization

$$\hat{H} = \hat{H}_0 + \hat{H}_I, \quad \hat{H}_I = -\vec{d}\cdot\vec{E},$$

$$\begin{aligned}\hat{H}_0 \;=\; & \sum_{i=1}^{3}\sum_{l=-M}^{M}\hbar\omega_i\hat{\sigma}_{ii}^{l} + \hbar\omega_s\sum_{l=-M}^{M}\hat{a}_{s,l}^{\dagger}\hat{a}_{s,l} + \hbar\sum_{l,l'}\omega_{ll'}\hat{a}_{s,l}^{\dagger}\hat{a}_{s,l'} \\ & +\hbar\omega_i\sum_{l=-M}^{M}\hat{a}_{i,l}^{\dagger}\hat{a}_{i,l} + \hbar\sum_{l,l'}\omega_{ll'}\hat{a}_{i,l}^{\dagger}\hat{a}_{i,l'},\end{aligned}$$

$$\begin{aligned}\hat{H}_I = -\hbar\sum_{l=-M}^{M}\Big\{ & \Omega_a(t)\hat{\sigma}_{01}^{l\dagger}e^{ik_a z_l - i\omega_a t} + \Omega_b(t)\hat{\sigma}_{32}^{l\dagger}e^{-ik_b z_l - i\omega_b t} \\ & + g_s\sqrt{2M+1}\hat{\sigma}_{12}^{l\dagger}\hat{a}_{s,l}e^{-ik_s z_l} + g_i\sqrt{2M+1}\hat{\sigma}_{03}^{l\dagger}\hat{a}_{i,l}e^{ik_i z_l} + h.c.\Big\}\end{aligned}$$

Maxwell-Bloch equations: atomic part

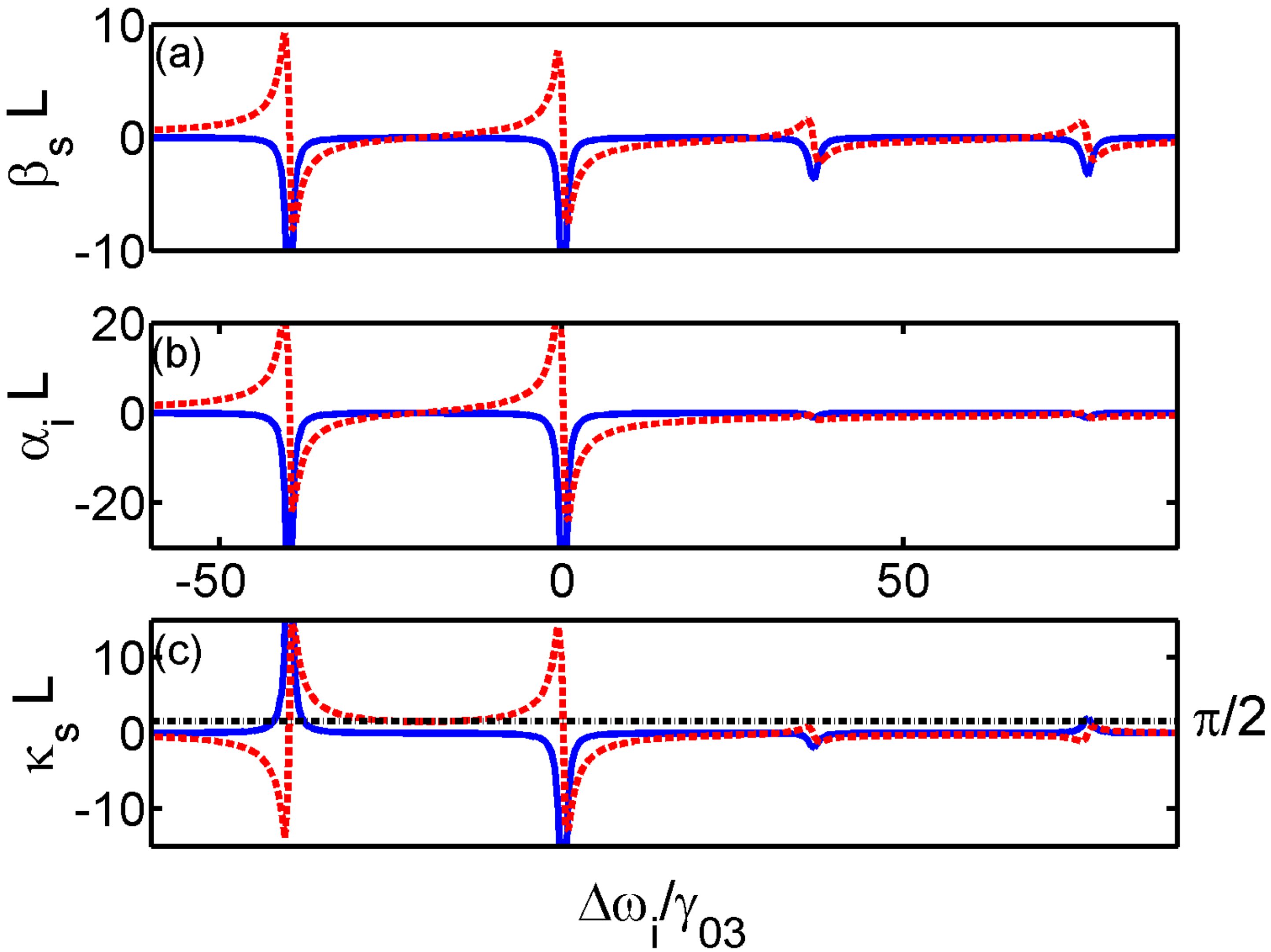
$$
\begin{aligned}
\frac{\partial}{\partial\tau}\tilde{\sigma}_{01} &= (i\Delta_1 - \frac{\gamma_{01}}{2})\tilde{\sigma}_{01} + i\Omega_a(\tilde{\sigma}_{00} - \tilde{\sigma}_{11}) + ig_s^*\tilde{\sigma}_{02}E_s^- - ig_i\tilde{\sigma}_{13}^\dagger E_i^+, \\
\frac{\partial}{\partial\tau}\tilde{\sigma}_{12} &= (i\Delta\omega_s - \frac{\gamma_{01}+\gamma_2}{2})\tilde{\sigma}_{12} - i\Omega_a^*\tilde{\sigma}_{02} + ig_s(\tilde{\sigma}_{11} - \tilde{\sigma}_{22})E_s^+ + iP^*\Omega_b\tilde{\sigma}_{13}, \\
\frac{\partial}{\partial\tau}\tilde{\sigma}_{02} &= (i\Delta_2 - \frac{\gamma_2}{2})\tilde{\sigma}_{02} - i\tilde{\sigma}_{12}\Omega_a + ig_s\tilde{\sigma}_{01}E_s^+ + iP^*\tilde{\sigma}_{03}\Omega_b - iP^*g_i\tilde{\sigma}_{32}E_i^+, \\
\frac{\partial}{\partial\tau}\tilde{\sigma}_{11} &= -\gamma_{01}\tilde{\sigma}_{11} + \gamma_{12}\tilde{\sigma}_{22} + i\Omega_a\tilde{\sigma}_{01}^\dagger - i\Omega_a^*\tilde{\sigma}_{01} - ig_s\tilde{\sigma}_{12}^\dagger E_s^+ + ig_s^*\tilde{\sigma}_{12}E_s^-, \\
\frac{\partial}{\partial\tau}\tilde{\sigma}_{22} &= -\gamma_2\tilde{\sigma}_{22} + ig_s\tilde{\sigma}_{12}^\dagger E_s^+ - ig_s^*\tilde{\sigma}_{12}E_s^- + i\Omega_b\tilde{\sigma}_{32}^\dagger - i\Omega_b^*\tilde{\sigma}_{32}, \\
\frac{\partial}{\partial\tau}\tilde{\sigma}_{33} &= -\gamma_{03}\tilde{\sigma}_{33} + \gamma_{32}\tilde{\sigma}_{22} - i\Omega_b\tilde{\sigma}_{32}^\dagger + i\Omega_b^*\tilde{\sigma}_{32} + ig_i\tilde{\sigma}_{03}^\dagger E_i^+ - ig_i^*\tilde{\sigma}_{03}E_i^-, \\
\frac{\partial}{\partial\tau}\tilde{\sigma}_{13} &= (i\Delta\omega_i - i\Delta_1 - \frac{\gamma_{01}+\gamma_{03}}{2})\tilde{\sigma}_{13} - i\Omega_a^*\tilde{\sigma}_{03} - iPg_s\tilde{\sigma}_{32}^\dagger E_s^+ + iP\Omega_b^*\tilde{\sigma}_{12} \\
&\quad + ig_i\tilde{\sigma}_{01}^\dagger E_i^+, \\
\frac{\partial}{\partial\tau}\tilde{\sigma}_{03} &= (i\Delta\omega_i - \frac{\gamma_{03}}{2})\tilde{\sigma}_{03} - i\Omega_a\tilde{\sigma}_{13} + iP\Omega_b^*\tilde{\sigma}_{02} + ig_i(\tilde{\sigma}_{00} - \tilde{\sigma}_{33})E_i^+, \\
\frac{\partial}{\partial\tau}\tilde{\sigma}_{32}^\dagger &= (-i\Delta_b - \frac{\gamma_{03}+\gamma_2}{2})\tilde{\sigma}_{32}^\dagger - iP^*g_s^*\tilde{\sigma}_{13}E_s^- + i\Omega_b^*(\tilde{\sigma}_{22} - \tilde{\sigma}_{33}) + iP^*g_i\tilde{\sigma}_{02}^\dagger E_i^+
\end{aligned}
$$

Maxwell-Bloch equations: field part

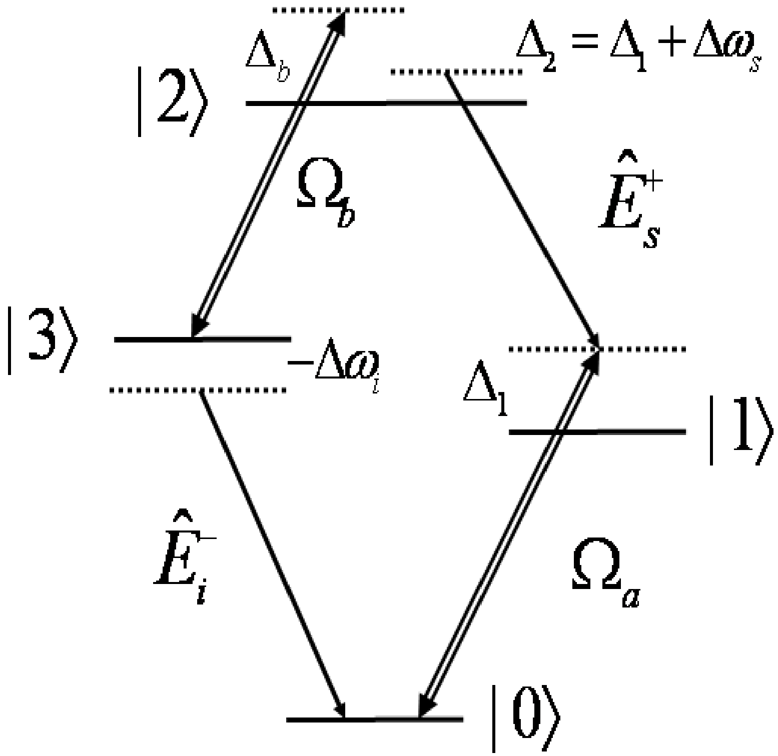
$$\frac{\partial}{\partial z}E_s^+ = \beta_s E_s^+ + \kappa_s E_i^+$$
$$\frac{\partial}{\partial z}E_i^+ = \kappa_i E_s^+ + \alpha_i E_i^+.$$

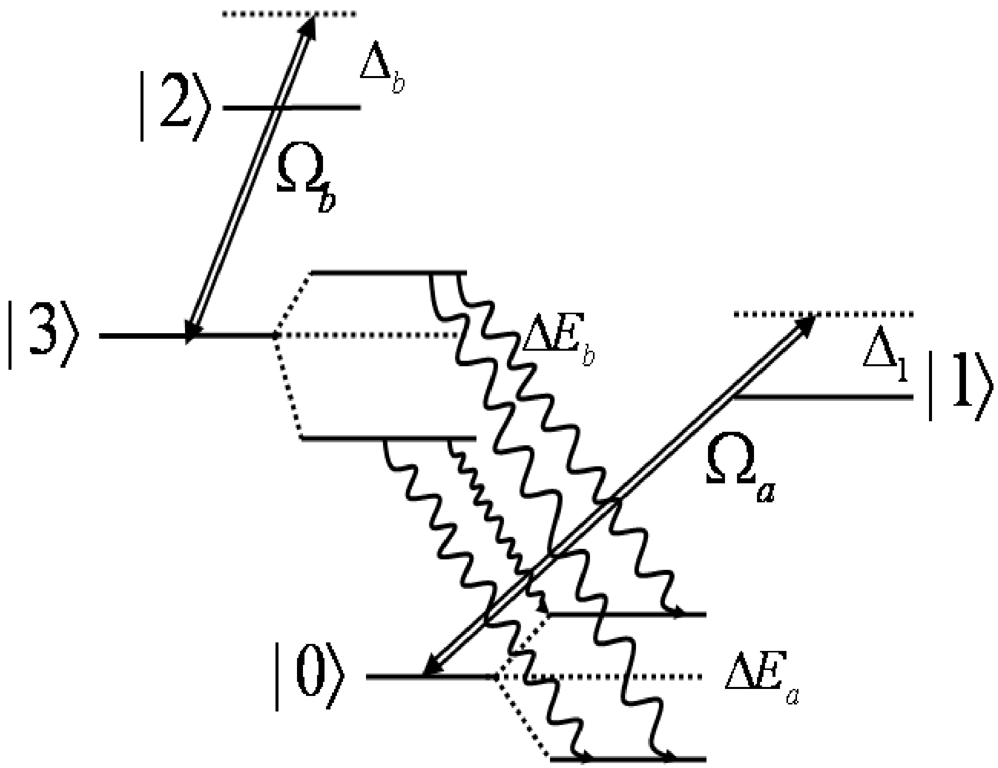
$$\eta_{\mathrm{d}} = \left|\frac{E_s^+(L)}{E_i^+(0)}\right|^2 = \left|\frac{\kappa_s}{2w}e^{(\alpha_i+\beta_s)L/2}(e^{wL} - e^{-wL})\right|^2$$
$$T_{\mathrm{d}} = \left|\frac{E_i^+(L)}{E_i^+(0)}\right|^2 = \left|\frac{e^{(\alpha_i+\beta_s)L/2}}{2w(w+q)}[\kappa_s\kappa_i e^{wL} + (q+w)^2 e^{-wL}]\right|^2.$$

Self- and cross-coupling coefficients



Diamond configuration: dressed state picture



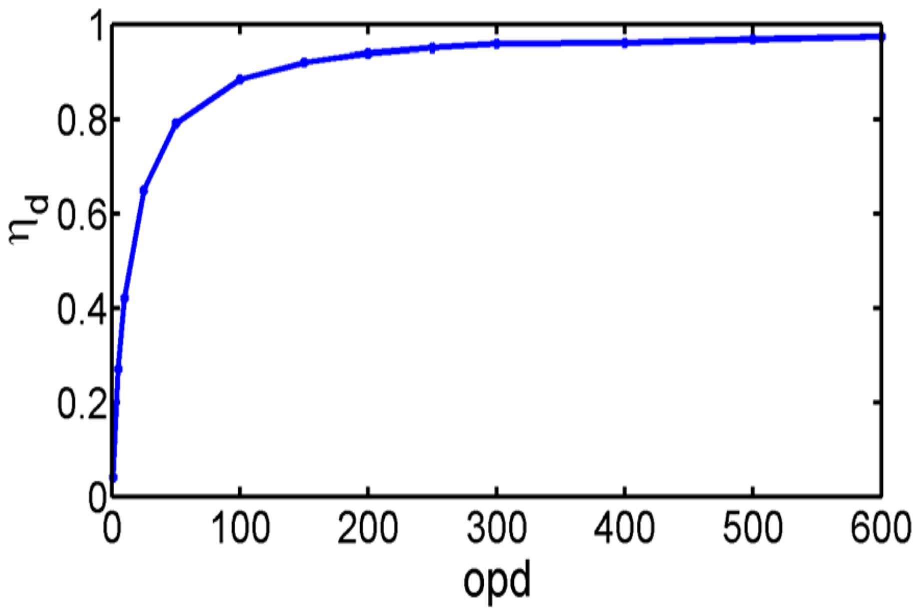


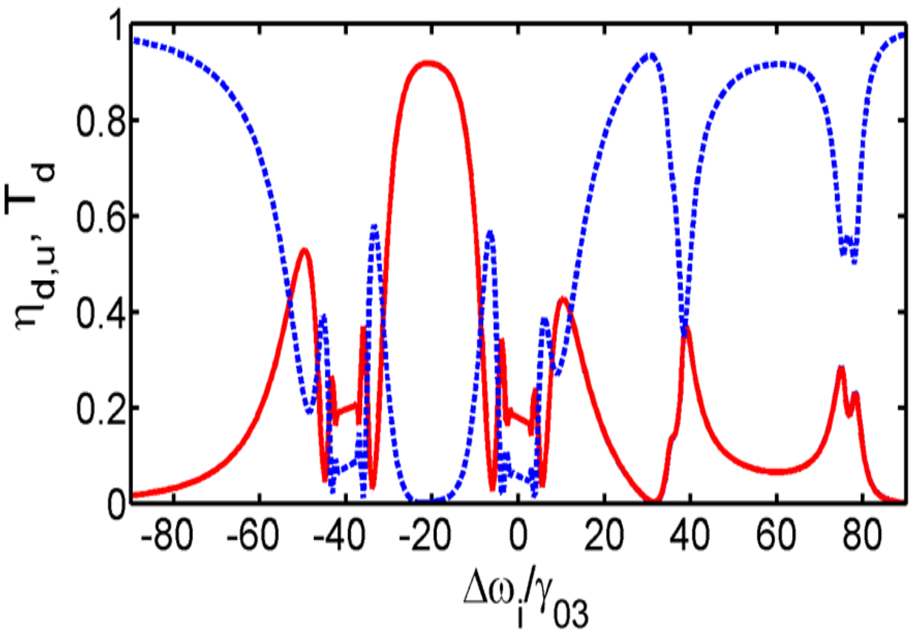
Optimal frequency conversion

Five variational parameters:

$$\Omega_a, \Omega_b, \Delta_1, \Delta_b, \Delta\omega_i.$$

$\mathrm{opd} = 150$





H. H. Jen and T. A. B. Kennedy, *PRA* 82, 023815 (2010)

Pulse conversion: numerical solution

Spatial and temporal integrations:

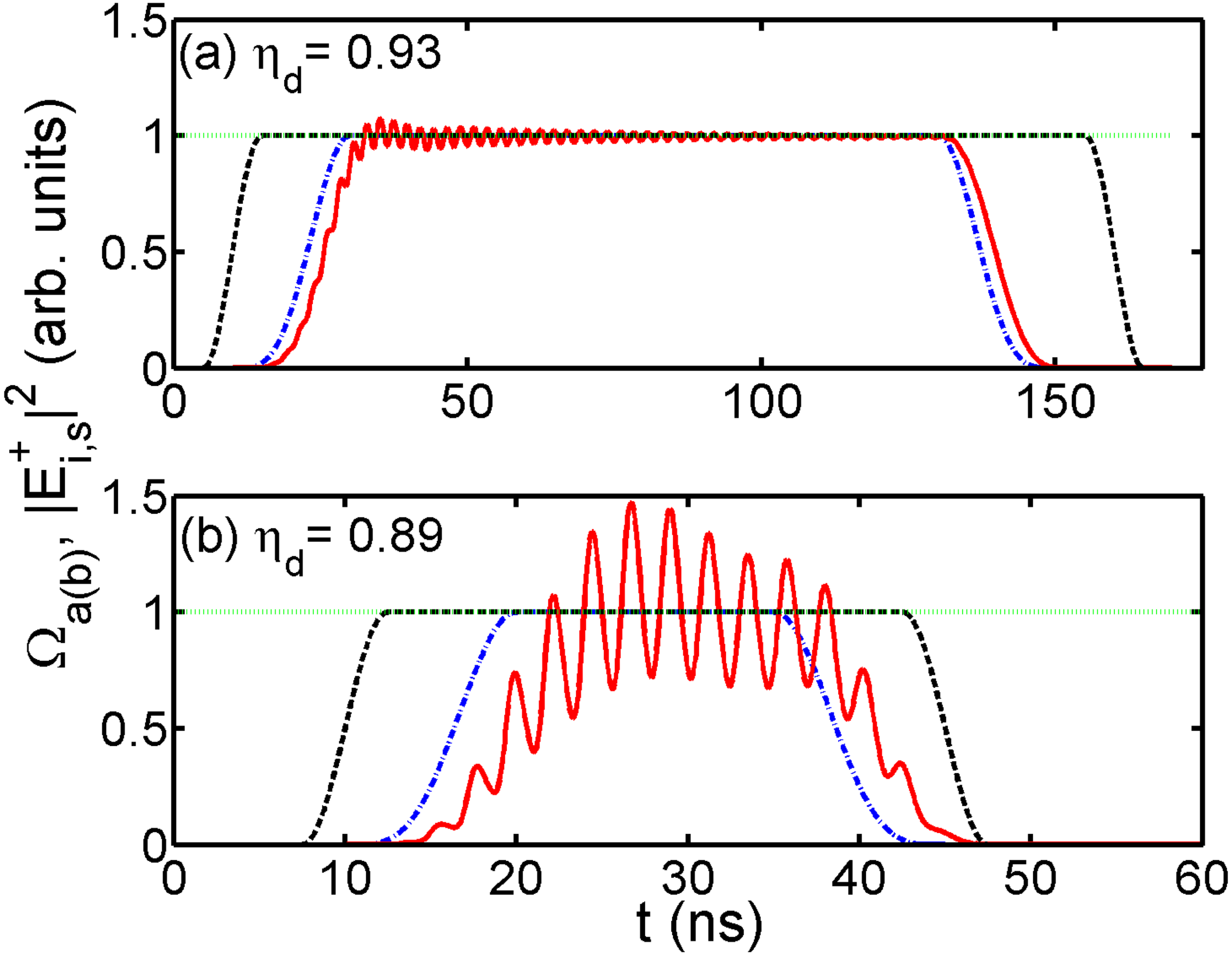
semi-implicit difference (midpoint) method.

Atomic ensemble in experiment:

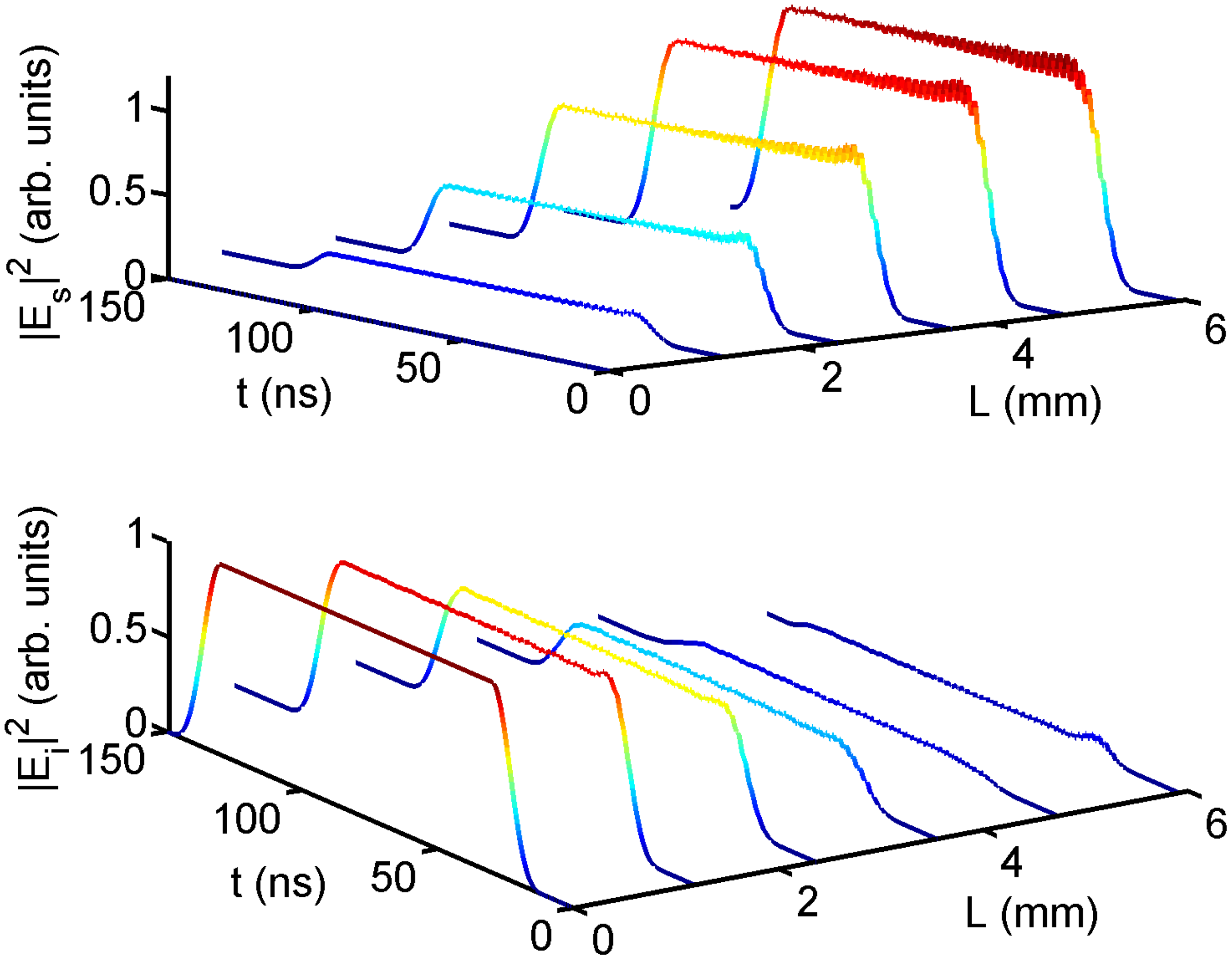
$\rho = 1.7 \times 10^{11}\mathrm{cm}^{-3}, L = 6\mathrm{mm}.$

$$\eta_{\mathrm{d}} = \frac{\int |E_s^+(z = L, \tau)|^2 d\tau}{\int |E_i^+(z = 0, \tau)|^2 d\tau}.$$

Down-converted pulse conversion



Down-converted pulse conversion



$$\eta_d = 0.93,$$

$$\text{opd} = 150.$$

Heisenberg-Langevin equations: quantum fluctuations

$$\frac{\partial}{\partial z}E_s^+ = \beta_s E_s^+ + \kappa_s E_i^+ + \hat{f}_s, \quad \left\langle \tilde{\mathcal{F}}_i^\dagger(t,z) \right\rangle = \left\langle \tilde{\mathcal{F}}_i(t',z') \right\rangle = 0$$

$$\frac{\partial}{\partial z}E_i^+ = \kappa_i E_s^+ + \alpha_i E_i^+ + \hat{f}_i \quad \left\langle \tilde{\mathcal{F}}_i^\dagger(t,z)\tilde{\mathcal{F}}_j(t',z') \right\rangle = \tfrac{L}{N}\delta(t-t')\delta(z-z')\hat{D}_{ij}$$

$$\begin{aligned}\hat{f}_s &= \frac{iNg_s^*}{cD}[(T_{03}+\frac{|\Omega_a|^2}{T_{13}}+\frac{|\Omega_b|^2}{T_{02}})\tilde{\mathcal{F}}_{12}+(\frac{\Omega_a^*\Omega_b}{T_{02}}+\frac{\Omega_a^*\Omega_b}{T_{13}})\tilde{\mathcal{F}}_{03} \\ &+\frac{i\Omega_b}{T_{13}}(T_{03}+\frac{|\Omega_b|^2-|\Omega_a|^2}{T_{02}})\tilde{\mathcal{F}}_{13}+\frac{i\Omega_a^*}{T_{02}}(-T_{03}+\frac{|\Omega_b|^2-|\Omega_a|^2}{T_{13}})\tilde{\mathcal{F}}_{02}]+\tilde{\mathcal{F}}_s,\end{aligned} \tag{6.21}$$

$$\begin{aligned}\hat{f}_i &= \frac{iNg_i^*}{cD}[(\frac{\Omega_a\Omega_b^*}{T_{02}}+\frac{\Omega_a\Omega_b^*}{T_{13}})\tilde{\mathcal{F}}_{12}+(T_{12}+\frac{|\Omega_a|^2}{T_{02}}+\frac{|\Omega_b|^2}{T_{13}})\tilde{\mathcal{F}}_{03} \\ &+\frac{i\Omega_a}{T_{13}}(-T_{12}+\frac{|\Omega_b|^2-|\Omega_a|^2}{T_{02}})\tilde{\mathcal{F}}_{13}+\frac{i\Omega_a^*}{T_{02}}(T_{12}+\frac{|\Omega_b|^2-|\Omega_a|^2}{T_{13}})\tilde{\mathcal{F}}_{02}]+\tilde{\mathcal{F}}_i\end{aligned} \tag{6.22}$$

Quantum fluctuations

$$
\begin{aligned}
\text{(i)}\ \hat{D}_{12,12} &= \gamma_{01}\langle\tilde{\sigma}_{22}\rangle \approx \gamma_{01}\tilde{\sigma}_{22,s} = 0; \\
\hat{D}_{12,03} &= \hat{D}_{12,02} = 0; \\
\hat{D}_{12,13} &= \gamma_{01}\langle\tilde{\sigma}_{23}\rangle \approx \gamma_{01}\tilde{\sigma}_{23,s} = 0; \\
\hat{D}_{12,s} &= \hat{D}_{12,i} = 0; && (6.34) \\
\text{(ii)}\ \hat{D}_{03,03} &= \gamma_{32}\langle\tilde{\sigma}_{22}\rangle \approx \gamma_{32}\tilde{\sigma}_{22,s} = 0; \\
\hat{D}_{03,02} &= \hat{D}_{03,13} = 0; \\
\hat{D}_{03,s} &= \hat{D}_{03,i} = 0; && (6.35) \\
\text{(iii)}\ \hat{D}_{02,02} &= \hat{D}_{02,13} = 0; \\
\hat{D}_{02,s} &= \hat{D}_{02,i} = 0; && (6.36) \\
\text{(iV)}\ \hat{D}_{13,13} &= \gamma_{01}\langle\tilde{\sigma}_{33}\rangle + \gamma_{32}\langle\tilde{\sigma}_{22}\rangle \approx \gamma_{01}\tilde{\sigma}_{33,s} + \gamma_{32}\tilde{\sigma}_{22,s} = 0; \\
\hat{D}_{13,s} &= \hat{D}_{13,i} = 0; && (6.37) \\
\text{(V)}\ \hat{D}_{s,s} &= \hat{D}_{s,i} = 0; && (6.38) \\
\text{(Vi)}\ \hat{D}_{i,i} &= 0; && (6.39)
\end{aligned}
$$

Outlook and conclusion

- Multi-particle entanglement. Quantum channel capacity.
- Long distance quantum communication.
- Advanced architecture for quantum computation.
- Fidelity and efficiency.
- Collective excitations via a Rydberg blockade mechanism.
- Hybrid systems.
- A demonstration of quantum correlation with telecom wavelength conversion, which provides low-loss quantum network communication.

Thank you for your attention!