Optical Waveform Synthesis

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AMO Summer School
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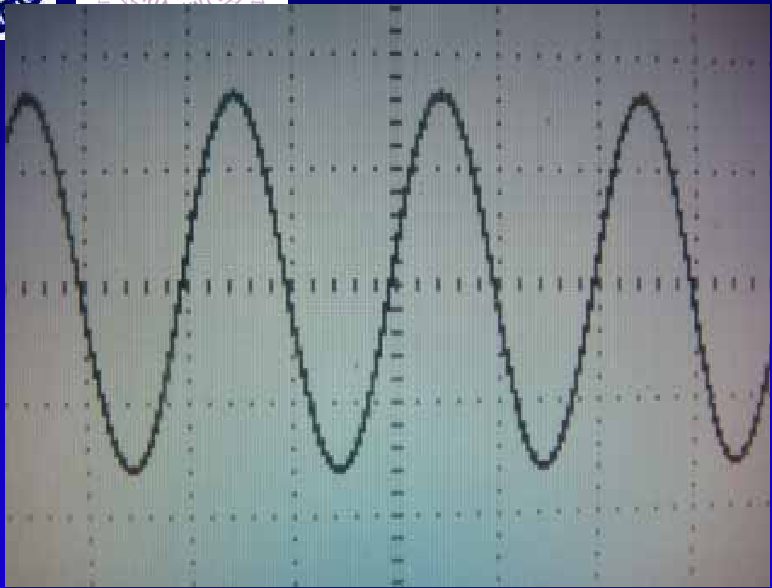
Why we are interested in it - motivation

What is the approach - basic concept

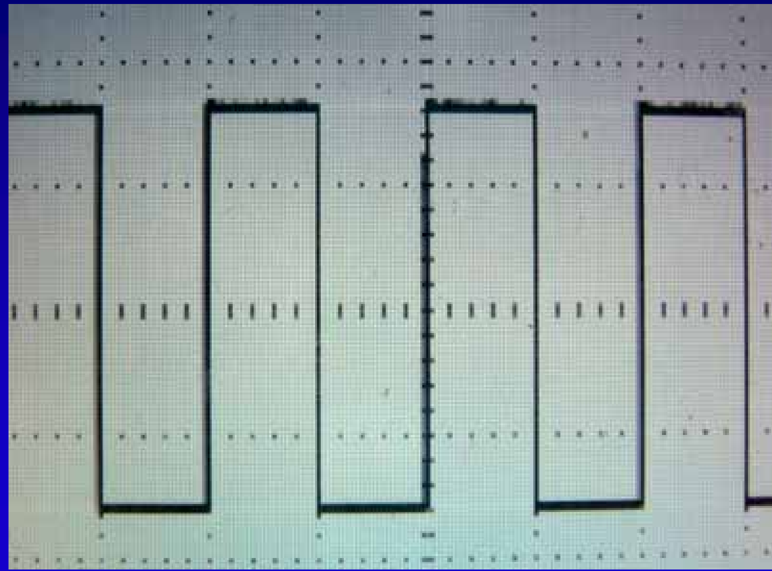
How it is done and what have been done - results

What can be done with these waveforms - application

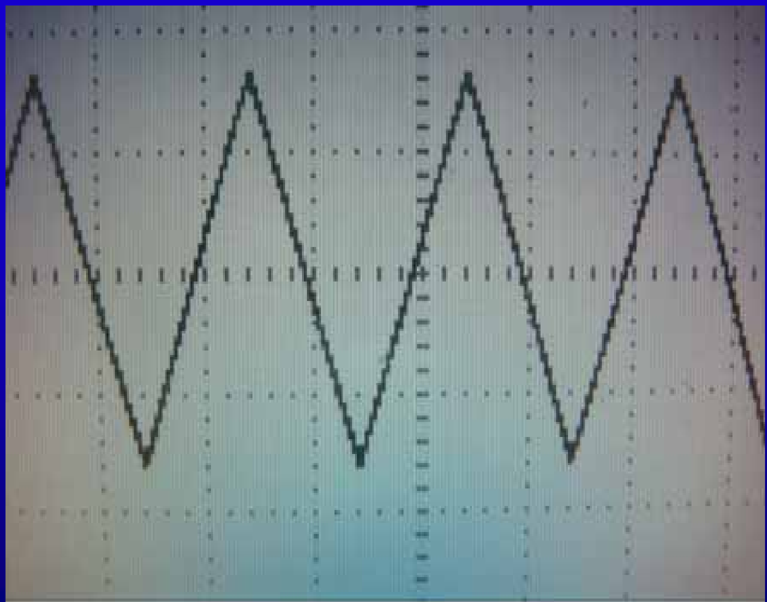
A function generator is used in many physical science laboratories to generate a wide variety of synthesized electrical signals such as the sine, square, triangle and ramp functions. These signals are used for testing and diagnostic applications.



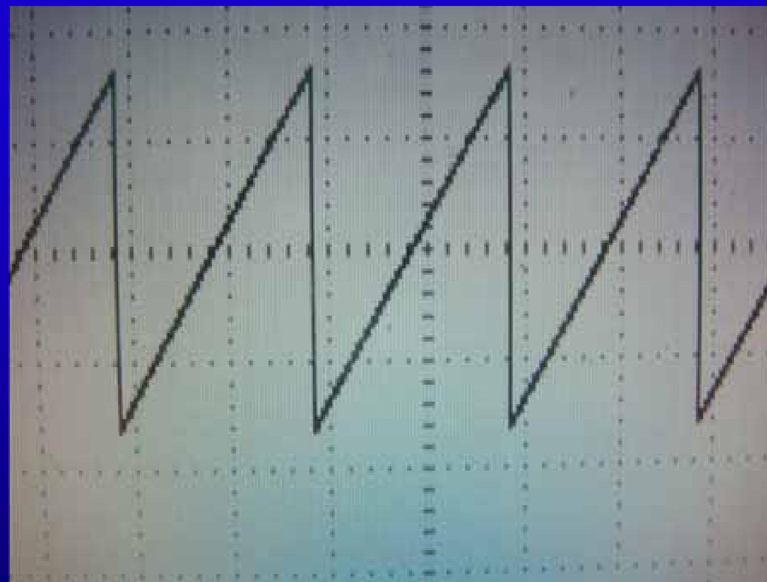
sine



square



triangle



sawtooth

Patented Jan. 6, 1942 2,268,872

UNITED STATES PATENT OFFICE

2,268,872

VARIABLE FREQUENCY OSCILLATION GENERATOR

William R. Hewlett, Palo Alto, Calif., assignor to Hewlett-Packard Company, Palo Alto, Calif., a co-partnership consisting of William R. Hewlett and David Packard

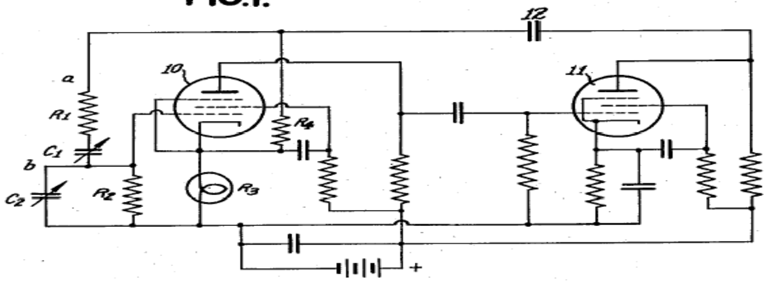
Application July 11, 1939, Serial No. 283,801

Jan. 6, 1942. W. R. HEWLETT 2,268,872

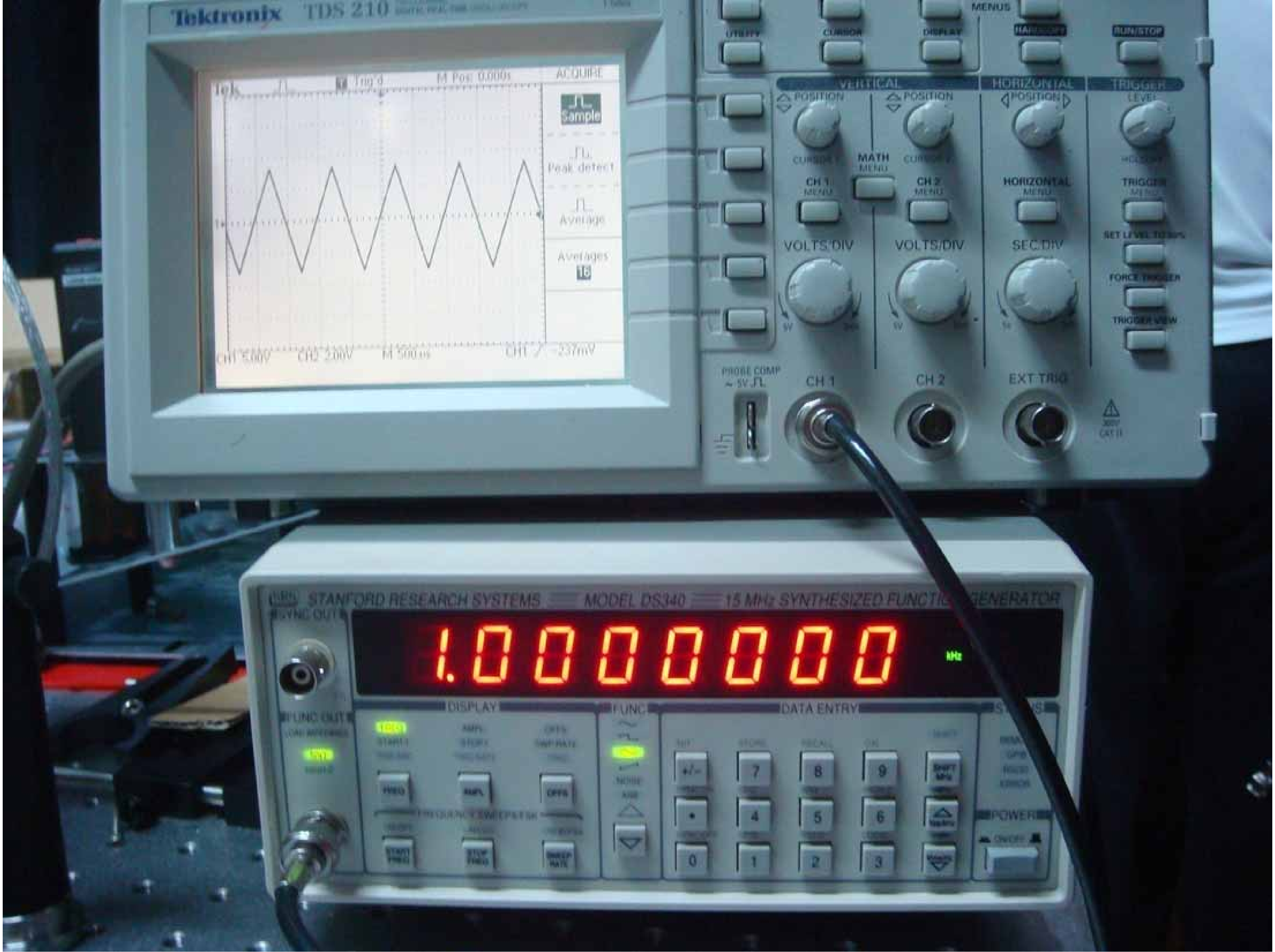
VARIABLE FREQUENCY OSCILLATION GENERATOR

Filed July 11, 1939

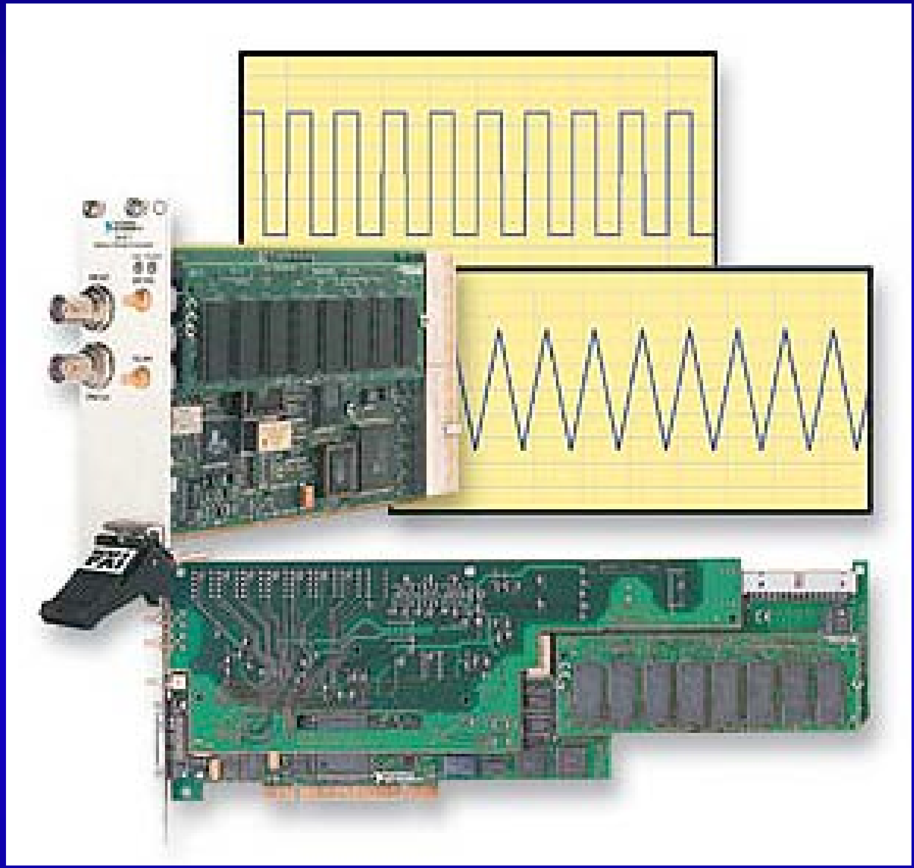
FIG.1.



HP 200A


Tektronix TDS 210
Trig'd
M Pos: 0.000s
ACQUIRE
Sample
Peak detect
Average
Averages
16
CH1 5.00V
CH2 2.00V
M 500us
CH1 -237mV
VERTICAL
HORIZONTAL
TRIGGER
VOLTS/DIV
SEC/DIV
CH 1
CH 2
EXT TRIG
STANFORD RESEARCH SYSTEMS
MODEL DS340
15 MHz SYNTHESIZED FUNCTION GENERATOR
1.0000000
kHz
DISPLAY
FUNC
DATA ENTRY
POWER

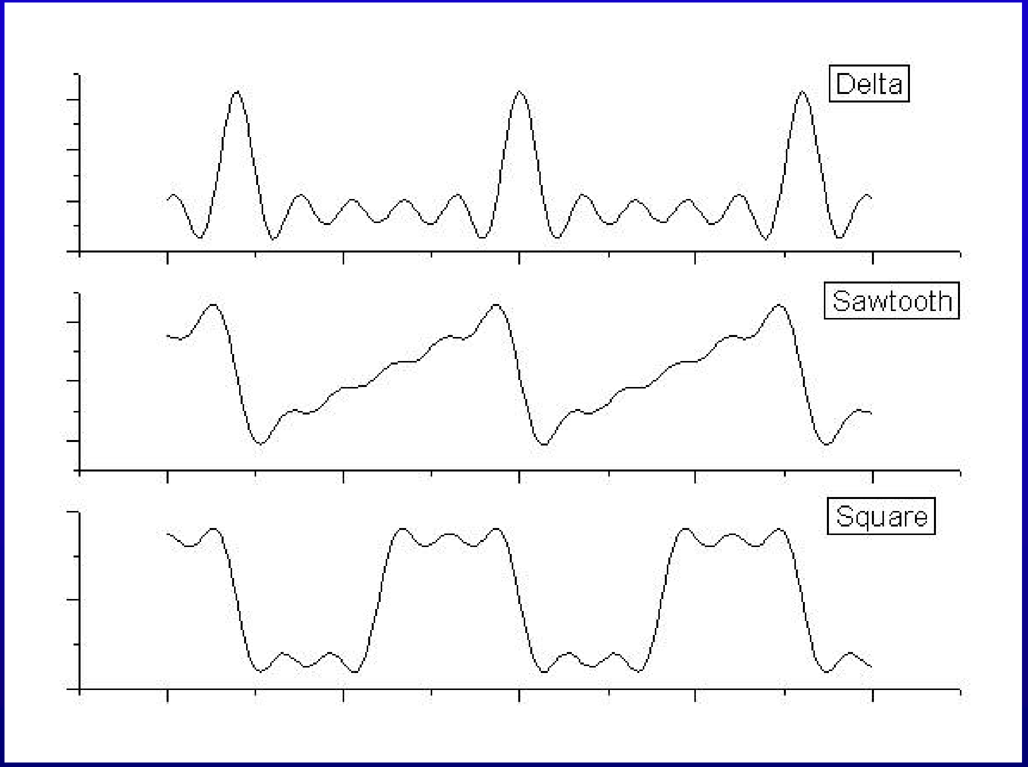
RF and Microwave function generators



Their speed are limited by the speed of IC circuits to <100GHz.

Synthesize various forms of ultrashort pulses

in the optical frequency (10^{15} cps) regime



atto- [A toh]

(Danish or Norwegian: eighteen; a decimal prefix used in the international metric system for measurements)

This prefix is used in the metric [decimal] system as quintillionth [U.S.] and trillionth part [U.K.], 10^{-18} [0.000 000 000 000 000 001]. The metric symbol for **atto-** is **a**.

Prefix for small and large numbers:

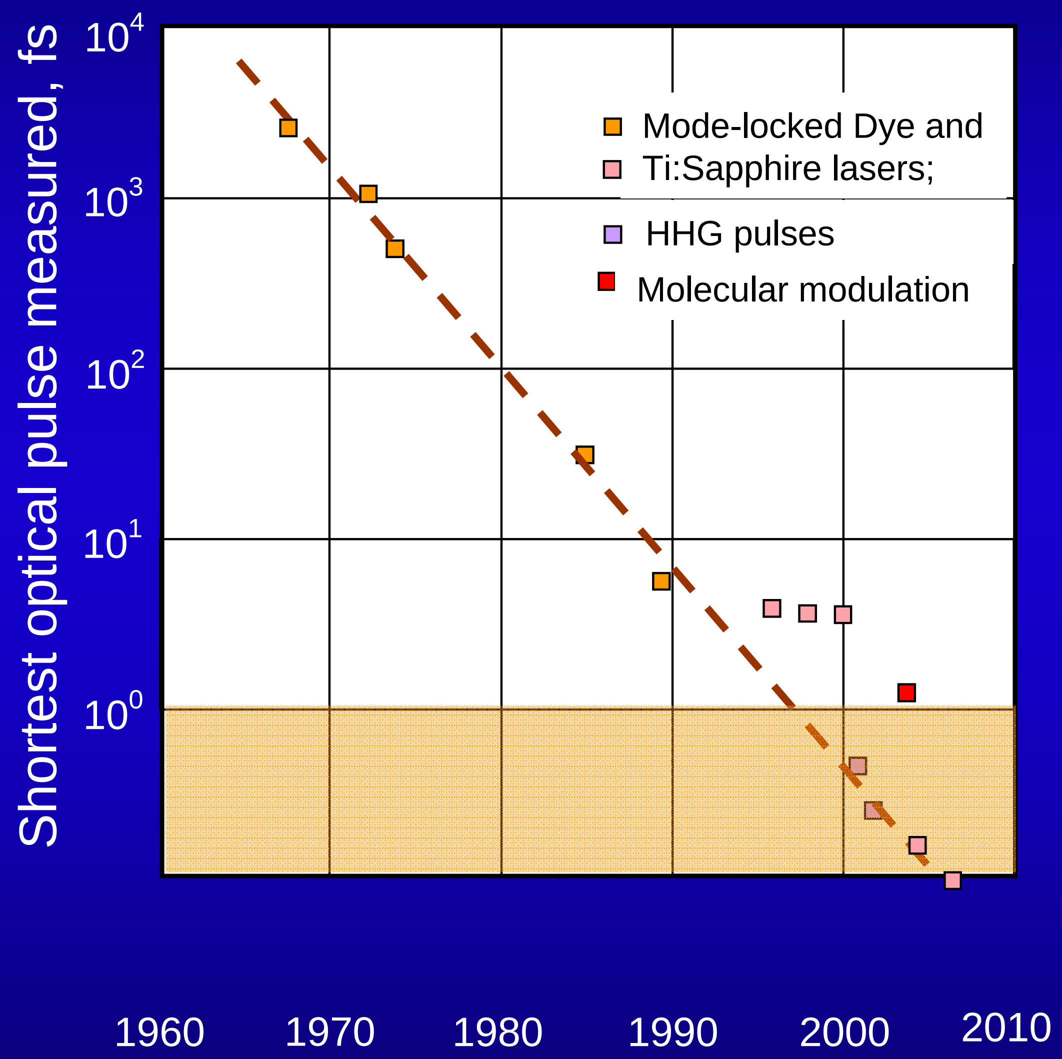
micro	10^{-6}	mega	10^{6}
nano	10^{-9}	giga	10^{9}
pico	10^{-12}	tera	10^{12}
femto	10^{-15}	peta	10^{15}
atto	10^{-18}	exa	10^{18}
zepto	10^{-21}	zetta	10^{21}
yotta	10^{-24}	yocto	10^{24}

300 nm = 1 fs

single-cycle pulse ≡ attosecond pulse

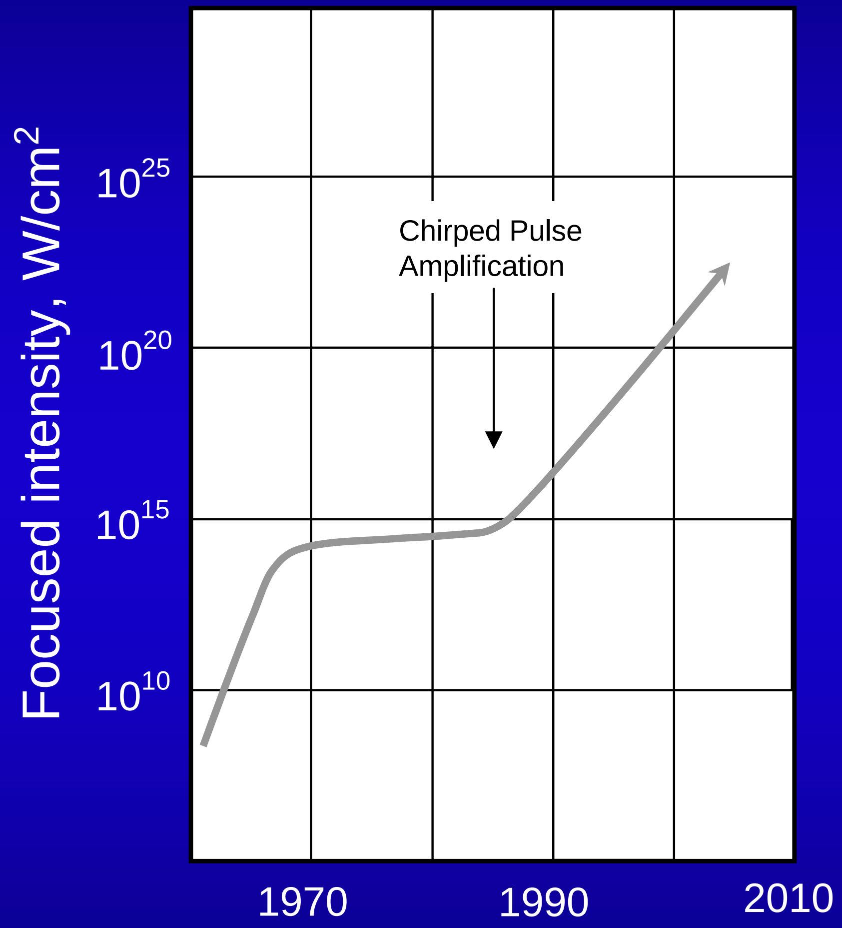
Frontier in optical physics

Laser pulses get shorter over the years



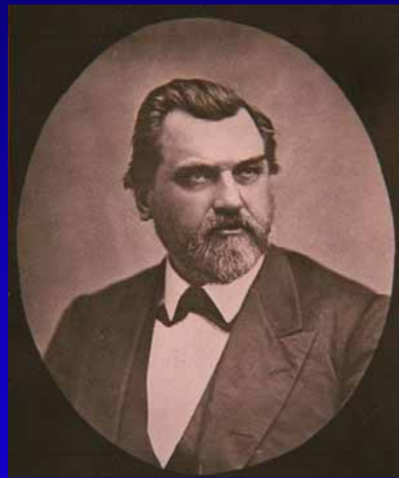
Ultrafast science

Peak intensity increased

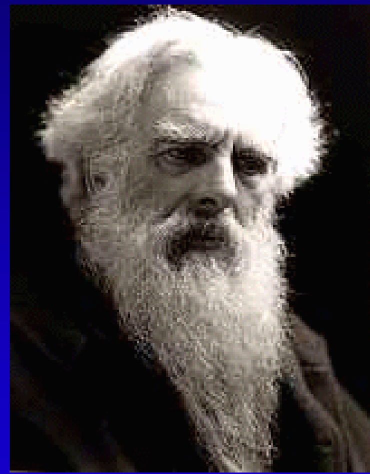


High field physics

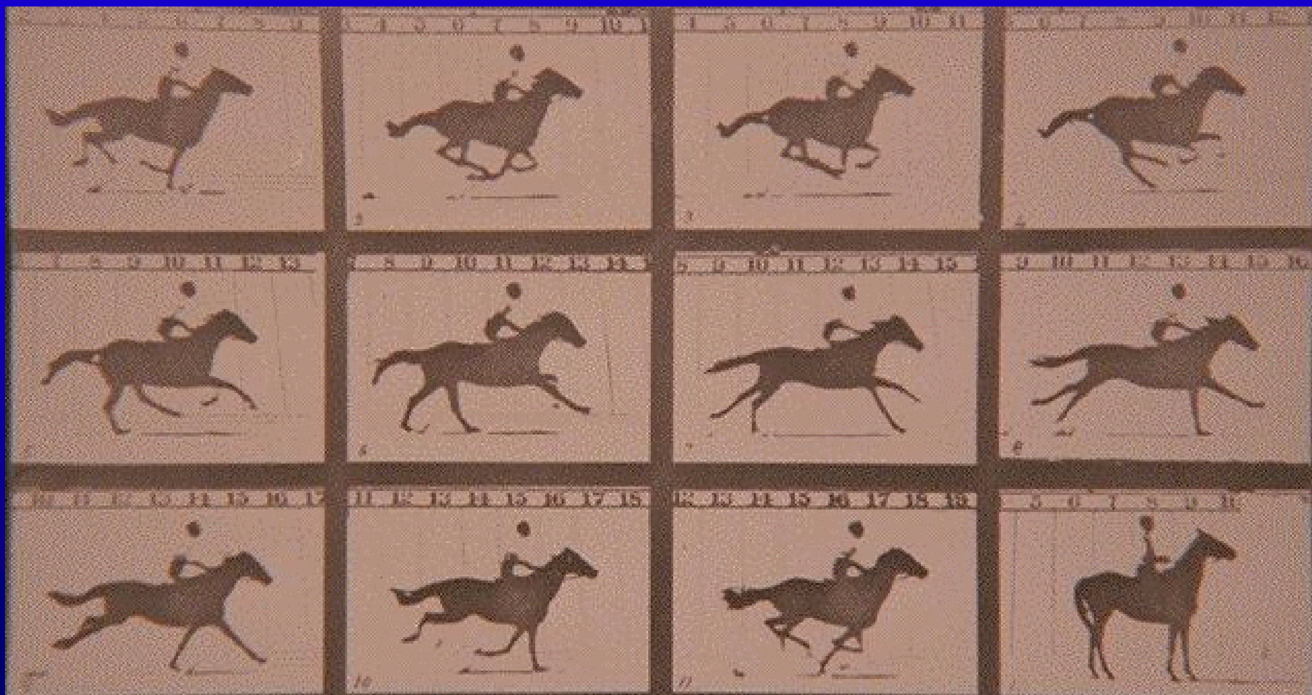
Bet: Do all four hooves of a trotting horse ever simultaneously leave the ground?



Leland Stanford



Eadweard Muybridge

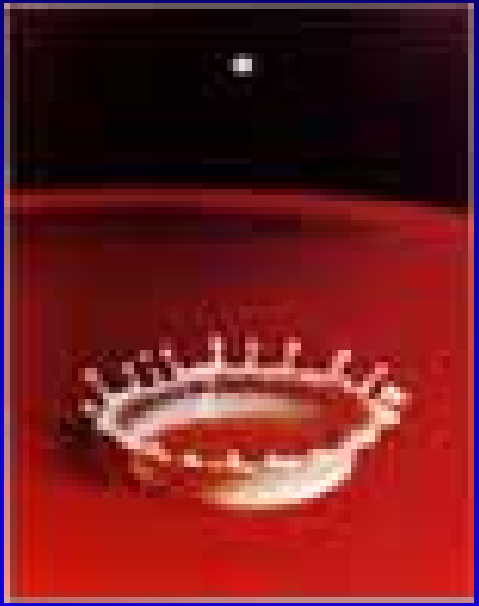


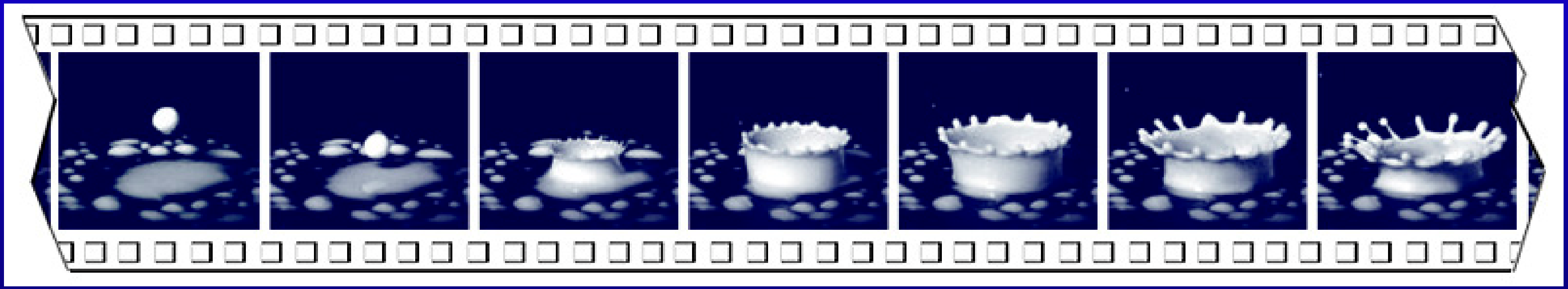
The "Trotting Horse" Controversy
Palo Alto, CA 1872

Time Resolution:
1/60th of a second

Birth of strobe photography

Harold
Edgerton
MIT, 1942





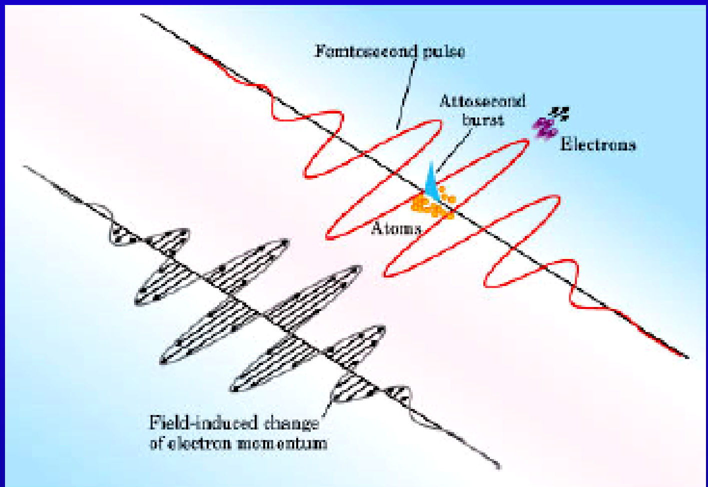
Time scale: a few microseconds

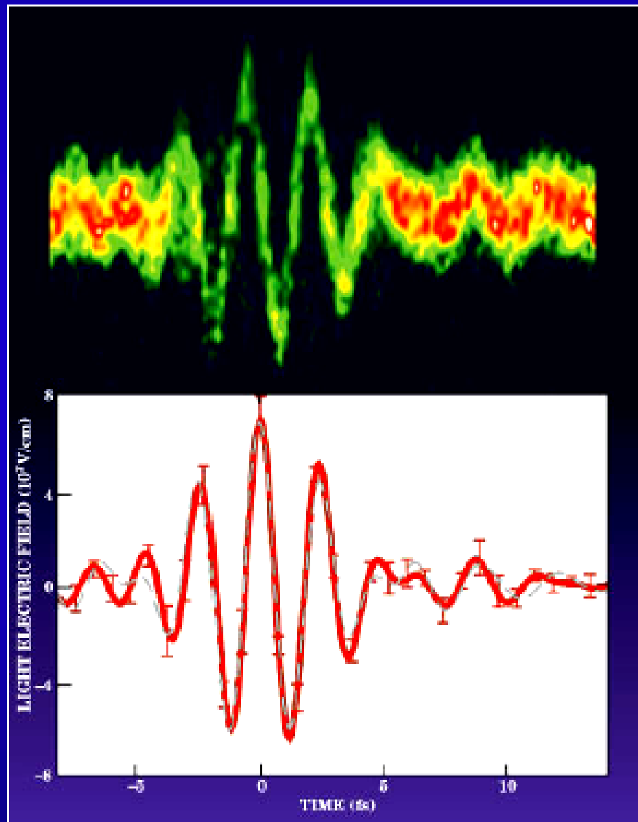
PHYSICS TODAY October 2004

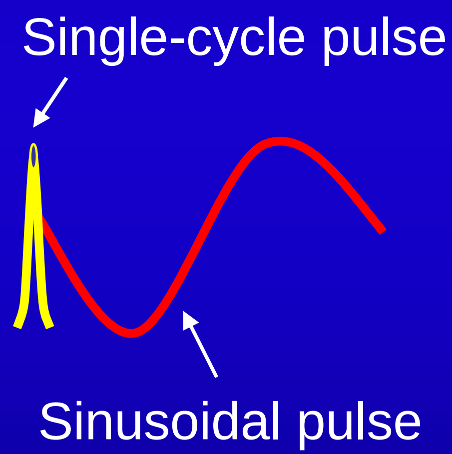
Search and Discovery

Attosecond Bursts Trace the Electric Field of Optical Laser Pulses

The familiar textbook sketch of light's oscillating electric field can now be drawn directly from measurements.

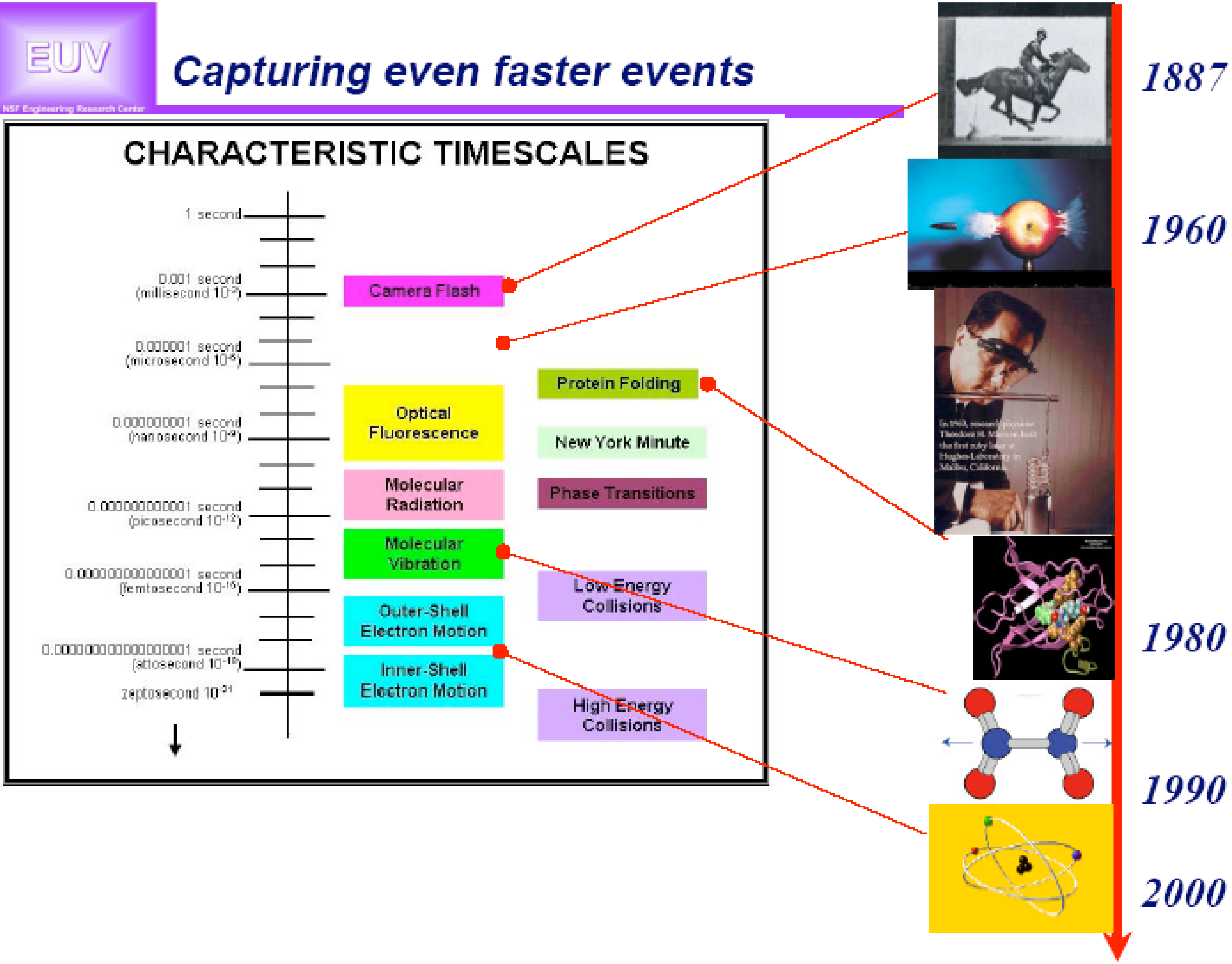






EUV
An NSF Engineering Research Center

Capturing even faster events



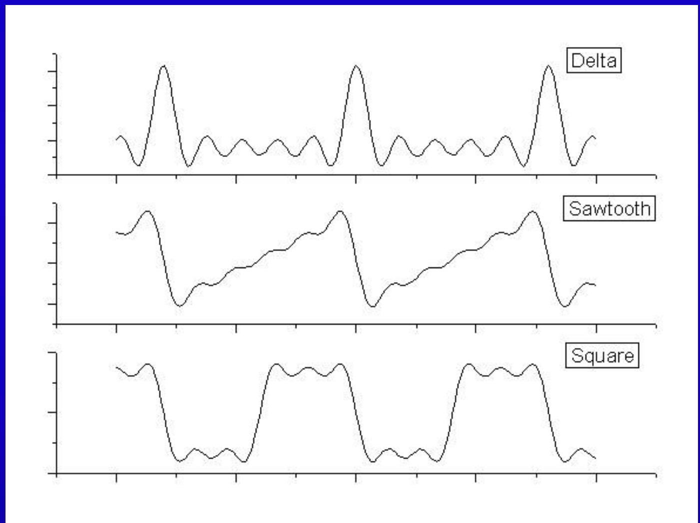
國立清華大學 光電工程研究所

Wouldn't it be nice if we can have complete control of the motion of electrons in space and in matter, or just about anywhere?

Electrons are charged particles. Their motion follows changes in the electric field. To control electron motion, we want electric fields of an arbitrary shape that we can produce at will.

GOAL

Develop an Optical Function Generator



With these waveforms we hope that we can do using lightwaves what others have done with microwaves:

Control free/bound electron motion with light

LIGHTWAVE ELECTRONICS

How do we do it?

Basic Concepts

Joseph Fourier

Fourier was obsessed with the physics of heat and developed the Fourier series and transform to model heat-flow problems.

Joseph Fourier 1768 - 1830

MÉMOIRE

SUR LA

PROPAGATION DE LA CHALEUR

DANS LES CORPS SOLIDES,

PAR M. FOURIER (¹).

Présenté le 21 décembre 1807 à l'Institut national.

Nouveau Bulletin des Sciences par la Société philomathique de Paris, t. I, p. 112-116, n° 6; mars 1808. Paris, Bernard.

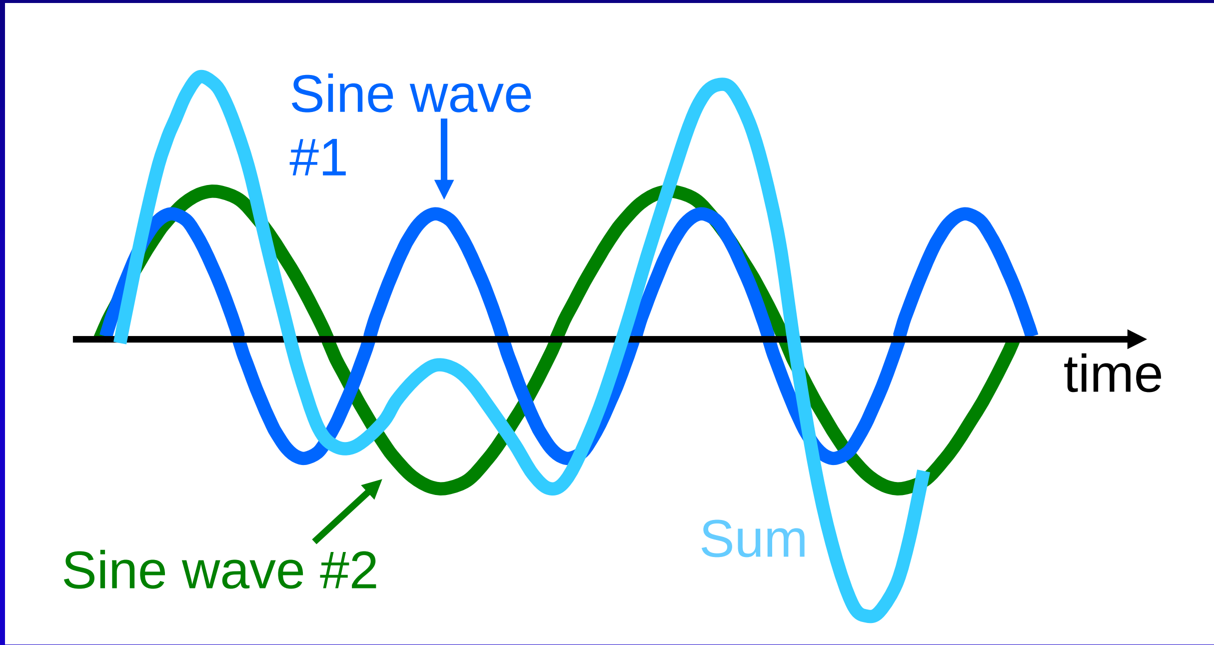
A revolutionary article

> $$\varphi(y) = a\cos\frac{\pi y}{2} + a'\cos 3\frac{\pi y}{2} + a''\cos 5\frac{\pi y}{2} + \cdots.$$
>
> Multiplying both sides by $\cos(2i+1)\frac{\pi y}{2}$, and then integrating from $y = -1$ to $y = +1$ yields:
>
> $$a_i = \int_{-1}^{1} \varphi(y)\cos(2i+1)\frac{\pi y}{2}\,dy.$$

—Joseph Fourier, Mémoire sur la propagation de la chaleur dans les corps solides, pp. 218--219.[1]

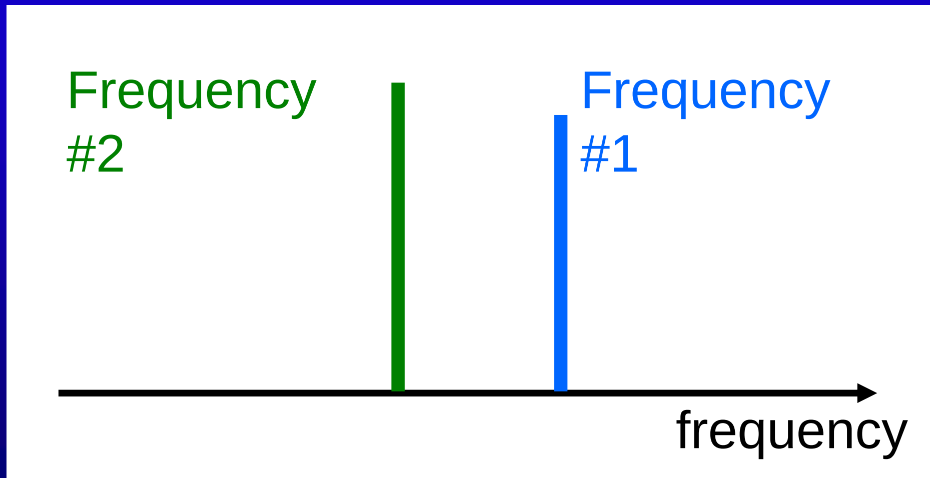
In these few lines, which are surprisingly close to the modern formalism used in Fourier series, Fourier unwittingly revolutionized both mathematics and physics. Although similar trigonometric series were previously used by Euler, d'Alembert, Daniel Bernoulli and Gauss,

Many waves are sums of sinusoids.



•Consider the sum of two sine waves of different frequencies:

The spectrum of the sum of the two sine waves:



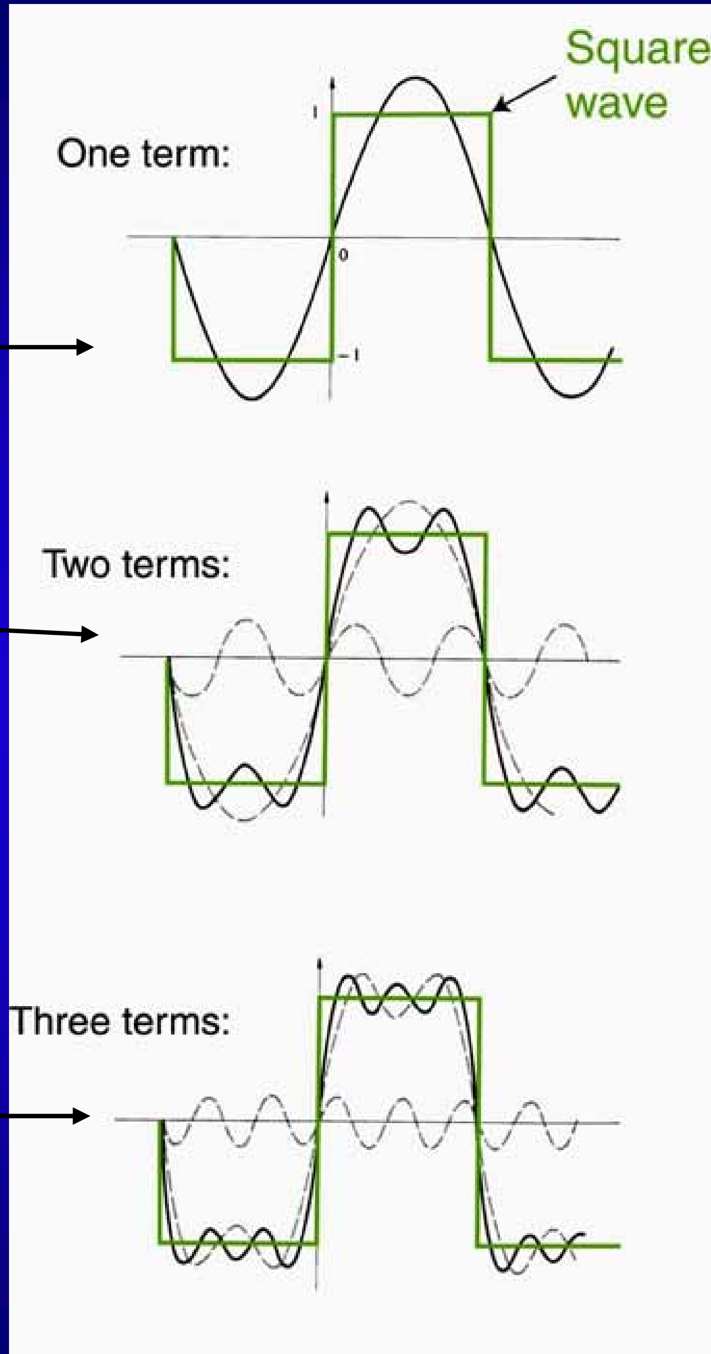
Fourier decomposing functions

- Here, we write a
- **square wave** as
- a sum of sine waves.

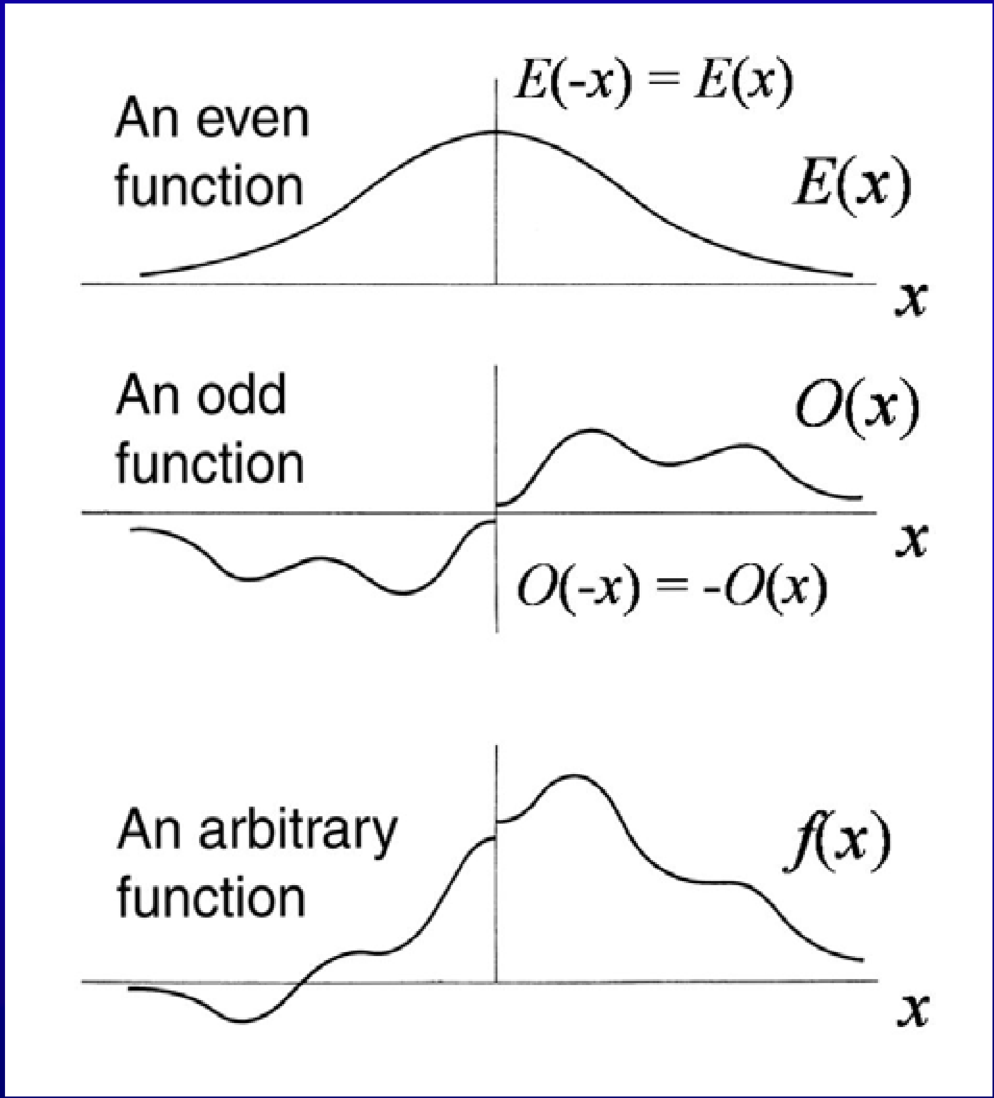
$\sin(\omega t)$

$\sin(3\omega t)$

$\sin(5\omega t)$



Any function can be written as the sum of an even and an odd function.



$$F(x) = E(x) + O(x)$$

Fourier series

So if $f(t)$ is a general function, neither even nor odd, it can be written:

even component odd component

$$f(t) = \frac{1}{\pi}\sum_{m=0}^{\infty} F_m \cos(mt) + \frac{1}{\pi}\sum_{m=0}^{\infty} F'_m \sin(mt)$$

where

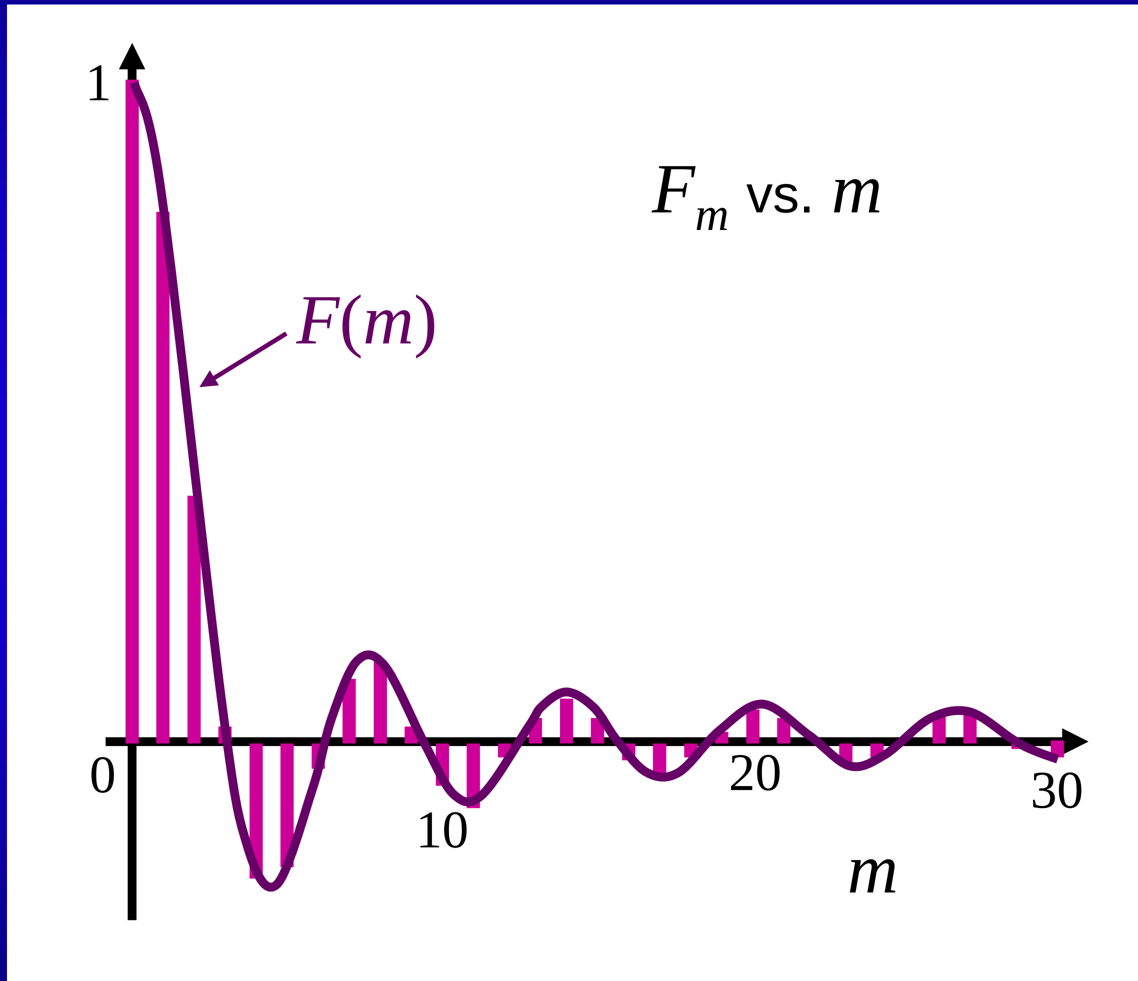
$$F_m = \int f(t)\cos(mt)\,dt$$

and

$$F'_m = \int f(t)\sin(mt)\,dt$$

We can plot the coefficients of a Fourier series.

Let the integer m become a real number and let the coefficients F_m become a function $F(m)$.



We really need two such plots, one for the cosine series and another for the sine series.

The Fourier transform

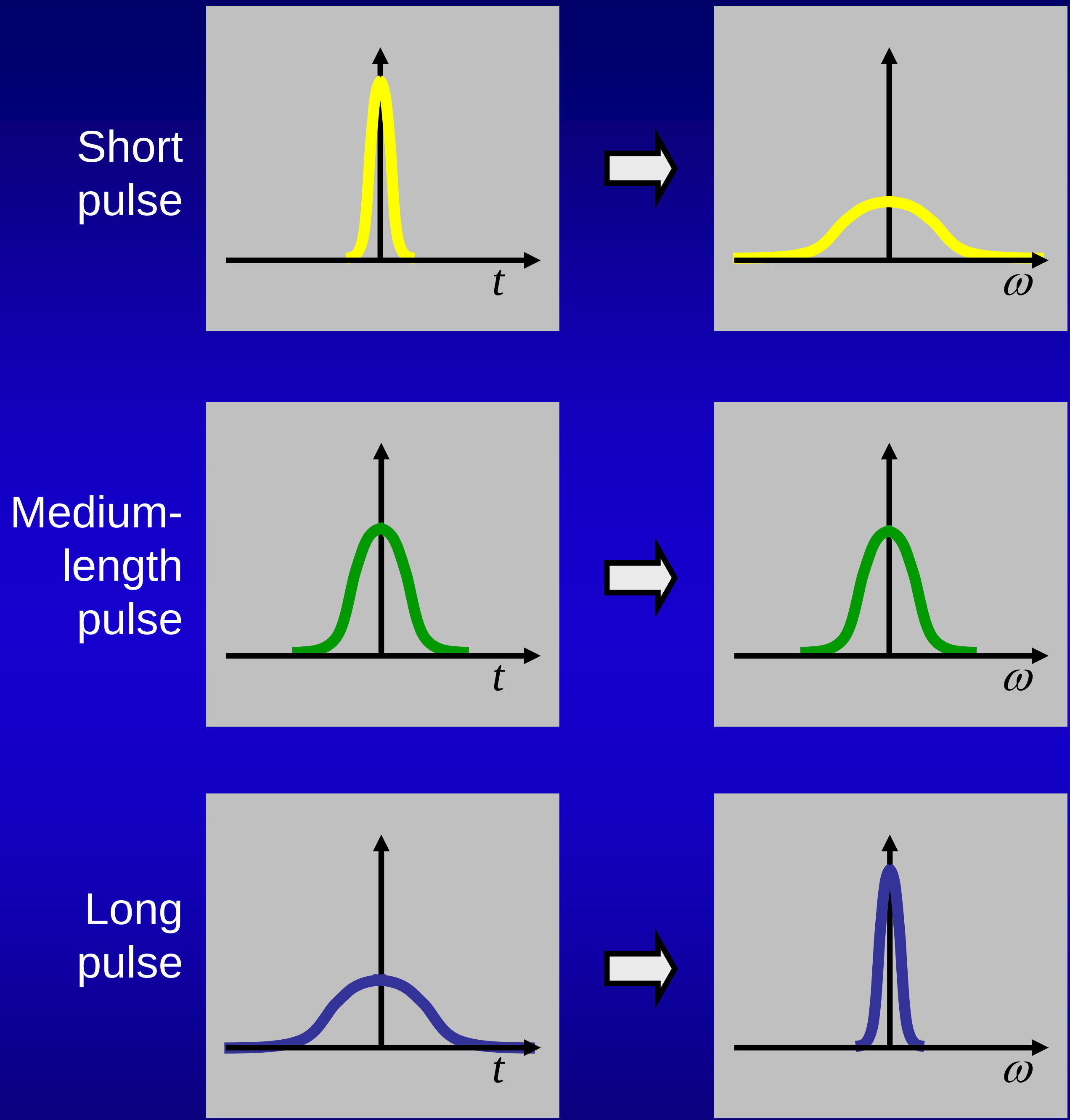
$$F(\omega) = \int_{-\infty}^{\infty} f(t)\exp(-i\omega t)\,dt$$

$F(\omega)$ is called the Fourier Transform of $f(t)$. **It contains equivalent information to that in $f(t)$.** We say that $f(t)$ lives in the **time domain**, and $F(\omega)$ lives in the **frequency domain**.

$F(\omega)$ is just another way of looking at a function or wave.

The shorter the pulse, the broader the spectrum!

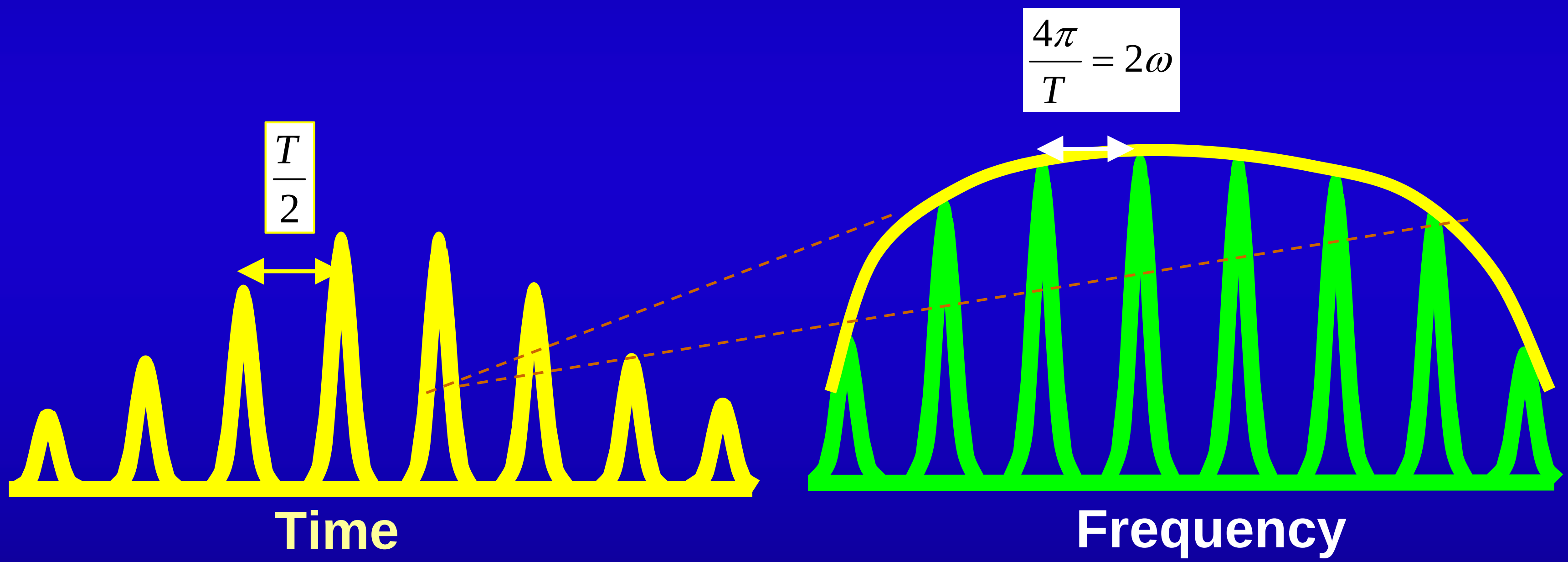
This is the essence of the Uncertainty Principle!



Correlation between time and frequency

$$x(t-t_0) \xleftrightarrow{FT} e^{-j\omega t_0} X(\omega)$$

Fourier transform: $$X(\omega) = \int_{-\infty}^{\infty} x(t) e^{-i\omega t} d\omega$$



2.6 fs ⟷ ***12,820 cm^{-1} , 780 nm***

Periodic waveforms are sums of sinusoidal waves

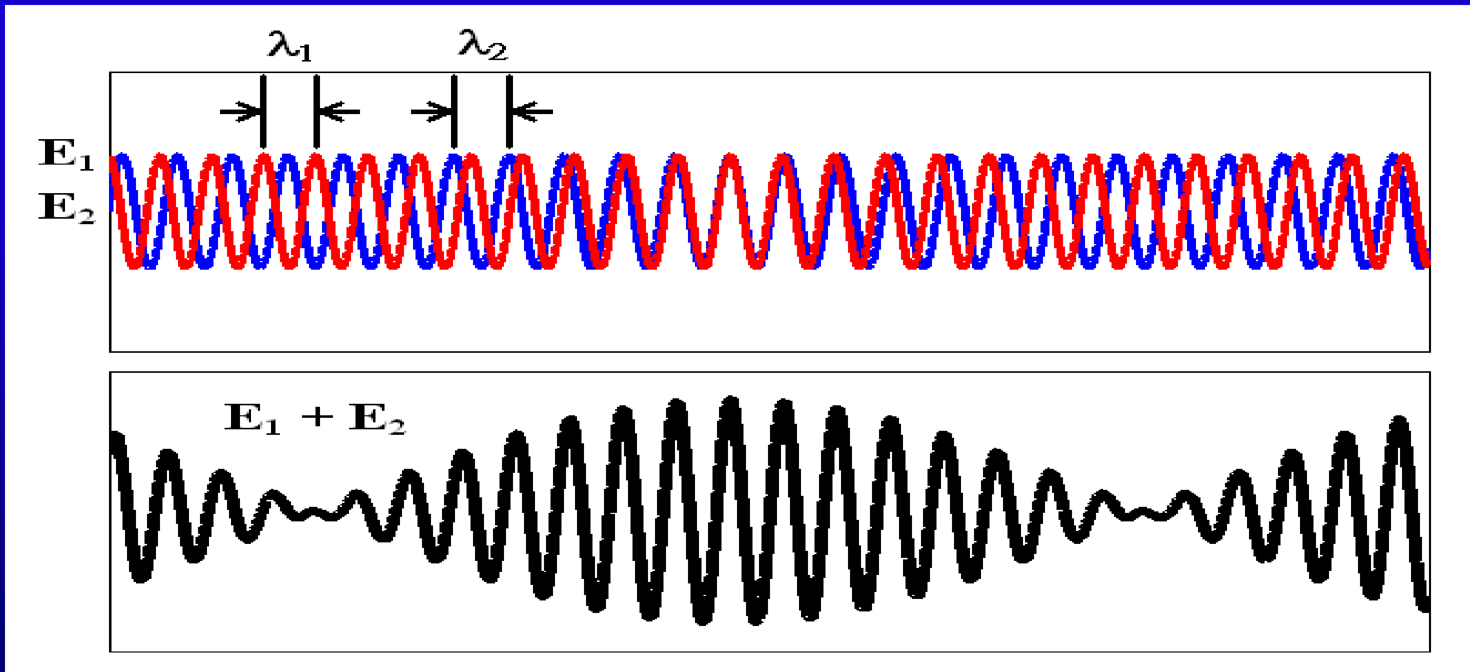
(a) Monochromatic light: sinusoidal wave

$$E(t) = A(t)\cos(\omega t + \phi)$$

(b) Beating of two waves, ω_1 and ω_2

$$E(t) = A_1(t)\cos(\omega_1 t + \phi_1) + A_2(t)\cos(\omega_2 t + \phi_2)$$

(b)

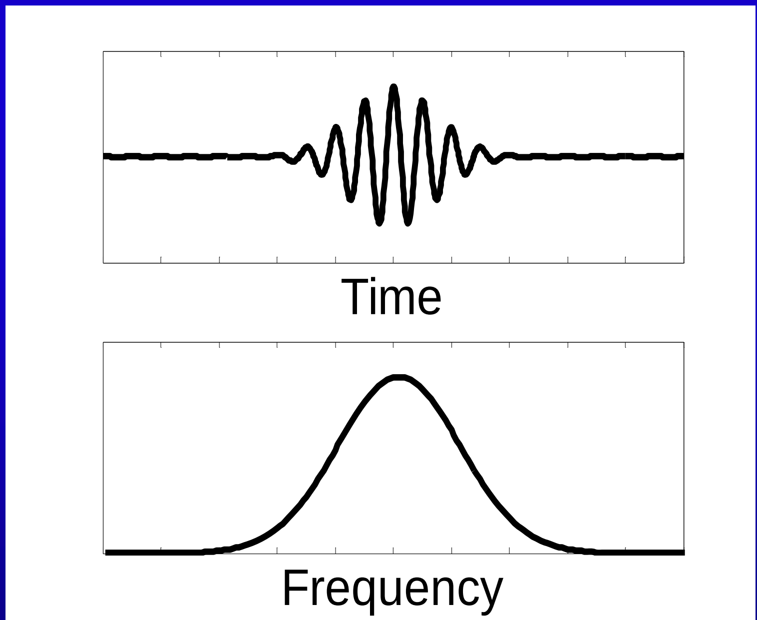


What is a single cycle optical pulse

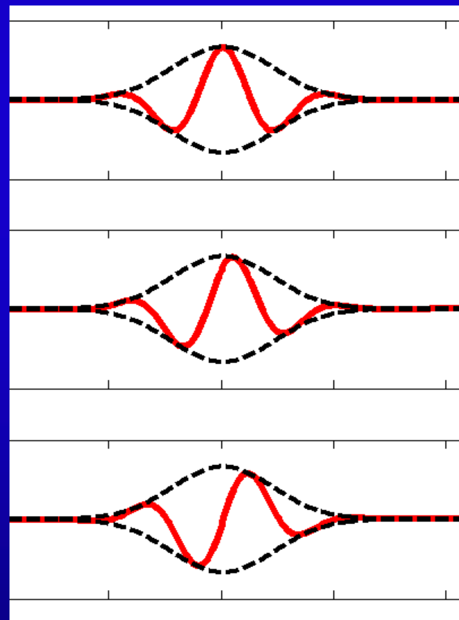
(c) Many waves propagating to form a wave packet (left)

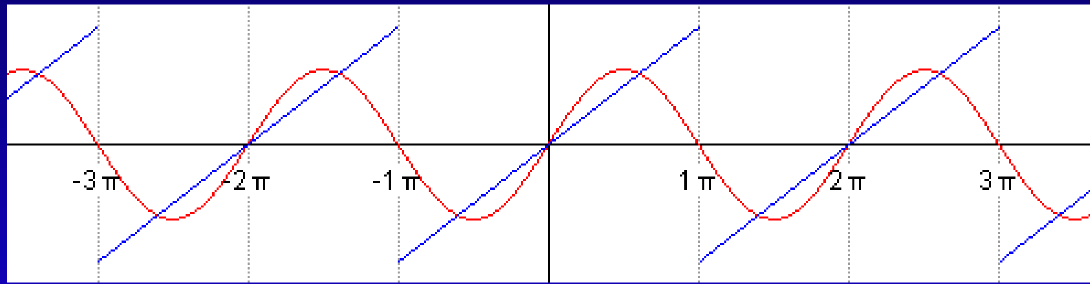
(d) Ultimate wavepacket is a single-cycle and sub-cycle pulse train (right)

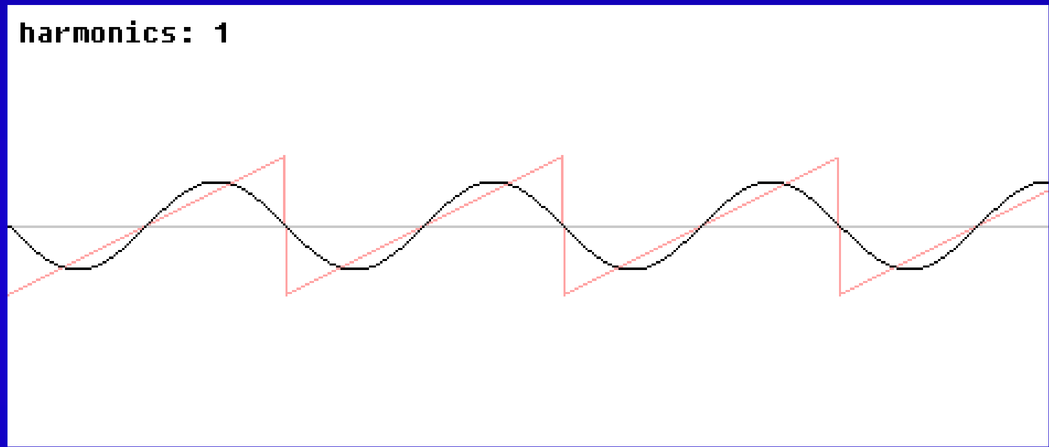
$$E(t) = \sum_n E_n(t) = \sum_n A_n(t)\cos(\omega_n t + \phi_n)$$

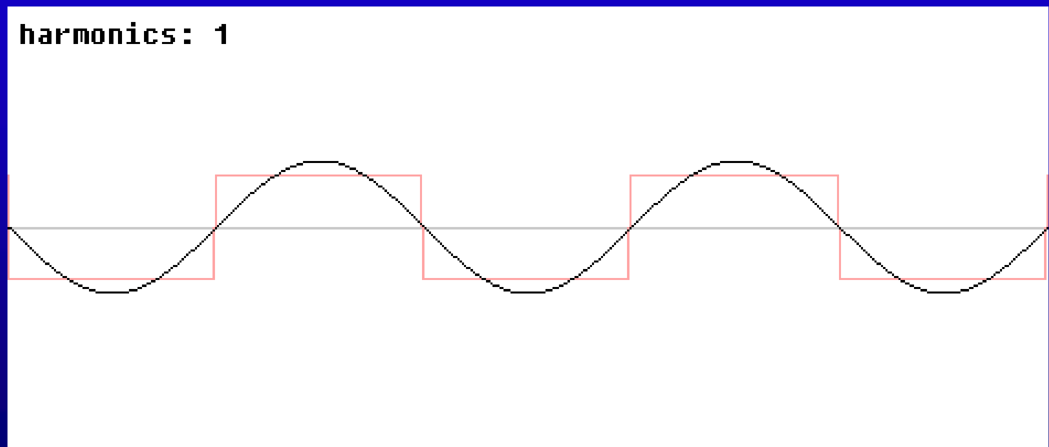


(c)

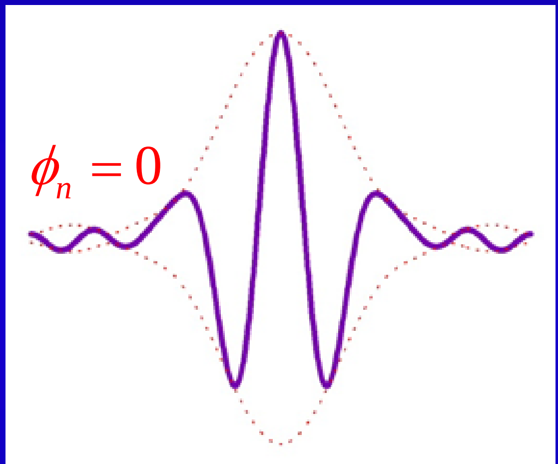

(d)


-3 π
-2 π
-1 π
1 π
2 π
3 π

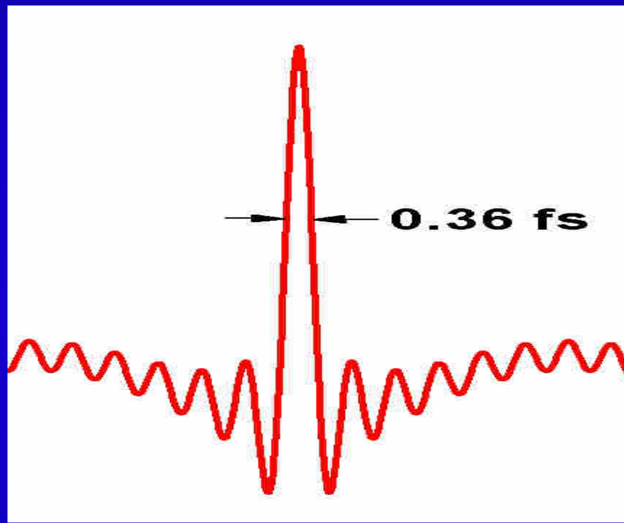

harmonics: 1


harmonics: 1

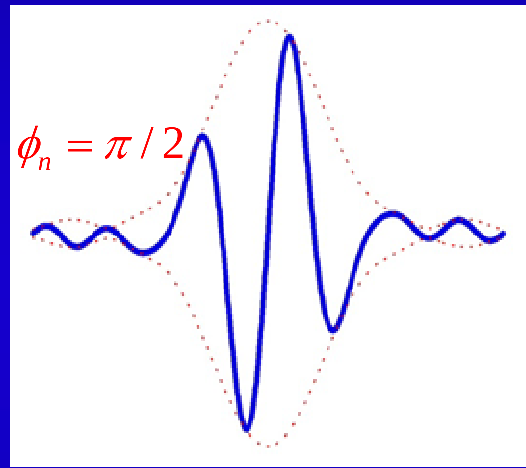
Single-cycle and sub-cycle waveforms



cosine pulse



half-cycle cosine pulse

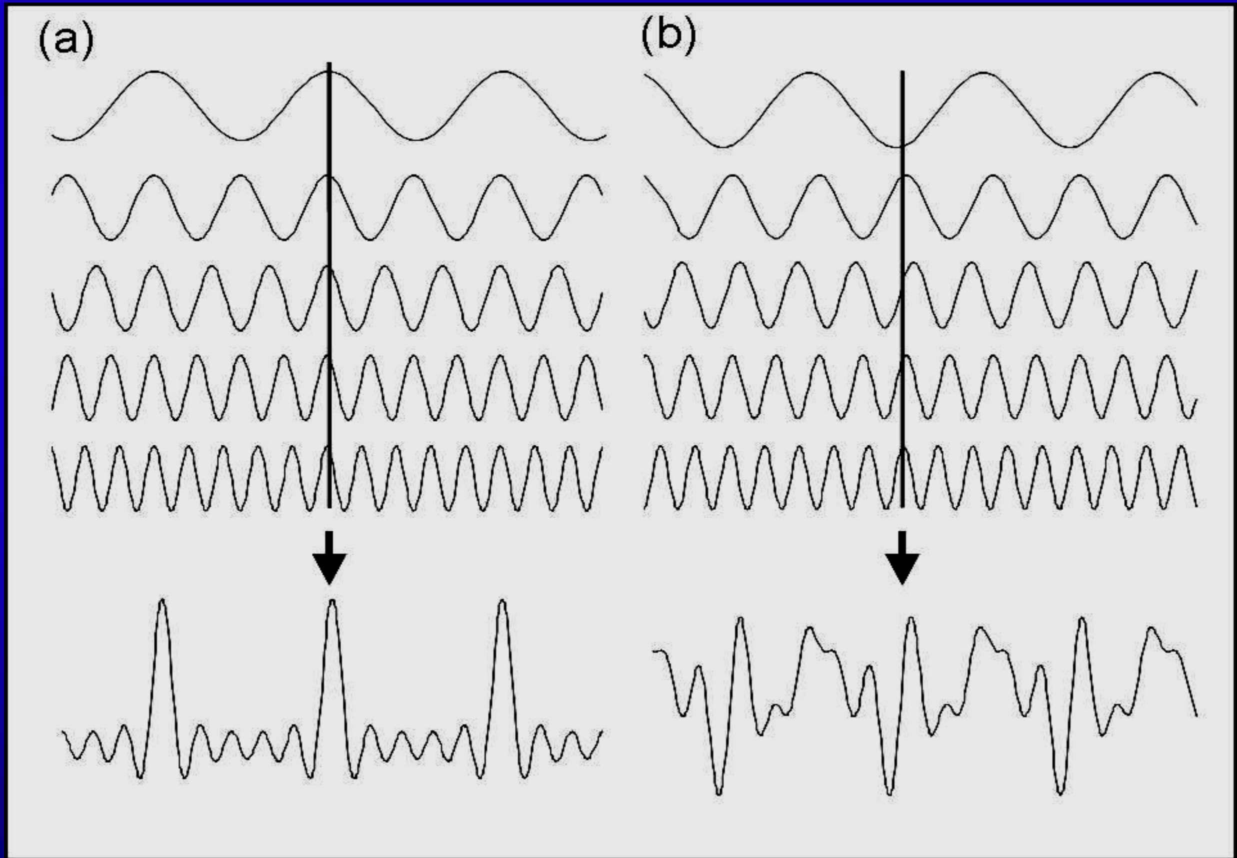


sine pulse

Phase coherence

$$E(t)=\sum_n E_n(t)=\sum_n A_n(t)\cos(\omega_n t+\phi_n)$$

(a) In phase $\phi_n = n\phi_o$ (b) Random phases



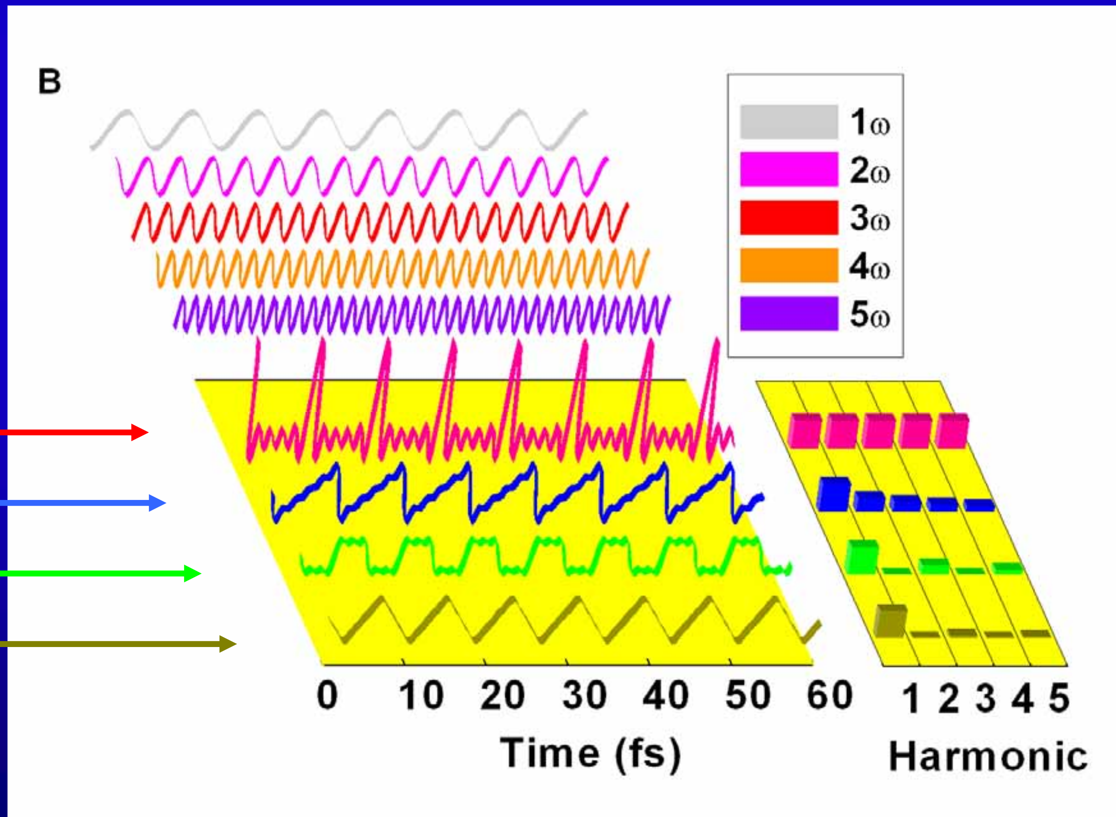
Our approach: Fourier Synthesis

Periodic waveforms are sums of sinusoidal waves

$$E(t) = \sum_n E_n(t) = \sum_n A_n(t)\cos(\omega_n t + \phi_n)$$

$\omega_n = n\,\omega_m$

single cycle cosine
saw tooth
square
triangle



Waveform	Formula	Normalized amplitude	CEP
Cosine	$\cos(n\omega t)$	1	NA
Sine	$\cos(n\omega t+\pi/2)$	1	NA
Single-cycle cosine	$\sum\cos(n\omega t)$, n=integer	{1, 1, 1, 1, 1, ...}	0
Single-cycle sine	$\sum\cos(n\omega t+\pi/2)$, n=integer	{1, 1, 1, 1, 1, ...}	$\pi/2$
Sawtooth	$(2/\pi)\sum\cos(n\omega t+\pi/2)/n$, n=integer	{1/1, 1/2, 1/3, 1/4, 1/5, ...}	$\pi/2$
Triangular	$(8/\pi^2)\sum\cos(n\omega t)/n^2$, n=odd	{1/1, 0, 1/9, 0, 1/25, ...}	0
Square	$(4/\pi)\sum(-1)^{(n-1)/2}\cos(n\omega t)/n$, n=odd	{1/1, 0, -1/3, 0, 1/5, ...}	0

Tasks

1. Generate a broad (prefer octave spanning) spectrum or comb of frequencies (with equal spacing)

2. Align the phases of the comb frequencies

3. Determine that the synthesized field is correct

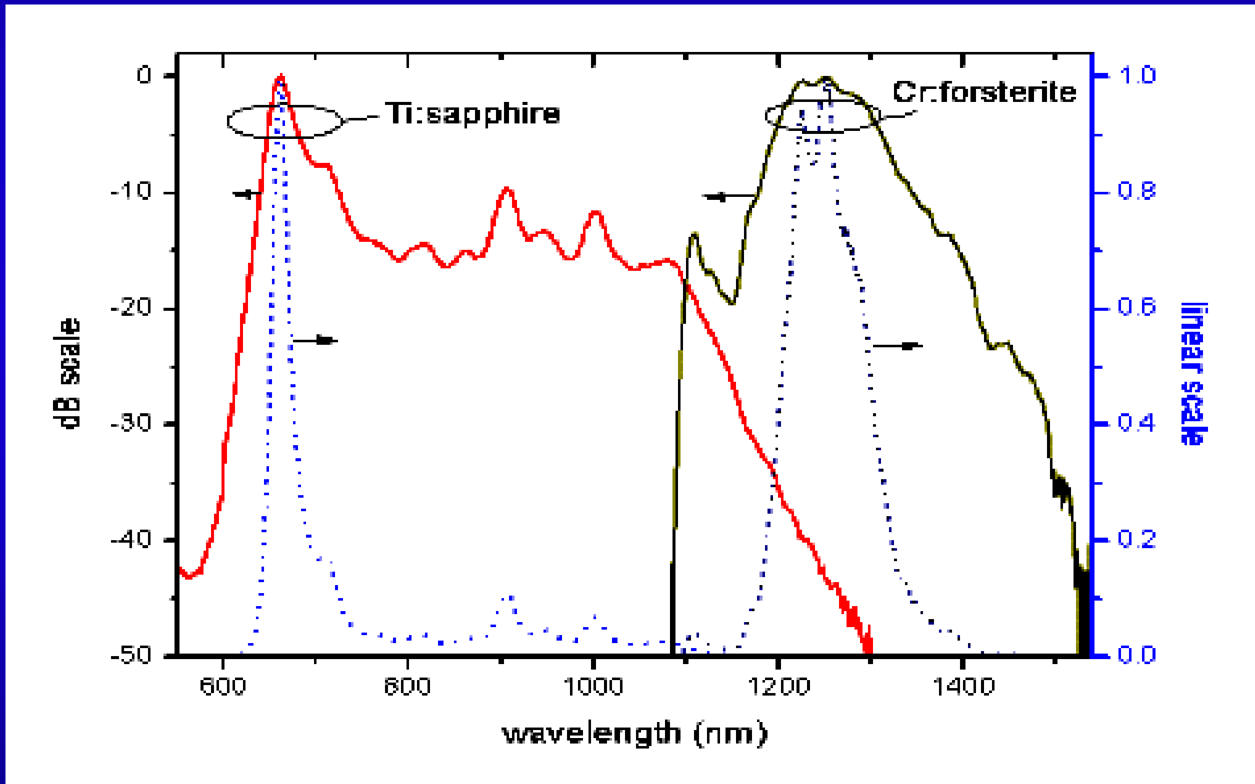
Challenge: these must be done in the optical domain

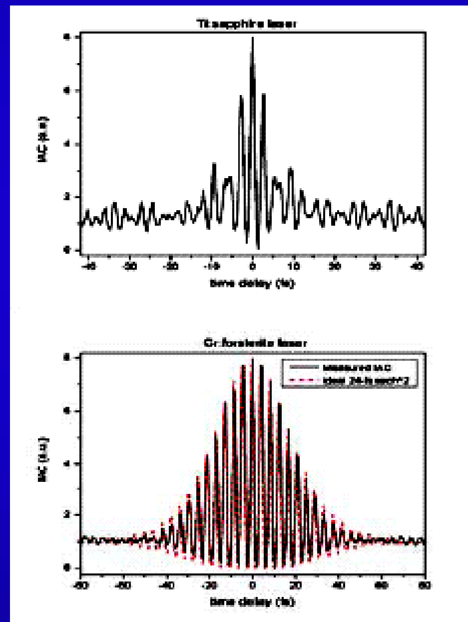
that is ~100 THz, ~ 1 femtosecond

Methods to produce optical waveforms

Synthesize optical pulses

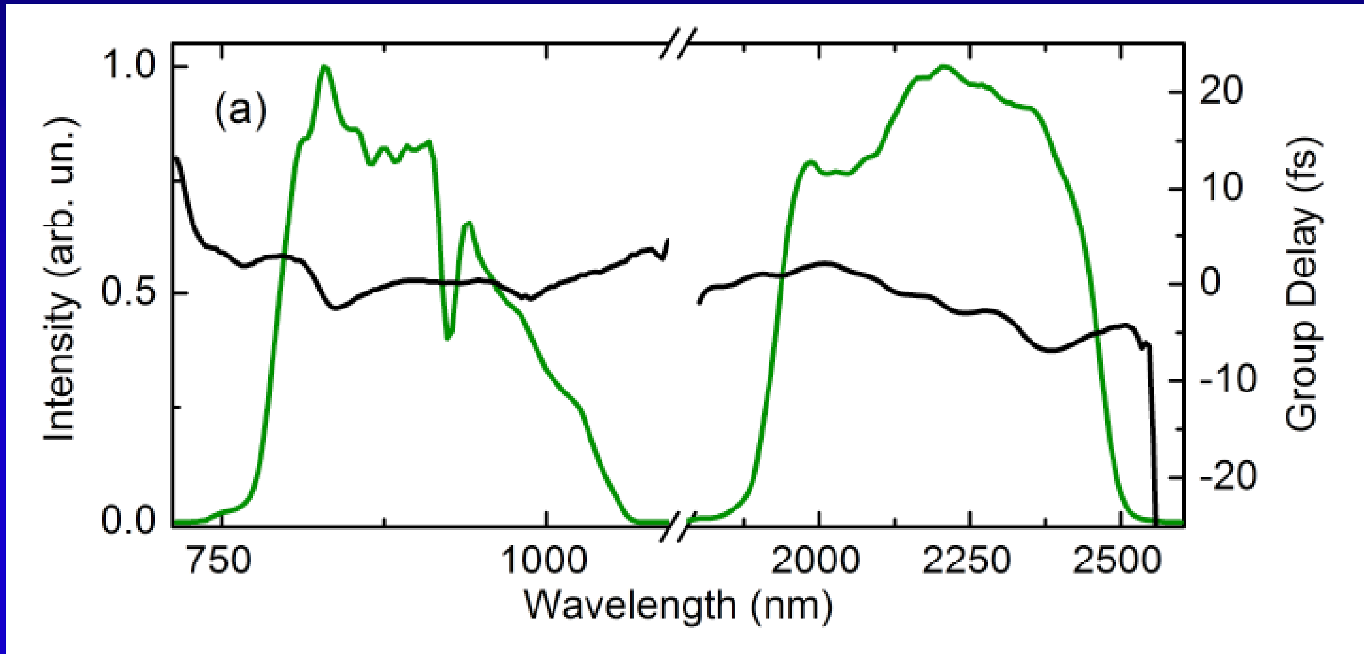
Isolated single cycle pulses -- extremely broadband lasers

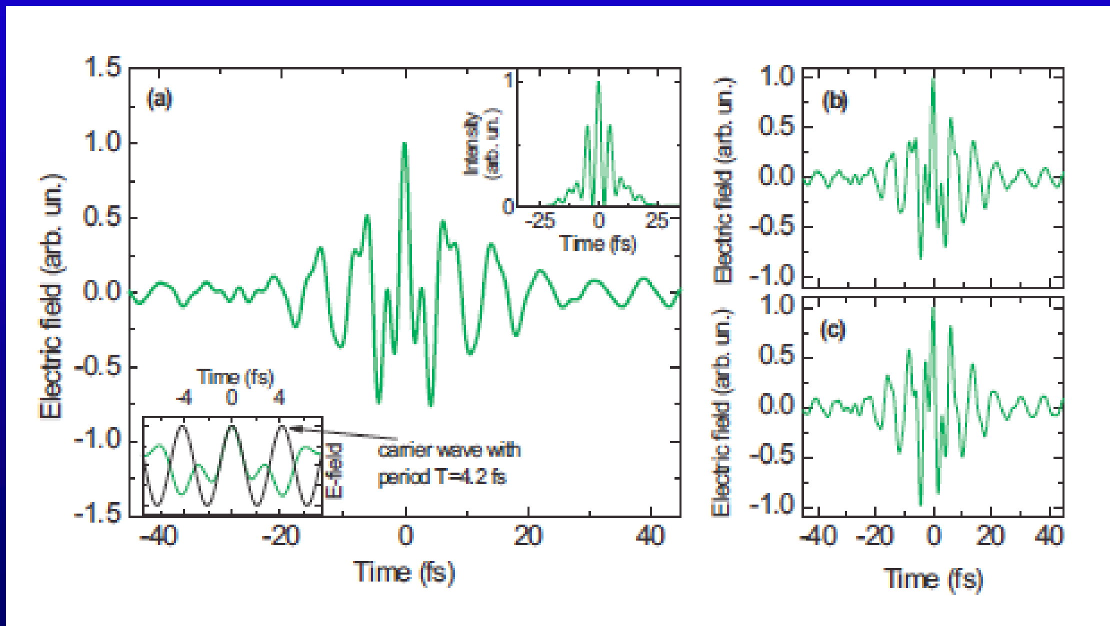


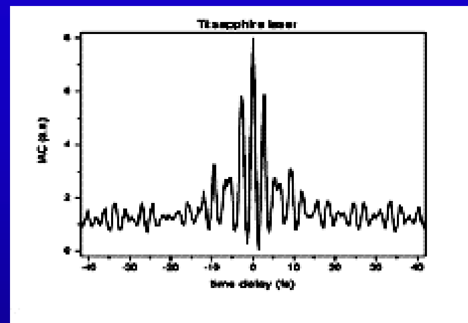


Franz Kartner, MIT

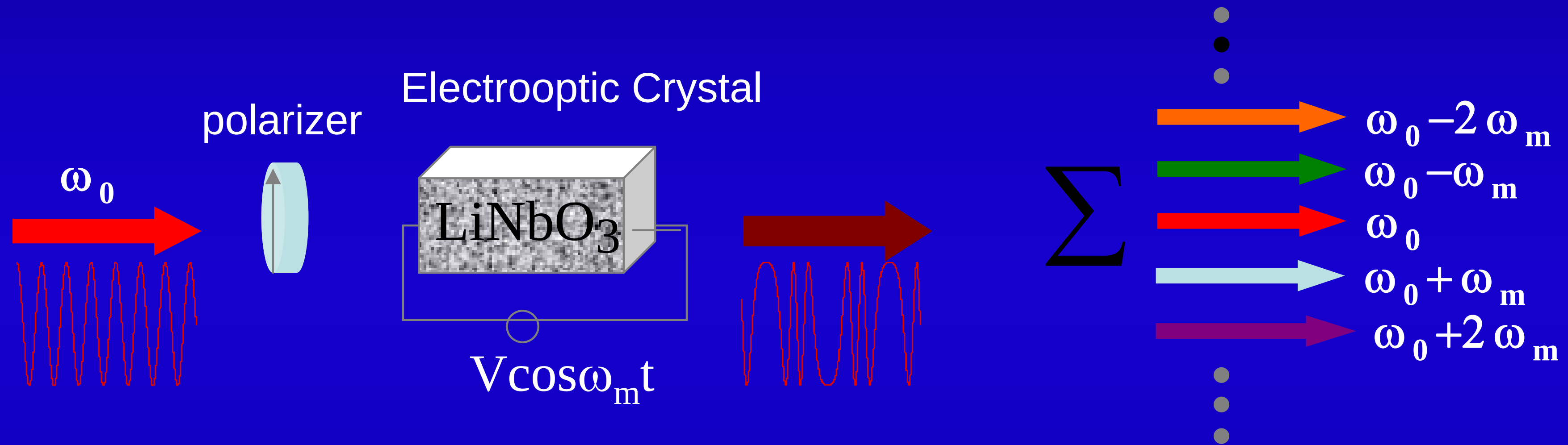
Huang et al., Nature Photonics, August, 2011



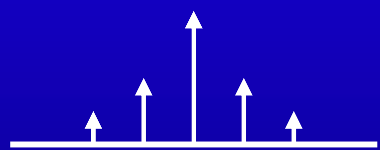




Frequency comb expansion by modulation



Refractive Index $n = n_0 + \delta \cos \omega_m t$

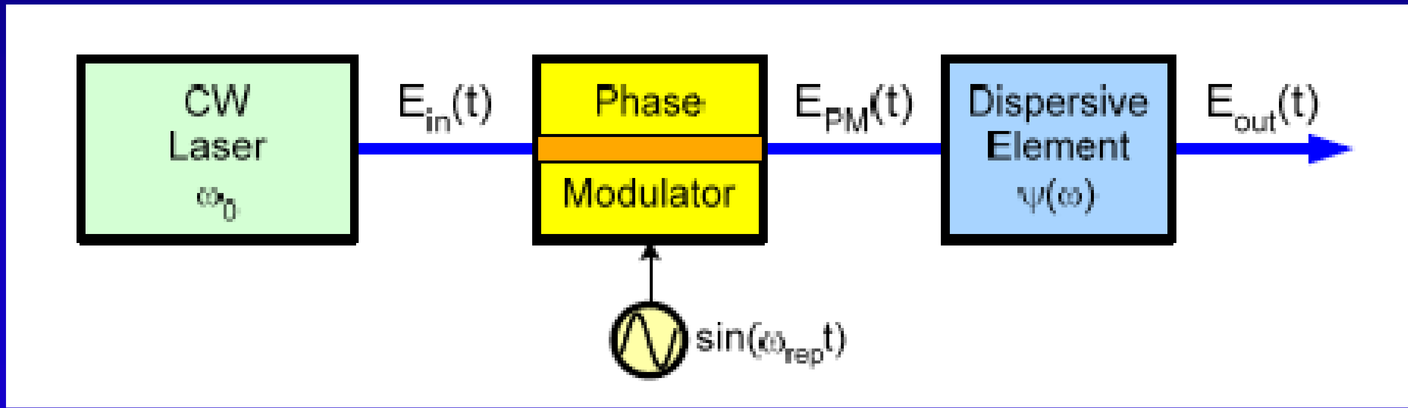


$$\omega_q = \omega_0 + q\omega_m$$

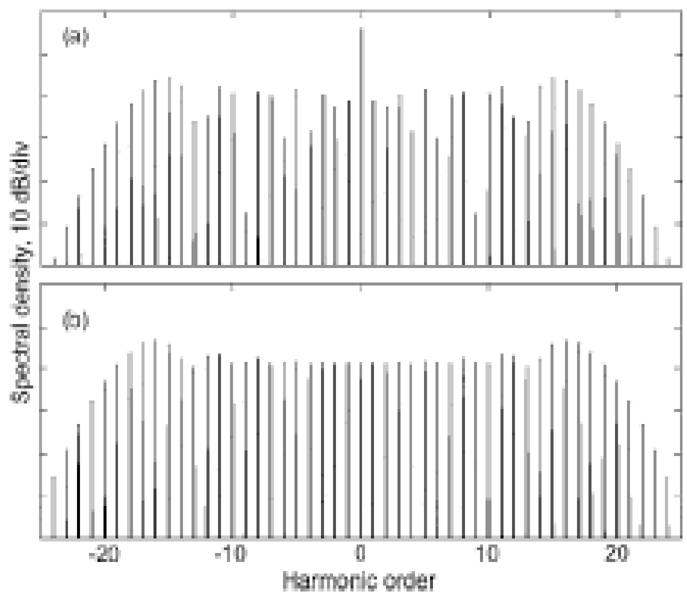
q^{th} sideband

modulation frequency

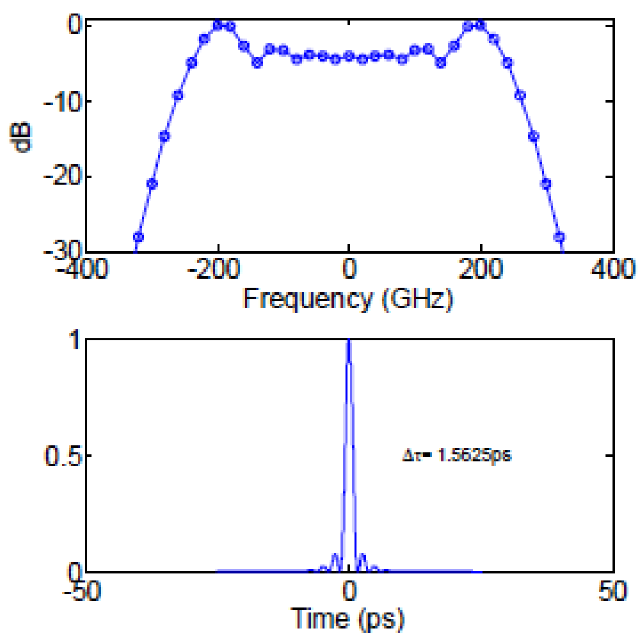
CW frequency comb produced by electro-optic modulation



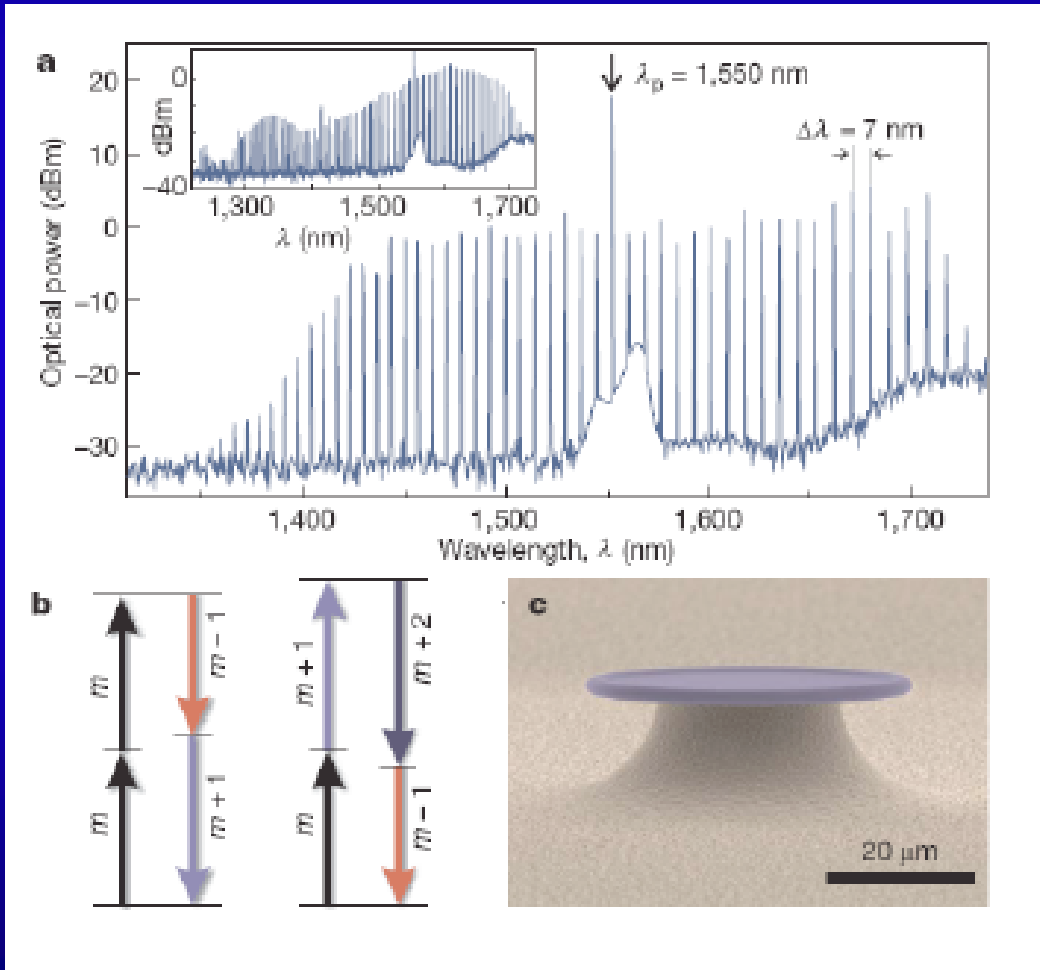
Mode spacing
Tens of GHz



T. Sakamoto et.al, Opt. Lett. **32**, 1515 (2007).



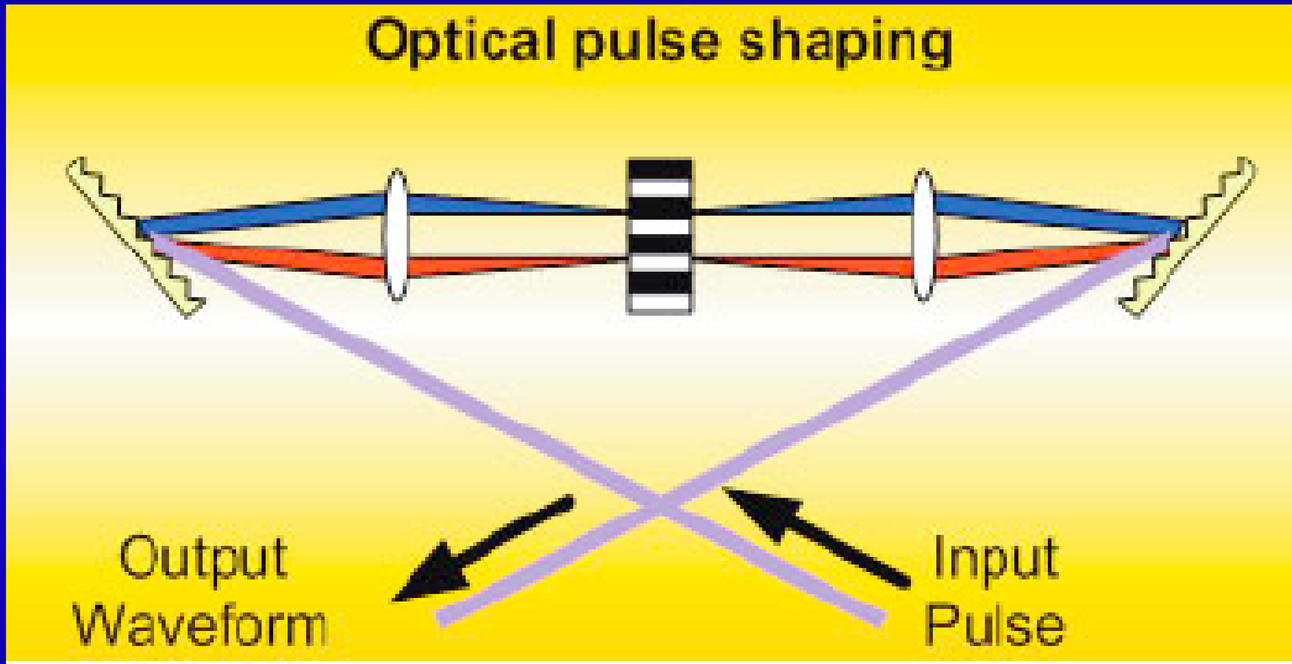
CW comb produced by four wave Kerr mixing in a micro-toroidal ring



Mode spacing
~1 THz

P. Del'Haye et al.,Nature, 2007

Waveform synthesis by line-by-line pulse shaping

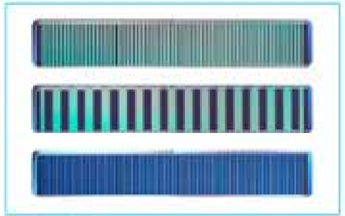


Andrew Weiner

Excellence through light.

JENOPTIK
JENA
Laser.Optik.
Systeme.

Spatial Light Modulator SLM-S640d.



Examples for patterns.

SLM-S 640d with Reflexive Mirror.

The Liquid Crystal Modulator SLM-S640d is an excellent tool for modulation of femto second laser pulses in wave length ranges from 430 to 1600 nm.

The simultaneous modulation of phase and amplitude independent from each other is achieved by the alignment of two connected liquid crystal modulators in a 4f-arrangement or in a cpa-system (chirped pulse amplification).
Any of the 1280 strips of the single masks can actively be controlled with 12 bit resolution.

The large active area of 64 x 10 mm offers the use of an increased laser pulse output power.
An extensive LabView™ instruction set as well as several demo-programmes for calibration and control of the modulators facilitate the effective work.

Using the SLM-S640d the user is offered the possibility to optionally mount a mirror.
Thus the same modulator can be operated both in transmissive and in reflexive configurations.

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光電工程研究所

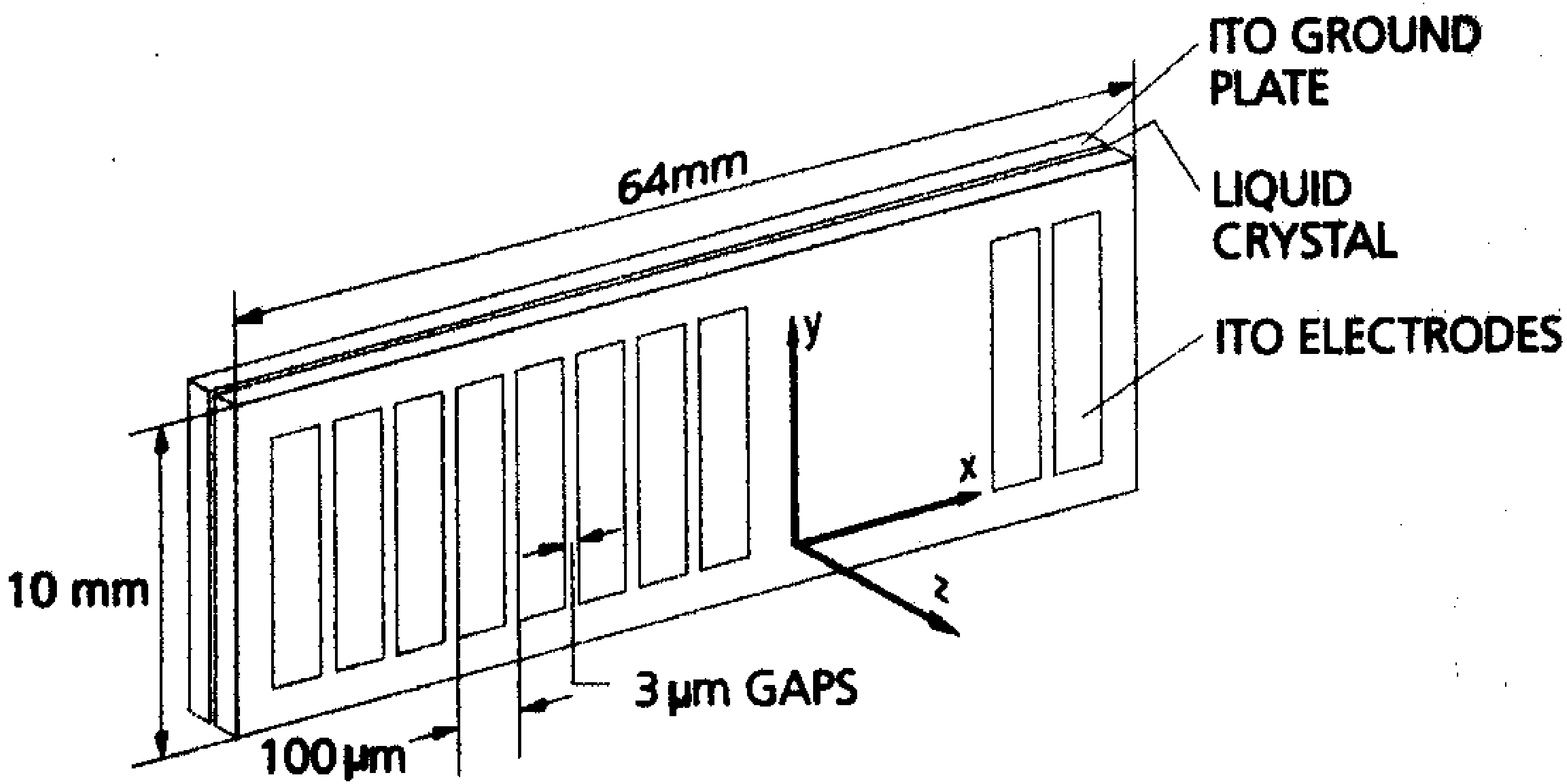


Fig.3.2 Layout of the optical active area

Shaped envelopes using CW frequency combs

Laser & Photon. Rev. **2**, No. 4 (2008)

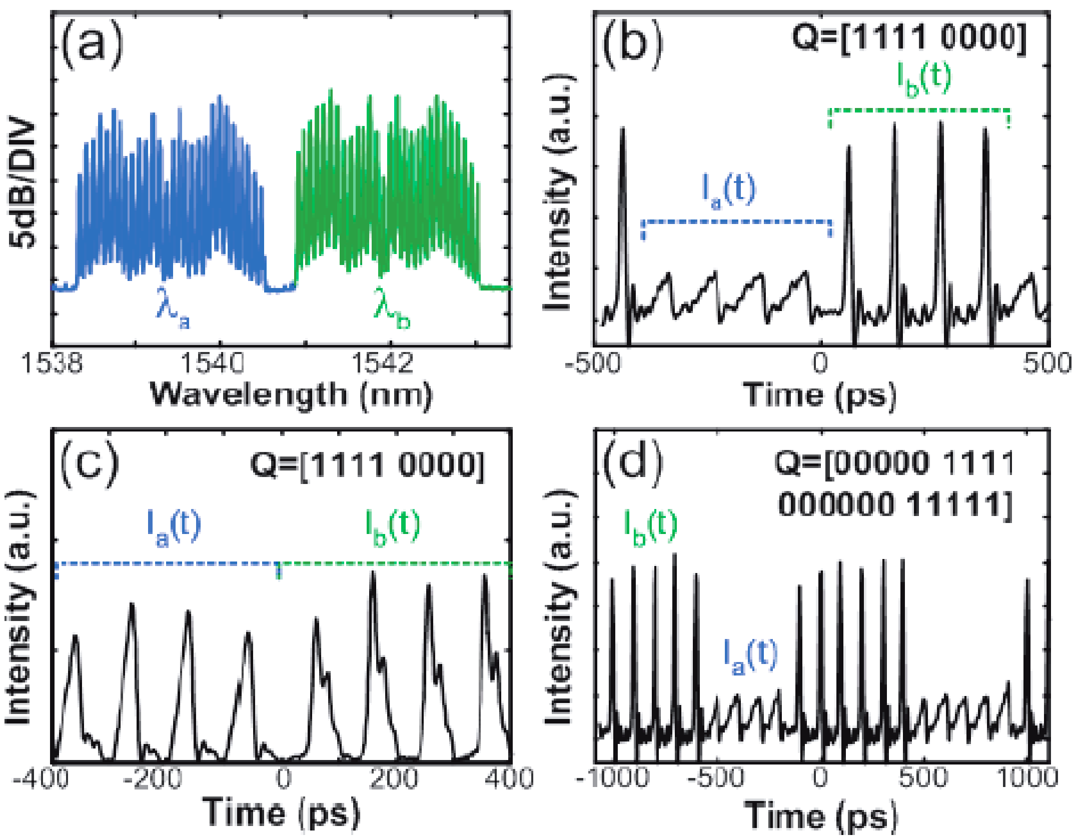
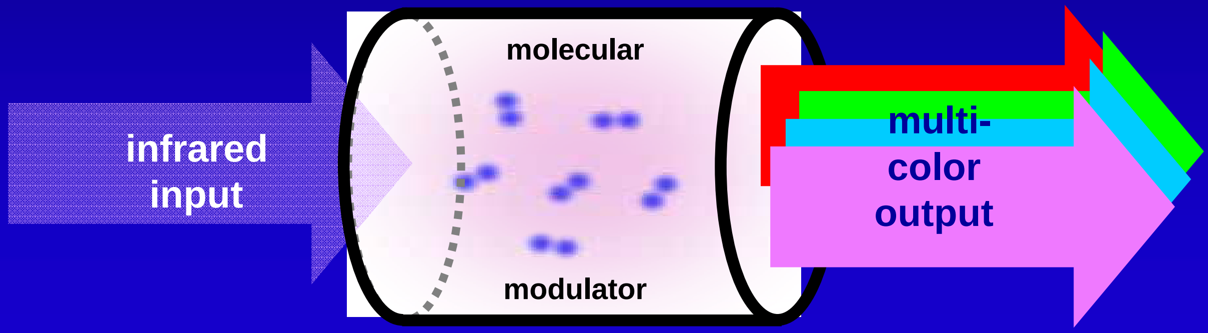


Figure 25 (online color at: www.lpr-journal.org) (a) 28-line

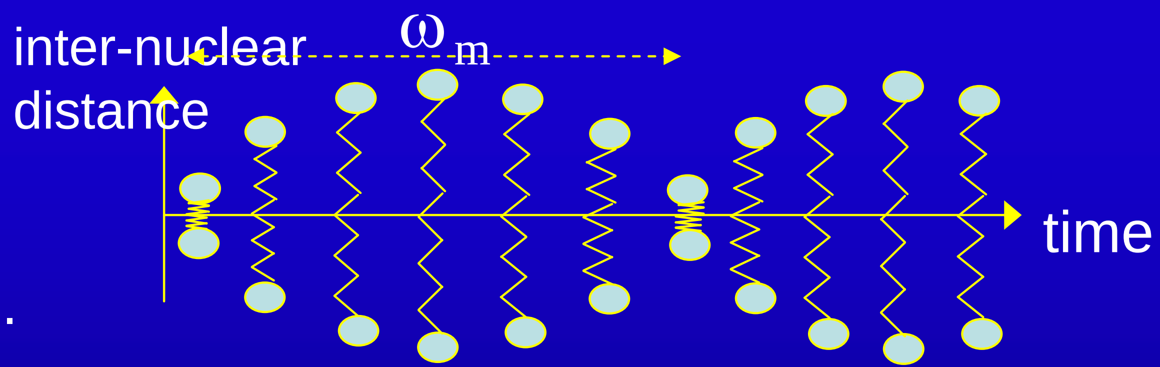
Bandwidth expansion by molecular modulation



Refractive Index

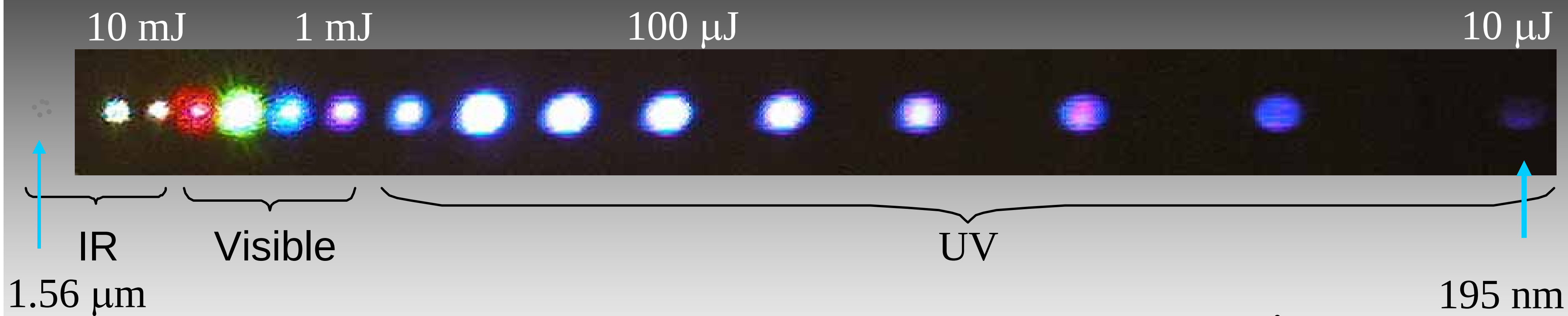
$$n = n_0 + \delta \cos \omega_m t$$

$\omega_q = \omega_0 + q\omega_m$ q= -2,-1,0,1,2,3….



S. E. Harris and A. V. Sokolov, Phys. Rev. A **55**, R4019 (1997);
S. E. Harris and A. V. Sokolov, Phys. Rev. Lett. **81**, 2894 (1998).

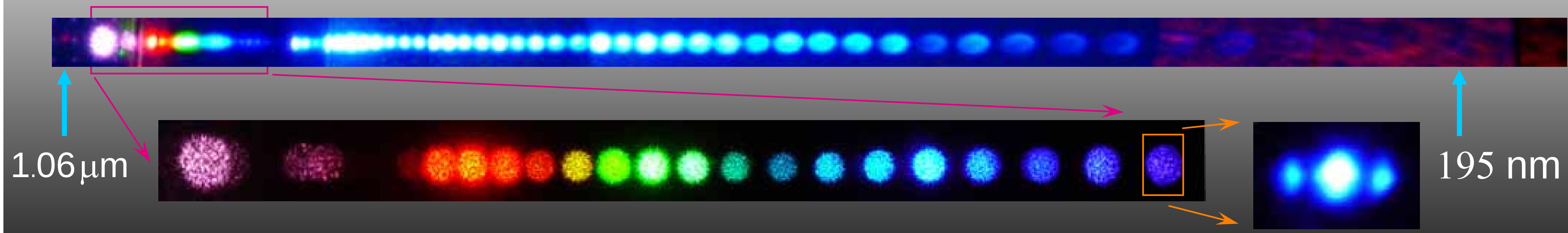
D_2 Vibration Spectra: 16 sidebands, spaced by 2994 cm^{-1}



Harris group:
Phys. Rev. A (R)(1997)
Phys. Rev. Lett. 81 (1998)
Opt. Lett. 24 (1999)
Phys. Rev. Lett. 84 (2000)
Phys. Rev. Lett. 85 (2000)
Phys. Rev. A 63 (2001)
Phys. Rev. Lett. 91 (2003)
Phys. Rev. Lett. 93 (2005)

H_2 Rotation Spectra: 29 sidebands, spaced by 587 cm^{-1}

Multiplicative Spectra: ~ 200 sidebands, spaced by < 587 cm^{-1}



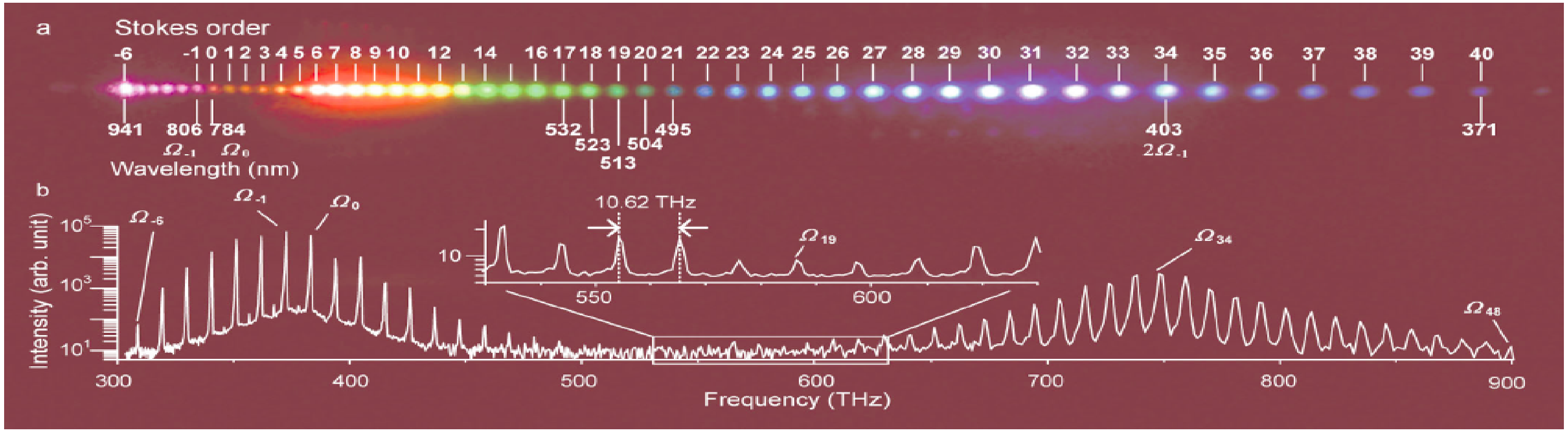
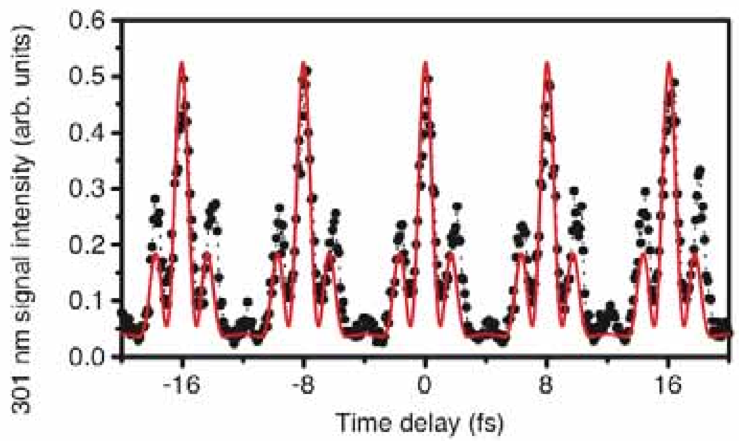


FIG. 2 (color). Spectra for octave-spanning Raman comb produced with three frequency laser radiations. (a) The photograph taken with a digital camera after dispersing the emission with a prism. (b) The spectrum taken with an OMA.

243602-3

Katsuragawa, PRL 101, 243602 (2008)



Commensurate comb

Constant CEP

W.-J.Chen et al. PRL 100, 163906 (2008)

Use the first two terms of the Fourier series

$n = 1, 2$ Then, $\phi_m = \phi_2 - \phi_1$

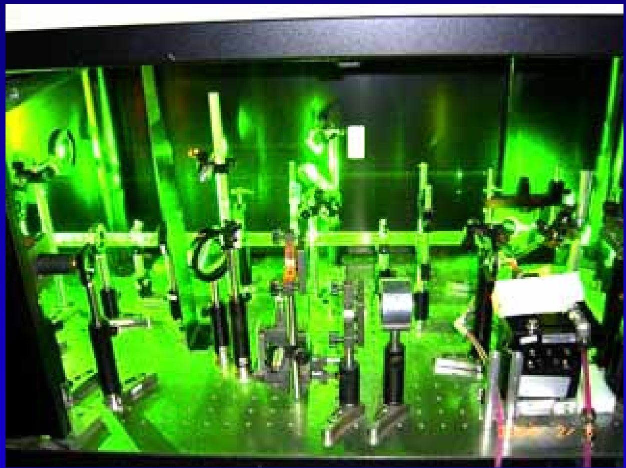
$$\phi_0 = 2\phi_1 - \phi_2$$

since $\omega_2 = 2\omega_1$ then, $\phi_2 = 2\phi_1 + \xi$

$$\begin{aligned}\phi_0 &= 2\phi_1 - \phi_2 \\ &= 2\phi_1 - (2\phi_1 + \xi) \\ &= \xi\end{aligned}$$

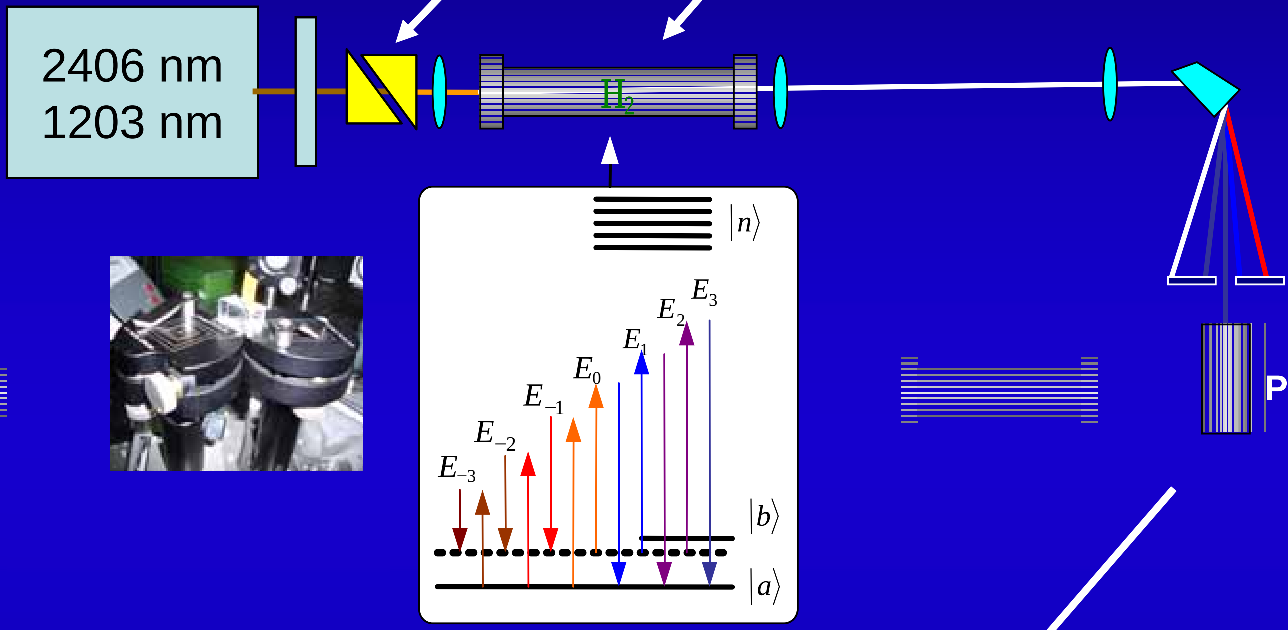
a constant we can choose and control

ω_1 is doubly significant. The period of the waveform is determined by the fundamental frequency and its shape is dominated by the first few terms of the Fourier series.



IR Laser at 2406 nm
Pulse duration ~ 2-5 ns
Peak power ~2-5 MW

Instantaneous electric field
10 MV/cm

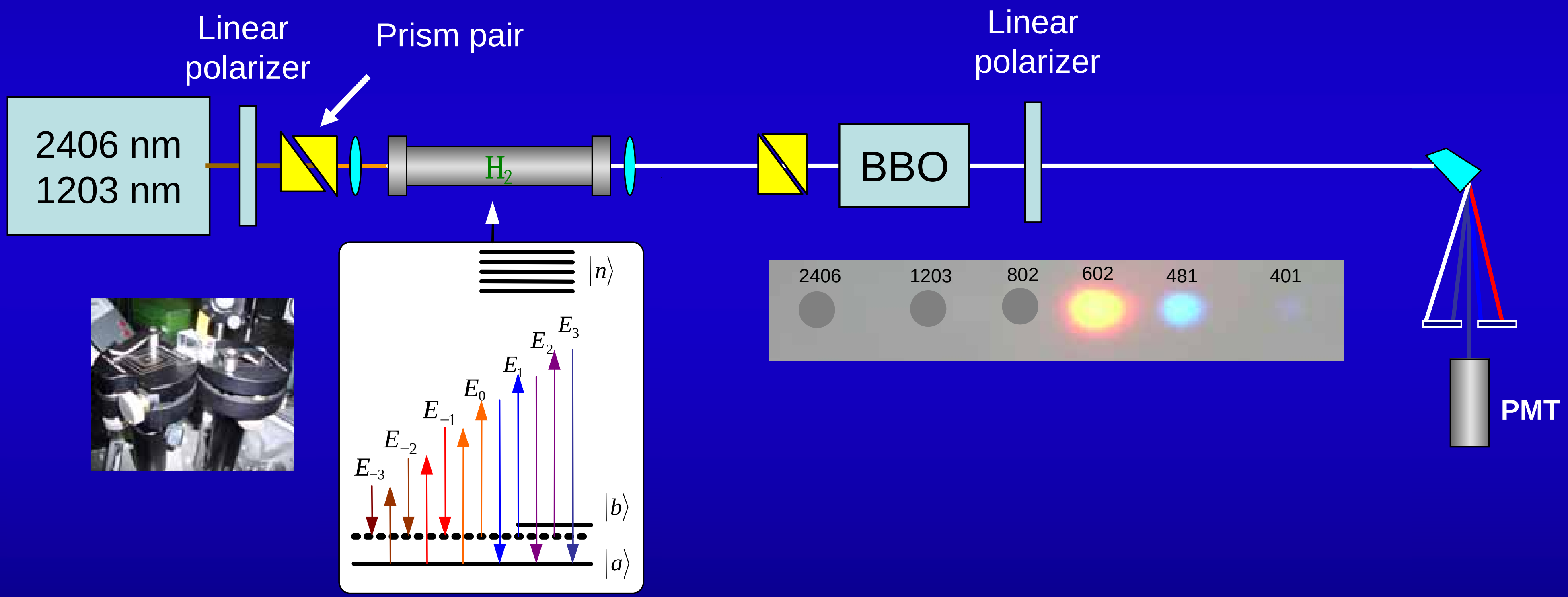


Raman sideband generation

Total spectral span > 4 octaves (70,000 cm^{-1})
(~500 as)

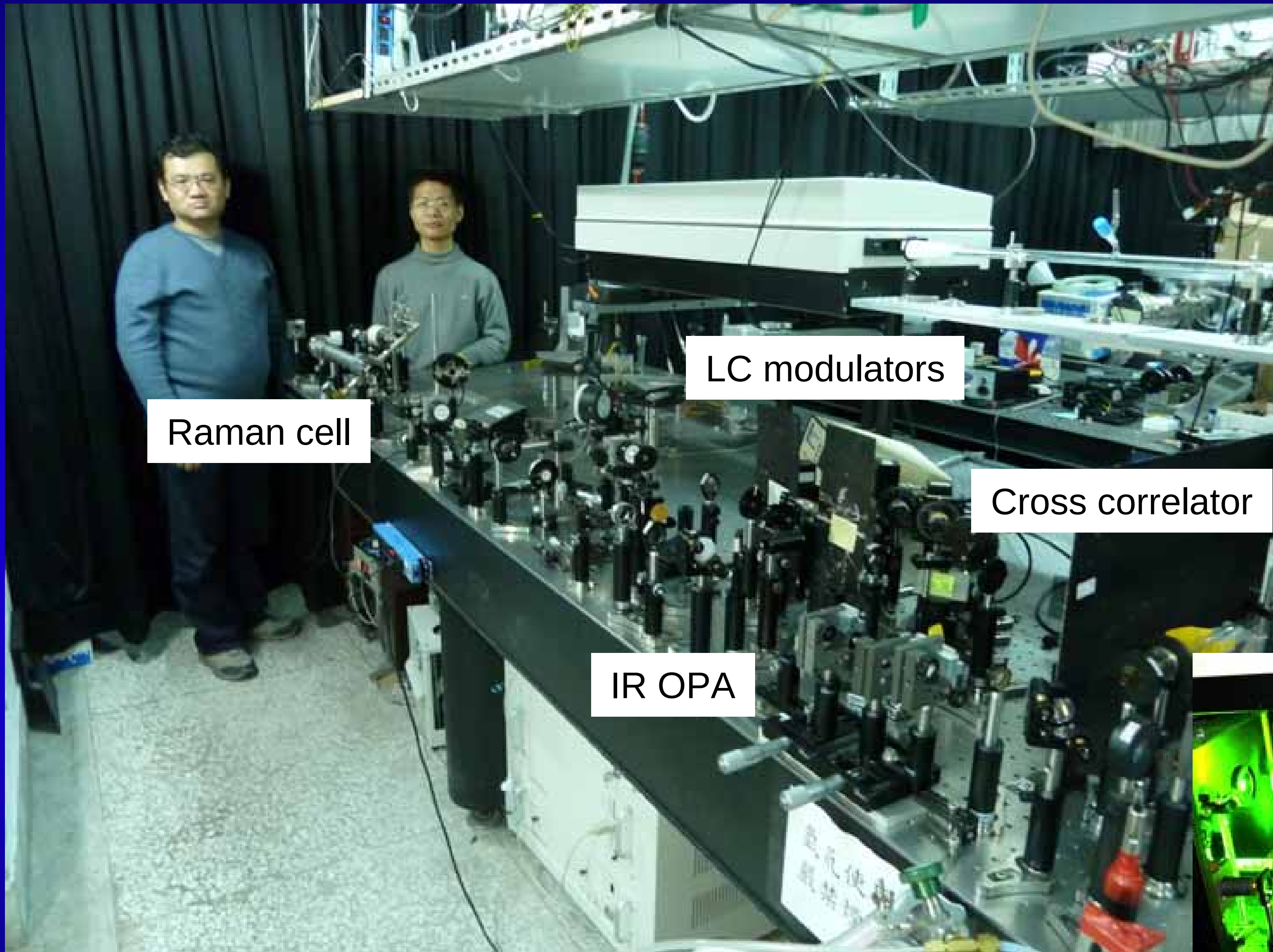
λ_1= 2406 nm = Raman resonance - $\delta\lambda$

λ_2= second harmonic = 1203 nm



Raman sideband generation

Optical waveform synthesizer set up

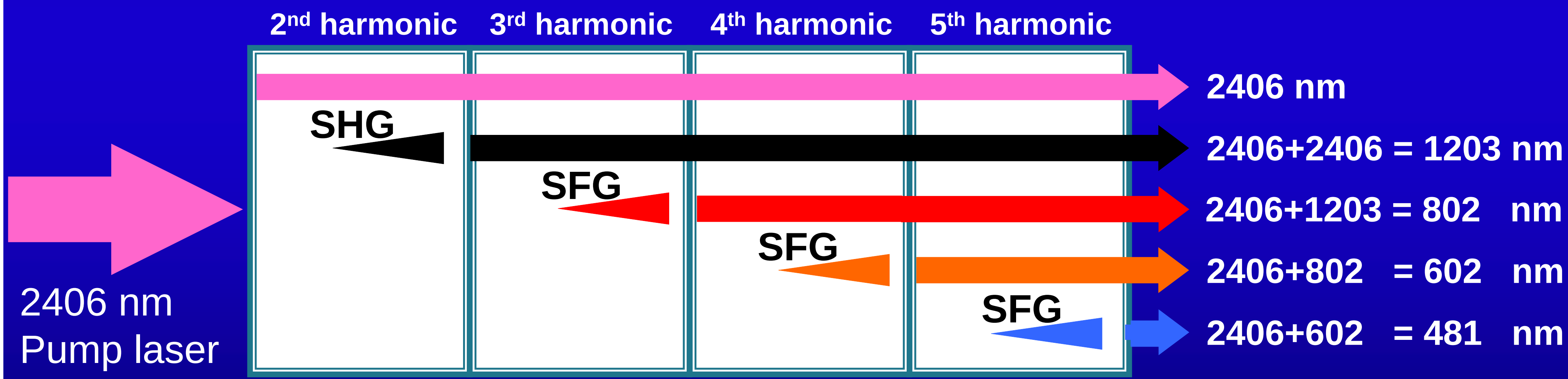


Harmonic frequency combs contain the fundamental frequency component that sets the periodicity and have multi-octave spectral width with a mode spacing of tens to hundreds of THz. These are ideal conditions for arbitrary optical waveform synthesis.

Multiple harmonic generation in a monolithic crystal

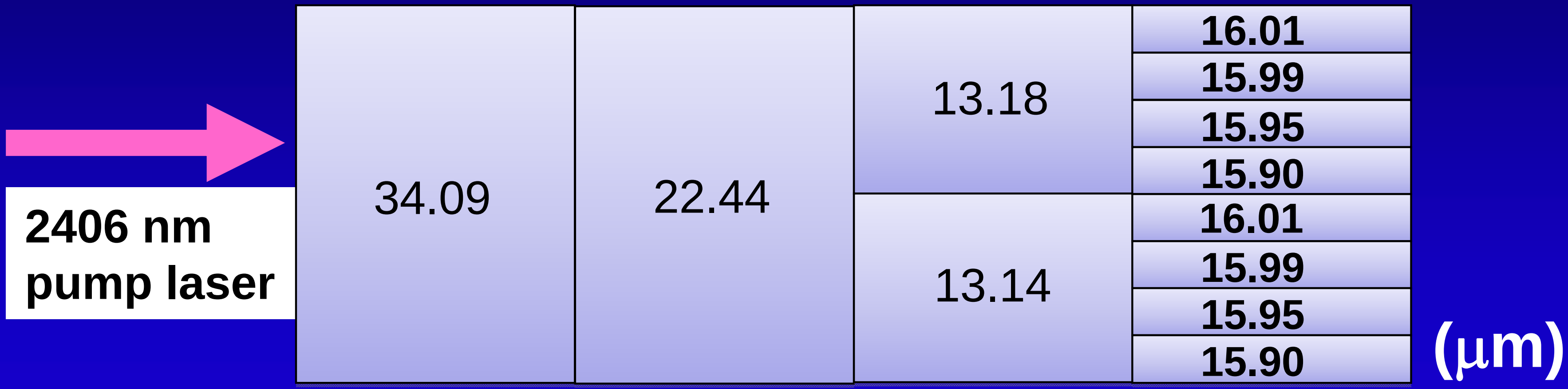
$$\omega_n = \omega_0 + n\omega_m$$

$$\phi_n = \phi_0 + n\phi_m$$



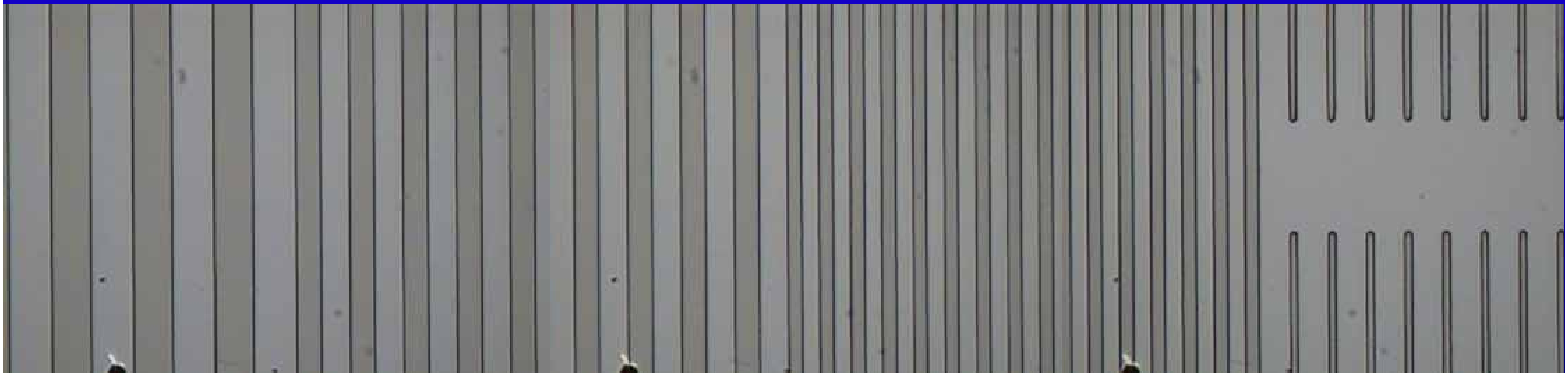
Optics Letters Nov 2009

The poled domain periods in the crystal

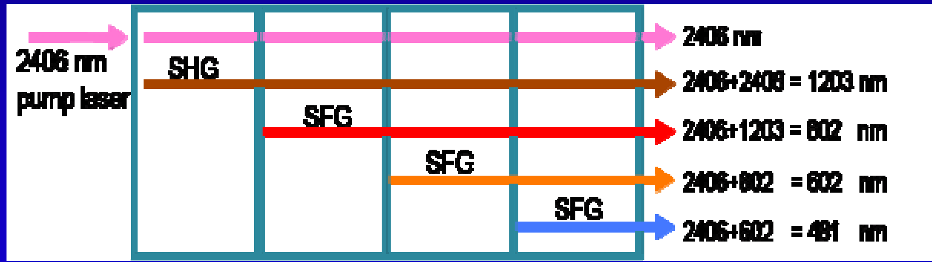


Cascaded QPM has a tight (10 nm) tolerance on domain widths

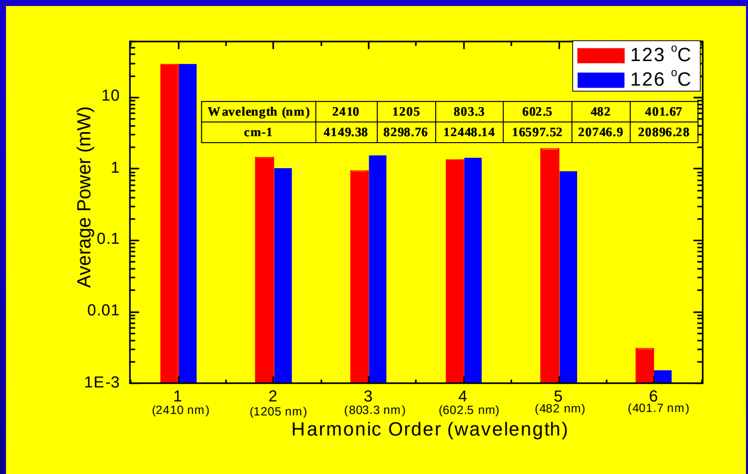
Photolithography



Multiple harmonics $\omega_n = n\omega_1$ are also generated in a QPM Xtal:



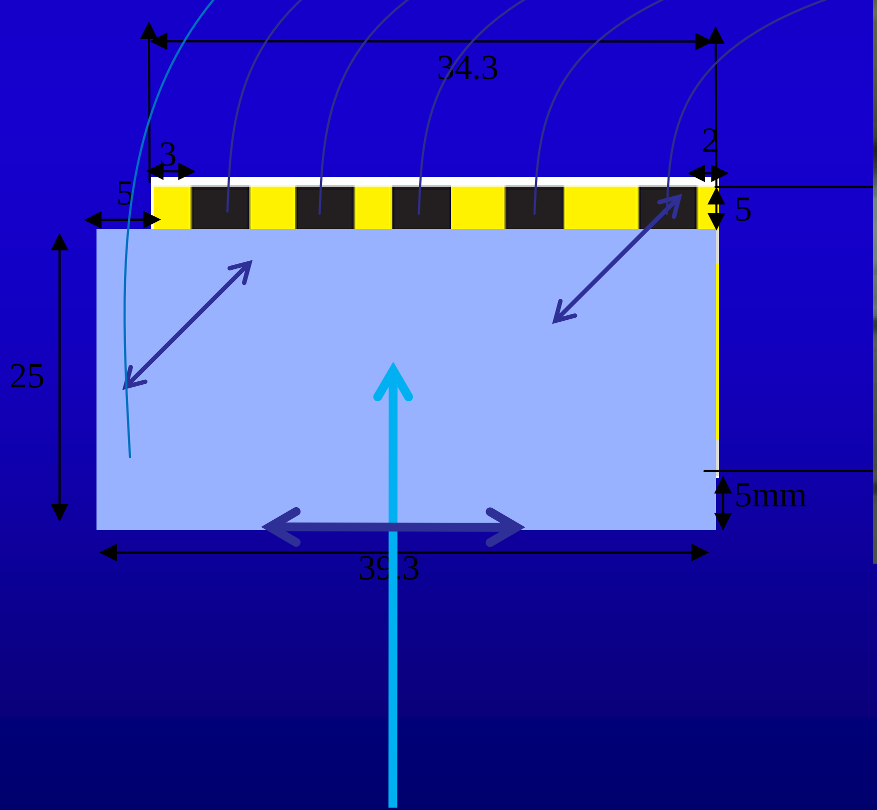
Cascaded QPM processes for multi-harmonics generation

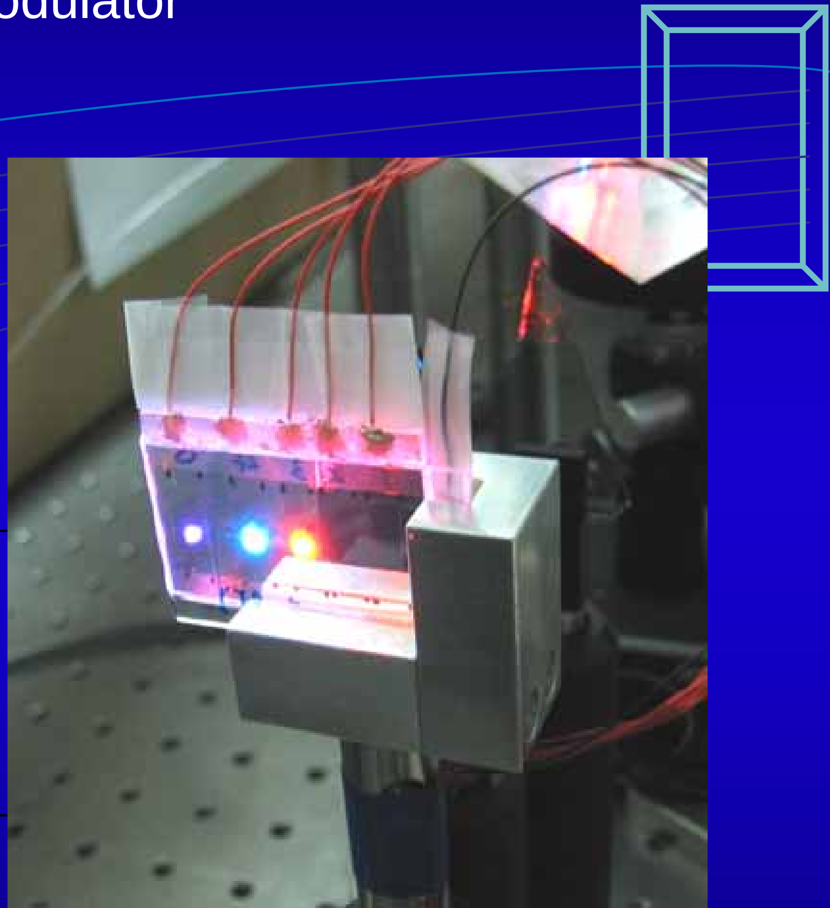


Up to the 7th harmonic has been observed

20% of the fundamental power is converted to the harmonics

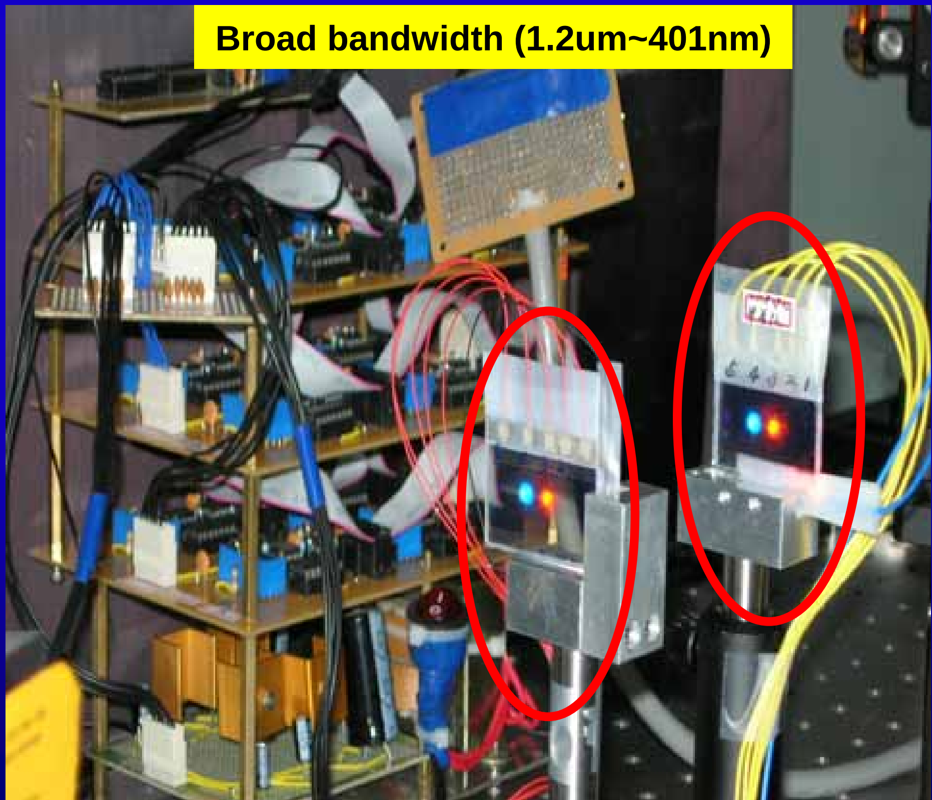
Liquid crystal spatial light modulator



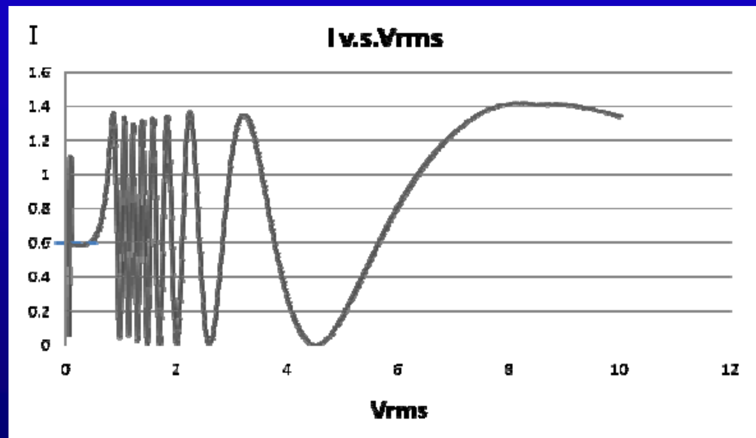


The essential component: spatial liquid crystal modulators

For this idea to work we must have modulators that are broadband and can be configured to modulate either the phase or the amplitude of the input beam. We custom-make these modulators to match the size and spacing of our beam components. The LC material is E7 from Merck.



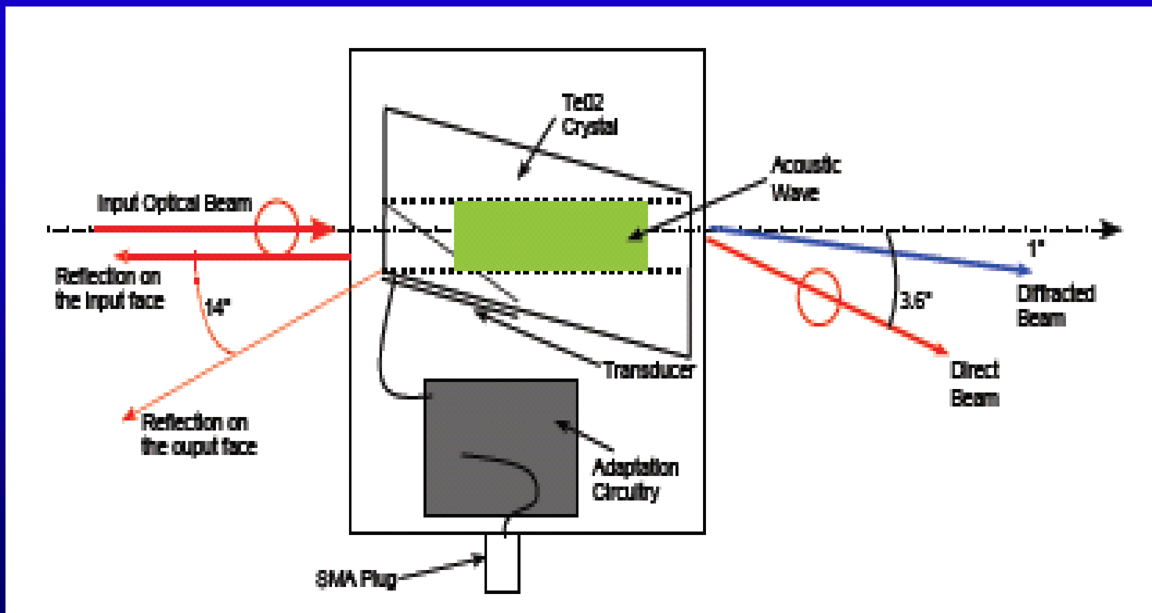
Now, our home-made liquid crystal modulator can cover wavelength from 2.4um to 401nm. Modulator for wavelength short than 344nm is now under testing.



quasi-longitudinal vs transverse control

ACOUSTO OPTIC PROGRAMMABLE DISPERSIVE FILTER (AOPDF)

Acousto-optic amplitude and phase modulation



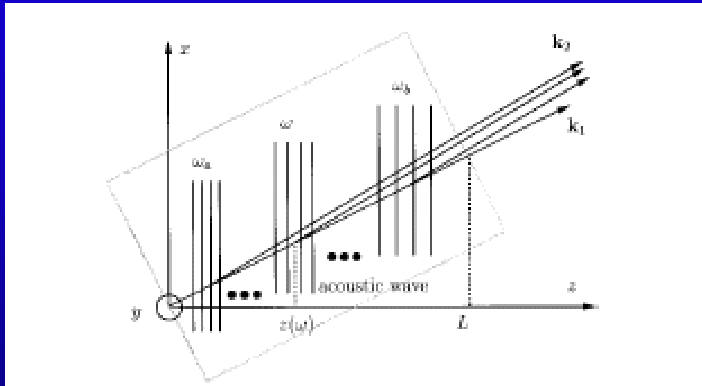
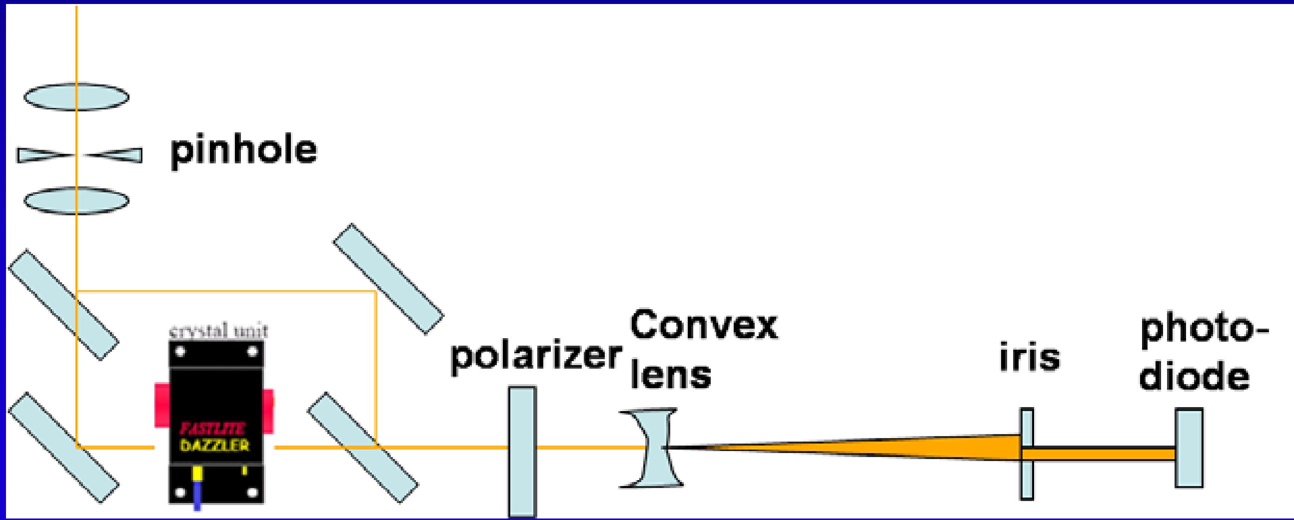
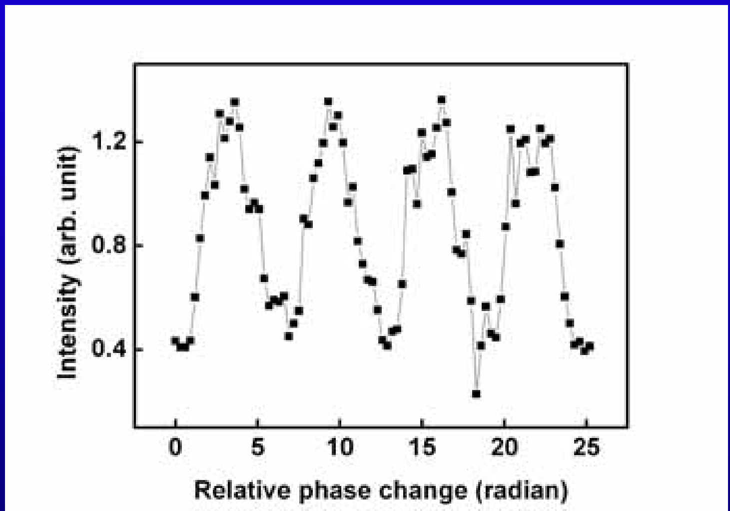
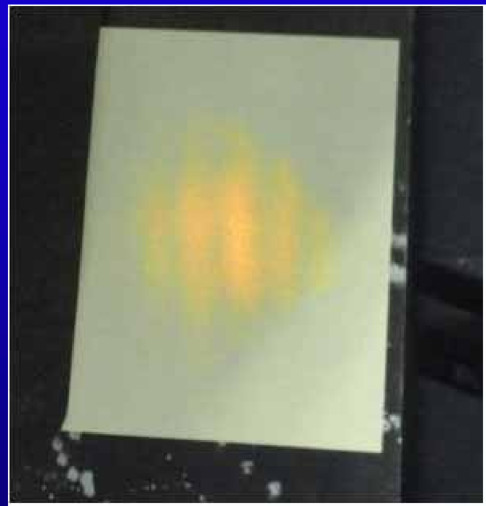


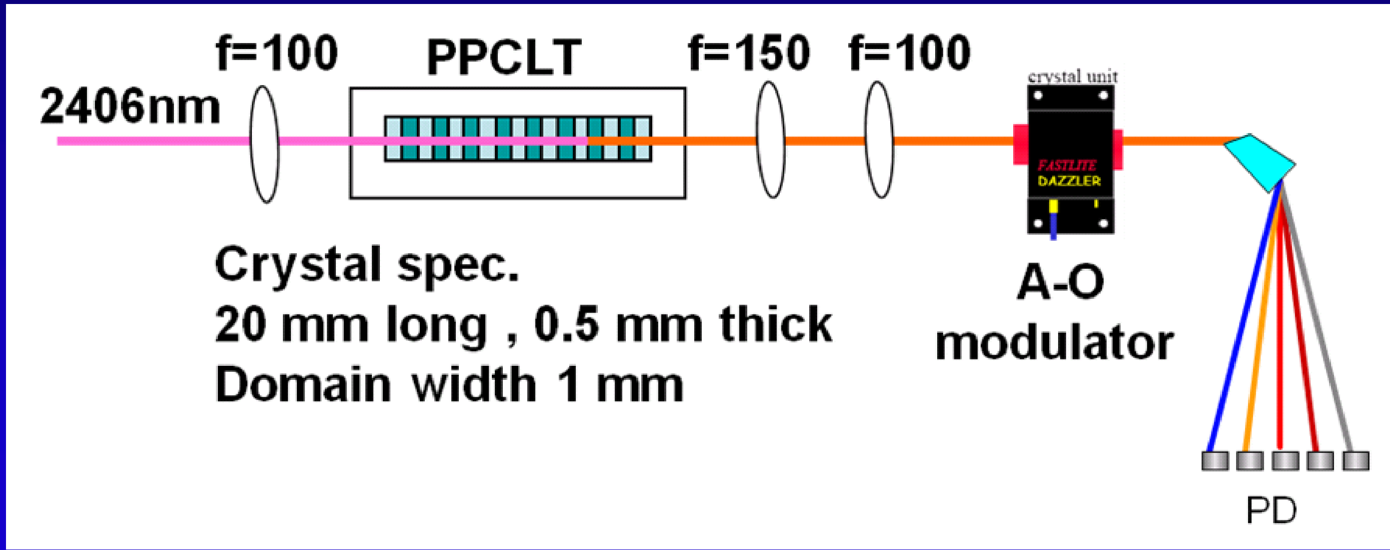
Fig. 5. Representation of the quasi-collinear acousto-optic interaction geometry.

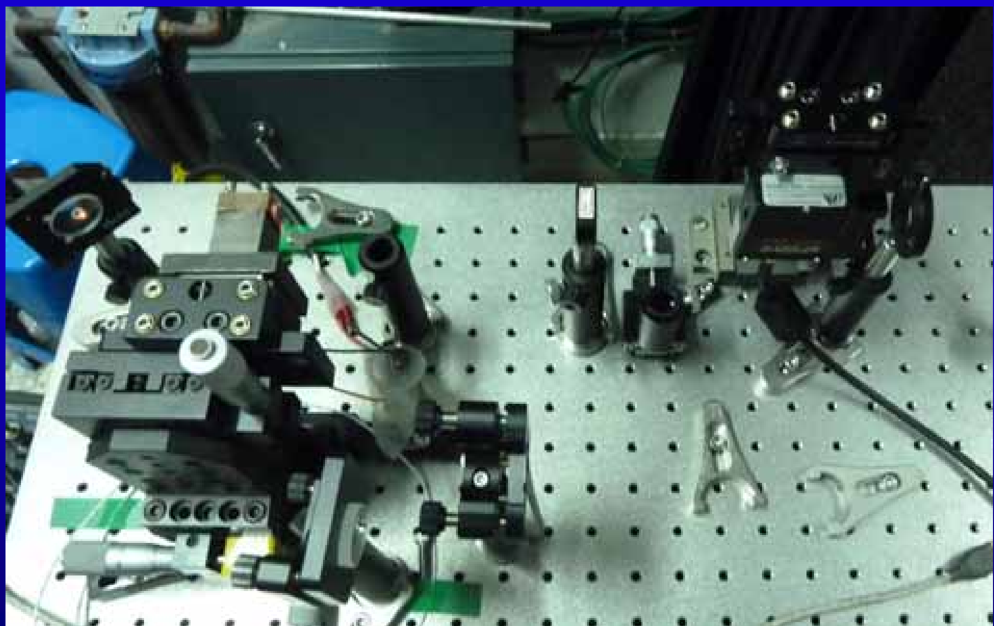
Mach-Zehnder interferometer to test and adjust phases



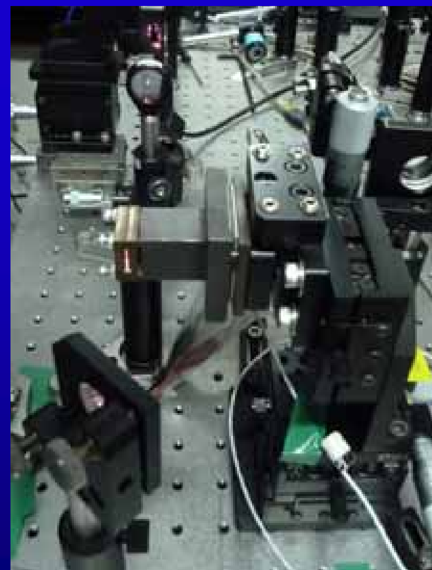








Breadboard set up



UB-AOM

Comparison of ways of generating a frequency comb for waveform synthesis

1. Broad continuous laser spectrum

   isolated pulses -- important for pump-probe experiments
   poor shaping resolution -- cannot get arbitrary shapes easily

2. Electro-optic modulation

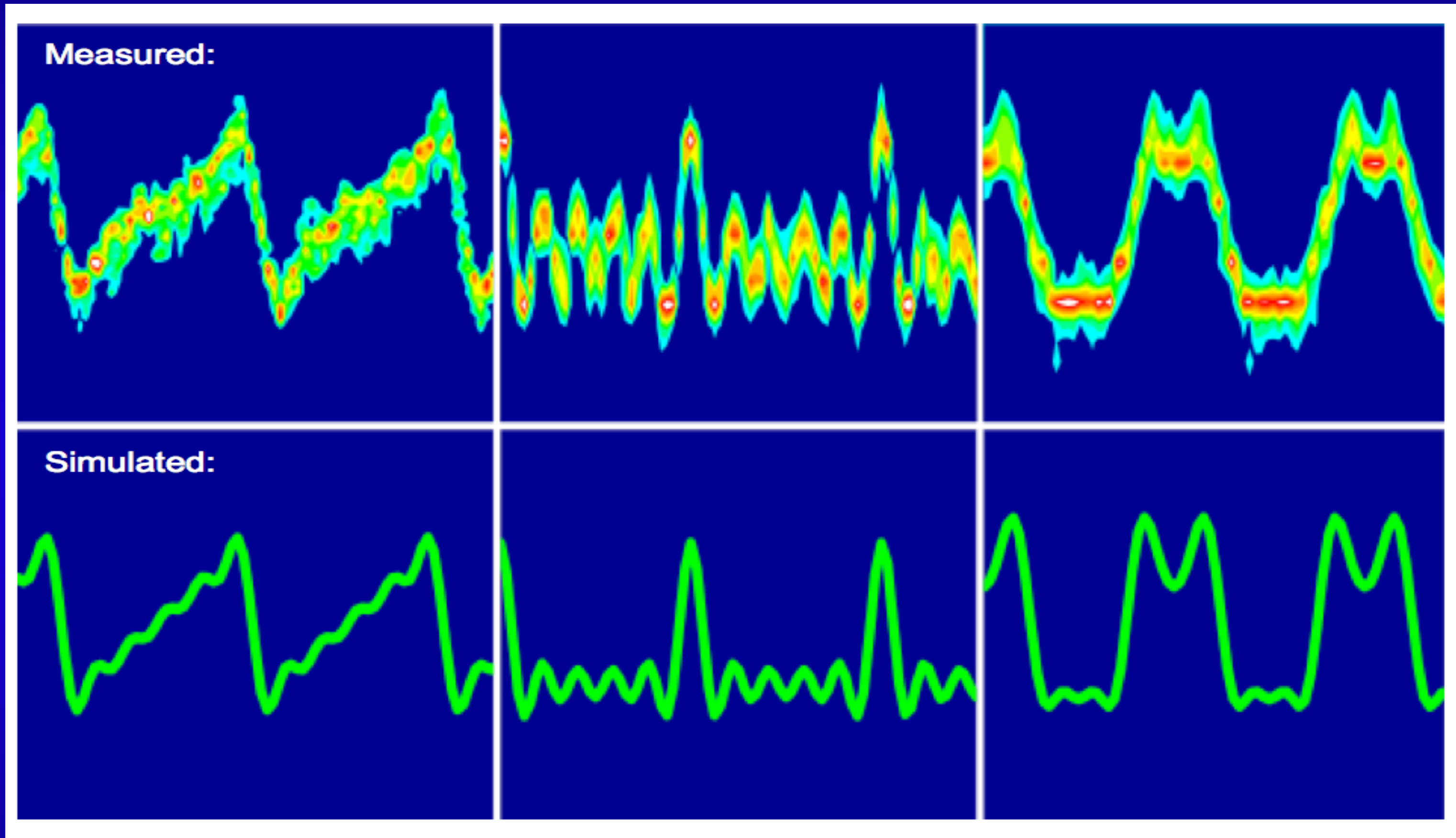
   tens to hundreds of comb lines -- high waveform shaping fidelity
   insufficient bandwidth -- shaping of envelopes only

3. Harmonic comb

   multi-octave comb
   field shaping achieved
   limited number of lines

Synthesized waveforms

Instantaneous field waveforms synthesized with 5 harmonic components

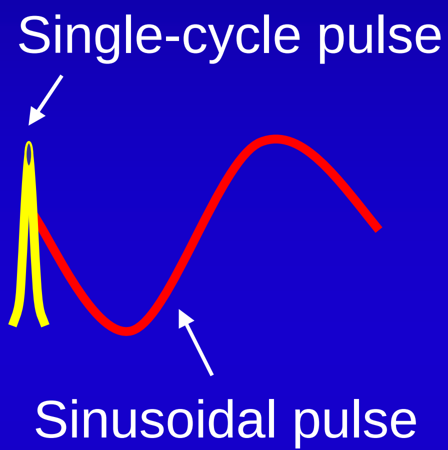


Saw tooth Single-cycle cosine Square wave

Peak to peak spacing: 8.02 fs
Narrowest field FWHM: 843 as
Envelope width: 1.5 fs

Chan et al, Science 2011

All optical waveform characterization



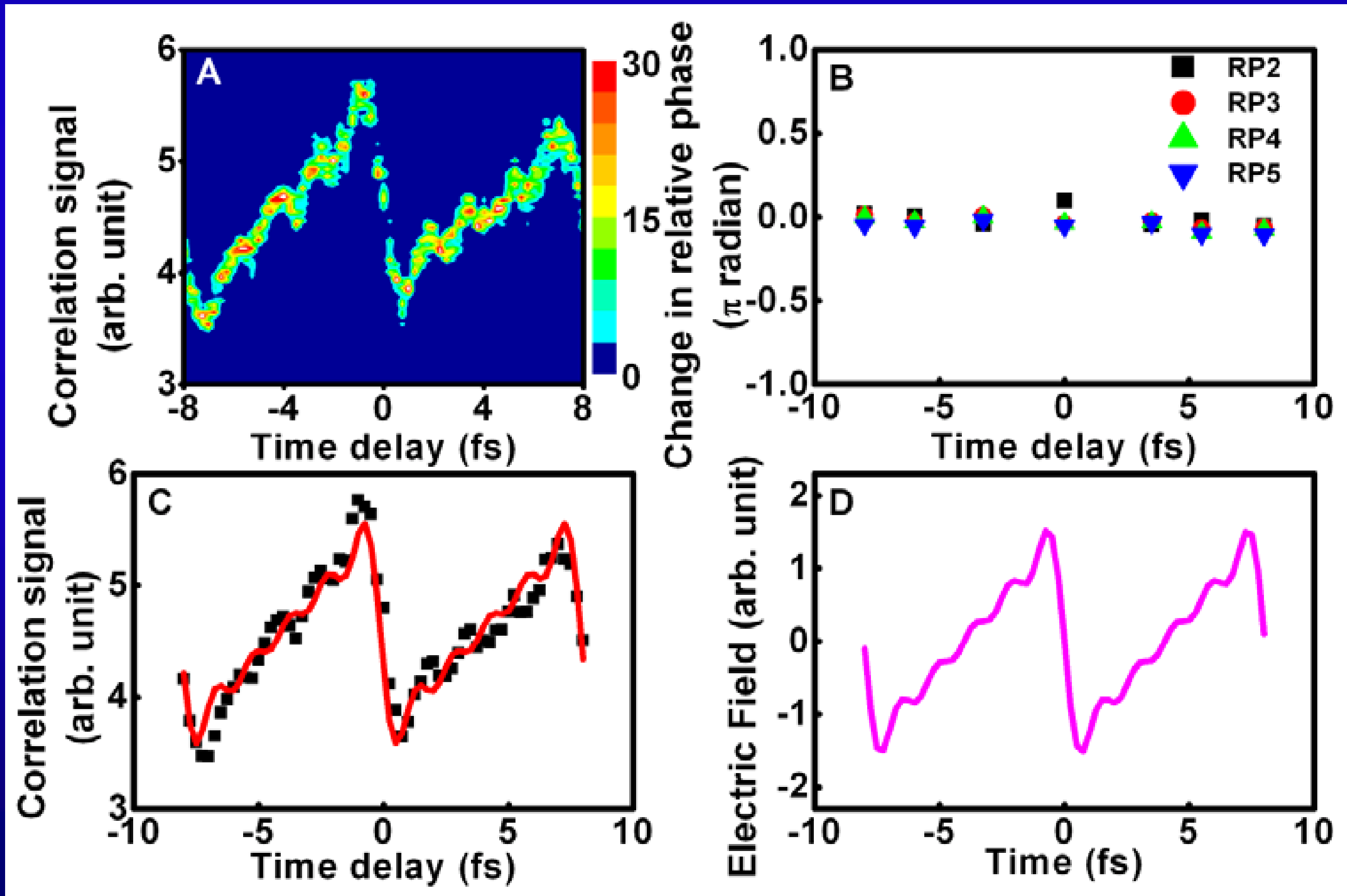
Shaper-Assisted Linear Cross-Correlation for optical field retrieval

SALC^2

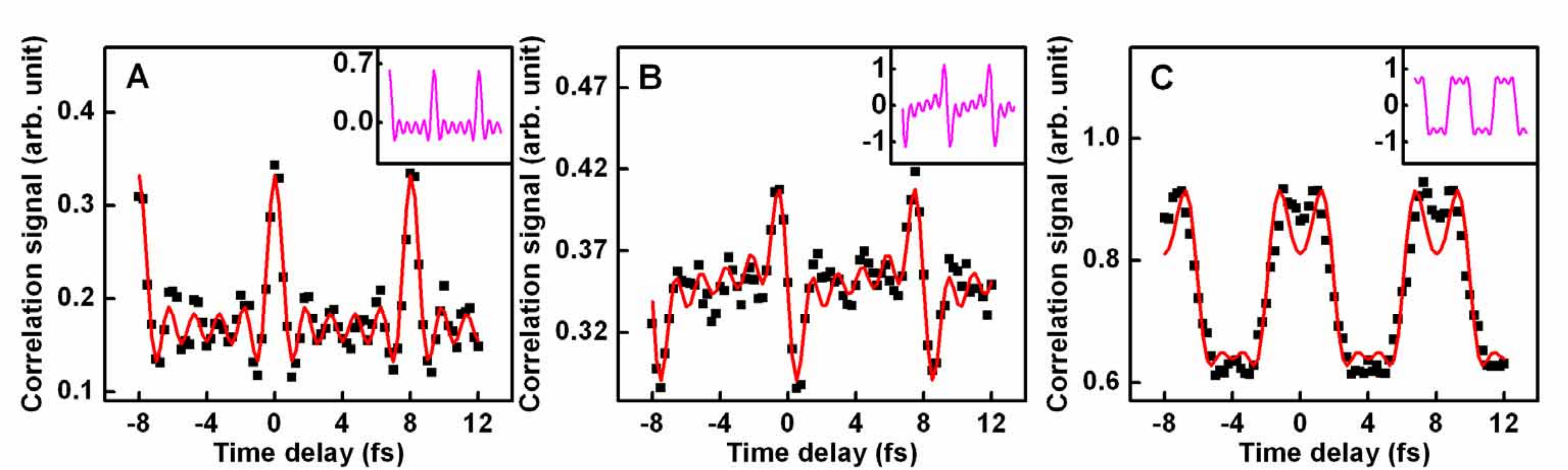
A. Galler and T. Feurer, Appl. Phys. B 90, 427-430 (2008).

Sawtooth waveform synthesized with 5 harmonic components

relative amplitudes {1,1/2,1/3,1/4,1/5} and CEP = $-\pi/2$



Instantaneous field waveforms synthesized with 5 harmonic components

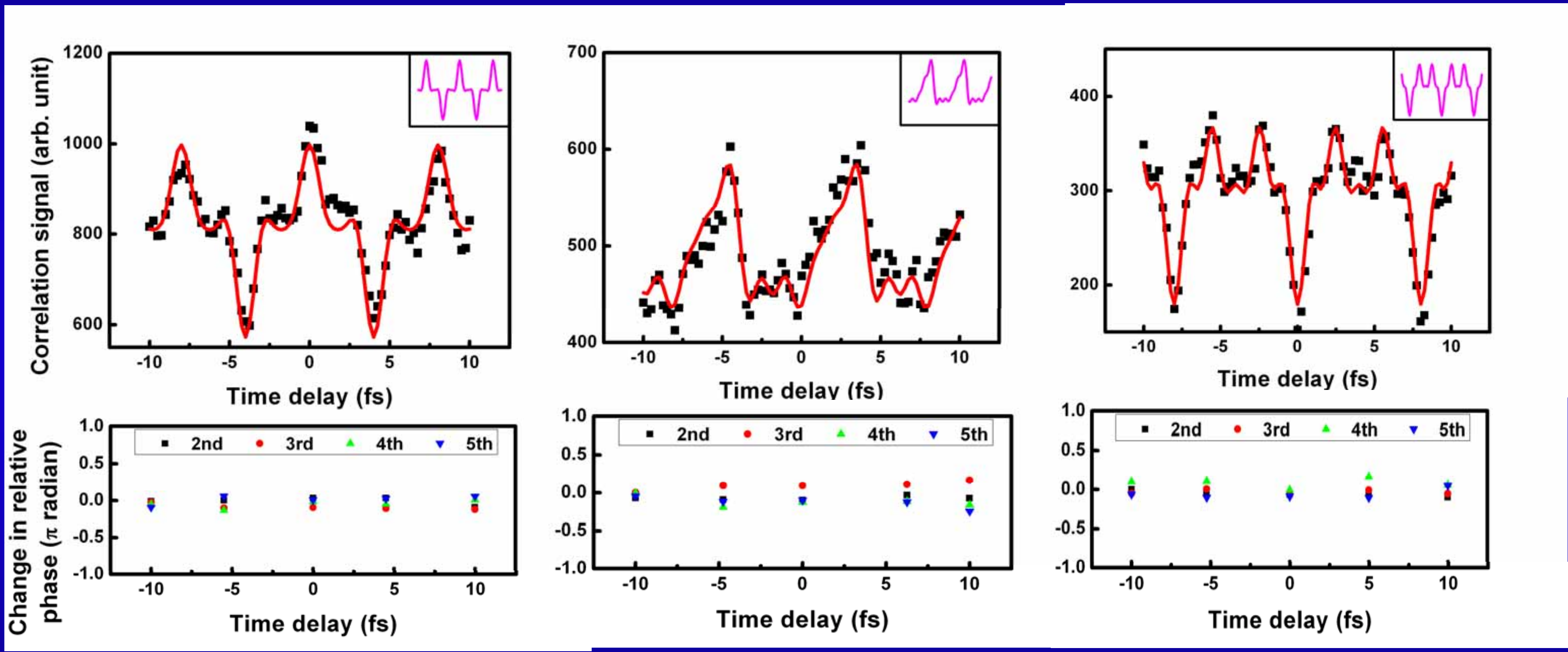


Single-cycle cosine　　Single-cycle sine　　Square wave

Peak to peak spacing: 8.02 fs
Narrowest field FWHM: 843 as
Envelope width: 1.5 fs

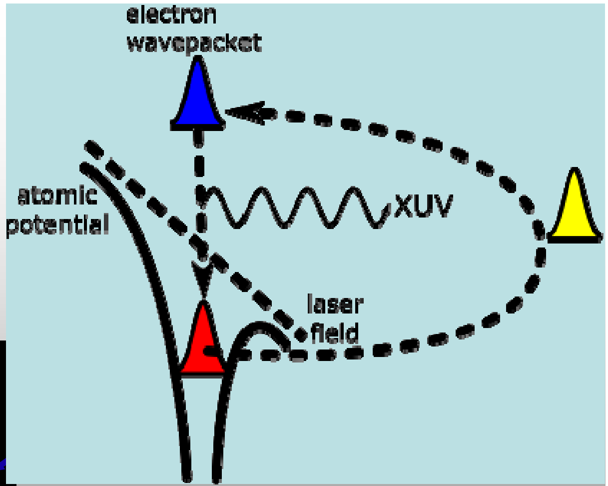
Chan et al, Science 2011

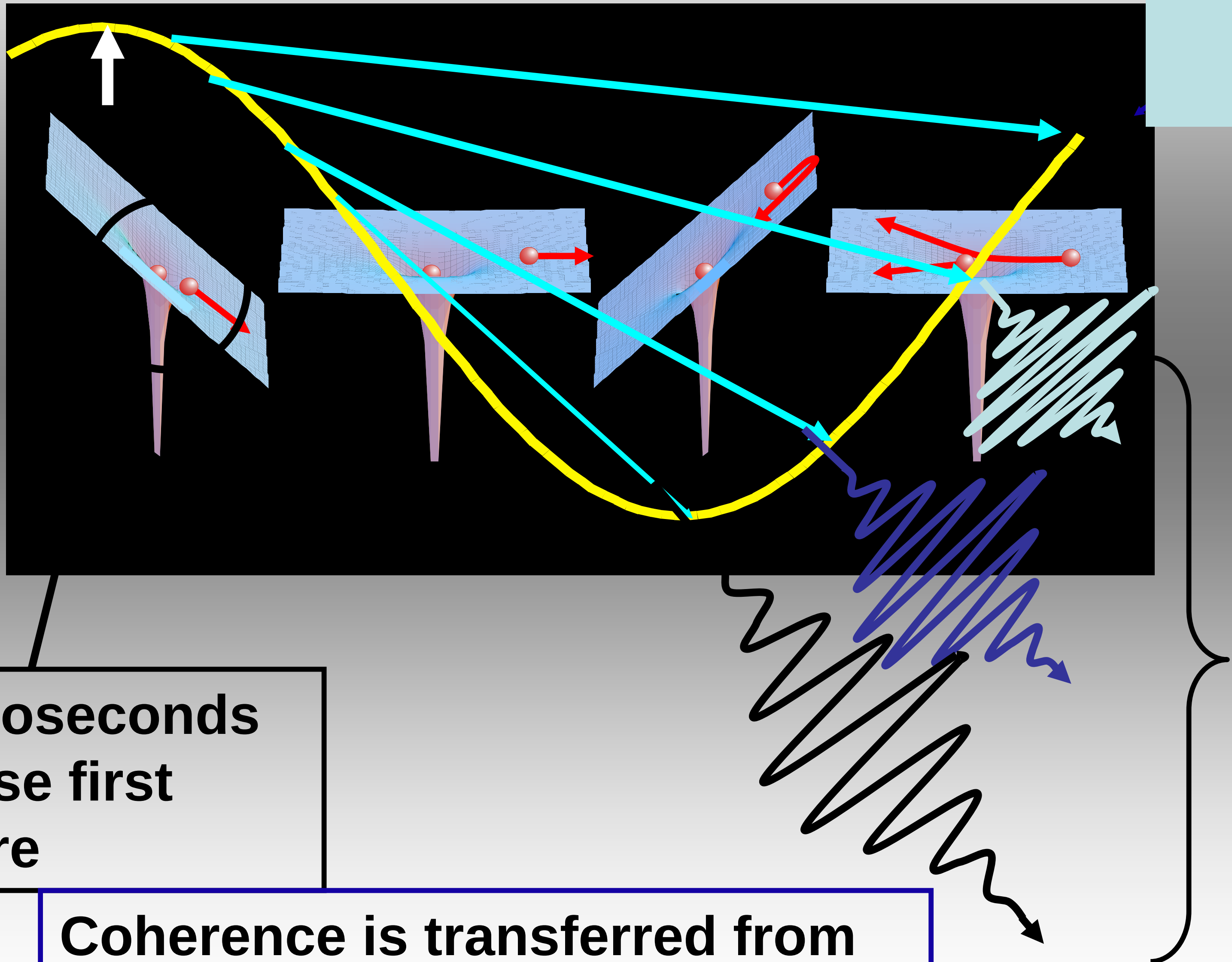
Arbitrary waveforms (correlation traces shown)



Application of arbitrary waveforms to high harmonic generation (HHG)

The key attosecond idea



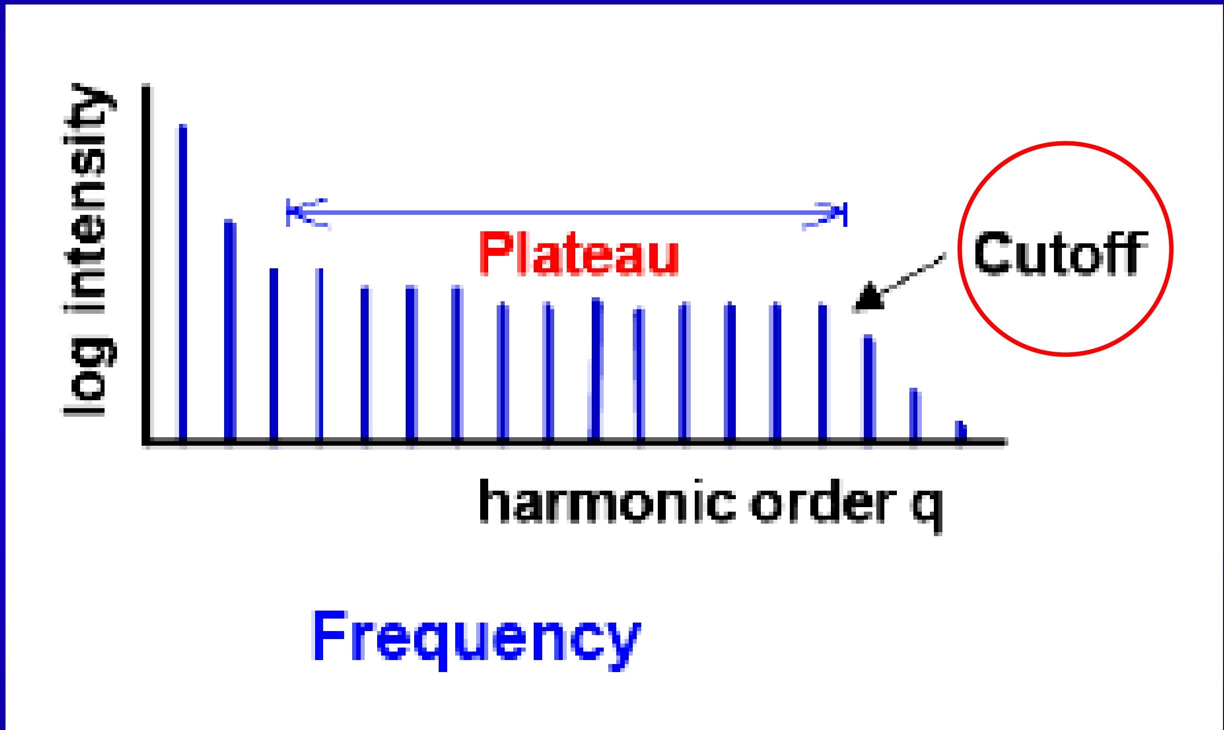


Attoseconds arise first here

Coherence is transferred from *photon to electron to photon* to.

Generating attosecond pulses

High-order harmonic generation



$$h\upsilon_{cutoff} = I_p + 3.2U_p$$

$$U_p = e^2 E_a^{\,2} / 4m\omega^2$$

Ponderomotive potential

Sinusoidal field is not optimized for HHG

PRL 102, 063003 (2009) PHYSICAL REV

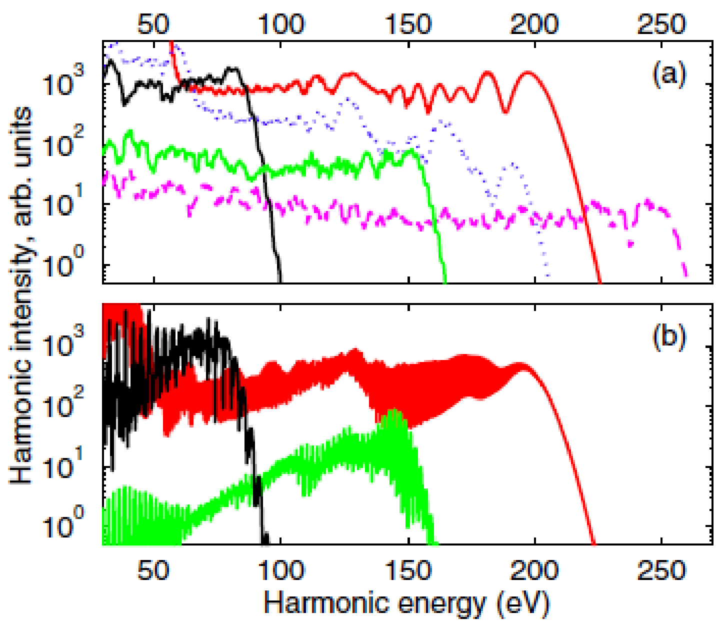
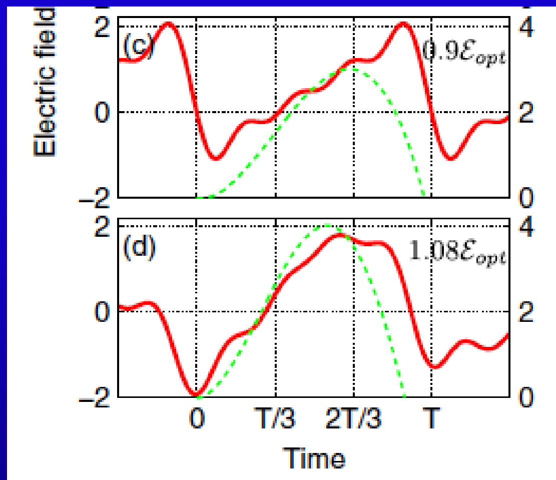


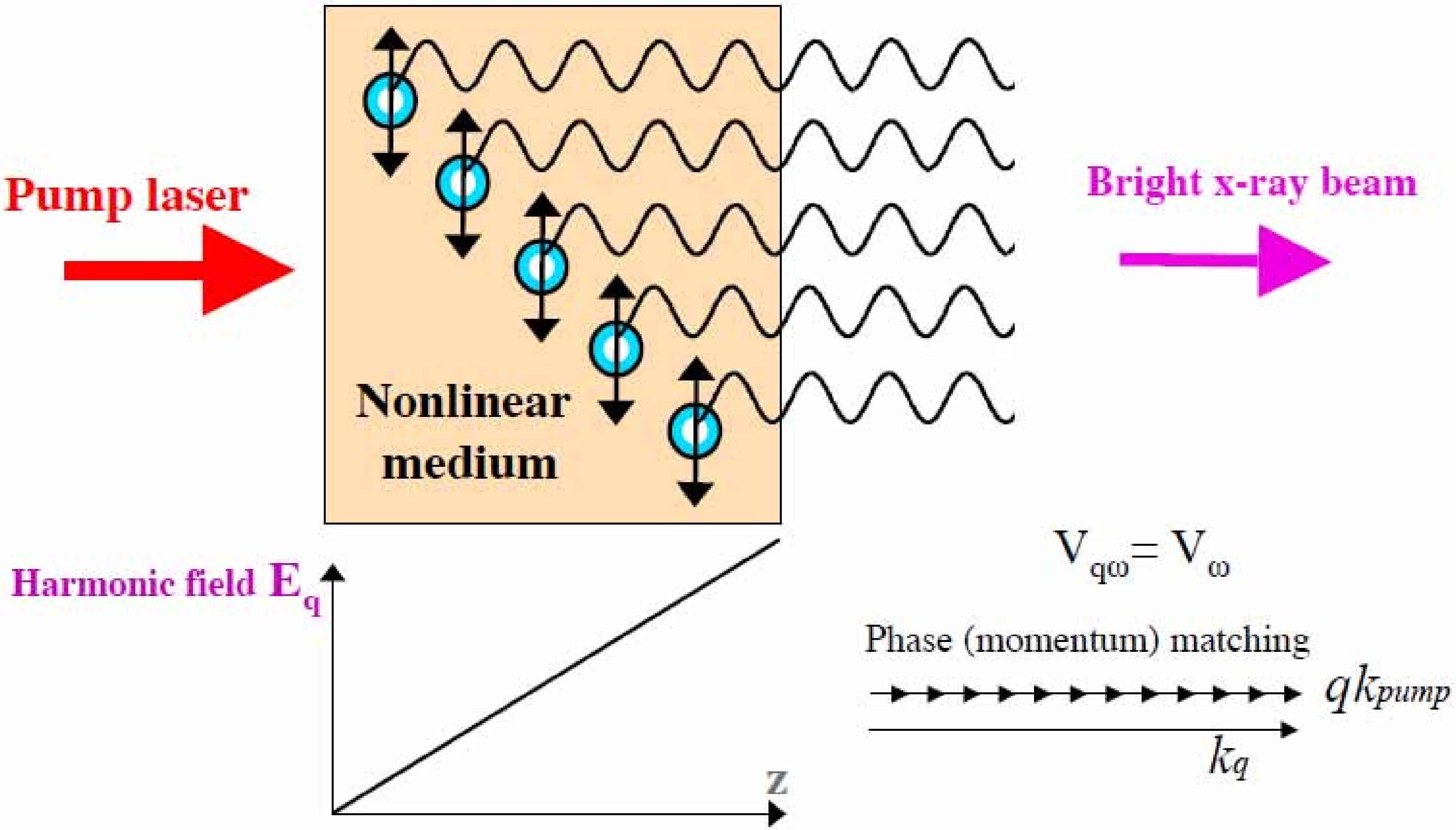
FIG. 2 (color online). Calculated HHG spectra generated by pure sinusoidal fields with wavelengths of 800 nm plotted as a solid black line, 1200 nm as a solid light gray (green) line, and 1600 nm as a dashed (pink) line; the optimized waveform plotted as solid dark gray (red) line; and a field composed solely of the 800 nm and 1600 nm components of the optimized field plotted in a dotted (blue) line. All spectra were generated using pulses of the same duration and pulse energy, normalized to a 16 fs FWHM 800 nm pulse with a peak intensity of 3×10^{14} W cm^{-2}. (a) Single active electron calculations. For clarity a 5 eV wide moving average has been used. (b) Full propagation calculations after radial integration.

Black:
sinusoidal field

Red:
optimized field

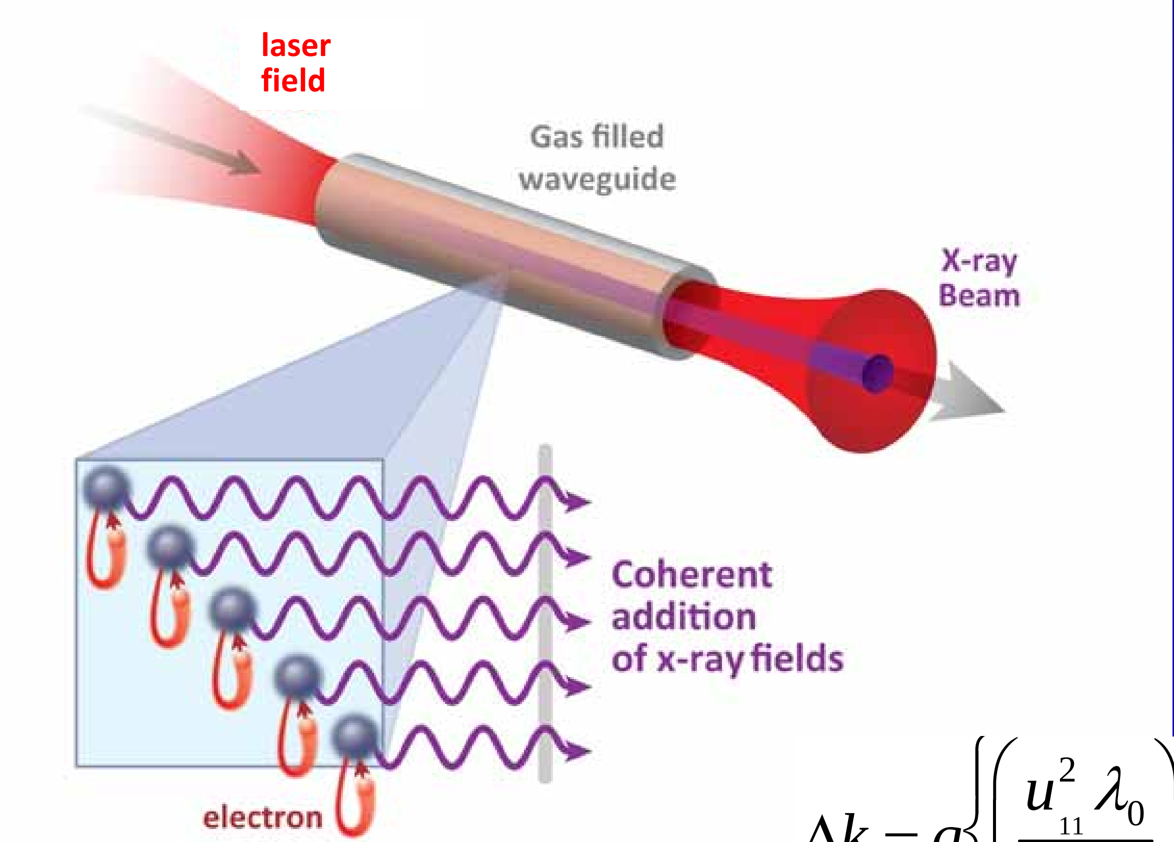


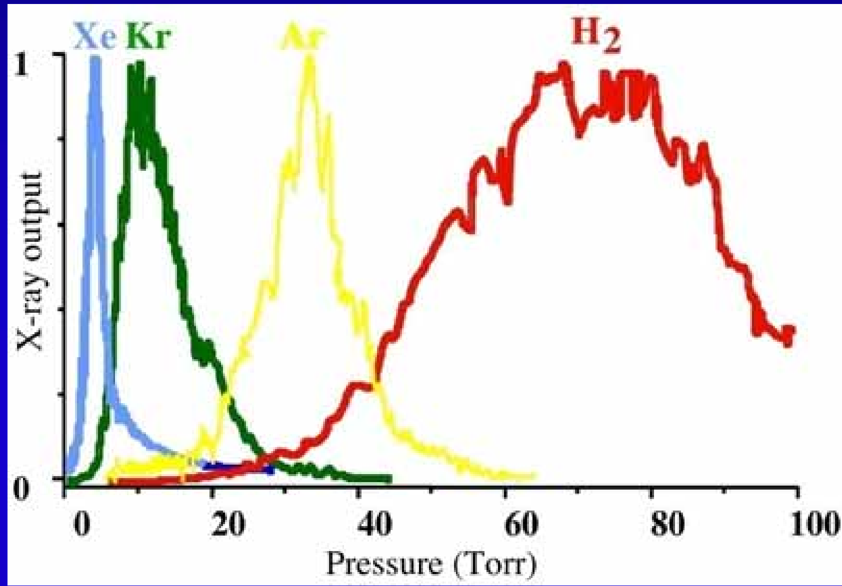
Second challenge - phase matching for bright x-rays



Phase matching - the emission from all atoms interfere constructively!

Phase matching of high harmonic beams





$\Delta k = q k_{laser} - k_{HHG} = 0$

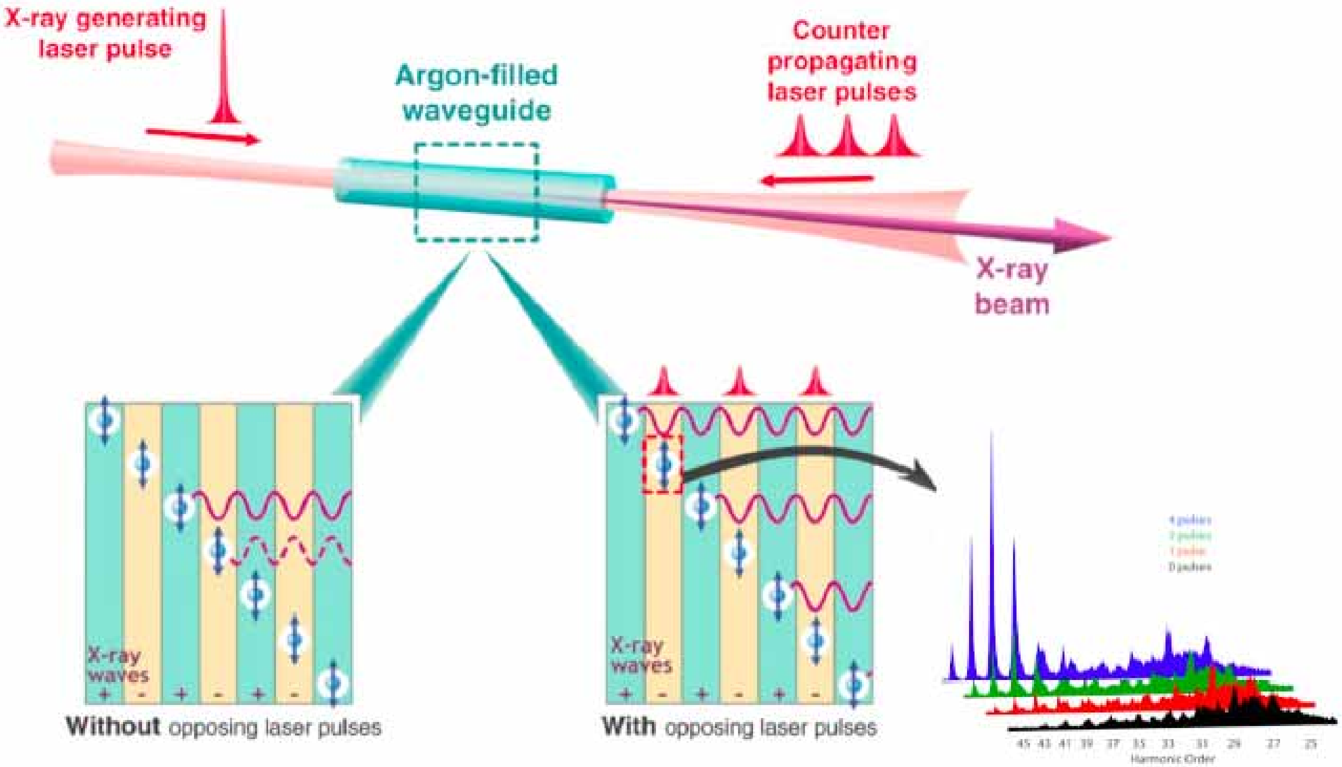
$V_{Laser}=V_{X\text{-}ray}=C$

$$\Delta k = q\left\{\left(\frac{u_{11}^2 \lambda_0}{4\pi a^2}\right) - P\left((1-\eta)\frac{2\pi}{\lambda_0}\Delta\delta - \eta\left[N_{atm} r_e \lambda_0\right]\right)\right\}$$

Waveguide **Neutrals** **Plasma**

國立清華大學 光電工程研究所

All-optical quasi phase matching



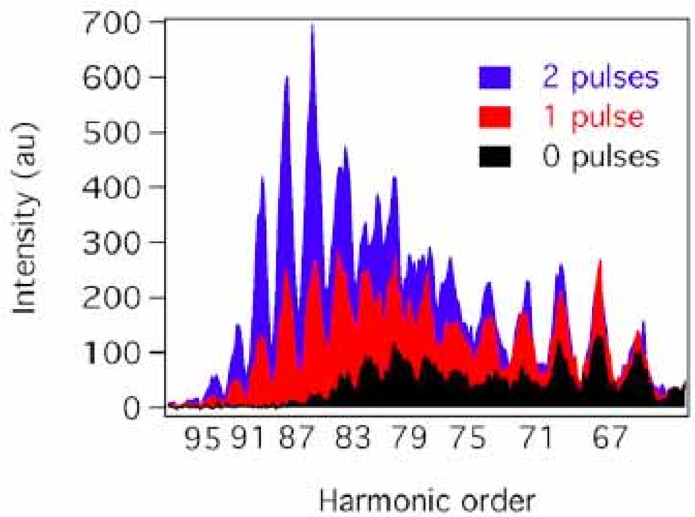
*PRL **98**, 123904 (2007); Nature Phys. **3**, 270 (2007); Science **317**, 775 (2007); CThX1*

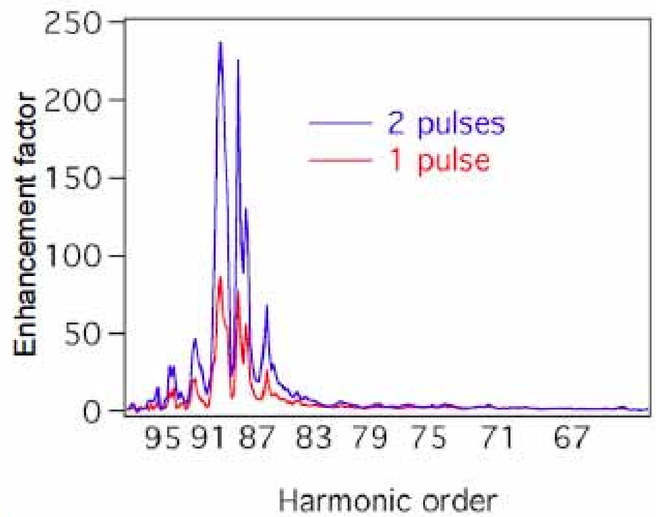
EUV

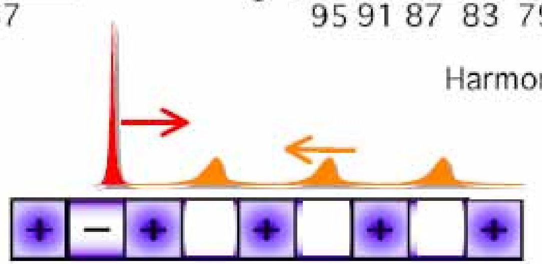
An NSF Engineering Research Center

All-optical quasi phase matching at 70eV and 140 eV

JILA NIST CU







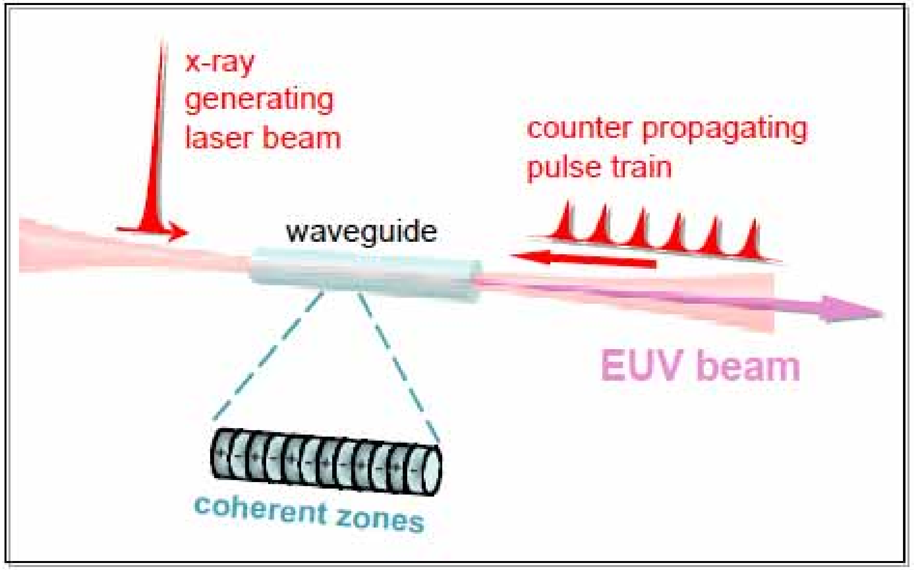
- L_c~1/q so coherence length changes with harmonic order
- Observe *selective* enhancement HHG when $L_c \approx$ pulse separation!
- No enhancement where conventional phase matching possible

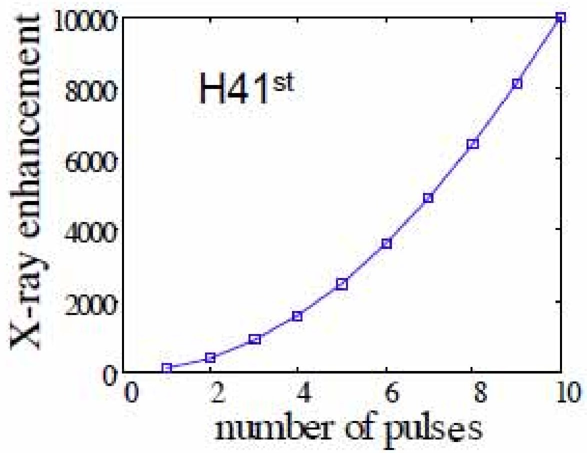
Lytle et al, Optics Letters 33, 174 (2008); Zhang et al, Nature Physics 3, 270 (2007)

How far can we go?

- **HHG in Helium in the water window**
  - **At a pressure of 5 torr, L_c ~100 μm**
  - **Absorption depth @ 300 eV: 10 *meters***
  - ***Possible enhancement:***

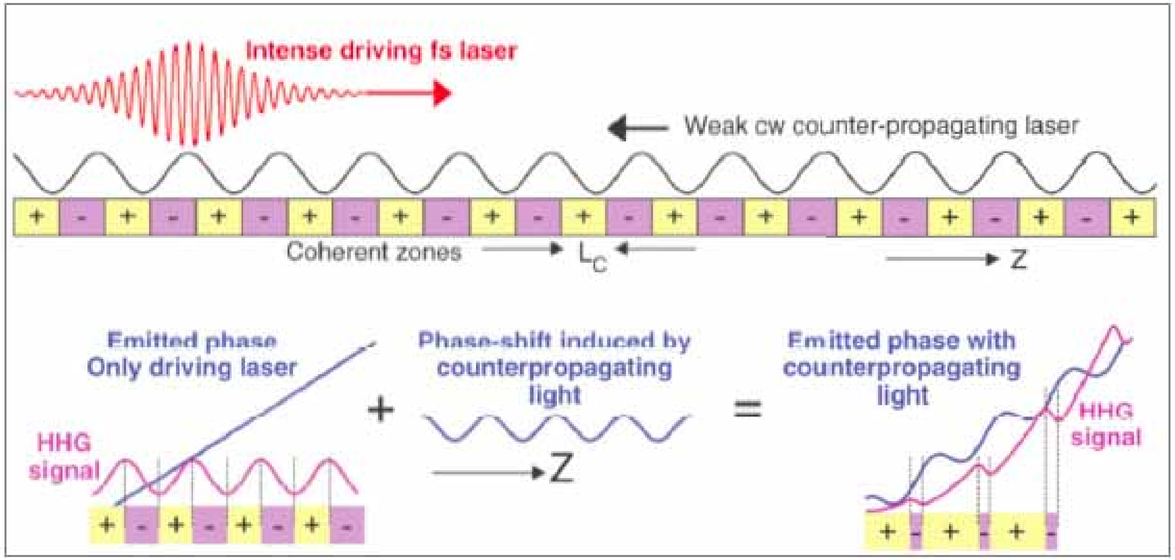
$$\frac{L_{abs}}{L_c} = \left(\frac{10}{10^{-4}}\right)^2 \sim 10^{10}$$





Phase matching at keV photon energies

- All-optical QPM solves phase matching problem even at keV energies!
- Use weak cw, mid-IR, counter-propagating beam in highly ionized regime where coherence lengths are very short (≈ microns)
- Use capillary discharge to reduce defocusing loss at high laser intensity (*J. Rocca*)



Grating-Assisted Phase Matching
Cohen et al.,PRL ***99****, 053902 (2007)*

SUMMARIZE

Why we are interested in it - Frontier research in optical physics

What is the approach - Fourier synthesis

How it is done and what have been done - harmonic frequency comb + AM/PM + $SALC^2$

What can be done with these waveforms - attosecond science and HHG process modification