AMO 2011
Introduction to bright matter-wave solitons: Formation, dynamics and open questions

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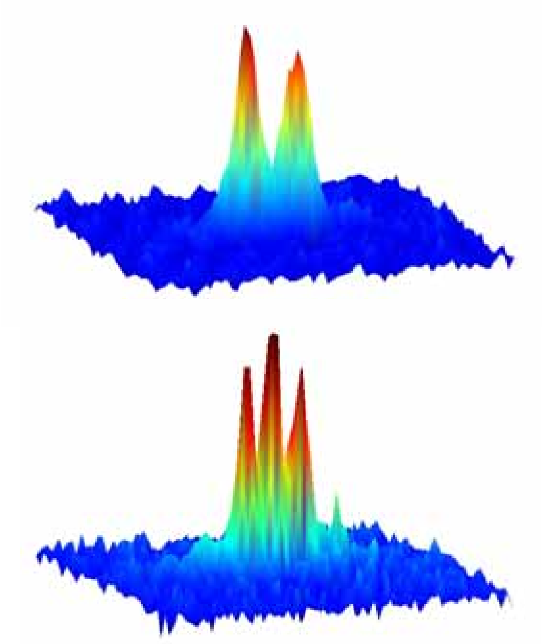

Durham University

EPSRC
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European Science Foundation
Setting science agendas for Europe

Simon Cornish group at Durham, UK

Outline

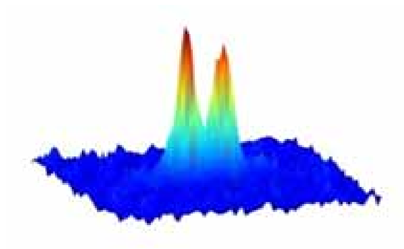
❶ Introduction to solitons
- *Necessary background*

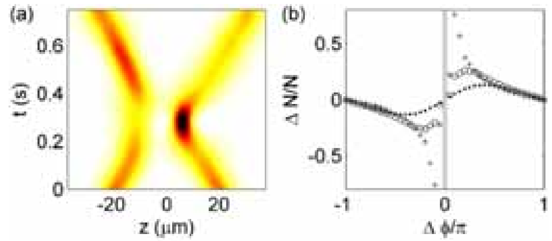
❷ BEC collapse & matter-wave solitons
- *Basic physics*
- *The past experiments*

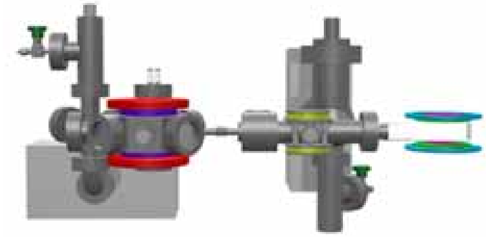
❸ Soliton dynamics (theory)
- *Numerical simulations*
- *Binary soliton collisions*

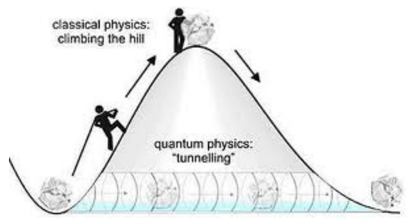
❹ Our experimental progress
- *Current status and next steps*

❺ Future goals
- *Atom surface studies*
- *Quantum reflection with solitons*



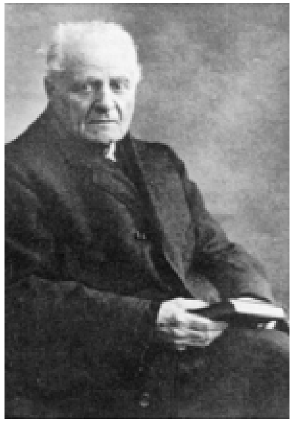


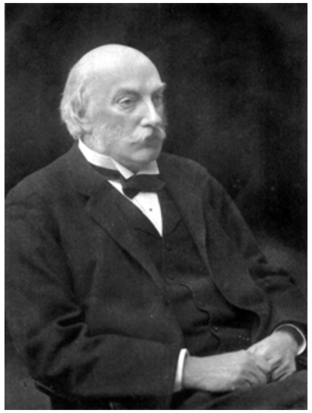




What is a soliton?

- A wave packet that preserves its shape as it propagates.
- First observation by John Russell, 1834 on Union Canal, Scotland



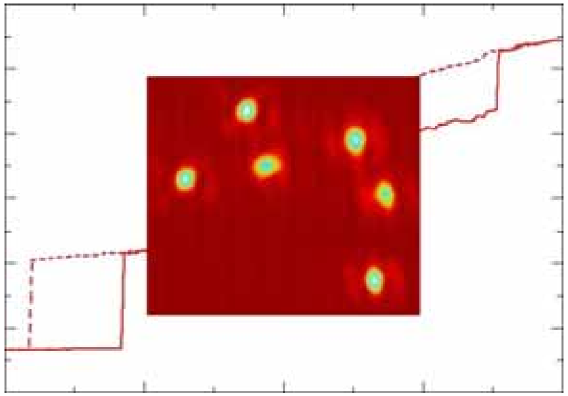


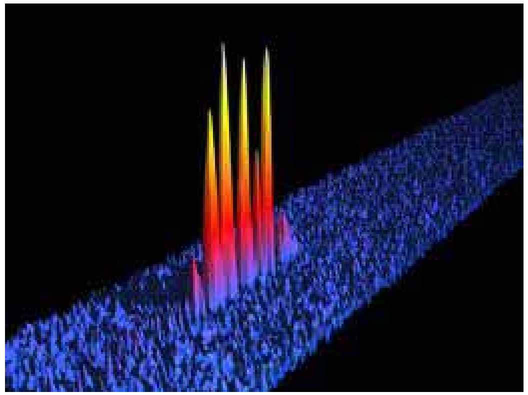
Solitons exist in many differing physical systems each with their specialised equations, eg:

- Water waves
- Coupled pendula
- Light waves
- Matter waves

Korteweg- de Vries

sine-Gordon





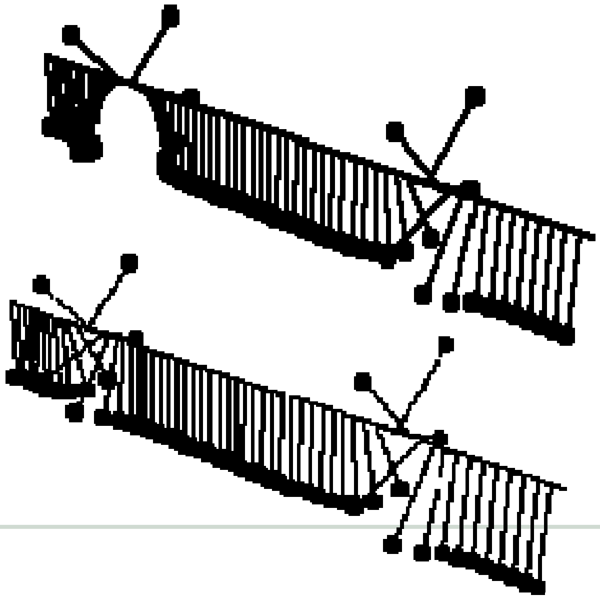
Non-linear Schrodinger (NLSE) ⇔ (GPE) Gross-Pitaevskvii

The common factor:

Dispersion (tendency to spread) is counteracted by nonlinearity (causes narrowing or self-attraction).

Coupling between these maintains shape, and sometimes size (topological solitons).

Sine-Gordon
Elastic band dispersive (flattens)
Gravity attractive (sharpens)



The GPE and NLSE equations in 1D

Gross-Pitaevskii equation (1D)

$$-\frac{\hbar^2}{2m}\frac{\partial^2\psi(z,t)}{\partial z^2}+V(z)\psi(z,t)\pm g_{1D}\left|\psi(z,t)\right|^2\psi(z,t)=i\hbar\frac{\partial\psi(z,t)}{\partial t}$$

Kinetic energy term | **External potential term** | **Nonlinear interaction term** | **Total energy**

Non-linear Schrodinger equation
(optical spatial and/or temporal solitons)

$$-\frac{1}{2}\frac{\partial^2 A(z,x)}{\partial x^2}-\left|A(z,x)\right|^2 A(z,x)=i\frac{\partial A(z,x)}{\partial z}$$

Temporal optical soliton

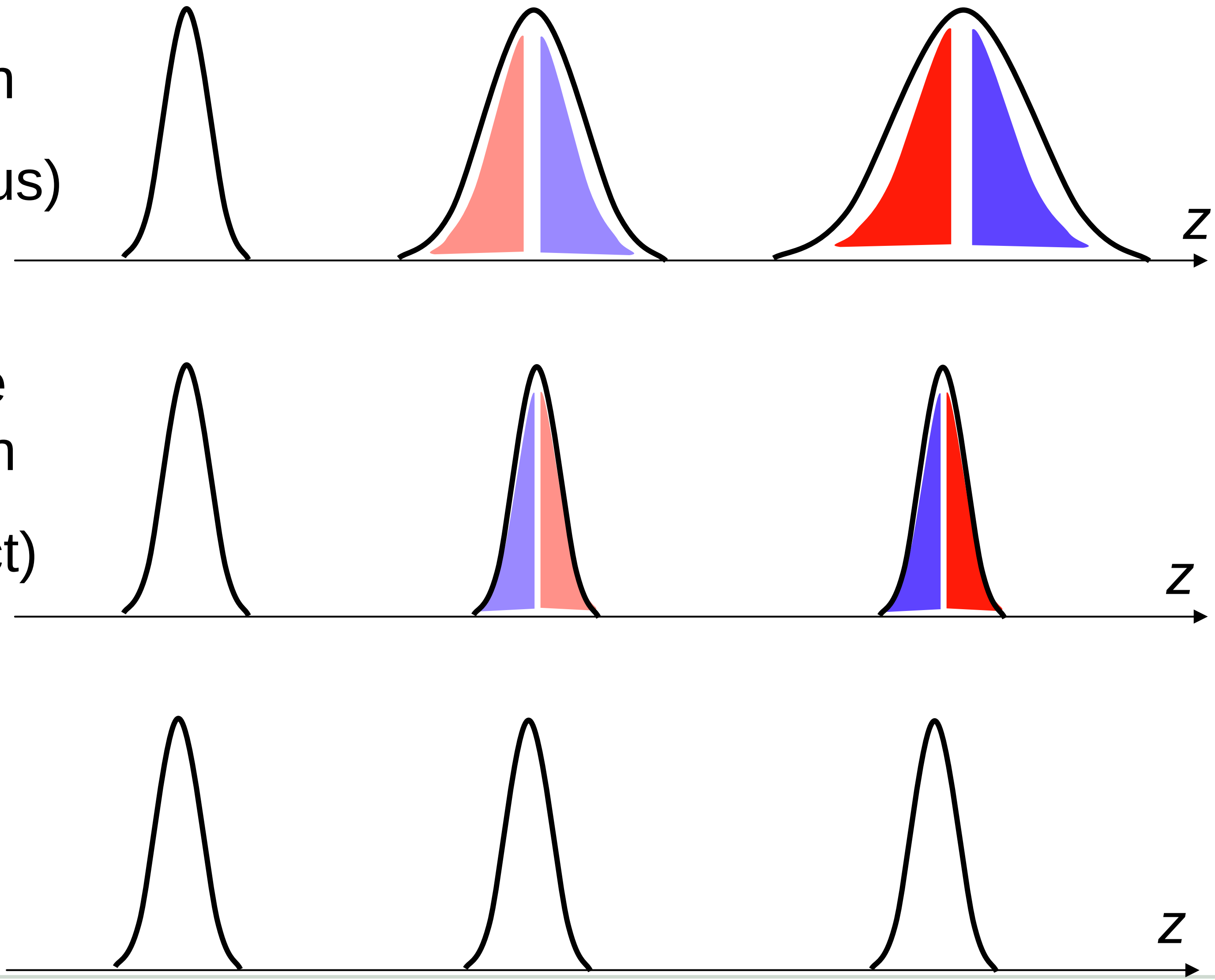
Dispersion

(anomalous)

Self phase modulation

(Kerr effect)

Temporal optical soliton



The GPE/NLSE 1d equation solutions

$$-\frac{\hbar^2}{2m}\frac{\partial^2\psi(z,t)}{\partial z^2} \pm g_{1D}\left|\psi(z,t)\right|^2\psi(z,t) = i\hbar\frac{\partial\psi(z,t)}{\partial t}$$

This 1d equation has exact solutions (Brown Pethick BEC text)

Bright, static, N=1

$$\psi(z,t) = b\,\mathrm{sech}(az)\exp(-i\mu t/\hbar)$$

$$a^2 = \frac{-2m\mu}{\hbar^2} \qquad b^2 = \frac{2\mu}{g_{1D}}$$

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$$-\frac{\hbar^2}{2m}\frac{\partial^2\psi(z,t)}{\partial z^2} \pm g_{1D}\left|\psi(z,t)\right|^2\psi(z,t) = i\hbar\frac{\partial\psi(z,t)}{\partial t}$$

This 1d equation has exact solutions (Brown Pethick BEC text)

Bright, moving, speed *u* N=1

$$\psi(z,t) = b\,\mathrm{sech}(a(z-ut))\exp(-i(\mu t - muz)/\hbar)$$

$$a^2 = \frac{-2m\left(\mu - mu^2/2\right)}{\hbar^2} \qquad b^2 = \frac{\left(\mu - mu^2/2\right)}{g_{1D}}$$

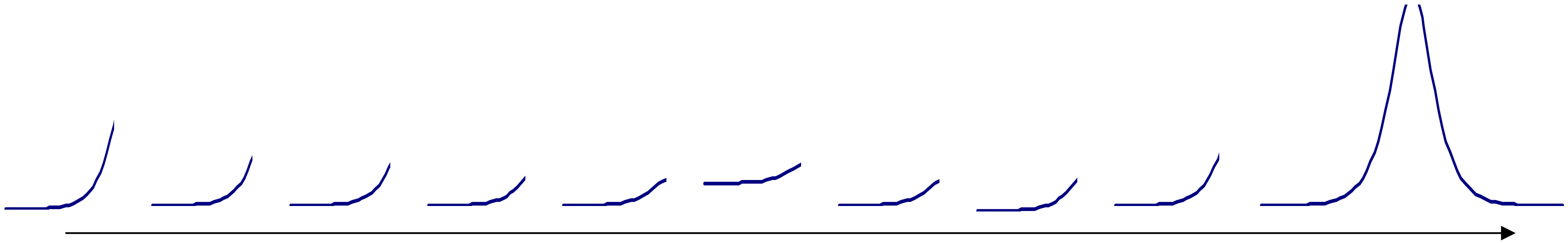
The GPE/NLSE 1d equation solutions

$$-\frac{\hbar^2}{2m}\frac{\partial^2\psi(z,t)}{\partial z^2} \pm g_{1D}\left|\psi(z,t)\right|^2\psi(z,t) = i\hbar\frac{\partial\psi(z,t)}{\partial t}$$

This 1d equation has exact solutions (Brown Pethick BEC text)

Bright,
moving,
speed *u*
N=2

$$\psi(z,t) = \text{complicated!}$$



The GPE/NLSE 1d equation solutions

$$-\frac{\hbar^2}{2m}\frac{\partial^2\psi(z,t)}{\partial z^2} \pm g_{1D}\left|\psi(z,t)\right|^2\psi(z,t) = i\hbar\frac{\partial\psi(z,t)}{\partial t}$$

This 1d equation has exact solutions (Brown Pethick BEC text)

Dark, speed *u* N=1

$$\psi(z,t) = \sqrt{n_0}\left[\frac{i\mu}{s} + \sqrt{\left(1-\frac{u^2}{s^2}\right)}\tanh\left(\frac{x-ut}{\sqrt{2}\xi}\right)\right]\exp(-i\mu t/\hbar)$$

$$s = \left(n_0 g_{1D}/m\right)^{1/2} \qquad \xi = \frac{\hbar^2}{2mng_{1D}\left(1-\left(u/s\right)^2\right)^{1/2}}$$

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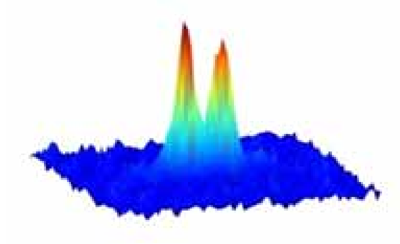
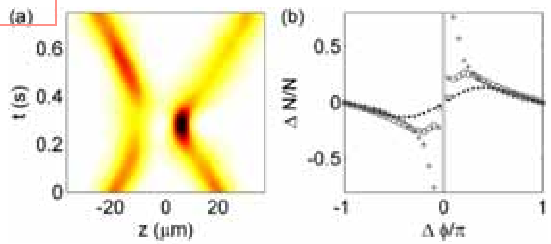
1d



❷ BEC collapse & matter-wave solitons
- *Basic physics*
- *The past experiments*

3d & quasi-1d



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- *Numerical simulations*
- *Binary soliton collisions*

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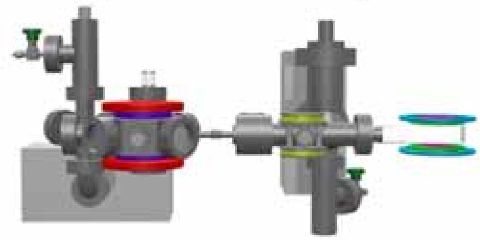
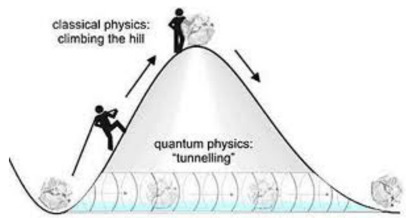


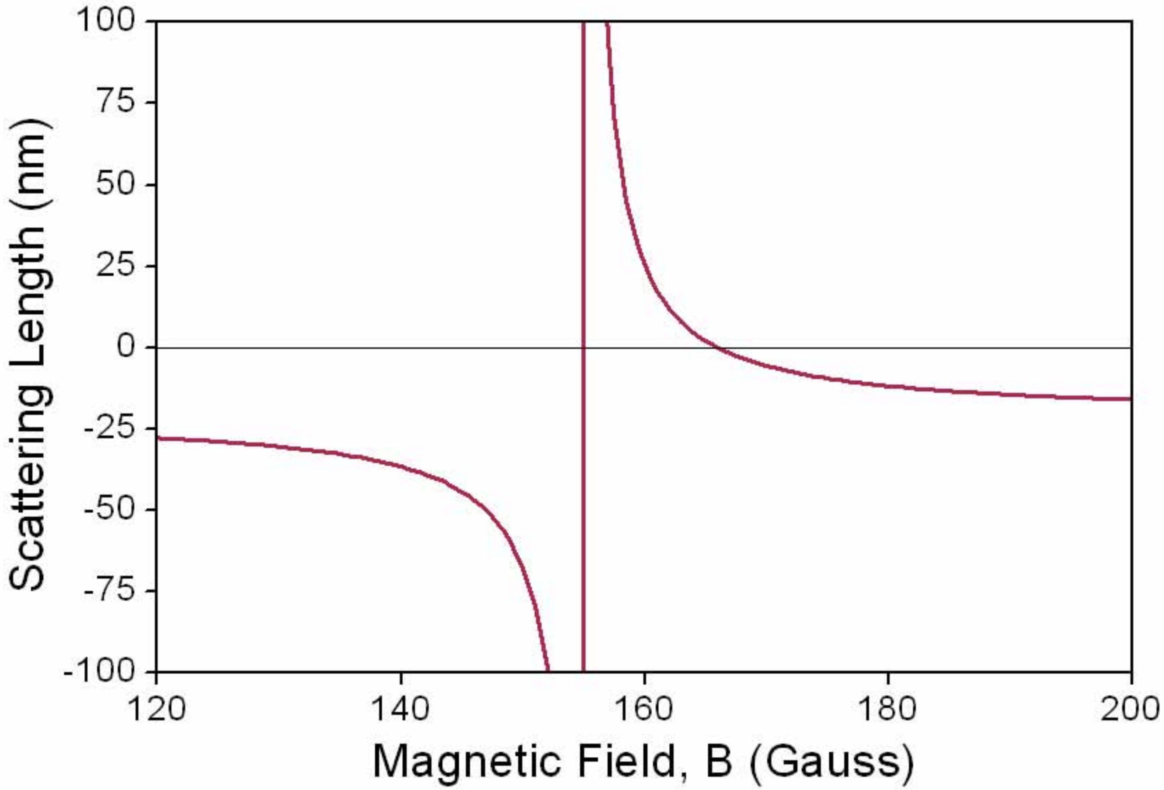
❺ Future goals
- *Atom surface studies*
- *Quantum reflection with solitons*



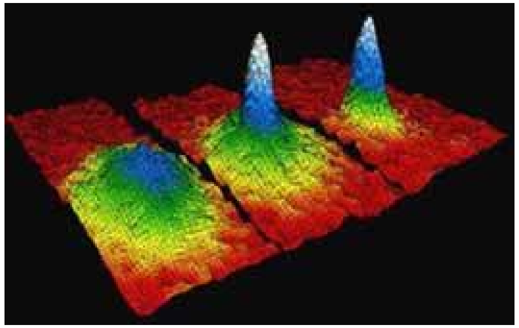
Feshbach resonances and scattering length

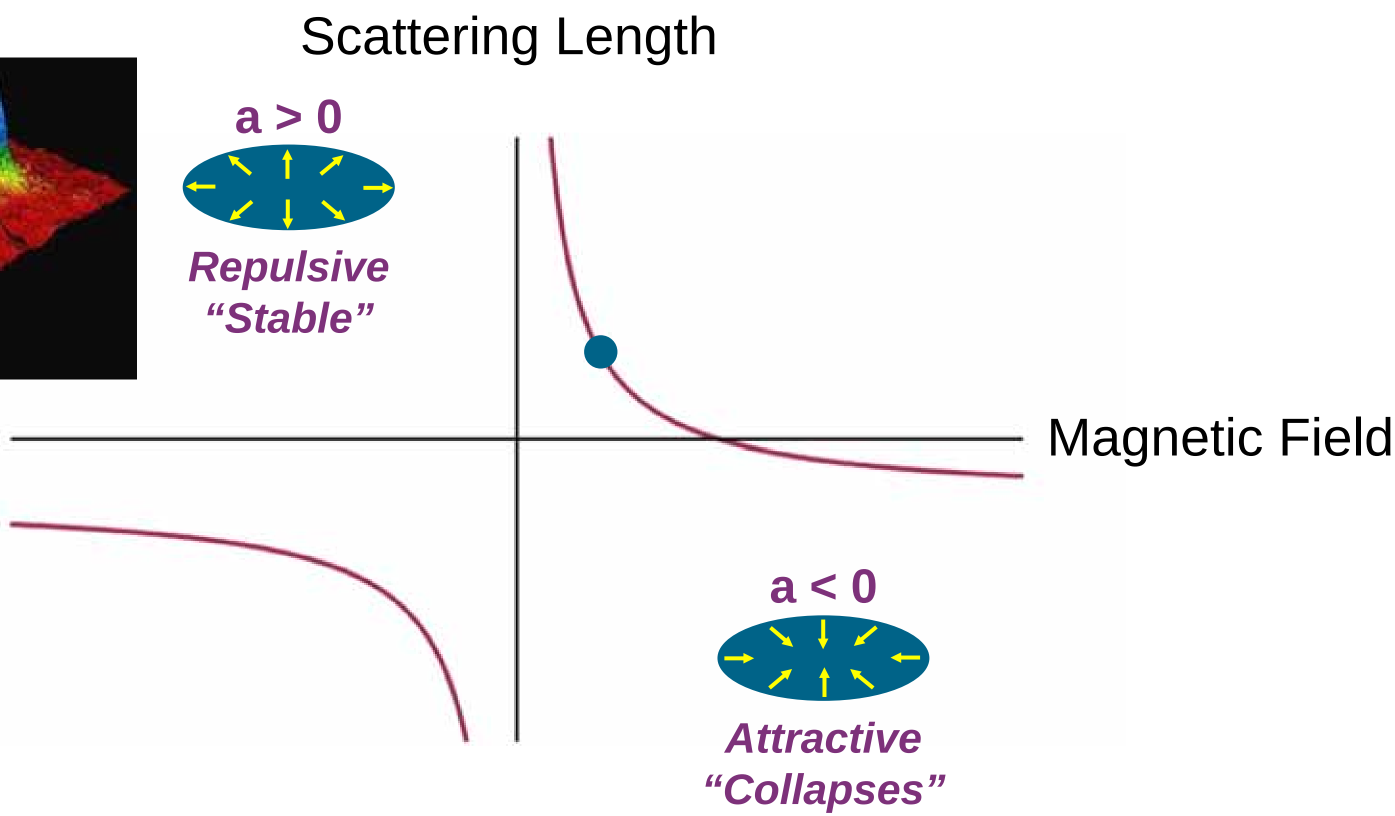
$$\left(-\frac{\hbar^2}{2m}\nabla^2 + V(\mathbf{r},t) + \frac{4\pi N\hbar^2 a}{m}\left|\psi(\mathbf{r},t)\right|^2\right)\psi(\mathbf{r},t) = i\hbar\frac{\partial\psi(\mathbf{r},t)}{\partial t}$$

^{85}Rb F = 2, m_F = -2



A simple question



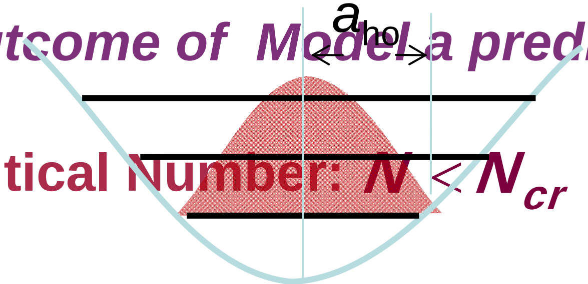


BEC with attractive interactions: variational model

Gaussian Model (isotropic 3d):

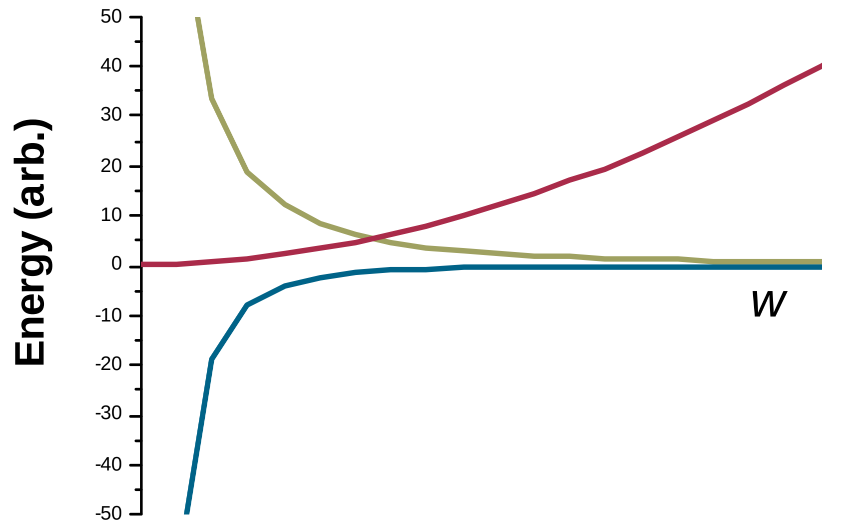
$$\varphi(r) = \left(\frac{N}{\pi^{3/2} (w a_{ho})^3} \right)^{1/2} \exp\left(\frac{-r^2}{2 (w a_{ho})^2} \right)$$

Outcome of Model: a prediction:

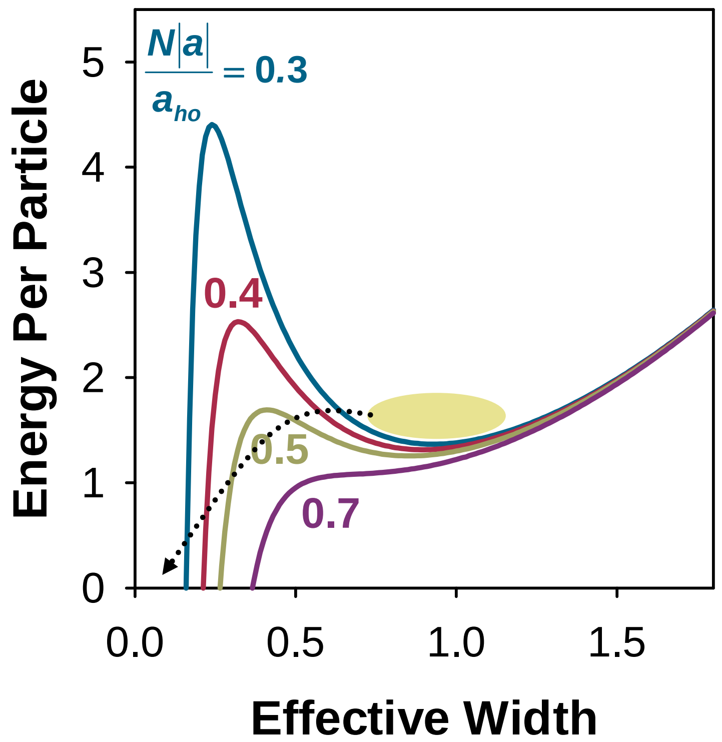


Critical Number: $N < N_{cr} = k \frac{a_{ho}}{|a|}$

$\text{KE} \sim \frac{1}{w^2}$ $\quad \text{PE} \sim w^2$ $\quad \text{Int} \sim -\frac{N|a|}{a_{ho}} \frac{1}{w^3}$



Parker *et al.*, J. Phys. B: **40** 3127 (2007)



Ruprecht *et al.*, PRA **51** 4704 (1995)

3d Gross-Pitaevskii equation

Gross-Pitaevskii equation (3d)

$$\underbrace{-\frac{\hbar^2}{2m}\nabla^2\psi(\mathbf{r},t)}_{\text{Kinetic energy term}}+\underbrace{V(\mathbf{r},t)\psi(\mathbf{r},t)}_{\text{External potential term}}+\underbrace{\frac{4\pi\hbar^2 a}{m}\left|\psi(\mathbf{r},t)\right|^2\psi(\mathbf{r},t)}_{\text{Nonlinear interaction term}}=\underbrace{i\hbar\frac{\partial\psi(\mathbf{r},t)}{\partial t}}_{\text{Total energy}}$$

Kinetic energy term **External potential term** **Nonlinear interaction term** **Total energy**

Attractive interactions compensate dispersion

Soliton solutions exist in 1D: $\psi(z,t)=\frac{1}{\sqrt{2l_z}}\,\mathrm{sech}\left(\frac{(z-vt)}{l_z}\right)\exp(-i\mu t/\hbar)$

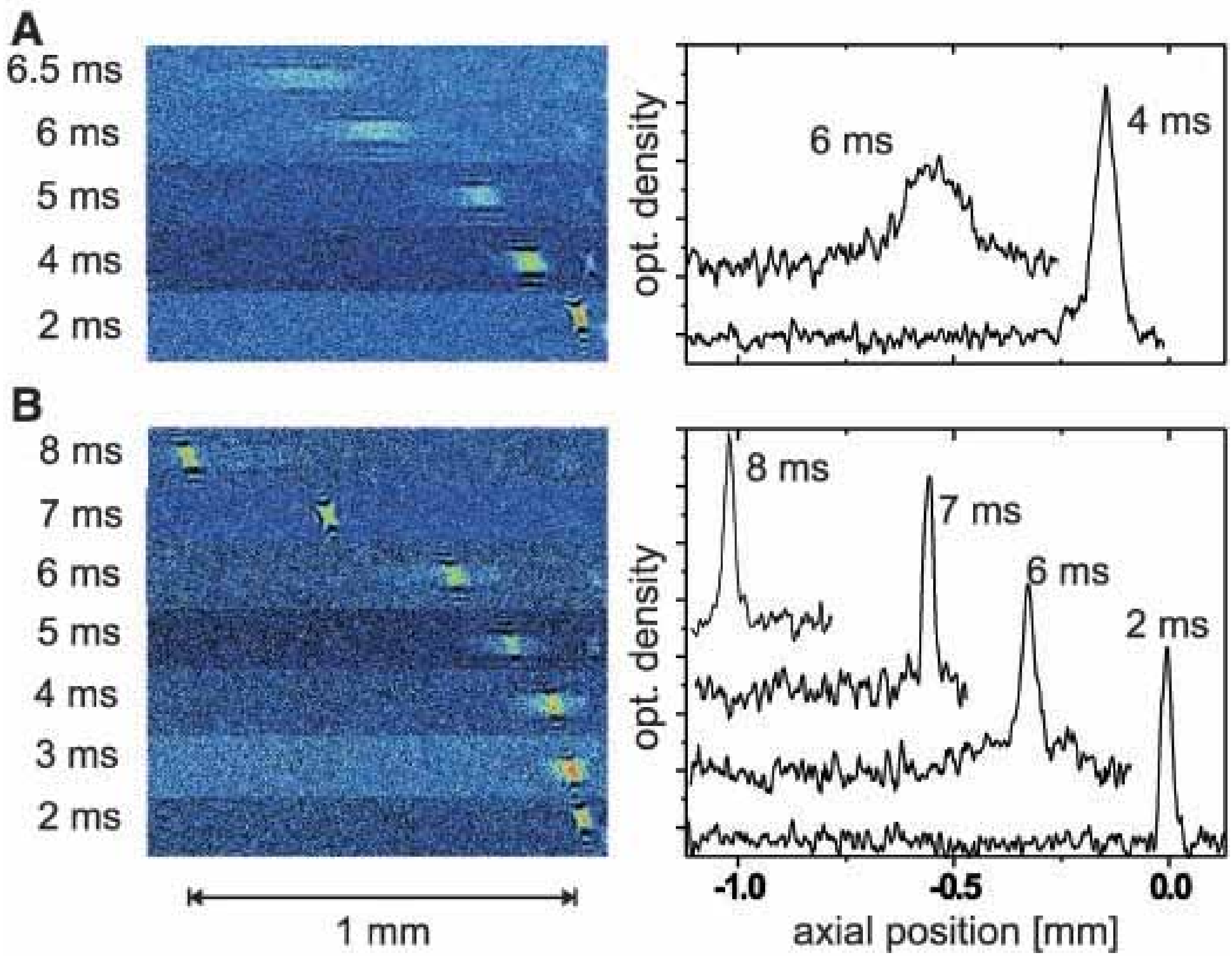
And in quasi-1d?

Quasi-1D: Bright matter-wave solitons, ENS (2002)

^{7}Li: Switch to a<0 and release into quasi-1D waveguide:

Ideal BEC gas
a ~ 0

Soliton
a = -0.21 nm

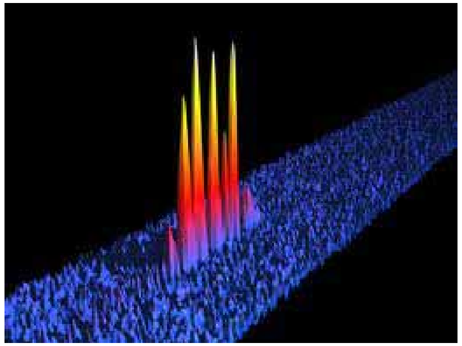


Khaykovich *et al.*, Science **296**, 1290 (2002)

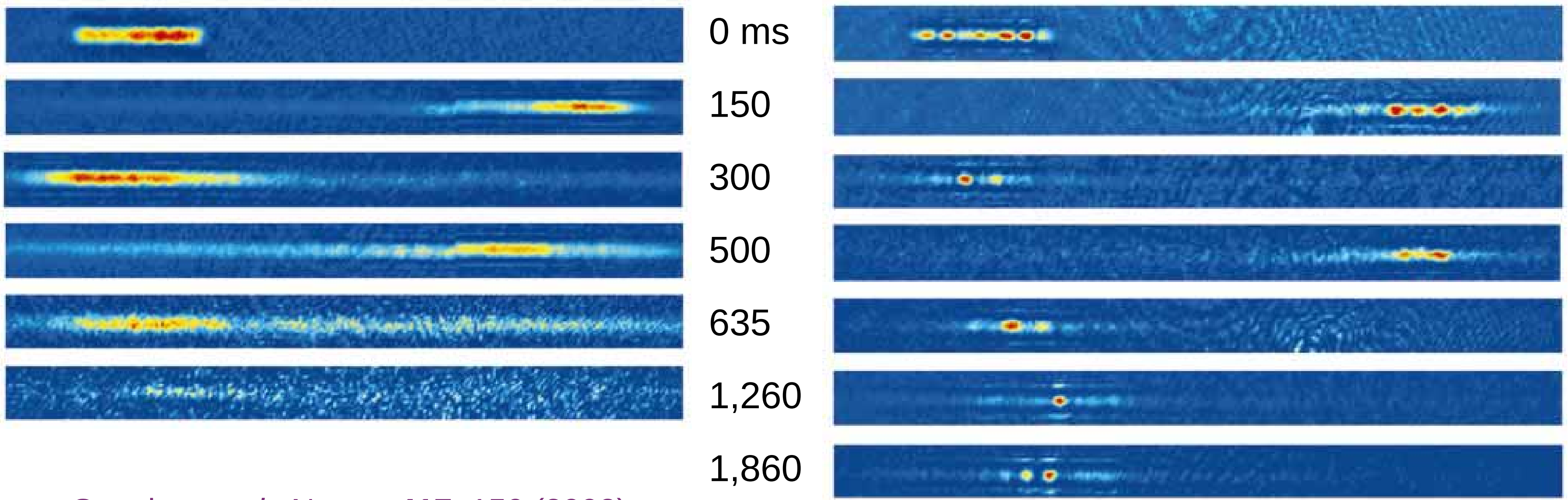
Quasi-1D: Bright matter-wave solitons, RICE (2002)

^{7}Li: Switch to a<0 and release into quasi-1D waveguide:

Repulsive BEC
$a > 0$

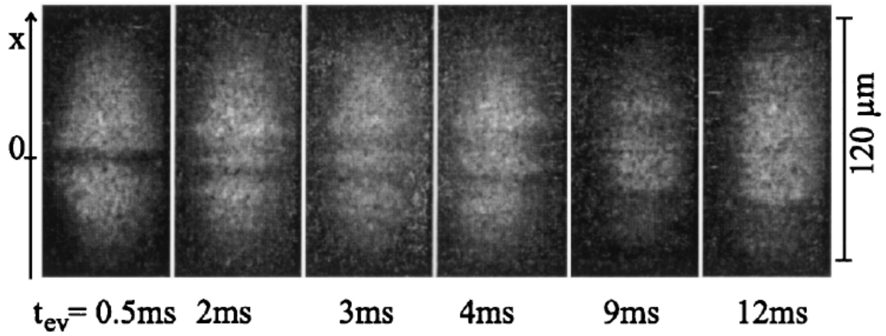


Attractive BEC
$a < 0$

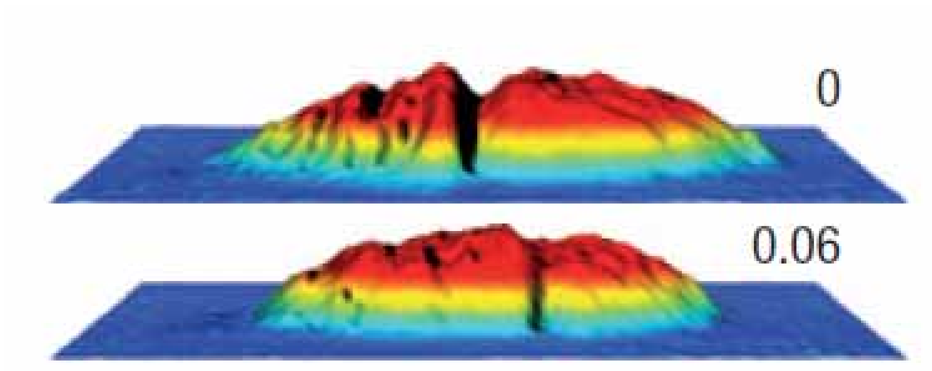


Strecker *et al.*, Nature **417**, 150 (2002)

Dark matter-wave solitons



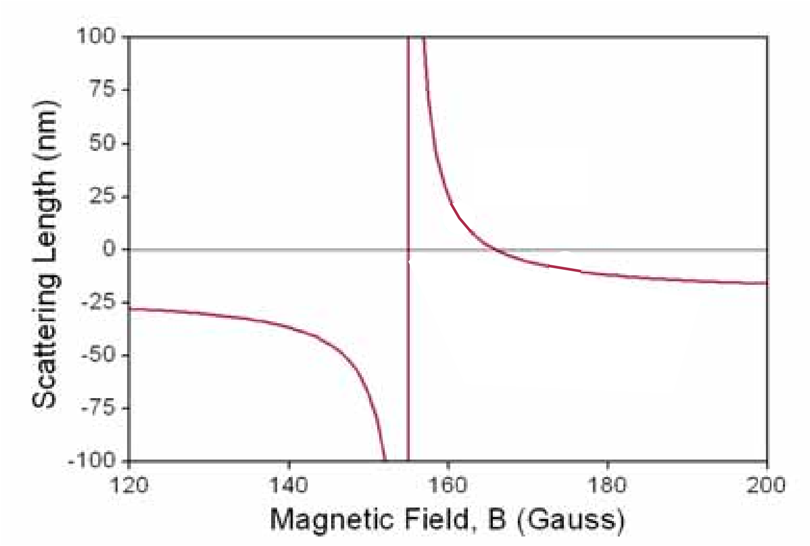
PRL **83**, 5198 (1999) - Hannover



Nature Physics **4**, 496 (2008) - Hamburg

The JILA Experiment (3d)

^{85}Rb F = 2, m_F = -2



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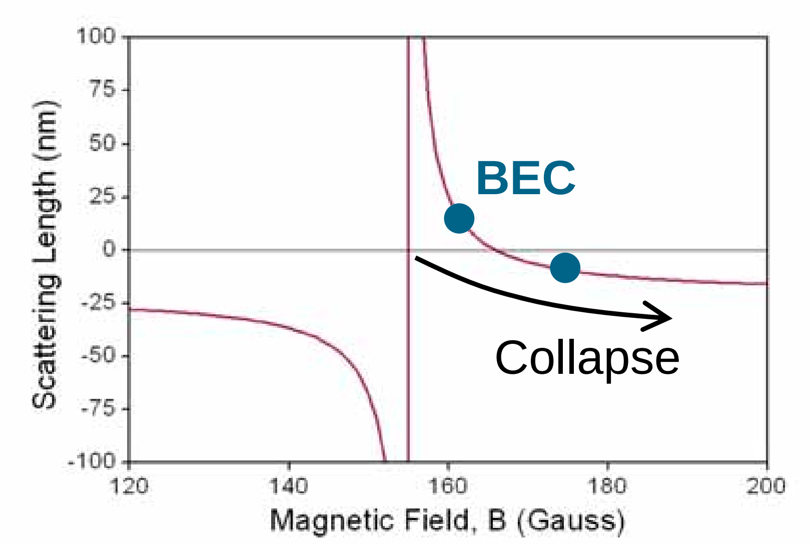
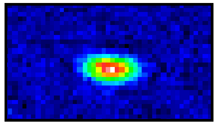


1. Form stable BEC



$$N_{BEC} \leq 15000(600)$$
$$N_{Thermal} < 500$$

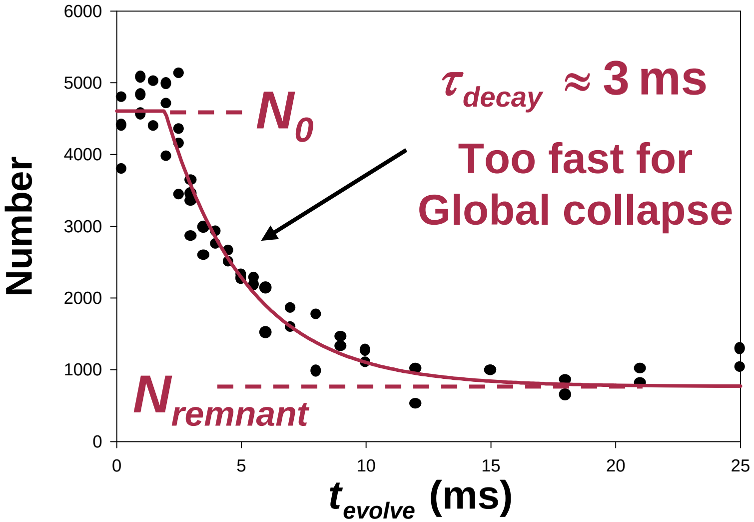
2. Move to a<0 - collapse

Violate $N_{cr} = k_{cr} \frac{a_{ho}}{|a|}$

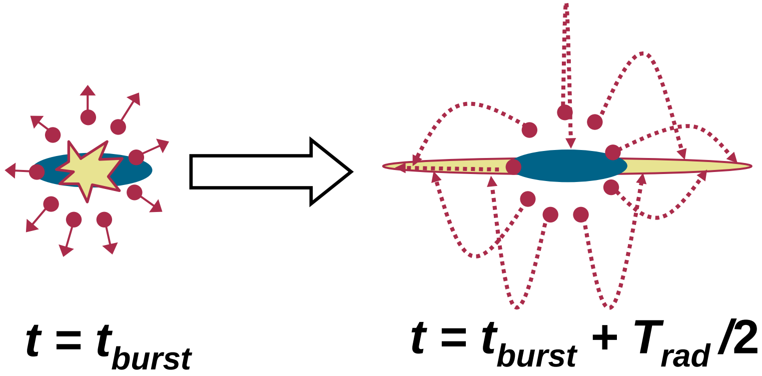
3. Wait and observe

Observation of collapse dynamics

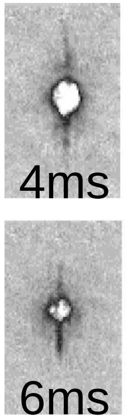
Rapid Loss of Atoms:

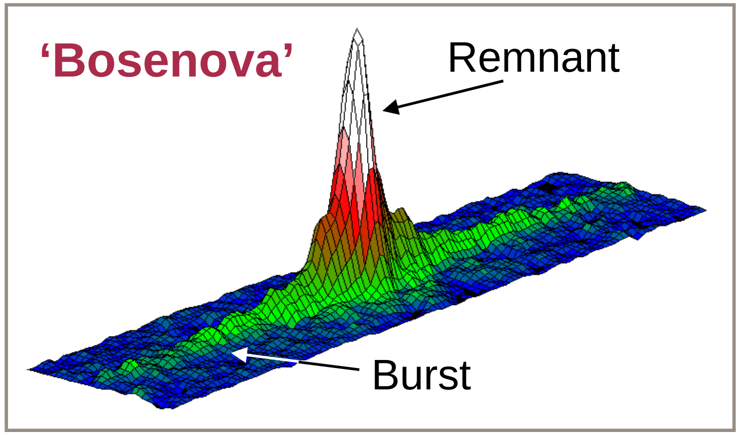


Explanation: Burst of ‘HOT’ Atoms



Jets:



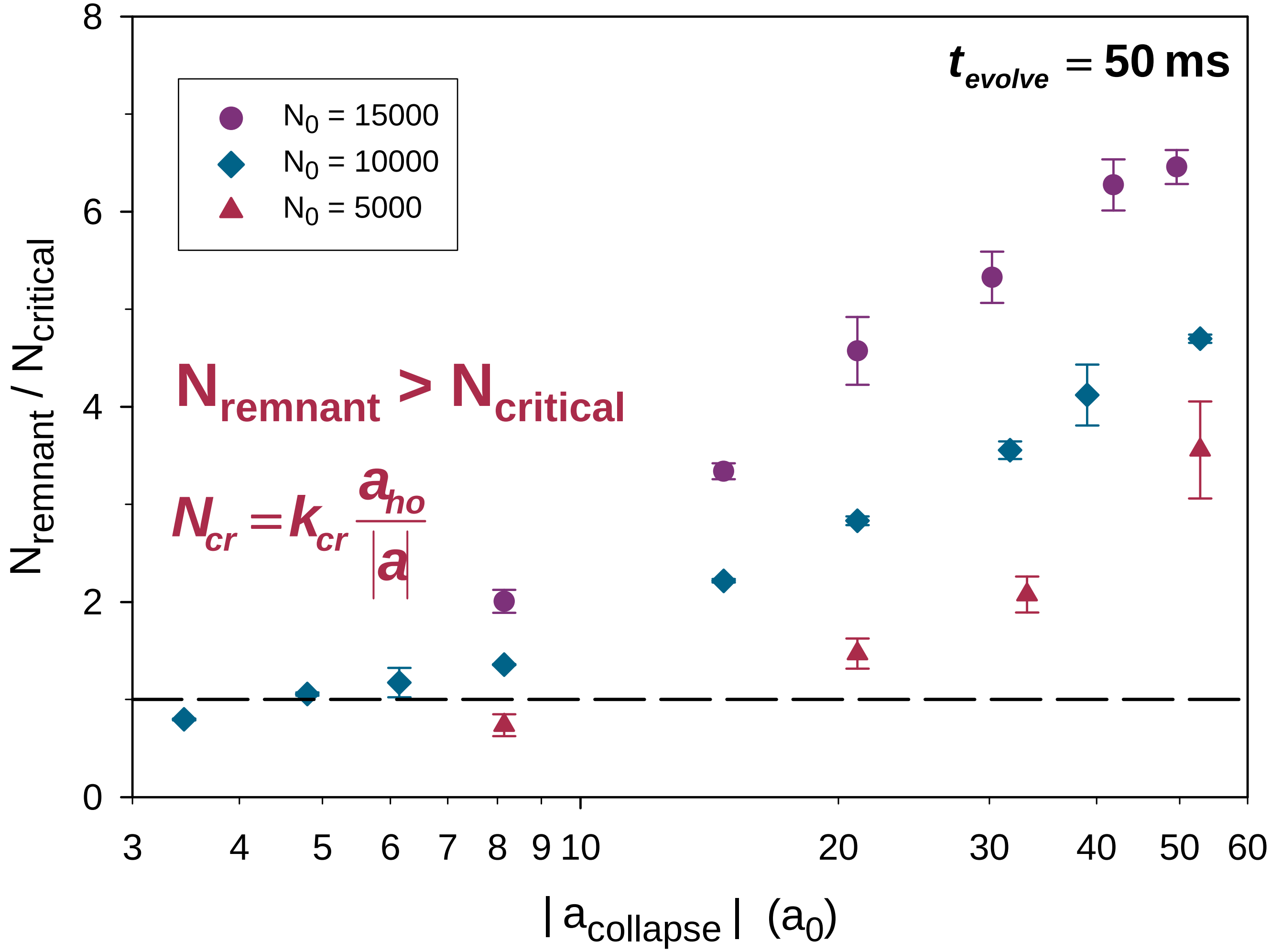


Donley *et al.*, Nature 412, 295 (2001)

Remnant number – a mystery?

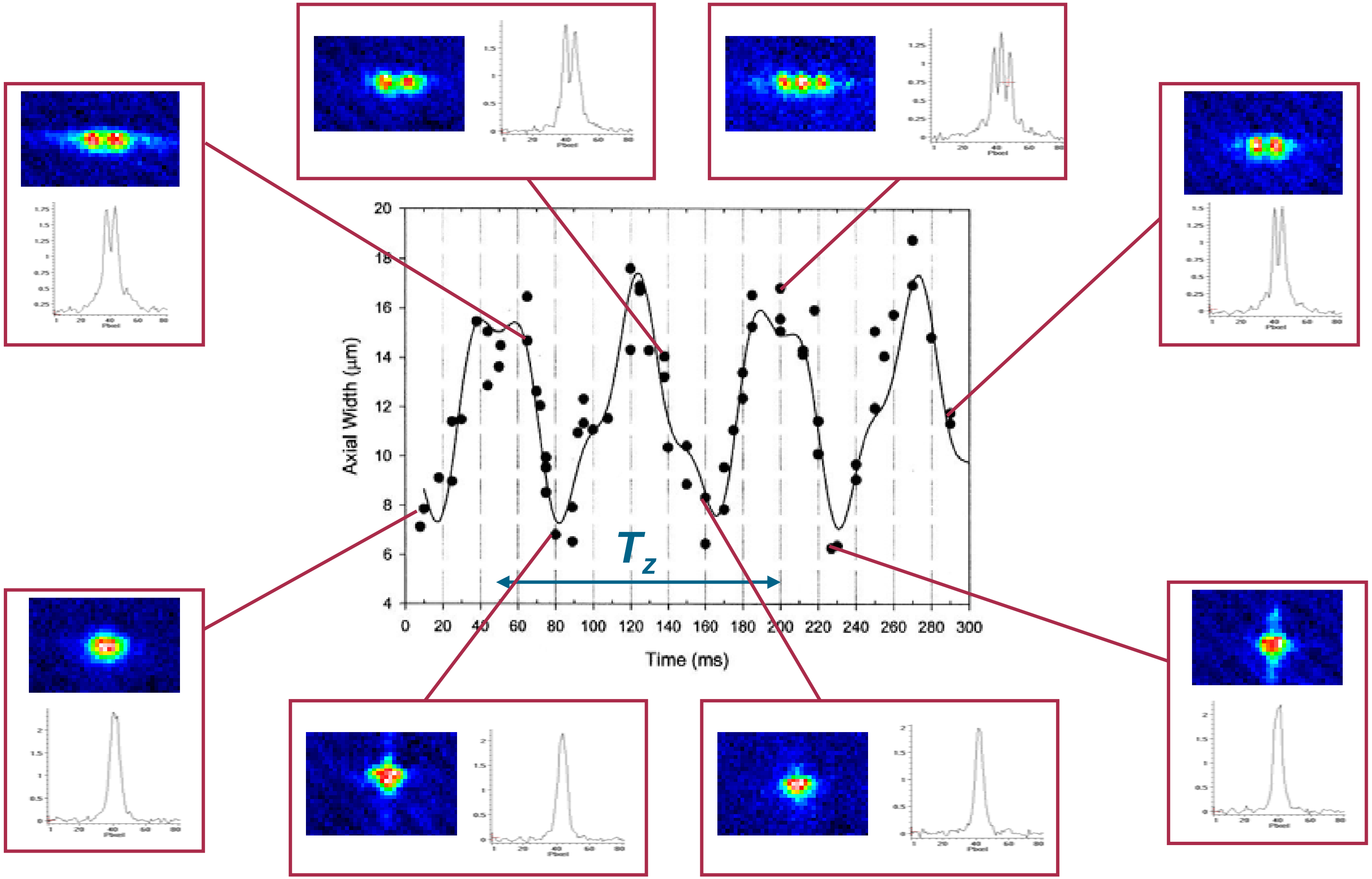
Cornish *et al.*, PRL **96**, 170401 (2006)

GPE gives *onset* of collapse, but not $N_{remnant}$?



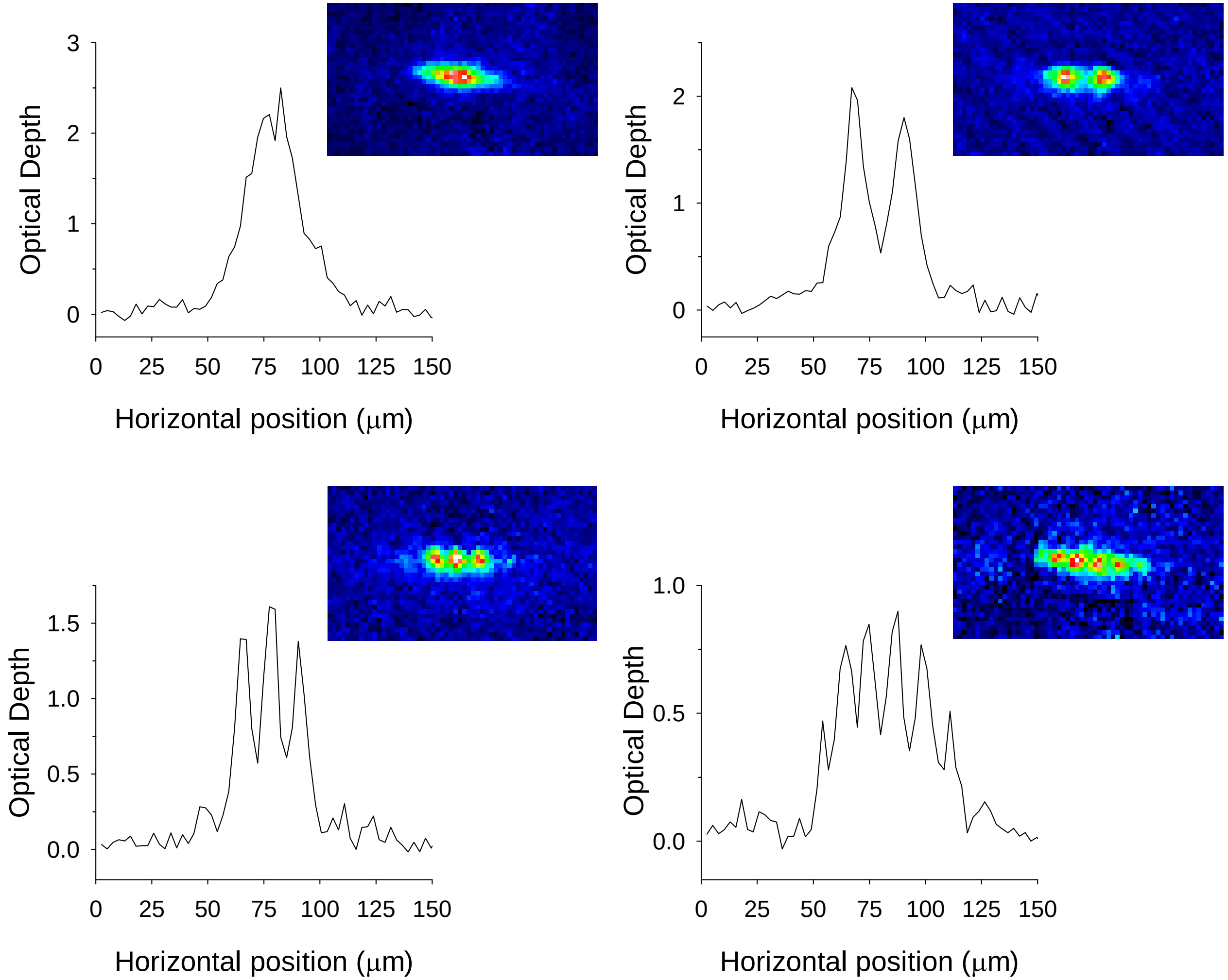
Remnant is highly excited

Switch to a<0 and hold in a 3D trap (ω_r/ω_z = 2.57):



Solitons are the solution!

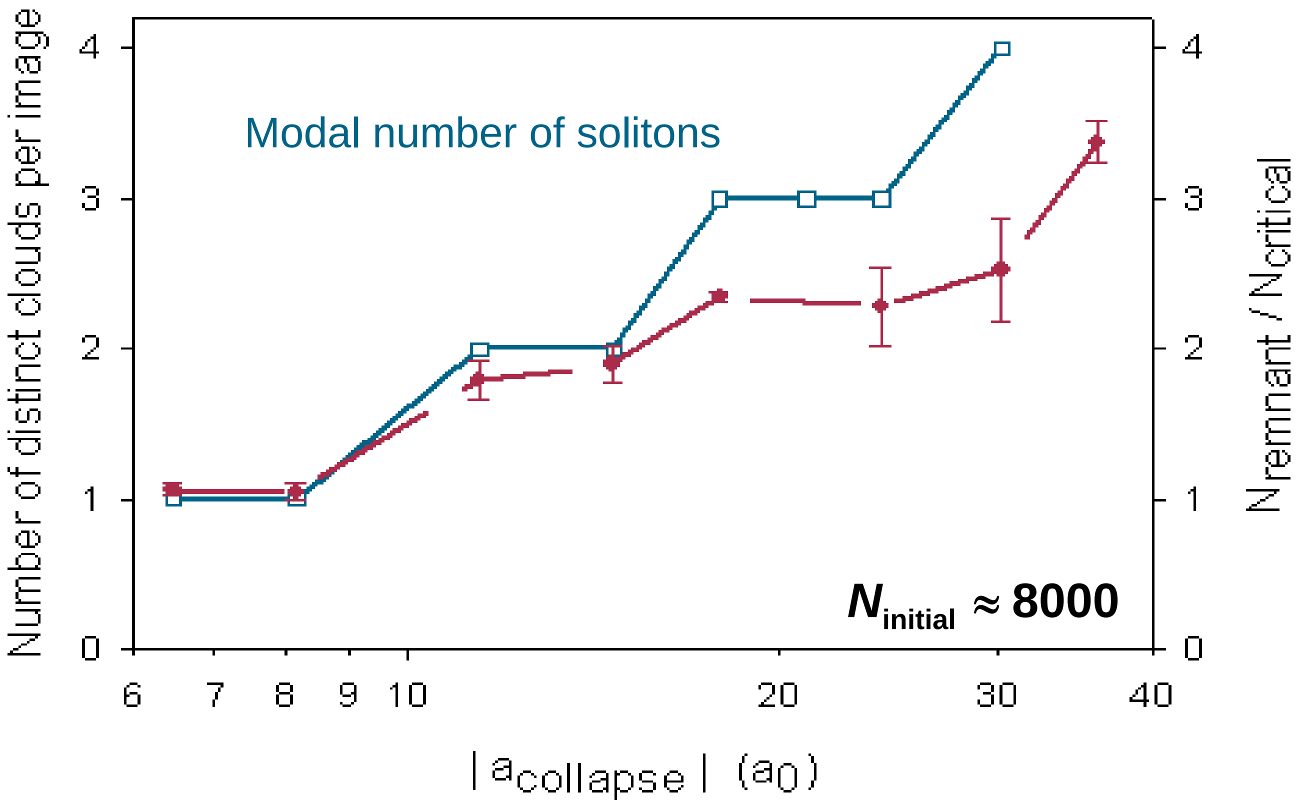
Cornish *et al.*, PRL **96**, 170401 (2006)



Number of atoms per soliton

Cornish *et al.*, PRL **96**, 170401 (2006)

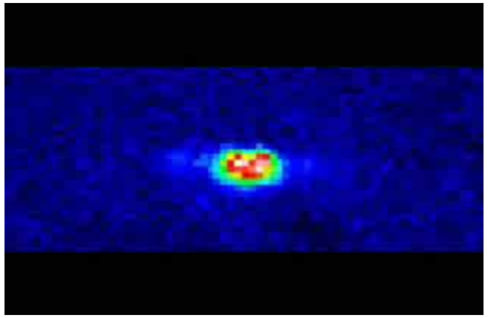
Number of atoms in each soliton < $N_{critical}$ **- Solitons STABLE**

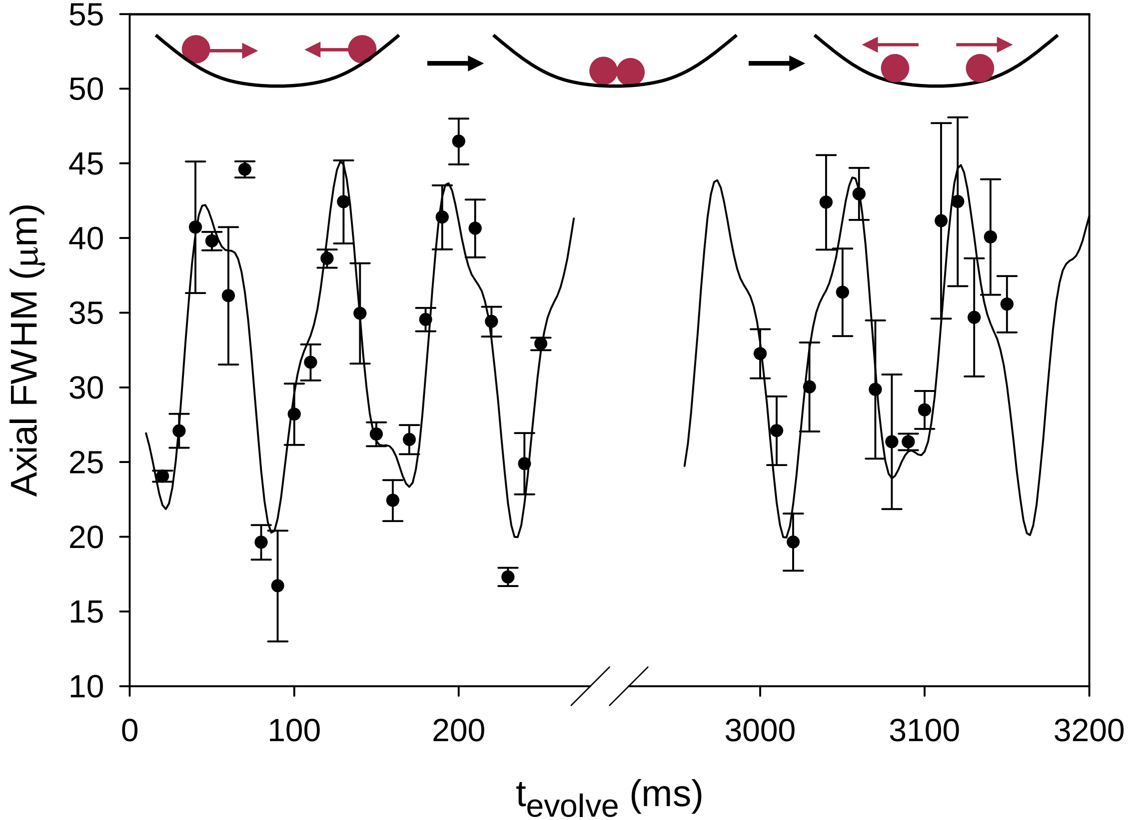


Explains puzzle of $N_{remnant} > N_{critical}$

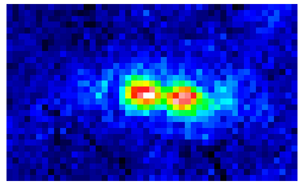
Two solitons oscillating in the trap

Cornish *et al.*, PRL **96**, 170401 (2006)

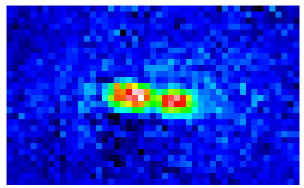




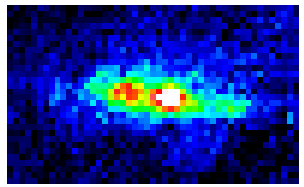
210 ms



1140 ms



3110 ms



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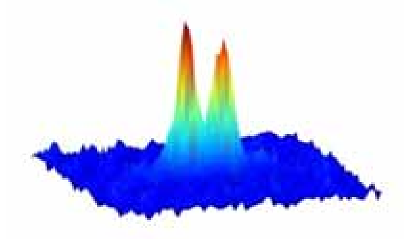
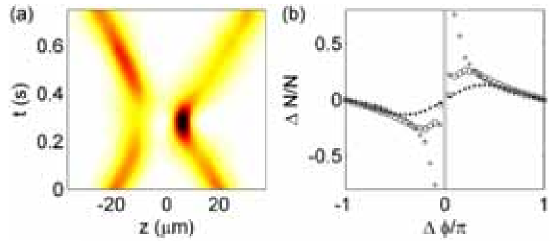
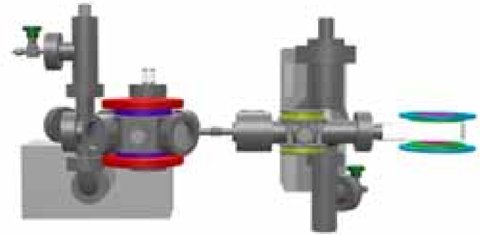
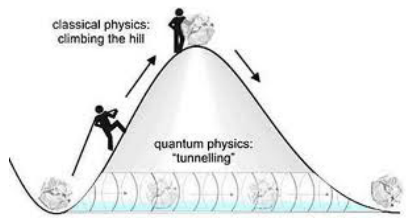








Durham University

3D GPE simulation – soliton collisions

$$\left(-\frac{\hbar^2}{2m}\nabla^2 + V(\mathbf{r},t) + \frac{4\pi N\hbar^2 a}{m}\left|\psi(\mathbf{r},t)\right|^2\right)\psi(\mathbf{r},t) = i\hbar\frac{\partial\psi(\mathbf{r},t)}{\partial t}$$

3D GPE simulation – soliton collisions

$$\left(-\frac{\hbar^2}{2m}\nabla^2 + V(\mathbf{r},t) + \frac{4\pi N\hbar^2 a}{m}\left|\psi(\mathbf{r},t)\right|^2\right)\psi(\mathbf{r},t) = i\hbar\frac{\partial\psi(\mathbf{r},t)}{\partial t}$$

Investigate binary soliton collisions:

1. **Interaction strength**

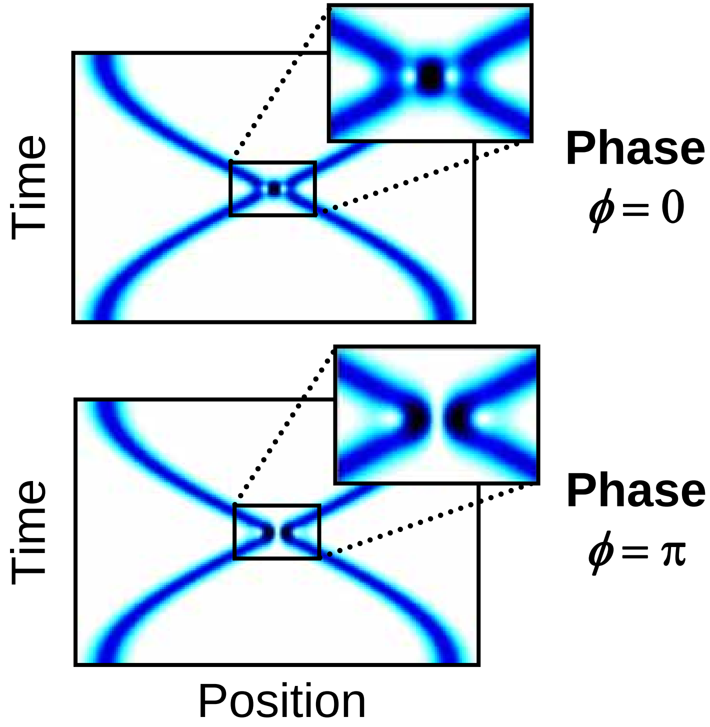
   - Determines inherent stability

2. **Relative velocity**

   - Determines interaction time
   - Compare to collapse time

3. **Relative phase**

   - Affects density during collision

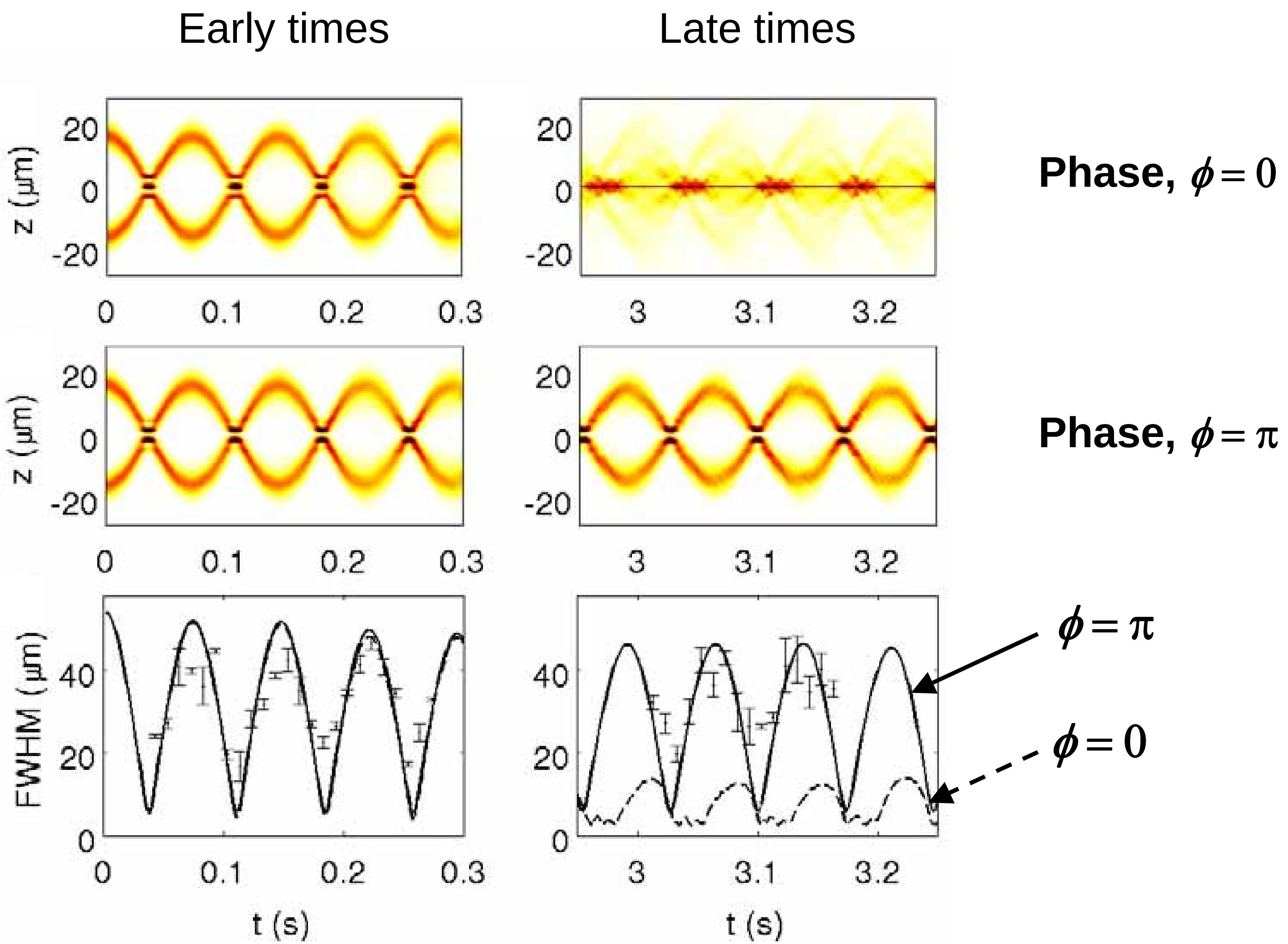


GPE simulation – relative phase

$N = 1750$

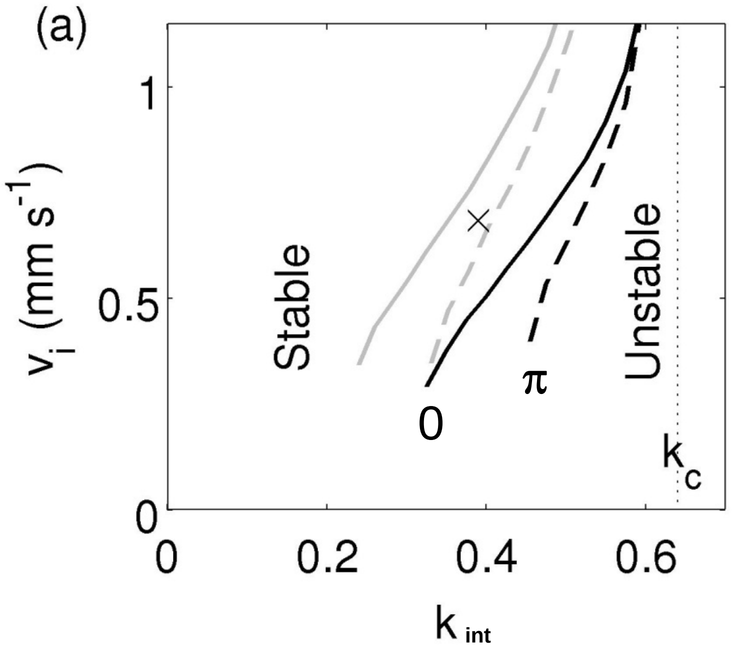
$\nu_r = 17.3\,\text{Hz}$

$\nu_z = 6.8\,\text{Hz}$

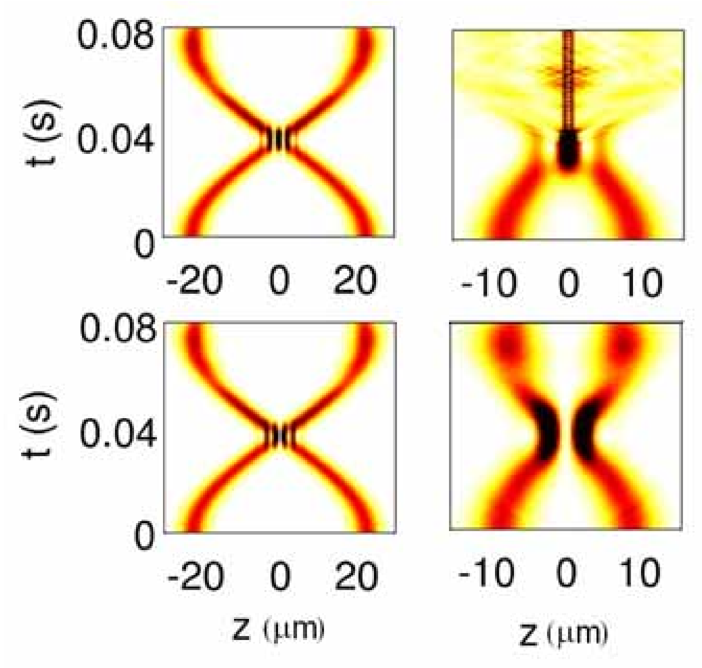


Incident velocity at "impact" important

1 collision
40 collisions



v_i=1mms^{-1} (k_{int} = 0.4) v_i=0.4mms^{-1} (k_{int} = 0.4)



Phase, $\phi = 0$

Phase, $\phi = \pi$

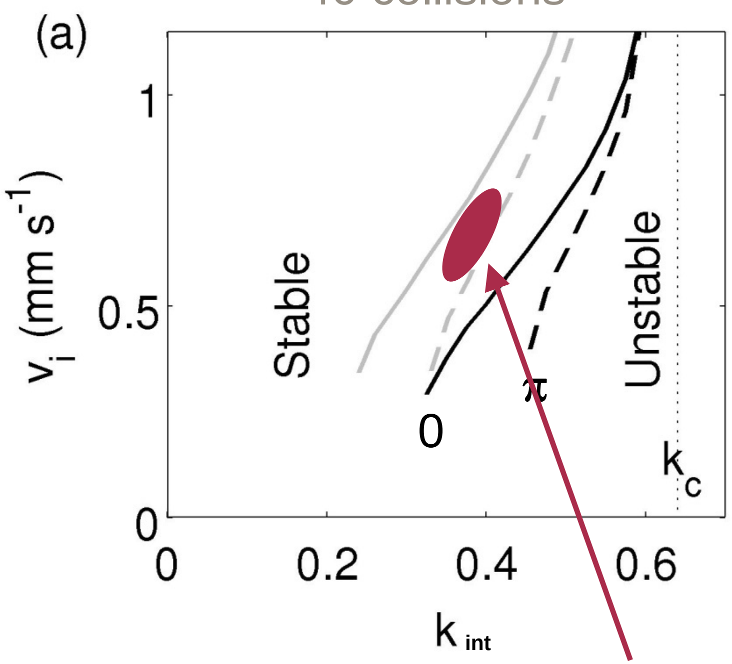
$$k_{int} = \frac{N|a_s|}{a_r}$$

Parker *et al.*, J. Phys. B: **41** 045303 (2008)

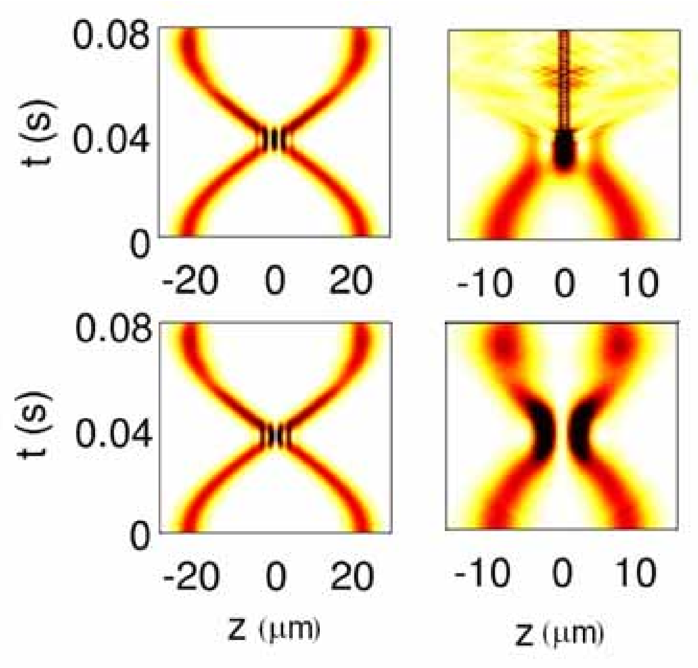
Incident velocity at "impact" important

1 collision

40 collisions



v_i=1mms^{-1} (k_{int} = 0.4) v_i=0.4mms^{-1} (k_{int} = 0.4)



Phase, $\phi = 0$

Phase, $\phi = \pi$

$$\left(k_{int} = \frac{N|a_s|}{a_r} \right)$$

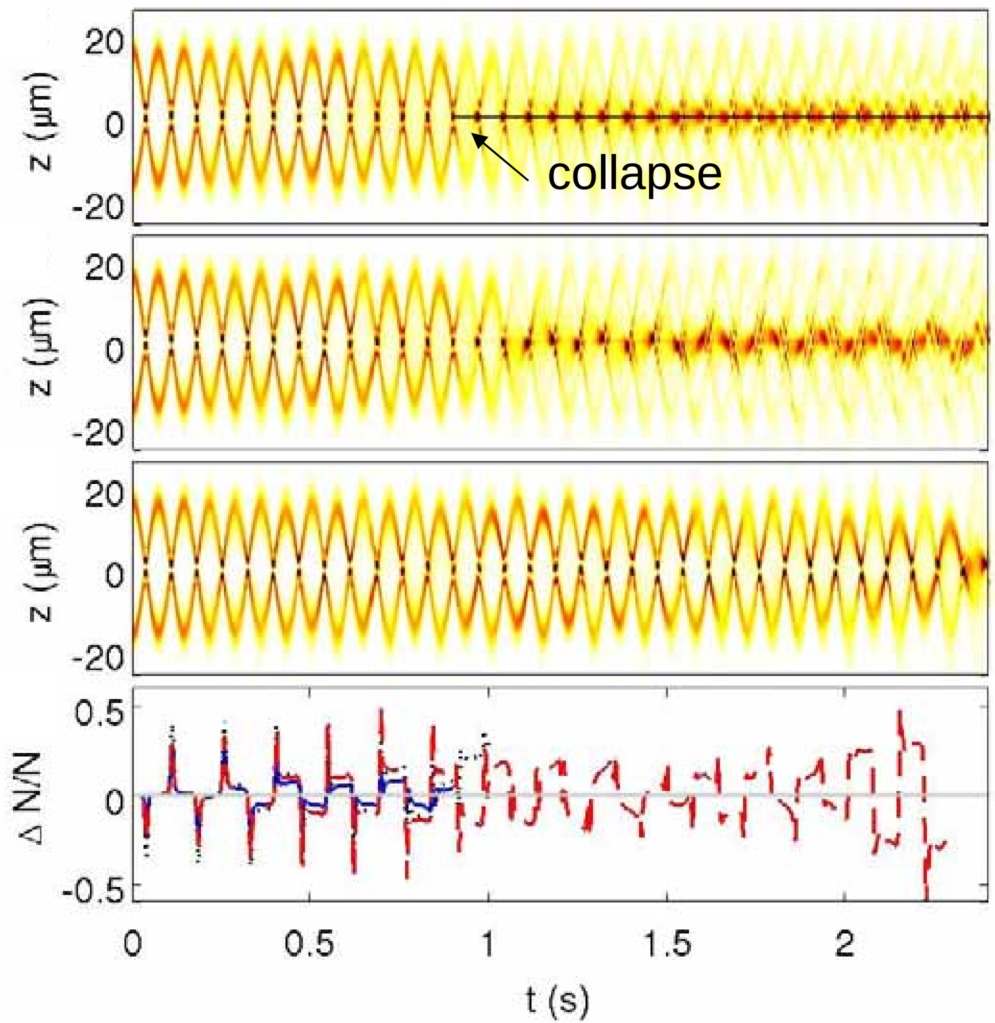
Experiment:

- Stable for π phase even for 40 collisions
- Unstable for 0 phase

Parker *et al.*, J. Phys. B: **41** 045303 (2008)

Intermediate Relative Phases

(k_{int} = 0.4)



Phase, $\phi = \pi/4$

Phase, $\phi = \pi/2$

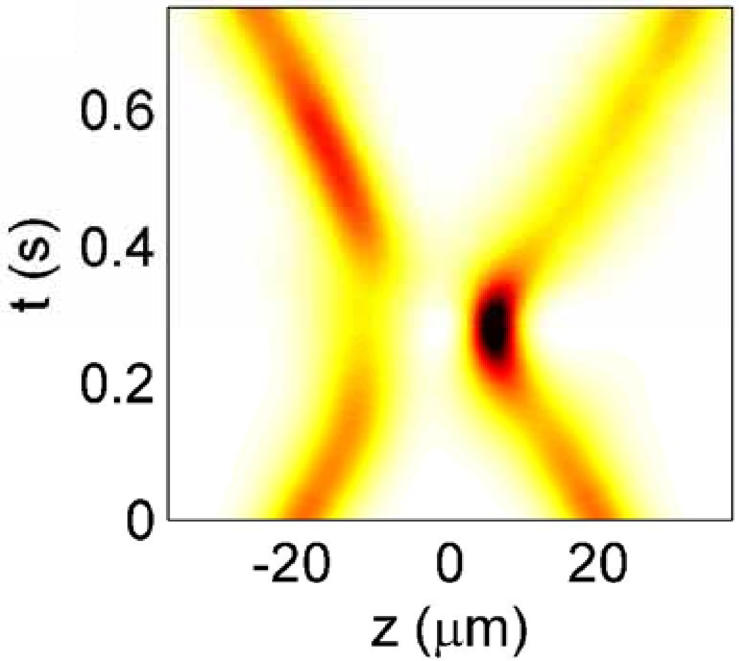
Phase, $\phi = 3\pi/4$

Population Difference

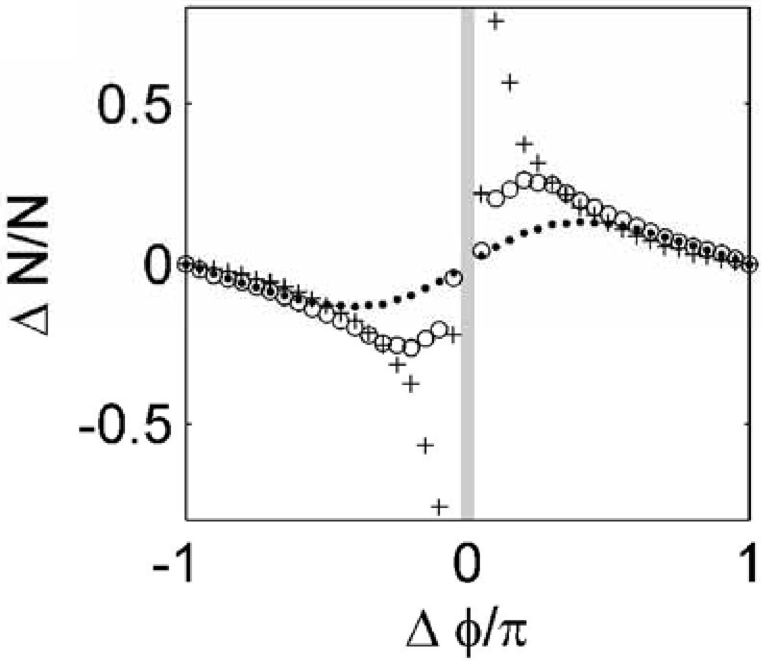
$\pi/4$
$\pi/2$
$3\pi/4$

Population Transfer & Relative Phase

k_{int}=0.4
v_i=0.05mms^{-1}
$\Delta\phi=\pi/2$



k_{int}=0.4, v_i = 0.025 mms^{-1} (crosses)
= 0.05 mms^{-1} (circles)
= 0.1 mms^{-1} (dots)



Basis for soliton interferometry!

Parker *et al.*, J. Phys. B: **41** 045303 (2008)

Open questions.

Trap geometry and critical number?

Formation mechanism?

Modulation instability of condensate wavefunction

What role does 3-body loss play?

Origin of relative phase and remarkable stability?

The GPE model requires this phase but..

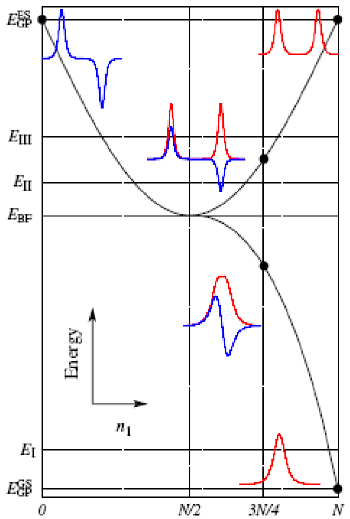
We have no *direct* evidence of this phase!

Is the GPE model correct for this scenario?

Is the GPE model correct?

More sophisticated theories:

1. Many-body theory

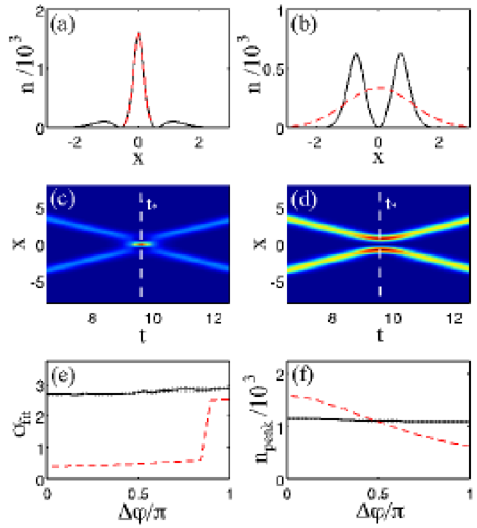


Fragmented states - *Fragmentons*

Streltsov *et al.* PRL **100**, 130401 (2008)

?

2. Quantum dynamical simulations



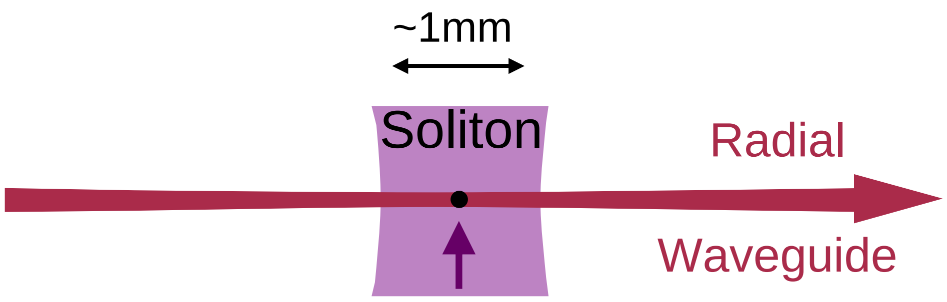
Quantum fluctuations leads to repulsive collisions

Dabrowska-Wuster *et al.* arXiv:0812.0493

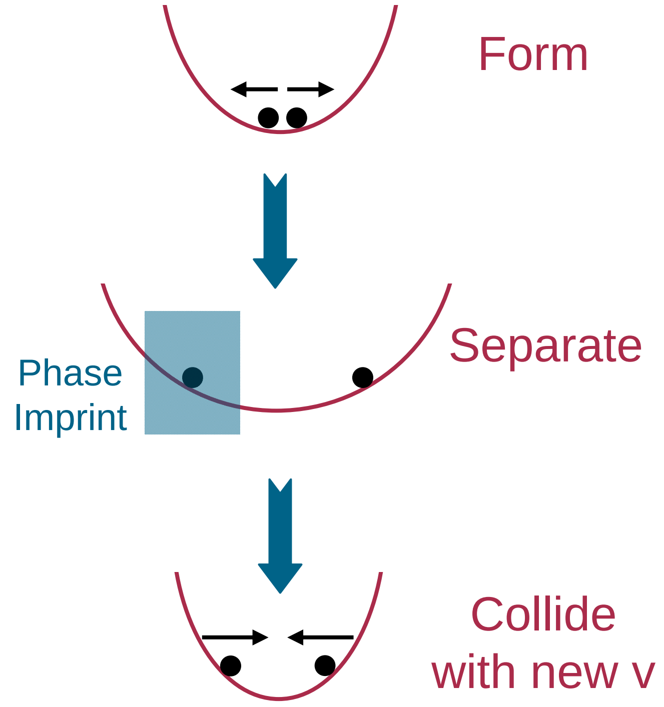
Proposed soliton studies at Durham

Experimental geometry:

- Dipole trap with adjustable geometry
- B uniform over region of experiment



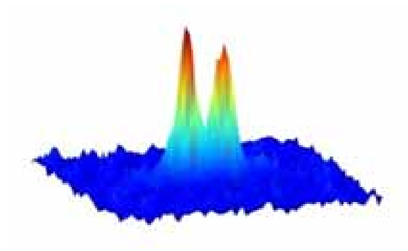
Controlled binary collisions:



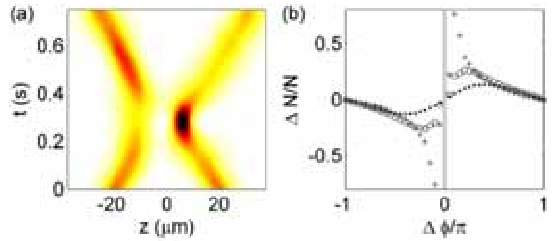
Tests population transfer and initial pi phase.

Outline

❶ Introduction to solitons
- *Necessary background*

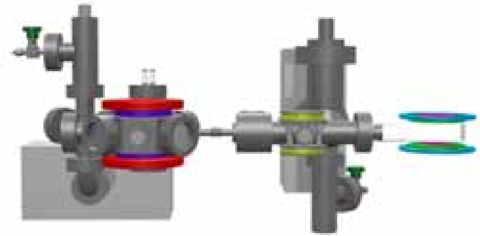


❷ BEC collapse & matter-wave solitons
- *Basic physics*
- *The past experiments*

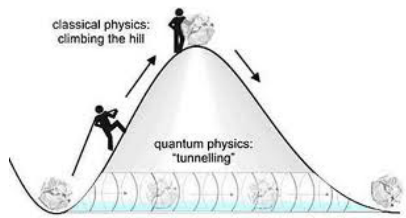


❸ Soliton dynamics (theory)
- *Numerical simulations*
- *Binary soliton collisions*

❹ Our experimental progress
- *Current status and next steps*



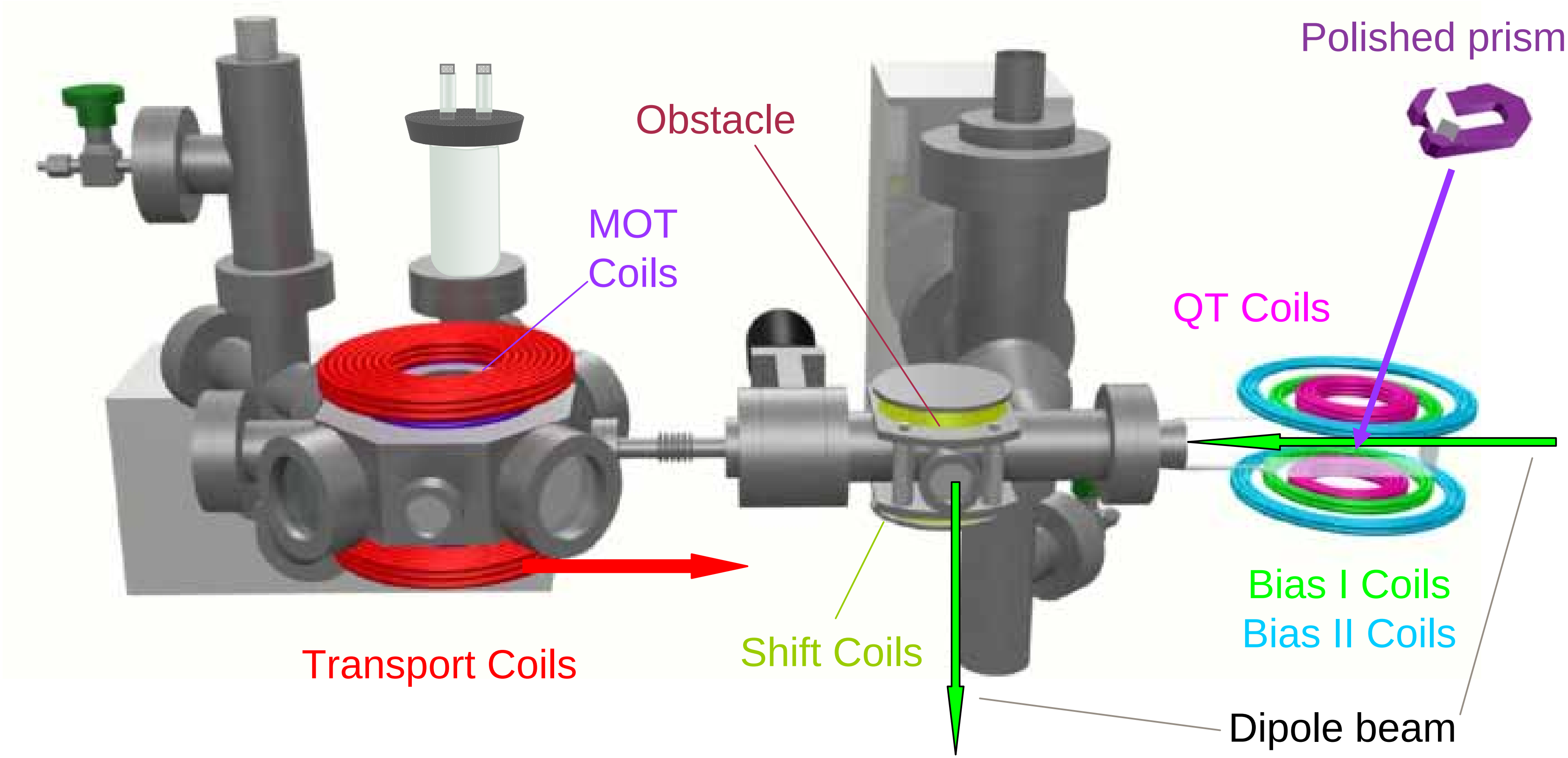
❺ Future goals
- *Atom surface studies*
- *Quantum reflection with solitons*



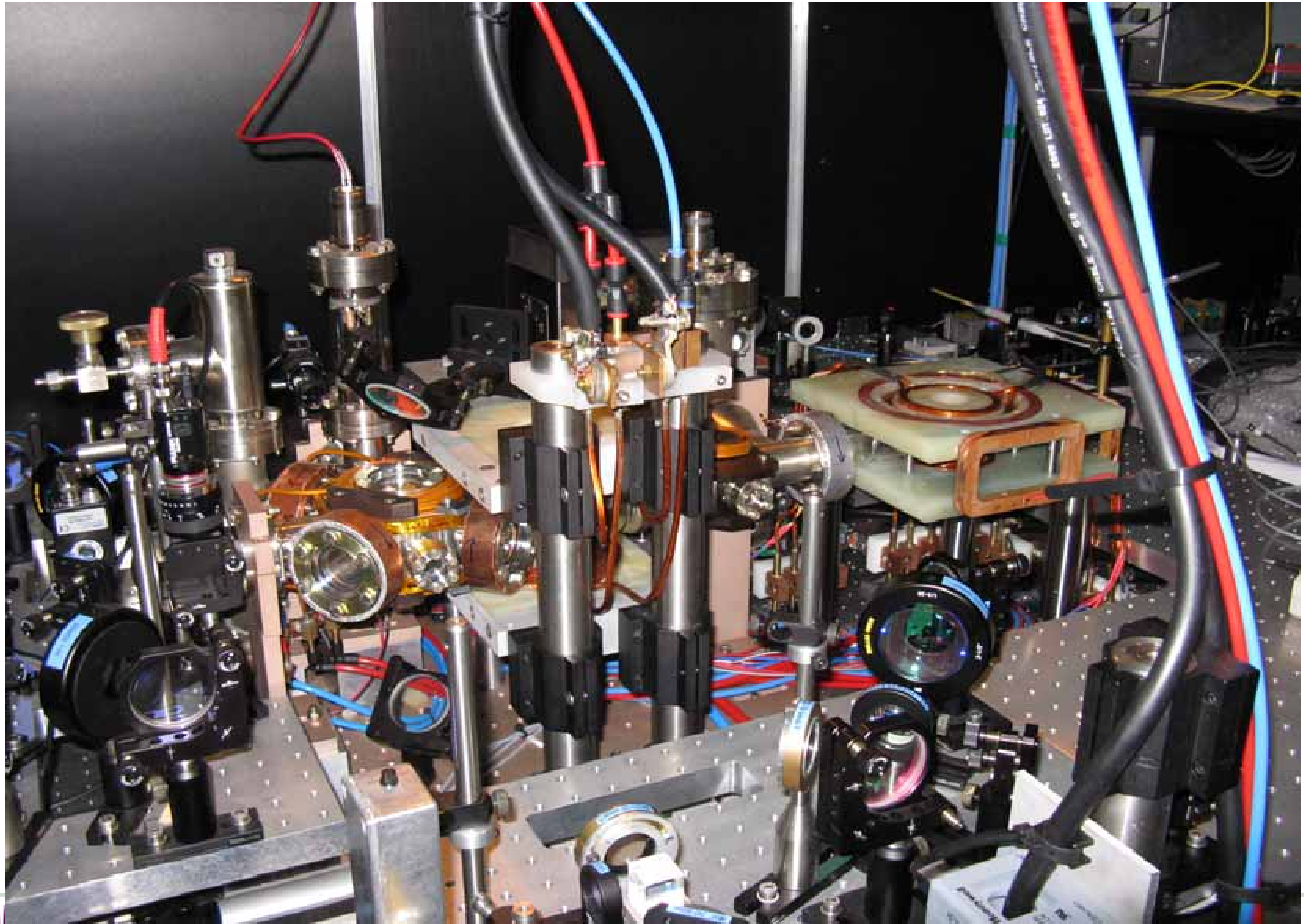
Durham University

Our apparatus: schematic

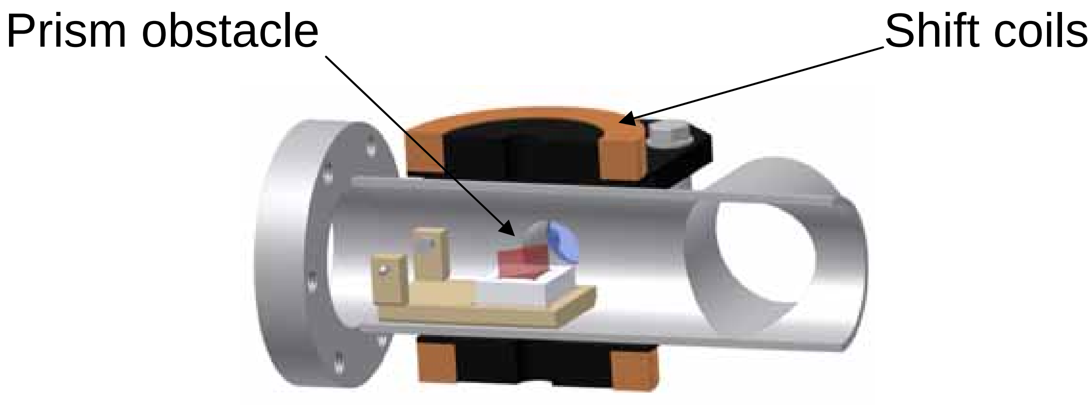
Rubidium 87 and 85



Our apparatus: Photo

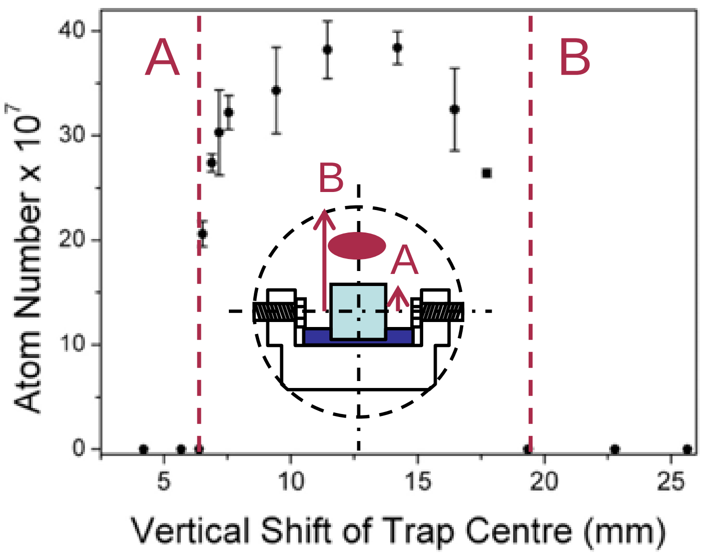


Special details I: obstacle prism

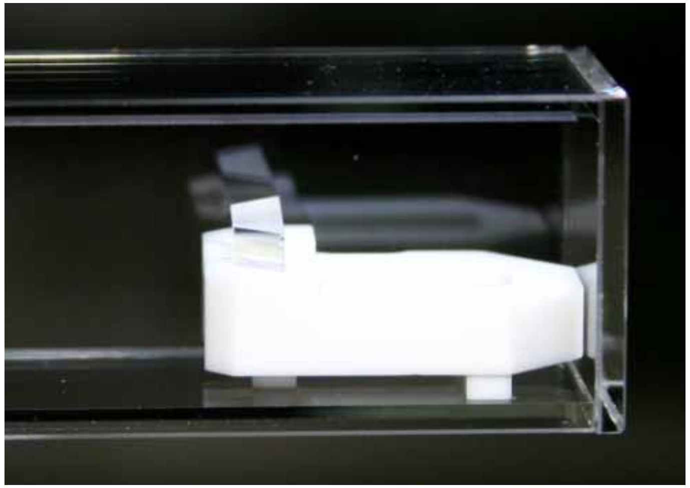


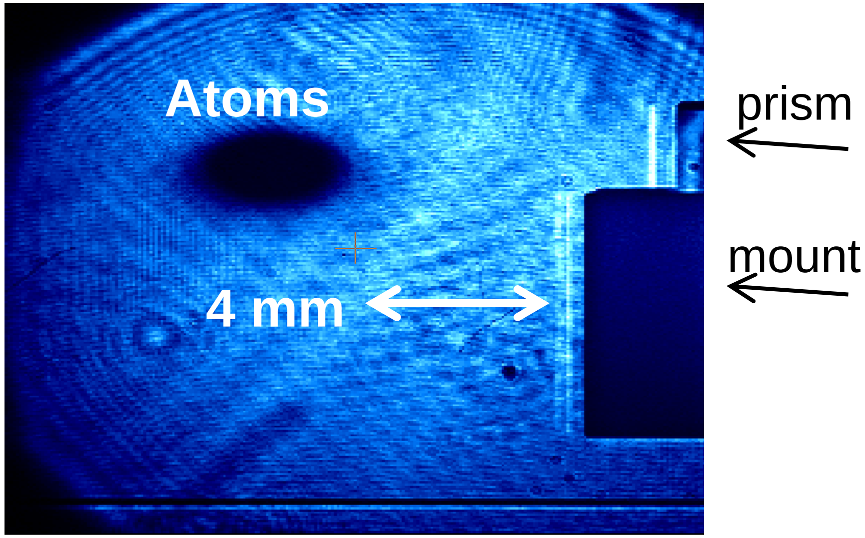
Transport performance

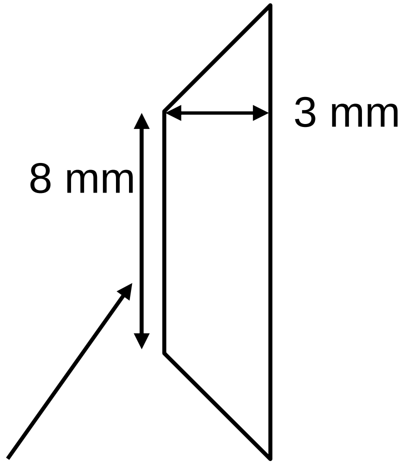
Gradient = 180 Gcm^{-1}
Acceleration = 2 ms^{-2}
Speed = 0.2 ms^{-1}
Distance = 0.52 m
Transfer ~ 70 %



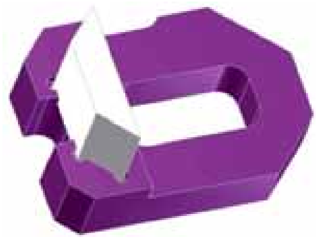
Special details I: Super-polished prism





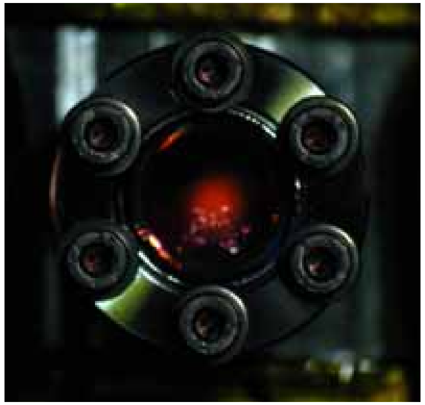


Roughness < 1nm RMS
over 2 x 5 mm



Stages of the soliton lab experiment

MOT Load ~ 10^9 in 30s

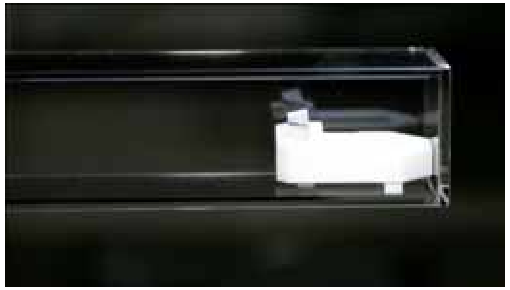


→ $F = 2$, $m_F = -2$

Transport trap

→

Science cell quadrupole trap $N > 3e8$, $T \sim 200\ \mu K$



↓ RF induced evaporative cooling

$N \sim 4e7$
$T \sim 35\ \mu K$
$\rho \sim 1e\text{-}4$

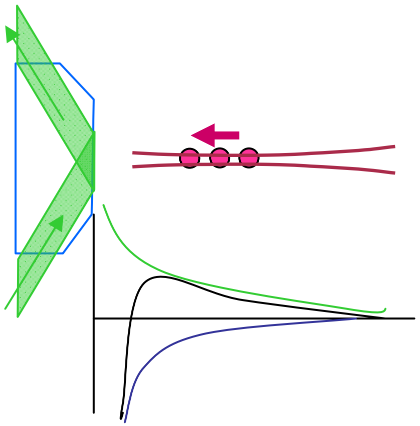
? Hybrid optical-magnetic dimple trap

? Sympathetic cooling of ^{85}Rb by ^{87}Rb

? Degenerate Raman sideband cooling

more evap cooling

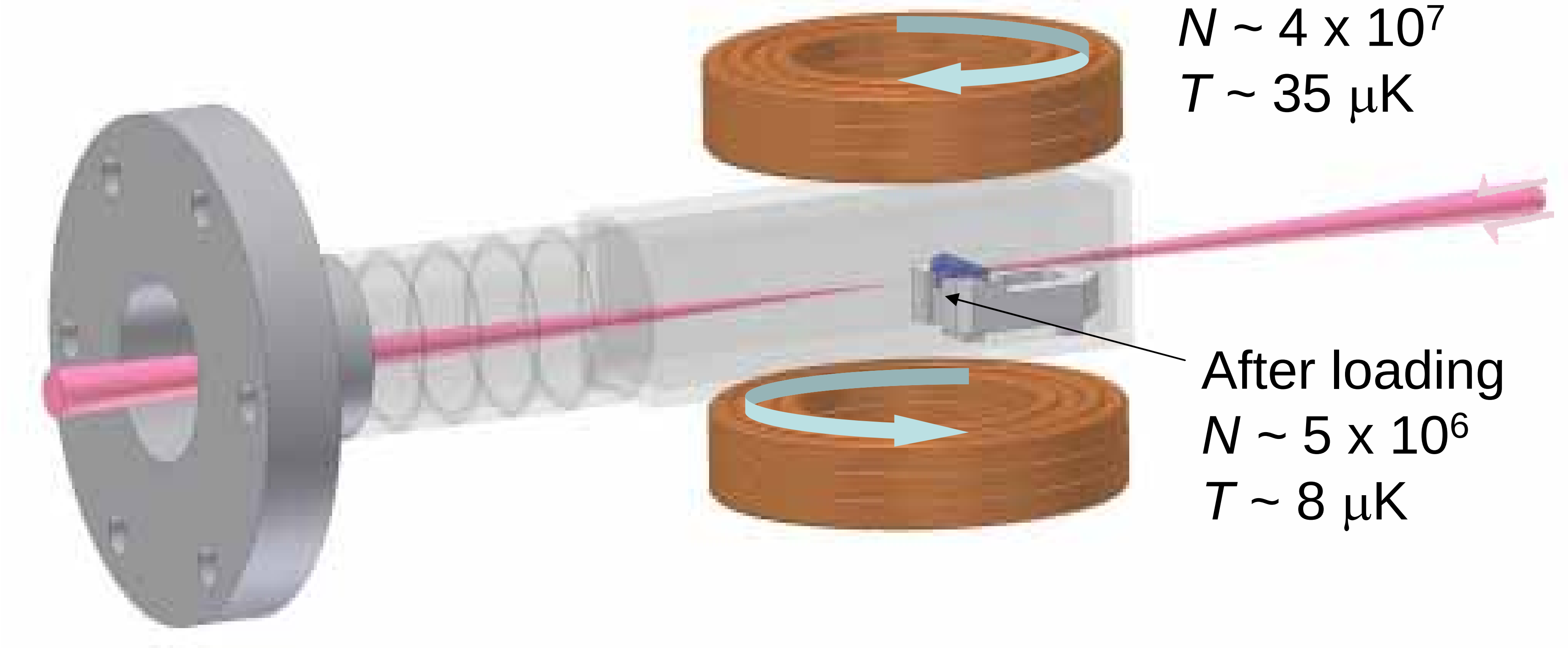
BEC & solitons
Atom-surface studies



Special details II: hybrid trap

Hybrid trap = focussed beam + quadrupole

Before loading
$N \sim 4 \times 10^7$
$T \sim 35\ \mu K$

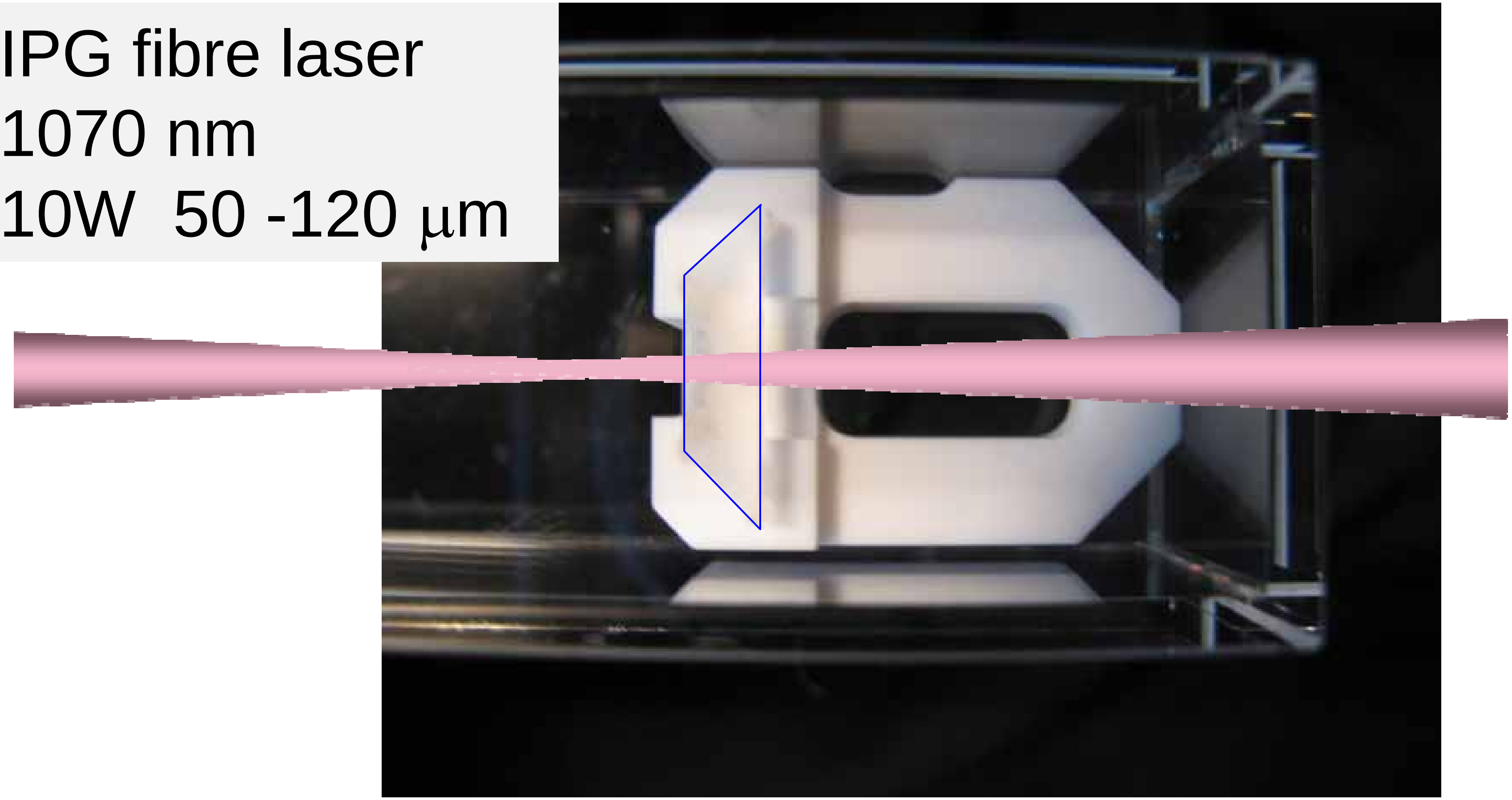


After loading
$N \sim 5 \times 10^6$
$T \sim 8\ \mu K$

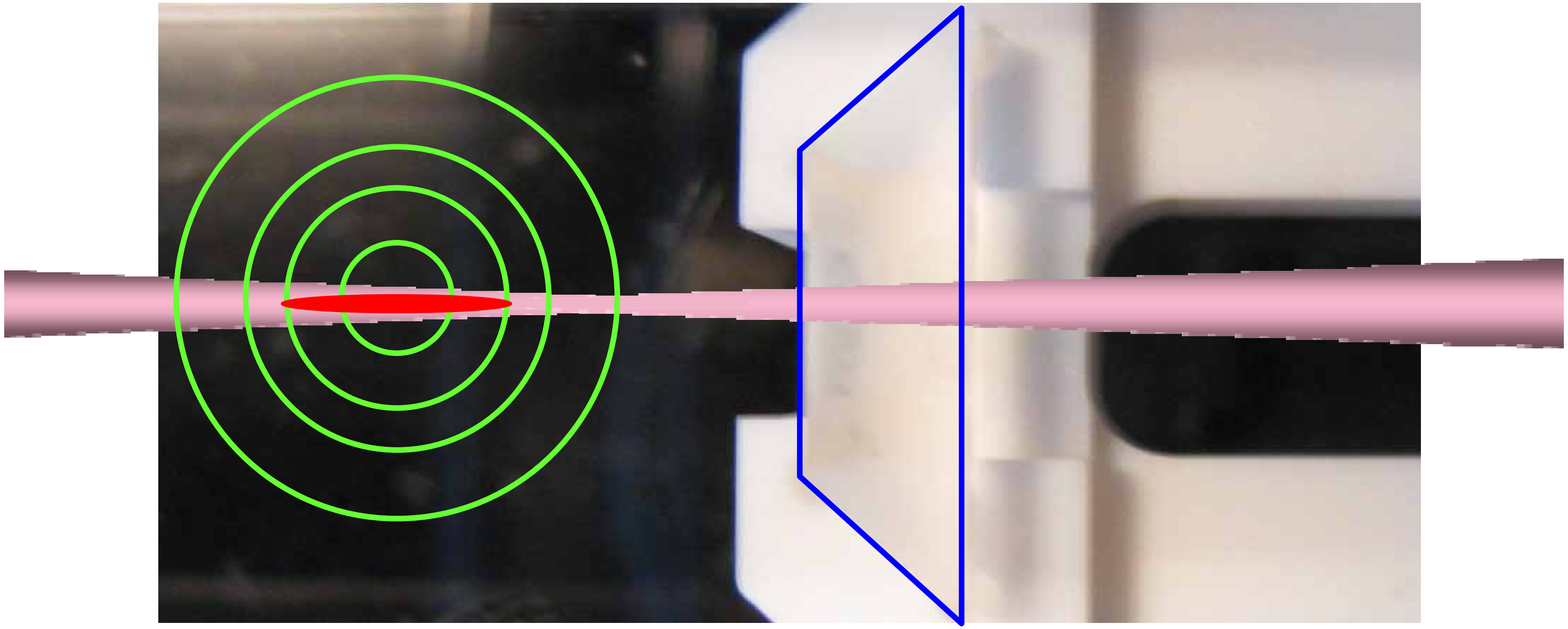
Y.-J Lin *et al.*, Phy. Rev A: **79** 063631 (2009)

Bird’s eye view

IPG fibre laser
1070 nm
10W 50 -120 μm

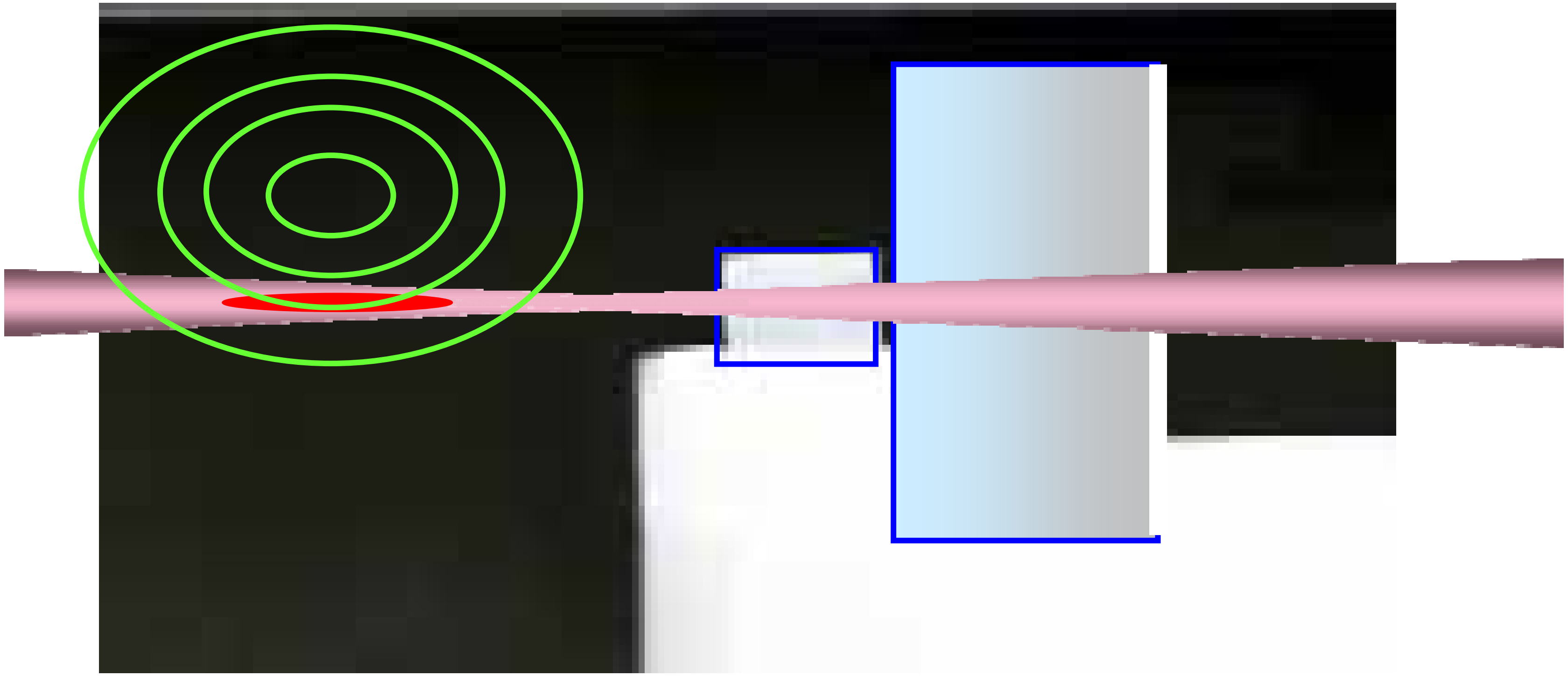


Hybrid potential construction

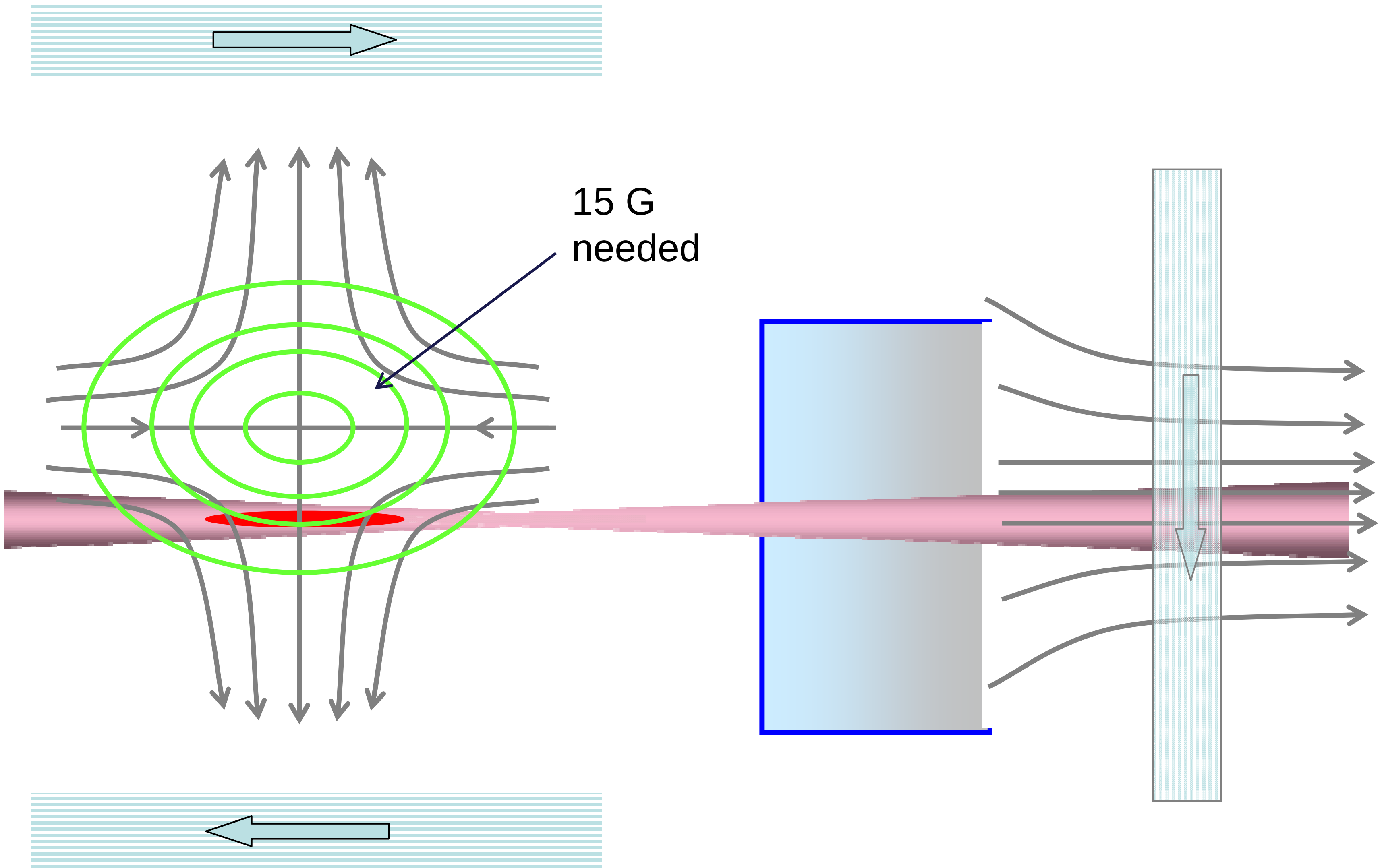


$$U_{\mathrm{dip}}(\mathbf{r}) \cong \frac{3c^2}{\omega_0^{\,3}} \frac{\Gamma}{\Delta} \frac{P}{w_0^{\,2}} \left[1 - \frac{2r^2}{w_0^{\,2}} - \frac{z^2}{z_R^{\,2}}\right] + \mathrm{H.O.}$$

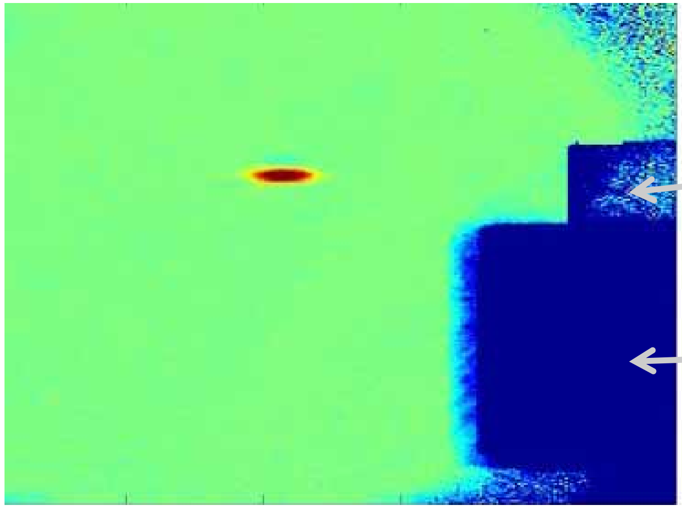
Side view

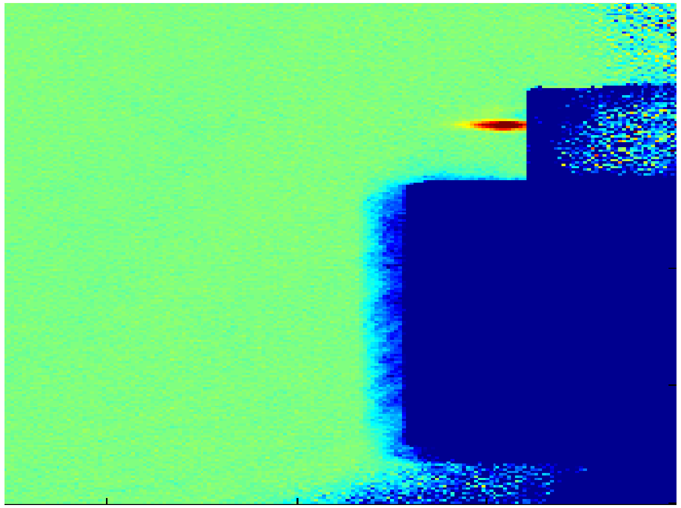


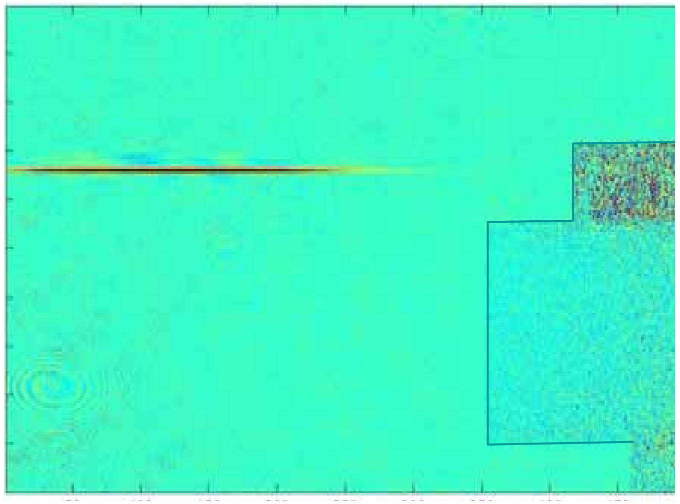
Hybrid potential construction

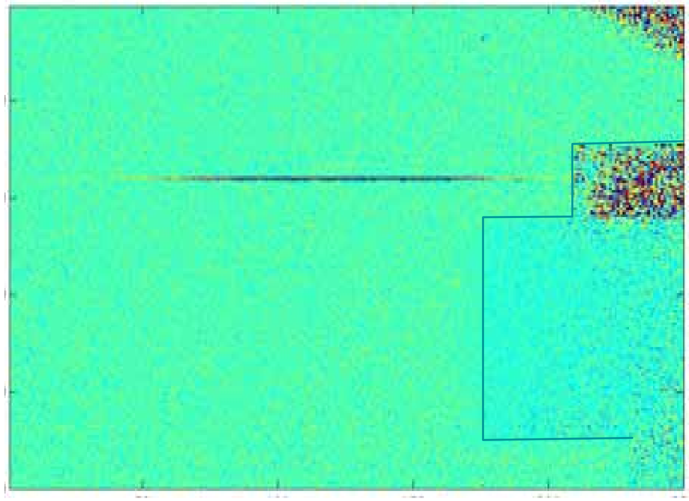


Example images

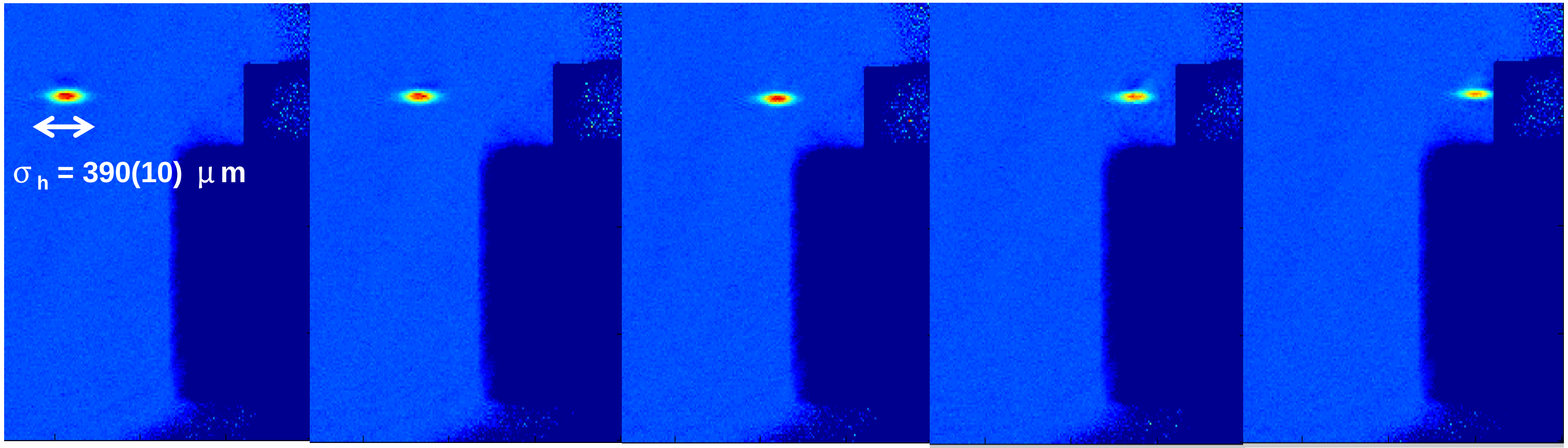








Special details III: guided atoms near a surface

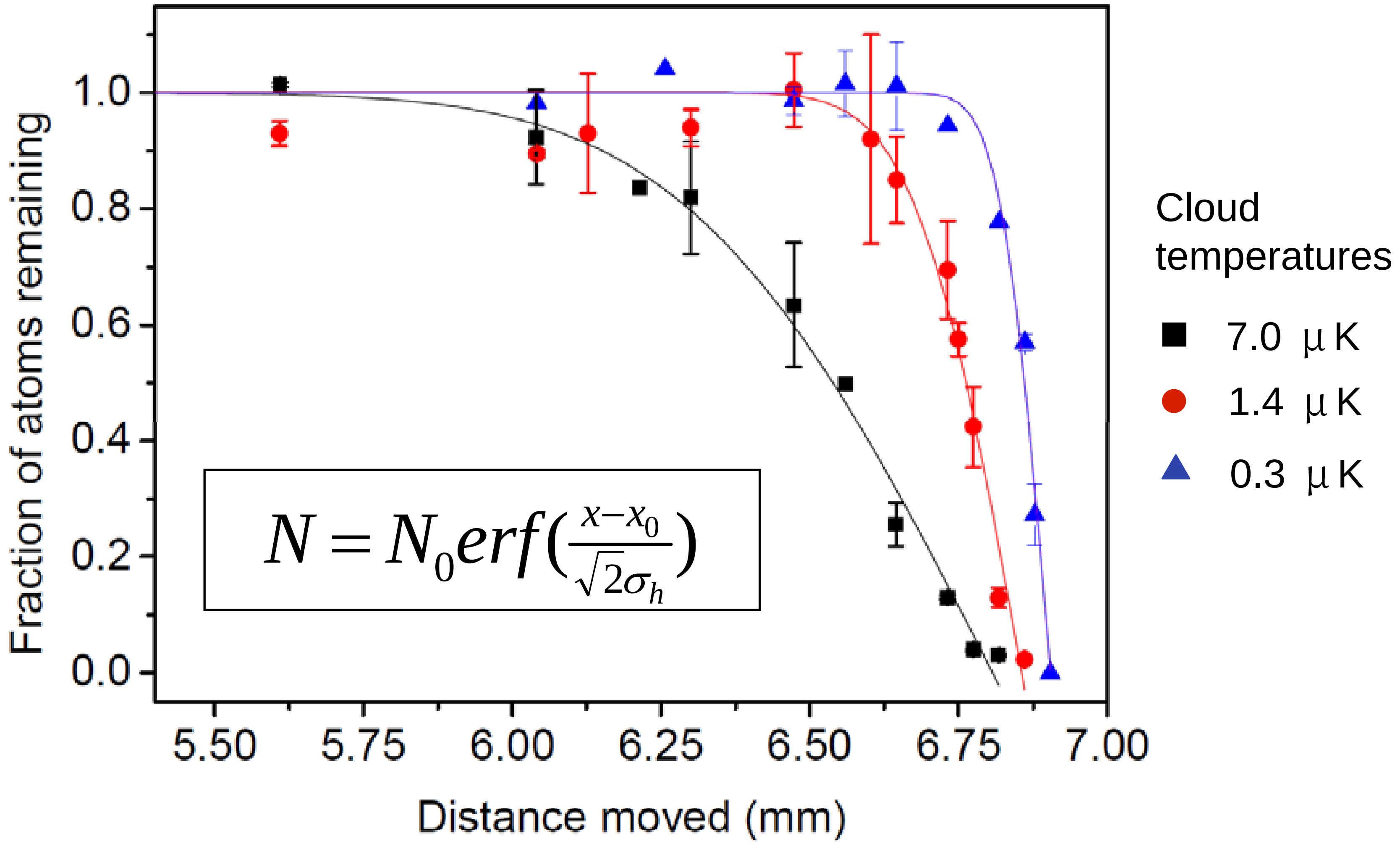


$\omega_{rad} = 2\pi \times 480$ Hz $\omega_{ax} = 2\pi \times 10$ Hz

P = 2.5 W at 1030nm   waist = 55 μm

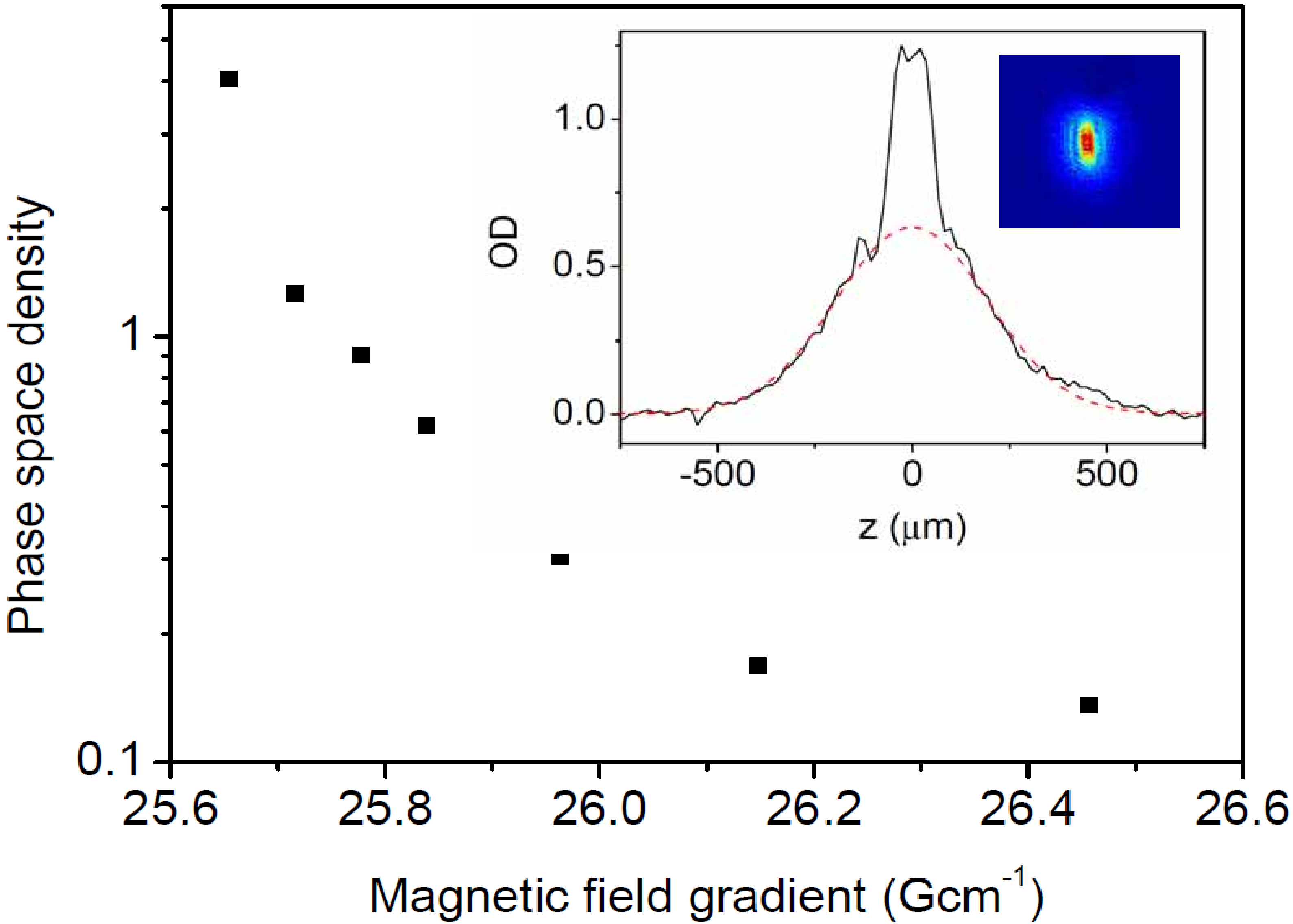
Measure size and temperature 300 nK with surface at 300 K!

Special details III: guided atoms near a surface



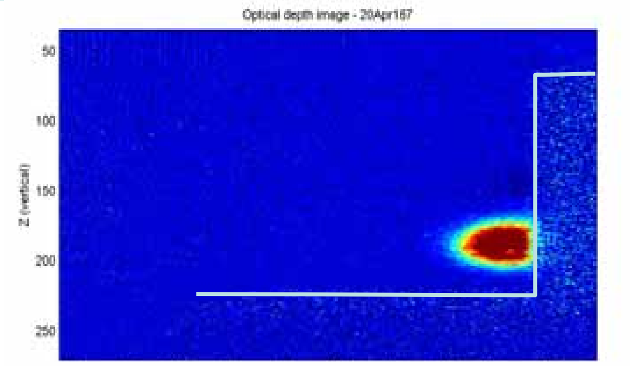
BEC using surface

A.L. Marchant et al submitted to New J. Phys



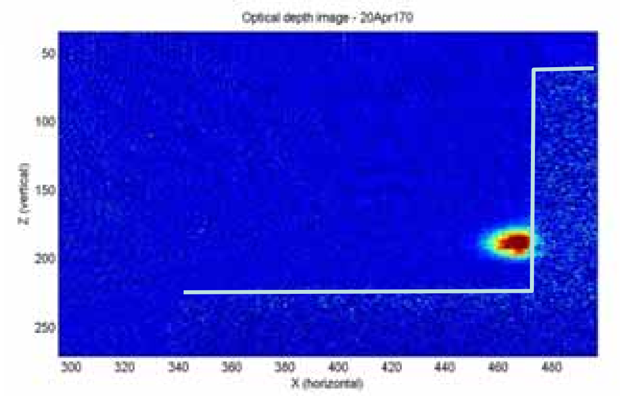
^{87}Rb BEC using surface

$T \sim 4\ \mu K$



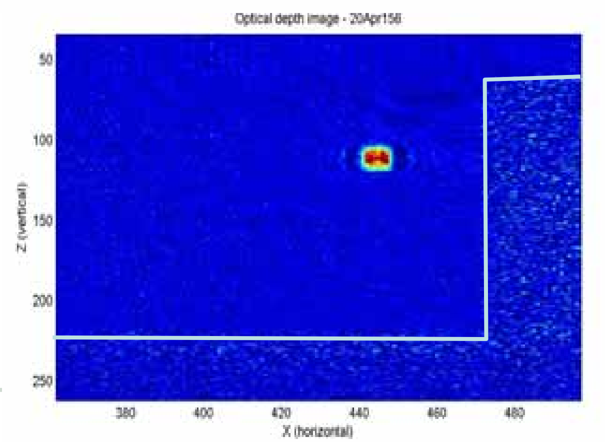
TOF 16 ms

$T \sim 1\ \mu K$



TOF 16 ms

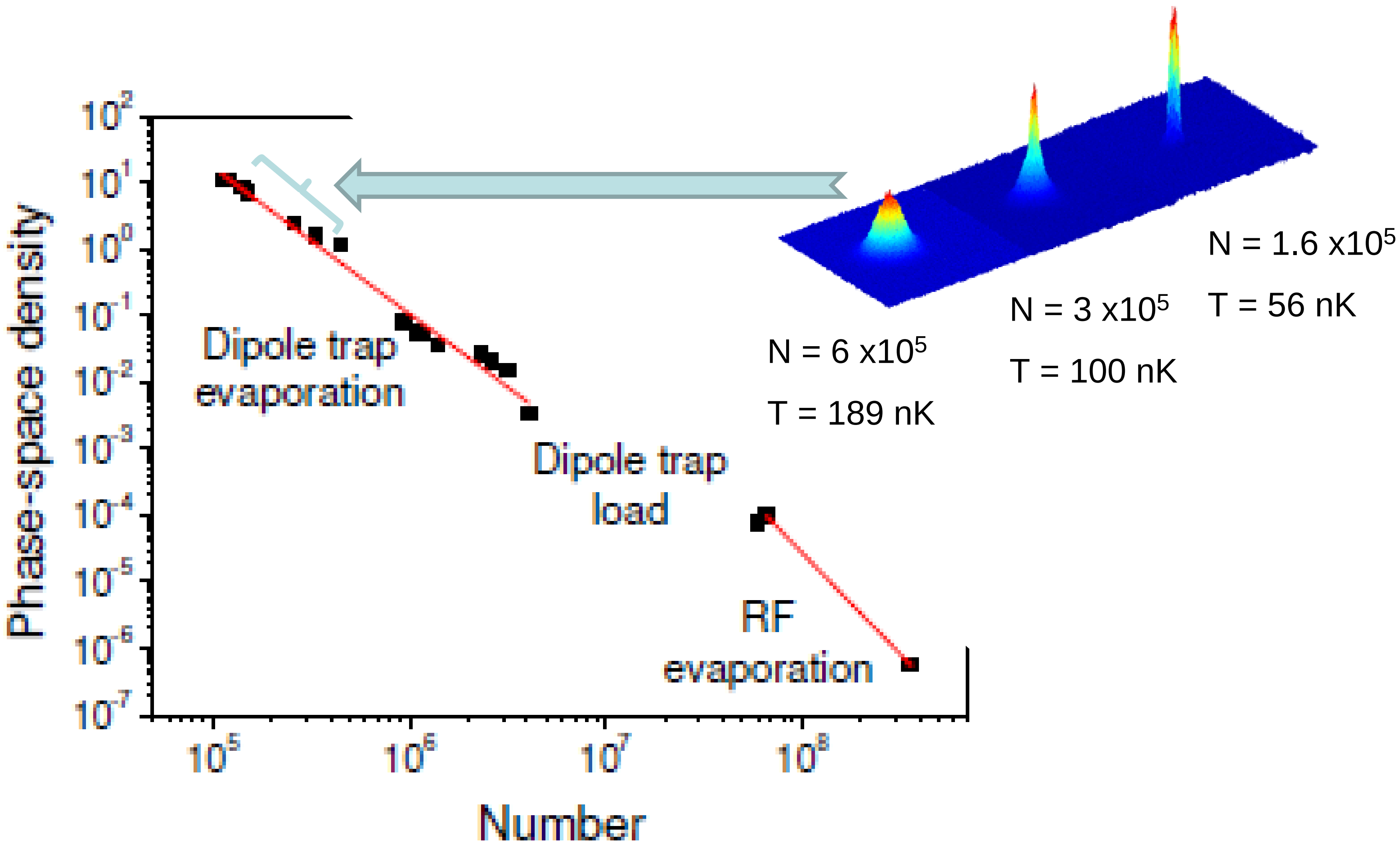
$T \sim 100$ nK



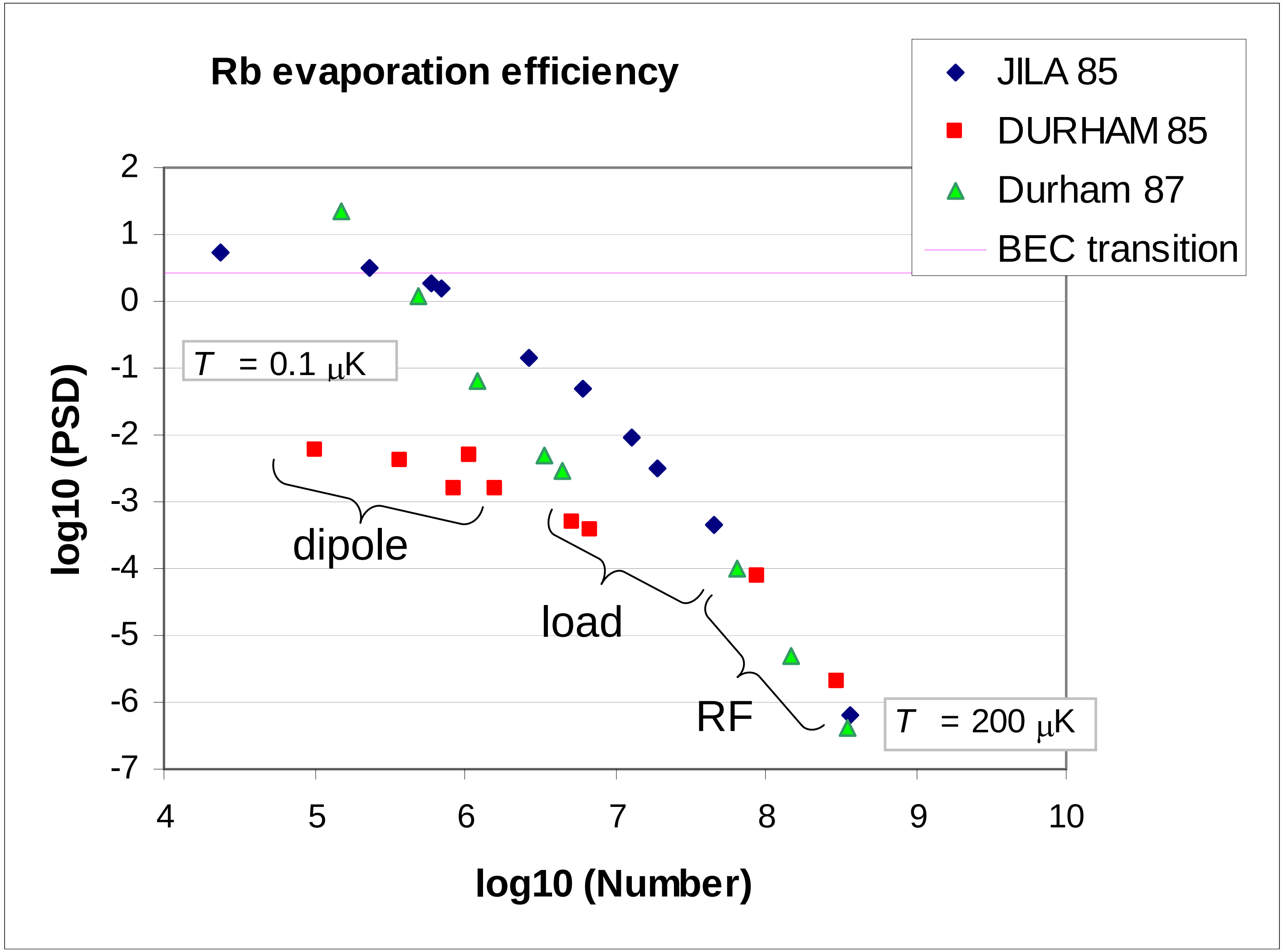
levitated TOF 40 ms

Durham University

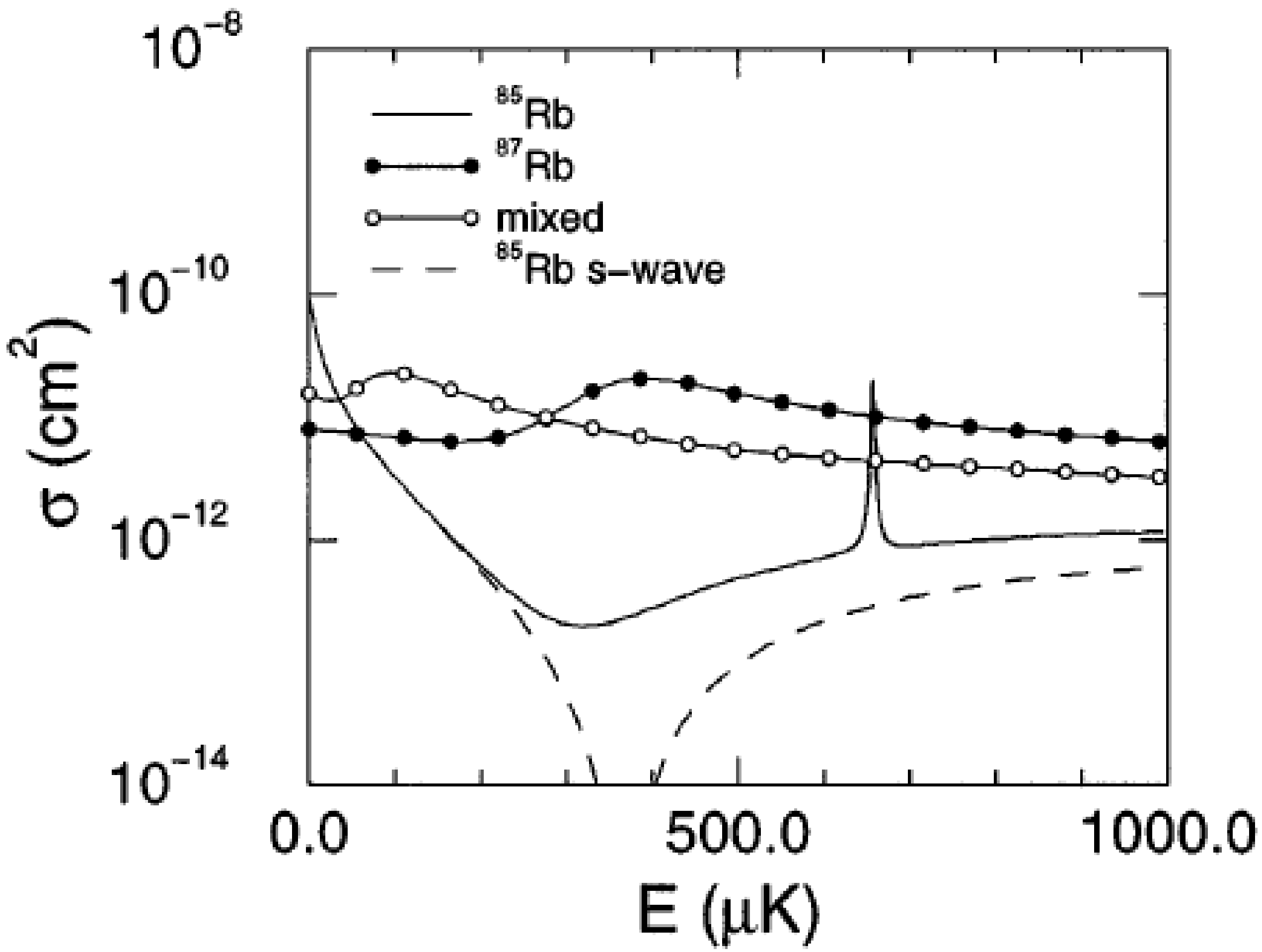
PSD v. N for hybrid trap ^{87}Rb



^{87}Rb and 85Rb evaporation efficiency compared



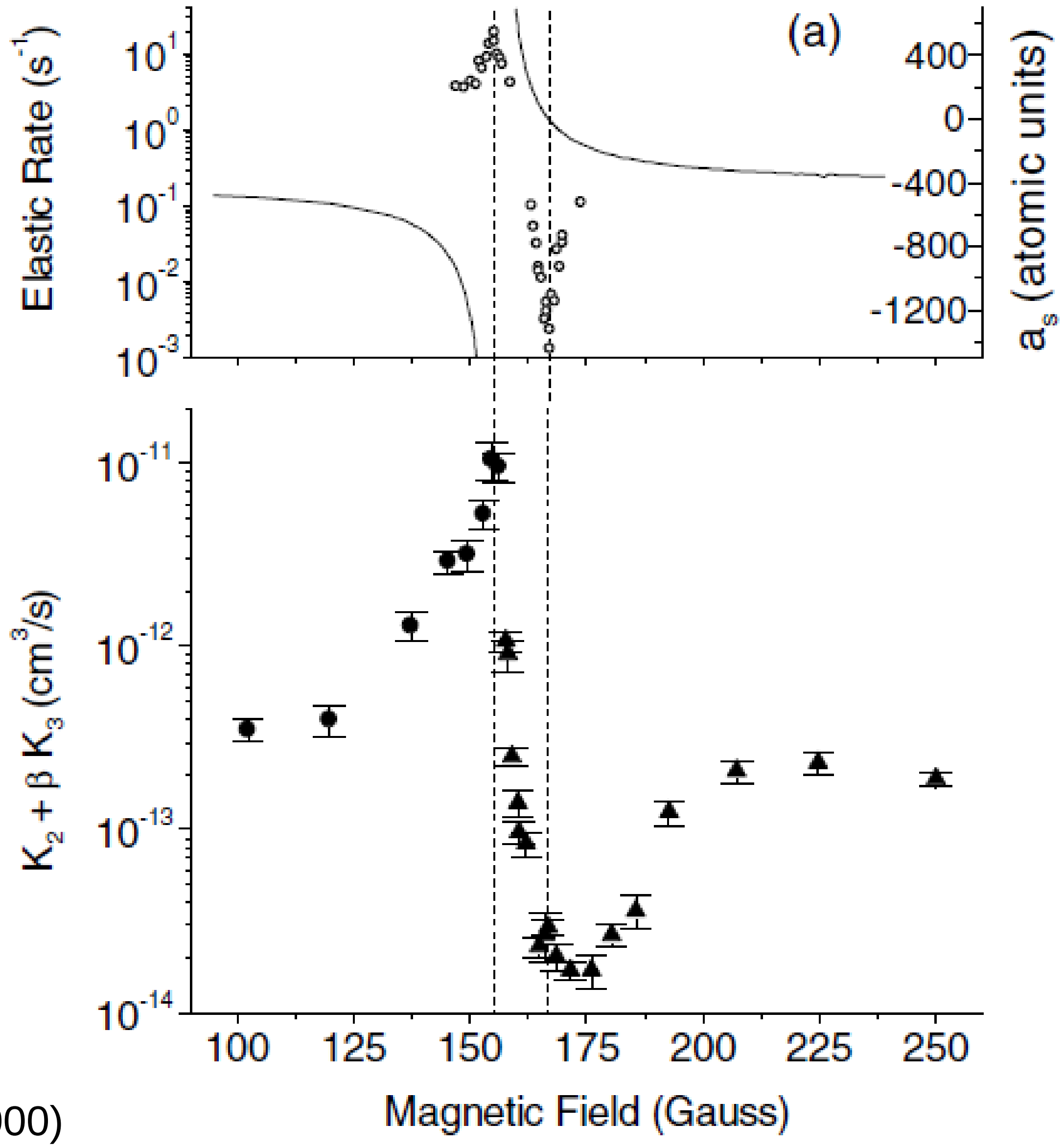
^{85}Rb problems



J.P Burke PRL **80**, 2097 (1988)

^{85}Rb inelastic rates

$$\frac{\Gamma_{\text{inelastic }85}}{\Gamma_{\text{inelastic }87}} \geq 10^4$$



J.L Roberts PRL **85**, 728 (2000)

Dipole trap parameters

$$U_{\mathrm{dip}}(\mathbf{r}) \cong \frac{3c^2}{\omega_0^{\ 3}} \frac{\Gamma}{\Delta} \frac{P}{w_0^{\ 2}} \left[1 - \frac{2r^2}{w_0^{\ 2}} - \frac{z^2}{z_R^{\ 2}} \right] + \mathrm{H.O.}$$

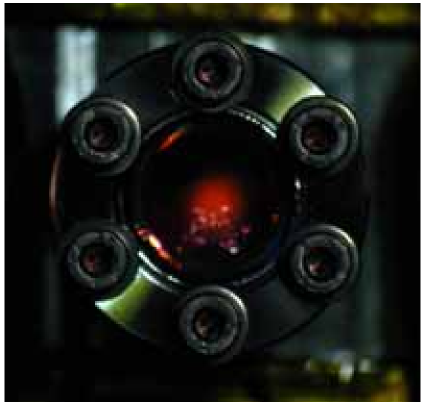
Volume $\sim w_0^{\ 3}$

Trap frequencies $\sim \dfrac{P^{1/2}}{w_0^{\ 2}}$

Trap depth $\sim \dfrac{P}{w_0^{\ 2}}$

Stages of the soliton lab experiment

MOT Load ~ 10^9 in 30s

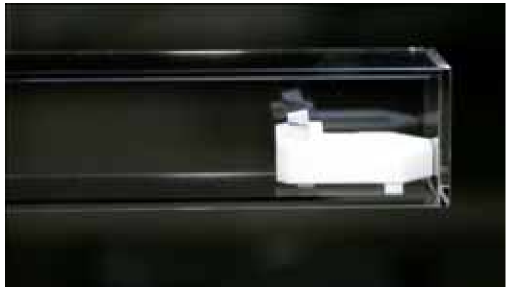


$F = 2$
$m_F = -2$

Transport trap

Science cell
quadrupole trap

N > 3e8
T ~ 200 μK



RF induced evaporative cooling

N ~ 5e7
T ~ 35 μK
ρ ~ 1e-4

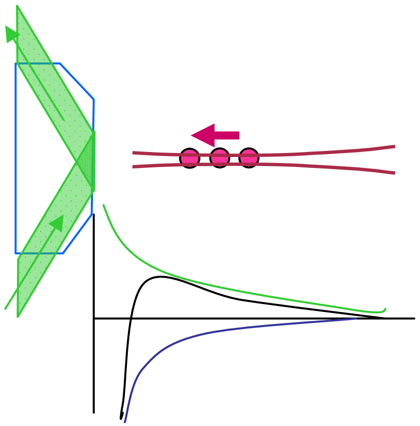
Hybrid optical-magnetic dimple trap ?

Sympathetic cooling of ^{85}Rb by ^{87}Rb ?

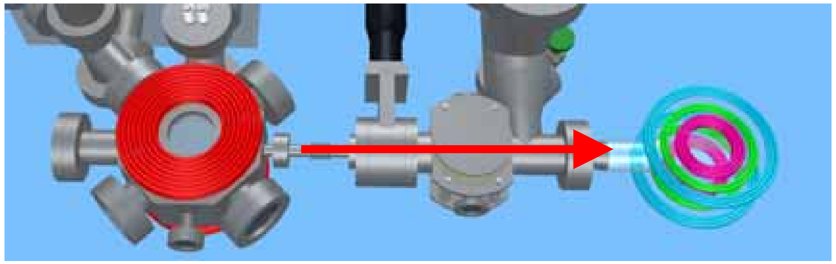
Degenerate Raman sideband cooling ?

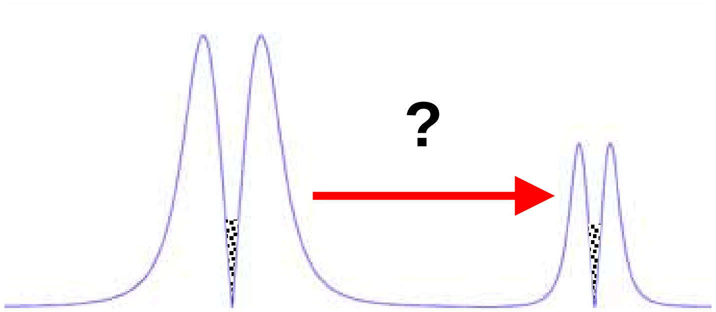
more evap cooling

BEC & solitons
Atom-surface studies



Special details IV: Merging quadrupole traps





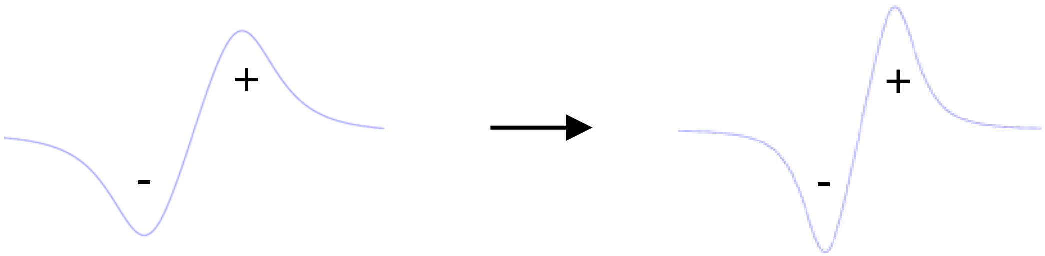
- Building on work by Jesper Bertelsen. *PRA* **75**, 013404 (2007)

Why do we multiple load?

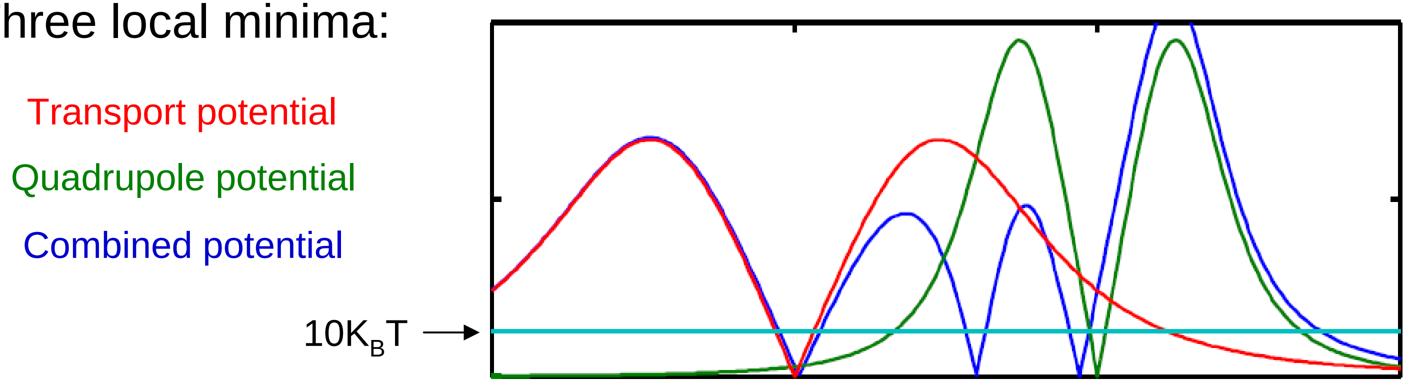
- Demonstrate a novel method of loading ^{85}Rb and ^{87}Rb with two lasers, instead of four.
- Two species mixing *PRA* **64**, 021402(R) (2001)
- Interspecies Feshbach studies and molecule formation. *PRL* **97**, 180404 (2006)
- Sympathetic cooling.

Merging two quadrupole traps

- Merging two *different* coils:

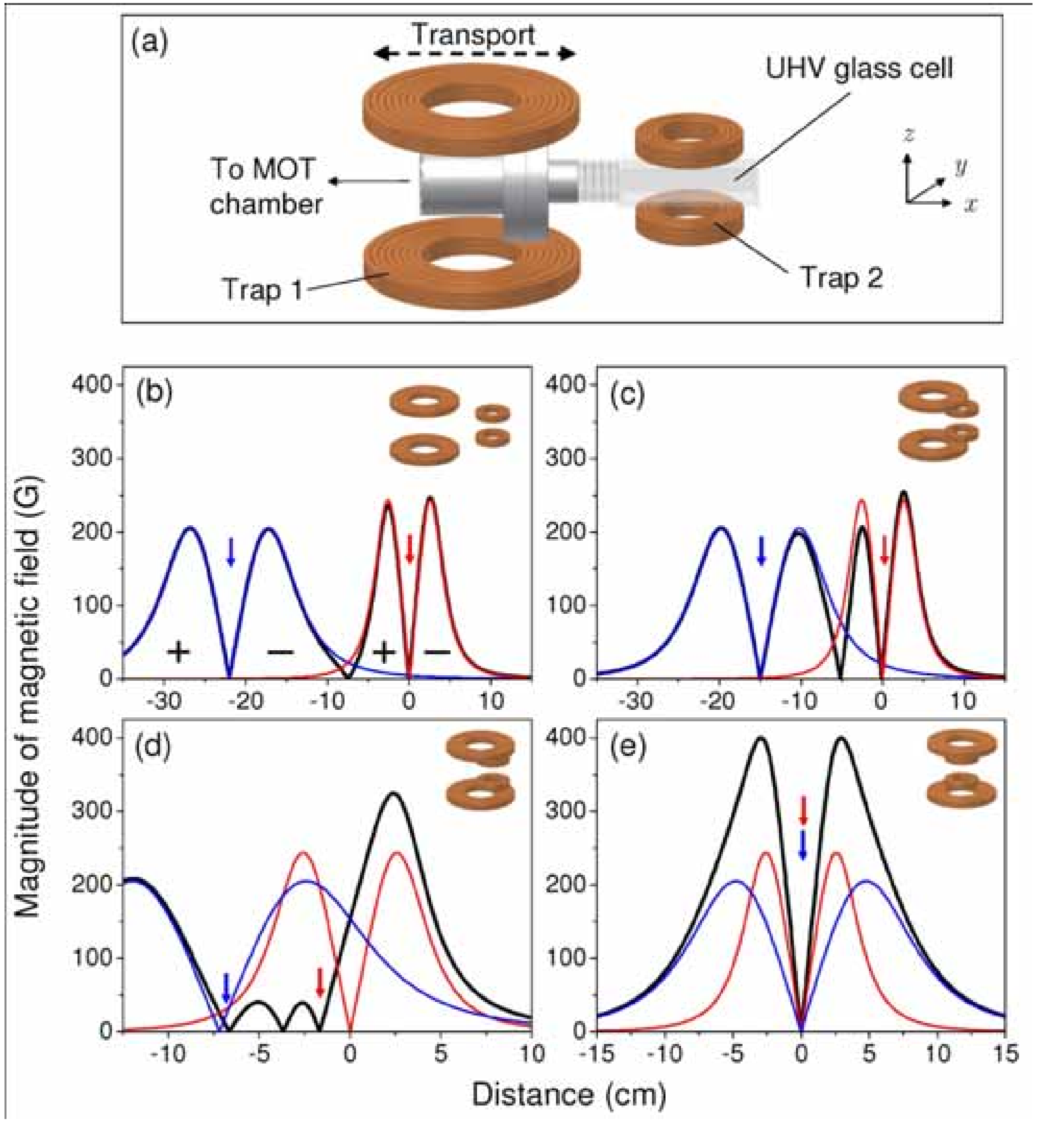


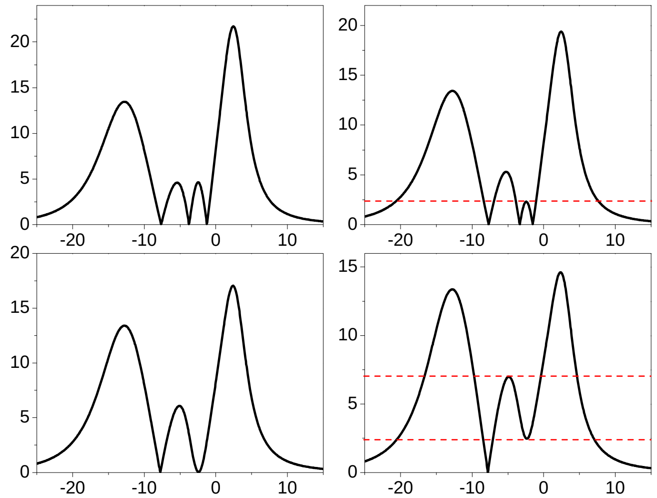
- Sign of the field very important – destructive interference.
- Three local minima:



- Adjust gradient ratio to change barrier heights and minima positions.

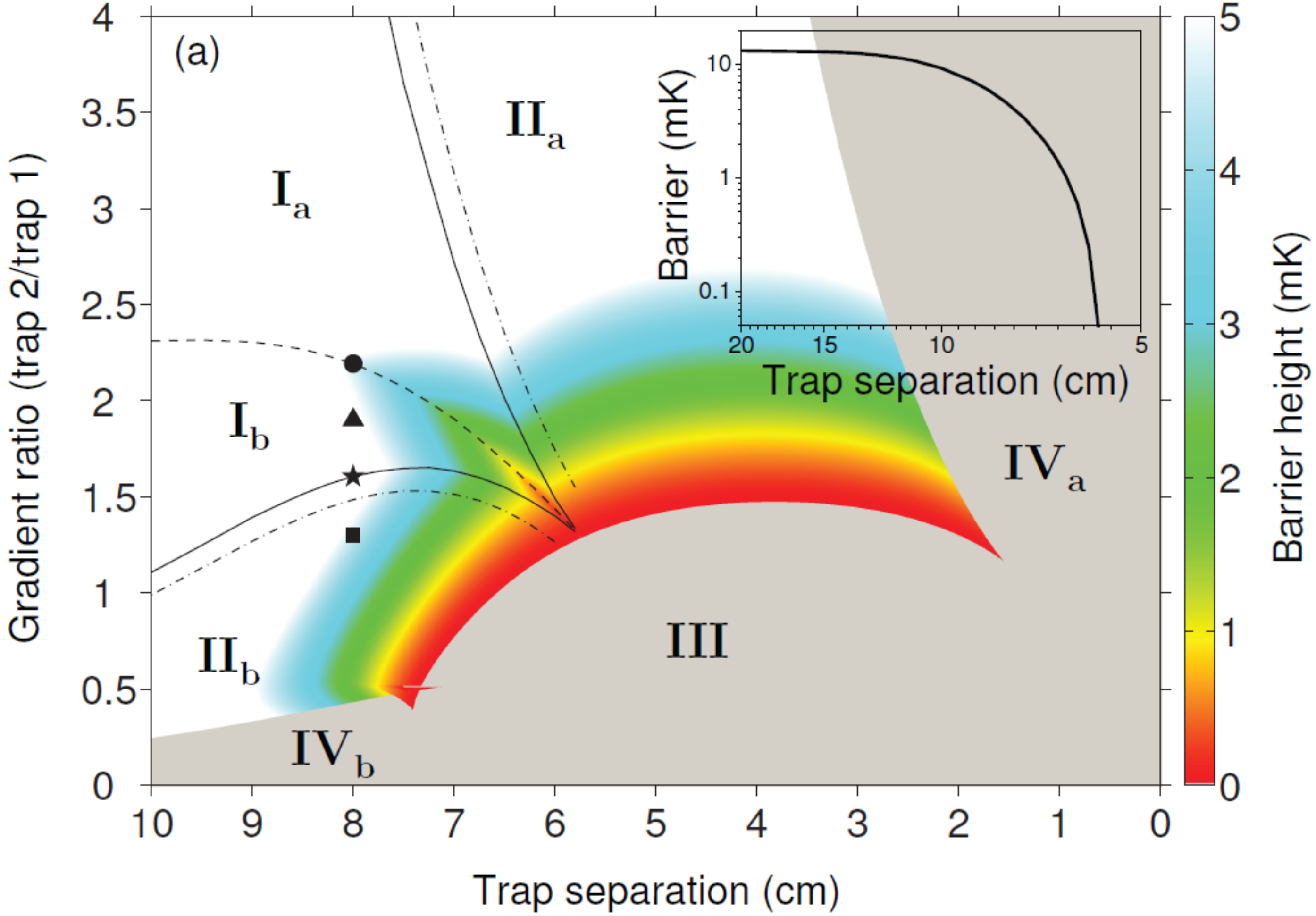
Controlling the barrier heights



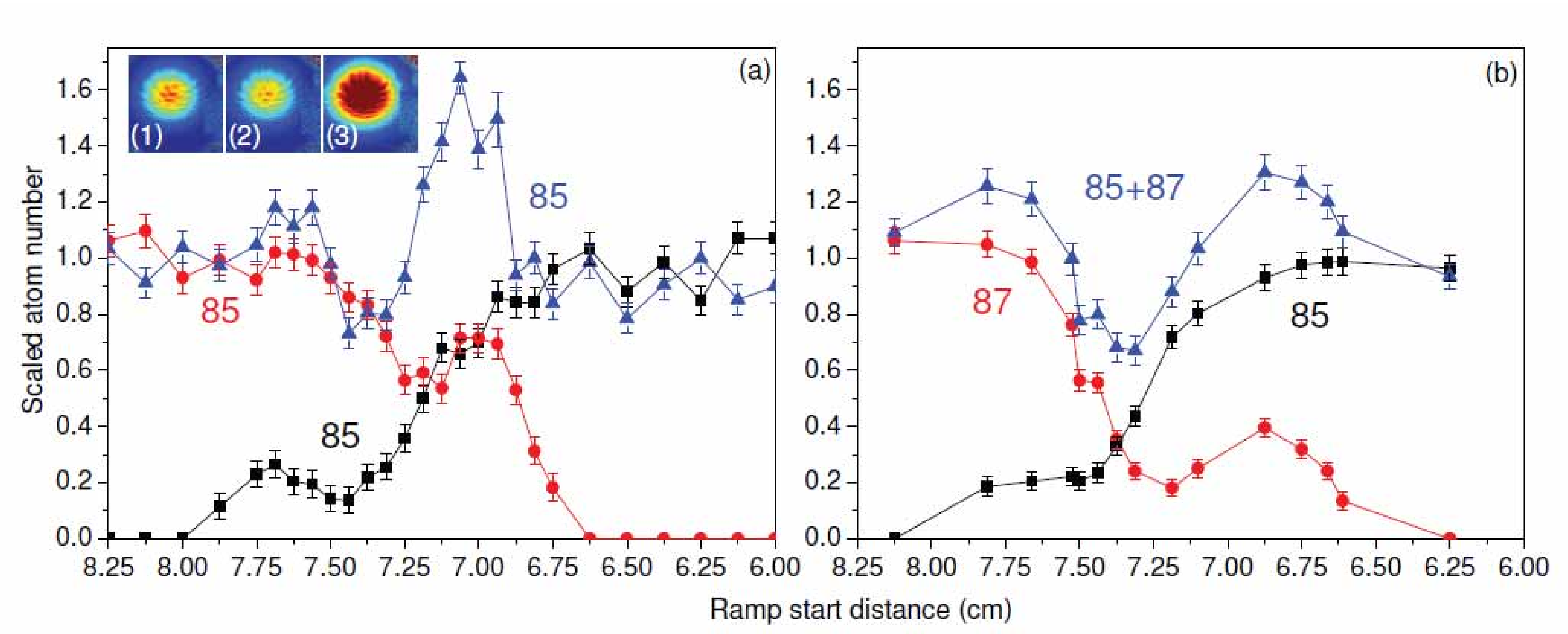


How to do it: A general recipe

S.Haendel et al, PRA **85**, 053633 (2011)



Magnetic merging results at Durham



Outline

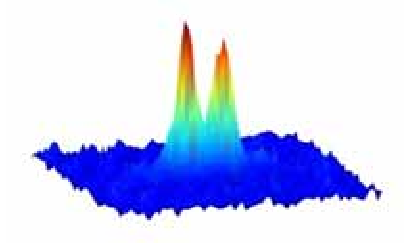
❶ Introduction to solitons
- *Necessary background*

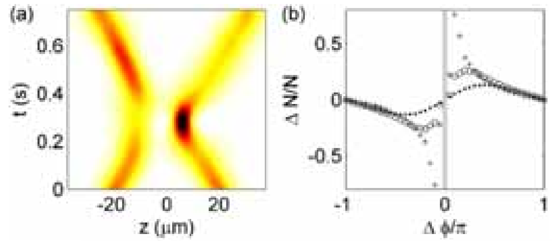
❷ BEC collapse & matter-wave solitons
- *Basic physics*
- *The past experiments*

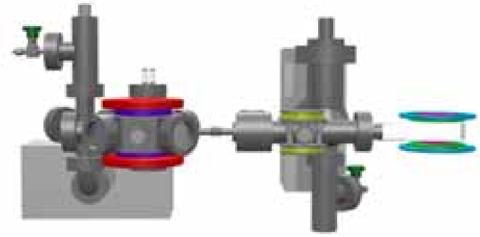
❸ Soliton dynamics (theory)
- *Numerical simulations*
- *Binary soliton collisions*

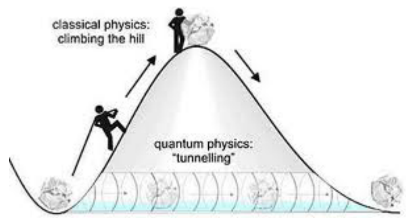
❹ Our experimental progress
- *Current status and next steps*

❺ Future goals
- *Atom surface studies*
- *Quantum reflection with solitons*









Why measure surface forces?

Geraci *et al*. PRD **78**, 022002 (2008)

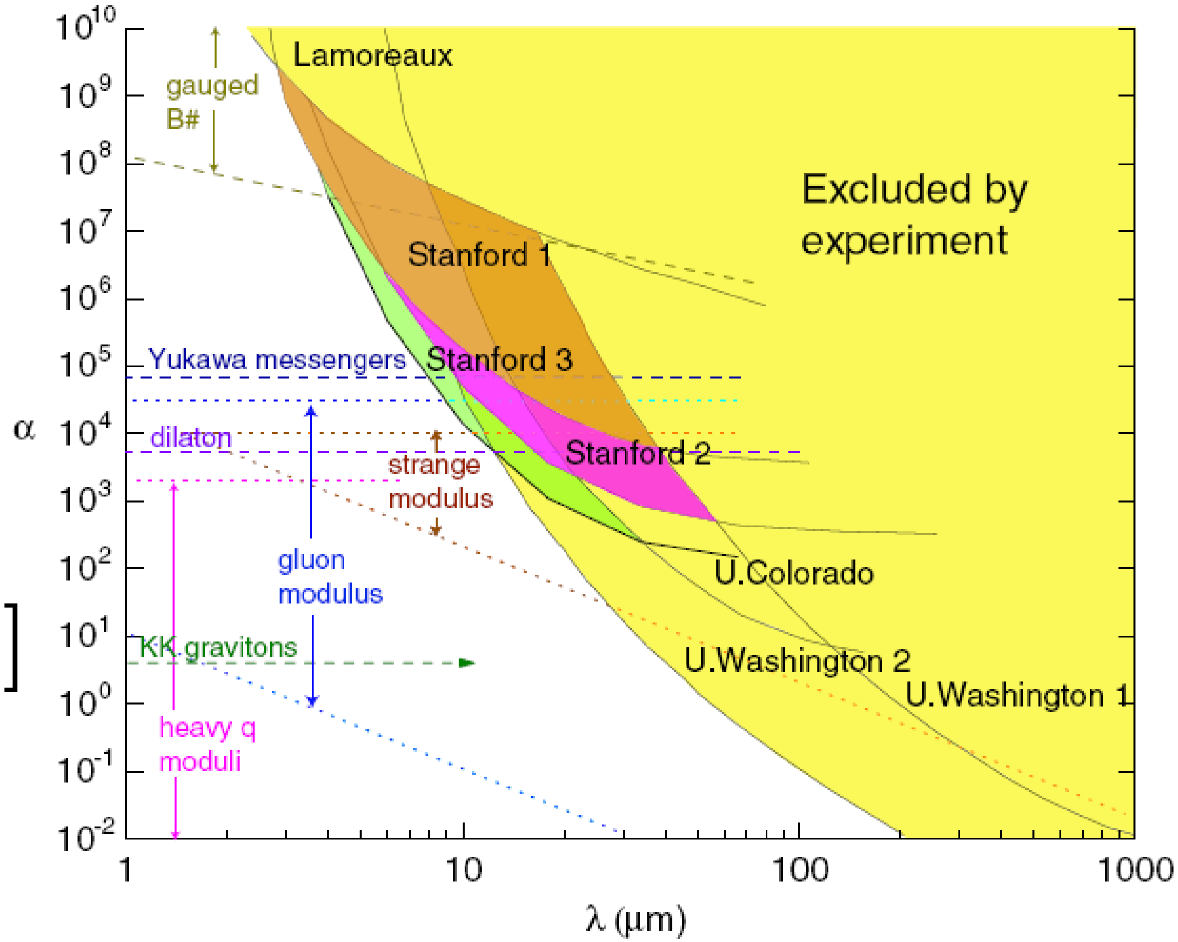
Short range tests of gravity are sensitive to new physics that manifests itself as deviations from Newtonian inverse square law.

Parameterize as Yukawa potential:

$$V(r) = -\frac{G m_1 m_2}{r}\left[1 + \alpha\, e^{-r/\lambda}\right]$$

Strength relative to gravity

Range of new force



Surface measurements

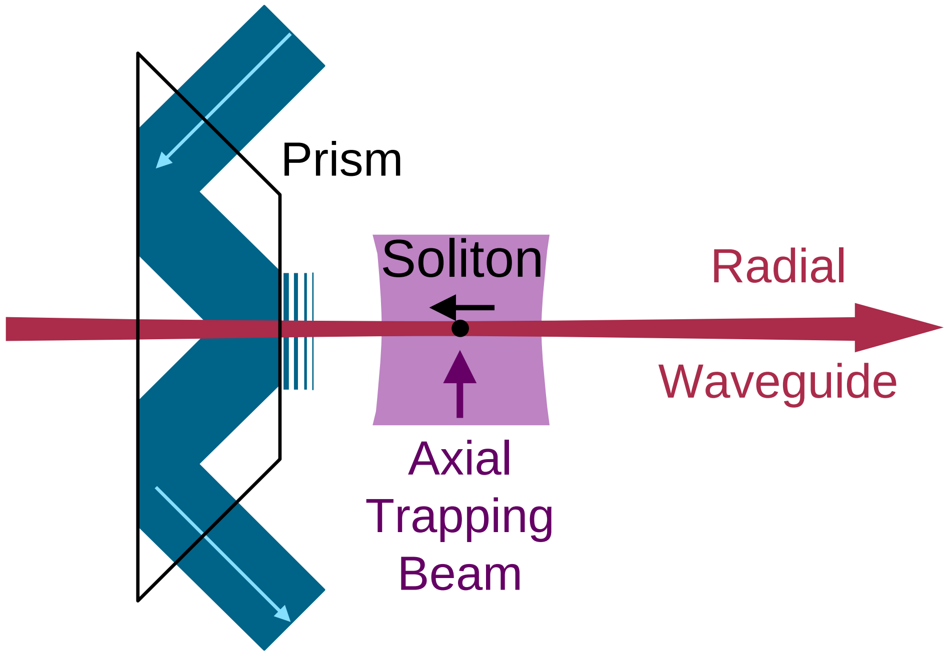
λ is the wavelength of the dominant atomic transition

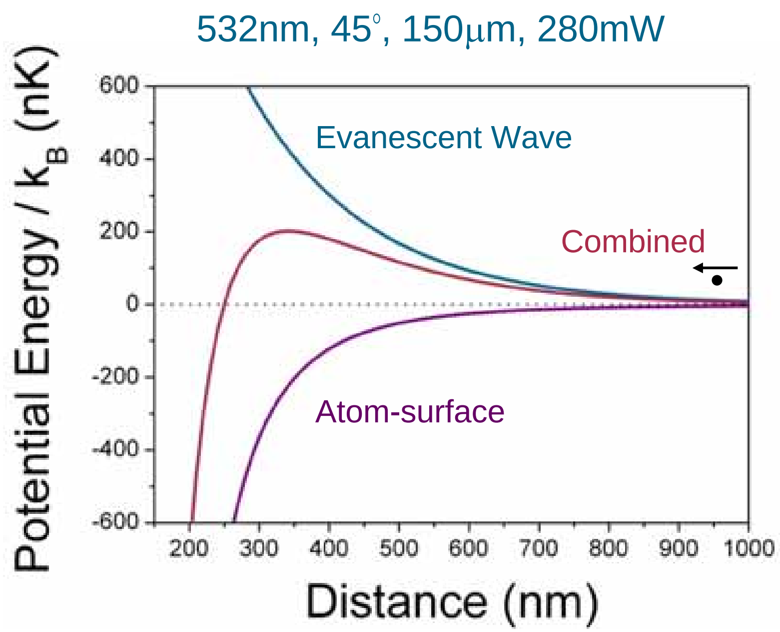
For z < λ/2π : van der Waals interaction

$$U(z) = -\frac{C_3}{z^3}$$

For z > λ/2π: Casimir-Polder result

$$U(z) = -\frac{C_4}{z^4}$$

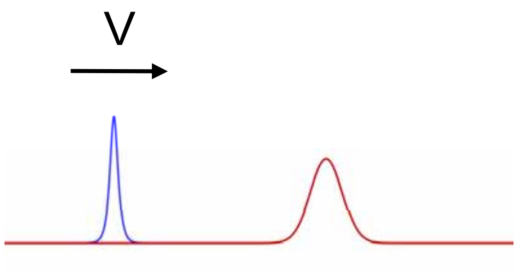




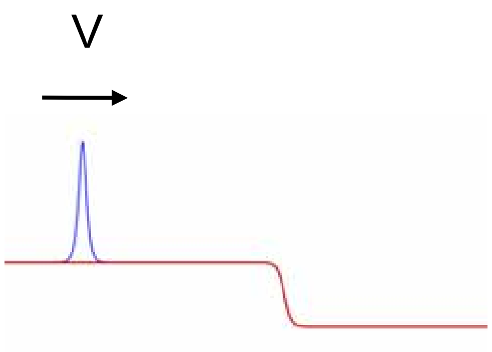
Solitons: well-localised, self stable structures
Optical trap: precise control over incident velocity
Quantum reflection with no evanescent field

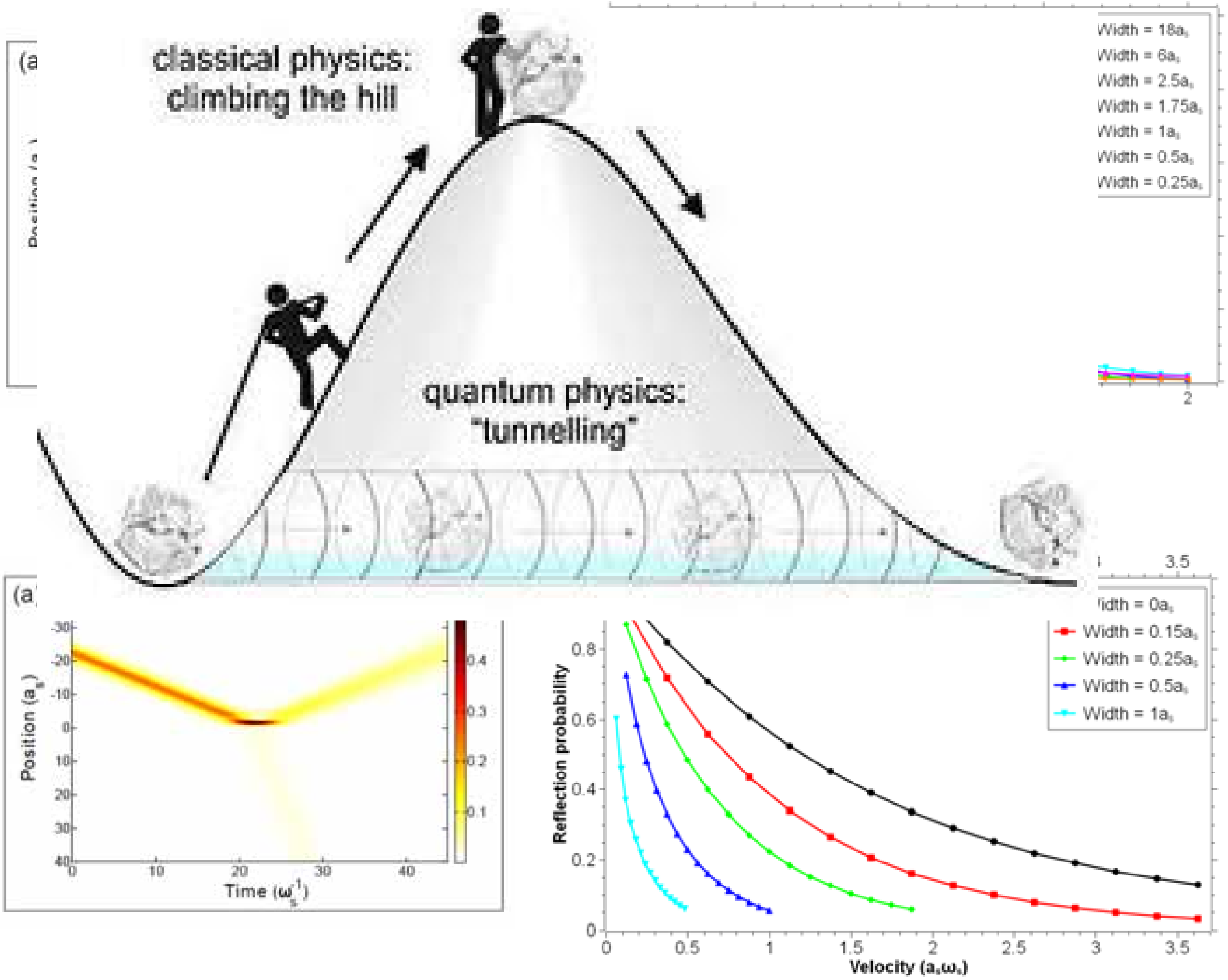
Quantum reflection introduction

Quantum tunnelling



Quantum reflection





Quantum Reflection - overview

Requires the local wave vector $k = \sqrt{\left(k_\infty^2 - 2mU/\hbar^2\right)}$

To change by more than k over a distance of 1/k *i.e.* $\frac{1}{k^2}\frac{dk}{dr} > 1$

For a surface, in the low energy limit ($k \to 0$):

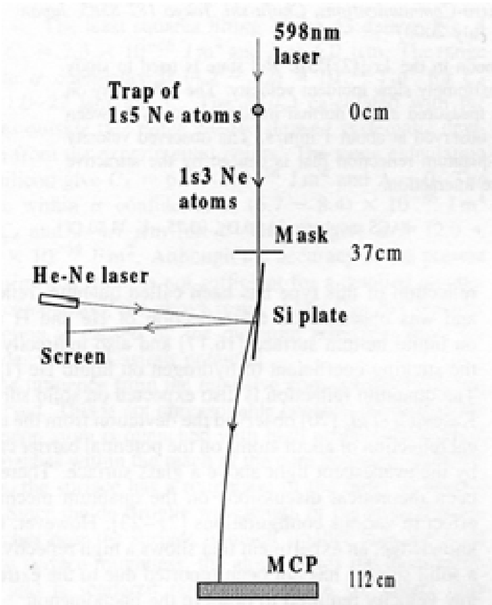
$$R \approx 1 - 2\beta_4 k \qquad C_4 = \frac{\beta_4^2 \hbar^2}{2m}$$

Friedrich *et al.*, PRA. **65**, 032902 (2002)

For appreciable reflection:

Weak potential:
- H,He on He surface

Low velocity:
- Grazing incidence

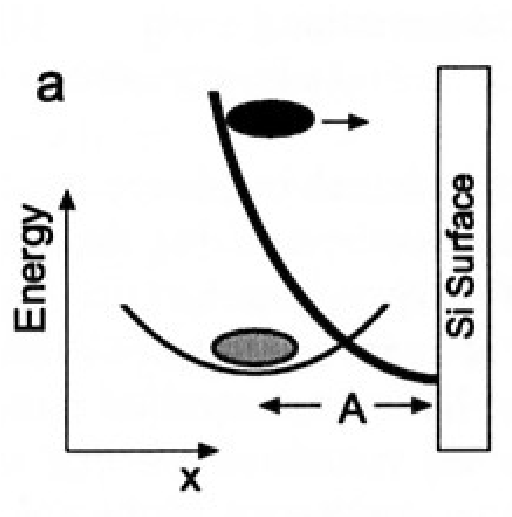


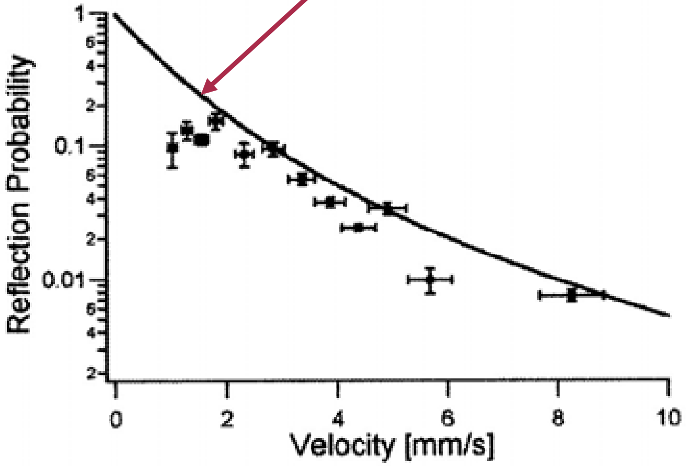
Shimuzu, PRL **86**, 987 (2001)

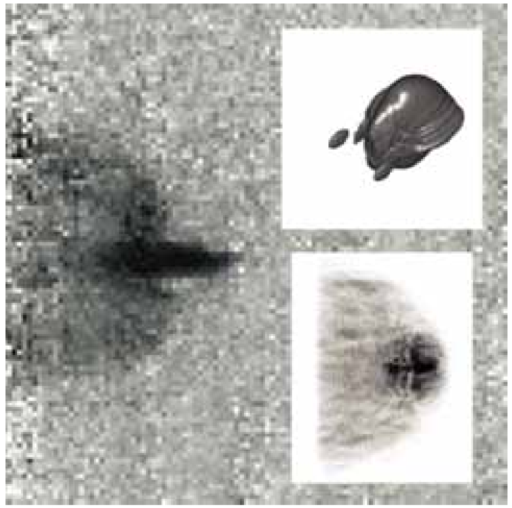
Quantum Reflection of a Na BEC

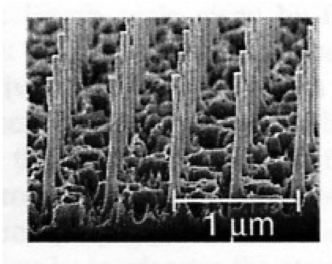
Pasquini *et al.*, Phys. Rev. Lett. **93**, 223201 (2004)

Anomalous Quantum Reflection
Scott *et al.*, Phys. Rev. Lett. **95**, 073201 (2005)







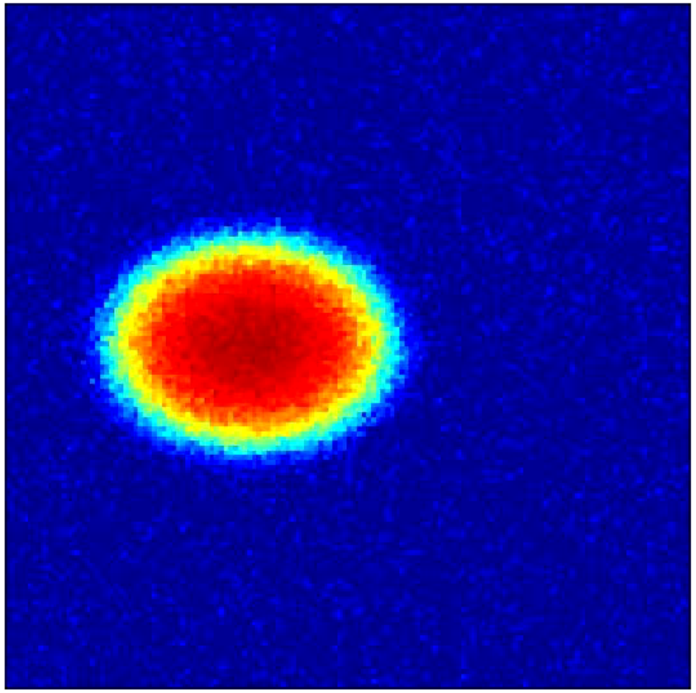


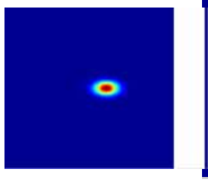
Disruption of reflected condensate
Scott *et al.*, Phys. Rev. A. 74, 053605 (2006)

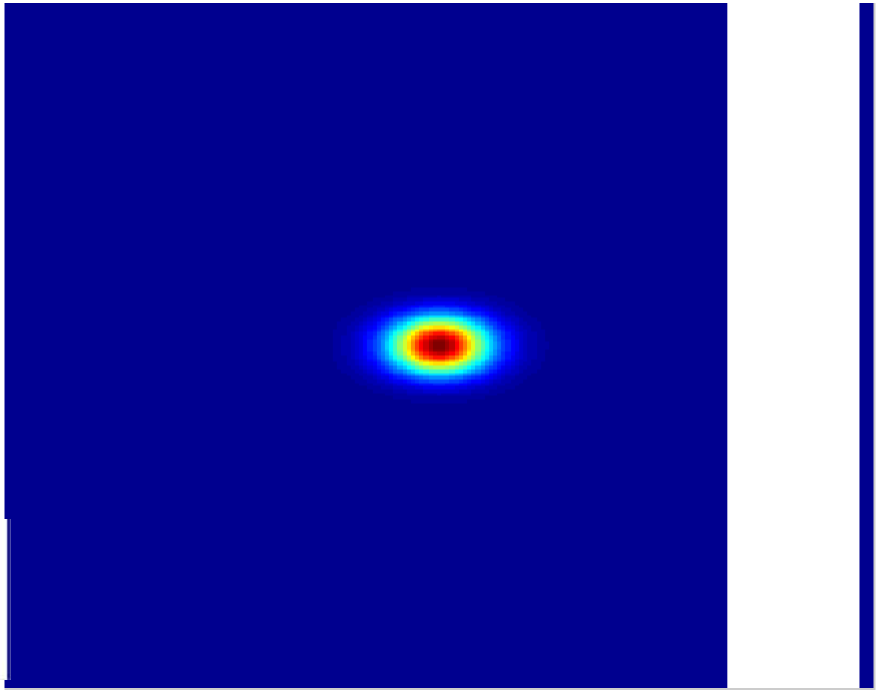
Quantum Reflection –
Simulation of launch from 3D harmonic trap

MIT - Na BEC
10^6 atoms, diameter ~ 150 μm
$v = 3\ \mathrm{ms}^{-1}$

^{85}Rb soliton
1500 atoms, diameter ~ 5 μm
$v = 0.85\ \mathrm{ms}^{-1}$







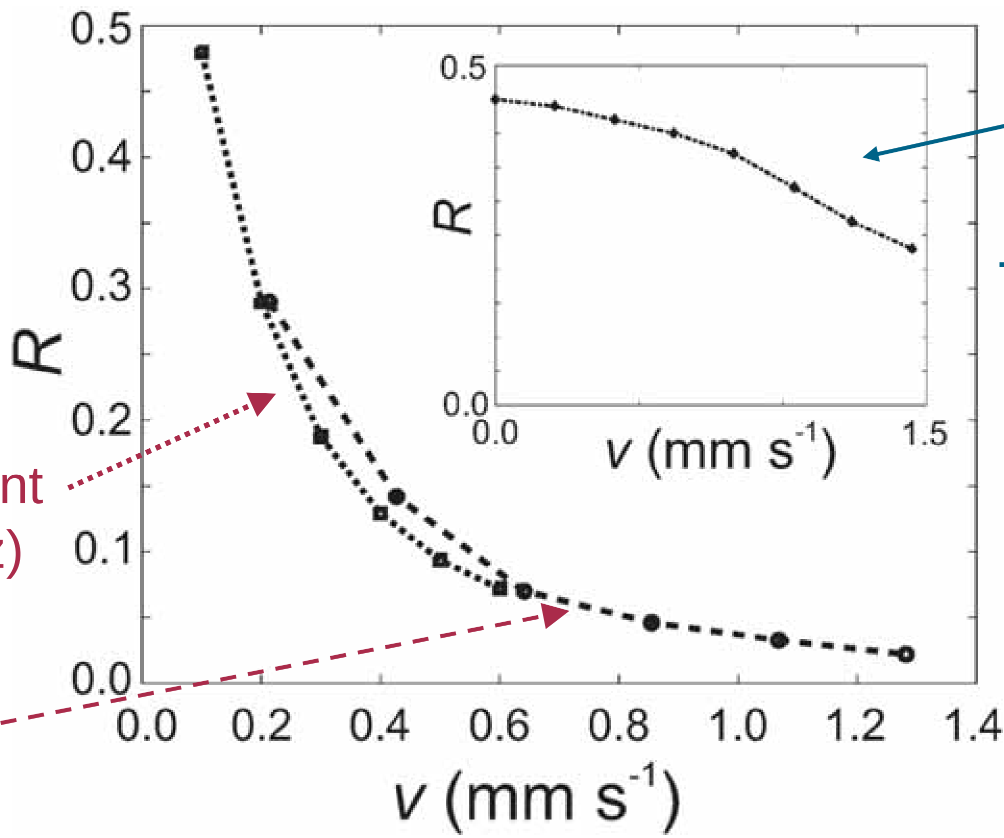
Movies courtesy of Scott, Judd, Fromhold - Nottingham

Quantum Reflection – Probability for Si surface

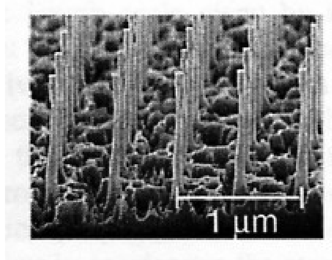
85Rb solitons
Solid surface

Radial confinement only (ν_r = 17.3Hz)

With axial confinement (ν_z = 6.8Hz)

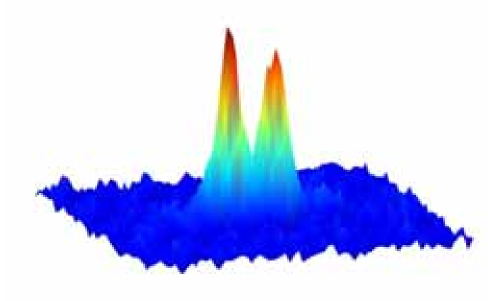


Na BEC
as in MIT
experiments
- pillared surface

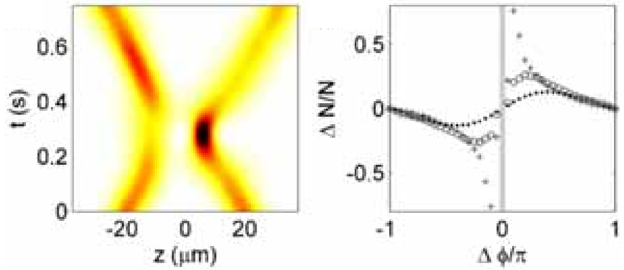


Summary – bright solitons in a variable trap

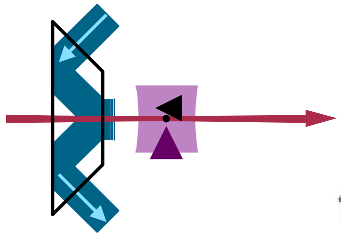
- Introduction to solitons
- Feshbach resonance to control interactions.
- Soliton formation for a<0.
- Explains “large condensate remnants”.



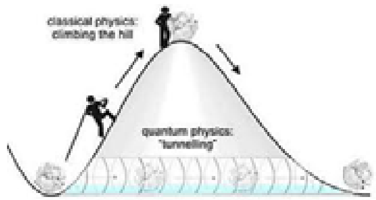
- Phase and velocity important for stability.
- Population transfer and interferometry.
- Open questions need experiments



- Proposed experiments
- Probe atom surface interactions.
- Quantum reflection of solitons.
- Our apparatus and progress so far.



- **Solitons**
    - **Understand, control, apply.**



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