TRAPPED ION QUANTUM INFORMATION

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Outline

- Introduction to Quantum Information
  - motivation
  - preliminary & terminologies
  - physical implementation
- Ion Trap QC/QS
  - ion trap physics
  - laser manipulation of ions
  - implementation of a quantum entangling gate
  - application: simulation of quantum magnetism
- Outlook
  - current status and perspective

Also reference to...

Implementation of quantum computation and simulation with trapped ions

Guin-Dar Lin 林俊達
University of Michigan
2010 NCKU AMO Summer School
September 2nd, 2010

Quest for quantum information processing

Qubit = Two-level quantum system

classical bit: 0 or 1
quantum bit: $\alpha|0\rangle + \beta|1\rangle$; α, β are *c*-numbers.

The Quest in building a quantum processor:
create a general ***entangled*** state of N qubits:

$$a_0|00\ldots0\rangle + a_1|00\ldots1\rangle + \ldots + a_N|11\ldots1\rangle$$

Motivation:
Moore's law?
Harvest the power of quantum coherence and entanglement
New paradigm for information processing

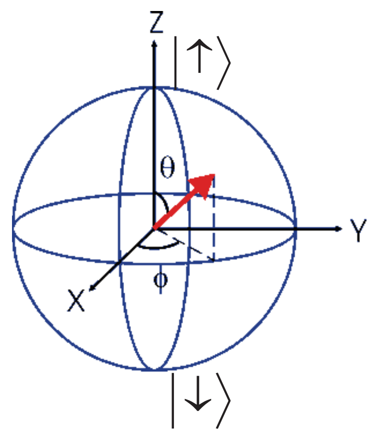
Qubit/spin: basic unit for QC/QS

Qubit = quantum two-level system
= spin-1/2 particle

$$I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad \sigma_z = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \quad \sigma_x = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \quad \sigma_y = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}$$

Choices of qubit:
any quantum two-level system
conveniently accessible to us.

e.g.
Photons (flying qubit)
Atoms (material qubit)



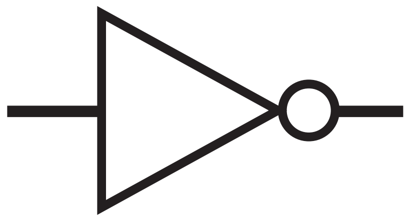
$$|\psi\rangle = \cos\frac{\theta}{2}|\downarrow\rangle + e^{i\phi}\sin\frac{\theta}{2}|\uparrow\rangle$$

Through out this talk, we mix-use the notations: $\{|0\rangle, |1\rangle\}$ and $\{|\downarrow\rangle, |\uparrow\rangle\}$

QC ~ built upon a universal set of quantum logic gates

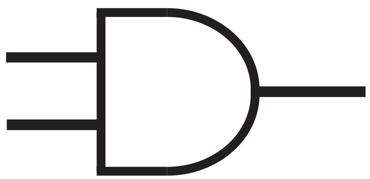
Classical

1-bit NOT



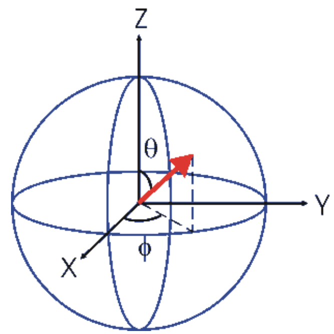
$0 \rightarrow 1$
$1 \rightarrow 0$

2-bit AND

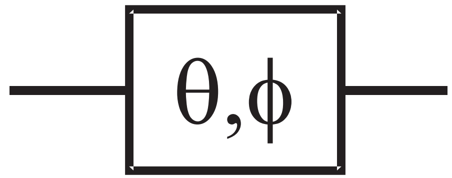


$00 \rightarrow 0$
$01 \rightarrow 0$
$10 \rightarrow 0$
$11 \rightarrow 1$

Quantum

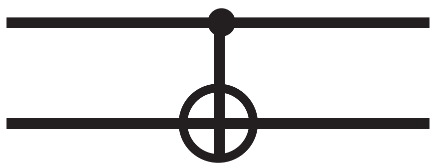


1-qubit gate (rotation)



$|0\rangle \rightarrow \cos\theta\ |0\rangle + e^{i\varphi} \sin\theta\ |1\rangle$
$|1\rangle \rightarrow \cos\theta\ |1\rangle - e^{-i\varphi} \sin\theta\ |0\rangle$

2-qubit XOR (controlled-NOT, or CNOT)



$|0\rangle\ |0\rangle \rightarrow |0\rangle\ |0\rangle$
$|0\rangle\ |1\rangle \rightarrow |0\rangle\ |1\rangle$
$|1\rangle\ |0\rangle \rightarrow |1\rangle\ |1\rangle$
$|1\rangle\ |1\rangle \rightarrow |1\rangle\ |0\rangle$

superposition → entanglement via CNOT

$[|0\rangle + |1\rangle]|0\rangle \rightarrow |0\rangle\ |0\rangle + |1\rangle\ |1\rangle$

CNOT Gate vs. Phase Gate

$\lvert\downarrow\downarrow\rangle$	CNOT	$\lvert\downarrow\downarrow\rangle$
$\lvert\downarrow\uparrow\rangle$	$\longrightarrow$	$\lvert\downarrow\uparrow\rangle$
$\lvert\uparrow\downarrow\rangle$		$\lvert\uparrow\color{red}{\uparrow}\rangle$
$\lvert\uparrow\uparrow\rangle$		$\lvert\uparrow\color{red}{\downarrow}\rangle$

Suppose we have a two-qubit phase gate…

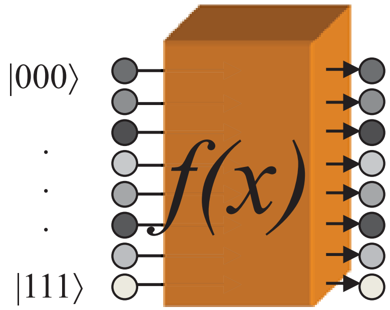
$\lvert\downarrow\downarrow\rangle$	PG	$\lvert\downarrow\downarrow\rangle$
$\lvert\downarrow\uparrow\rangle$	$\longrightarrow$	$\lvert\downarrow\uparrow\rangle$
$\lvert\uparrow\downarrow\rangle$		$\color{red}{-}\lvert\uparrow\downarrow\rangle$
$\lvert\uparrow\uparrow\rangle$		$\lvert\uparrow\uparrow\rangle$

One can realize a CNOT gate by the following steps:

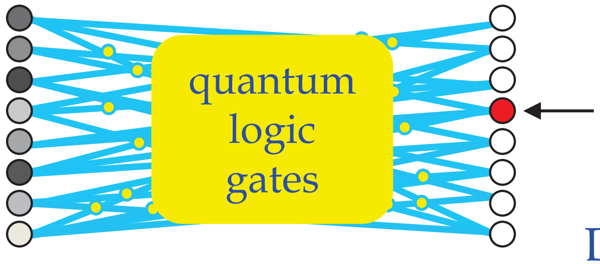
$$\lvert\uparrow_a\uparrow_b\rangle \xrightarrow{R_b(x,\frac{\pi}{2})} \lvert\uparrow\rangle_a \frac{\lvert\uparrow\rangle_b \pm \lvert\downarrow\rangle_b}{\sqrt{2}} \xrightarrow{P.G.} \lvert\uparrow\rangle_a \frac{\lvert\uparrow\rangle_b \color{red}{\mp} \lvert\downarrow\rangle_b}{\sqrt{2}} \xrightarrow{R_b(-x,\frac{\pi}{2})} \lvert\uparrow_a \color{red}{\downarrow_b}\rangle$$

What's different about quantum computation?

Superposition parallel processes 2^N inputs

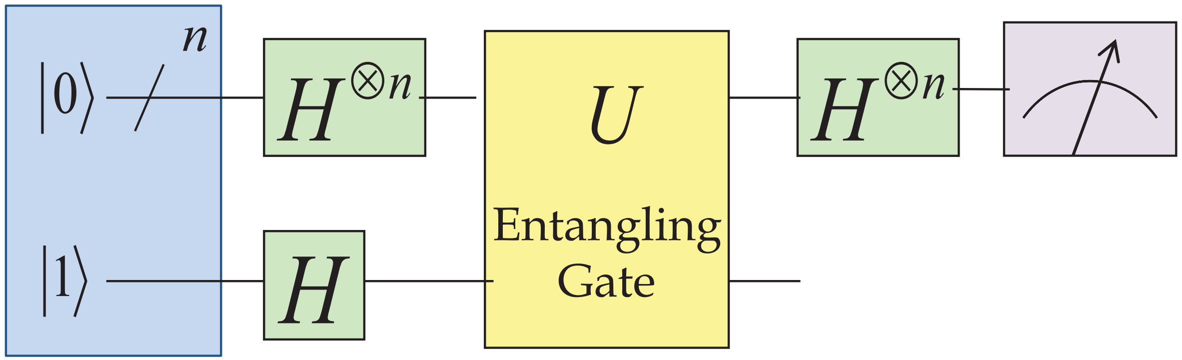


Quantum interference enhances correct outcomes and suppresses erroneous outcomes.



D. Deutsch (1985)

Superposition and entanglement can speed up certain algorithms.



Solve for $f : \{0, 1\} \longrightarrow \{0, 1\}$

Deutsch-Jozsa (Cleve et al.) algorithm: deterministic and requires only one quiry

Requirements for quantum compution

1. Scalable system of qubits: $a|0\rangle + b|1\rangle;\ |a|^2 + |b|^2 = 1.$
2. A qubit-specific measurement capability
3. Initialization: $|0,0,0,\cdots\rangle$
4. A "universal" set of quantum gates
5. Decoherence times τ >> gate operation time

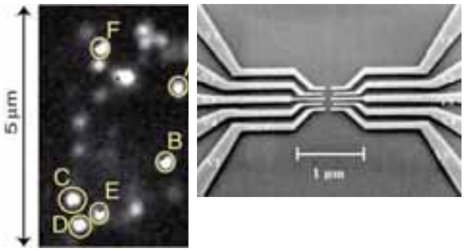
$$a|0\rangle + b|1\rangle \xrightarrow{\tau} |a|^2 |0\rangle\langle 0| + |b|^2 |1\rangle\langle 1|.$$

Two additional criteria for quantum communication

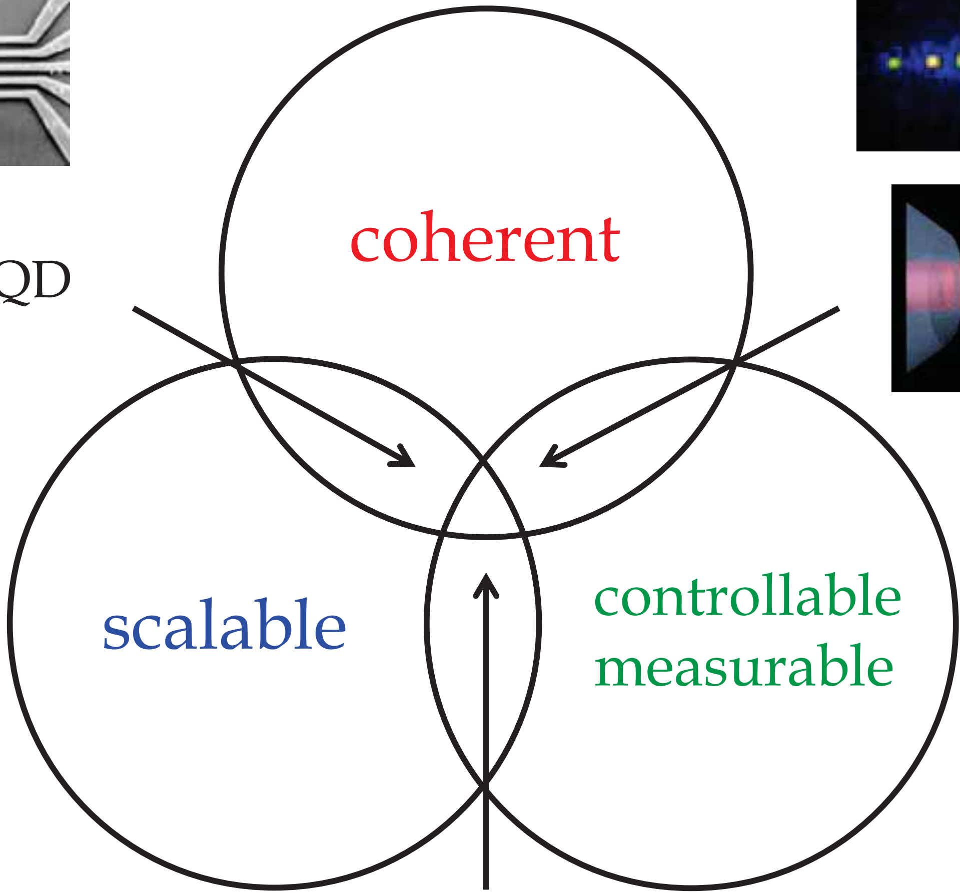
6. Interconvert stationary and flying qubits
7. Transmit flying qubits between distant locations

D. P. DiVincenzo, Fortschr. Phys. **48** 9-11, 771-783(2000)

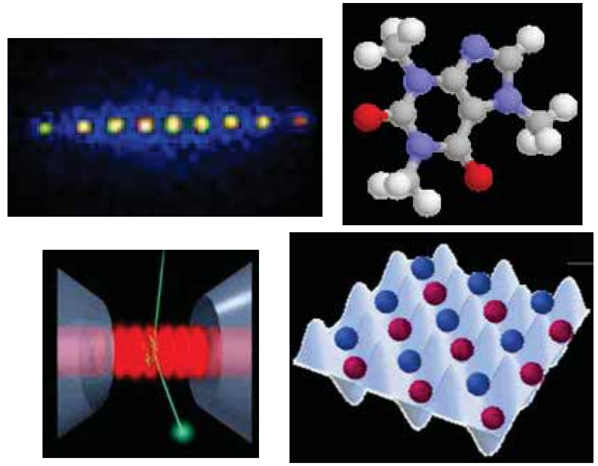
physical implementations of a QC/QS



Nuclear spin in QD
Diamond NVC

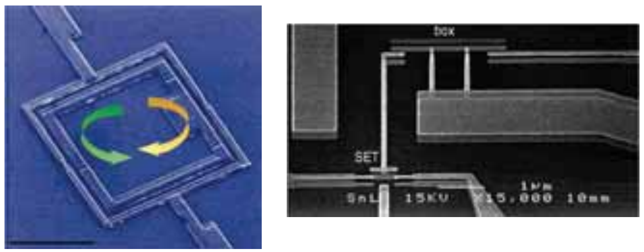


Bottom-up



Trapped ions
Atoms in OL
Cavity QED
molecule NMR

Top-down



Cooper pair box
SQUID

Ion quantum computer

- Advantages
  - Long storage time (⟶ hours to months)
  - Long coherence time (⟶ seconds to minutes)
  - Strong controlled interactions (⟶ kHz to MHz)
- State-of-the-art quantum entangling gates
  - Cirac-Zoller gate
  - Milburn-Schneider-James gate (σ_z gate)
  - Molmer-Sorensen gate (σ_x gate)
  - Ultrafast gate (multi-mode coherent control)
  - Laserless gate (B field gradient)
- Perspectives
  - Scalable to a large quantum system (currently < 20 ions)
  - Improve fidelity: toward fault-tolerant QC (infidelity < 10^{-3})
    - fidelities of initialization, state detection, single- and two-qubit gates…

What is an entangled state?

E.g., two particles are entangled if their complete quantum state cannot be expressed as the product of the quantum states of each individual one. I.e.,

$$|\psi\rangle_{a,b} \neq |\phi\rangle_a |\varphi\rangle_b$$

Product state:

$$|\phi\rangle_{a,b} = |1\rangle_a |1\rangle_b$$

$$|\phi\rangle_{a,b} = (\alpha|1\rangle + \beta|0\rangle)_a \cdot (\gamma|1\rangle + \delta|0\rangle)_b$$

Entangled states:

$$|\psi\rangle_{a,b} = |1\rangle_a |0\rangle_b + |0\rangle_a |1\rangle_b$$

$$|\psi\rangle_{a,b} = |1\rangle_a |1\rangle_b + |1\rangle_a |0\rangle_b + |0\rangle_a |1\rangle_b - |0\rangle_a |0\rangle_b$$

Peris-Horodeck criterion, and O. Ghne and G. Toth, Phys Rep **474,** 1 (2009)…

Entangled state generation

Strategy I. delete some constituent states

$$|\phi\rangle_{a,b} = \left(|1\rangle_a + |0\rangle_a\right)\cdot\left(|1\rangle_b + |0\rangle_b\right)$$

$$|\phi'\rangle_{a,b} = |1\rangle_a|1\rangle_b + |1\rangle_a|0\rangle_b + |0\rangle_a|1\rangle_b + |0\rangle_a|0\rangle_b$$

Strategy II. alter the phase (sign) of some constituent states

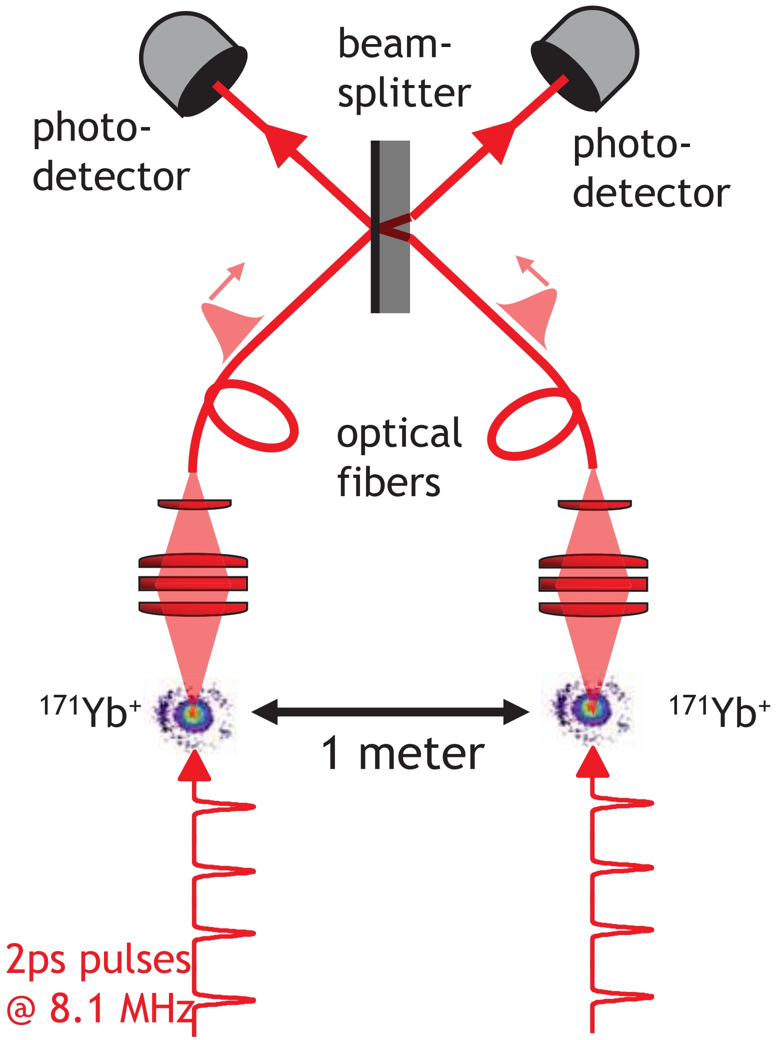
$$|\phi\rangle_{a,b} = |1\rangle_a|1\rangle_b + |1\rangle_a|0\rangle_b + |0\rangle_a|1\rangle_b + |0\rangle_a|0\rangle_b$$

$$\Big\Downarrow \ \hat{U}$$

$$|\phi'\rangle_{a,b} = |1\rangle_a|1\rangle_b + \quad |1\rangle_a|0\rangle_b + \quad |0\rangle_a|1\rangle_b + |0\rangle_a|0\rangle_b$$

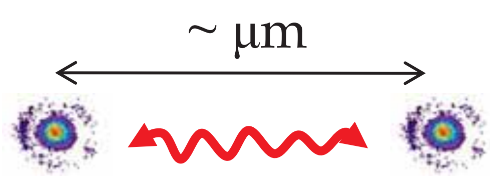
Entanglement generation

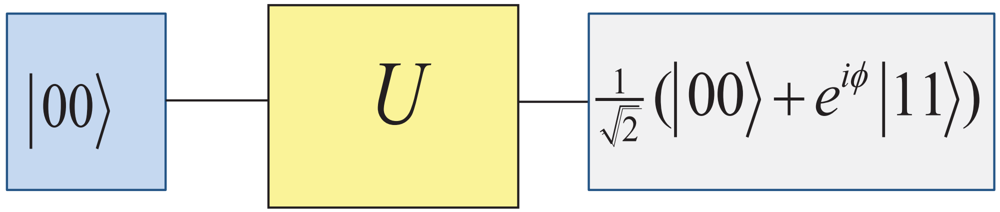
Strategy I



Moehring *et al.*, Nature **449**, 68 (2007)

Strategy II





$$U(\tau) = \exp\{-\frac{i}{\hbar} H\tau\},$$

where $H = J\sigma_1^z \sigma_2^z.$

Entanglement vs. entangling gate

Strategy I. deletion

$$|11\rangle \Rightarrow \times$$
$$|10\rangle \Rightarrow |10\rangle$$
$$|01\rangle \Rightarrow |01\rangle$$
$$|00\rangle \Rightarrow \times$$

Measurement based:
probabilistic

No outcome for $|11\rangle$ & $|00\rangle$

Not a gate!

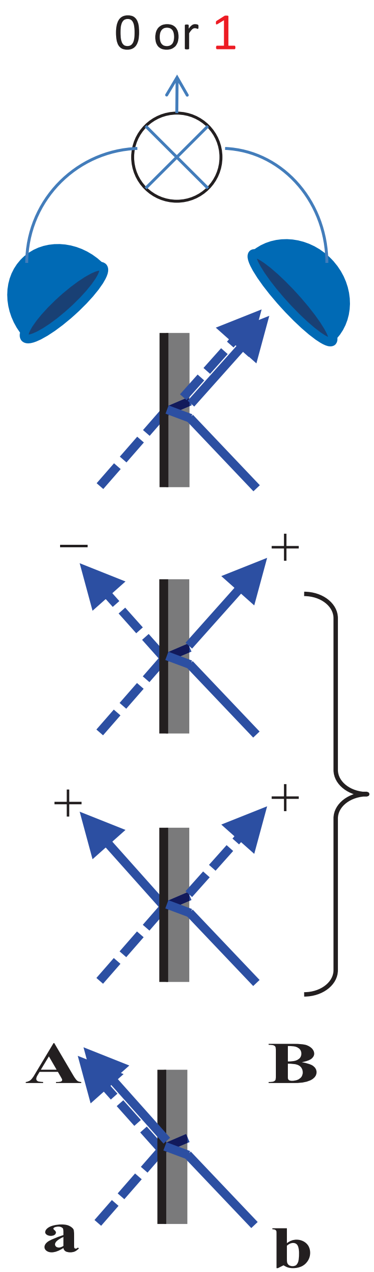
Strategy II. spin-dependent phase shift

$$|11\rangle \Rightarrow |11\rangle$$
$$|10\rangle \Rightarrow i|10\rangle$$
$$|01\rangle \Rightarrow i|01\rangle$$
$$|00\rangle \Rightarrow |00\rangle$$

Interaction based:
deterministic

A phase gate

measurement induced entanglement



Hong-Ou-Mandel Interference
- photon bunching for identical photons

photon bunching $\begin{cases} |r,r\rangle_{a,b} \\ |b,b\rangle_{a,b} \end{cases} \Rightarrow \begin{cases} |0,rr\rangle_{A,B} + e^{i\theta} |rr,0\rangle_{A,B} \\ |0,bb\rangle_{A,B} + e^{i\theta} |bb,0\rangle_{A,B} \end{cases}$

destructive interference of paths, no outcome ... unless two photons are not identical

$$\begin{cases} |r,b\rangle_{a,b} \\ |b,r\rangle_{a,b} \end{cases} \Rightarrow \begin{cases} |0,rb\rangle_{A,B} + e^{i\theta} |rb,0\rangle_{A,B} \\ |r,b\rangle_{A,B} + e^{i\theta} |b,r\rangle_{A,B} \end{cases}$$

↓

coincidence photon detection

Y.H. Shih and C.O. Alley, Proc. 2nd Int'l Symp. Found. Quant. Mech, Tokyo (1986)
Hong, Ou, and Mandel, *Phys. Rev. Lett.*, **59**, 2044 (1987)
Y.H. Shih and C.O. Alley, *Phys. Rev. Lett.* **61**, 2921 (1988)

Preservation of Entanglement

Strategy I. deletion

$$|11\rangle \Rightarrow \text{✗}$$

$$|10\rangle \Rightarrow |10\rangle$$

$$|01\rangle \Rightarrow |01\rangle$$

$$|00\rangle \Rightarrow \text{✗}$$

Measurement based:
probabilistic

Entanglement is destroyed
upon measurement!

Strategy II. spin-dependent phase shift

$$|11\rangle \Rightarrow |11\rangle$$

$$|10\rangle \Rightarrow i|10\rangle$$

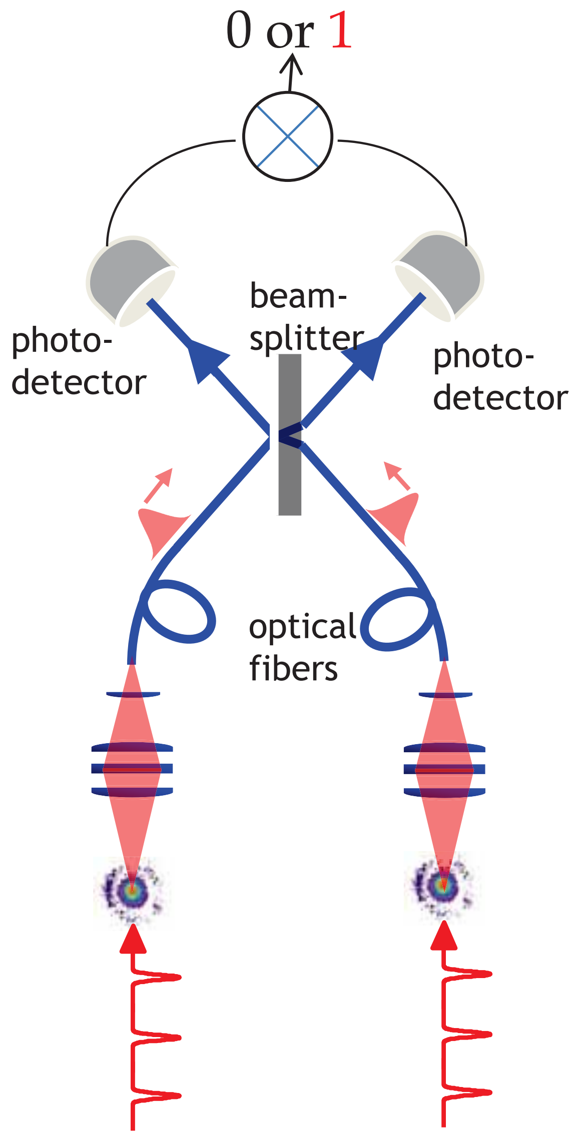
$$|01\rangle \Rightarrow i|01\rangle$$

$$|00\rangle \Rightarrow |00\rangle$$

Interaction based:
deterministic

Entanglement is preserved.

heralded entanglement



$$|\Psi\rangle = (|\downarrow\rangle_1 |\text{blue}\rangle_1 + |\uparrow\rangle_1 |\text{red}\rangle_1) \otimes (|\downarrow\rangle_2 |\text{blue}\rangle_2 + |\uparrow\rangle_2 |\text{red}\rangle_2)$$

ion state

photon state

...upon coincidence photon detection

$$\Rightarrow |\downarrow\rangle_1 |\uparrow\rangle_2 - |\uparrow\rangle_2 |\downarrow\rangle_2$$

Preserve entangled state for two atomic qubits

Moehring *et al.*, Nature **449**, 68 (2007)

Olmschenk *et al.*, Science **323**, 486 (2009)

Maunz *et al.*, PRL **102**, 250502 (2009)

Pironi *et al.*, Nature **464**, 1021 (2010)

Hong, Ou, Mandel, PRL 59, 2044 (1987)

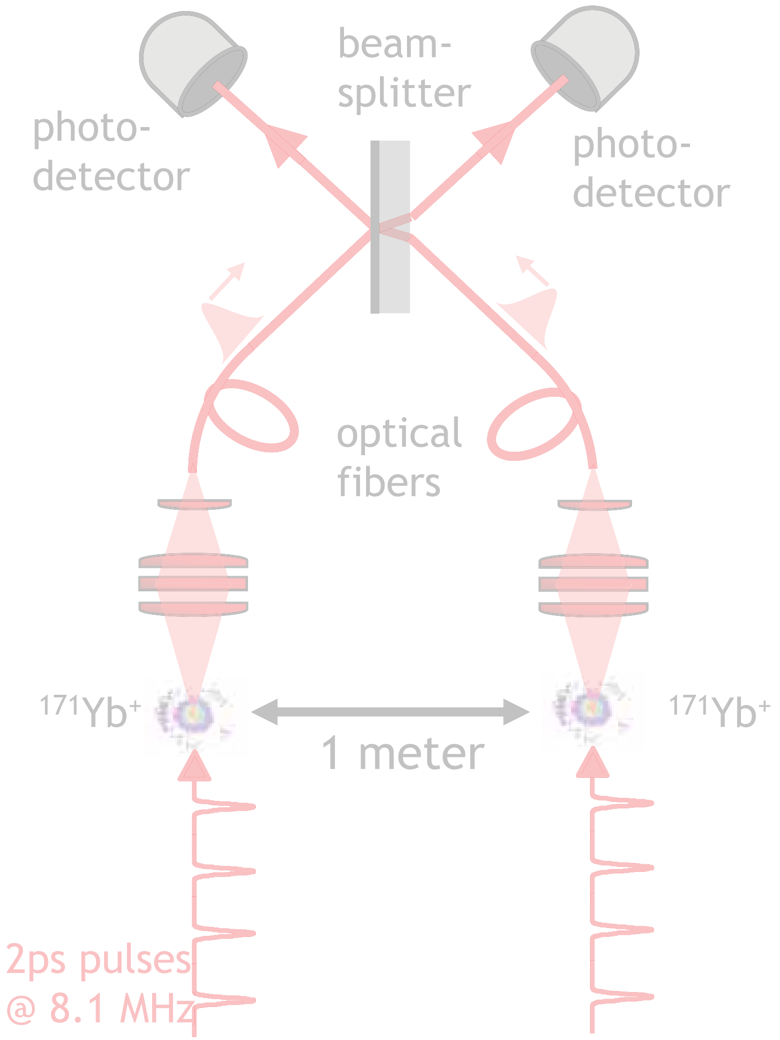
Y.H. Shih & C. O. Alley, PRL 61, 2921 (1988)

C. Simon and W. Irvine, PRL **91**, 110405 (2003)

L.-M. Duan, et. al., *Quant. Inf. Comp.* **4**, 165 (2004)

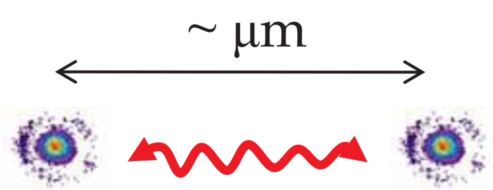
Entanglement generation

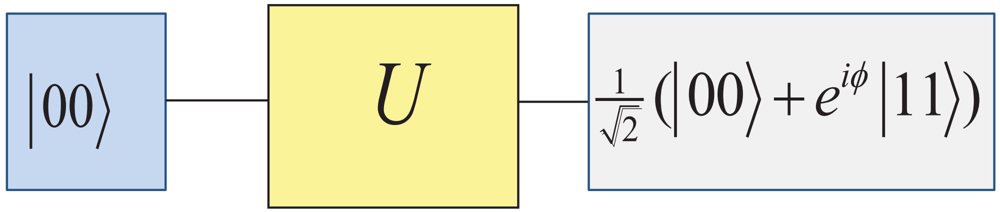
Strategy I



Moehring *et al.*, Nature **449**, 68 (2007)

Strategy II



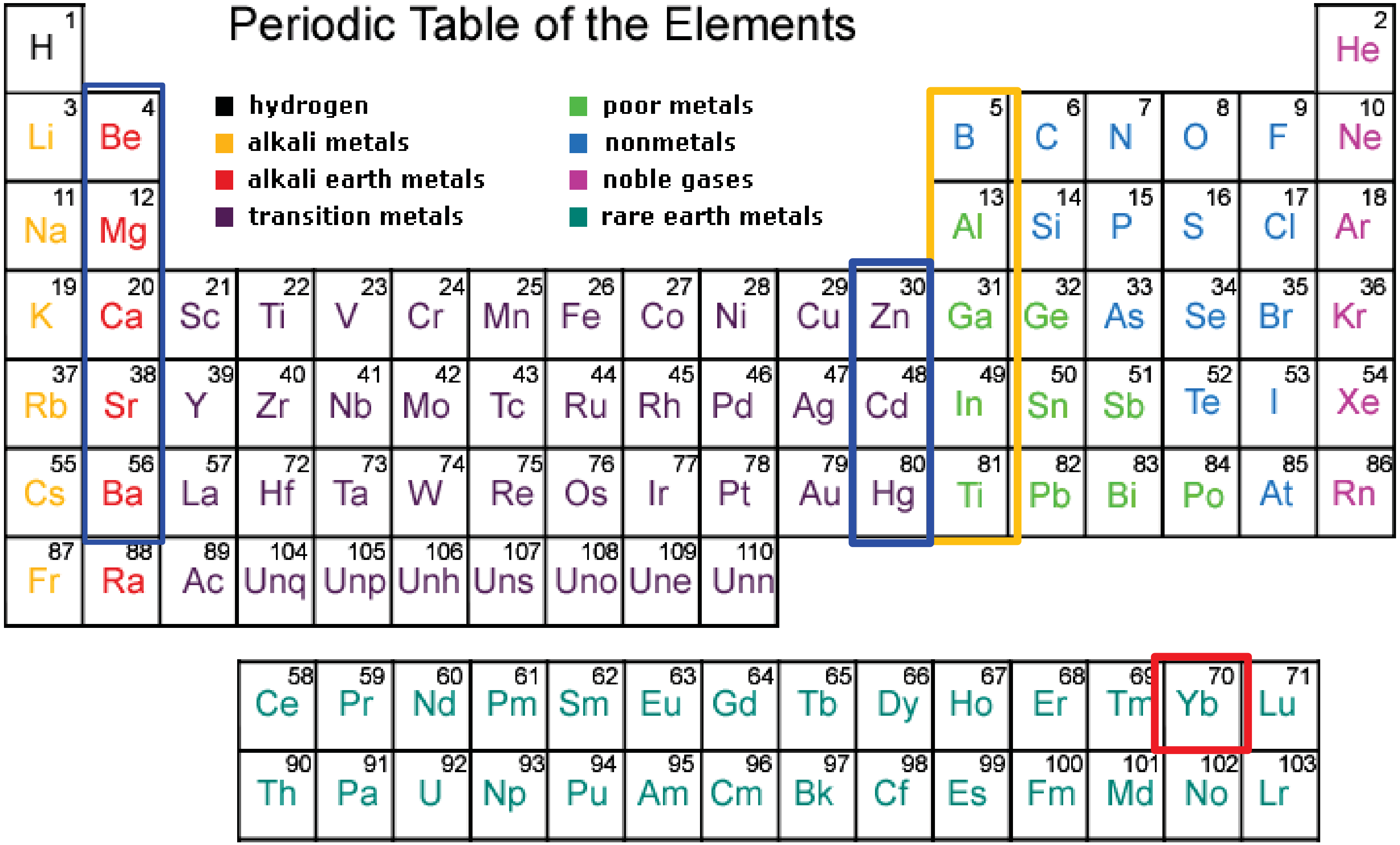


$$U(\tau) = \exp\{-\frac{i}{\hbar} H\tau\},$$

where $H = J\sigma_1^z \sigma_2^z.$

Ion trap physics

Which ion to trap?



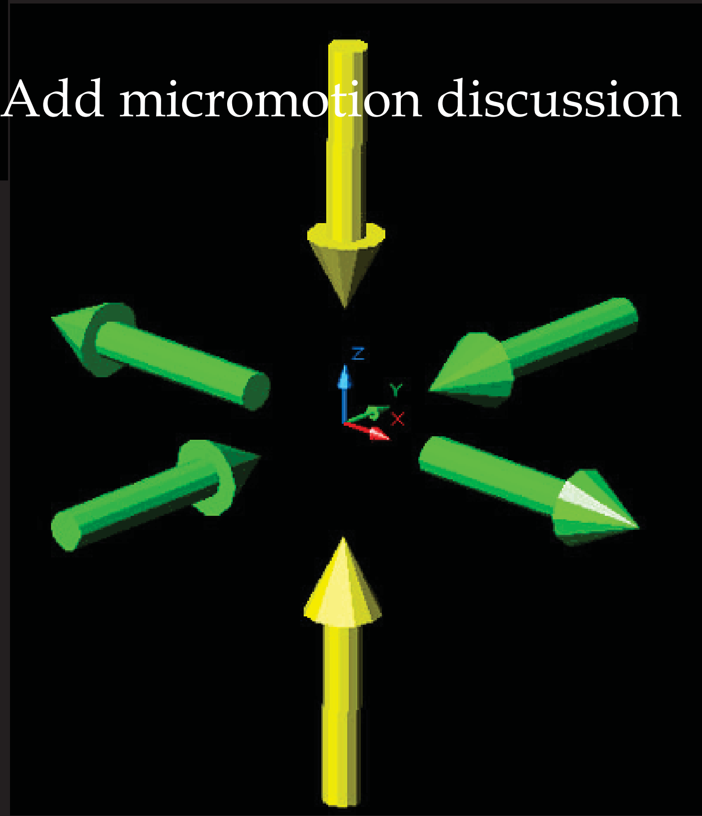
In this talk $^{171}Yb^{+}$ is used.

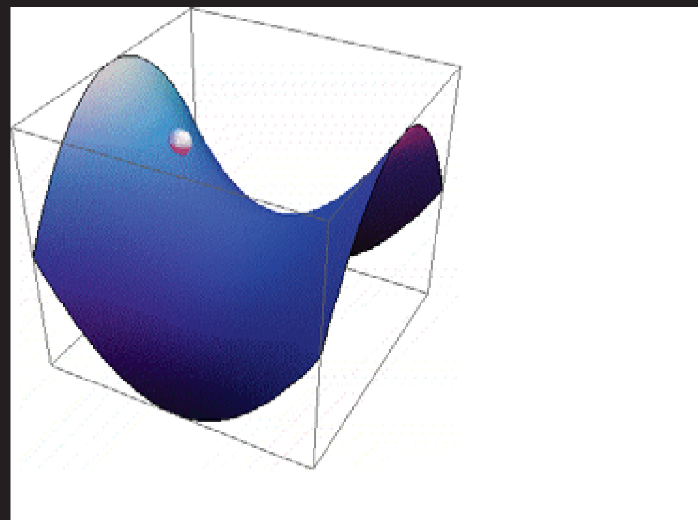
How to trap an ion?

Electric Field Vectors

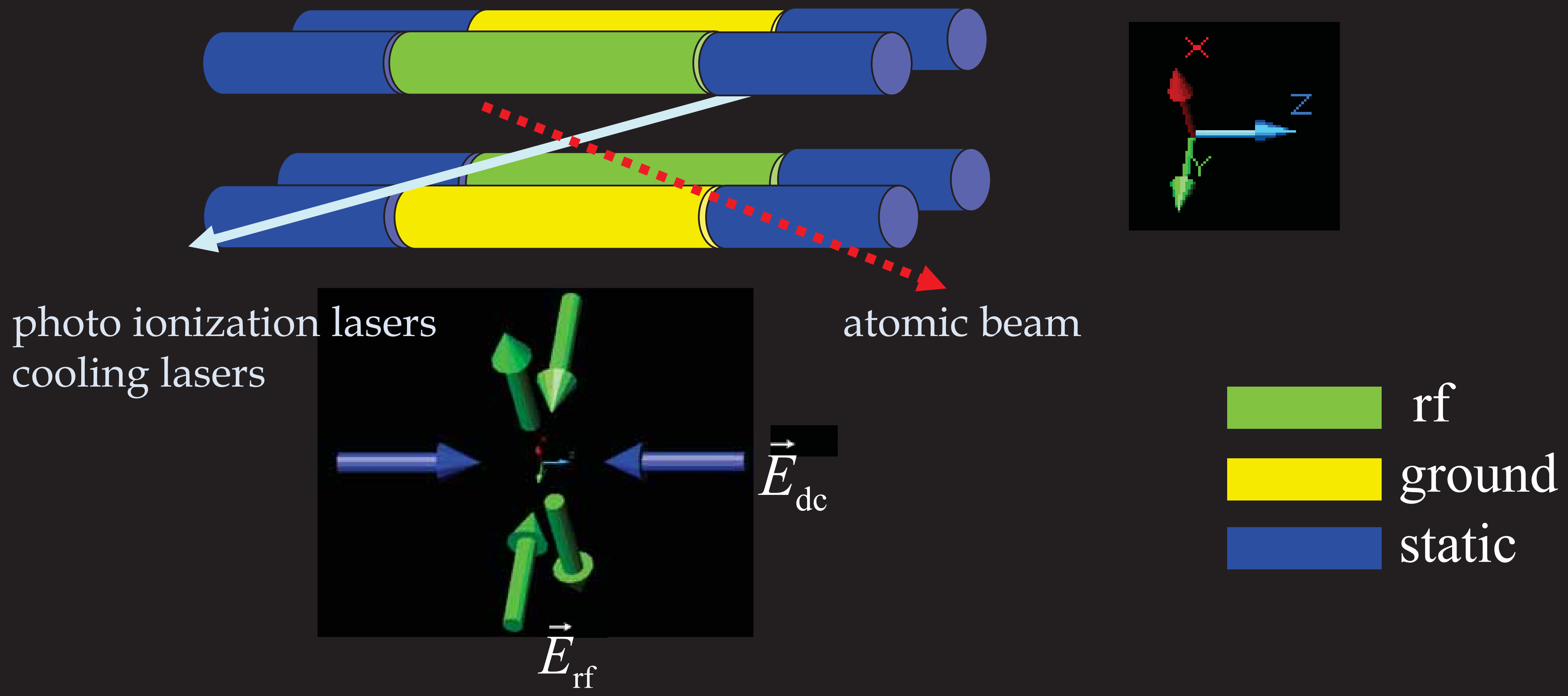
$$\vec{\nabla} \bullet \vec{E} = 0$$

Add micromotion discussion

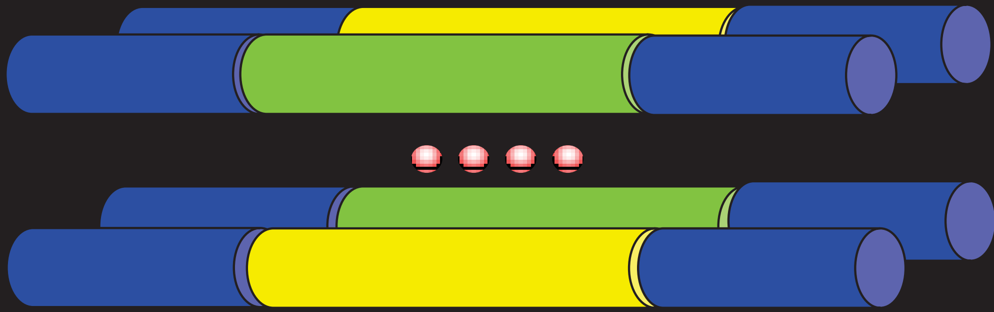


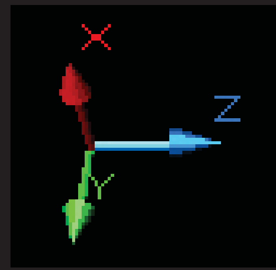


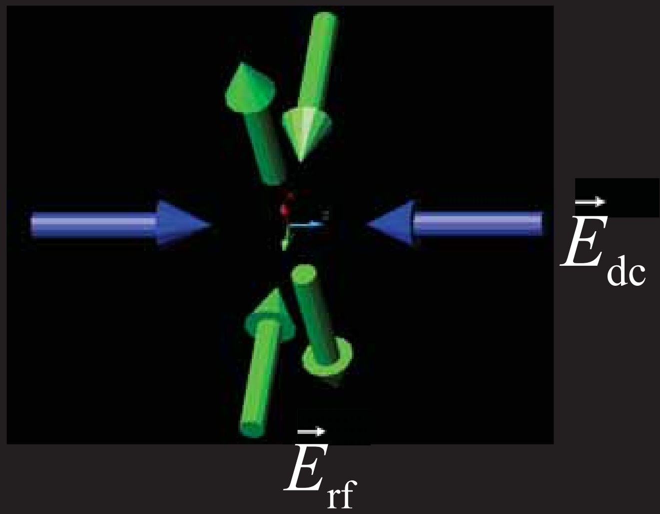
Linear Paul Trap



Linear Paul Trap

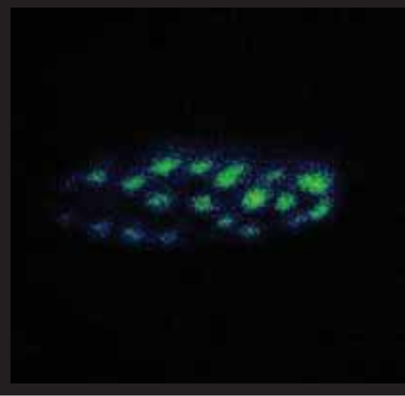




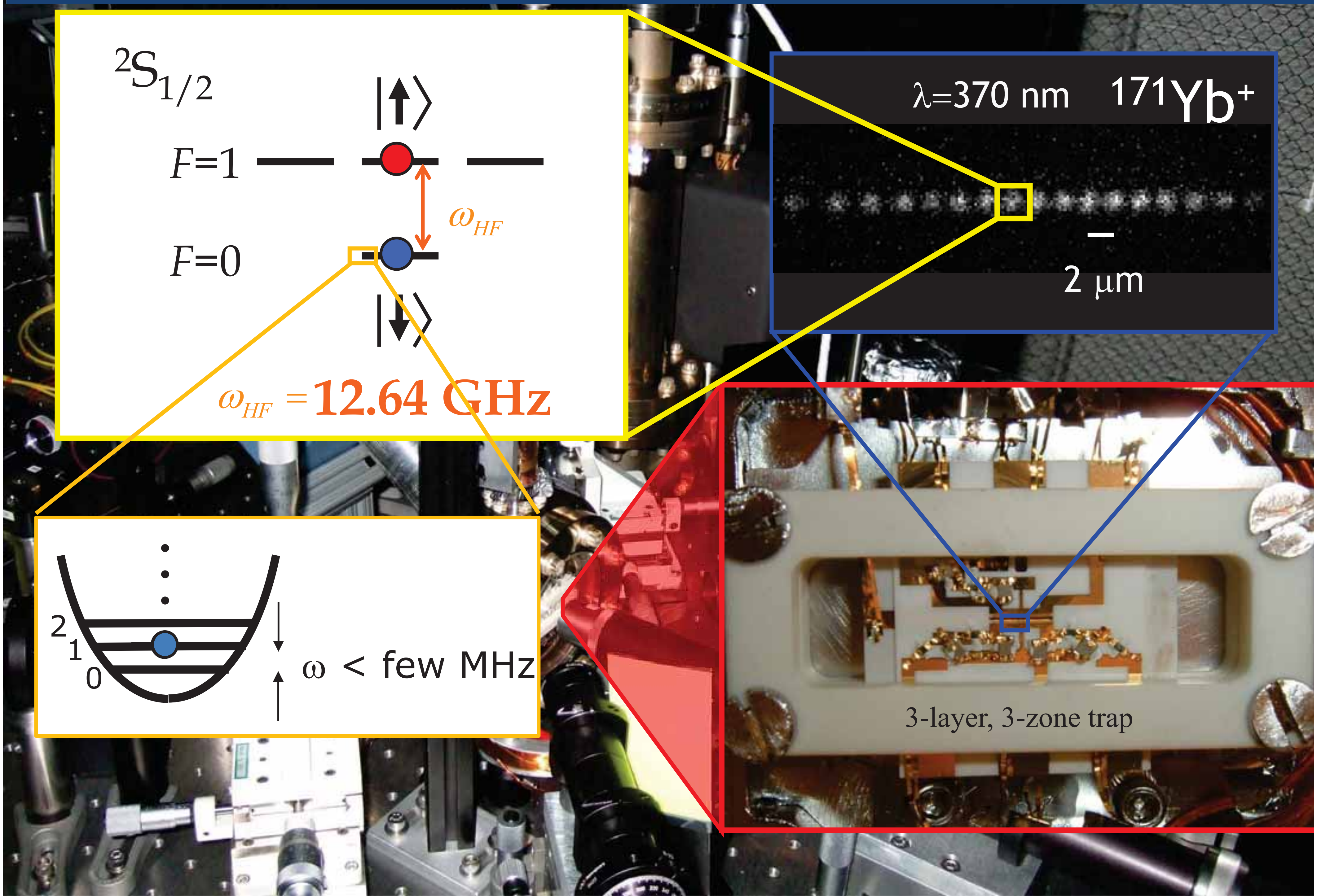


rf
ground
static

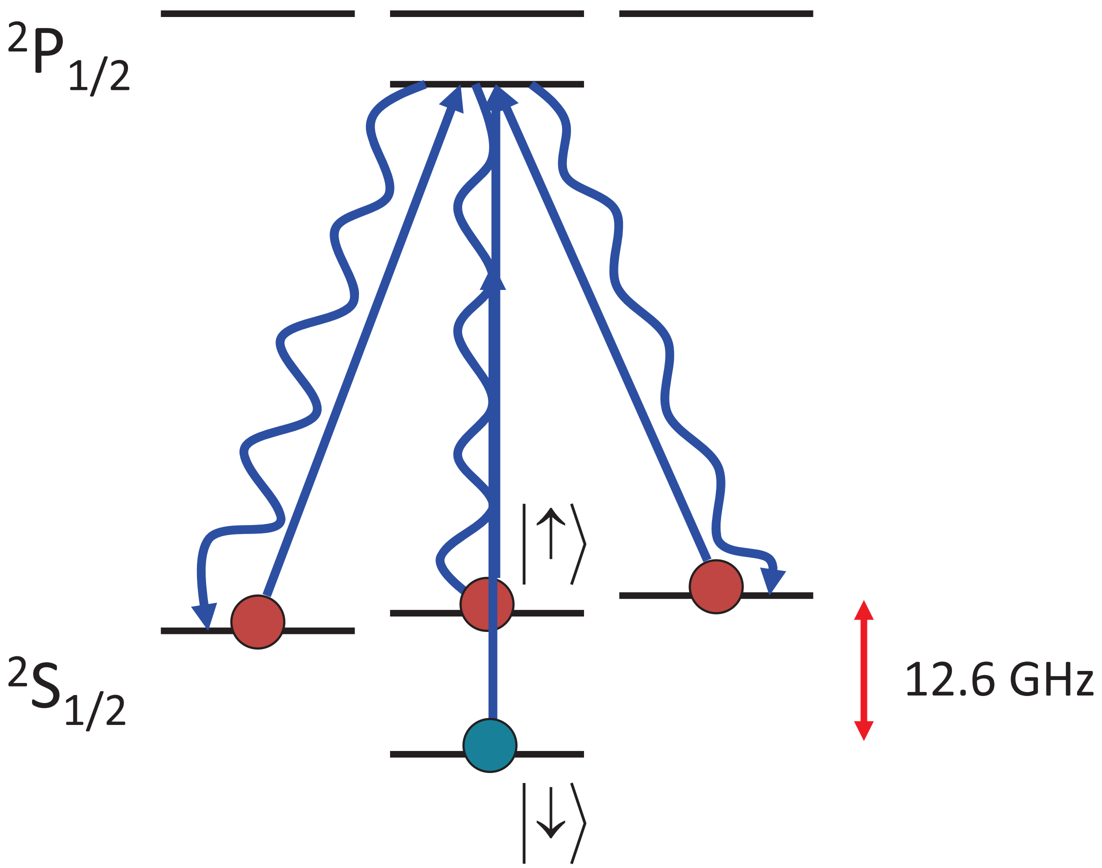
Gallery

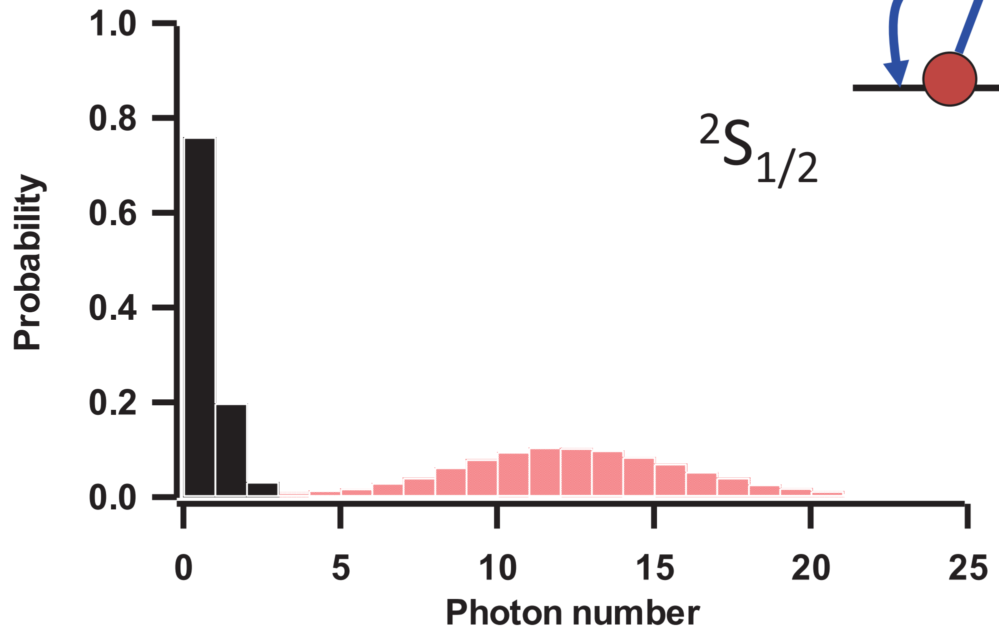


Trapped Ion QC / QS @ UMD



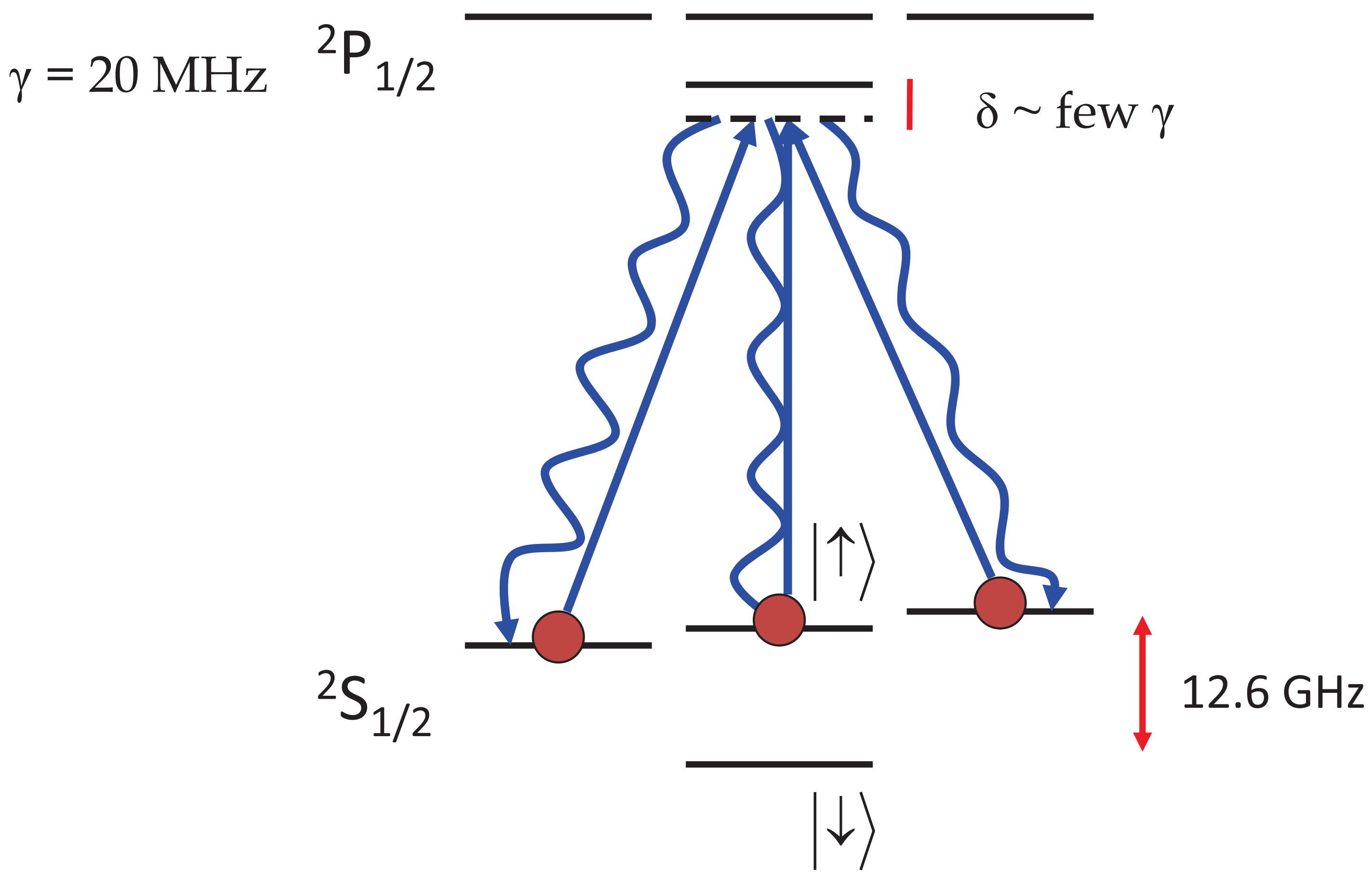
State detection



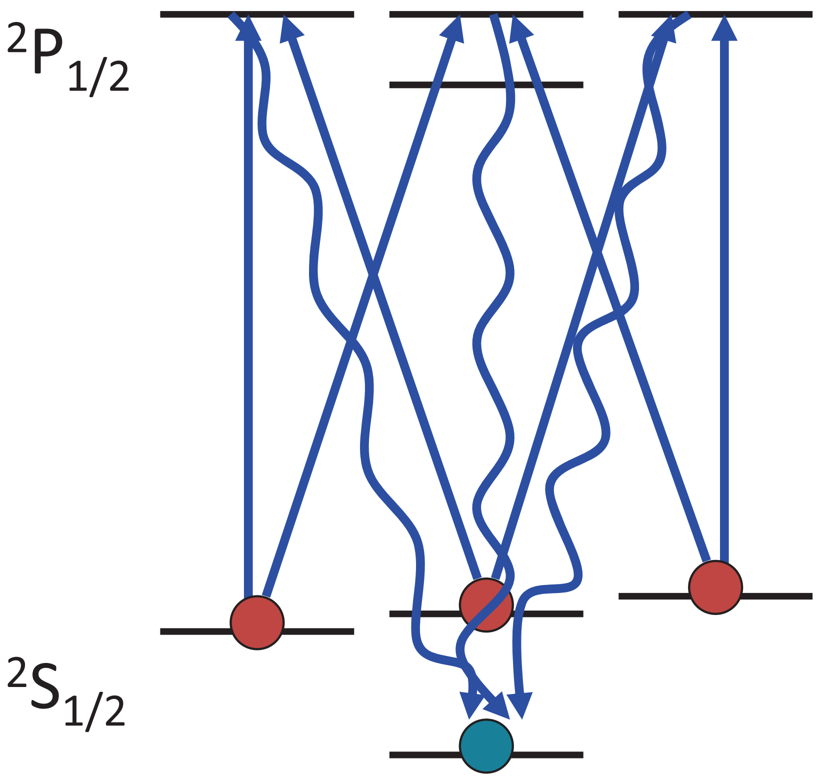


State detection fidelity ~ 97%

Laser cooling (Doppler cooling)

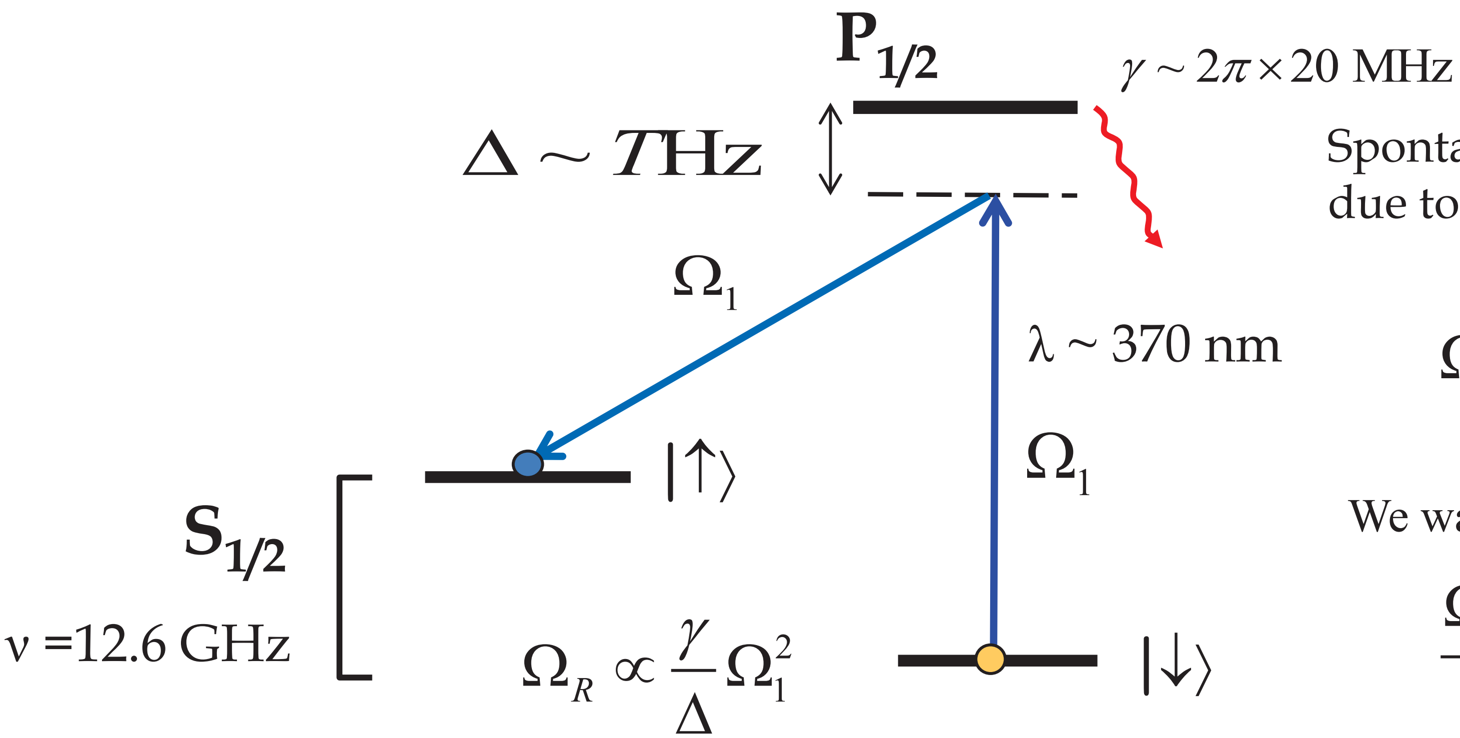


Qubit (spin) initialization



Coupling qubit states

Coupling two ground hyperfine states (qubit states) with a Raman transition (two-photon transition)



Spontaneous emission rate due to off-resonance excitation

$$\Omega_{spon} \propto \frac{\gamma^2}{\Delta^2}\Omega_1^2$$

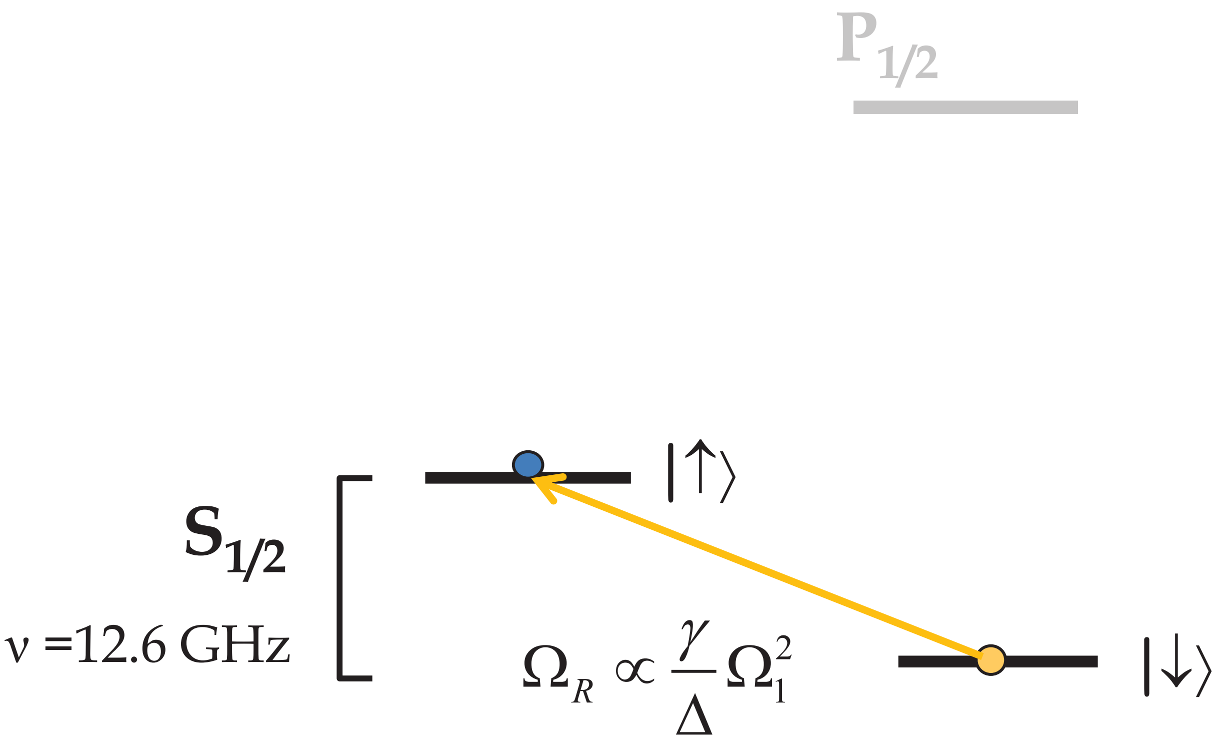
We want

$$\frac{\Omega_{spon}}{\Omega_R} = \frac{\gamma}{\Delta} << 1$$

The larger detuning (Δ), the better…but needs more laser power to keep the same coupling rate (Ω_R).

Coupling qubit states

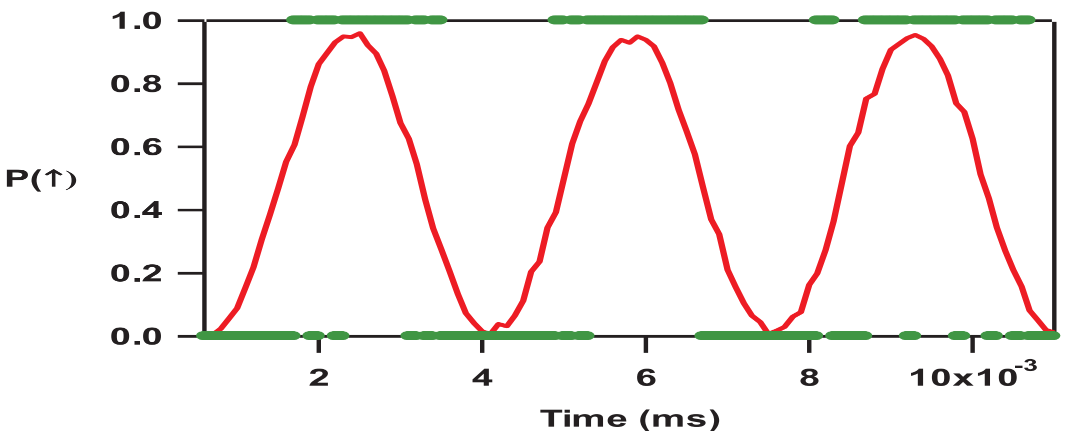
Coupling two ground hyperfine states (qubit states) with Raman transition (two-photon transition)

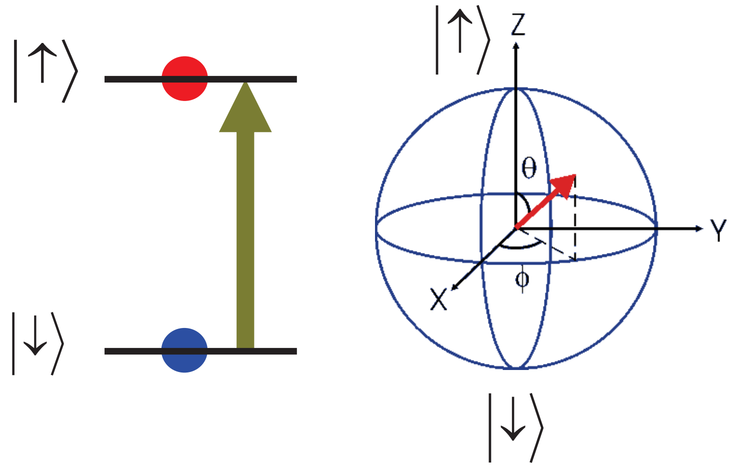


One can also use a microwave photon to do the job… but optical photon carries larger momentum, which will be used to control the ion motion.

Single qubit gate & spin coherence time measurement

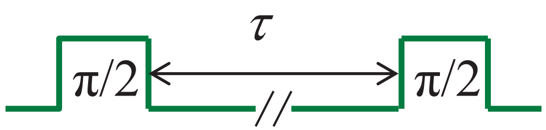
Rabi oscillation



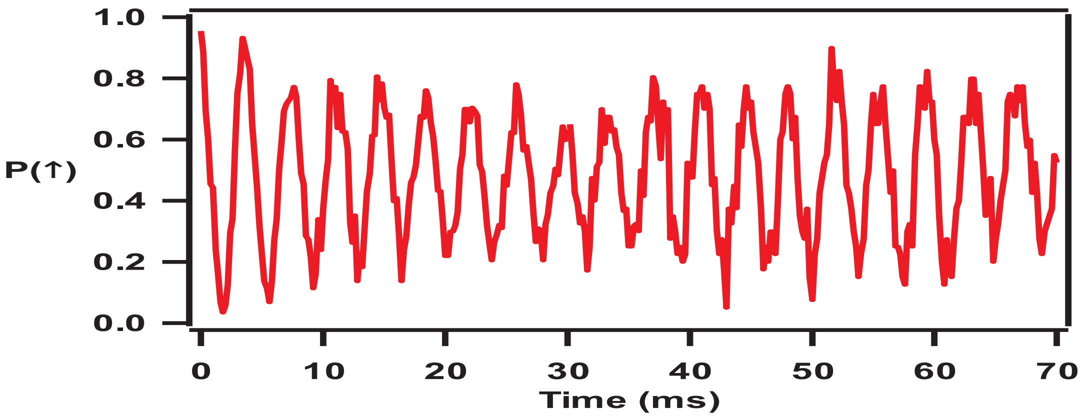


$$R_{\phi+\pi/2}(\theta)$$

Ramsey oscillation



Coherence time > 70 ms

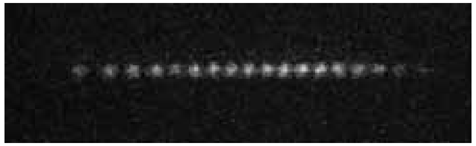


$$P(\uparrow) = \frac{1}{2} + \frac{1}{2}\sin(\delta\tau + \phi),$$

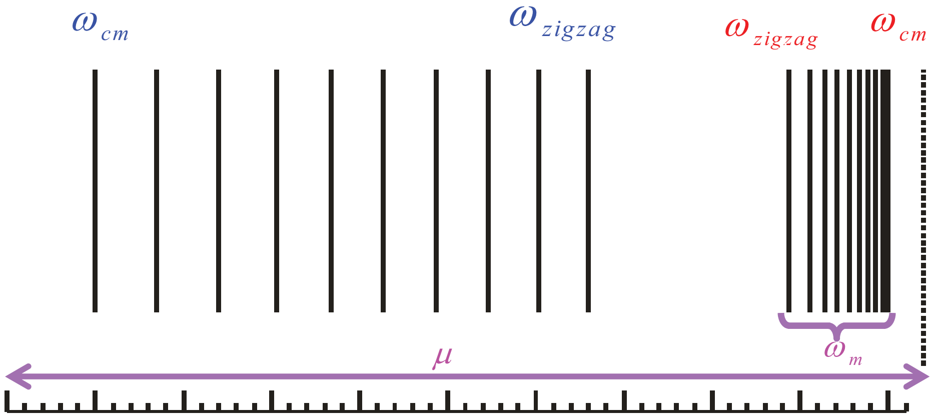
δ : detuning

ϕ : phase

Phonons: ion motional normal modes



$$H = \sum_i \frac{p_i^2}{2m} + \frac{1}{2}\sum_{i,j} x_i V_{ij} x_j$$
$$= \sum_k \hbar\omega_k (a_k^+ a_k + \tfrac{1}{2})$$



$e.g.\quad \omega_{com}^{transverse} / \omega_{com}^{axial} = 10$

cm = center of mass mode

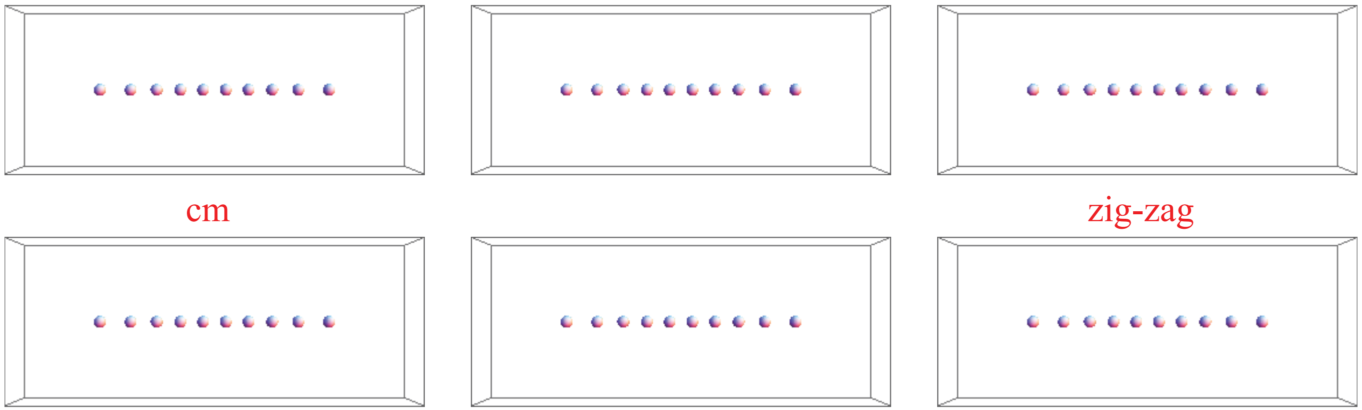
zig-zag

Axial

cm

zig-zag

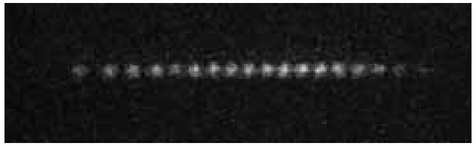
Transverse



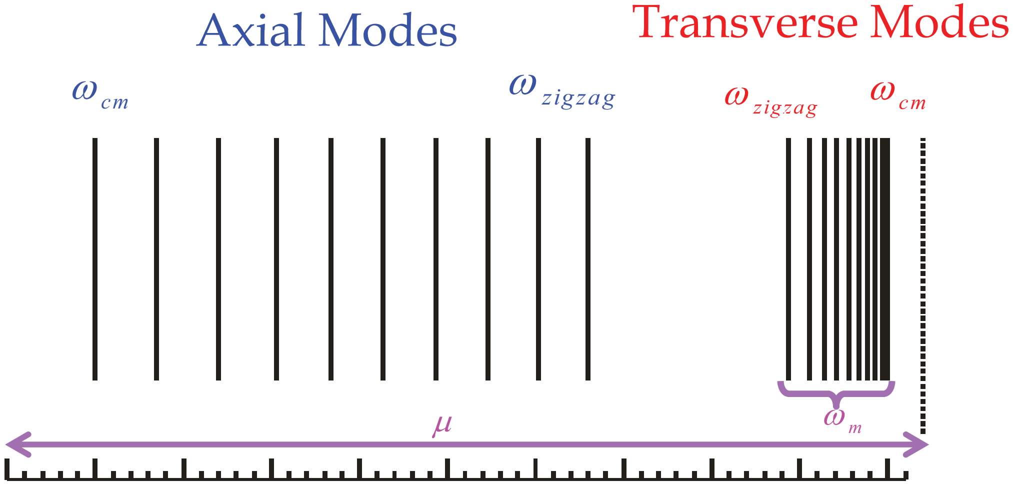
ion-ion spacing is ~μm, while the vibration amplitude is ~ nm.

James, APB **66**, 181 (1998); Zhu, Monroe, and Duan, PRL **97**, 050505 (2006).

Phonons: ion motional normal modes

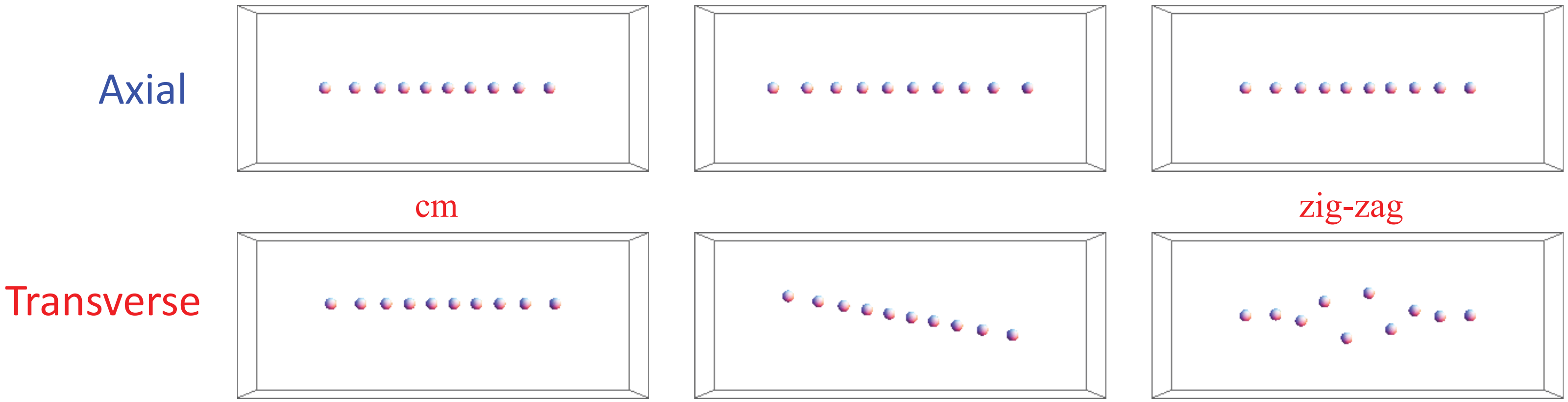


$$H = \sum_i \frac{p_i^2}{2m} + \frac{1}{2}\sum_{i,j} x_i V_{ij} x_j$$
$$= \sum_k \hbar\omega_k (a_k^+ a_k + \tfrac{1}{2})$$



$e.g.\ \ \omega_{com}^{transverse} / \omega_{com}^{axial} = 10$

cm = center of mass mode



ion-ion spacing is ~µm, while the vibration amplitude is ~ nm.

James, APB **66**, 181 (1998); Zhu, Monroe, and Duan, PRL **97**, 050505 (2006).

Spin-motion coupling

$$H = \hbar\omega_0\hat{\sigma}_z + \frac{\hat{p}^2}{2m} + \frac{1}{2}m\omega^2\hat{x}^2 - \hat{\mu}\cdot E(\hat{x})$$

$$\underbrace{\frac{\hat{p}^2}{2m} + \frac{1}{2}m\omega^2\hat{x}^2}_{\hbar\omega(a^+a+1/2)}$$

$$-\hat{\mu}\cdot E(\hat{x}) \rightarrow -\mu_0\cdot\frac{E_0}{2}(\hat{\sigma}_+ + \hat{\sigma}_-)(e^{ik\hat{x}-i\omega_L t} + e^{-ik\hat{x}+i\omega_L t})$$

ω_L: frequency of applied radiation

interaction frame; "rotating wave approximation"

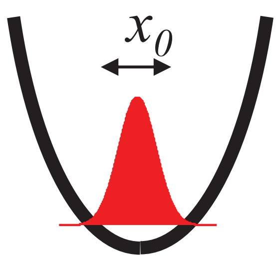
$$H = \hbar g(\hat{\sigma}_+ e^{ik\hat{x}-i\delta t} + \hat{\sigma}_- e^{-ik\hat{x}+i\delta t})$$

$\delta = \omega_L - \omega_0 =$ *detuning*

$k = 2\pi/\lambda =$ *wavenumber*

$$\hat{x} = x_0(ae^{-i\omega t} + a^+e^{i\omega t})$$

$$x_0 = \sqrt{\frac{\hbar}{2m\omega}}$$



$$H = \hbar g\left[\hat{\sigma}_+ e^{ikx_0(ae^{-i\omega t}+a^+e^{i\omega t})-i\delta t} + \hat{\sigma}\_e^{-ikx_0(ae^{-i\omega t}+a^+e^{i\omega t})+i\delta t}\right]$$

$$= \hbar g\{[\hat{\sigma}_+ e^{-i\delta t}(1+ikx_0(ae^{-i\omega t}+a^+e^{i\omega t})+H.O.]$$

$$+H.C.\}$$

stationary terms arise in H at particular values of δ:

$$\delta = 0 \qquad H_0 = \hbar g(\hat{\sigma}_+ + \hat{\sigma}_-) \xrightarrow{kx_0\sqrt{n+1}<<1} \langle\downarrow,n|H_0|\uparrow,n\rangle = \hbar g$$

"CARRIER"

$$\delta = -\omega \qquad H_{-1} = \hbar g(kx_0)(\sigma a^+ + \sigma a^+) \xrightarrow{kx_0\sqrt{n+1}<<1} \langle\downarrow,n+1|H_{+1}|\uparrow,n\rangle = \hbar g\sqrt{n+1}$$

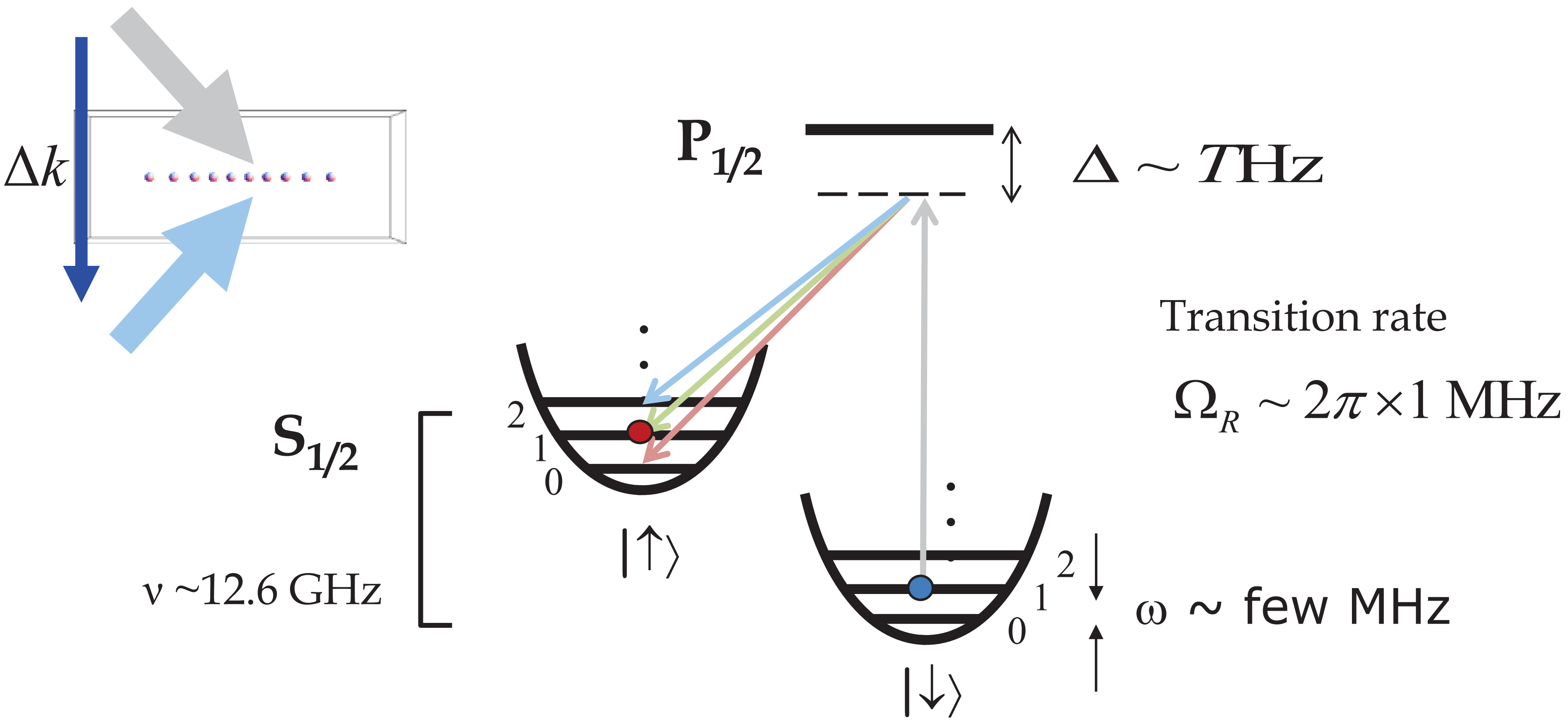
"1ST RED SIDEBAND"

$$\delta = +\omega \qquad H_{+1} = \hbar g(kx_0)(\hat{\sigma}_+ a + \hat{\sigma}\_a^+) \xrightarrow{kx_0\sqrt{n+1}<<1} \langle\downarrow,n-1|H_0|\uparrow,n\rangle = \hbar g\sqrt{n}$$

"1ST BLUE SIDEBAND"

$kx_0\sqrt{n+1}<<1$ "Lamb-Dicke Limit"

Coupling Spin to Motion



carrier $H_c = \frac{\hbar\Omega}{2}\sigma_+ + h.c.$

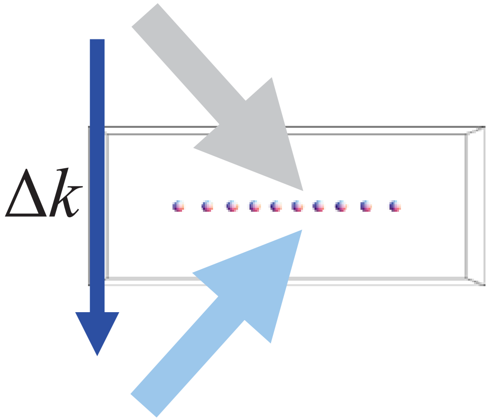
blue sideband $H_{bsb} = \frac{\hbar\eta\Omega}{2}\sigma_+ a^\dagger + h.c.$

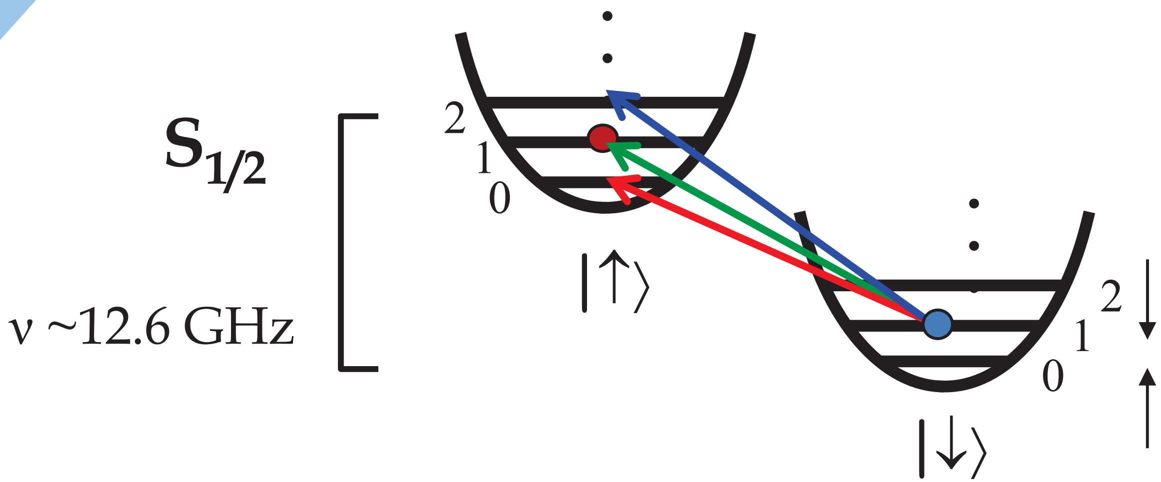
red sideband $H_{rsb} = \frac{\hbar\eta\Omega}{2}\sigma_+ a + h.c.$

Lamb-Dicke Parameter

$$\eta = \Delta k x_0 \sqrt{n} \propto \frac{x_0}{\lambda} << 1$$

Coupling Spin to Motion





ω ~ few MHz

Transition rate

$$\Omega_R \sim 2\pi \times 1\ \mathrm{MHz}$$

carrier $$H_c = \frac{\hbar\Omega}{2}\sigma_+ + h.c.$$

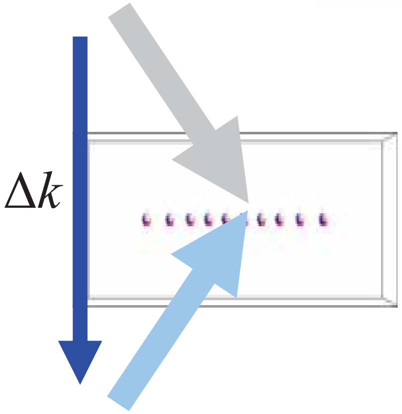
blue sideband $$H_{bsb} = \frac{\hbar\eta\Omega}{2}\sigma_+ a^\dagger + h.c.$$

red sideband $$H_{rsb} = \frac{\hbar\eta\Omega}{2}\sigma_+ a + h.c.$$

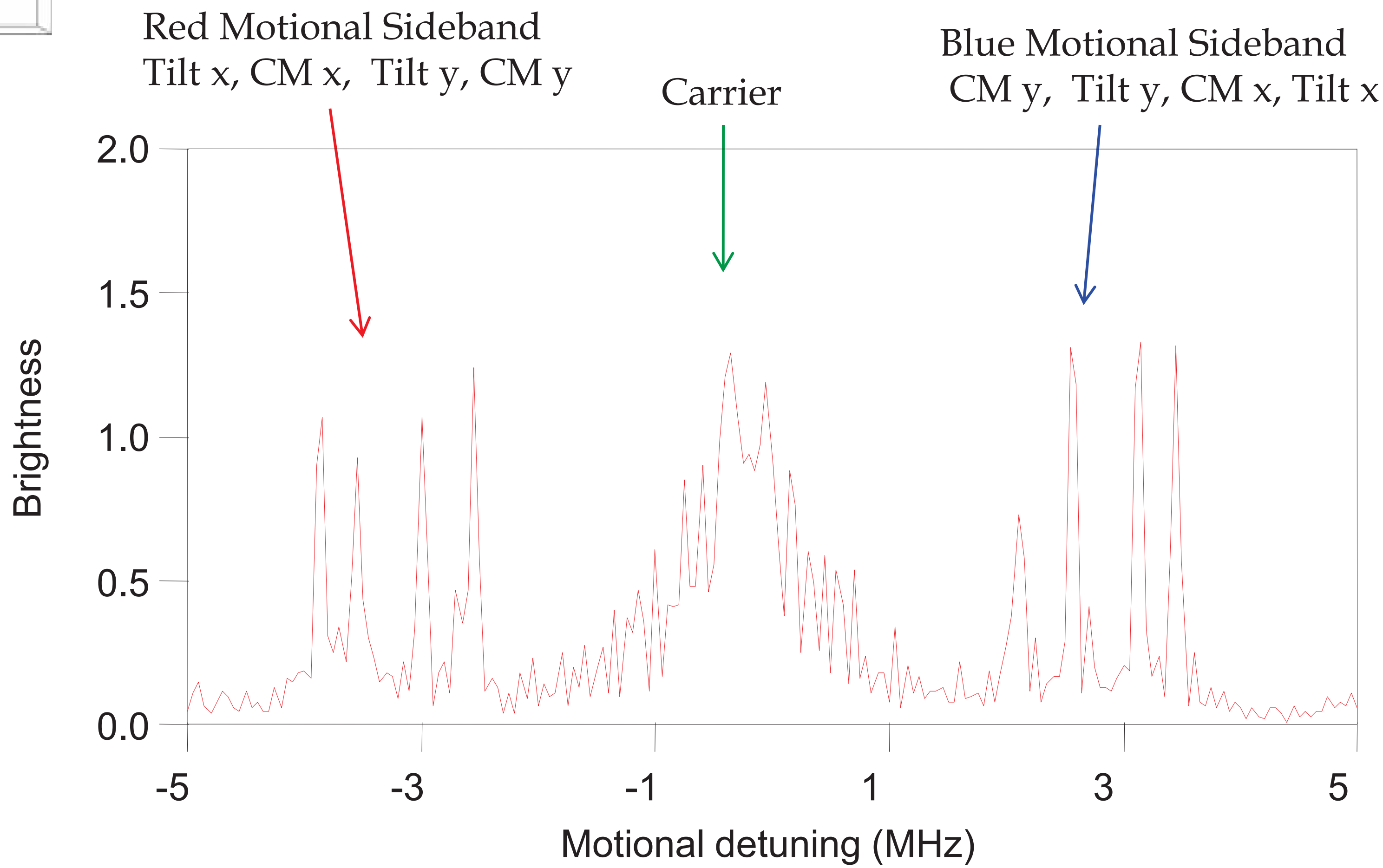
Lamb-Dicke Parameter

$$\eta = \Delta k x_0 \sqrt{n} \propto \frac{x_0}{\lambda} << 1$$

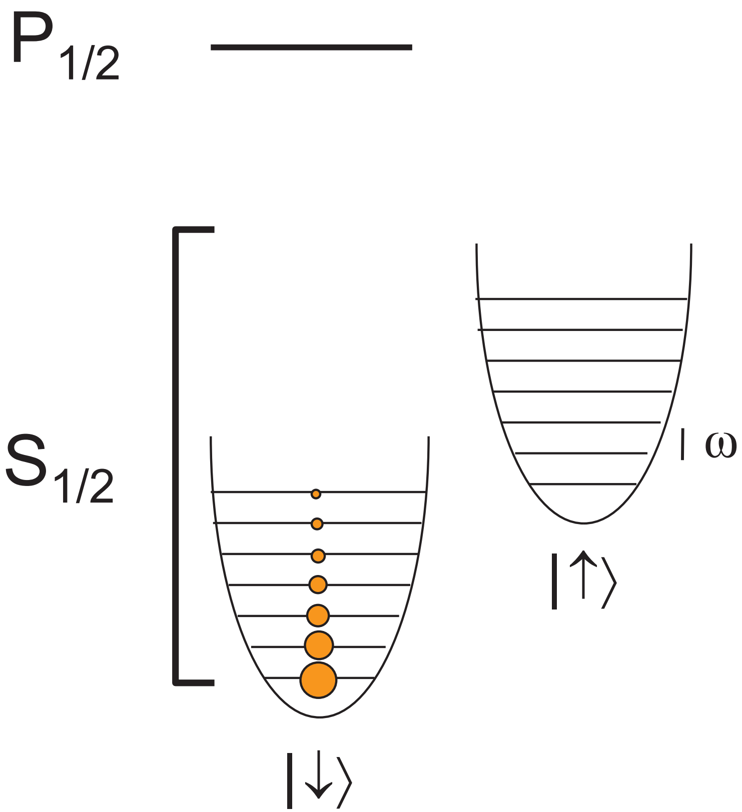
Ion normal mode spectrum



Two ions, transverse modes



1st Step: Doppler Cooling

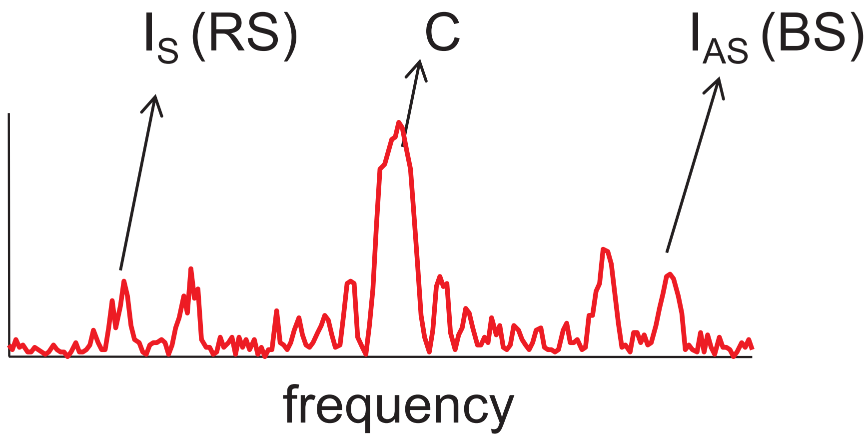


$$T_D = \frac{1}{k_B}\frac{\hbar\gamma}{2}$$

$$\langle n \rangle = \frac{\gamma}{2\omega}$$

γ → P state linewidth

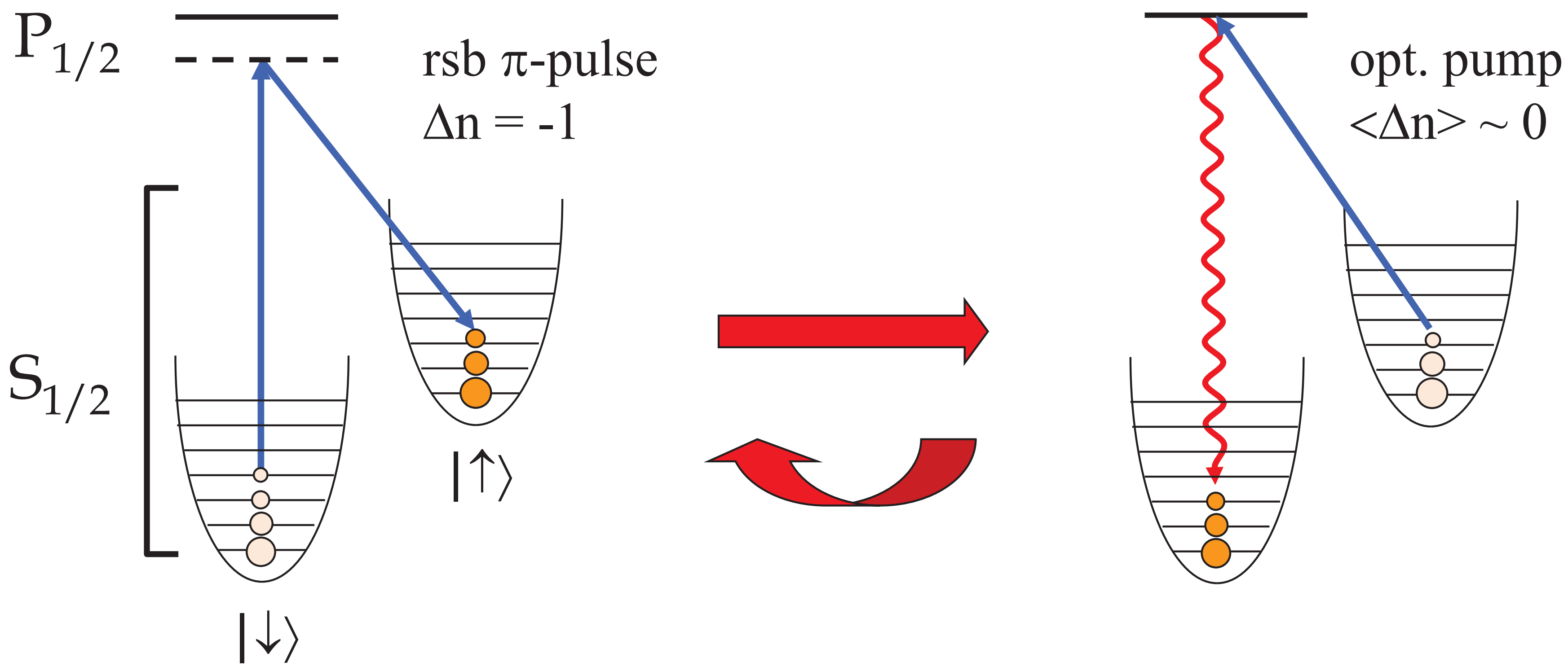
ω → vib. mode freq.



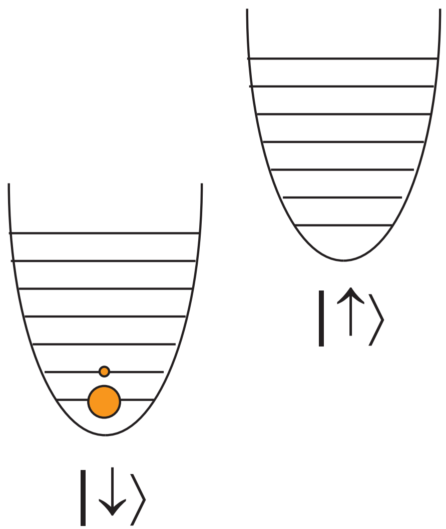
Thermometry:

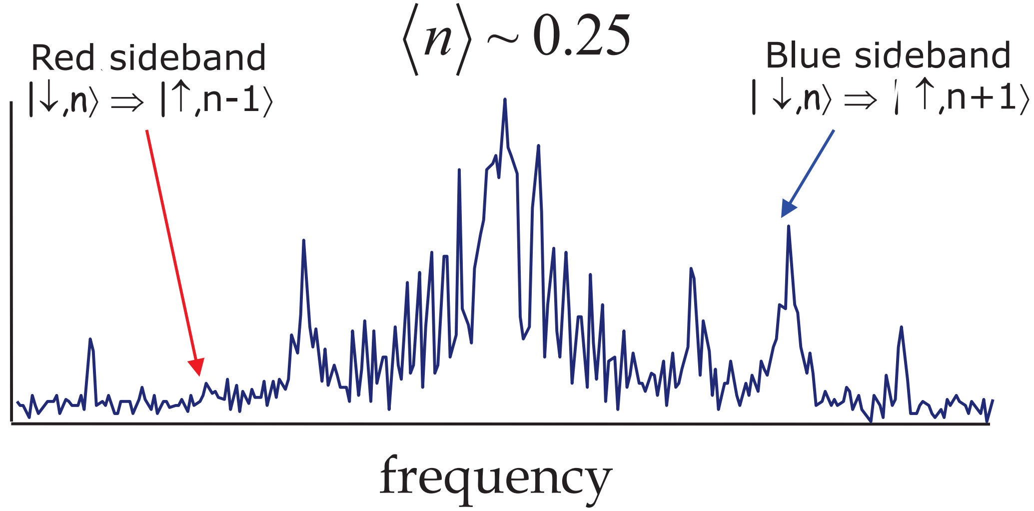
$$\frac{I_S}{I_{AS}} = \frac{\langle n \rangle}{1+\langle n \rangle}$$

2^{nd} Step: Raman Sideband Cooling



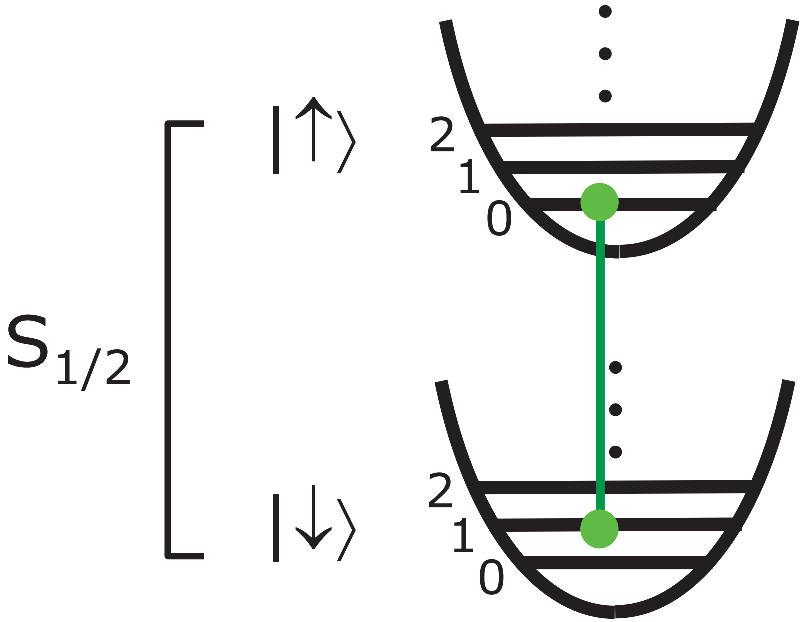
after Raman
Sideband cooling:





excitation on 1st lower ("red") motional sideband (n=0)

$$\hat{H}_{rsb} = \frac{\hbar\Omega_r}{2}\hat{\sigma}_+\hat{a} + h.c.$$



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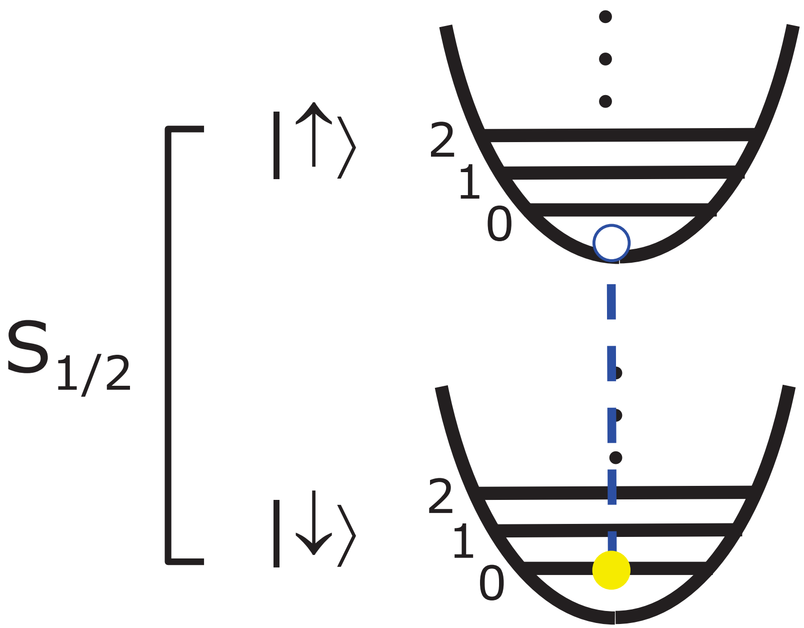

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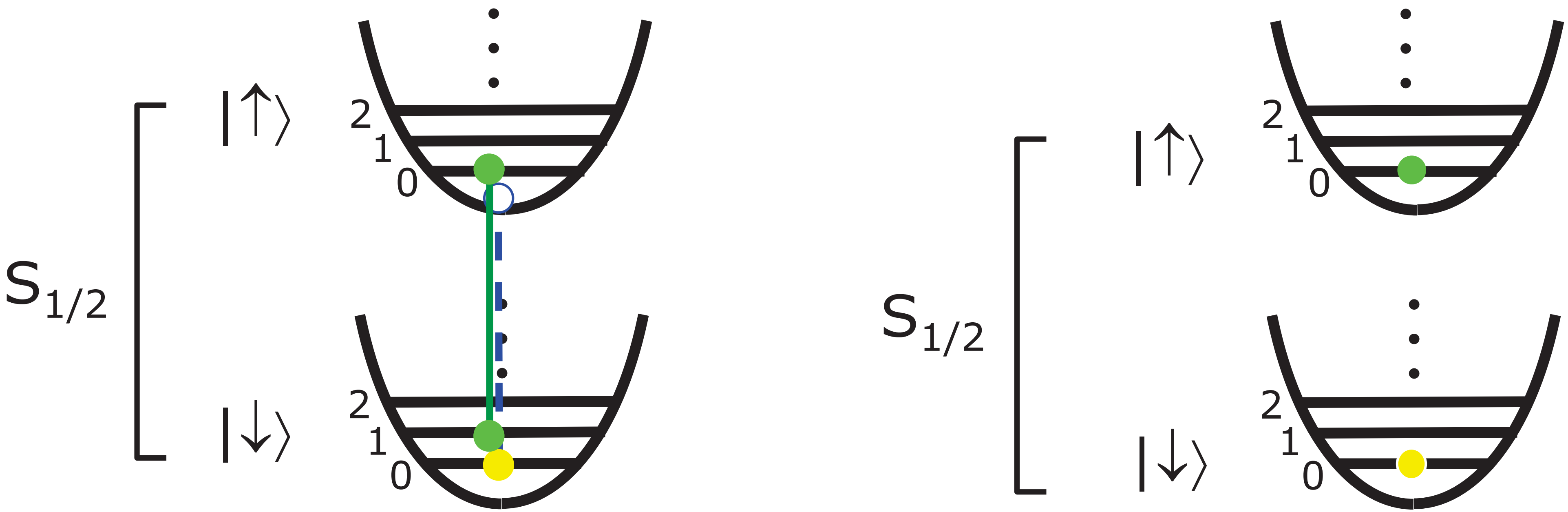

State mapping: $(\alpha|\downarrow\rangle + \beta|\uparrow\rangle)\,|0\rangle_m$

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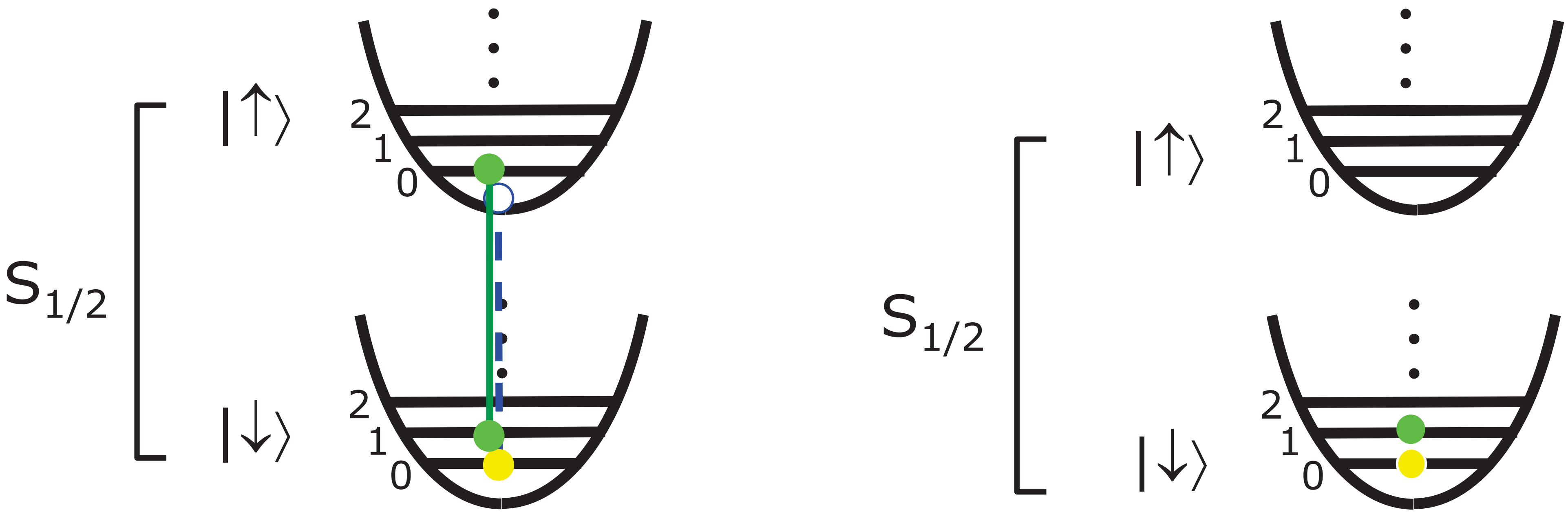
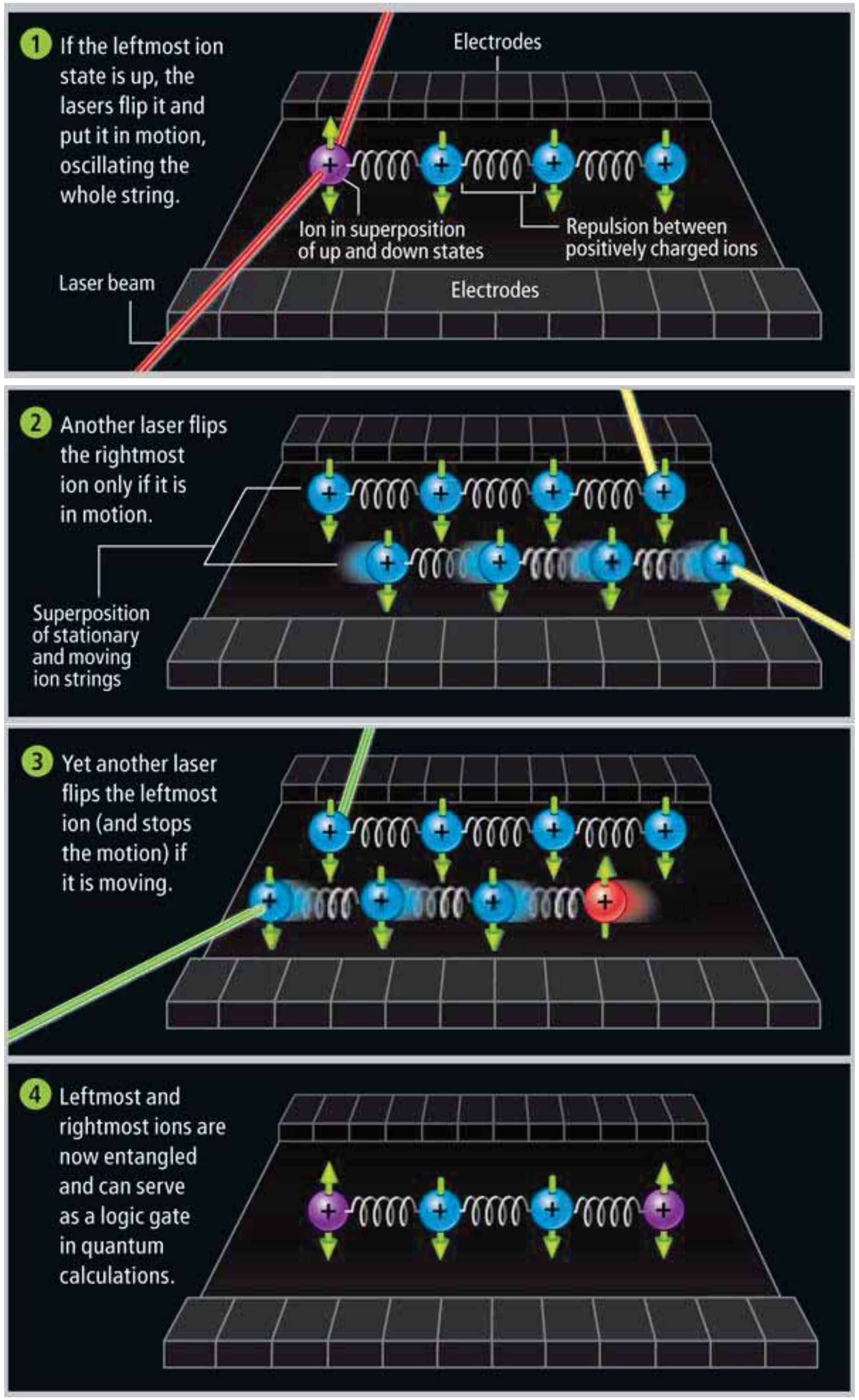


State mapping: $(\alpha|\downarrow\rangle + \beta|\uparrow\rangle)\,|0\rangle_m \rightarrow |\downarrow\rangle\,(\alpha|0\rangle_m + \beta|1\rangle_m)$

Cirac and Zoller, PRL **74**, 4091 (1995)

Scientific American, July 21, 2008



laser cool to rest (n=0), map jth qubit to phonon

flip kth qubit if phonon present

map phonon back to jth qubit

entangled state!

Cirac and Zoller, PRL **74**, 4091 (1995)
Schmidt-Kaler *et al.*, Nature **422**, 408 (2003)

Implementation of a geometric phase gate

Geometric Phase I

$$\hat{A}(\alpha) = \alpha a^{+} - \alpha^{*} a$$

Displacement Op.: $D(\alpha) = e^{\hat{A}(\alpha)}$

Coherent state: $\left|\alpha(t)\right\rangle = D(\alpha)\left|0\right\rangle$

$$D(\beta)\left|\alpha(t)\right\rangle = e^{i\,\mathrm{Im}(\alpha^{*}\beta)}\left|\alpha+\beta\right\rangle = D(\beta)D(\alpha)\left|0\right\rangle$$

$$D(\alpha)D(\beta) = D(\alpha+\beta)e^{i\,\mathrm{Im}(\alpha\beta^{*})}$$

Here we have used *Baker-Campbell-Hausdorff* relation

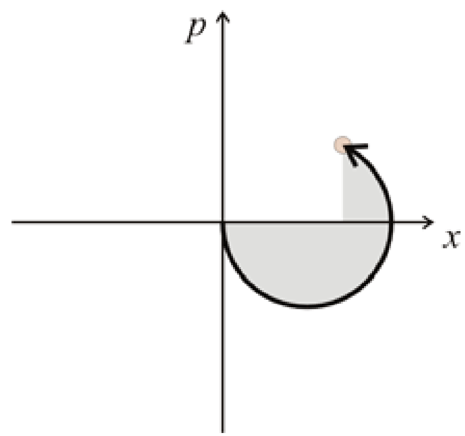
$$e^{A}e^{B} = e^{C}, \text{ where } C = A + B + \tfrac{1}{2}[A,B] + \tfrac{1}{12}([A,[A,B]] + [[A,B],B]) + \ldots$$

Geometric Phase II

$$H(t) = \hbar[\gamma(t)a^{+} - \gamma^{*}(t)a]$$

$$U(t) = \lim_{n\to\infty}\prod_{k=1}^{n}\exp\{-\frac{i}{\hbar}H(t_k)\Delta t\} = \exp\{i\Phi(t)\}D(\alpha(t)),$$

where $\Delta t = t/n, t_k = k\Delta t,$ and



$$\alpha(t) = \int_0^t dt'\gamma(t'),$$

$$\Phi(t) = \mathrm{Im}\int_0^t dt'\gamma(t')\int_0^{t'} dt''\gamma^{*}(t'').$$

Forced harmonic oscillator

$$H = \hbar\omega(a^{+}a + \frac{1}{2}) + f^{*}(t)x_0 a + f(t)x_0 a^{+}$$

$$H_I(t) = f^{*}(t)x_0 a e^{-i\omega t} + f(t)x_0 a^{+} e^{i\omega t}$$

$$f(t) = Fe^{-i(\omega-\delta)t}/2$$

$$H_I(t) = \frac{F^{*}x_0}{2} a e^{-i\delta t} + \frac{Fx_0}{2} a^{+} e^{i\delta t}$$

$$U(t) = \exp\{-\frac{i}{\hbar}(\int_0^t H(t')dt' + \tfrac{1}{2}\int_0^t dt'\int_0^{t'} dt''[H(t'), H(t'')] + \cdots)\}$$

$$\alpha(t) = -\frac{i}{\hbar}\int_0^t \frac{Fx_0}{2} e^{i\delta t'} dt' = \frac{Fx_0}{2\hbar\delta}(1 - e^{i\delta t})$$

Entangling Ions via Spin-Dependent Force

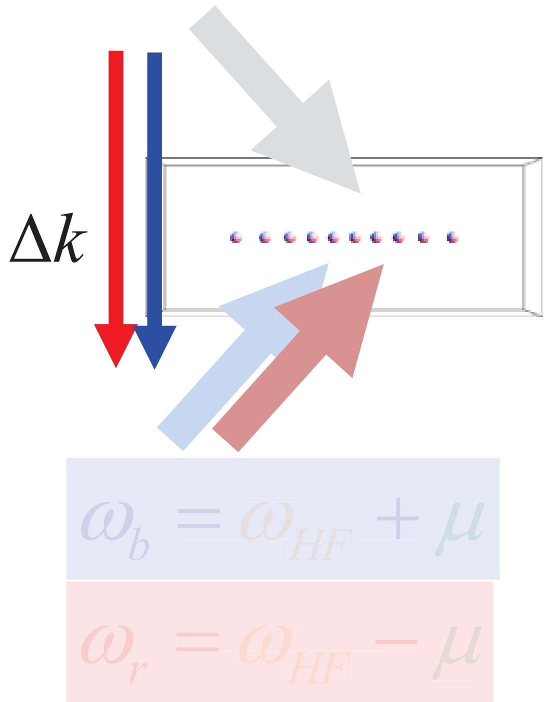
Bichromatic Raman lasers create spin-dependent force:

$$H = -F\hat{x}(|\uparrow\rangle\langle\uparrow| - |\downarrow\rangle\langle\downarrow|) = Fx_0(\hat{a} + \hat{a}^+)\hat{\sigma}_{z(\text{or } x,y)}$$

$$H(t) = \tfrac{1}{2}\hbar\Omega\sum_{i,m}\sigma_i^z[a_m e^{-i\delta_m t} + a_m^+ e^{i\delta_m t}]$$

rsb bsb

Bichromatic



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$$H = -F\hat{x}(|\uparrow\rangle\langle\uparrow| - |\downarrow\rangle\langle\downarrow|) = Fx_0(\hat{a}+\hat{a}^{+})\hat{\sigma}_{z(\text{or } x,y)}$$

$$H(t) = \tfrac{1}{2}\hbar\Omega\sum_{i,m}\sigma_i^z[a_m e^{-i\delta_m t} + a_m^{+}e^{i\delta_m t}]$$

$$U(\tau) = \mathbf{T}\exp\{-\frac{i}{\hbar}\int_0^{\tau} H(t)dt\}$$

Magnus expansion

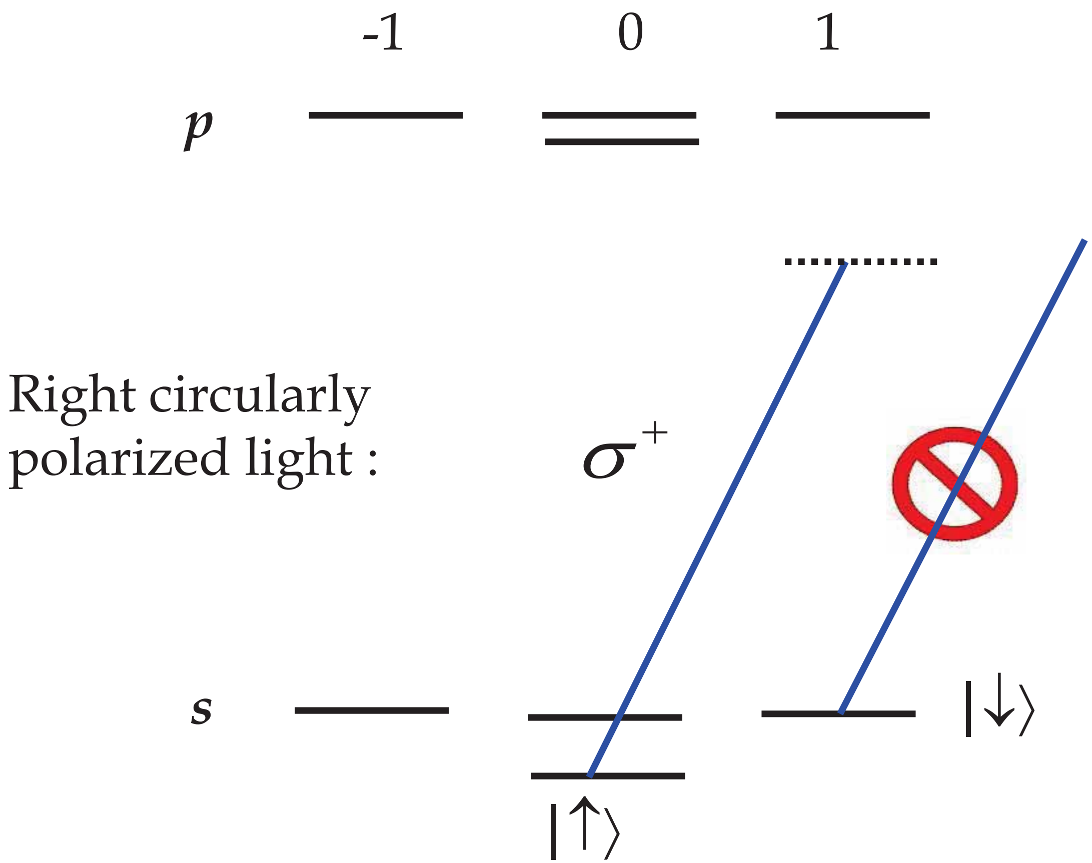
$$= \exp\{-\frac{i}{\hbar}\int_0^{\tau} H(t)dt - \frac{i}{2\hbar}\int_0^{\tau} dt_1\int_0^{t_1} dt_2[H(t_1),H(t_2)] + \cdots\}$$

$$\sim \exp\{-i\tilde{\Omega}\tau\sigma_1^z\sigma_2^z\}$$

$$(|11\rangle + |10\rangle + |01\rangle + |00\rangle) \xrightarrow{U(\tau)} |11\rangle + i|10\rangle + i|01\rangle + |00\rangle$$

Implement spin-dependent force: σ_z

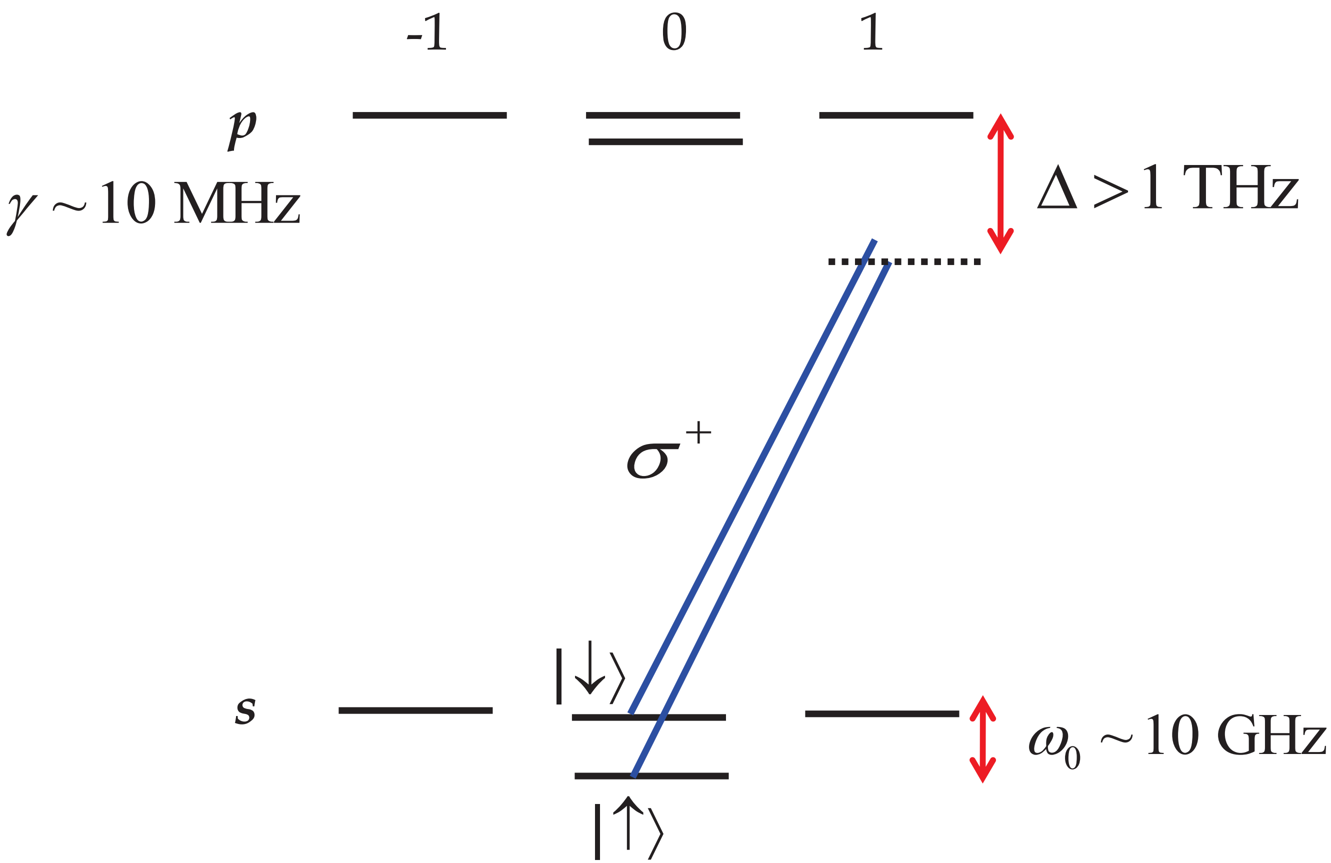
$$H = -Fx_0 \left|\uparrow\right\rangle\left\langle\uparrow\right| = \frac{Fx_0}{2}(1-\sigma_z) \sim \frac{\hbar\Omega}{2}\sigma_z$$



Qubits subject to 1st order Zeeman shift.

Implement spin-dependent force: σ_z

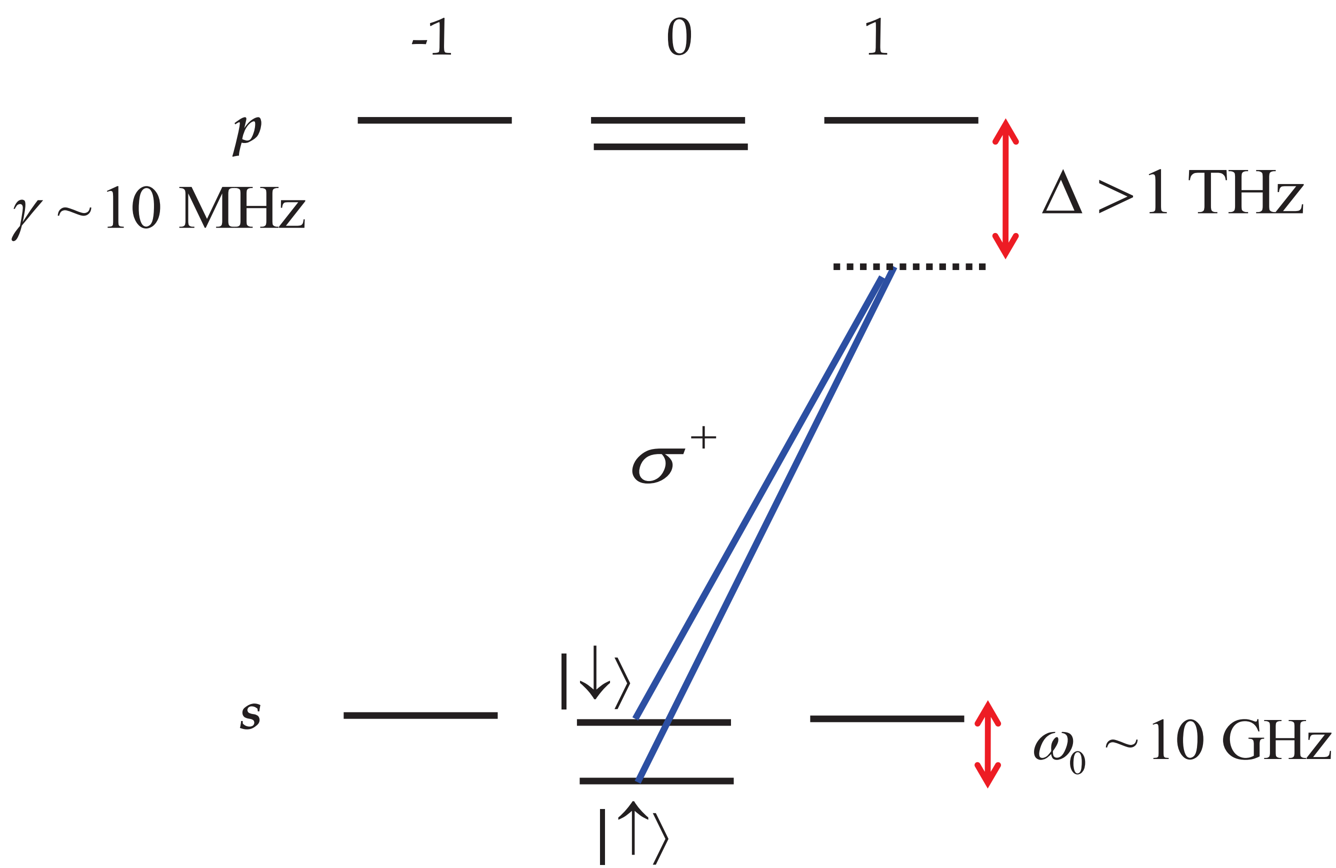
$$H = -Fx_0 \left|\uparrow\right\rangle\left\langle\uparrow\right| = \frac{Fx_0}{2}(1-\sigma_z) \sim \frac{\hbar\Omega}{2}\sigma_z$$



Don't work for clock states…
with dipole allowed transitions!

Implement spin-dependent force: σ_x

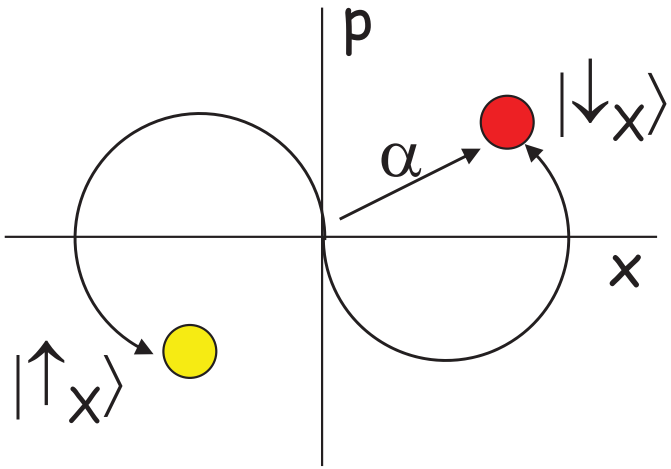
$$H = -Fx_0 \left|\uparrow\right\rangle\left\langle\uparrow\right| = \frac{Fx_0}{2}(1-\sigma_x) \sim \frac{\hbar\Omega}{2}\sigma_x$$



Apply spin-dependent force and flip the spin simultaneously: σ_z force in the x basis

σ_X force on a single ion

$$H = -\frac{Fx_0}{2}\sigma_x(ae^{i\delta t + i\phi} + a^+ e^{-i\delta t - i\phi})$$



$|\uparrow_x\rangle \sim |\downarrow_Z\rangle + |\uparrow_Z\rangle$
$|\downarrow_x\rangle \sim |\downarrow_Z\rangle - |\uparrow_Z\rangle$

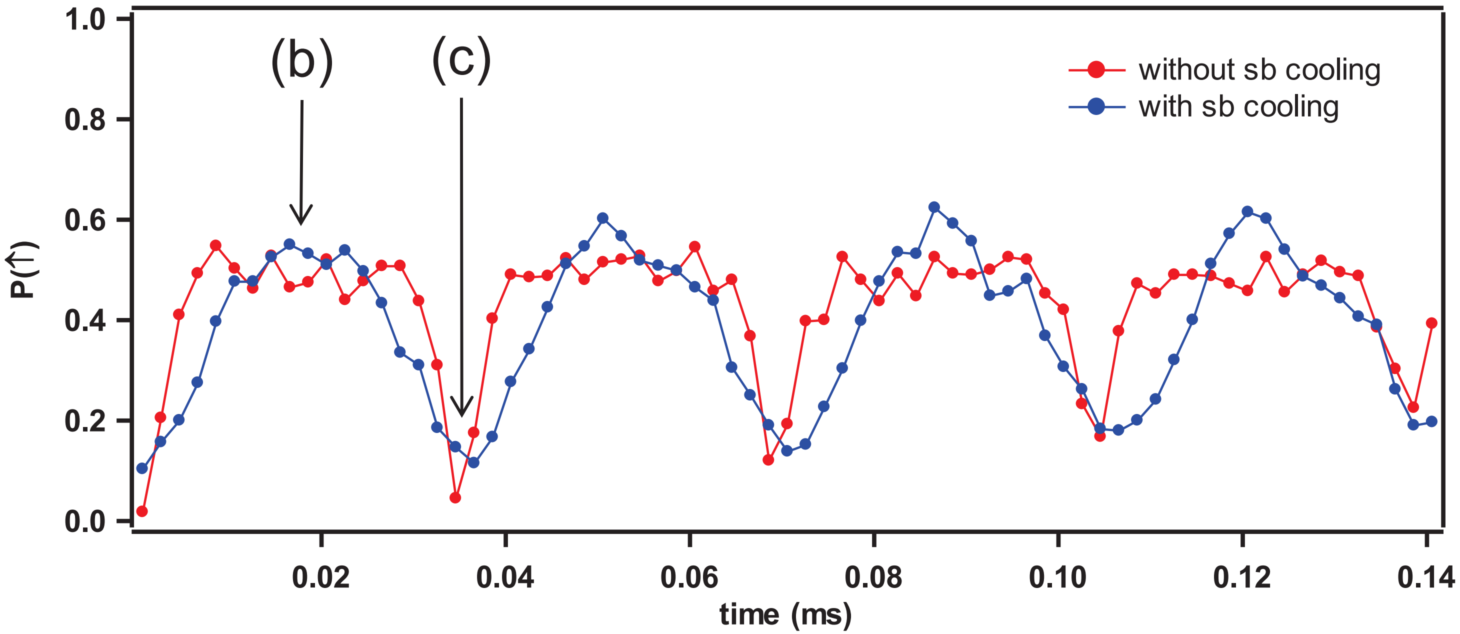
Initial in $|\Psi\rangle = |\downarrow_Z\rangle = (|\uparrow_X\rangle - |\downarrow_X\rangle)\,|\text{motion}=0\rangle$

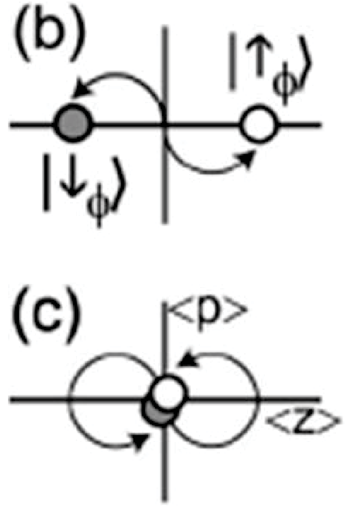
Apply bichromatic force:

$|\Psi\rangle = |\downarrow_X\rangle|\alpha\rangle + |\uparrow_X\rangle\,|-\alpha\rangle$

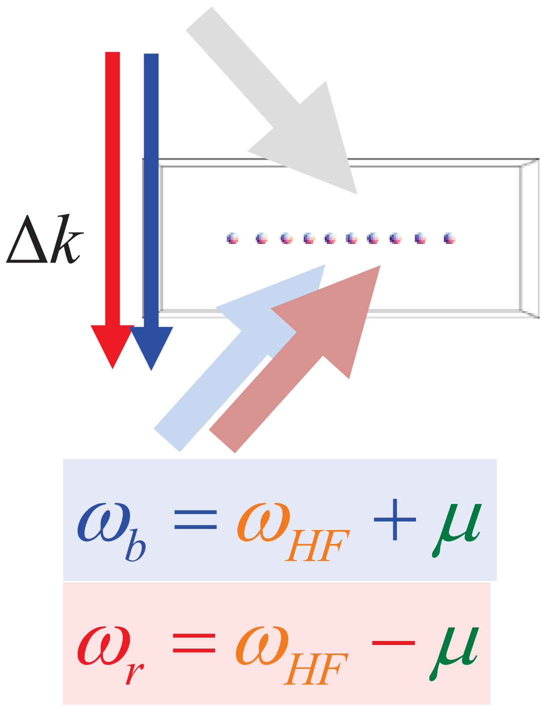
$= |\downarrow_Z\rangle(|\alpha\rangle - |-\alpha\rangle) + |\uparrow_Z\rangle\,(|\alpha\rangle + |-\alpha\rangle)$

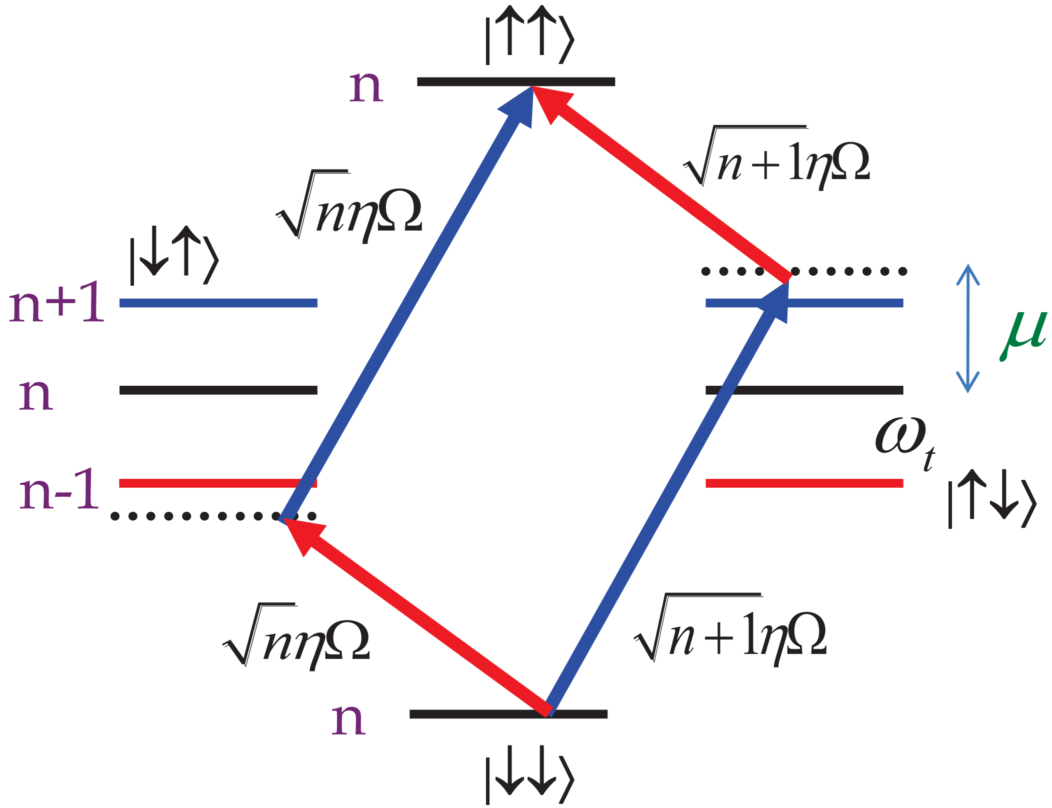
Spin dependent force – time scan





σ_x force on two ions: Molmer-Sorensen ($\sigma_x \otimes \sigma_x$) Gate

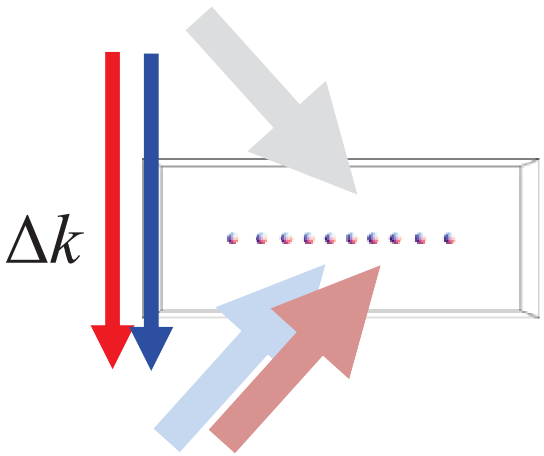




$$\tilde{\Omega} = \frac{\eta^2\Omega^2}{\mu - \omega_t}(n + 1 - n)$$

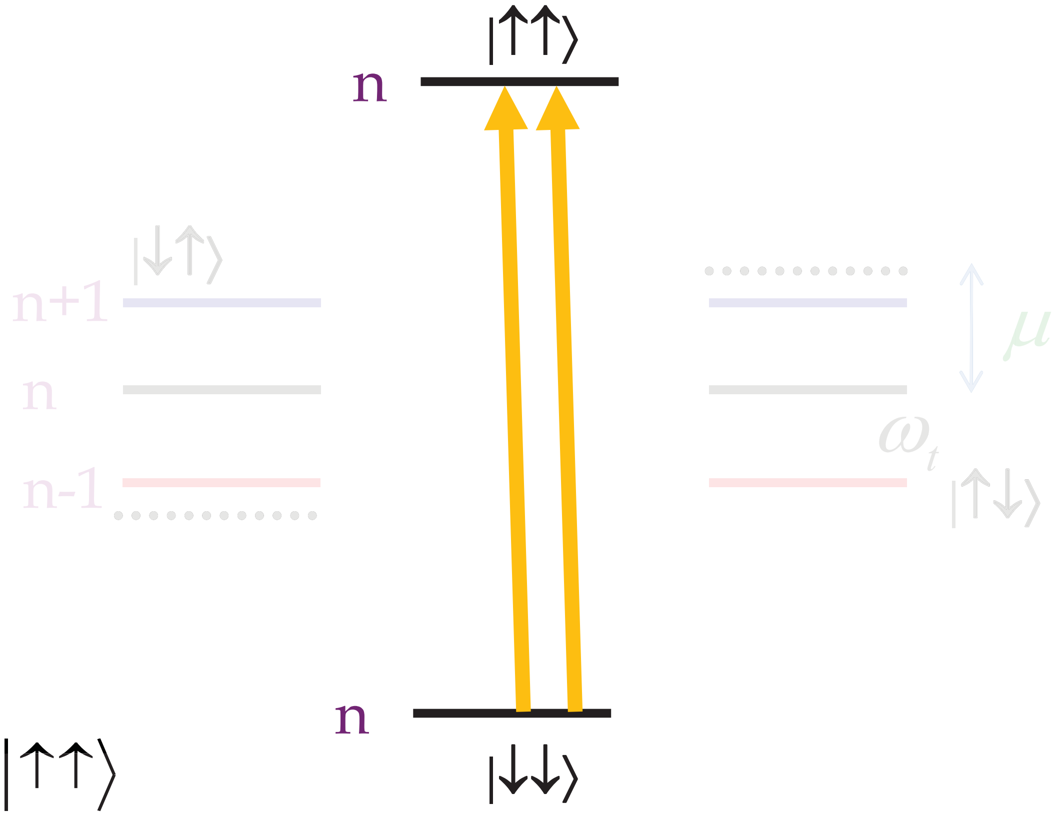
Molmer & Sorensen, PRL 82, 1971 (1999) & PRL 82, 1835 (1999)

σ_x force on two ions: Molmer-Sorensen ($\sigma_x \otimes \sigma_x$) Gate



$$\omega_b = \omega_{HF} + \mu$$

$$\omega_r = \omega_{HF} - \mu$$



$$|\downarrow\downarrow\rangle \Rightarrow \cos(\frac{\tilde{\Omega}T}{2})|\downarrow\downarrow\rangle + e^{i\phi}\sin(\frac{\tilde{\Omega}T}{2})|\uparrow\uparrow\rangle$$

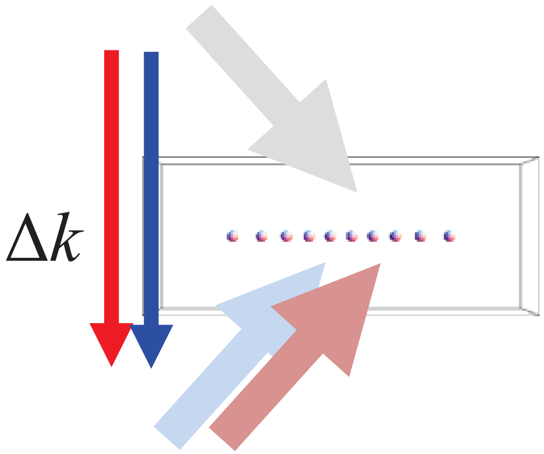
choose $\frac{\tilde{\Omega}T}{2} = \frac{\pi}{4}$, then

$$|\downarrow\downarrow\rangle \Rightarrow \frac{1}{\sqrt{2}}|\downarrow\downarrow\rangle + e^{i\phi}\frac{1}{\sqrt{2}}|\uparrow\uparrow\rangle$$

$$\tilde{\Omega} = \frac{\eta^2\Omega^2}{\mu - \omega_t}(\cancel{n} + 1 - \cancel{n})$$

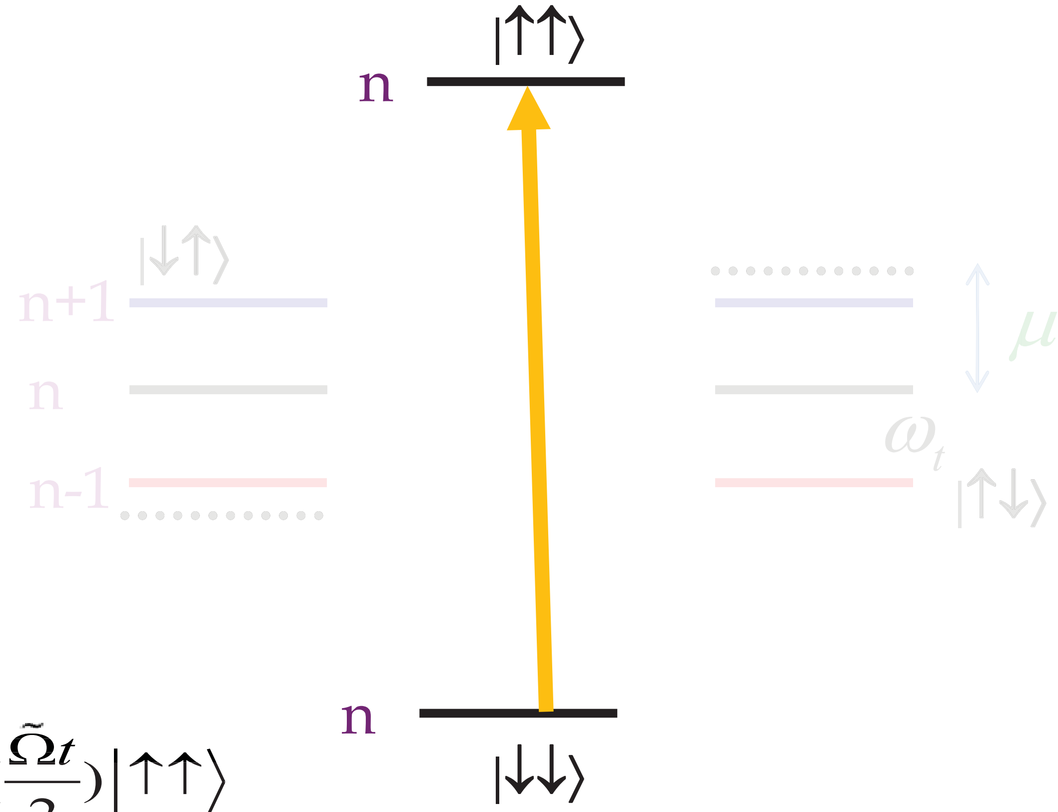
Molmer & Sorensen, PRL 82, 1971 (1999) & PRL 82, 1835 (1999)

σ_x force on two ions: Molmer-Sorensen ($\sigma_x \otimes \sigma_x$) Gate



$$\omega_b = \omega_{HF} + \mu$$

$$\omega_r = \omega_{HF} - \mu$$



Generally…

$$|\downarrow\downarrow\rangle \Rightarrow \cos(\frac{\tilde{\Omega}t}{2})|\downarrow\downarrow\rangle + e^{i\phi}\sin(\frac{\tilde{\Omega}t}{2})|\uparrow\uparrow\rangle$$

$$|\uparrow\uparrow\rangle \Rightarrow \cos(\frac{\tilde{\Omega}t}{2})|\uparrow\uparrow\rangle + e^{i\phi}\sin(\frac{\tilde{\Omega}t}{2})|\downarrow\downarrow\rangle$$

$$|\downarrow\uparrow\rangle \Rightarrow \cos(\frac{\tilde{\Omega}t}{2})|\downarrow\uparrow\rangle + e^{i\phi'}\sin(\frac{\tilde{\Omega}t}{2})|\uparrow\downarrow\rangle$$

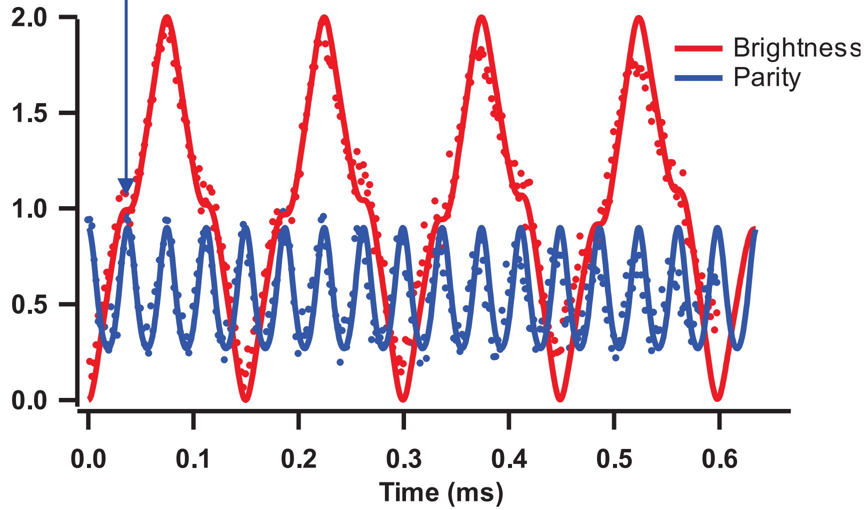
$$|\uparrow\downarrow\rangle \Rightarrow \cos(\frac{\tilde{\Omega}t}{2})|\uparrow\downarrow\rangle + e^{i\phi'}\sin(\frac{\tilde{\Omega}t}{2})|\downarrow\uparrow\rangle$$

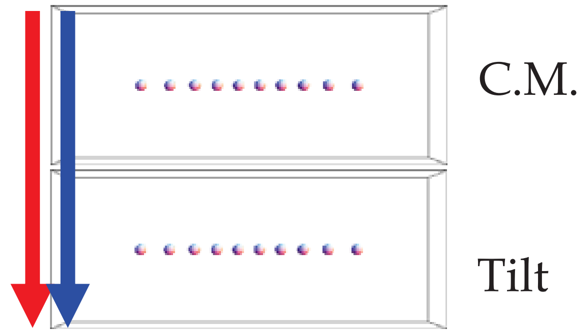
$$\tilde{\Omega} = \frac{\eta^2\Omega^2}{\mu - \omega_t}$$

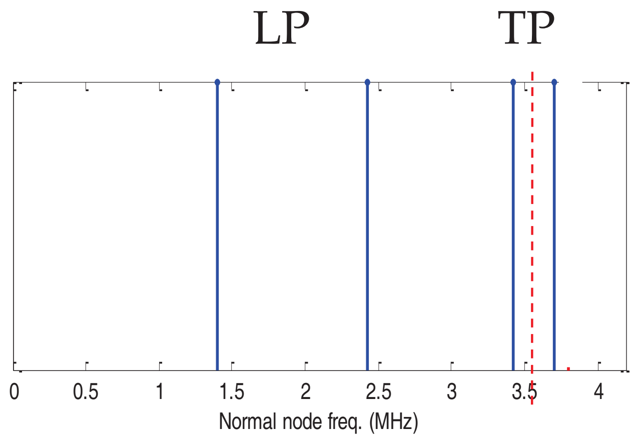
Molmer & Sorensen, PRL 82, 1971 (1999) & PRL 82, 1835 (1999)

Two-mode Molmer–Sorensen gate

$$\frac{1}{2}\left(|\uparrow\uparrow\rangle + e^{i\phi}|\downarrow\downarrow\rangle\right)$$







Brightness

$$B = P(\uparrow\downarrow) + P(\downarrow\uparrow) + 2P(\uparrow\uparrow)$$

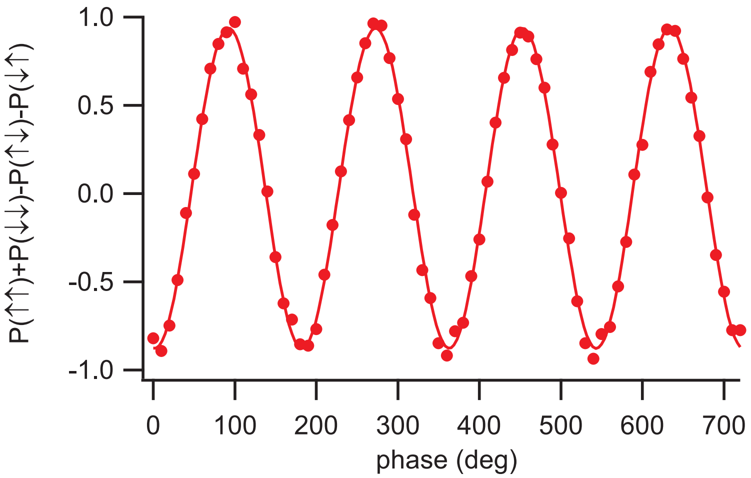
Parity

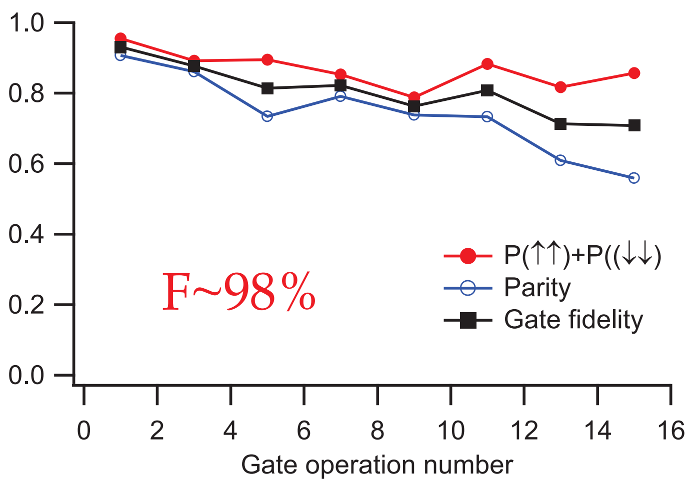
$$\pi = P(\uparrow\uparrow) + P(\downarrow\downarrow) - P(\uparrow\downarrow) - P(\downarrow\uparrow)$$

Gate fidelity: F~98%!

Fidelity of two-mode M-S gate

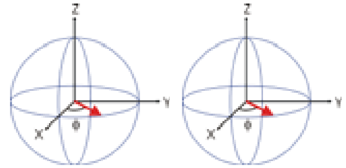
$$\pi = P(\uparrow\uparrow) + P(\downarrow\downarrow) - P(\uparrow\downarrow) - P(\uparrow\downarrow)$$





$$D \times F^{15} = 0.71, \text{ with } D = 0.97.$$

Parity oscillation

e.g. $|\uparrow\uparrow\rangle \Rightarrow |$  $\rangle$

$$|\psi_{ideal}\rangle = \frac{1}{\sqrt{2}}\left(|\downarrow\downarrow\rangle + e^{i\phi}|\uparrow\uparrow\rangle\right)$$

$$\rho = \rho_{pure} + \rho_{mix}$$

$$F = \langle \psi_{ideal} | \rho | \psi_{ideal} \rangle$$

$$F = \frac{P(\uparrow\uparrow) + P(\downarrow\downarrow)}{2} + \frac{|\pi|}{2}$$

Quantum simulation of frustrated Ising spins

Modeling quantum magnets

- Ising spins in transverse B field: $$H = \sum_{i,j} J_{ij}^{z} \sigma_i^{z} \sigma_j^{z} + B^{x} \sum_{i} \sigma_i^{x}$$

- XY model: $$H = \sum_{i,j} J_{ij} (\sigma_i^{x} \sigma_j^{x} + \sigma_i^{y} \sigma_j^{y})$$

- XXZ model : $$H = \sum_{i,j} J_{ij} (\sigma_i^{x} \sigma_j^{x} + \sigma_i^{y} \sigma_j^{y}) + J_{ij}^{z} \sigma_i^{z} \sigma_j^{z}$$

  (~ Bose-Hubbard under Holstein-Primakoff transformation)

- Possible Observations
  - Quantum phase transition
  - Spin frustration
  - Complex entangled states

- Provided tunable spin-spin interactions:
  - Strength,
  - Sign (ferro or anti-ferro),
  - Range,
  - Coupling graph (geometry).

Requirements for quantum simulation: analogue version

1. Scalable system of qubits: $a|0\rangle + b|1\rangle;\ |a|^2 + |b|^2 = 1.$

2. A qubit-specific measurement capability

3. Initialization: $|0,0,0,\cdots\rangle$

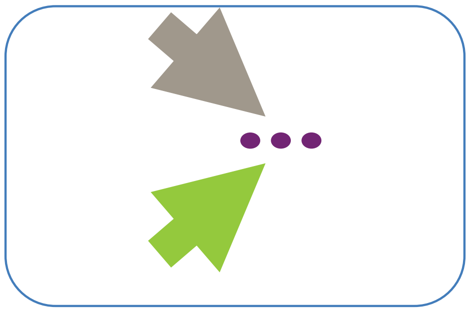
4. Global and always-on interactions (analogue version, no error accumulation)

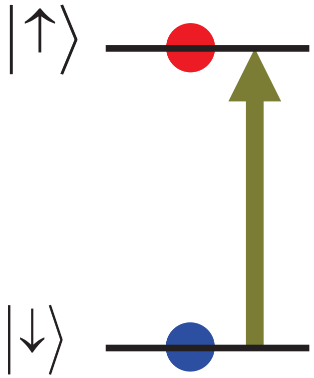
5. Decoherence can serve as environment for the studied system.

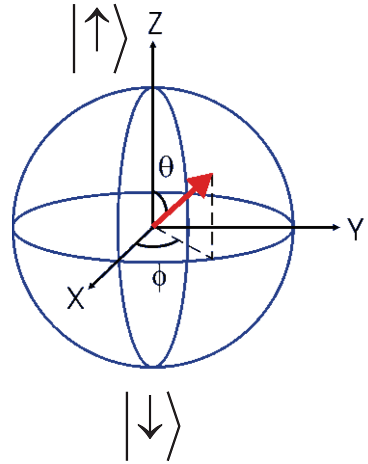
Easier, and closer to Feynman's original proposal for a QC.

Simulating a B field (single qubit rotation)

$$H_{XY} = B\sum_i \sigma_y^{(i)} + \sum_{i<j} J_{ij}\sigma_x^{(i)}\sigma_x^{(j)}$$

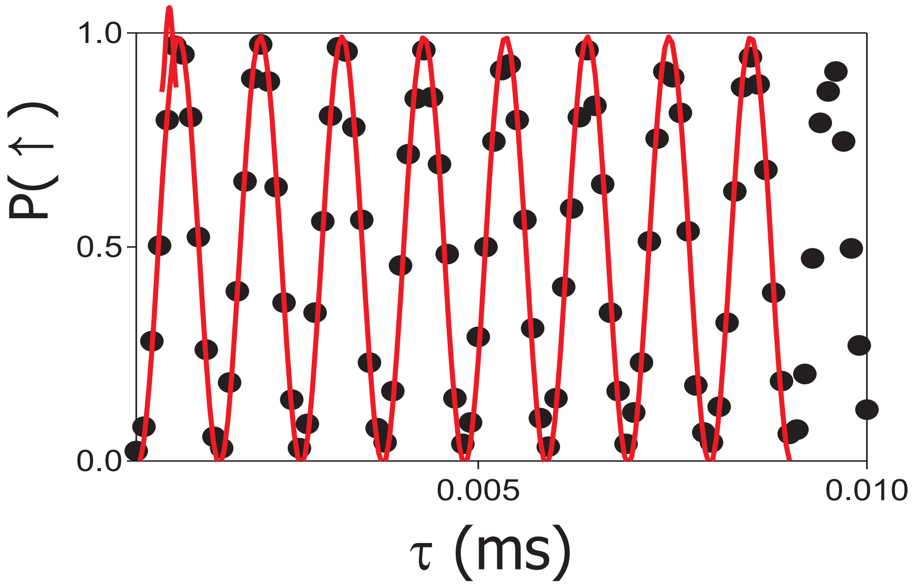




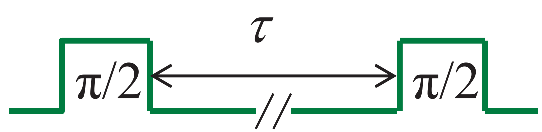


$$R_{\phi+\pi/2}(\theta)$$

Rabi oscillations



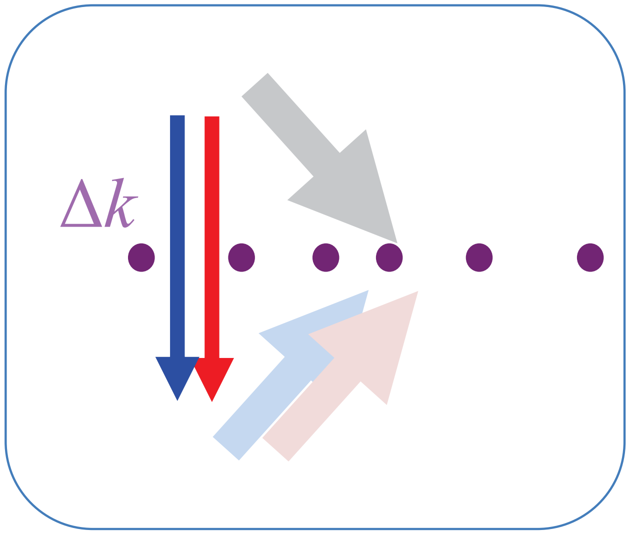
Ramsey oscillations

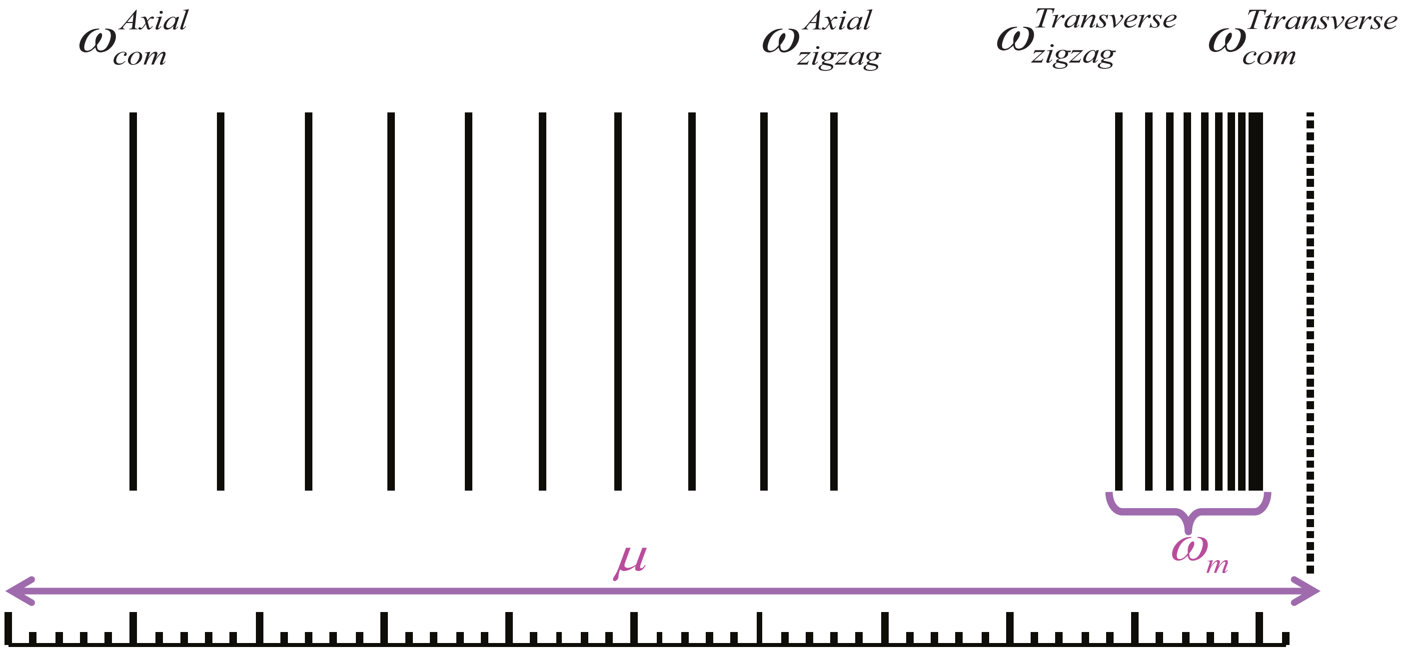


Coherence time > 70 ms

Simulating spin–spin interactions (J_{ij})

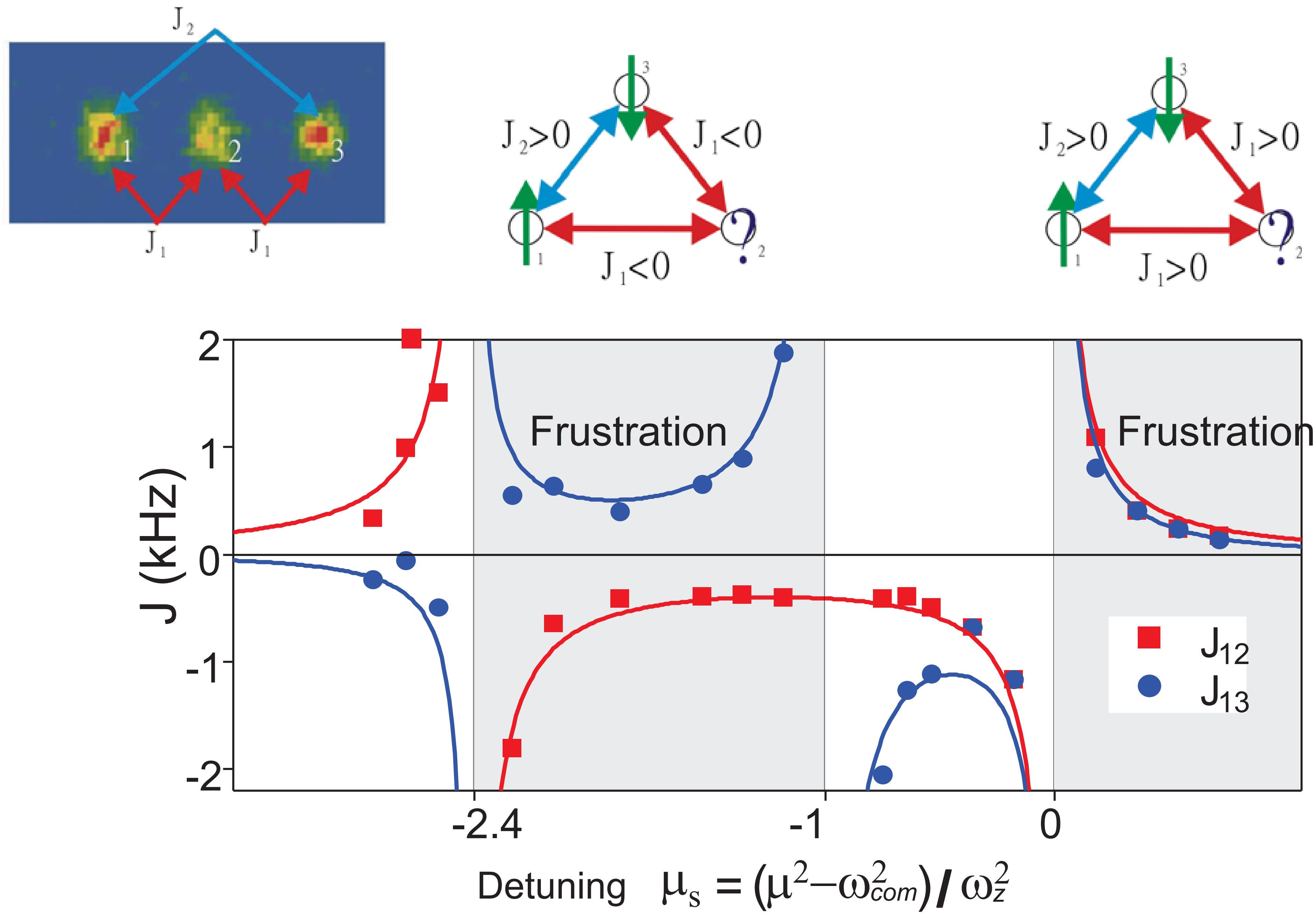
$$H = B\sum_i \sigma_y^{(i)} + \sum_{i<j} J_{ij}\sigma_x^{(i)}\sigma_x^{(j)}$$



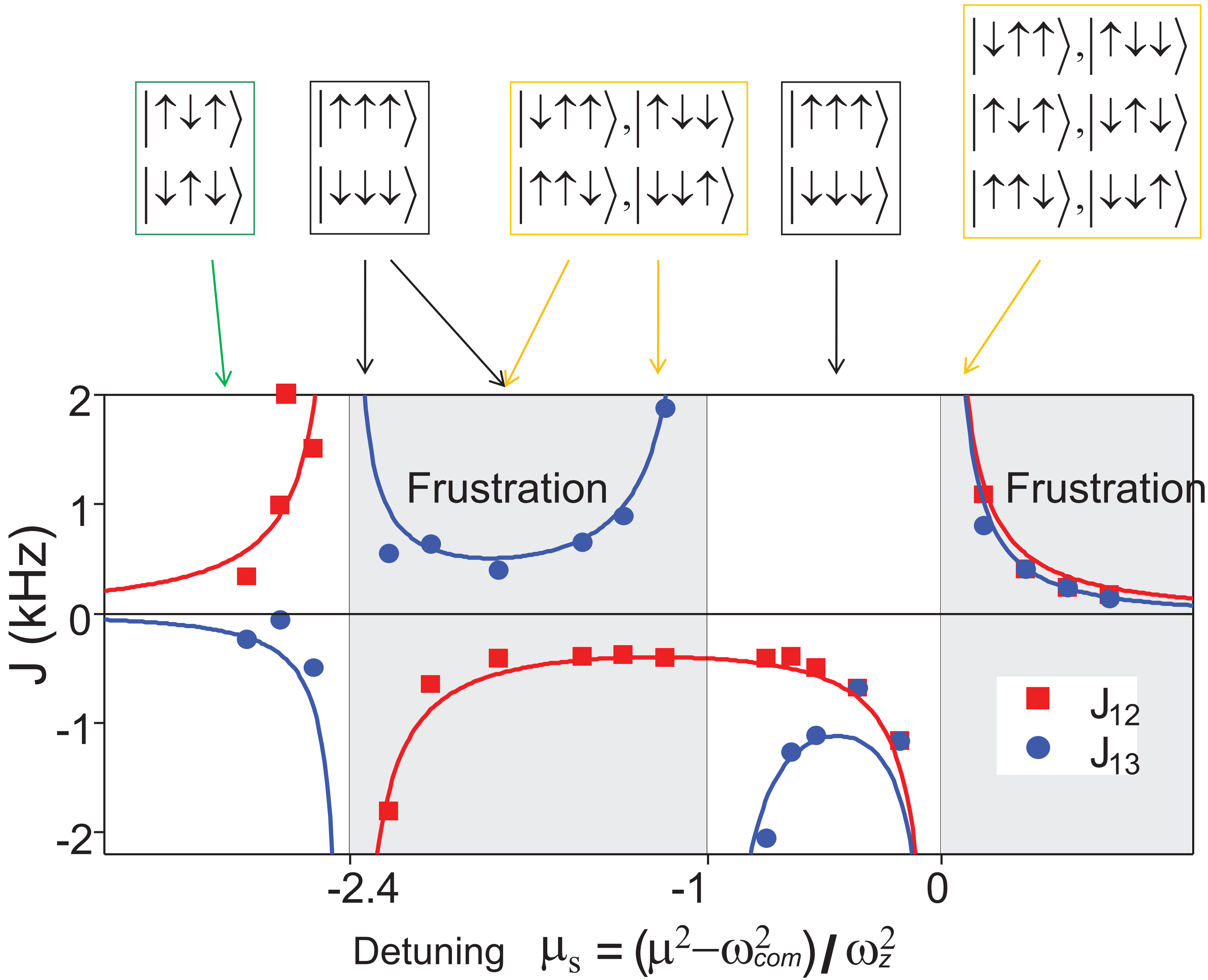


$$J_{i,j} = \sum_m \frac{\Omega_i \Omega_j \eta_i^m \eta_j^m \omega_m}{\mu^2 - \omega_m^2},$$

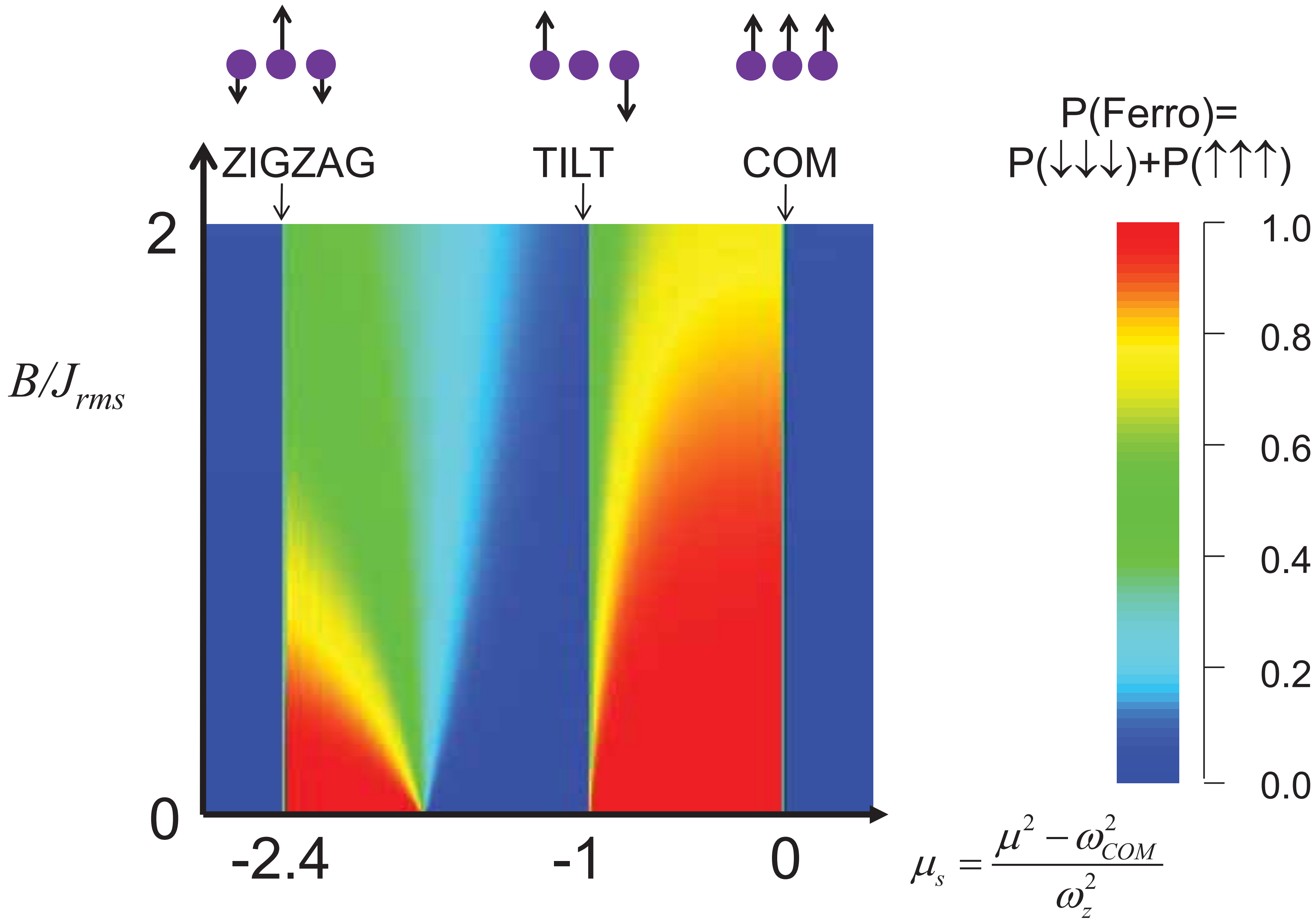
Spin frustration in triangular lattice



ground states vs. spin-spin couplings



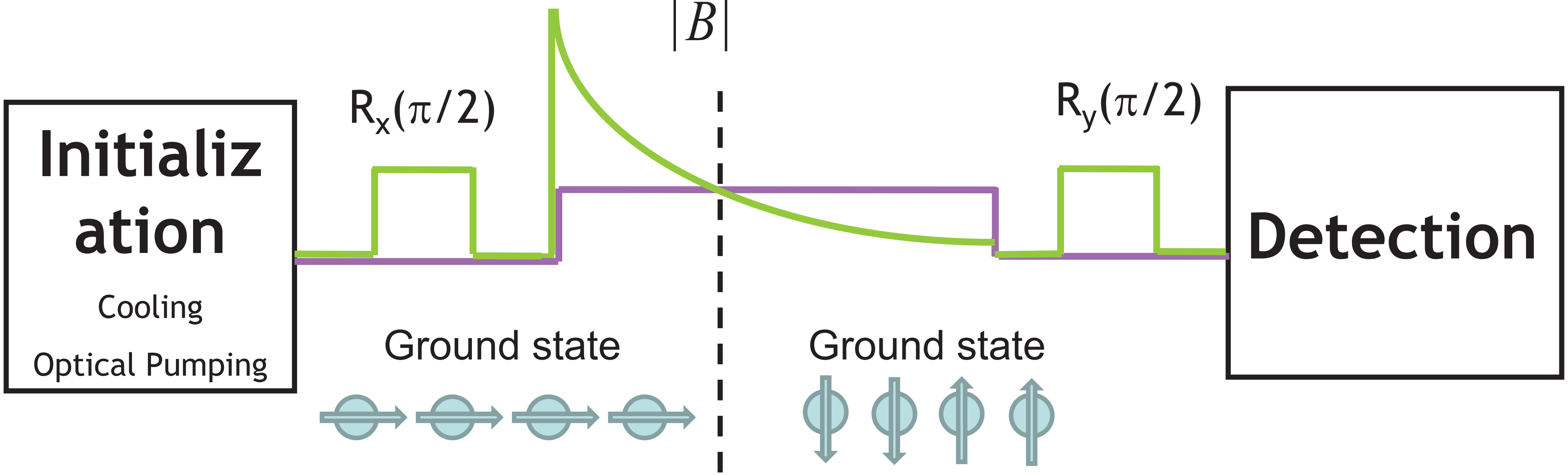
Exact Ground State (Theory) – Freericks and Duan



Quantum simulation: implementation

$$H(t) = \sum_{i \neq j} J_{ij} \sigma_x^{(i)} \sigma_x^{(j)} + B(t) \sum_i \sigma_y^{(i)}$$

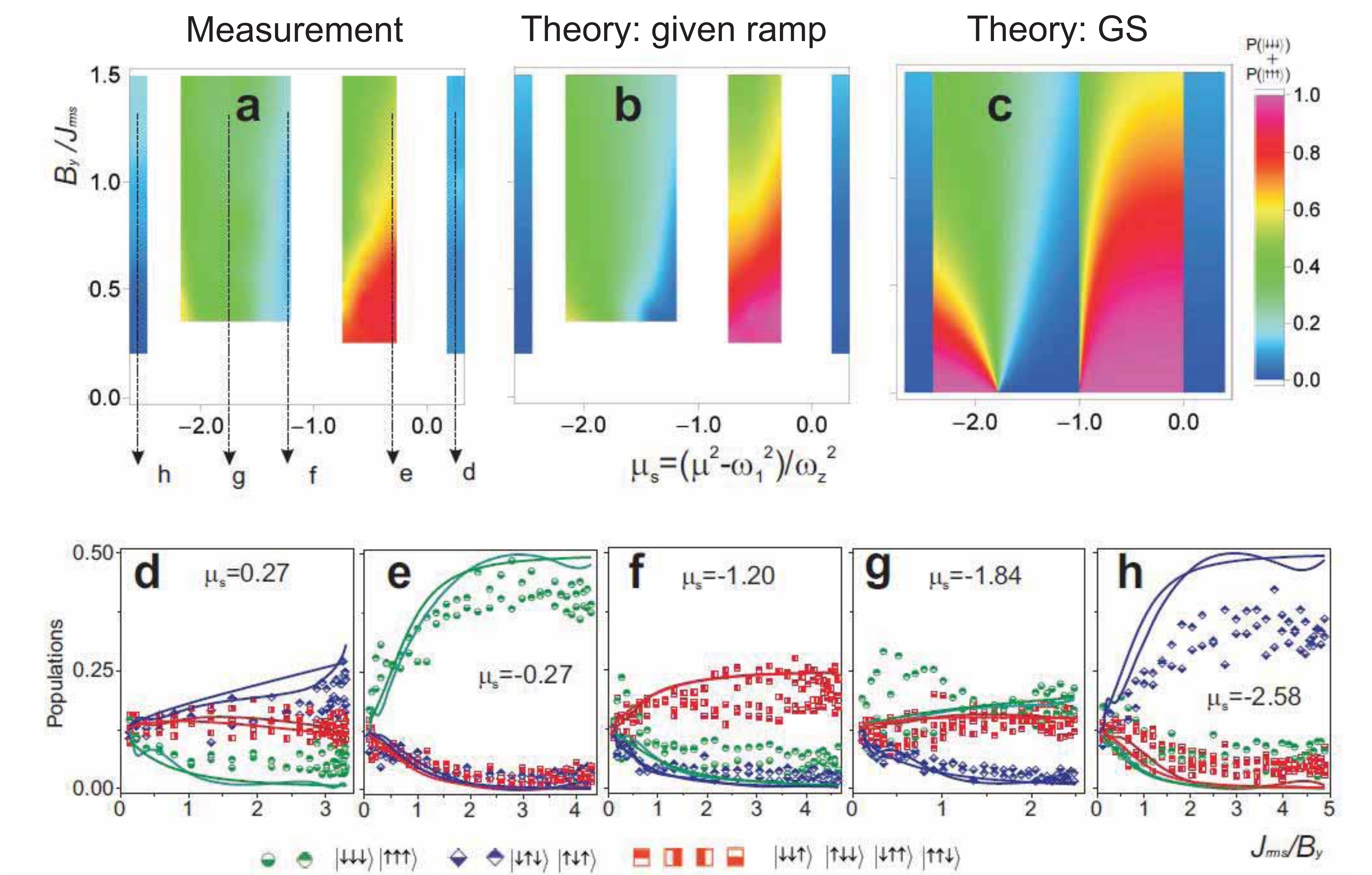
$$\left| \frac{J}{B} \right| \approx 1$$



$$|\psi(0)\rangle = |\downarrow_x \downarrow_x \downarrow_x \cdots\rangle \quad \xrightarrow{\text{Adiabatically following}} \quad |\psi(t)\rangle = \hat{T} e^{-\frac{i}{\hbar}\int H(t)dt} |\psi(0)\rangle$$

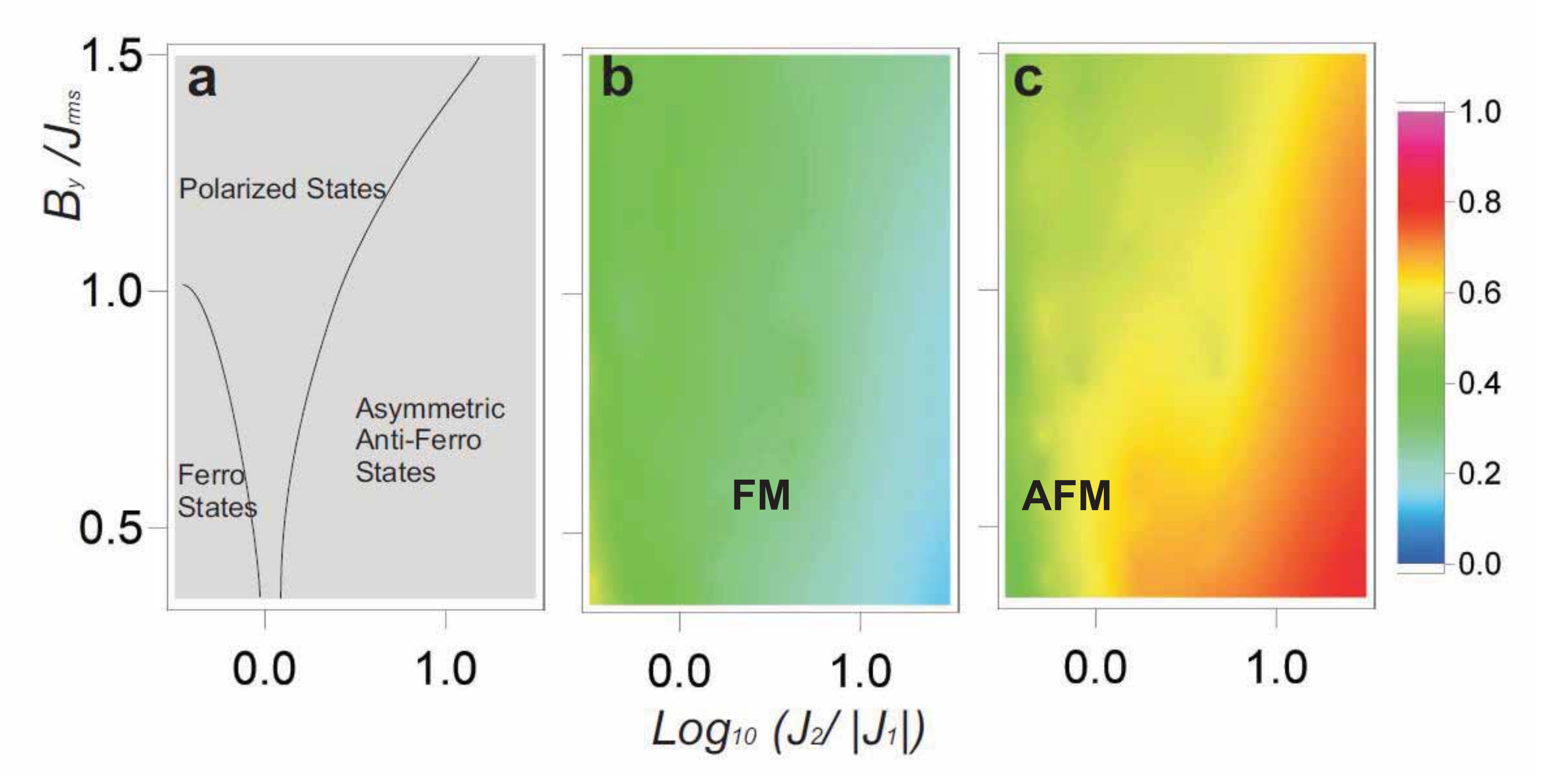
Lloyd, Science **273**, 1073 (1996) and **319** 1209 (2008). Farhi et al., Science **292**, 472 (2001);
(2 ion simulator) Friedenauer et al., Nat. Phys., **4**, 757 (2008)

Phase diagram measurement



Kim et al., Nature **465**, 590 (2010)

Universal phase diagram

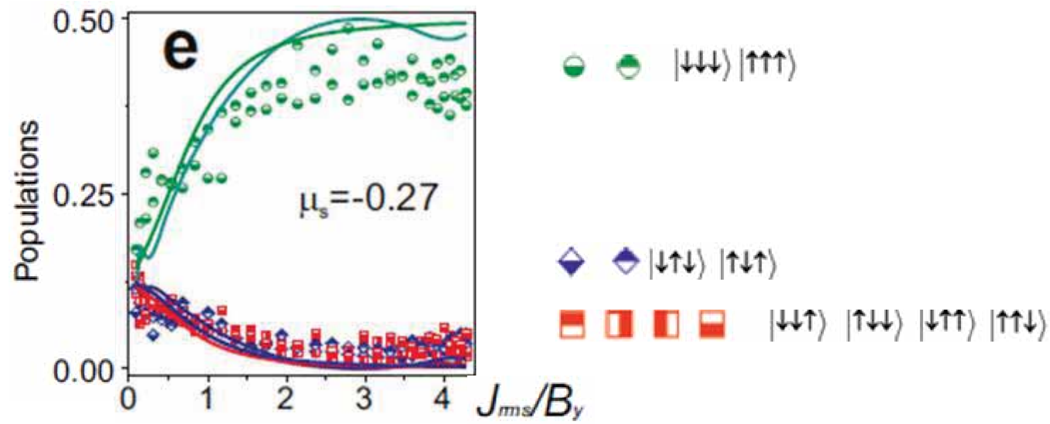


Ground state entanglement or entropy?

FM ground state:

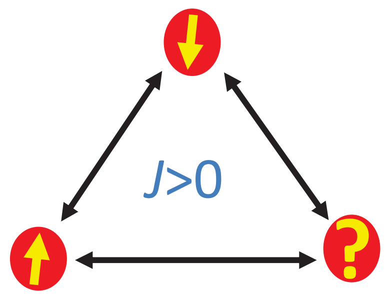
$$|\psi\rangle = |\uparrow\uparrow\uparrow\rangle - |\downarrow\downarrow\downarrow\rangle ?$$

or $$\rho = |\uparrow\uparrow\uparrow\rangle\langle\uparrow\uparrow\uparrow| + |\downarrow\downarrow\downarrow\rangle\langle\downarrow\downarrow\downarrow| ?$$



AFM frustrated ground state:

$$|\psi\rangle = |\uparrow\uparrow\downarrow\rangle + |\uparrow\downarrow\uparrow\rangle + |\downarrow\uparrow\uparrow\rangle - |\downarrow\downarrow\uparrow\rangle - |\downarrow\uparrow\downarrow\rangle - |\uparrow\downarrow\downarrow\rangle ?$$



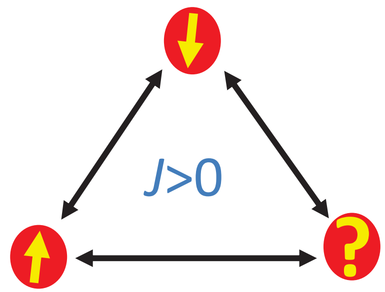
If we know the density matrix (8 x 8), we can know the underlying state. However, density matrix is hard to reconstruct.

Luckily there is a short cut…

Detecting the frustrated ground state: entanglement detection

FM ground state: $|\psi\rangle = |\uparrow\uparrow\uparrow\rangle - |\downarrow\downarrow\downarrow\rangle$?

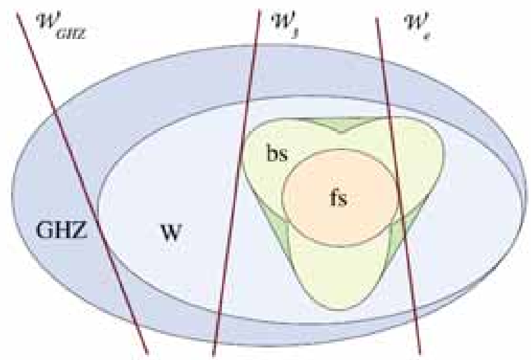
AFM frustrated ground state:



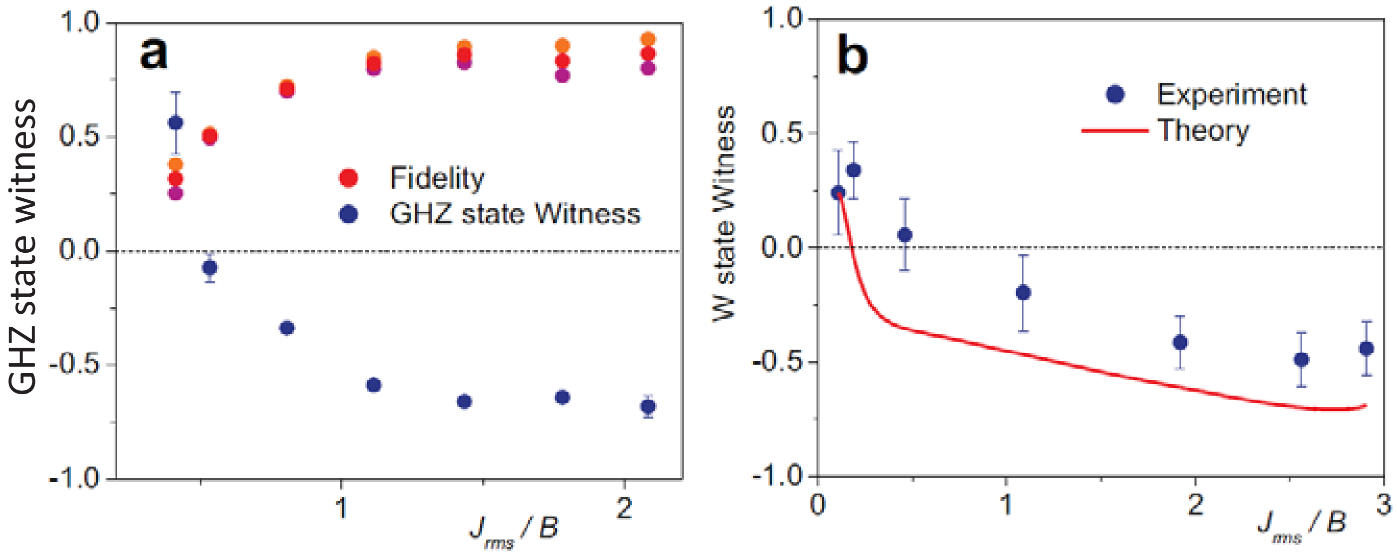
$$|\psi\rangle = |\uparrow\uparrow\downarrow\rangle + |\uparrow\downarrow\uparrow\rangle + |\downarrow\uparrow\uparrow\rangle - |\downarrow\downarrow\uparrow\rangle - |\downarrow\uparrow\downarrow\rangle - |\uparrow\downarrow\downarrow\rangle ?$$

GHZ state Witness = $\frac{9}{4} - J_z^2 - \sigma_\phi^{(1)}\sigma_\phi^{(2)}\sigma_\phi^{(3)}$

W state Witness = $\left(4+\sqrt{5}\right) - 2(J_y^2 + J_z^2), \quad J_\alpha = \frac{1}{2}\sum_i \sigma_\alpha^{(i)}$

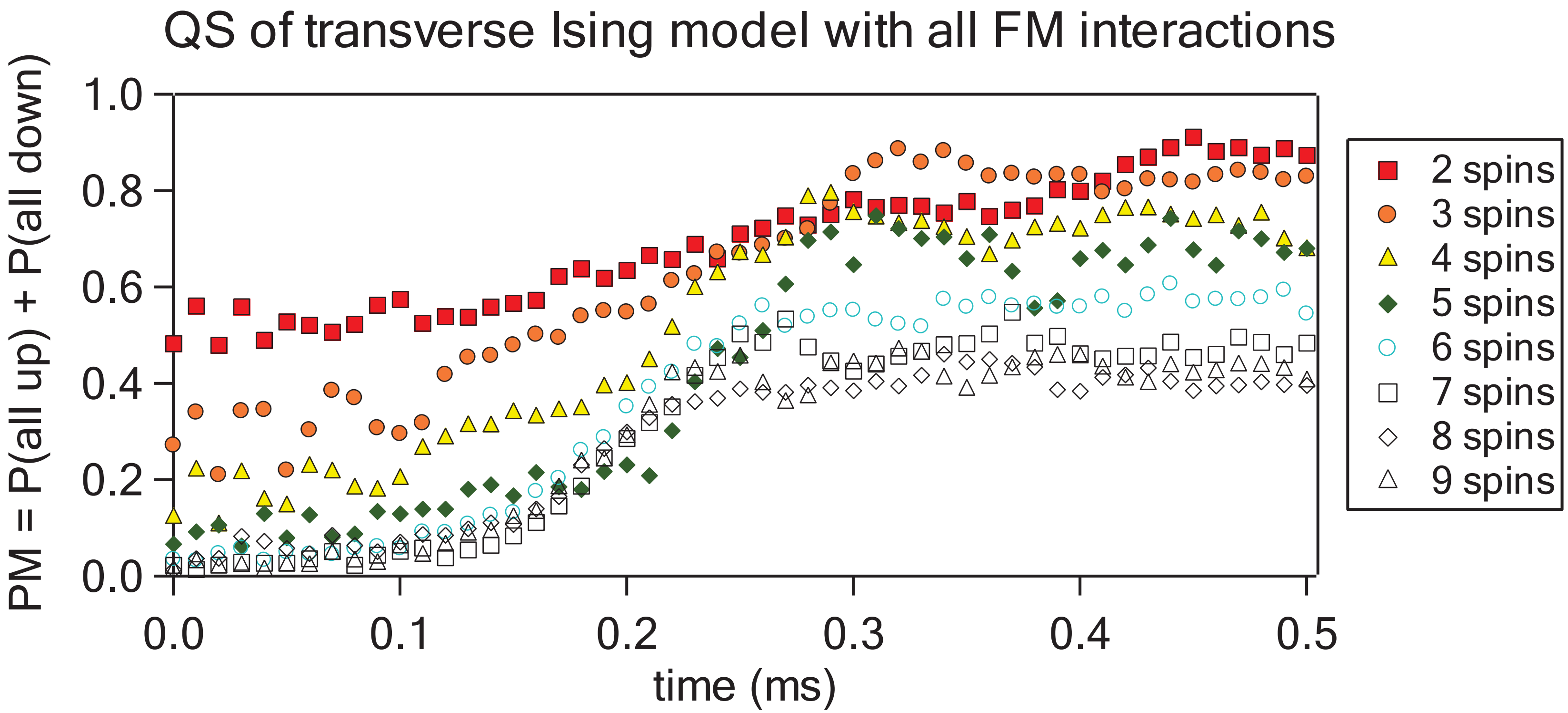


O. Ghne and G. Toth, Phys Rep **474,** 1 (2009).



Links frustration to ground state entanglement.

The more ions the better



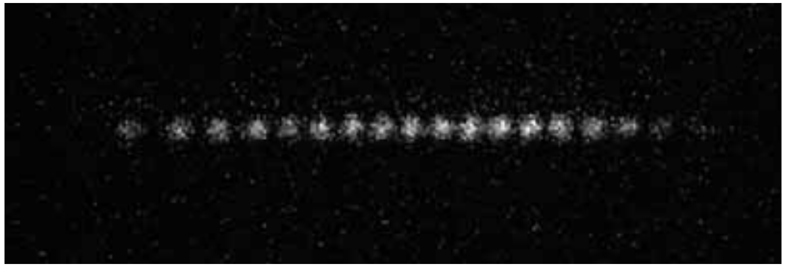
Sharper phase transition as # of spins increases.

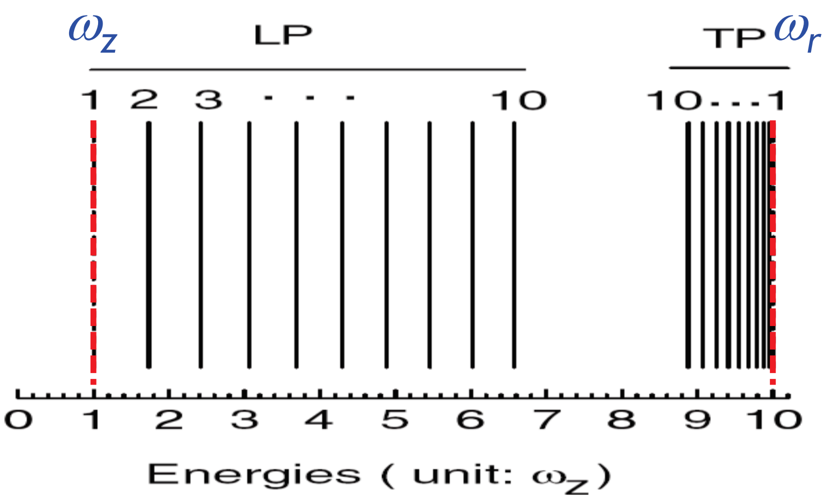
Islam et al., Nature Communication (2011)

Outlook

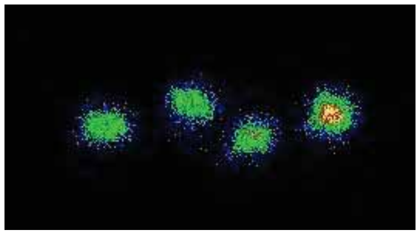
Scalability in a linear Paul trap

Harmonic external axial potential (ω_z)

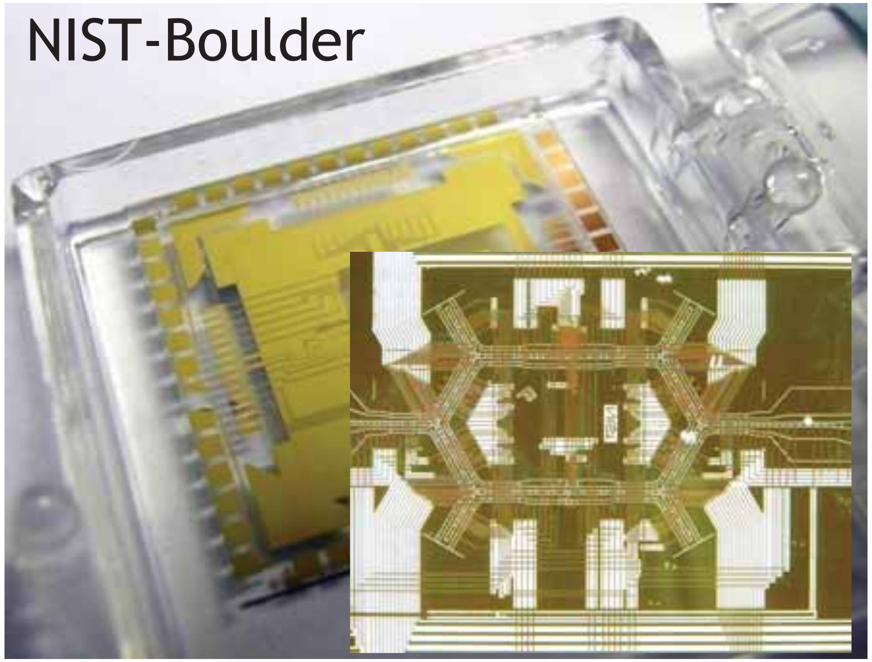


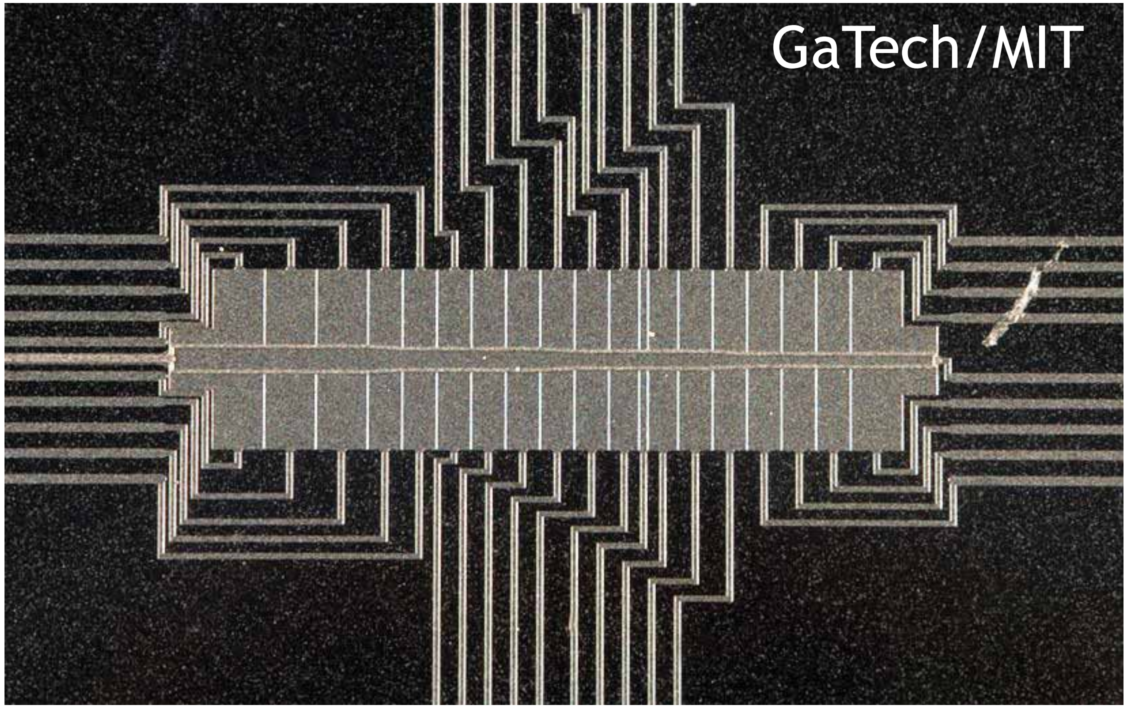


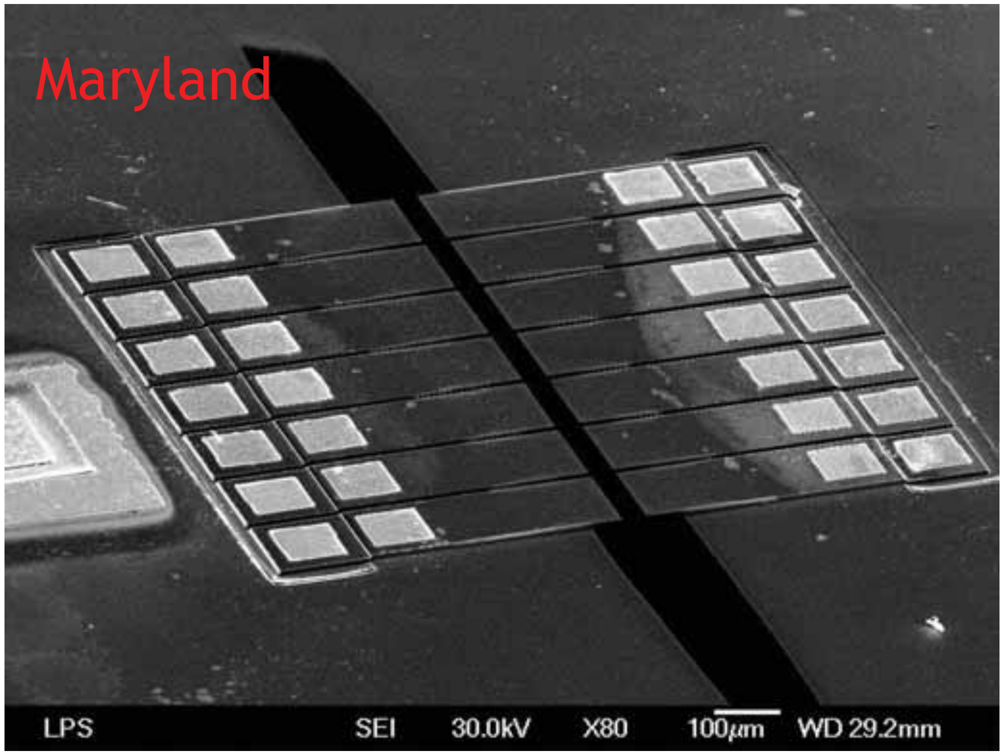
linear crystal: $\frac{\omega_r}{\omega_z} > 0.73 N^{0.86}$

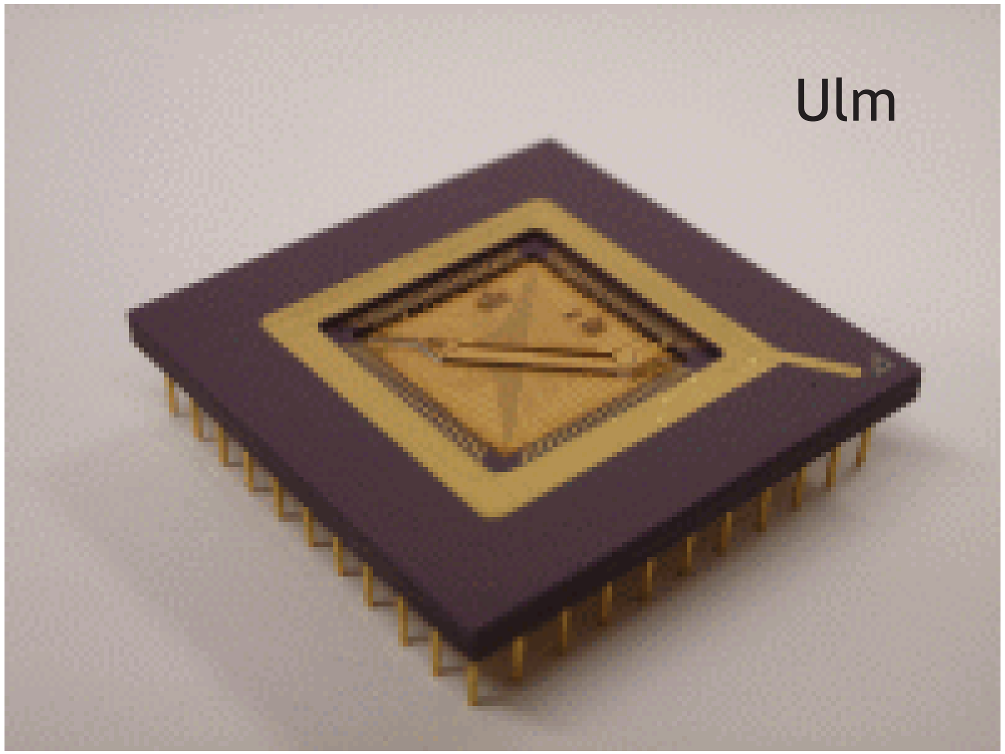


Ion Trap Chips





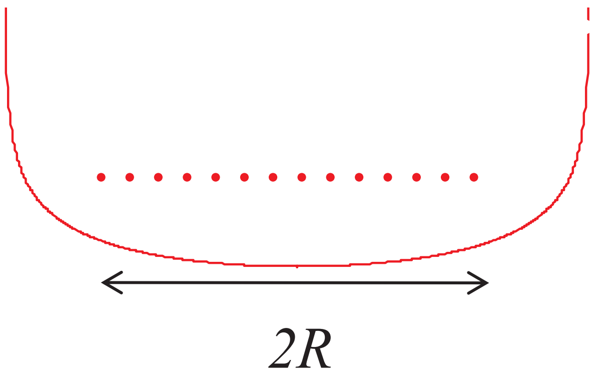




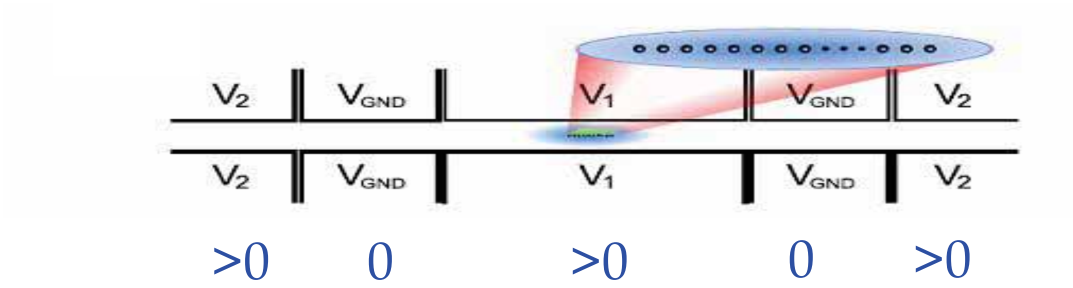
Scaling a single crystal to >> 10 ions

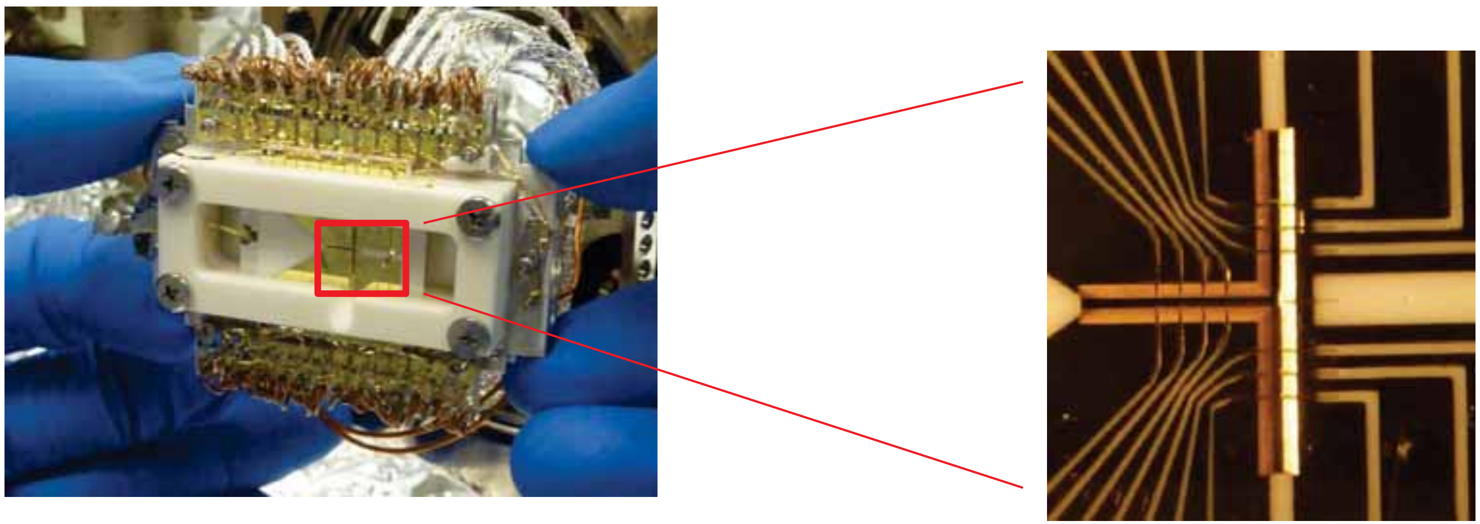
Uniformly-spaced ion crystal (spacing = s)

$$\omega_r > \sqrt{\frac{7\zeta(3)e^2}{2ms^3}}$$

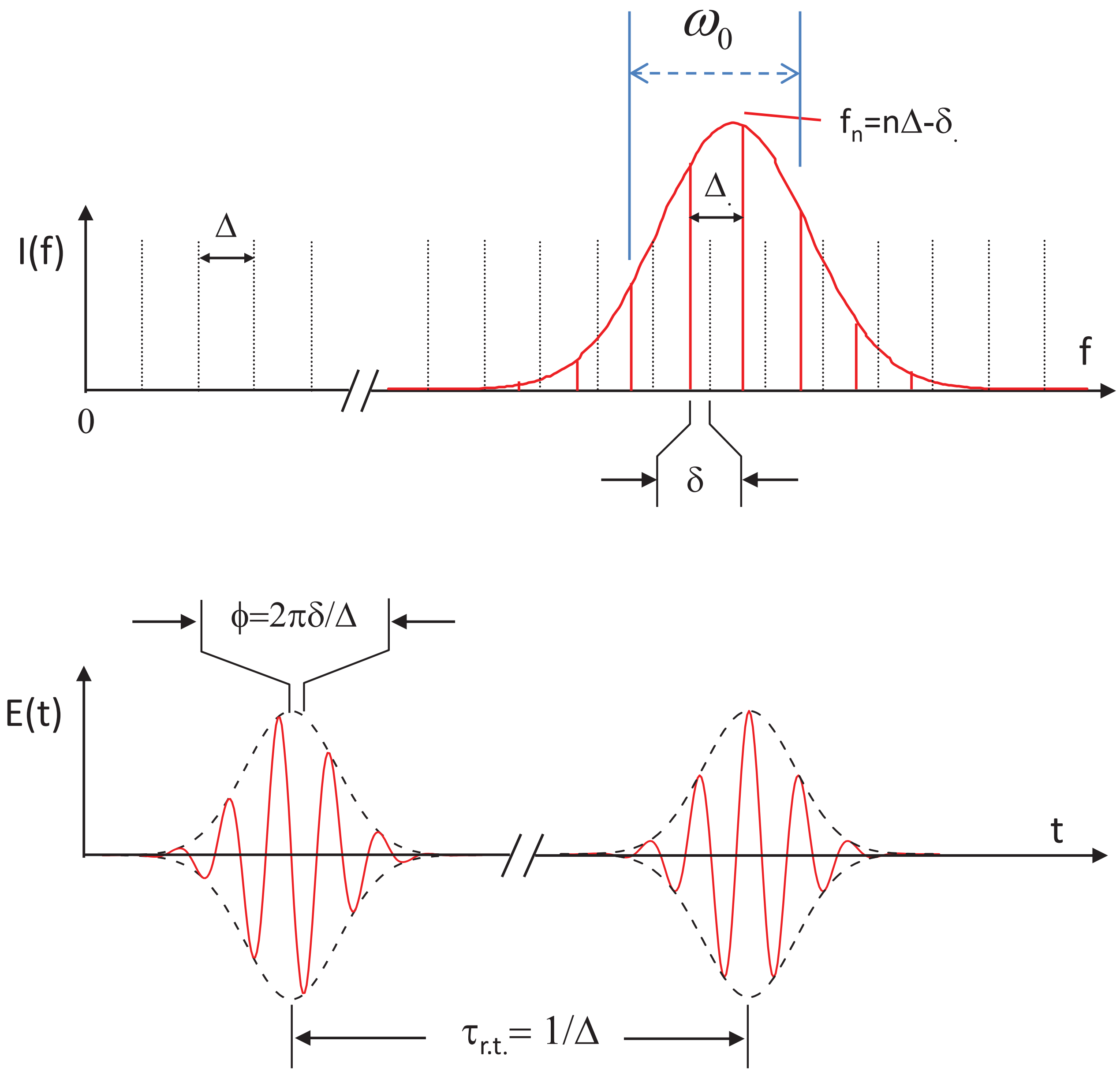


$$U(z) = U_0 \log\left(\frac{1}{1 - z^2/R^2}\right) \sim \alpha z^4$$ (quartic)





Raman transition with picosecond lasers



Δ = repetition rate=1/T

δ = Comb offset from harmonics of Δ

ϕ = Phase slip b/t carrier & envelope each round trip

Advantages:
Built-in phase locked frequency comb for Raman transitions.

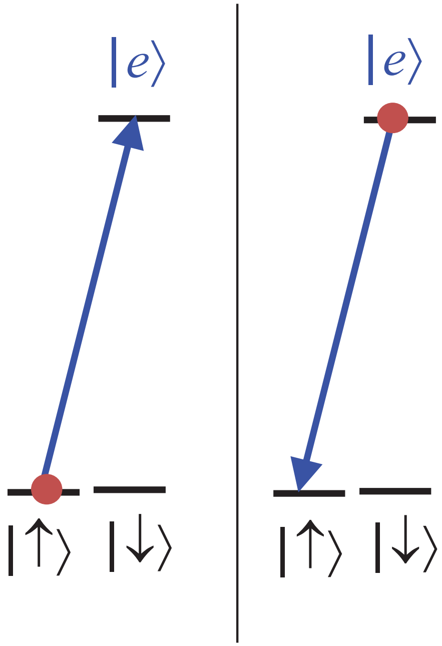
Requirements:

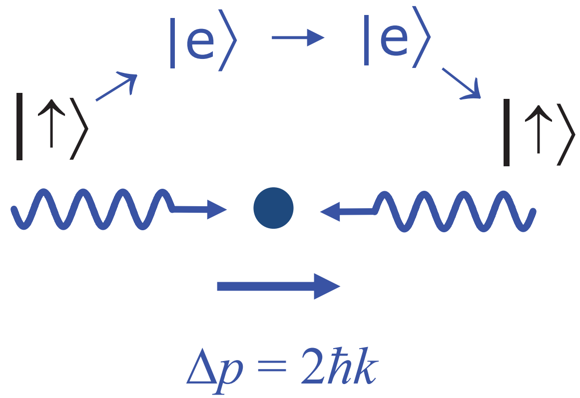
1. Bandwidth $\approx$ HF splitting
2. Lock carrier – envelop phase

D. J. Jones, S. A. Diddams, J. K. Ranka, A. Stentz, R. S. Windeler, J. L. Hall, S. T. Cundiff, Science 288, 635 (2000).

Impulsive (fast) spin-dependent kicks

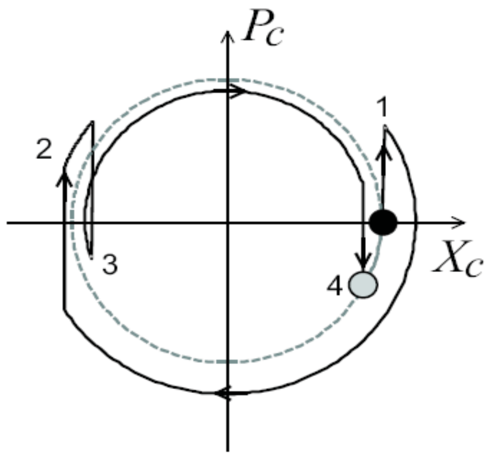
two sequential π-pulses





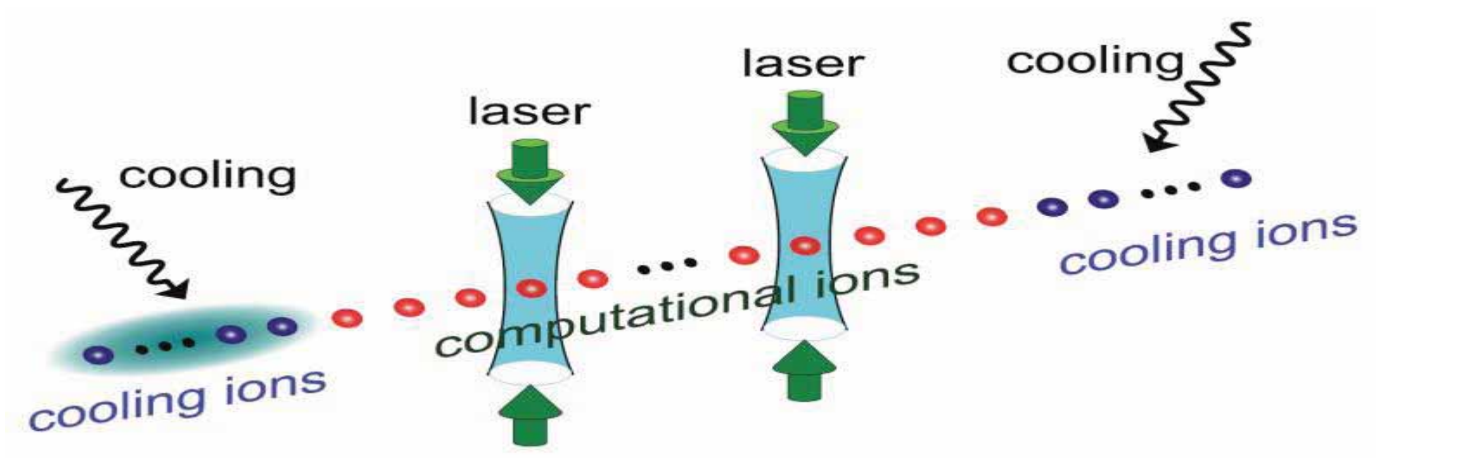
no speed limit!
no temperature limit!
(only require harmonicity)

eg: 4-pulse protocol

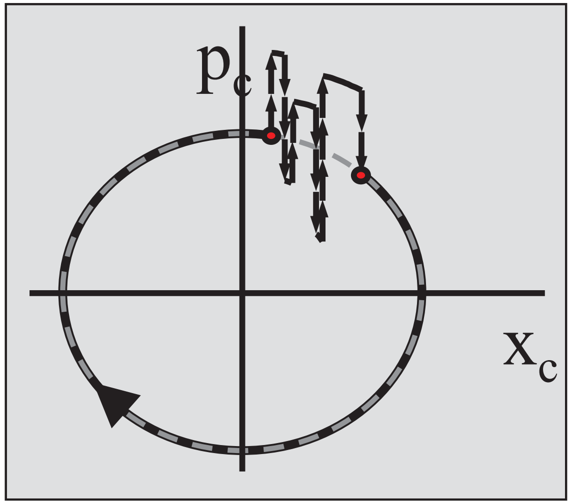


Garcia-Ripoll, Zoller, & Cirac, *PRL* **91**, 157901 (2003)

A quantum computer with ~100 qubits



G.-D. Lin, et al., EPL **86,** 60004 (2009)



• Optimal control: finding the right pulse sequence to bring the atom back to its initial state, regardless of the details of that initial state.

Garcia-Ripoll, Zoller, & Cirac, *PRL* **91**, 157901 (2003)
PRL **104**, 140501 (2010), PRL **105**, 090502 (2010)

Summary

- Quantum Information Processing

- Trapped ion QC/QS
  - Driving coherent dynamics with lasers
  - Implementing a quantum entangling gate
  - Engineering spin-spin interactions
  - Quantum simulator of a three-spin network

- Outlook
  - Scalable to larger number of qubits (spins)
  - Entangling (fast) gates with a ultrafast pulse laser.

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