Synthetic gauge fields for ultracold neutral atoms

Yu-Ju Lin[1,2]

Robert Compton[2], Karina Jimenez-Garcia[2], Trey Porto[2], William Phillips[2] and Ian Spielman[2]

[1] Present address: Institute of Atomic and Molecular Sciences, Academia Sinica, Taiwan

[2] Joint Quantum Institute, National Institute of Standards and Technology, Gaithersburg and University of Maryland

23 August, 2011

Outline

- Introduction of ultracold quantum gases

Review

✶ Bose-Einstein condensate (BEC)

✶ Light-atom coupling:
trapping, manipulation, probe imaging

Main topics:

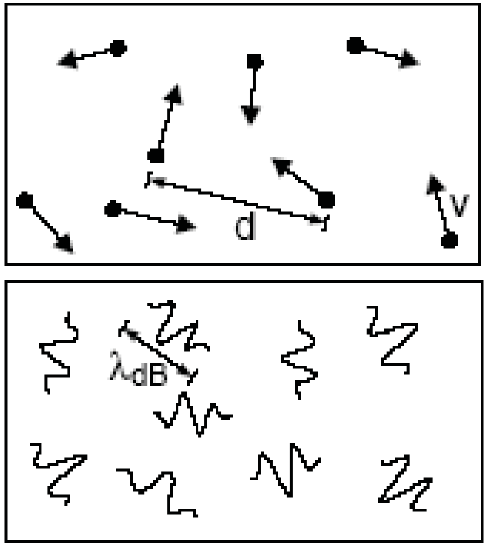
- To “charge” neutral atoms: synthetic gauge potentials A*

- Uniform A* $\rightarrow$ $\boxed{\vec{B}^* = \nabla \times \vec{A}^*}$

- Spin-dependent A* :
can make non-abelian gauge potentials and spin-orbit coupling

$$\left[A_i^*, A_j^*\right] \neq 0$$

Quantum gases

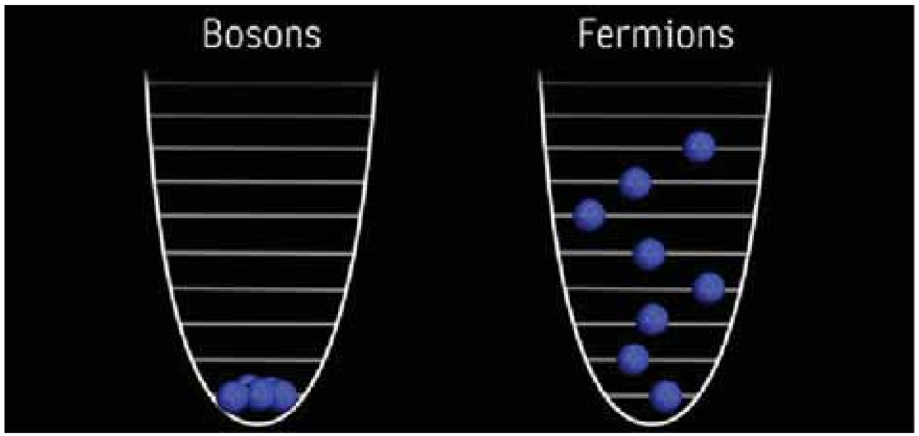


High Temperature T:
thermal velocity v
density d^{-3}
"Billiard balls"

Low Temperature T:
De Broglie wavelength
$\lambda_{dB}=h/mv \propto T^{-1/2}$
"Wave packets"

even lower T: $\frac{N}{V}\lambda_{dB}^{3} \geq 1$

quantum gases



Ultracold quantum gases

ultracold atoms: degenerate, non-classical gases $\frac{N}{V}\lambda_{dB}^3 \geq 1$

$\boxed{\text{(1) cold and dilute:}}$

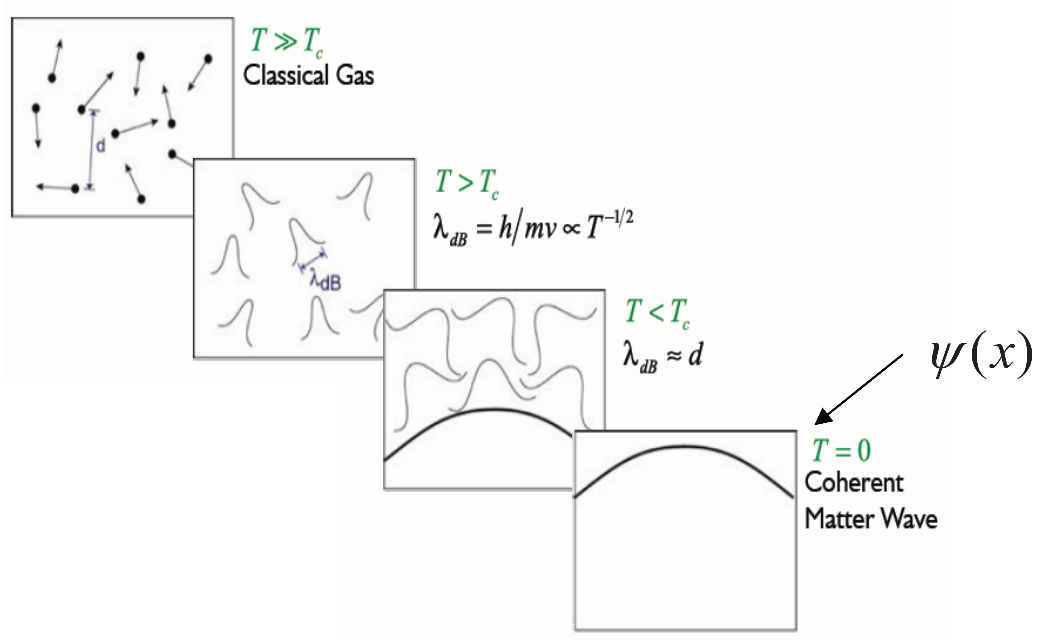
cold: s-wave scattering dominated: $\lambda_{dB} > R_{vdw}$
dilute: inter-particle spacing $d > R_{vdw}$
cold and dilute: reduced to contact interaction

$$V(r-r') = \frac{4\pi\hbar^2 a}{m}\delta(r-r'), \quad a = \text{s-wave scattering length}$$

Ex. Bose-Einstein condensate (BEC) , Degenerate Fermi gas (DFG)

General references:
1. Many body physics with ultracold atoms, Rev. Mod. Phys. **80**, 885 (2008).
2. Making, probing, and understanding Bose-Einstein condensates, arxiv cond-mat 9904034

Review: Bose-Einstein condensate (BEC)



- macroscopic occupation of a single-particle state $\varphi_0(x)$
  exist for weakly interacting bosons:
  described by the order parameter $\psi(x) = \sqrt{N}\varphi_0(x)$

- dynamics: Time dependent Gross-Pitaevskii Equation (TDGPE)

$$i\hbar \frac{\partial \psi(r,t)}{\partial t} = \frac{\hbar^2}{2m}\nabla^2\psi(r,t) + V(r)\psi(r,t) + \underline{g|\psi(r,t)}|^2\psi(r,t)$$

interaction energy

$$g = \frac{4\pi\hbar^2 a}{m}$$

Dynamics of BEC

Second quantization $\hat{\psi}(x) = \varphi_0(x)\hat{a}_0 + \sum_{i>0}\varphi_i(x)\hat{a}_i$

Bogoliubov approximation $\hat{a}_0 \to \sqrt{N}$

$$\hat{\psi}(x) = \underline{\sqrt{N}\varphi_0(x)} + \delta\hat{\psi}(x)$$

$\text{c number}: \langle \hat{\psi}(x) \rangle$

$\hat{\psi}(r) \to \hat{\psi}(r,t)$

Heisenburg equation of motion: $i\hbar\dfrac{\partial\hat{\psi}(r,t)}{\partial t} = \left[\hat{\psi}(r,t), \hat{H}\right]$

$$\hat{H} = \int dr\hat{\psi}^+(x)\left[-\frac{\hbar^2}{2m}\nabla^2 + V(r)\right]\hat{\psi}(x) + \frac{1}{2}\int drdr'\hat{\psi}^+(r)\hat{\psi}^+(r')V(r-r')\hat{\psi}(r')\hat{\psi}(r)$$

$$\hat{\psi}(r,t) = \underline{\psi(r,t)} + \delta\hat{\psi}(r,t)$$

c number: $\langle \hat{\psi}(r,t) \rangle$

$$i\hbar\frac{\partial\psi(r,t)}{\partial t} = \frac{\hbar^2}{2m}\nabla^2\psi(r,t) + V(r)\psi(r,t) + \underline{g|\psi(r,t)|^2}\psi(r,t)$$

interaction energy

Time dependent Gross-Pitaevskii Equation (TDGPE)

BEC in stationary ground state

Ground state: satisfies Gross-Pitaevskii Equation (GPE)

$$-\frac{\hbar^2}{2m}\nabla^2\psi(r) + V(r)\psi(r) + g|\psi(r)|^2\psi(r) = \mu\psi(r)$$

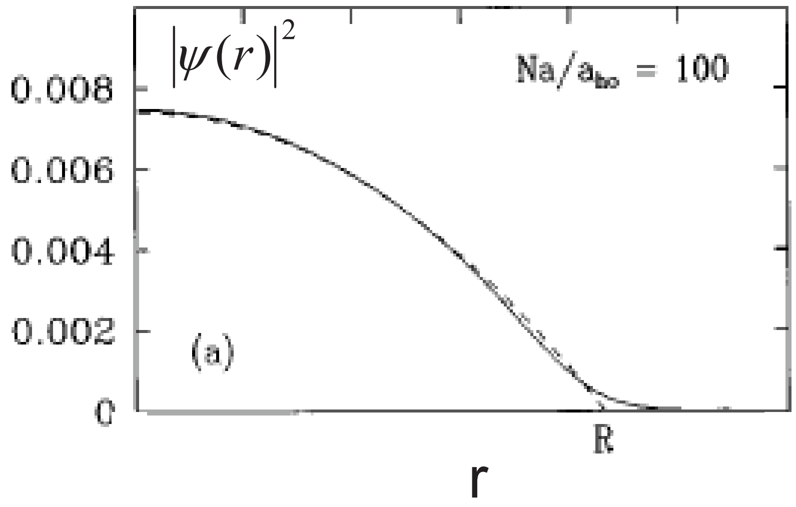
kinetic potential interaction energy

$g = \frac{4\pi\hbar^2 a}{m}$, μ = chemical potential

- GPE in the Thomas-Fermi (TF) limit: interaction >> kinetic energy

$$|\psi(r)|^2 = \frac{1}{g}[\mu - V(r)]$$

TF BEC $V(r) = \frac{1}{2}m\omega^2 r^2$



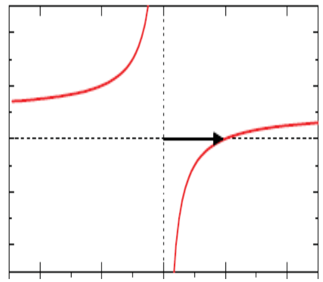
Ultracold quantum gases

ultracold atoms: degenerate, non-classical gases

√ (1) cold and dilute: contact interaction $V(r-r') = \frac{4\pi\hbar^2 a}{m}\delta(r-r')$

(2) tunable interaction: Feshbach resonance

a vs. magnetic field B



a= s-wave scattering length

* can achieve strong interaction limit a≫d, even for a dilute gas
* can study strongly-correlated states with a simple model of interaction

Ex. BEC-BCS (Bardeen-Cooper-Schrieffer) crossover for fermions

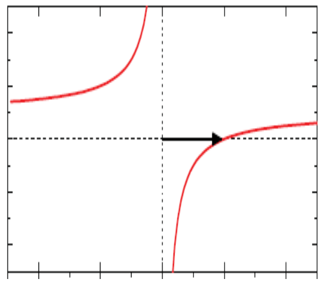
Ultracold quantum gases

ultracold atoms: degenerate, non-classical gases

(1) cold and dilute: contact interaction $V(r-r') = \frac{4\pi\hbar^2 a}{m}\delta(r-r')$

(2) tunable interaction: Feshbach resonance

a vs. magnetic field B



(3) nearly disorder free

precisely controlled magnetic and optical potentials

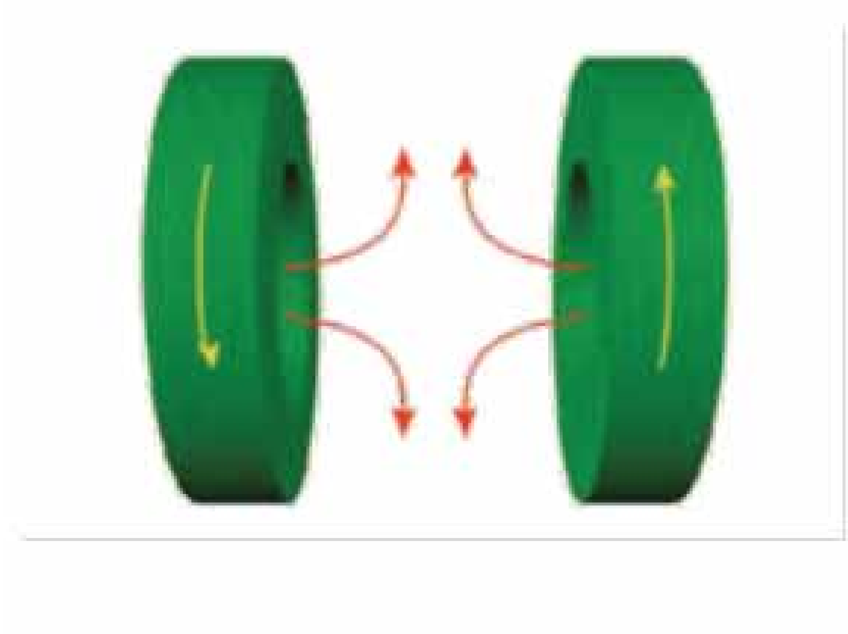
Magnetic potentials: Zeeman shift

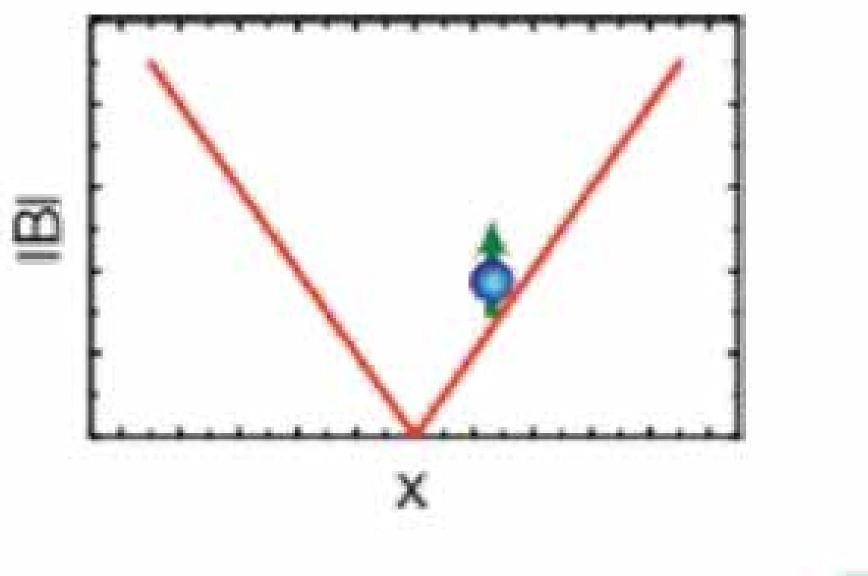
An atom in an external magnetic field

Energy $E = -\vec{\mu} \cdot \vec{B}$

Force $\vec{F} = -\vec{\mu} \cdot \nabla \vec{B}$

B(r)





Optical potentials : AC stark shift

Trapped atoms in light fields

Dipole moment $\vec{d} = \alpha\vec{E}$

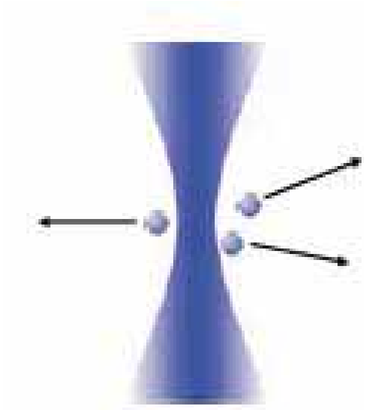
Energy $U_{dip} = -\vec{d}\cdot\vec{E}$

$\propto \alpha(\omega)\, I(r)$

Red detuning Δ<0

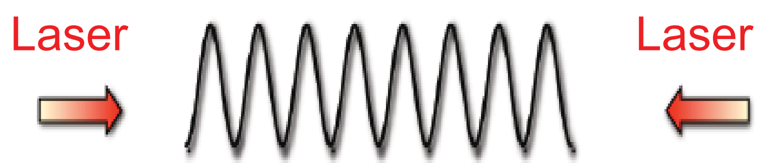
Atoms are trapped
in intensity maximum

Blue detuning Δ>0



Atoms are repelled
from intensity maximum

Optical lattice



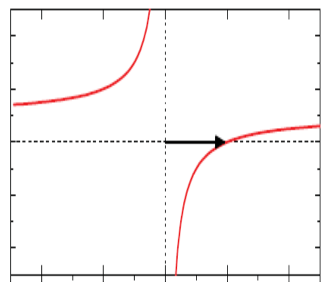
Ultracold quantum gases

ultracold atoms: degenerate, non-classical gases

✓ (1) cold and dilute: contact interaction $V(r-r') = \frac{4\pi\hbar^2 a}{m}\delta(r-r')$

✓ (2) tunable interaction: Feshbach resonance

a vs. magnetic field B



✓ (3) nearly disorder free

precisely controlled magnetic and optical potentials

→ ideal for quantum simulation:
model systems for condensed-matter physics

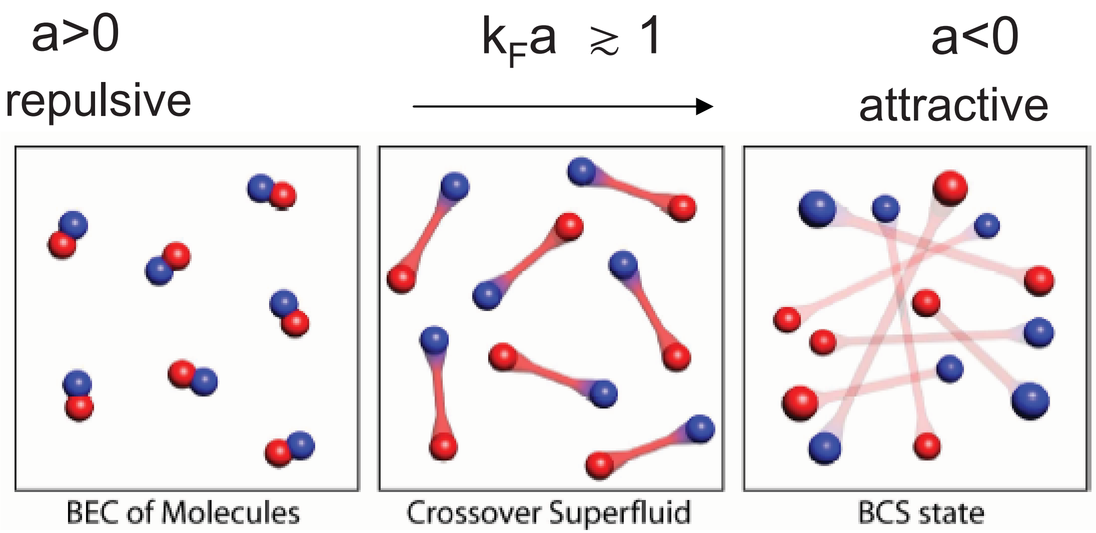
Ultracold atoms have realized iconic condensed matter systems

Use cold, degenerate gases: BEC (Bose-Einstein condensate) or degenerate Fermi gas

- superfluid → Mott-insulator transition: BEC in optical lattices

- low dimensional systems: 1D, 2D physics

- BEC-BCS (Bardeen-Cooper-Schrieffer) crossover:
  two-component Fermi gas, interaction tuned from repulsive → attractive

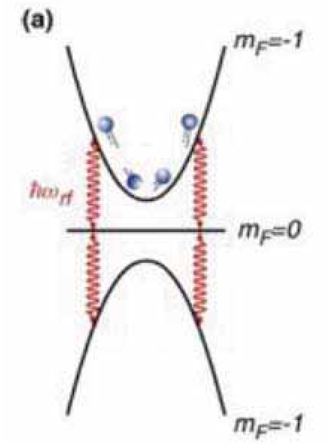


Ref: JILA, MIT, 2004

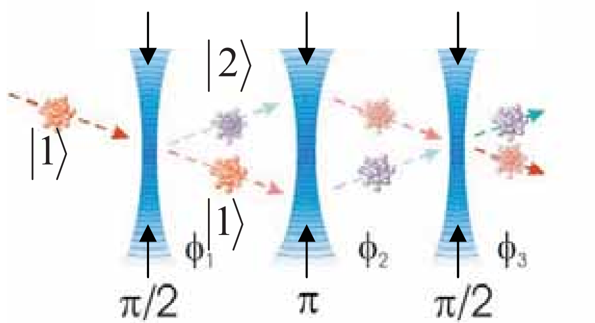
Review: Light-atom coupling

- For trapping, manipulating and probing atoms
- Dipole traps & optical lattices
- rf and Raman : couple between spin states

rf evaporation

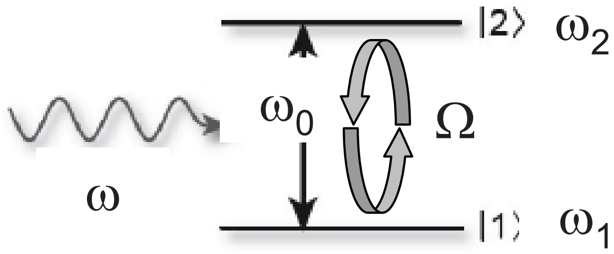


2-arm interferometer



- Probe imaging

Light-atom interaction: two-level system



Light-atom interaction:

$$\langle 2|H_I|1\rangle = \Omega\cos\omega t$$

detuning $\Delta = \omega - \omega_0$

Rotating wave approximation (RWA): $|\omega-\omega_0| \ll \omega_0$

$$\Omega\cos\omega t = \frac{\Omega}{2}\left(e^{-i\omega t} + e^{i\omega t}\right)$$

$$|\psi(t)\rangle = c_1(t)e^{-i\omega_1 t}|1\rangle + c_2(t)e^{-i\omega_2 t}|2\rangle$$

$$c_1(t) \approx 1 \qquad \text{for } c_1(0)=1,\ c_2(0)=0$$

$$c_2(t) \approx \frac{\Omega}{2}\left\{\frac{1-e^{i(\omega_0-\omega)t}}{\omega_0-\omega} + \frac{1-e^{i(\omega_0+\omega)t}}{\omega_0+\omega}\right\}$$

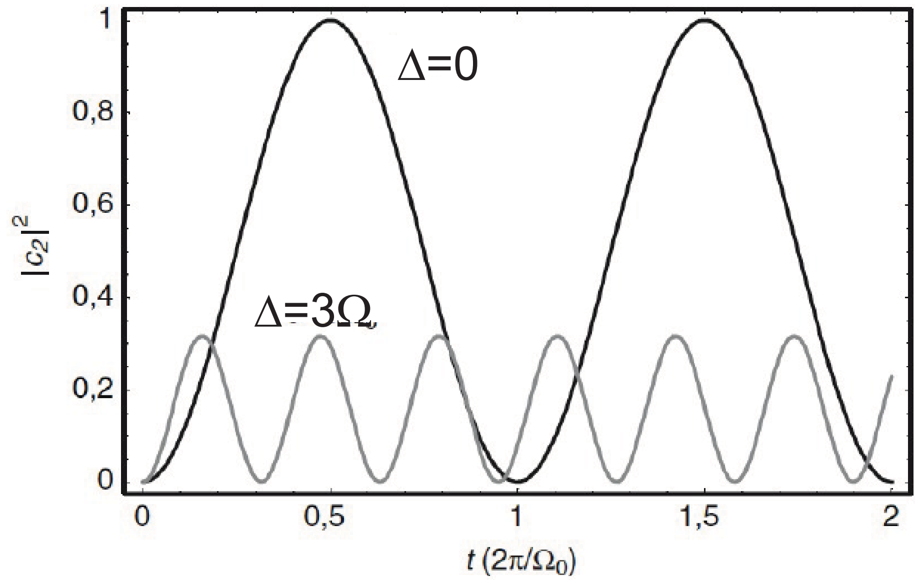
Expressed in rotating frame at ω:

H_{rot} independent of t !

$$i\hbar\begin{pmatrix}\tilde{c}_1\\ \tilde{c}_2\end{pmatrix} = H_{rot}\begin{pmatrix}\tilde{c}_1\\ \tilde{c}_2\end{pmatrix} = \begin{pmatrix}-\Delta/2 & \Omega/2\\ \Omega/2 & \Delta/2\end{pmatrix}\begin{pmatrix}\tilde{c}_1\\ \tilde{c}_2\end{pmatrix}$$

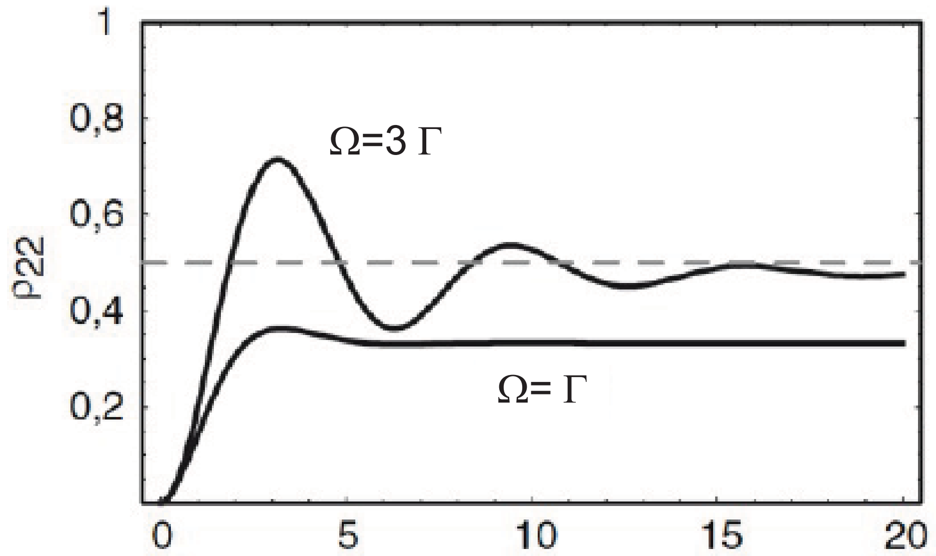
Light-atom interaction: two-level system

Rabi flopping: $c_1(t)=1$, $c_2(t)=0$



With damping:

$$|1\rangle \to |g\rangle, |2\rangle \to |e\rangle \quad \dot{\rho}_{22} = -\Gamma\rho_{22}$$



steady state:

$$\tilde{\rho}_{12} = \frac{1}{2}\frac{\Omega(\Delta + i\Gamma/2)}{\Delta^2 + (\Gamma/2)^2 + \Omega^2/2} \longrightarrow$$

$$\tilde{\rho}_{22} = \frac{1}{2}\frac{\Omega^2/2}{\Delta^2 + (\Gamma/2)^2 + \Omega^2/2} \longrightarrow$$

For $\Delta >> \Gamma, \Omega$:

$$U_{dip} = -d \cdot E = \hbar\frac{\Omega^2}{4\Delta}$$ (dipole trap)

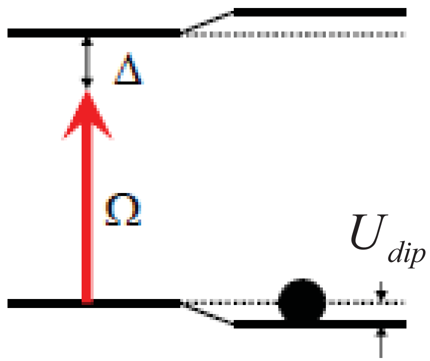
$$\Gamma_{sc} = \Gamma\frac{\Omega^2}{4\Delta^2}$$ (probe imaging)

Optical potentials : AC stark shift

Trapped atoms in light fields

Dipole moment $\vec{d} = \alpha \vec{E}$

Energy $U_{dip} = -\vec{d} \cdot \vec{E}$

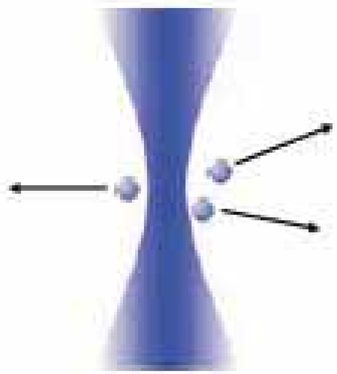


$$U_{dip} = \hbar \frac{\Omega^2}{4\Delta}$$

Red detuning Δ<0

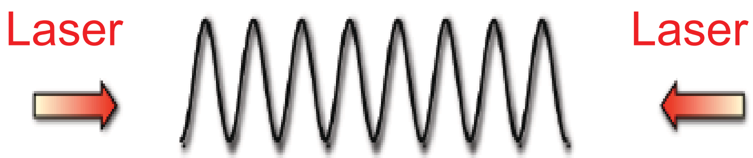
Atoms are trapped
in intensity maximum

Blue detuning Δ>0

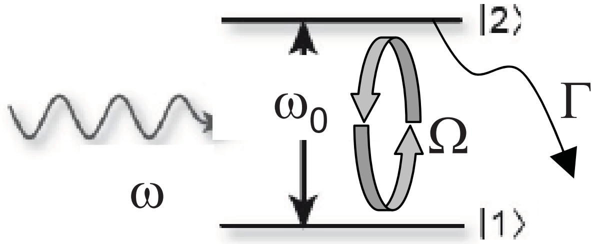


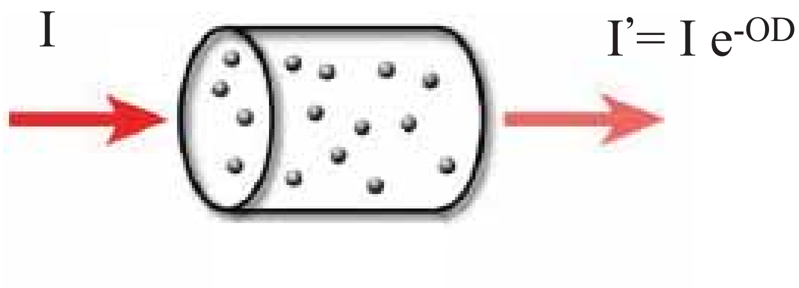
Atoms are repelled
from intensity maximum

Optical lattice



Probing atoms: absorption imaging





Absorption:

$$\text{Optical density OD} = \frac{I' - I}{I} \quad \text{for OD} << 1$$

$$\text{OD} = \frac{\text{total scattered photon}}{\text{incoming probe photon}} = \frac{N\Gamma_{sc}}{IA/\hbar\omega}$$

$$= \frac{N\sigma}{A}$$

on resonance: $\sigma = \sigma_0 = \frac{3}{2\pi}\lambda^2$

off resonance: $\sigma = \frac{\sigma_0}{1 + 4(\Delta/\Gamma)^2 + (I/I_{\mathrm{sat}})}$

Light-atom coupling parameter:
intensity: I/I_{sat} $\Omega = d*E$
detuning: Δ/Γ
$I_{sat} = \Gamma\, \hbar\omega / 2\sigma_0$

Γ_{sc}: scattered photon# per atom
σ : scattering cross section per atom

Time-of-Flight (TOF) imaging

- Switch off trap, free expansion

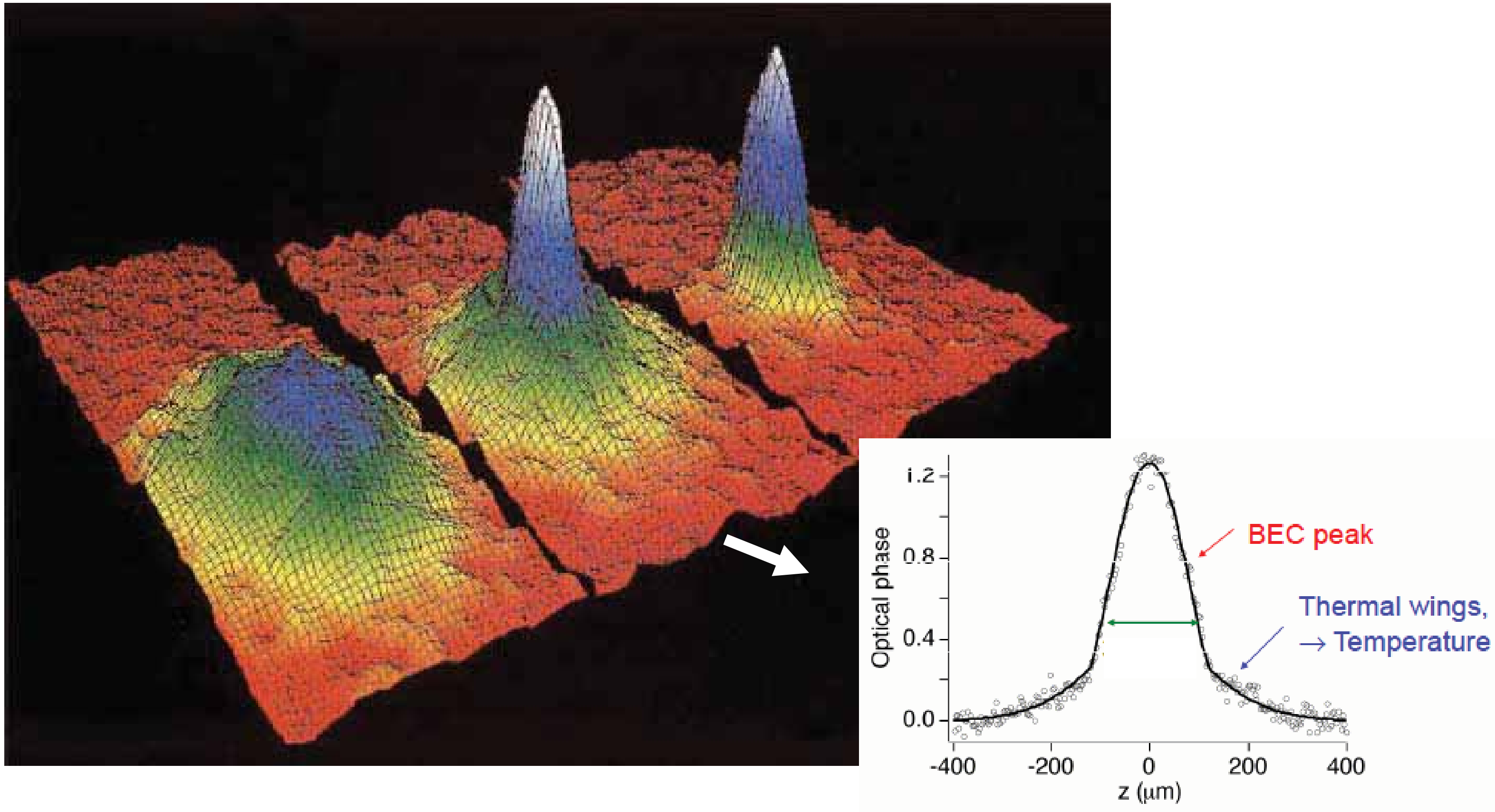
- measure k distribution : k mapped to x
  (1) ballistic expansion: no interaction during TOF
  (2) after long expansion $t \gg 1/\omega$

- thermal atoms: ballistic expansion
  BEC: superfluid expansion, interaction driven

Detection of BEC: bimodal distribution at T>0

thermal→ thermal+BEC→ ~ pure BEC

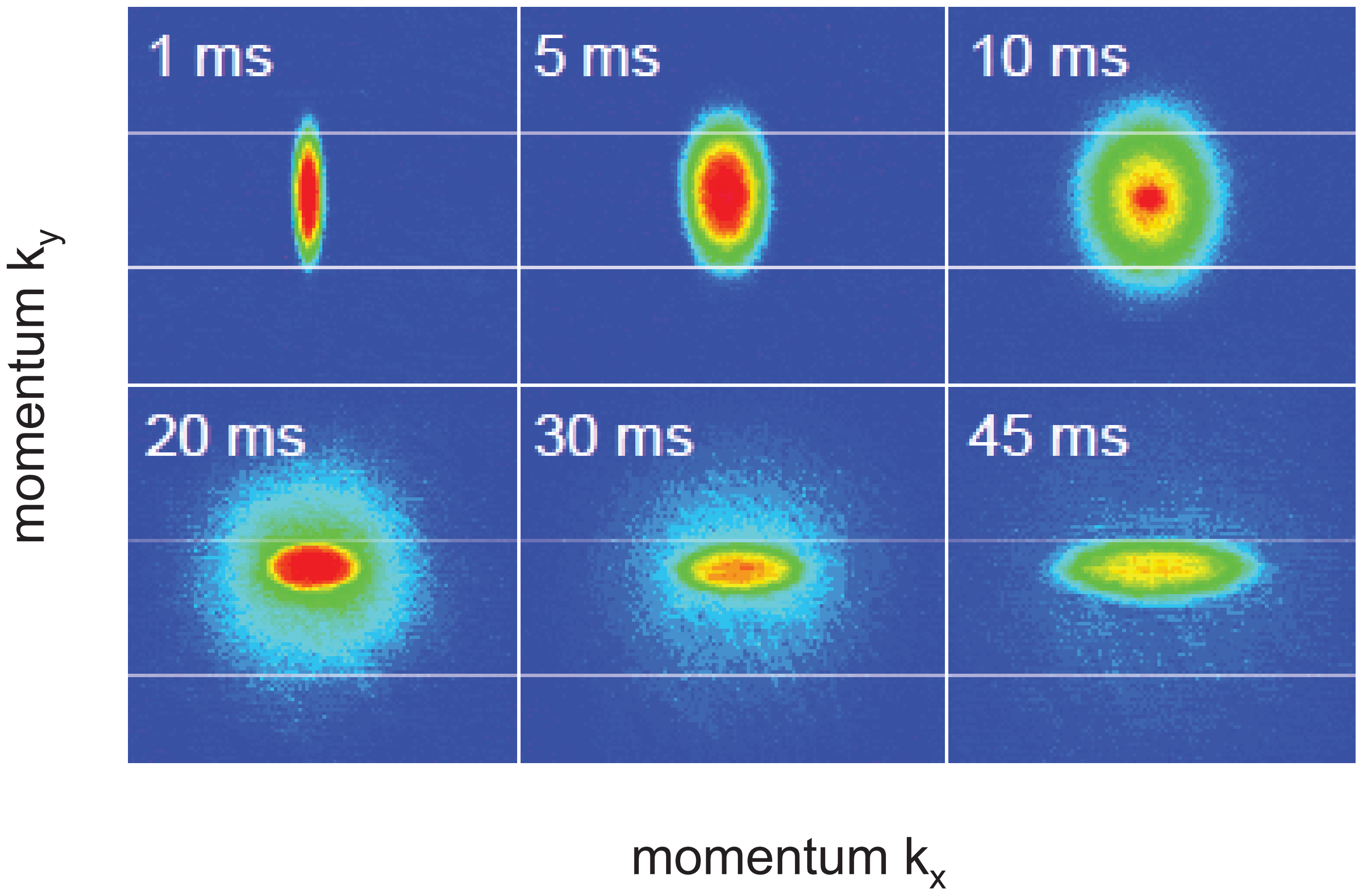
momentum k_y



momentum k_x

distribution in (k_x,k_y) space
long expansion time $t \gg 1/\omega$

Detection of BEC: anisotropic distribution

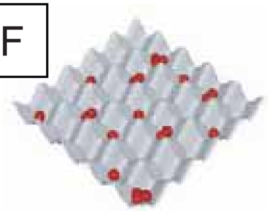


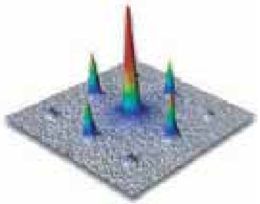
Ultracold atoms have realized iconic condensed matter systems

Use quantum gases: BEC (Bose-Einstein condensate) or degenerate Fermi gas

- superfluid → Mott-insulator transition: BEC in optical lattices

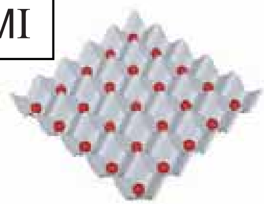
SF





x-space k-space

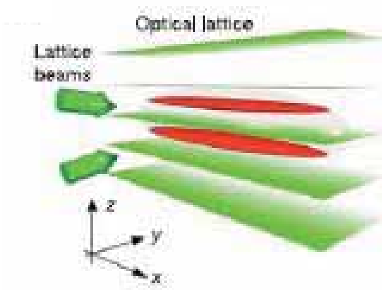
MI

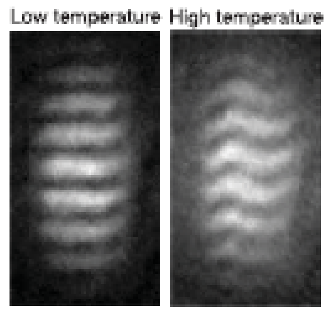


x-space k-space

Ref: Greiner et al., 2003

- low dimensional systems: 1D : Tonks-Giradeau gas 2D : BKT superfluid





(Berezinskii-Kosterlitz-Thouless superfluid)

Ref: D. S. Weiss, 2004

Ref: Jean Dalibard, 2006

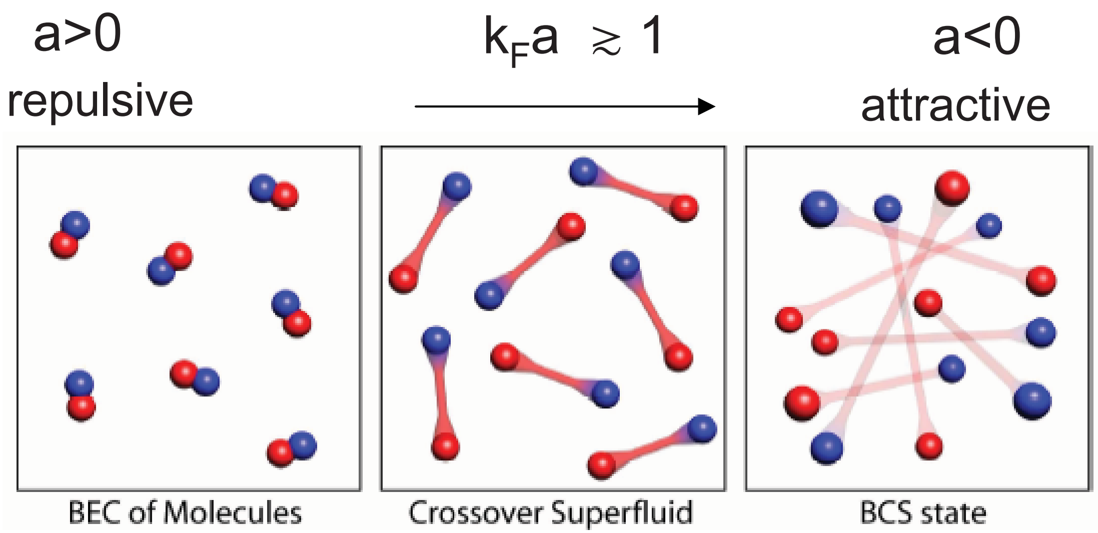
Ultracold atoms have realized iconic condensed matter systems

Use cold, degenerate gases: BEC (Bose-Einstein condensate) or degenerate Fermi gas

- superfluid → Mott-insulator transition: BEC in optical lattices

- low dimensional systems: 1D, 2D physics

- BEC-BCS (Bardeen-Cooper-Schrieffer) crossover:
  two-component Fermi gas, interaction tuned from repulsive → attractive

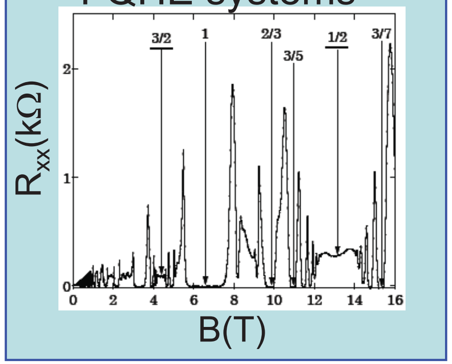


Ref: JILA, MIT, 2004

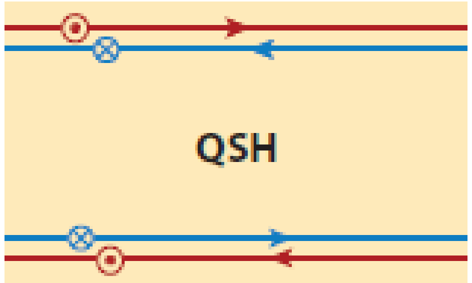
What we want to simulate with ultracold atoms

to "charge" neutral atoms by creating a " synthetic gauge potential A*"

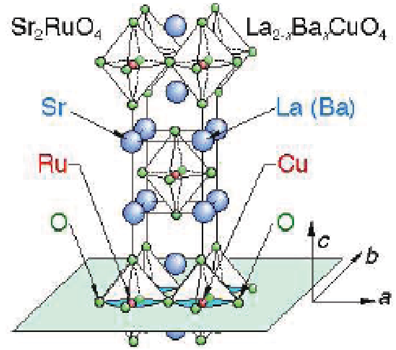
FQHE systems



Quantum spin Hall effects



p-wave superconductor



- new approach to generate large B* to study quantum-Hall physics:
(2D system and $\nu = N_{2D}/N_v \leq 1$ N_{2D}= atom#, N_v= # of flux quanta)

Advantages to study w/ ultracold atoms

- bosonic ν =1 state: w/ binary contact interaction,
nonabelian, for topological quantum computation

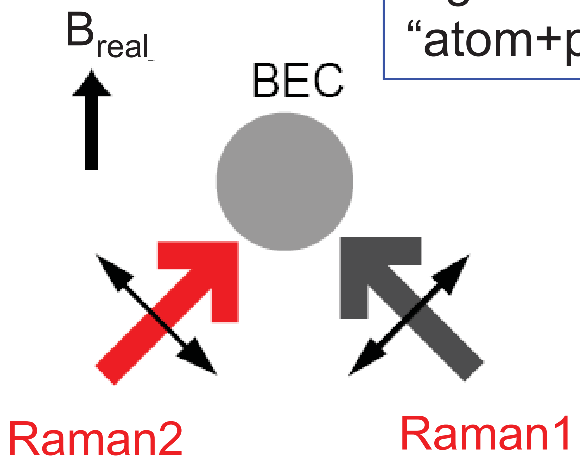
Ref: N. R. Cooper, 2008

- Spin-dependent A*: spin-orbit coupling
TR preserved topological insulators, topological superconductors

Outline: synthetic gauge potentials A*

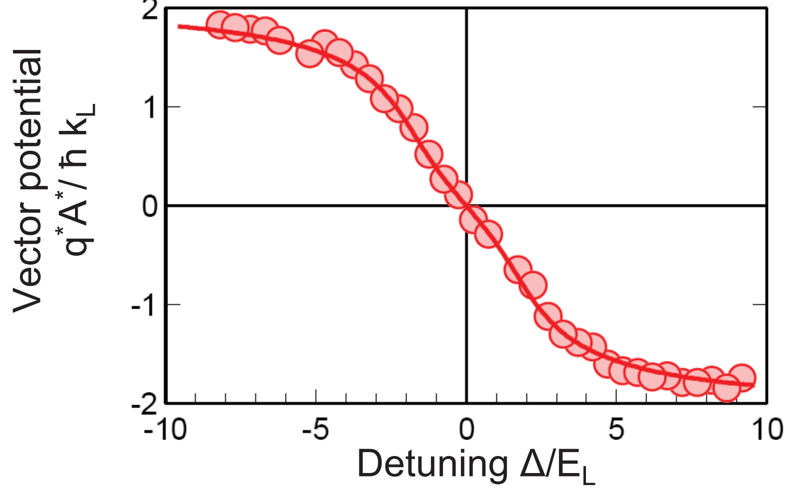
Raman-dressed BEC

eigenstate of
“atom+photon”

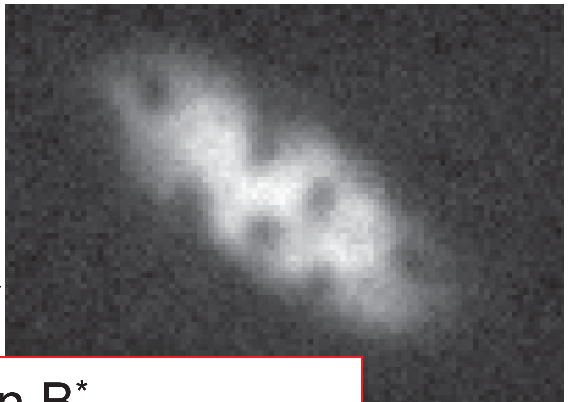


Synthetic Vector potential A*

$$H = \frac{\hbar^2}{2m^*}\left[(k_x - \frac{q^* A_x^*}{\hbar})^2 + k_y{}^2\right]$$

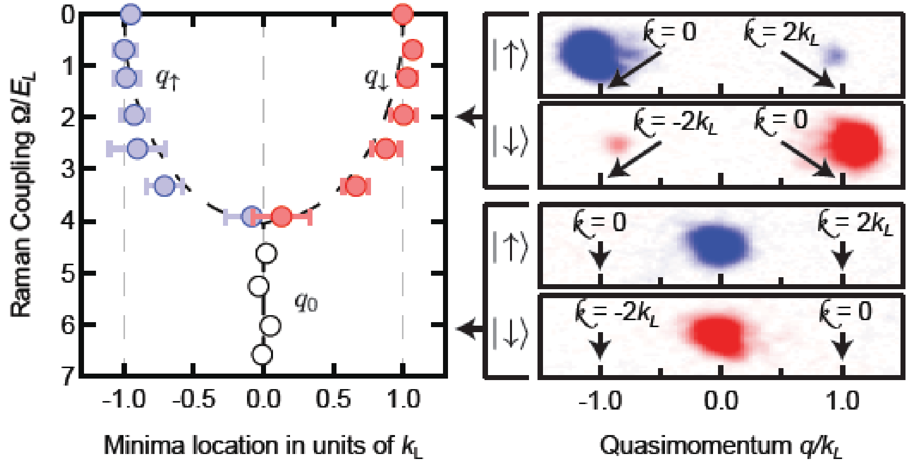


Magnetic field $B^* = \nabla \times A^*$



superfluid in B*
(like superconductor in B)

Spin dependent A*: spin-orbit coupling



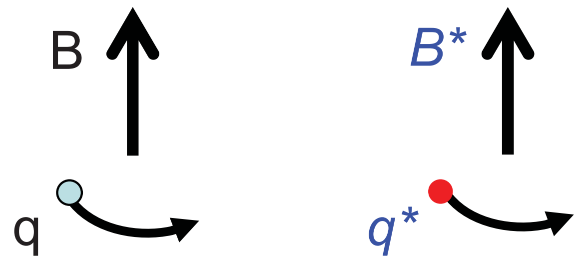
Introduction of gauge potential

- Optically induced vector gauge potential A^* for neutral atoms:

$$H = \frac{(p - q^* A^*)^2}{2m^*} + V(x)$$

$\rightarrow$ synthetic electric and magnetic fields $\quad E^* = -\frac{\partial A^*}{\partial t}, B^* = \nabla \times A^*$

- Create synthetic field B^* for neutral atoms:
  effective Lorentz force
  to simulate charged-particles in real magnetic fields



charge q $\qquad$ ^{87}Rb

- Light-induced potential to generate B^* in lab frame, no rotation of trap:
  (1) steady B^*, not metastable
  (2) easy to add optical lattices

B* in rotating frame:
Coriolis force↔Lorentz force

Traditional methods to create B^* : rotation

rotating neutral atoms

charge q in B

$\Omega \leftrightarrow qB/2m$

$F_{Coriolis} = 2m\Omega v_{rot}$

$F_{Lorentz} = qvB$

$$H_{rot} = \frac{\hbar^2}{2m}\left[(k_x - \frac{m\Omega y}{\hbar})^2 + (k_y + \frac{m\Omega x}{\hbar})^2\right] + V'(r)$$

$$V'(r) = \frac{1}{2}m(\omega^2 - \Omega^2)r^2$$

$$H_B = \frac{\hbar^2}{2m}\left[(k_x - \frac{qBy}{2\hbar})^2 + (k_y + \frac{qBx}{2\hbar})^2\right] + V(r)$$

$$V(r) = \frac{1}{2}m\omega^2 r^2$$

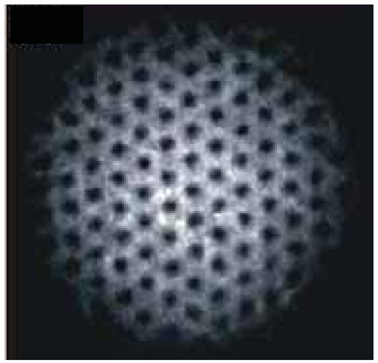
w/ mean field interaction

N_v vortices, $L/N = N_v/2$ (large N_v)

N_v vortices or flux quanta

(one vortex $\leftrightarrow$ $\Phi_0 = h/q$)

rotating BEC (experiment)



$\omega/2\pi \sim 10$Hz, $\Omega/\omega = 0.975$

$N \sim 10^6$ $R \sim 30\mu m$

Coddington et al., JILA, 2004

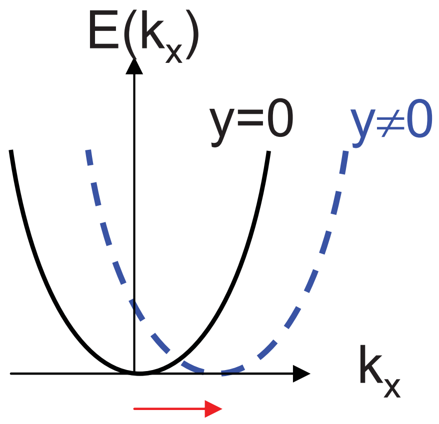
Principles (I)

- charged particle q in a real field $\vec{B} = B\hat{z}$, Landau gauge

$$H_B = \frac{\hbar^2}{2m}\left[(k_x - \frac{qA_x}{\hbar})^2 + k_y{}^2\right]$$

$$\delta k_x \equiv \frac{qA_x}{\hbar} = \frac{qBy}{\hbar}$$

$$\vec{B} = \nabla \times \vec{A}$$



- to simulate w/ laser-atom interaction

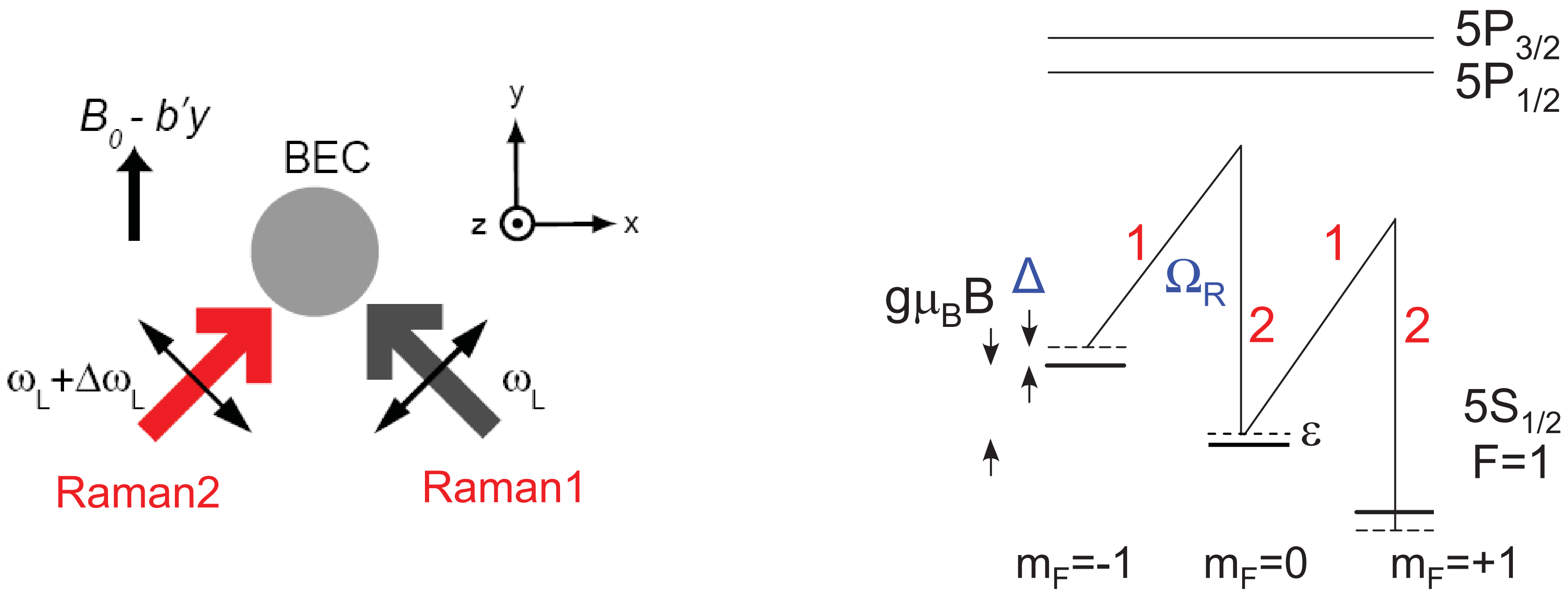
- laser photons : create δk_x= momentum shift along x

→ make $\delta k_x(\Delta)$ Δ=laser-atom detuning

→ make $\Delta=\Delta' y$: $\delta k_x(y)$

→ synthetic field $\frac{q^* B^*}{\hbar} = \frac{\partial(\delta k_x)}{\partial y}$ along z

Principles (II): Formalism of Light-atom Coupling



- controlled Raman detuning Δ= $\Delta\omega_L - g\mu_B B$, $B=B_0-b'y$ or $B=B_0(t)$

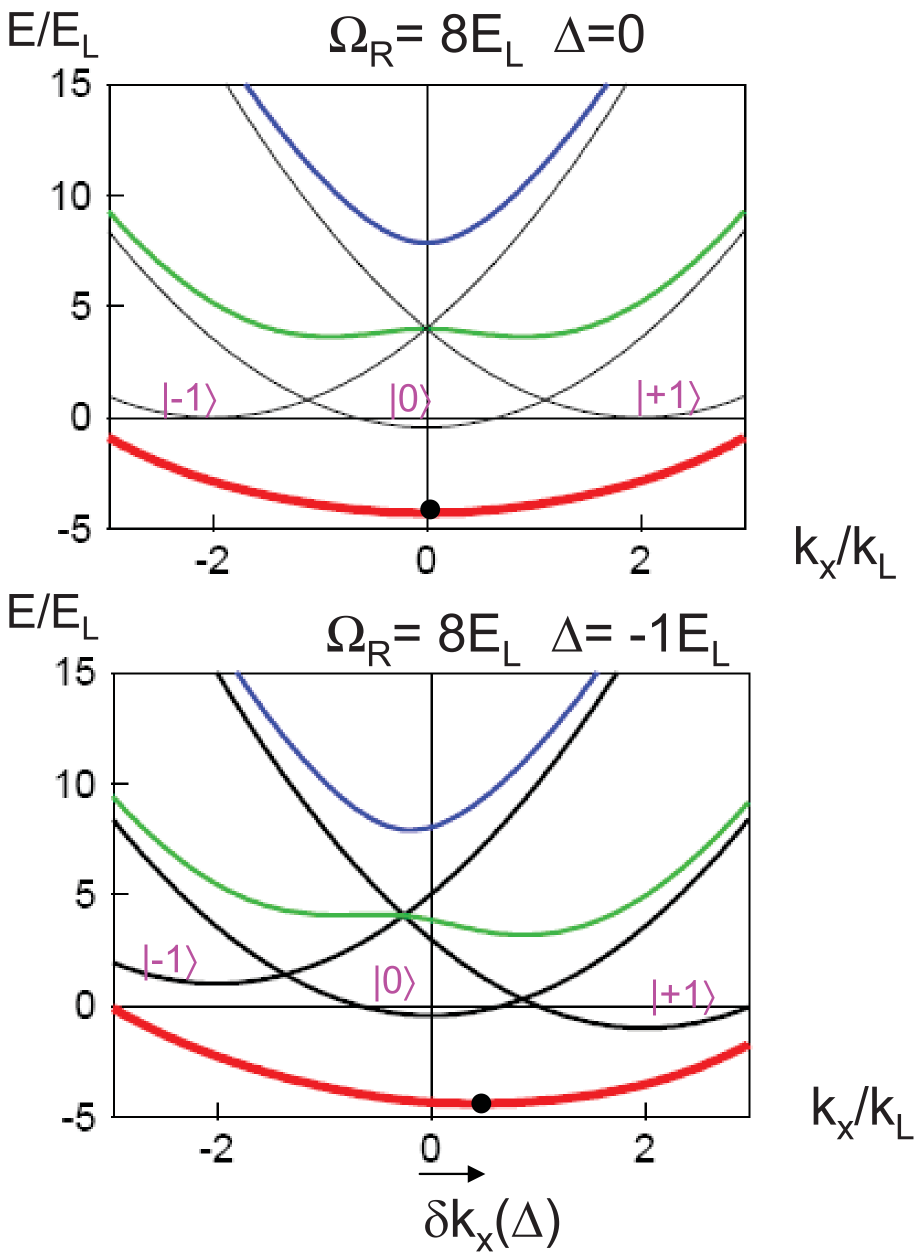
$$\hat{H} = \frac{\hbar^2 \hat{k}^2}{2m} + V(x) + \underline{H_{\text{int}}(\Omega_R, \Delta)} = \begin{array}{c} \begin{array}{ccc} |-1,k_x+2\rangle & |0,k_x\rangle & |+1,k_x-2\rangle \end{array} \\ \begin{pmatrix} (\hat{k}_x+2)^2-\Delta & \Omega_R/2 & 0 \\ \Omega_R/2 & \hat{k}_x^{\,2} - \varepsilon & \Omega_R/2 \\ 0 & \Omega_R/2 & (\hat{k}_x-2)^2+\Delta \end{pmatrix} \end{array} + k_y^{\,2} + V(x)$$

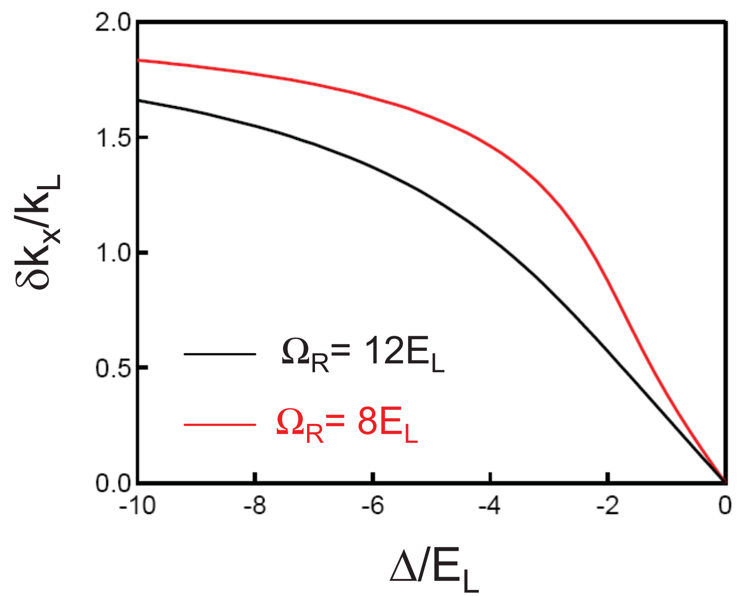
k (k_L), E(E_L)

$E_L = \hbar^2 k_L^{\,2}/2m$

- diagonalize $\rightarrow$ Raman-dressed state : eigenvalue= $E_0(\Delta) + (k_x - \delta k_x(\Delta))^2$

Principles (III):
Shift of momentum in dispersion relation



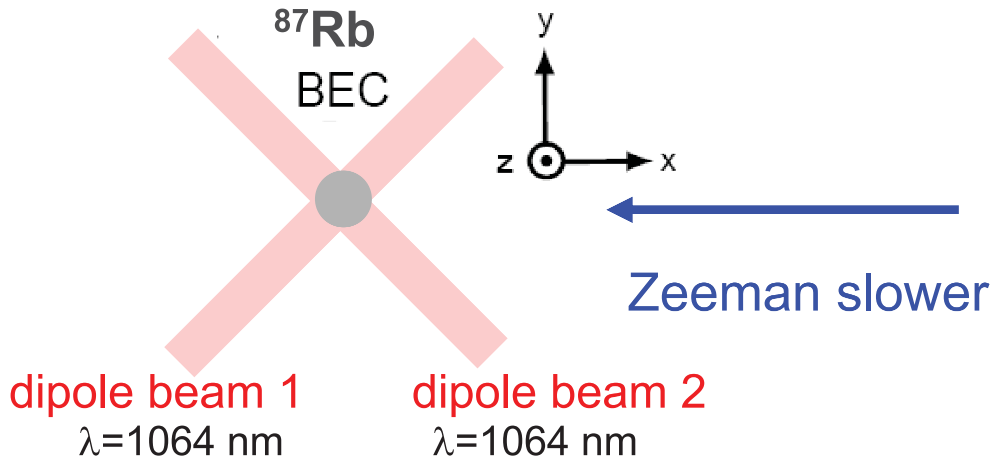


- the lowest energy dressed-state:

$$\delta k_x = \frac{q^* A_x^*}{\hbar} = \text{momentum shift}$$

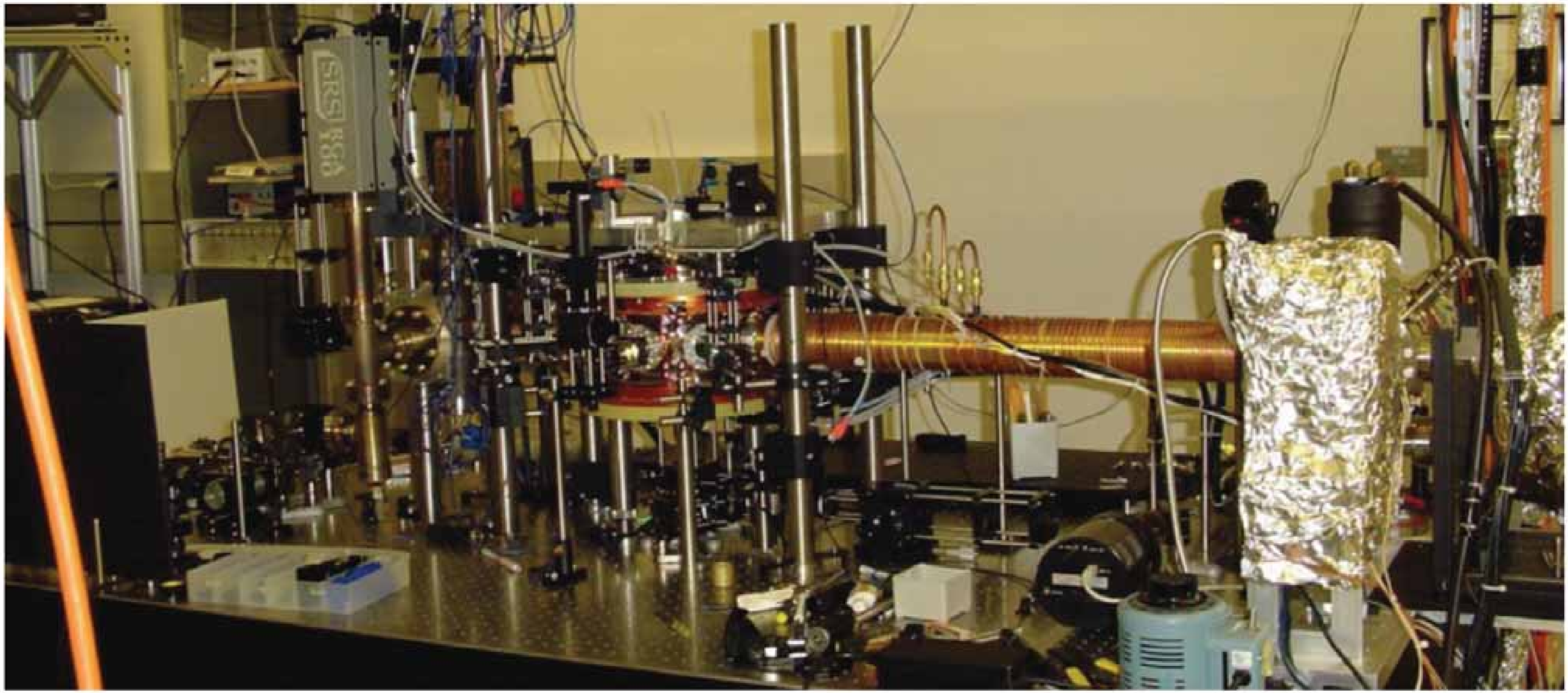
$$H_x = \frac{\hbar^2}{2m}\left[(k_x - \frac{q^* A_x^*}{\hbar})^2 \right]$$

Setup: BEC production

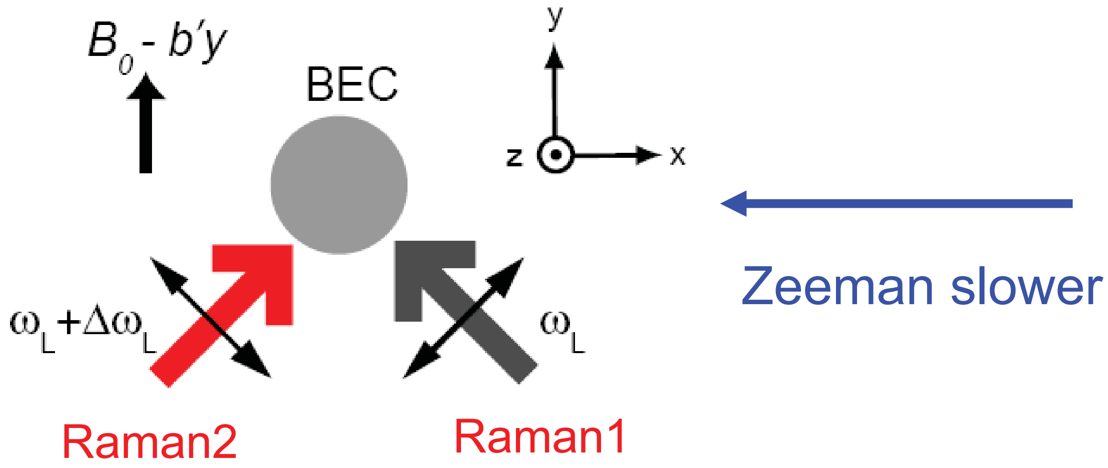


- load MOT from Zeeman slower: ~ 10^9 atoms in 3 s
- rf-evaporative cooling in a quadrupole magnetic trap for 3 s, $|F=1, m_F = -1\rangle$
- single beam optical dipole trap + weak magnetic trap:
  evaporate in hybrid potential for ~ 7 s $\rightarrow$ 2 x10^6 atoms in BEC
- load the BEC into the crossed dipole trap: 5 x10^5 atoms
- total cycle time~ 15 s

Actual experimental Setup

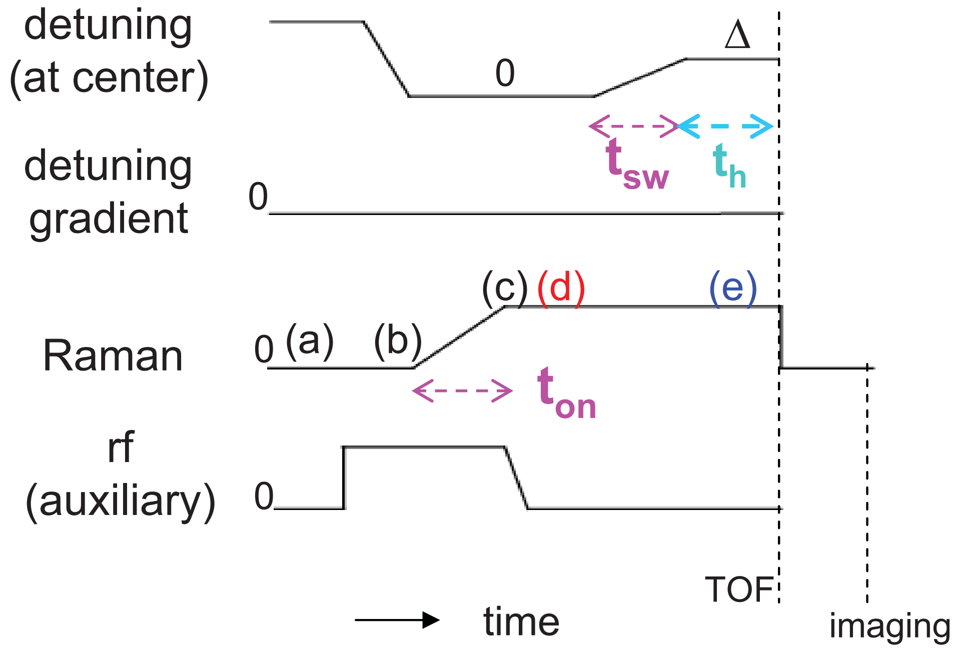


Setup: Raman-dressed BEC



- Bose-Einstein condensate in a crossed dipole trap, ^{87}Rb $|F=1,m_F= -1\rangle$
  N~ 5 x 10^5 every 15 s
- trap frequency: $(\omega_x, \omega_y, \omega_z) \approx 2\pi$ x (70,70,80) Hz
- for N $\rightarrow$ 2.5 x 10^5 in Raman-dressed state
  from decreasing dipole trap power to reduce heating from Raman beams
- Zeeman shift: linear = $g\mu_B B$= 2.71 MHz, quadratic=1.0 kHz
  λ= 801.7nm (D1=795nm, D2=780nm)

Adiabatic loading: uniform A* (I)



(a) Bare state, $|-1\rangle$

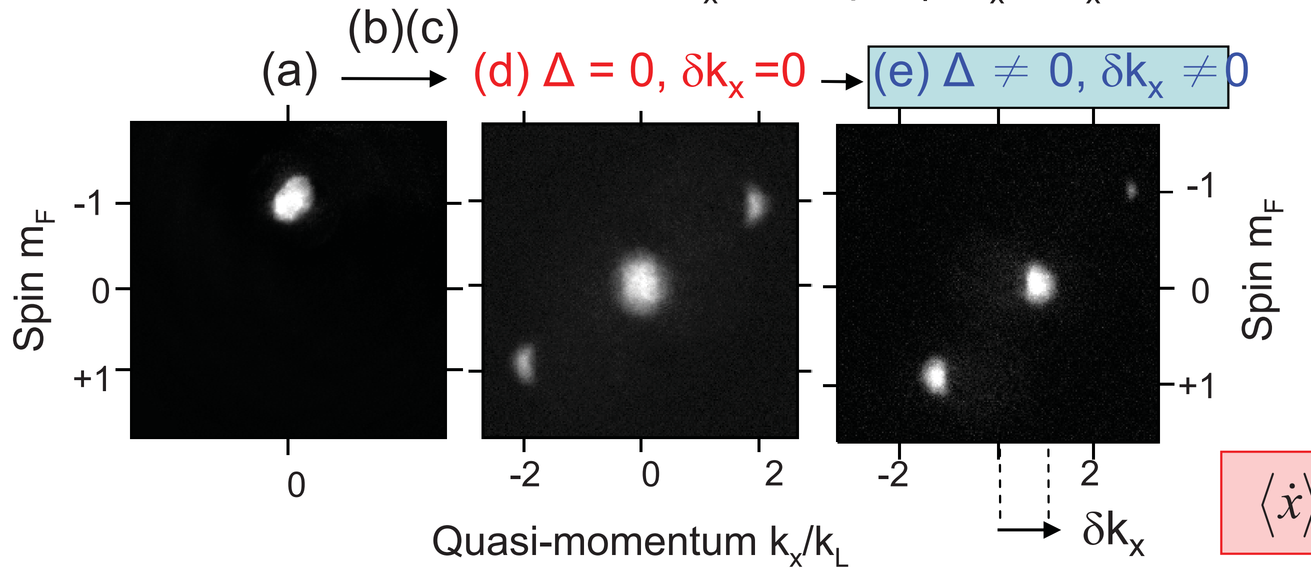
(b) rf-dressed state, $\delta k_x = 0$

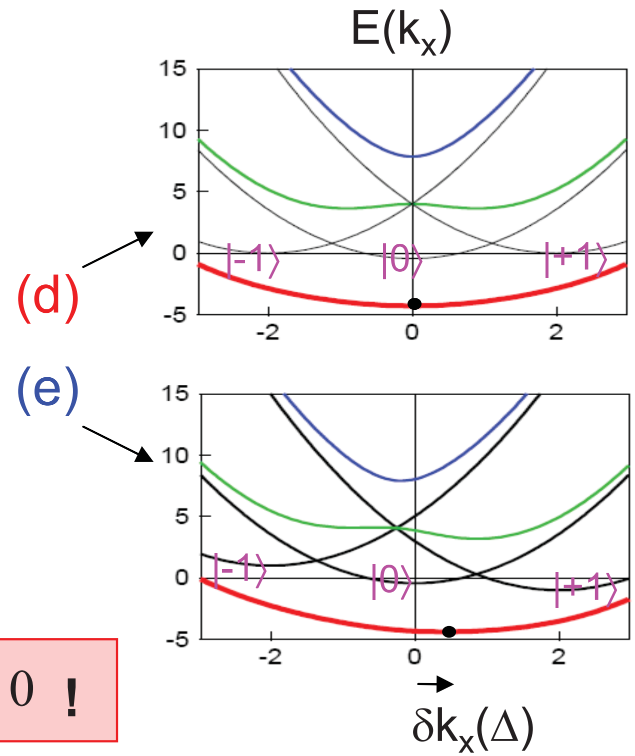
(c) rf+Raman-dressed state, $\delta k_x = 0$

(d) Raman-dressed state, $\delta k_x = 0$

(e) Raman-dressed state, $\delta k_x(\Delta) = q^* A^*/\hbar \neq 0$

Time-of-Flight images of $|-1, k_x+2\rangle$, $|0, k_x\rangle$, $|+1, k_x-2\rangle$
measure δk_x from spin $|0, k_x = \delta k_x\rangle$



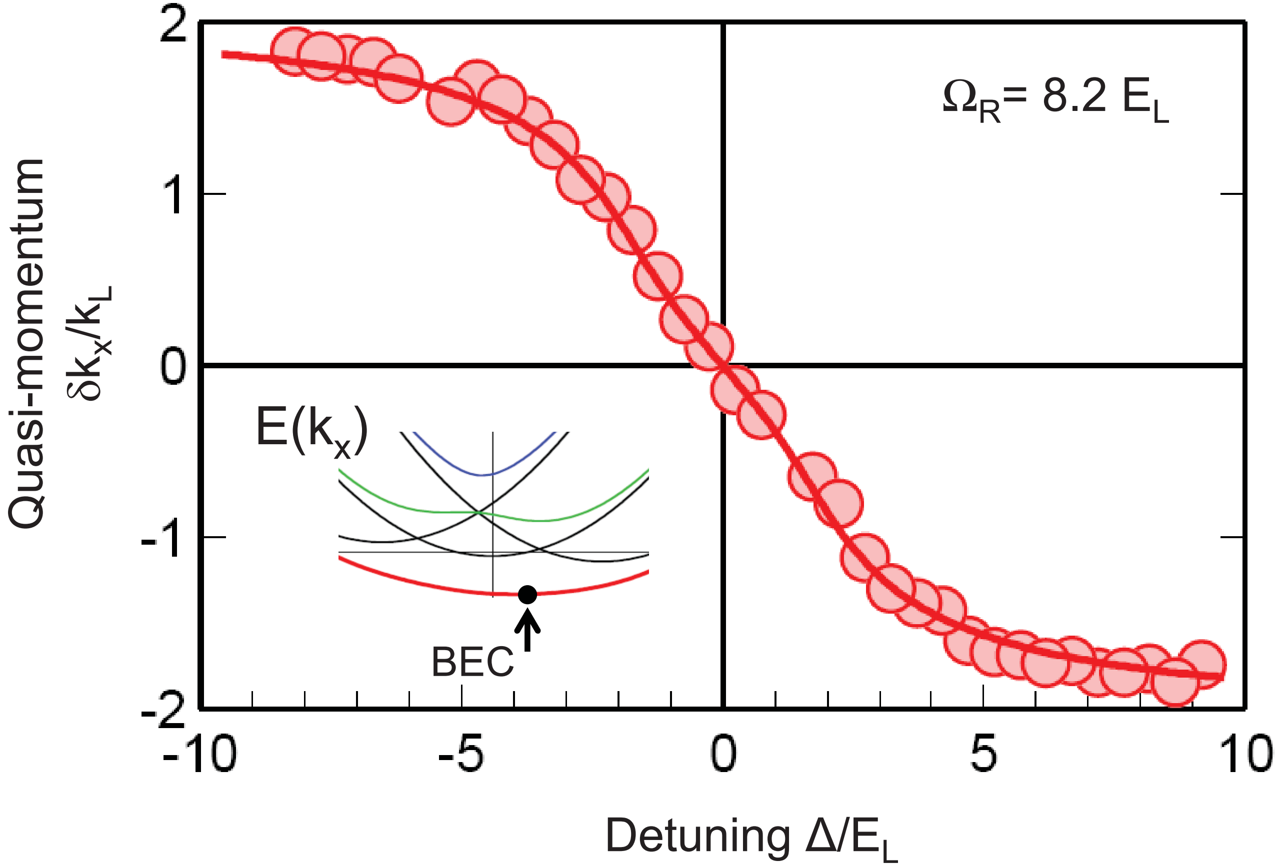


$$\langle \dot{x} \rangle_{m_F} = 0 \ \ !$$

Uniform vector potential A* vs. detuning Δ

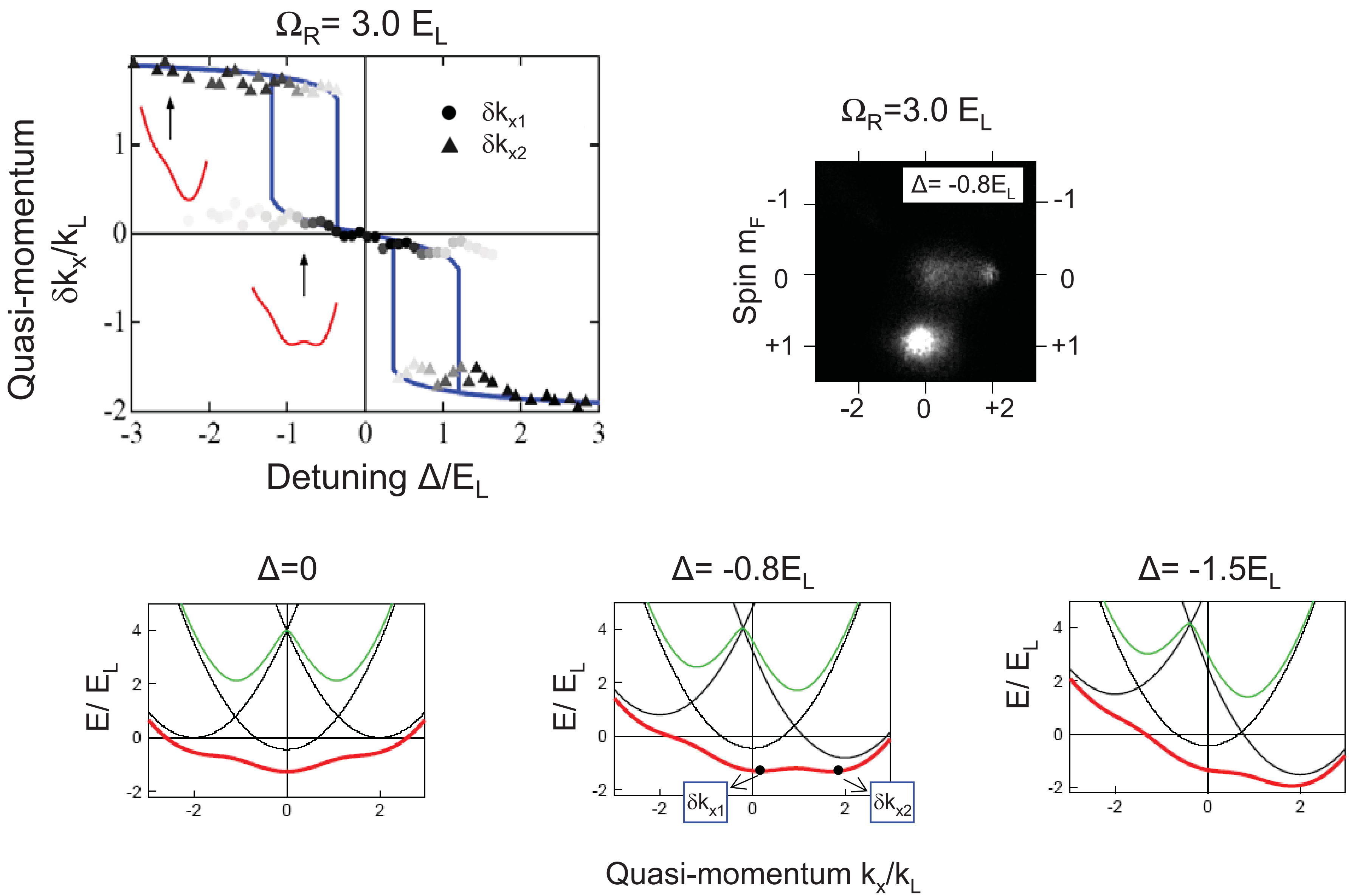
effective vector potential $q^*A^*/\hbar$ = measured quasi-momentum k_x
adiabatic loading at energy minimum $\rightarrow k_x = q^* A^* / \hbar$

vector potential $q^*A^*/\hbar$ vs. detuning Δ

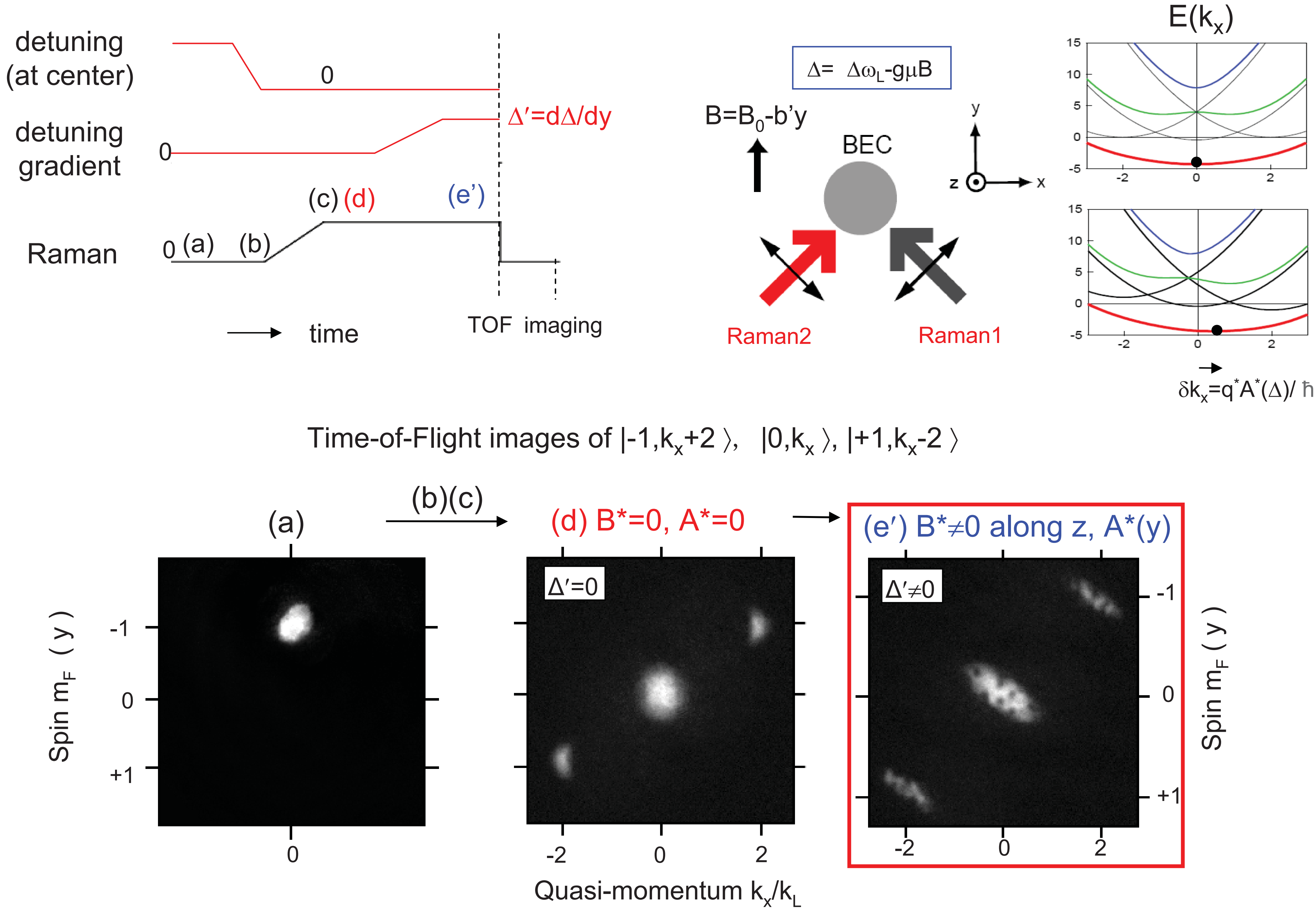


Uniform vector potential A* vs. detuning (II)

- for weak coupling $\Omega_R < 4.5\ E_L$, double energy minimum in k space

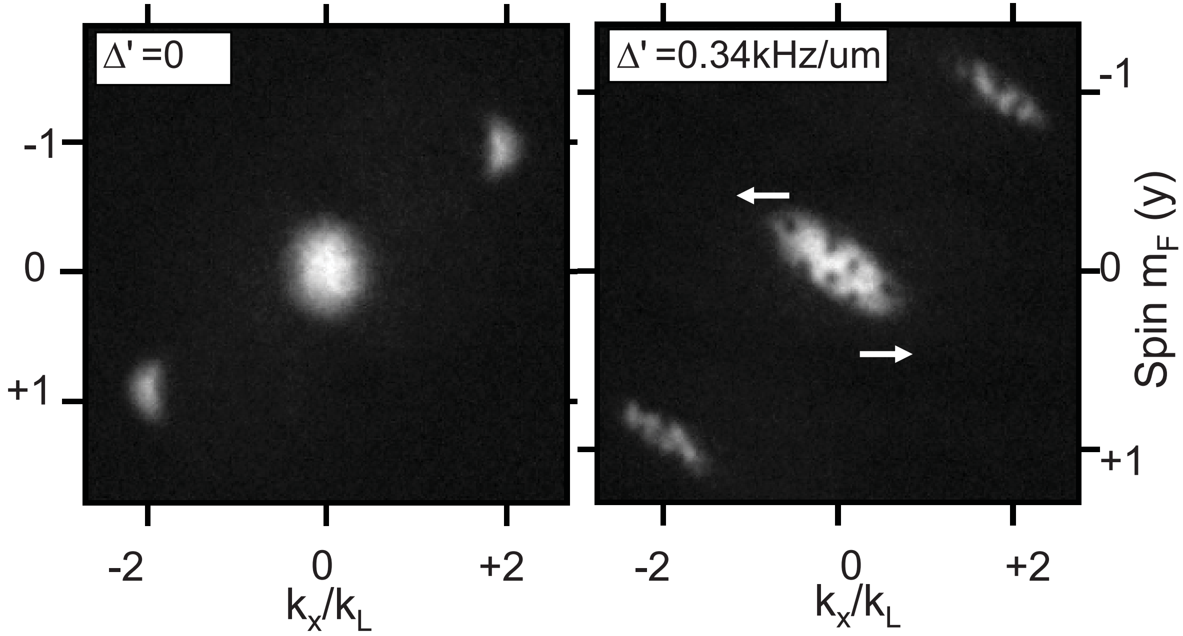


Synthetic field B*= ∇×A*



Raman-dressed BEC in synthetic B*

Spin and momentum projection, TOF=25.1ms



$$H_B = \frac{\hbar^2}{2m^*}\left(k_x - \frac{q^* B^* y}{\hbar}\right)^2 + \frac{\hbar^2 k_y{}^2}{2m} + V(x,y)$$

- vector potential $\delta k_x \equiv \frac{q^* A^*}{\hbar} = \frac{q^* B^* y}{\hbar}$

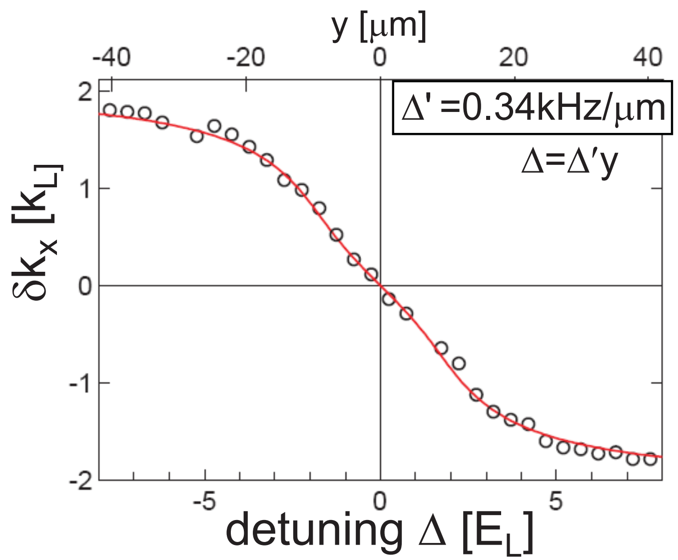
$$\vec{B}^* = \nabla \times \vec{A}^*$$

- magnetic field B*

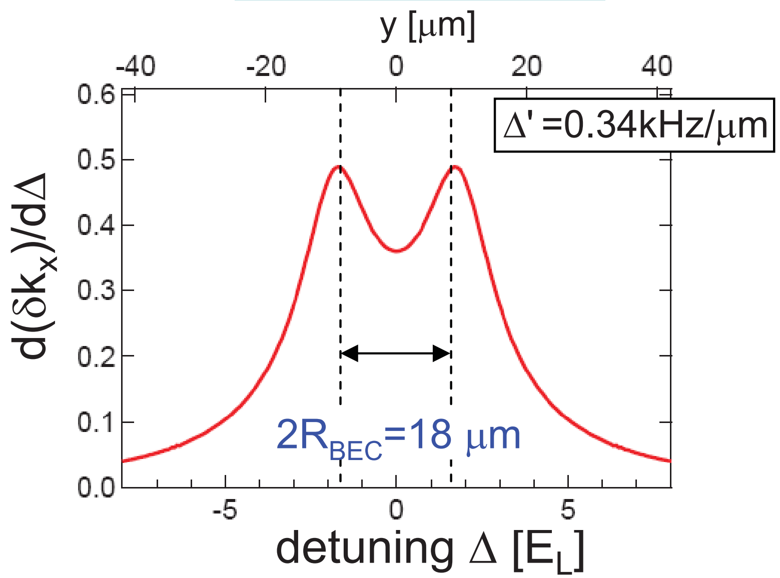
$$\frac{q^* B^*}{\hbar} = \frac{\partial(\delta k_x)}{\partial y} = \Delta' \frac{\partial(\delta k_x)}{\partial \Delta}$$

detuning gradient $\Delta' = \frac{d\Delta}{dy}$

A* vs. Δ or y



B* vs. Δ or y



Vortex number N_v vs. B^*

- For infinite system size:
  $N_v = \text{area} \times B^*/(h/q^*)$
  $= \Phi_{B^*}/(h/q^*)$

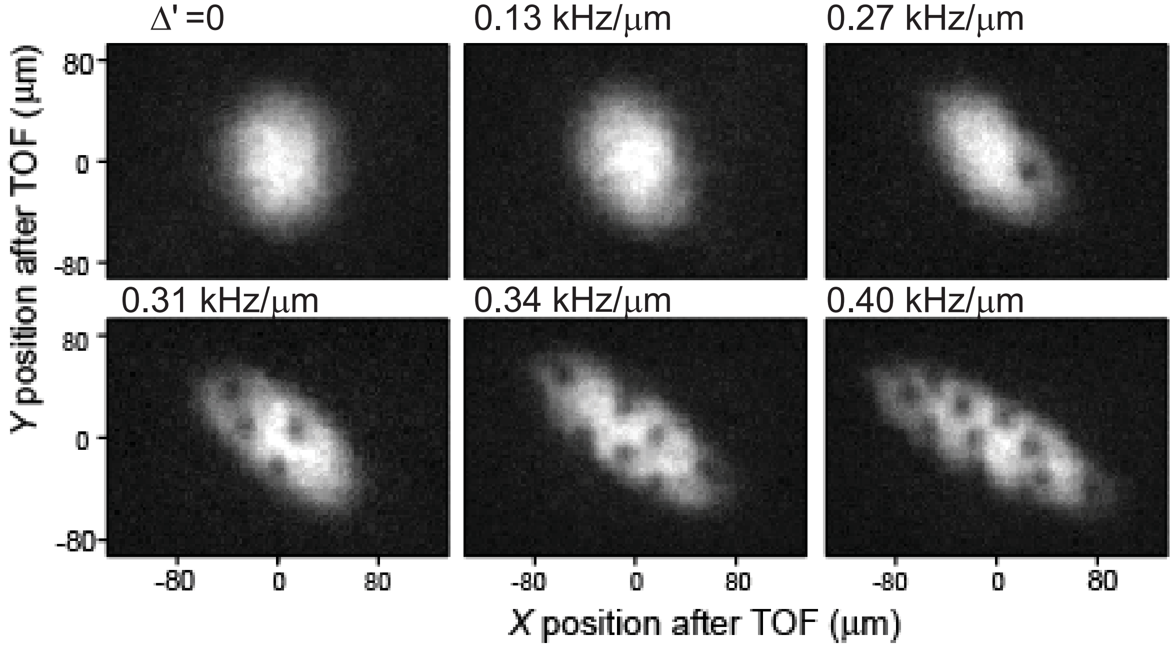
- threshold energy E_v to create $N_v=1$
  for finite system radius R: $E_v \propto 1/R^2$

$R_{BEC}=9\ \mu m$ (in situ)
$N \sim 1.4 \times 10^5$
$(\omega_x,\omega_y,\omega_z)/2\pi \approx (40,70,80)$ Hz

$$\frac{q^* B^*}{\hbar} = \Delta' \frac{\partial(\delta k_x)}{\partial \Delta}$$

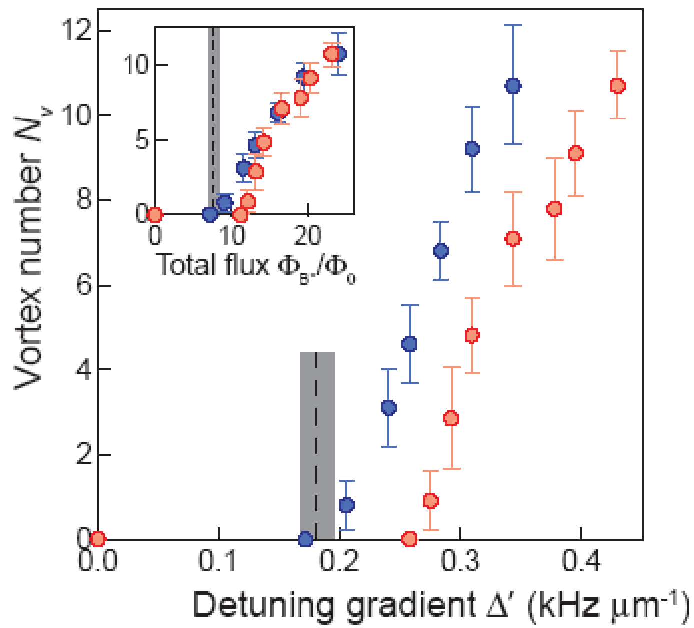
TOF images vs. gradient $\Delta' \propto B^*$

$\Omega_R = 8.2\ E_L$, $t_h = 0.6$s



Vortex number N_v vs. gradient Δ'

● $\Omega_R = 5.9\ E_L$ ● $\Omega_R = 8.2\ E_L$



Ref: Y.-J. Lin et al., Nature **462**, 628 (2009).

Conclusions and outlook

- Observing vortices in a Raman-dressed BEC: superfluid in a synthetic magnetic field B^*
- stable B* in lab frame, easy to add optical lattices
- outlook: optimize vortex density ($\propto B^*$) and trap geometry
- Hofstadter butterfly: add 2D lattices
- long term: large B^* in quantum Hall regime
  prepare 2D systems, w/ filling factor $\nu = N_{2D}/N_v \leq 1$

  1D lattice along z, compress along y, relax along x
  (small N_{2D},large B* and N_v)