

Graphene and 2010 Nobel Prize in Physics

石墨烯與2010年諾貝爾物理獎

Guang-Yu Guo (郭光宇)^{1,2}

¹Graduate Institute of Applied Physics,

National Chengchi University, Taiwan

(政治大學應用物理所)

²Physics Dept, National Taiwan University, Taiwan

(台灣大學物理系借調)

(A talk in CMRFG Winter Meeting, NCTS (South),

National Cheng Kung Univ., Jan. 15, 2011)

Plan of this Talk

I. 2010 Nobel Prize in Physics

1. The Prize
2. The Laureates

II. Introduction to graphene

1. Allotropes of carbon
2. Discovery and identification of graphene
3. Characteristics of graphene

III. Fascinating physics on graphene

1. Band structure and massless ultra-relativistic (Dirac) fermions
2. Anomalous **room temperature** quantum Hall effect
3. “**Big**” physics on a chip: **Klein paradox** and *zitterbewegung*
4. Specular Andreev reflection and tunneling conductance

IV. Possible applications of graphene

1. Ultrasmall and fast nanoelectronic devices, e.g., FET
2. Graphene-based spintronics and magnetoelectronics
3. Molecule sensor and graphene biodevices

I. 2010 Nobel Prize in Physics

1. The Prize



The Nobel Prize in 2010 Andre Geim, Konstantin Novoselov

The Nobel Prize in Physics 2010 was awarded jointly to Andre Geim and Konstantin Novoselov “*for groundbreaking experiments regarding the two dimensional material graphene*”.

[The official web site of the Nobel Prize:

http://nobelprize.org/nobel_prizes/physics/laureates/2010/#]

2. The Laureates



Andre Geim

Konstantin Novoselov

ANDRE GEIM

School of Physics and Astronomy
University of Manchester
Oxford Road
Manchester M13 9PL
UK

www.condmat.physics.manchester.ac.uk/people/academic/geim



Dutch citizen. Born 1958 in Sochi, Russia. Ph.D. 1987 from Institute of Solid State Physics, Russian Academy of Sciences, Chernogolovka, Russia. Director of Manchester Centre for Mesoscience & Nanotechnology, Langworthy Professor of Physics and Royal Society 2010 Anniversary Research Professor, University of Manchester, UK.

KONSTANTIN NOVOSELOV

School of Physics and Astronomy
University of Manchester
Oxford Road
Manchester M13 9PL
UK



www.condmat.physics.manchester.ac.uk/people/academic/novoselov

British and Russian citizen. Born 1974 in Nizhny Tagil, Russia. Ph.D. 2004 from Radboud University Nijmegen, The Netherlands. Professor and Royal Society Research Fellow, University of Manchester, UK.

Both Laureates think research should be fun. They have been working together for a long time now. Konstantin Novoselov, 36, began working for Andre Geim, 51, as a PhD-student in the Netherlands. He subsequently followed Geim to the United Kingdom. Both of them originally studied and began their careers as physicists in Russia. Now they are both professors at the University of Manchester.

Playfulness is one of their hallmarks. With the building blocks they have at their disposal they attempt to create something new, sometimes even by just allowing their brains to meander aimlessly. One always learns something in the process and, who knows, you may even hit the jackpot. That was the case seven years ago when they created a super sticky tape inspired by the Gecko lizard's ability to stick to even the smoothest surfaces. Prior to that, in 1997, Andre Geim managed to make a frog levitate in a magnetic field; an ingenious way of illustrating the principles of physics. The levitating frog awarded him the Ig nobel prize in 2000 intended to "make people laugh first and think second".

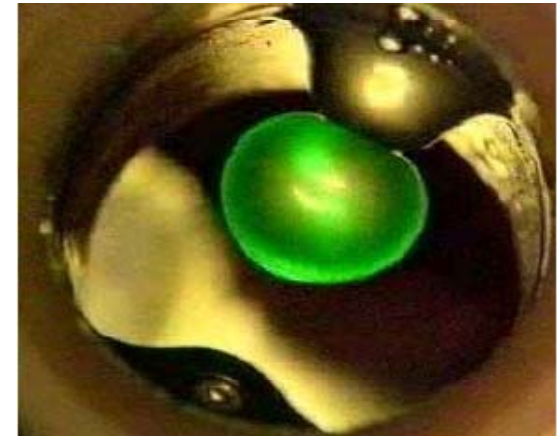
The Frog That Learned to Fly

(Molecular Magnetism and Levitation)

Seeing is believing:

A little frog (alive !) and a water ball levitate inside a Ø32mm vertical bore of a Bitter solenoid in a magnetic field of about 16 Tesla at the Nijmegen High Field Magnet Laboratory.

Radboud University



(Geim won the Ig Nobel Prize in 2000)

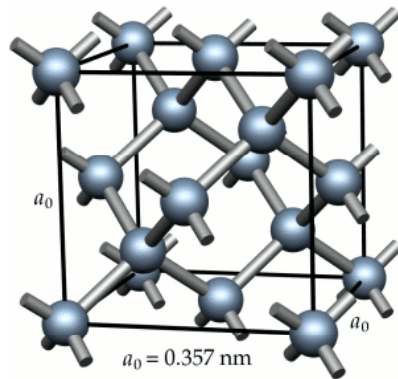
II. Introduction to graphene

1. Allotropes of carbon

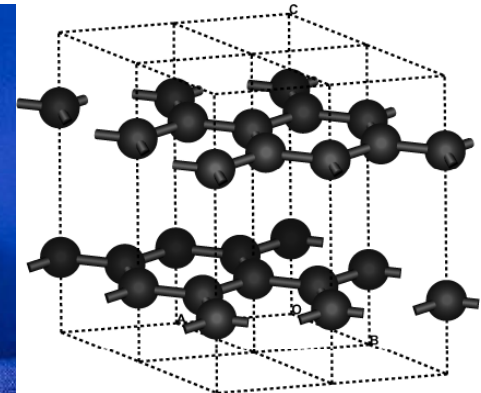
Carbon (C) is the most wonderful element in the world

1) Carbon is the *materia prima* for life and the basis of all organic chemistry.

2) Elemental carbon exists in many interesting forms, e.g.,



[Diamond: beautiful and hard;
3D sp^3 bond and insulator]



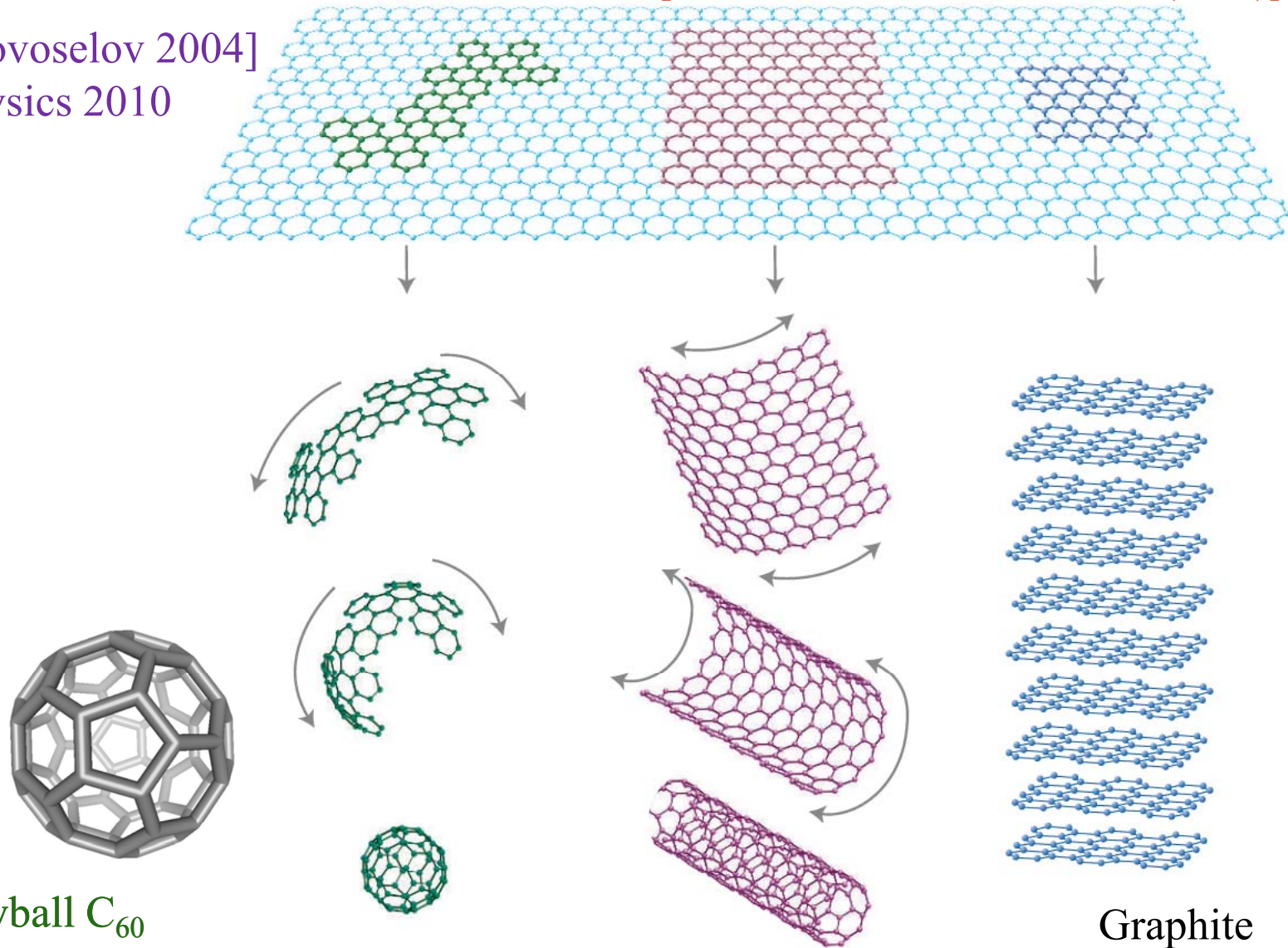
[Graphite: gray and soft; planar sp^2 bond and semimetal]

Graphene

[Geim, Novoselov 2004]

Nobel Physics 2010

[Geim, Novoselov, Nat. Mat. 6 (2007)]



Buckyball C₆₀

[Curl Jr., Kroto, Smalley 1985]

Nobel Chemistry 1996

Carbon nanotubes

[Ijima 1991]

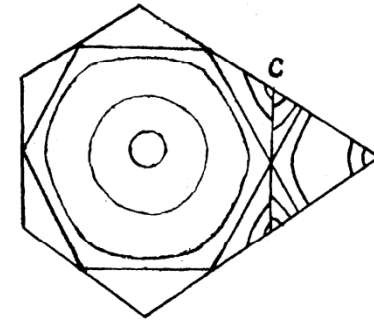
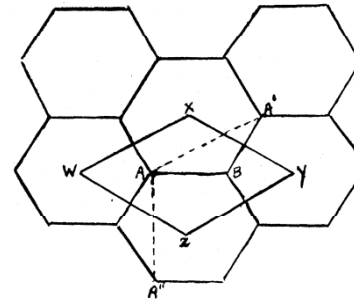
Graphite

2. Discovery and identification of graphene

Graphene flakes are produced everyday by millions of people since pencils were invented in 1564.

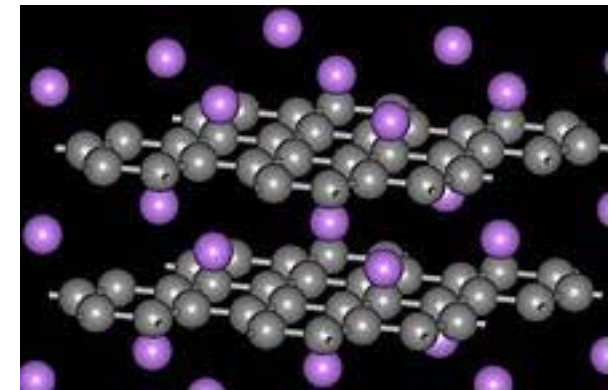
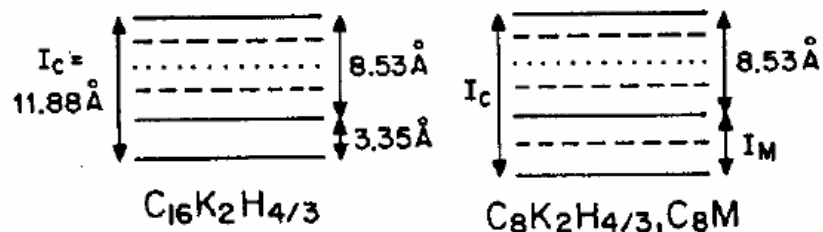
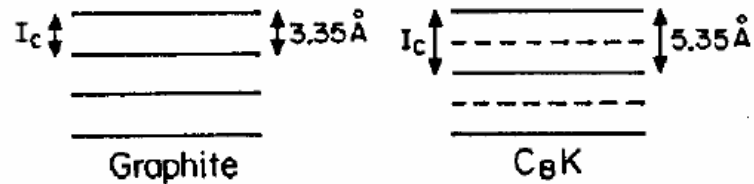


Graphene has been studied theoretically since 1947
[Wallace, PR 71 (1947) 622]



$$|E - E_c| \approx \sqrt{3}\pi\gamma_0 a |\mathbf{k} - \mathbf{k}_c|$$

Attempts to create graphite monolayers have also made since by chemical exfoliation such as intercalation of graphite (hot since 1970's).



Intercalated graphite KC_8 :
Superconductor, $T_c = 11.5 \text{ K}$.

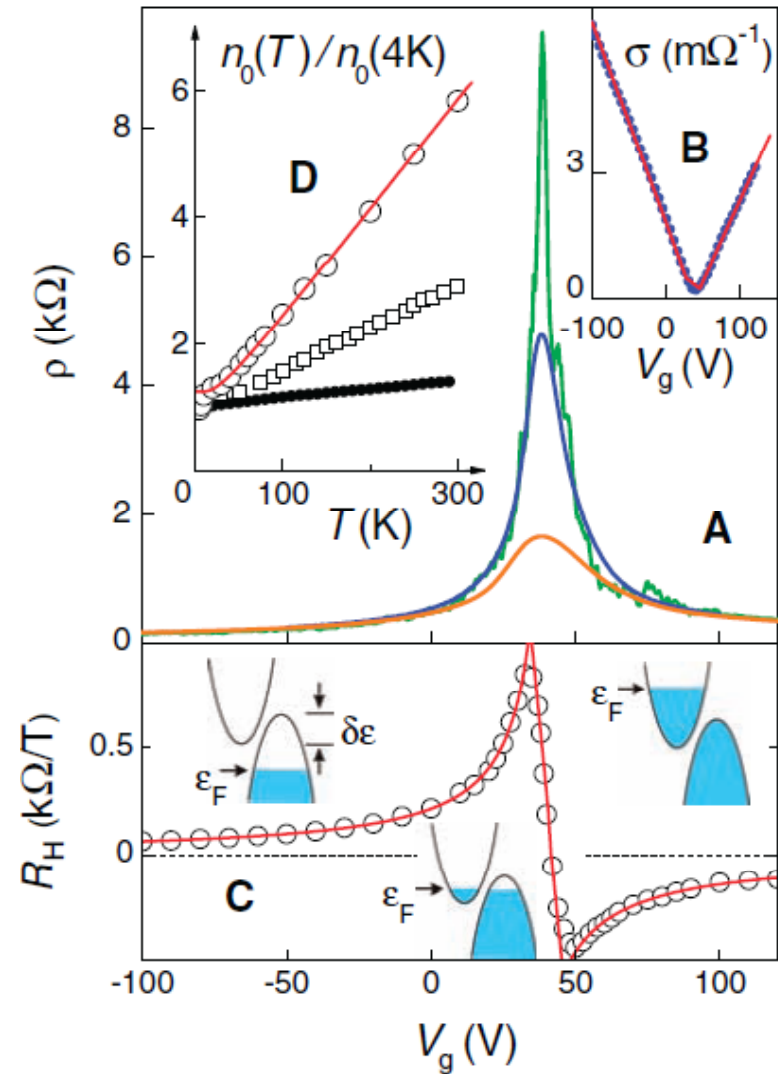
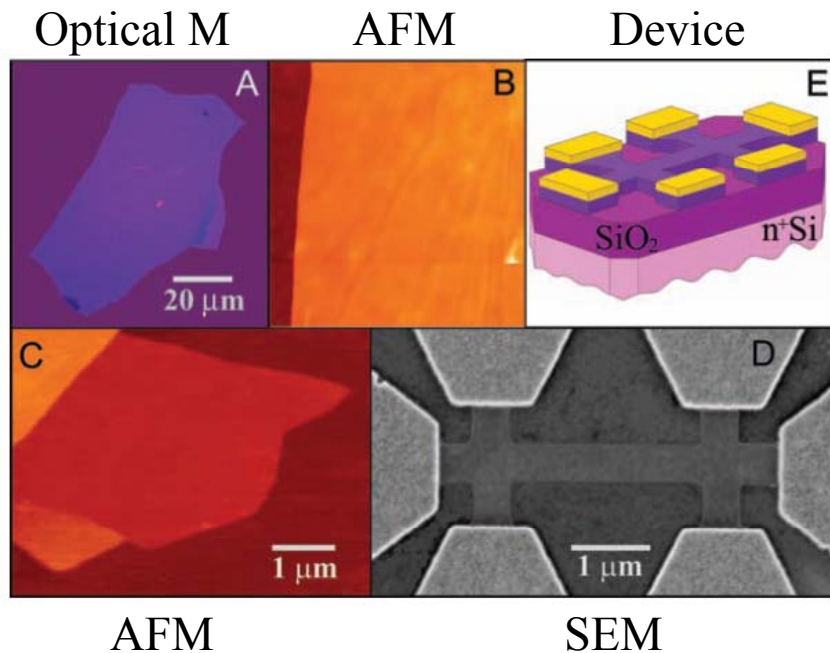
[Dresselhaus couple,
Adv. Phys. 51, 1 (2002)]

Isolation and identification of graphene (2004) (Nobel prize winning paper)

Electric Field Effect in Atomically Thin Carbon Films

K. S. Novoselov,¹ A. K. Geim,^{1*} S. V. Morozov,² D. Jiang,¹
Y. Zhang,¹ S. V. Dubonos,² I. V. Grigorieva,¹ A. A. Firsov²

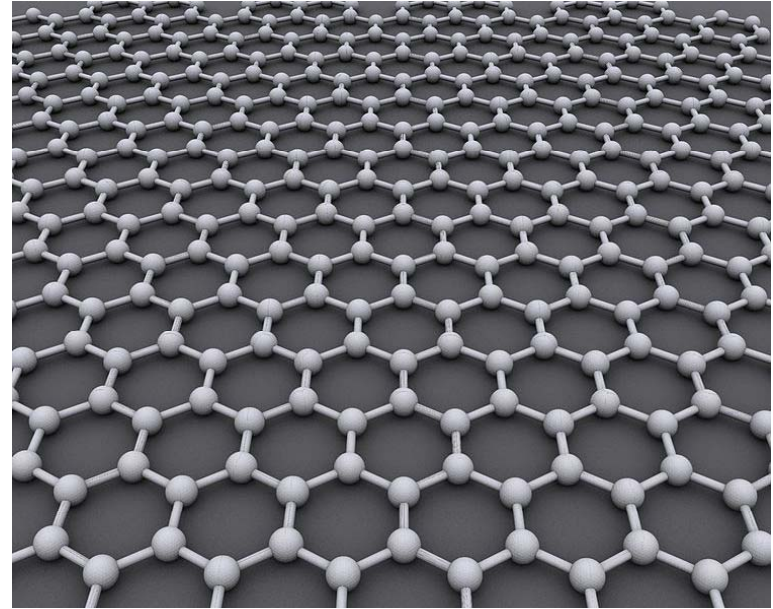
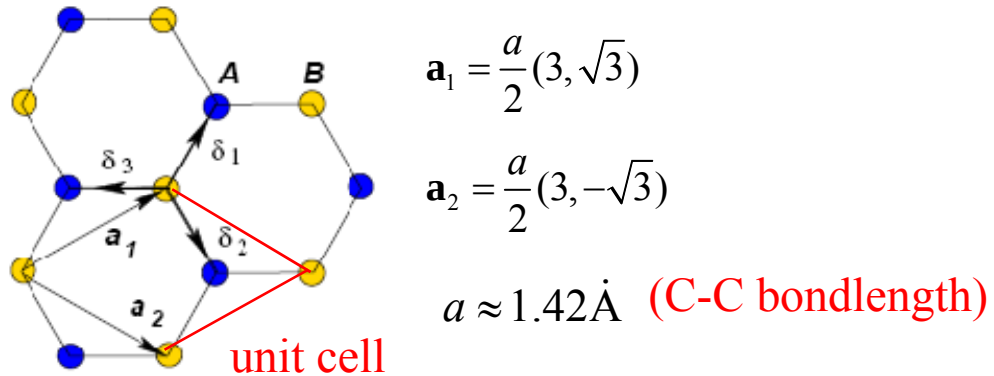
We describe monocrystalline graphitic films, which are a few atoms thick but are nonetheless stable under ambient conditions, metallic, and of remarkably high quality. The films are found to be a two-dimensional semimetal with a tiny overlap between valence and conduction bands, and they exhibit a strong ambipolar electric field effect such that electrons and holes in concentrations up to 10^{13} per square centimeter and with room-temperature mobilities of $\sim 10,000$ square centimeters per volt-second can be induced by applying gate voltage.



3. Characteristics of graphene

1) Atomic structure

A single atomic layer of 2D honeycomb lattice



2) Mechanical properties

[Lee et al., Science 321 (2008) 385]

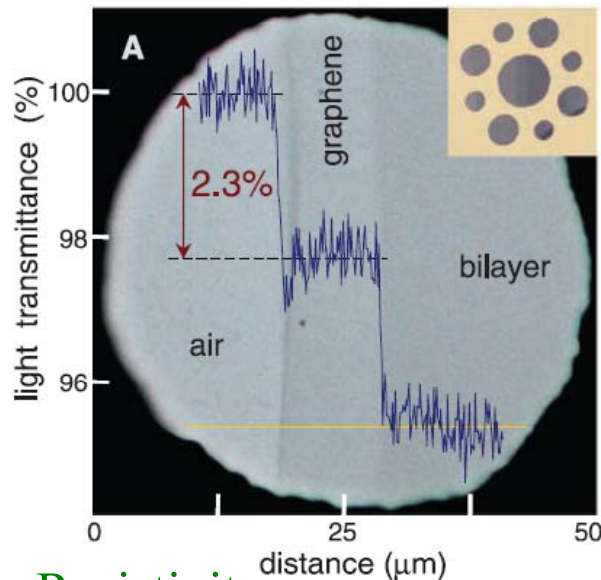
Tensile strength:

$420 \times 10^9 \text{ Pa}$ (graphene),

$150 \times 10^9 \text{ Pa}$ (nanotube),

$2 \times 10^9 \text{ Pa}$ (steel)

3) A transparent semimetal (conductor)



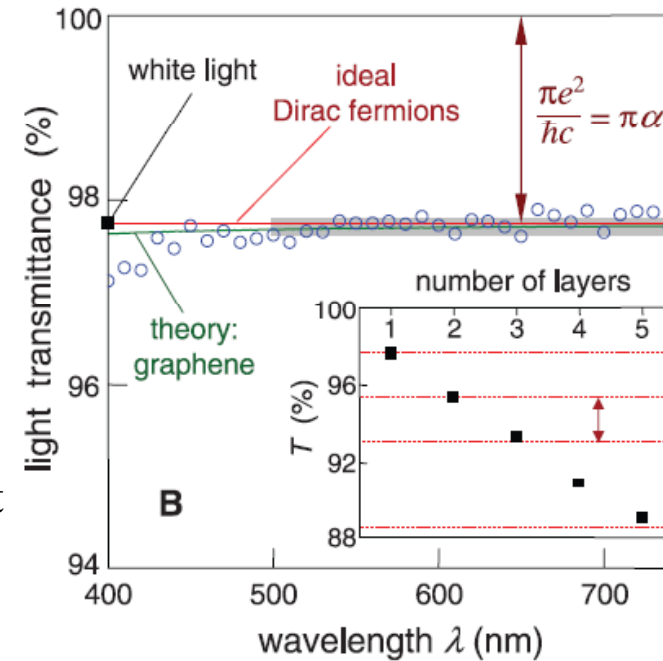
[Geim et al.,
Science 320 (2008)
1308]

Fine structure constant

$$\alpha \approx \frac{e^2}{\hbar c}$$

Resistivity

$\rho = 1.0 \times 10^{-6} \Omega \text{cm}$,
 $< \rho_{\text{Ag}} = 2.0 \times 10^{-6} \Omega \text{cm}$ (silver),
 lowest ρ substance at RT.



Mobility

$\mu \leq 20 \times 10^4 \text{ cm}^2 \text{V}^{-1} \text{s}^{-1}$,
 $\mu \leq 10 \times 10^4 \text{ cm}^2 \text{V}^{-1} \text{s}^{-1} > \text{(nanotube)}$
 $\mu \leq 0.14 \times 10^4 \text{ cm}^2 \text{V}^{-1} \text{s}^{-1}$ (silicon).

4) Thermal properties

Thermal conductivity

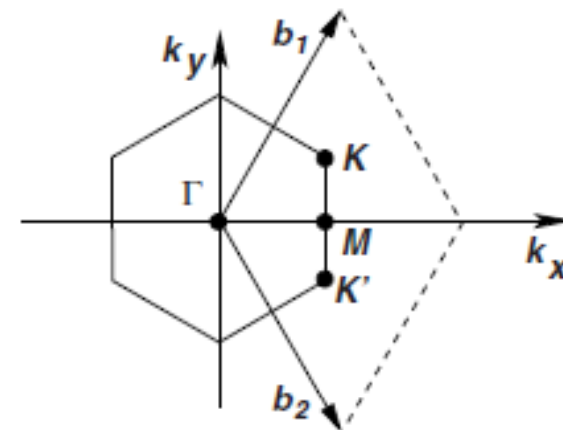
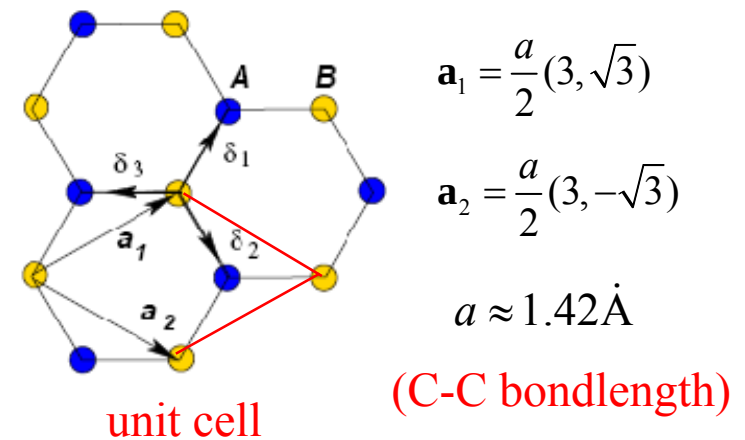
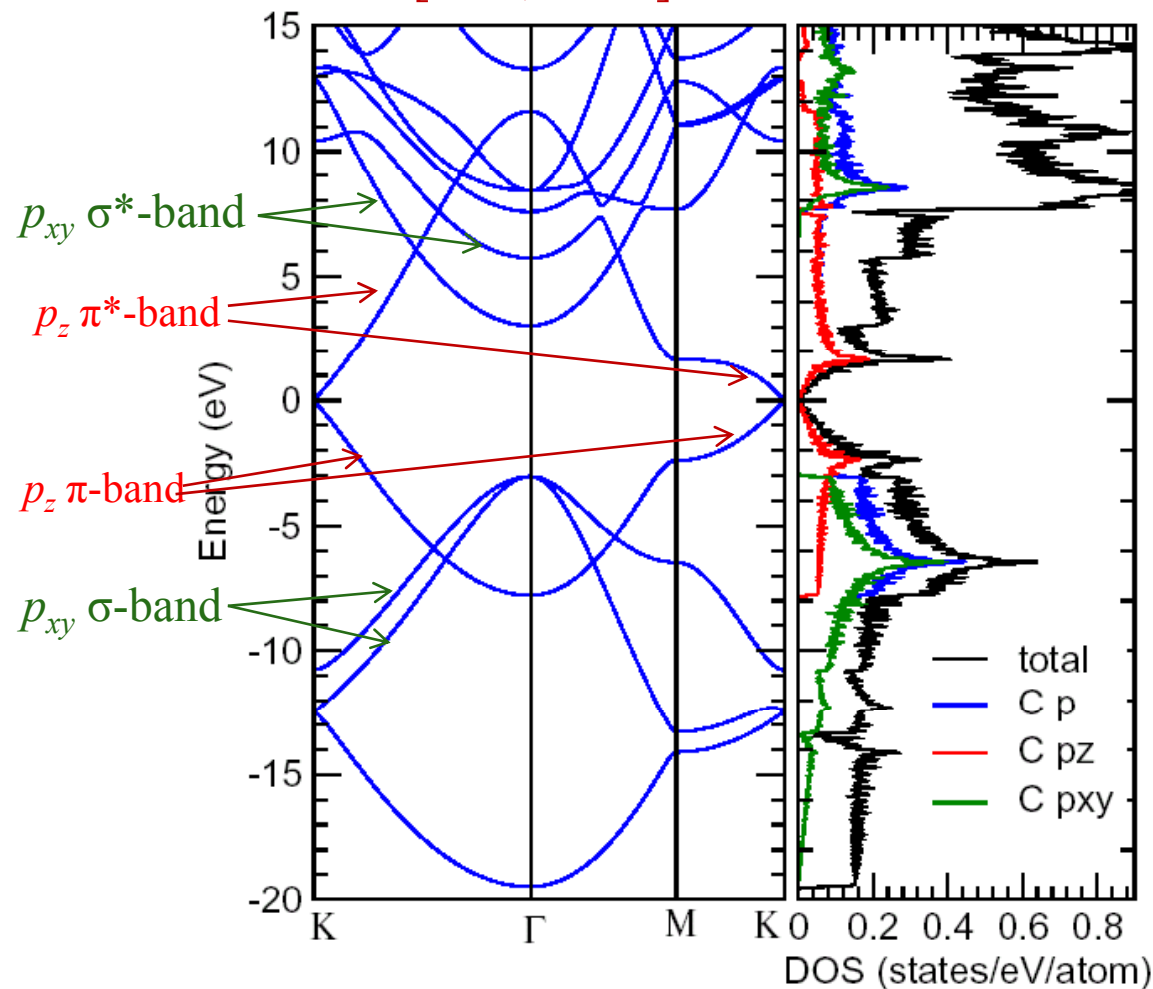
$\kappa \sim 5 \times 10^3 \text{ Wm}^{-1} \text{K}^{-1}$, $> 3.5 \times 10^3 \text{ Wm}^{-1} \text{K}^{-1}$ (nanotubes),
 $> 3.3 \times 10^3 \text{ Wm}^{-1} \text{K}^{-1}$ (diamond), $> 0.15 \times 10^3 \text{ Wm}^{-1} \text{K}^{-1}$ (silicon).

III. Fascinating physics on graphene

1. Band structure and massless ultra-relativistic fermions

(1) *Ab initio* band structure of graphene

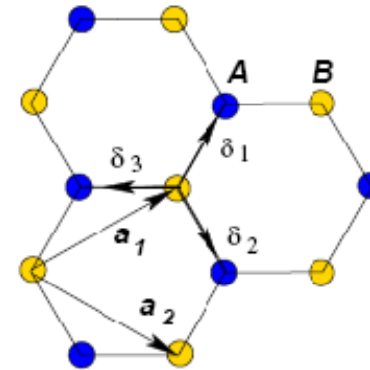
[Guo, 2003]



(2) 1-band (π) tight-binding Hamiltonian

$$H = -\gamma_0 \sum_{\langle i,j \rangle, \sigma} (a_{\sigma,i}^\dagger b_{\sigma,j} + \text{h.c.}) - \gamma_1 \sum_{\langle\langle i,j \rangle\rangle, \sigma} (a_{\sigma,i}^\dagger a_{\sigma,j} + b_{\sigma,i}^\dagger b_{\sigma,j} + \text{h.c.})$$

$$\gamma_0 \approx 2.7\text{eV}, \gamma_1 \approx 0.1\text{eV}.$$

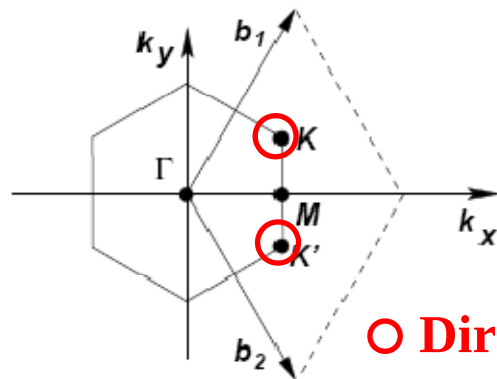


(3) Linear energy dispersions

[Wallace, PR 71 (1947) 622]

$$E_{\pm}(\mathbf{k}) = \pm\gamma_0\sqrt{3+f(\mathbf{k})} - \gamma_1 f(\mathbf{k})$$

$$f(\mathbf{k}) = 2\cos(\sqrt{3}k_y a) + 4\cos\left(\frac{\sqrt{3}}{2}k_y a\right)\cos\left(\frac{3}{2}k_x a\right)$$

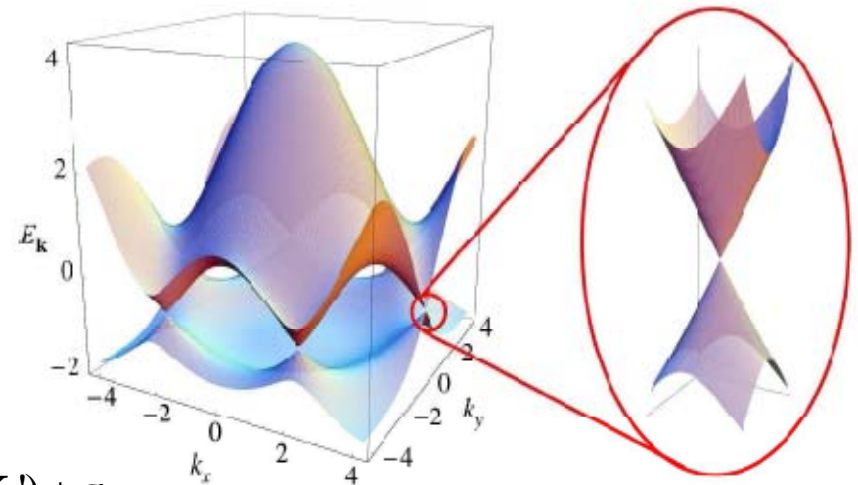


○ Dirac point

If $\gamma_1 \approx 0$ and $\mathbf{k} = \mathbf{K}(\mathbf{K}') + \mathbf{q}$,

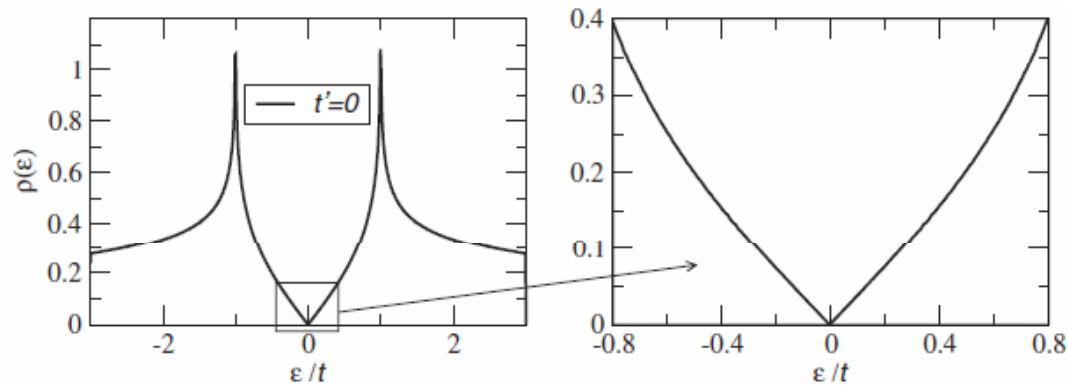
$$E_{\pm}(\mathbf{q}) \approx \pm v_F |\mathbf{q}| + O[(q/K)^2]$$

$$v_F \approx 1 \times 10^6 \text{ m/s}, c \approx 3 \times 10^8 \text{ m/s}.$$



[Castro Neto et al.,
RMP 81 (2008) 109]

Density of states

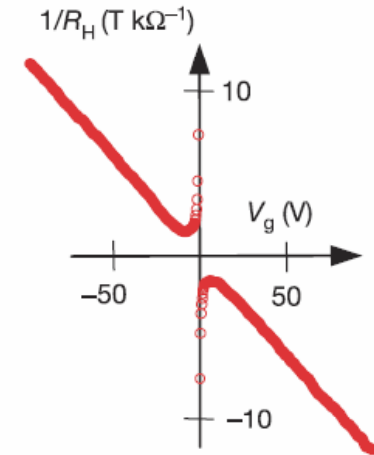


DOS

$$D(E) = \frac{3\sqrt{3}a^2}{\pi} \frac{|E|}{v_F^2}$$

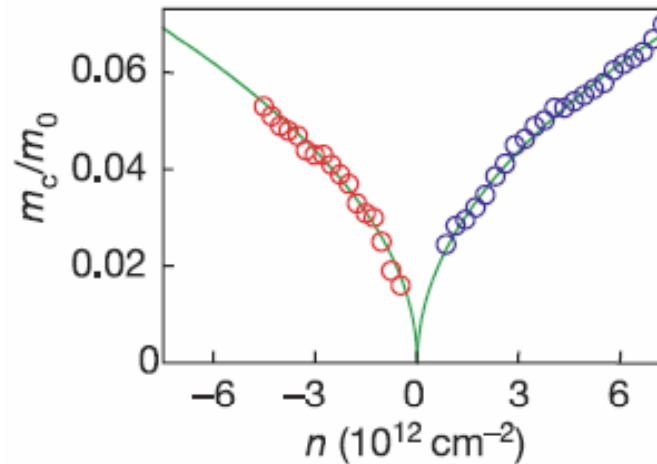
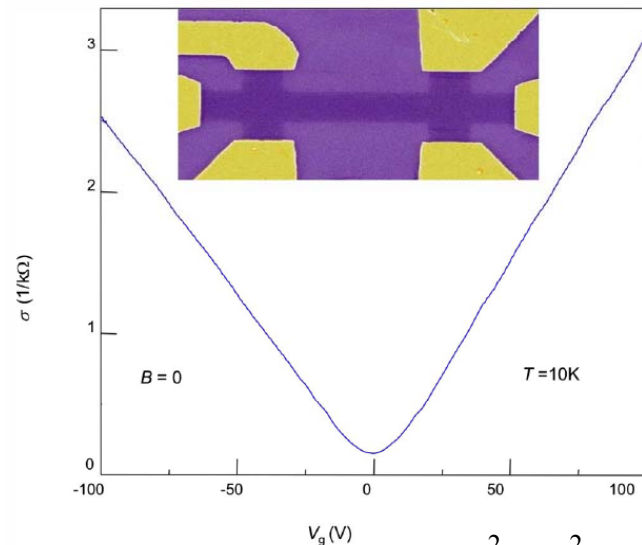
Carrier density

$$n(E) = \frac{E^2}{\pi v_F^2}$$



Conductivity and cyclotron mass (predictions confirmed)

[Novoselov et al., Nature 438 (2005) 197]



$$R_H = 1/ne$$

Cyclotron mass

$$m^* = \frac{1}{2\pi} \left[\frac{\partial A(E)}{\partial E} \right]_{E=E_F}$$

$$A(E) = \pi q(E)^2 = \pi \frac{E^2}{v_F^2}$$

$$m^* = \frac{k_F}{v_F} = \frac{\sqrt{\pi}}{v_F} \sqrt{n}$$

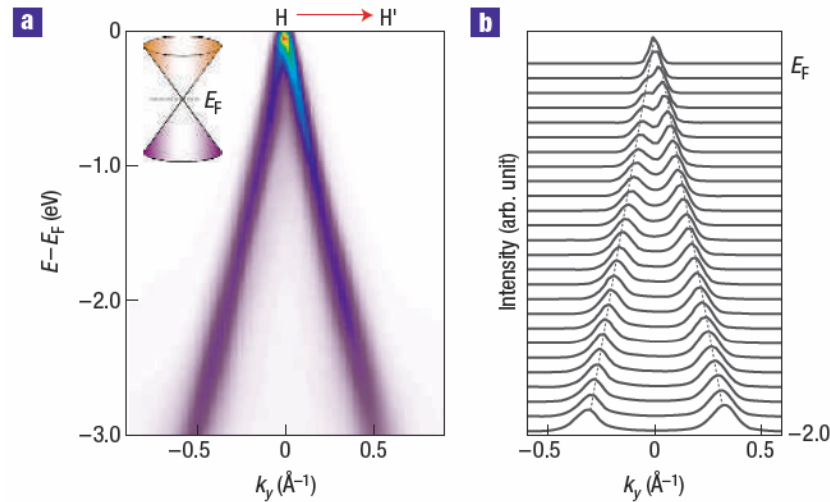
[Ashcroft & Mermin 1976]

Conductivity

$$\sigma_{xx} = 2 \frac{e^2}{h} \frac{\pi v_F^2}{u_0^2} n$$

$$v_F = c^* \approx 10^6 \text{ m/s}$$

Linear dispersion relations (predictions confirmed) Angle-resolved photoemission spectroscopy (ARPES)



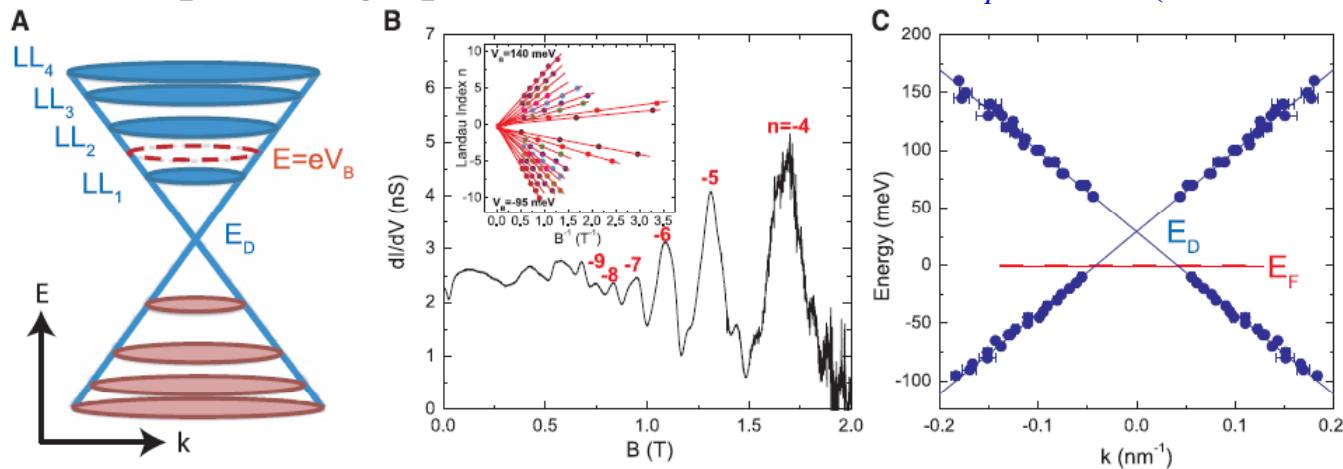
[Zhou et al., Nat. Phys. 2 (2006) 595]

$$v_F = c^* = (0.91 \pm 0.15) \times 10^6 \text{ m/s}$$

Scanning tunneling spectroscopy on epitaxial graphene on SiC

[Miller et al., Science 324 (2009) 924]

$$v_F = c^* = (1.070 \pm 0.006) \times 10^6 \text{ m/s}$$



(3) Chiral (Dirac) fermions

[*Condensed-matter simulation of a 3D anomaly*,
Semenoff, PRL 53 (1984) 2449]

By Fourier transform + algebra, the T-B
Hamiltonian can be written as

$$H = -iv_F \int dxdy \left[\hat{\Psi}_1^\dagger(\mathbf{r}) \sigma \cdot \nabla \hat{\Psi}_1(\mathbf{r}) + \hat{\Psi}_2^\dagger(\mathbf{r}) \sigma^* \cdot \nabla \hat{\Psi}_2(\mathbf{r}) \right]$$

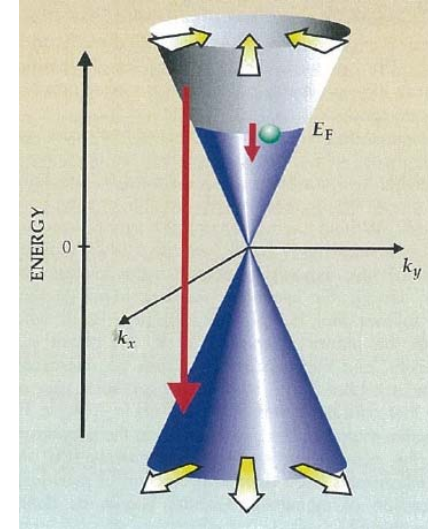
with $\sigma = (\sigma_x, \sigma_y)$, $\sigma^* = (\sigma_x, -\sigma_y)$ for (pseudo)spin,

$$\hat{\Psi}_i^\dagger = (a_i^\dagger, b_i^\dagger) \quad (i=1,2).$$

Wave function $\psi(\mathbf{r})$ near $K(K')$ obeys

$$-iv_F \sigma \cdot \nabla \psi(\mathbf{r}) = E \psi(\mathbf{r}) \quad \text{(2D Dirac Equ.)}$$

with eigenvalues $E = \pm v_F k$.



Wave function near $K(K')$ in $\mathbf{k}$ -space

$$\psi_{\pm, \mathbf{K}}(\mathbf{k}) = \frac{1}{\sqrt{2}} \begin{pmatrix} e^{-i\theta_{\mathbf{k}}/2} \\ \pm e^{i\theta_{\mathbf{k}}/2} \end{pmatrix} \text{ for } H_K = v_F \sigma \cdot \mathbf{k},$$

$$\psi_{\pm, \mathbf{K}'}(\mathbf{k}) = \frac{1}{\sqrt{2}} \begin{pmatrix} e^{i\theta_{\mathbf{k}}/2} \\ \pm e^{-i\theta_{\mathbf{k}}/2} \end{pmatrix} \text{ for } H_{K'} = v_F \sigma^* \cdot \mathbf{k},$$

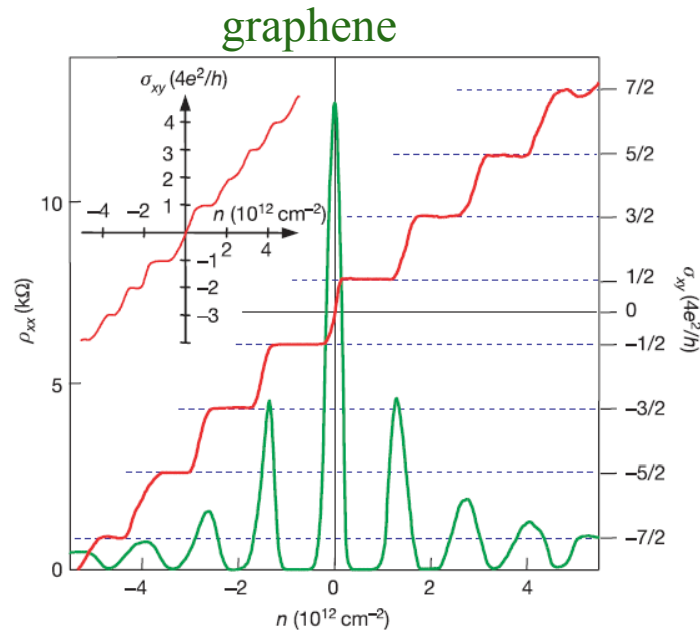
are eigenstates of the helicity operator

$$\hat{h} = \frac{1}{2} \sigma \cdot \frac{\mathbf{p}}{|\mathbf{p}|}, \text{ i.e.,}$$

$$\hat{h} \psi_{\pm, \mathbf{K}} = \pm \frac{1}{2} \psi_{\pm, \mathbf{K}}, \quad \hat{h} \psi_{\pm, \mathbf{K}'} = \mp \frac{1}{2} \psi_{\pm, \mathbf{K}}.$$

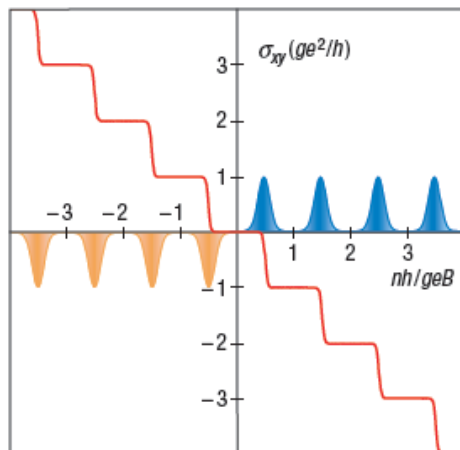
2. Anomalous room temperature quantum Hall effect

(1) Chiral quantum Hall effect in (monolayer) graphene $\Delta E \approx 2800\text{K}$



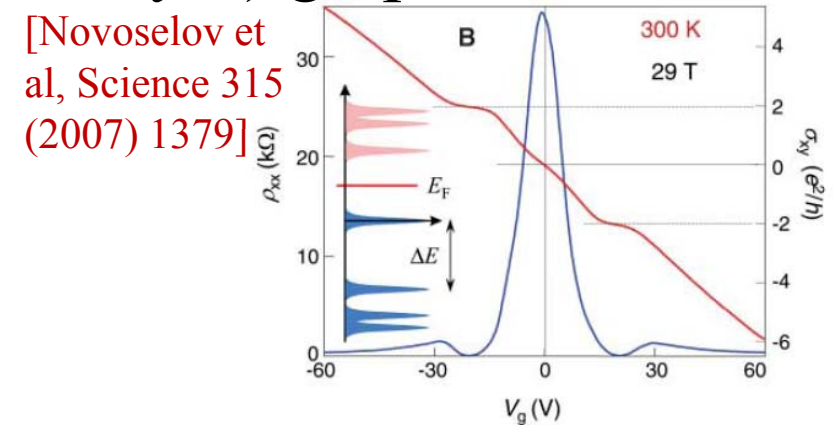
$$\sigma_{xy} = \pm 4 \frac{e^2}{h} \left(n + \frac{1}{2} \right)$$

[Novoselov et al.,
Nature 438 (2005)
197; Zhang et al,
Nature 438 (2005)
201]

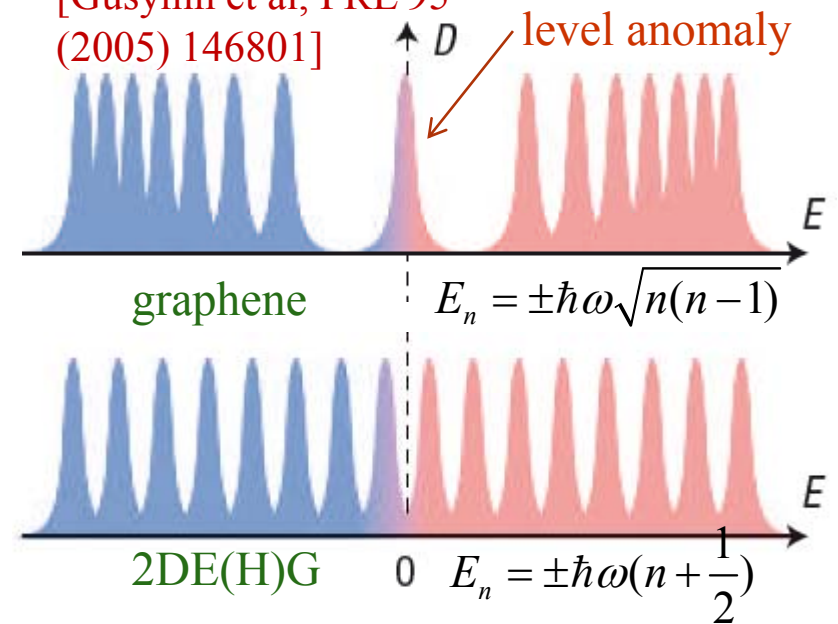


2DE(H)G

$$\sigma_{xy} = \pm 2 \frac{e^2}{h} n$$



[Gusynin et al, PRL 95 (2005) 146801]

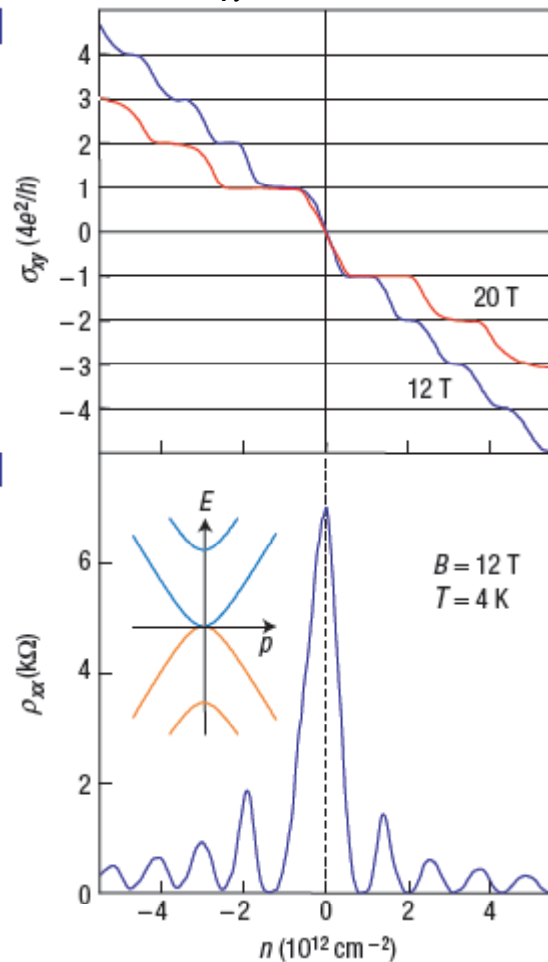


(2) Integer quantum Hall effect in bilayer graphenes

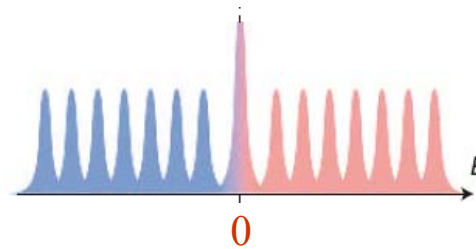
[Hsu, Guo, PRB 82 (2010) 165404]

AB bilayer graphene

$$\sigma_{xy} = \pm 4 \frac{e^2}{h} n, \quad (n = 1, 2, 3, \dots)$$



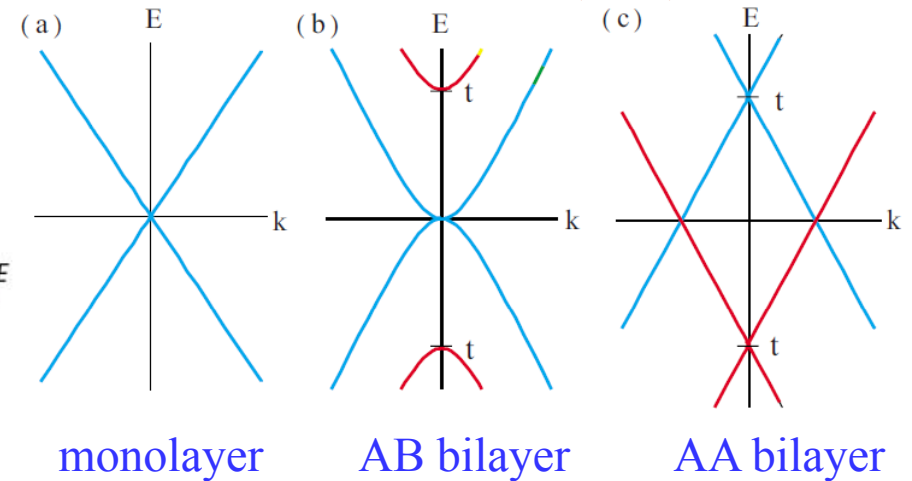
Landau level



$$E_n = \pm \hbar \omega \sqrt{2n}$$

[McCann et al., PRL 96 (2006) 086805]

Energy bands

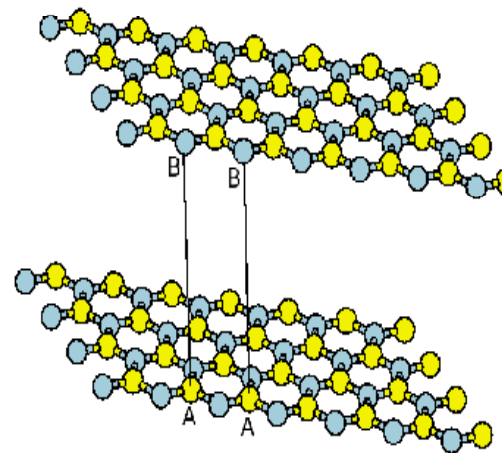


monolayer

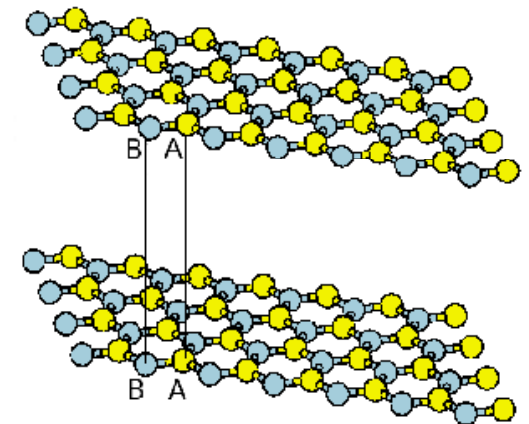
AB bilayer

AA bilayer

AB Stacking



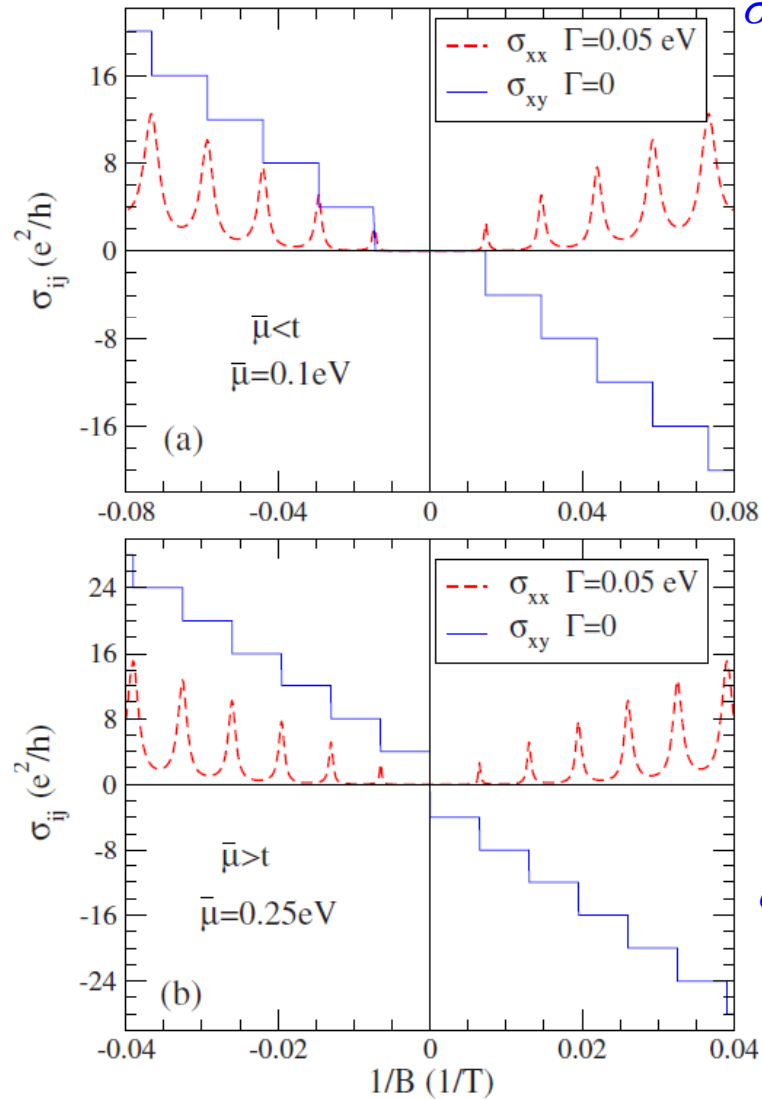
AA Stacking



[Novoselov et al., Nat.Phys. 2 (2006) 178]

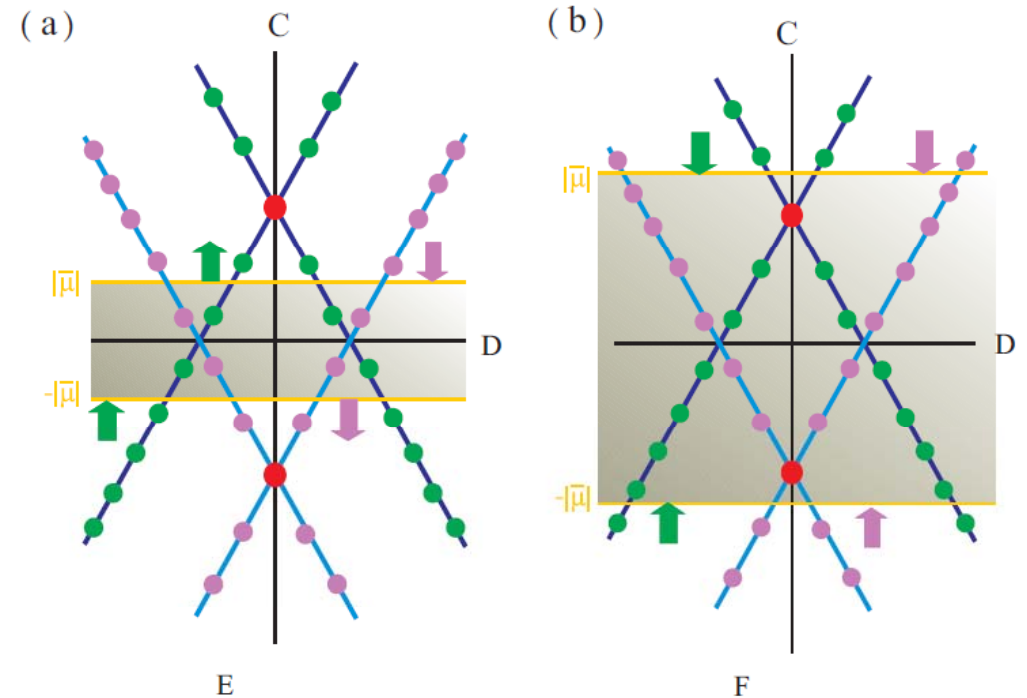
AA bilayer graphene

[Hsu, Guo, PRB 82 (2010) 165404]



$$\sigma_{xy} = \pm 4 \frac{e^2}{h} n, (n = 0, 1, 2, \dots)$$

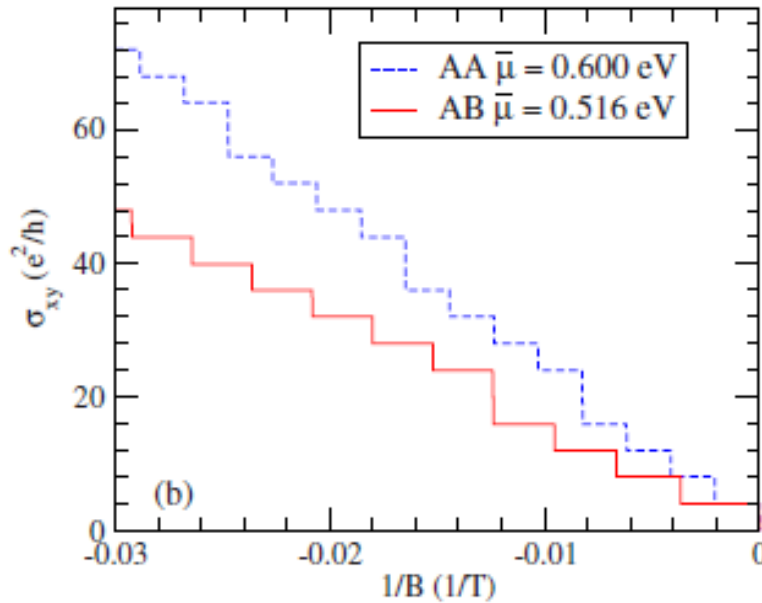
Landau level



$$E_n^{\mu\nu} = -\mu\sqrt{n}\hbar\omega - \nu t, \text{ with } \mu = \pm, \nu = \pm$$

$$\sigma_{xy} = \pm 4 \frac{e^2}{h} n, (n = 1, 2, 3, \dots)$$

$$|\mu| \neq \frac{\sqrt{n_2} + \sqrt{n_1}}{\sqrt{n_2} - \sqrt{n_1}} t, (n_2 > n_1, n_1 = 1, 2, 3, \dots; n_2 = 2, 3, 4, \dots)$$



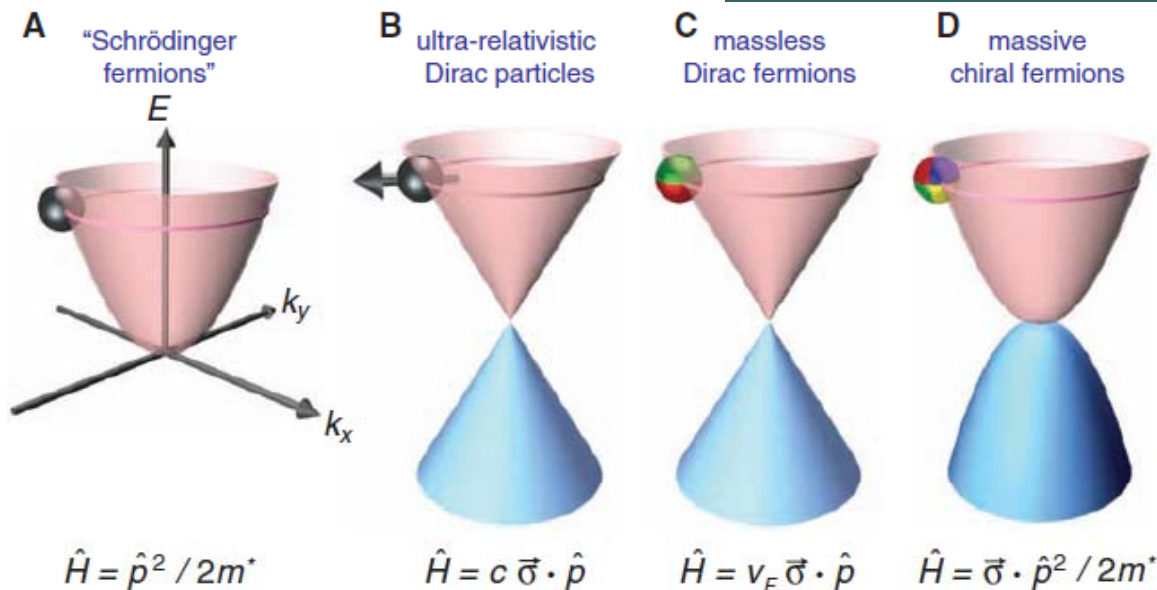
[Hsu, Guo, PRB 82 (2010) 165404]

$$\text{If } |\mu| = \frac{\sqrt{n_2} + \sqrt{n_1}}{\sqrt{n_2} - \sqrt{n_1}} t, (n_2 > n_1, n_1 = 1, 2, 3, \dots; n_2 = 2, 3, 4, \dots)$$

plateaus $\bar{\nu} = \pm 4(n_1 + n_2)n$, ($n = 1, 2, 3, \dots$), are absent.

e.g., $n_1 = 1, n_2 = 4$, no plateaus $\bar{\nu} = \pm 5, \pm 10, \pm 15, \dots$.

	2D electron (hole) gas	ML graphene	AB-bilayer graphene	AA-bilayer graphene
Steps (e^2/h)	2	4	4	4, 8
$\nu = 0$ plateau	Yes	No	No	Yes



Summary

[Geim, Science 324 (2009) 1530]

3. “Big” physics on a chip: Klein paradox and Zitterbewegung

1) Klein paradox (tunneling)

Barrier tunneling of Schroedinger particles

Transmission coefficient

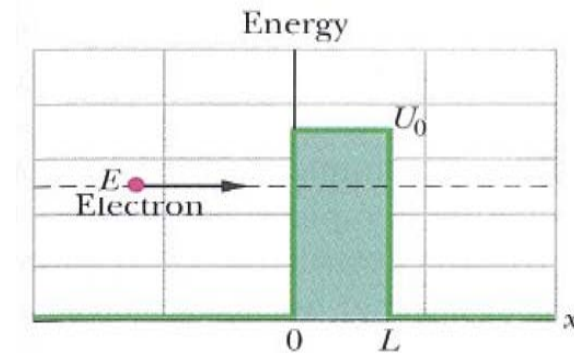
$$T \approx e^{-2kL}, \quad k = \sqrt{\frac{8\pi^2 m(U_0 - E)}{h^2}}$$

Klein paradox [Klein, Z. Phys. 53 (1929) 157]

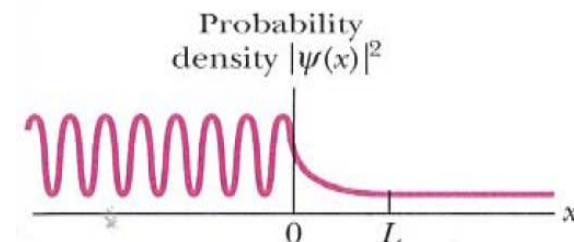
An incoming electron starts penetrating through a potential barrier if its height U_0 (V) exceeds the twice its rest energy, i.e., $U_0 \geq 2mc^2$ (a paradoxical implication of the Dirac equation).

Klein's gedanken experiment has never been carried out, owing to the extreme conditions involved, e.g., an electric field of 10^{16} Vcm^{-1} needed.

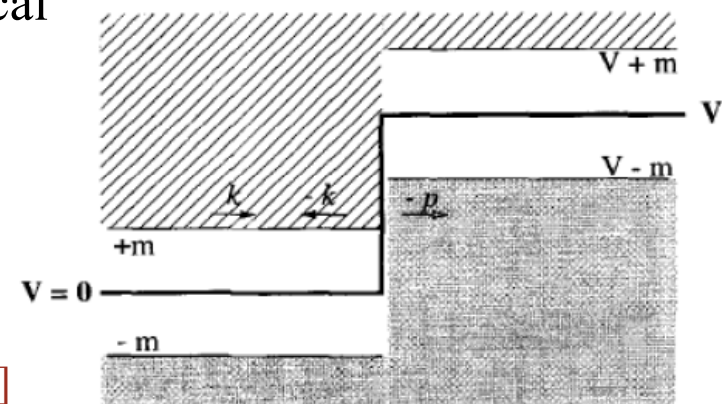
[Calogeros, Cont. Phys. 40 (1999) 313]



(a)



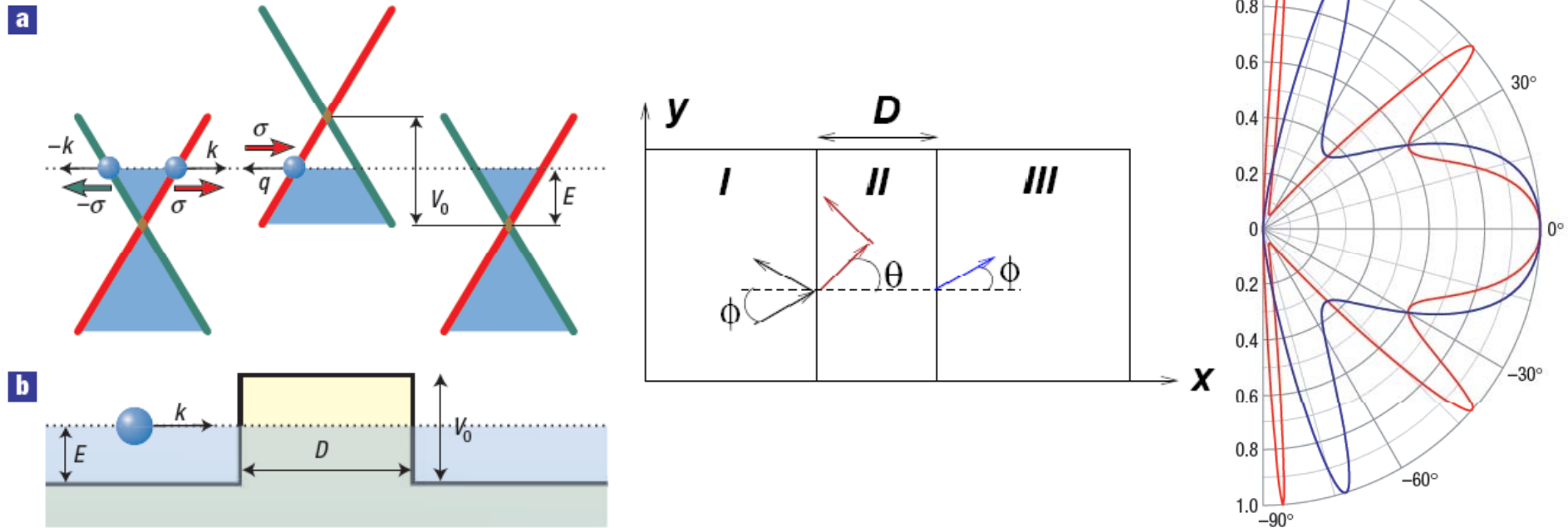
(b)



Klein's gedanken experiment on a graphene chip

[Katsnelson, et al., NatPhys 2 (2006) 620]

Transmission probability



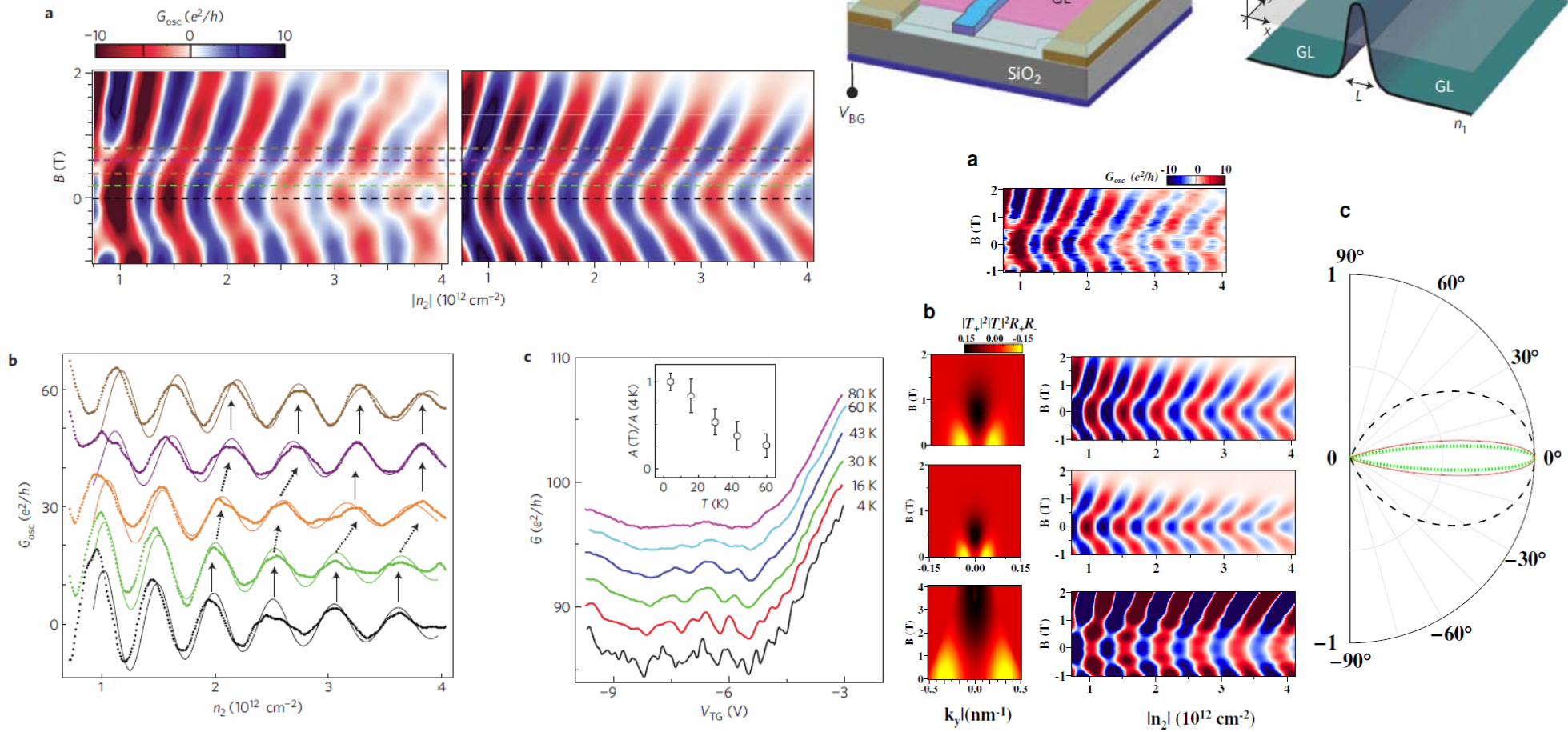
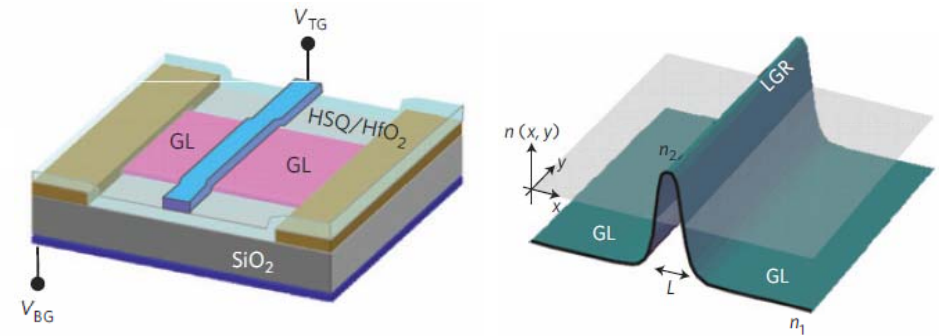
Transmission probability
$$T(\phi) = \frac{\cos^2 \theta \cos^2 \phi}{[\cos(Dq_x) \cos \phi \cos \theta]^2 + \sin^2(Dq_x)(1 + \sin \phi \sin \theta)^2}$$

Perfect tunneling is the consequence of masslessness and pseudospin conservation.

Experiments on graphene [weak and controversial evidence so far]

Quantum interference and Klein tunnelling in graphene heterojunctions

Andrea F. Young and Philip Kim★ [NatPhy 5 (2009) 222]



Evidence for Klein Tunneling in Graphene p - n Junctions

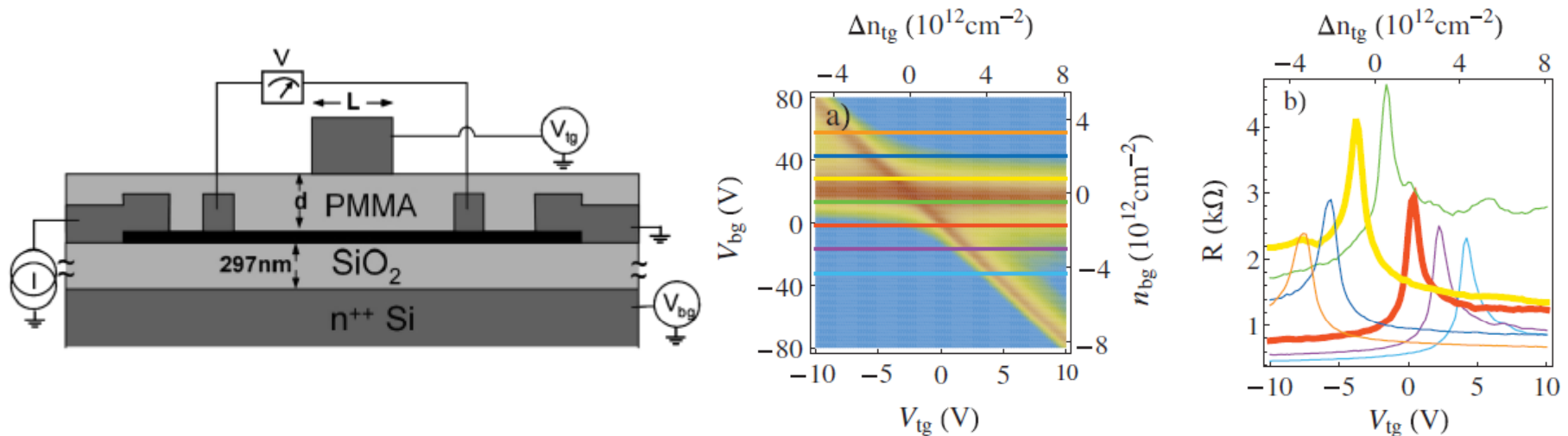
[PRL 102 (2009) 026807]

N. Stander, B. Huard, and D. Goldhaber-Gordon*

Department of Physics, Stanford University, Stanford, California 94305, USA

(Received 13 June 2008; published 16 January 2009)

Transport through potential barriers in graphene is investigated using a set of metallic gates capacitively coupled to graphene to modulate the potential landscape. When a gate-induced potential step is steep enough, disorder becomes less important and the resistance across the step is in quantitative agreement with predictions of Klein tunneling of Dirac fermions up to a small correction. We also perform magnetoresistance measurements at low magnetic fields and compare them to recent predictions.

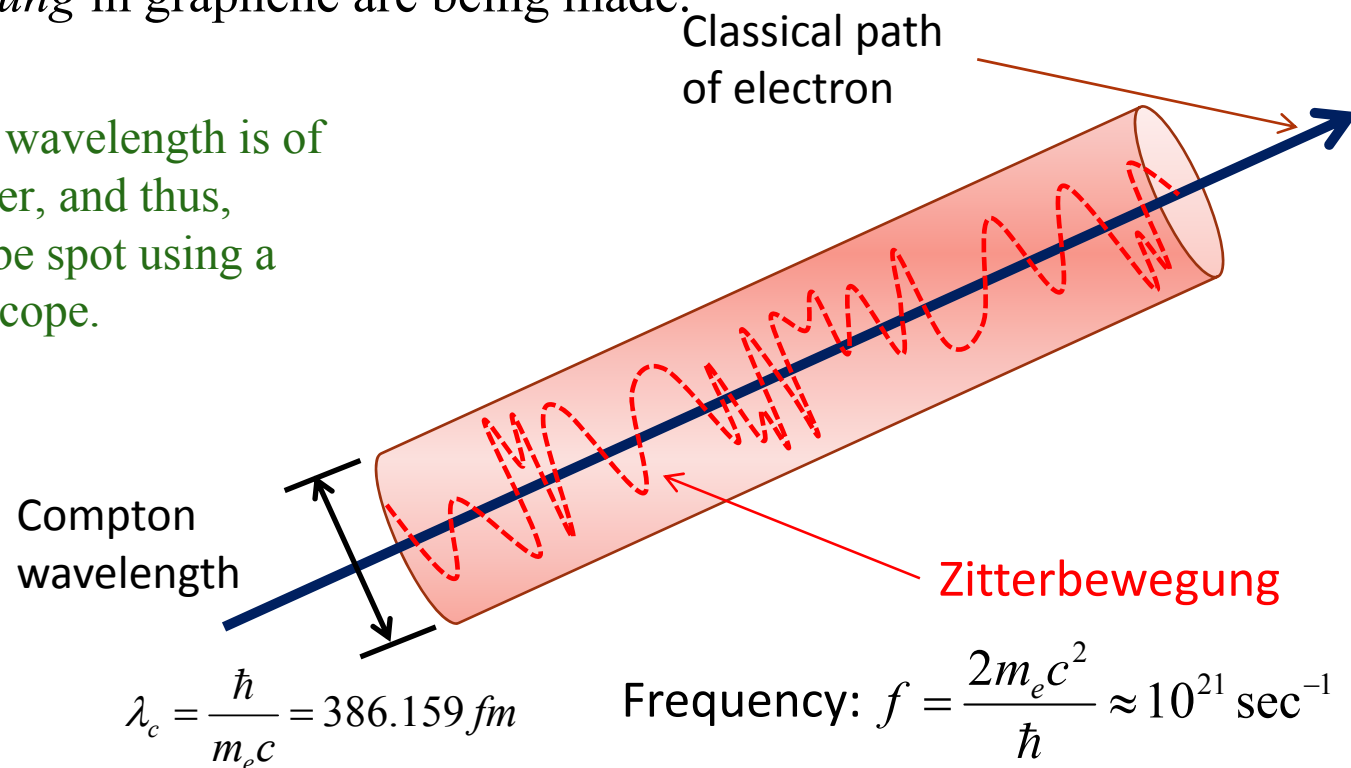


2) Zitterbewegung (trembling motion)

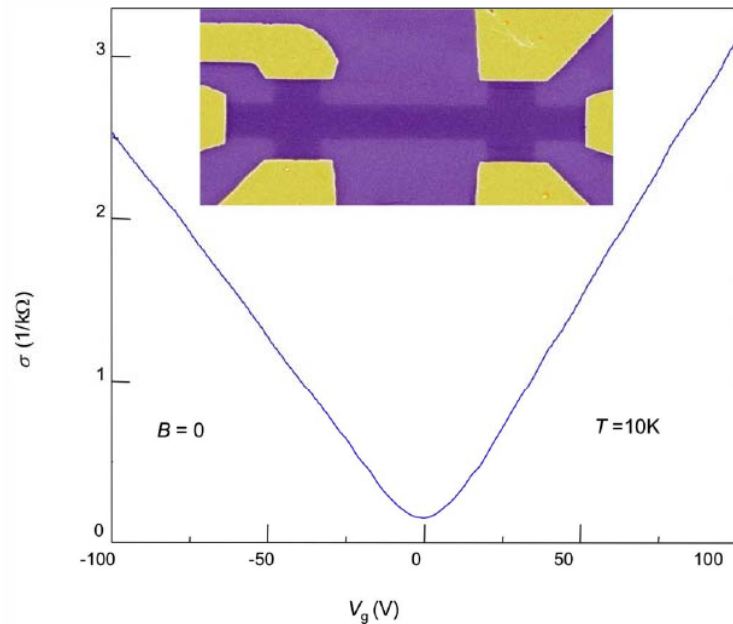
refers a rapid motion of a free Dirac (electron) particle pointed out by Schrodinger (1930), i.e., a fluctuation (at speed c) of the position of the particle around the median, due to an interference between positive and negative energy states (charge conjugate symmetry).

Zitterbewegung of a free particle has never been observed, but now attempts to observe *Zitterbewegung* in graphene are being made.

In graphene, Compton wavelength is of the order of a nanometer, and thus, zitterbewegung could be spot using a high-resolution microscope.



[Novoselov et al., Nature 438 (2005) 197]



Conductivity
$$\sigma_{xx} = 2 \frac{e^2}{h} \frac{\pi v_F^2}{u_0^2} n$$

It has been argued that the observed minimal conductivity is a manifestation of Zitterbewegung.

Zitterbewegung, chirality, and minimal conductivity in graphene

M.I. Katsnelson^a

Eur. Phys. J. B 51 (2006) 157.

Zitterbewegung effect is responsible for the minimal conductivity, which is explained as the impossibility to localize ultrarelativistic particles and to measure their coordinates. Zitterbewegung introduces the “intrinsic” disorder in the system which causes presence of a minimal conductivity even for ideal (without scattering) crystals at low temperatures. The relativistic “jittering” of electron could be interpreted in terms of classical physics as an interaction of electron with some potential caused by the presence of disorder in crystal.

4. Specular Andreev reflection and tunneling conductance

1) Unique specular Andreev reflection at graphene N-S interface

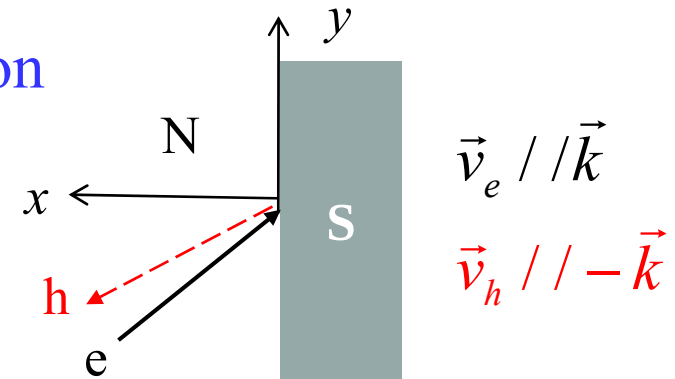
[Beenakker, PRL 97 (2006) 067007]

Normal metal-superconductor (N/S) junction

$$\begin{pmatrix} H - E_F & \Delta \\ \Delta^* & E_F - THT^{-1} \end{pmatrix} \begin{pmatrix} u \\ v \end{pmatrix} = \varepsilon \begin{pmatrix} u \\ v \end{pmatrix} \quad (\text{BdG equation})$$

$$H = -\frac{\hbar^2}{2m} \nabla^2 + U(\mathbf{r}),$$

T = time-reversal operator.



Andreev retroreflection

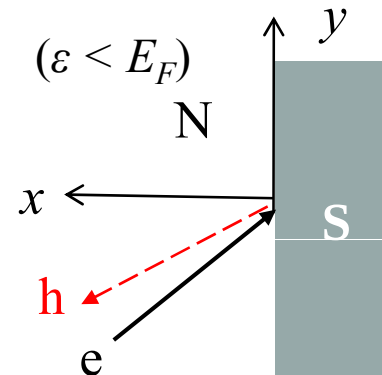
Graphene metal-superconductor junction

$$\begin{pmatrix} H_{\pm} - E_F & \Delta \\ \Delta^* & E_F - H_{\pm} \end{pmatrix} \begin{pmatrix} u^{\pm} \\ v^{\pm} \end{pmatrix} = \varepsilon \begin{pmatrix} u^{\pm} \\ v^{\pm} \end{pmatrix}$$

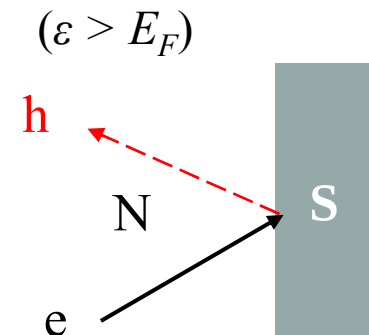
$$H_{\pm} = -i\hbar v_F (\sigma_x \partial_x \pm \sigma_y \partial_y) + U(\mathbf{r}),$$

$$\varepsilon = \sqrt{|\Delta|^2 + (E_F - U \pm \hbar v_F |\mathbf{k}|)^2}.$$

(Dirac-BdG equation)



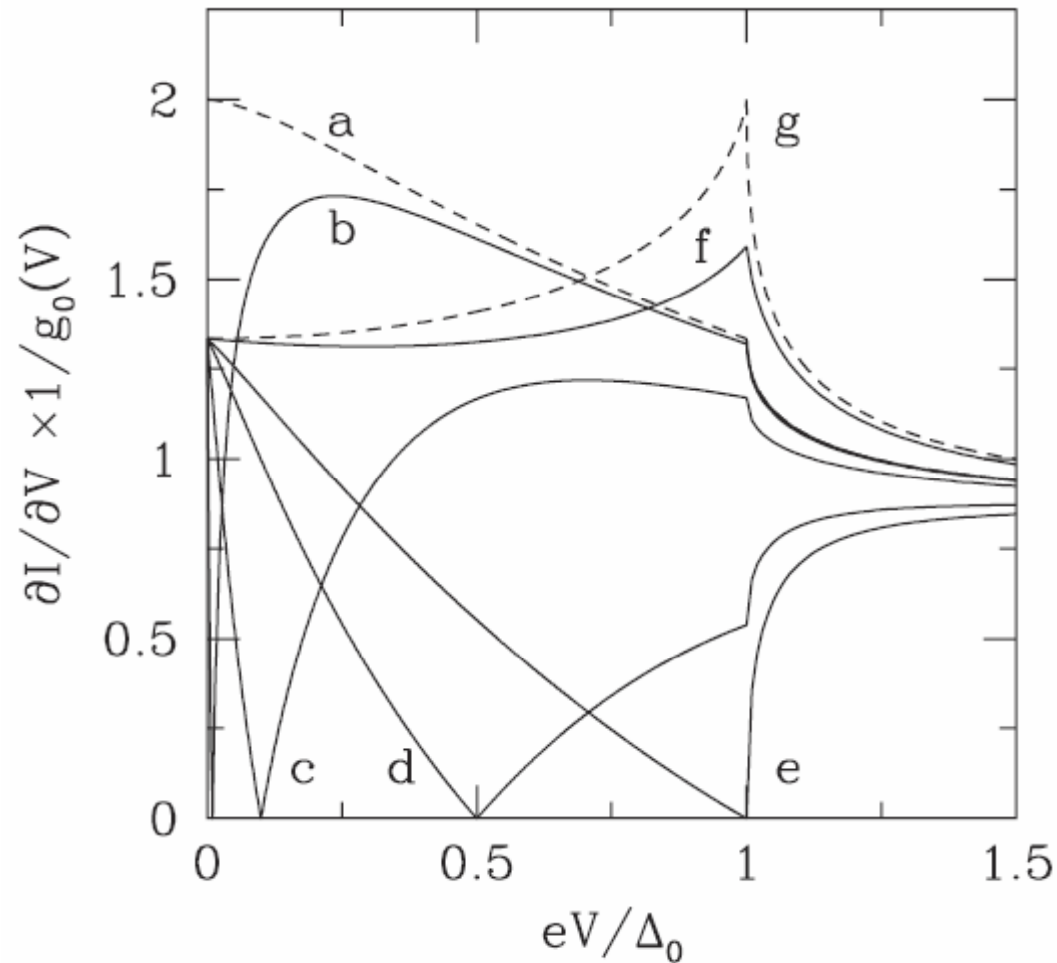
Andreev retroreflection



Specular Andreev reflection

Graphene metal-superconductor junction

[Beenakker, PRL 97 (2006) 067007]

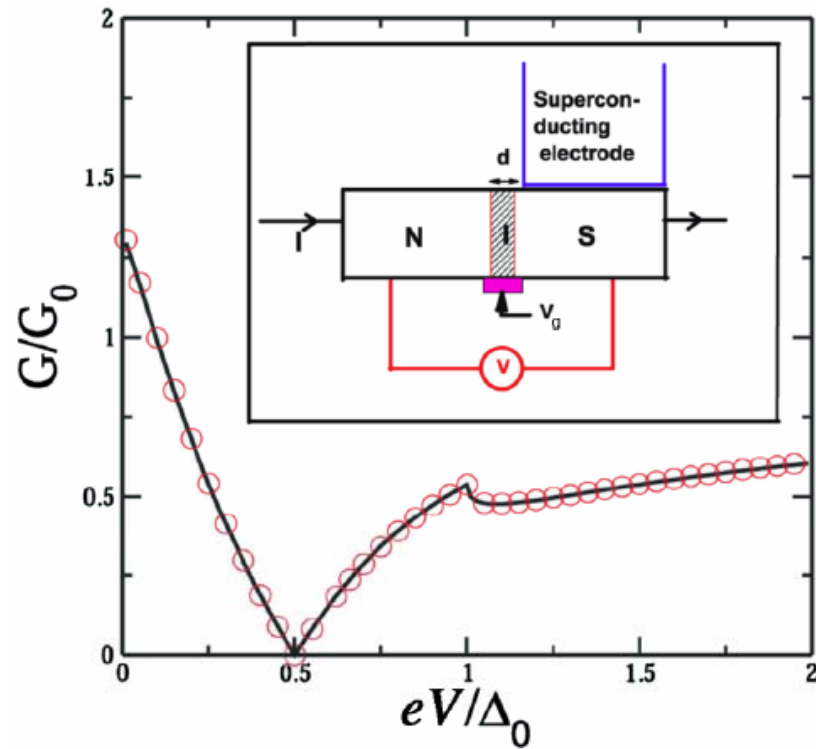


- $U_0 \gg E_F, \Delta_0, E$
- a. $E_F / \Delta_0 \rightarrow 0$
specular reflection predominant
 - b. $E_F / \Delta_0 = 0.01$
 - c. $E_F / \Delta_0 = 0.1$
 - d. $E_F / \Delta_0 = 0.5$
 - e. $E_F / \Delta_0 = 1$
 - f. $E_F / \Delta_0 = 10$
 - g. $E_F / \Delta_0 \rightarrow \infty$
retro-reflection predominant

2) Tunneling conductance of graphene NIS junctions

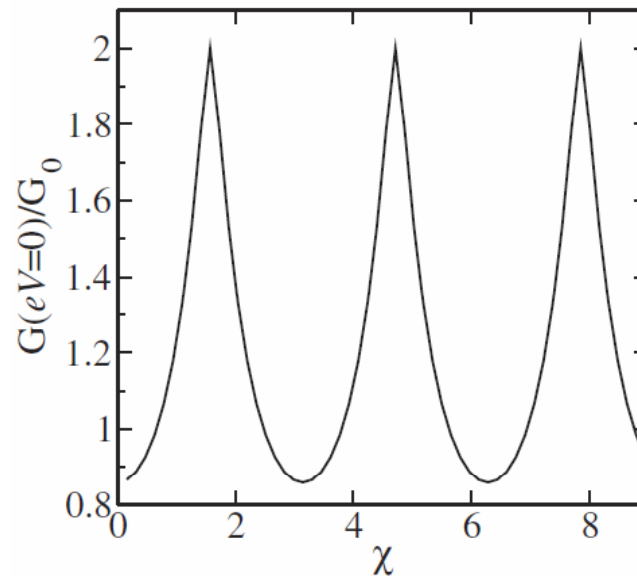
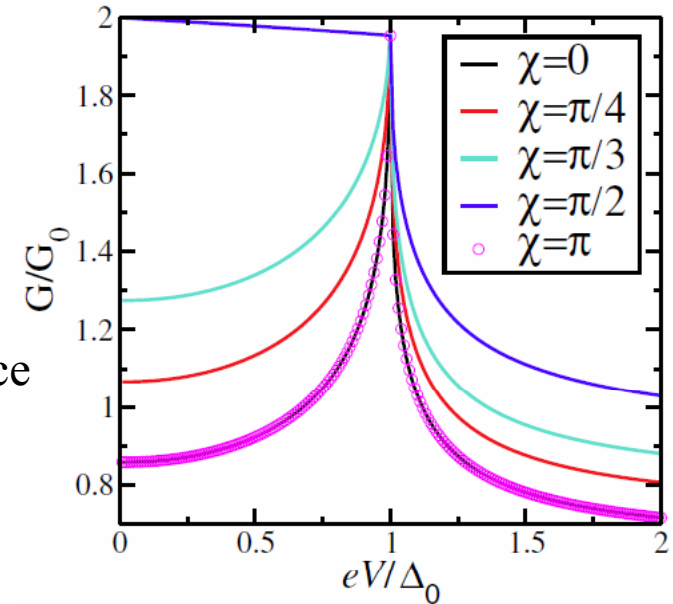
[Bhattacharjee, Sengupta, PRL 97 (2006) 217001]

Normal metal-insulator-superconductor
(NIS) junction



Effective barrier strength $\chi = 0$ (solid lines),
 $\chi = \pi/2$ (open circles).

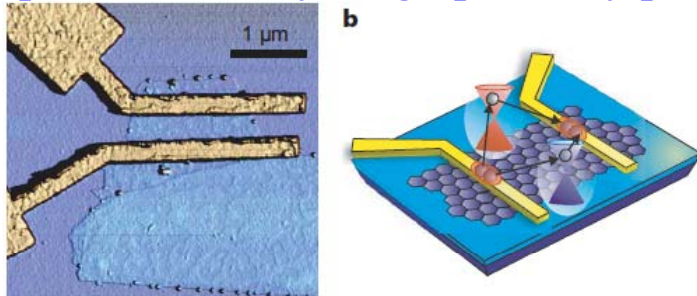
Conductance
vs bias



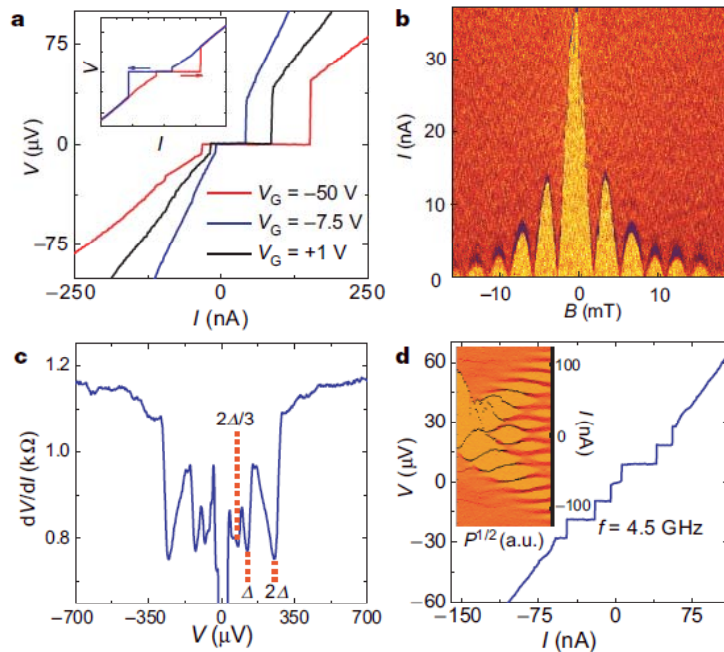
Conductance vs
barrier strength

3) Tunneling conductance of graphene FIS junctions

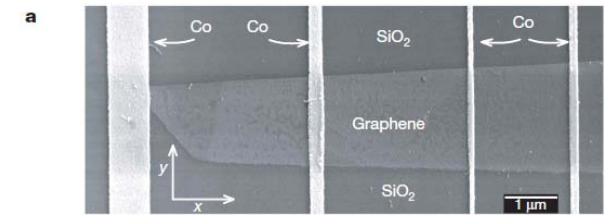
Superconductivity on graphene by proximity



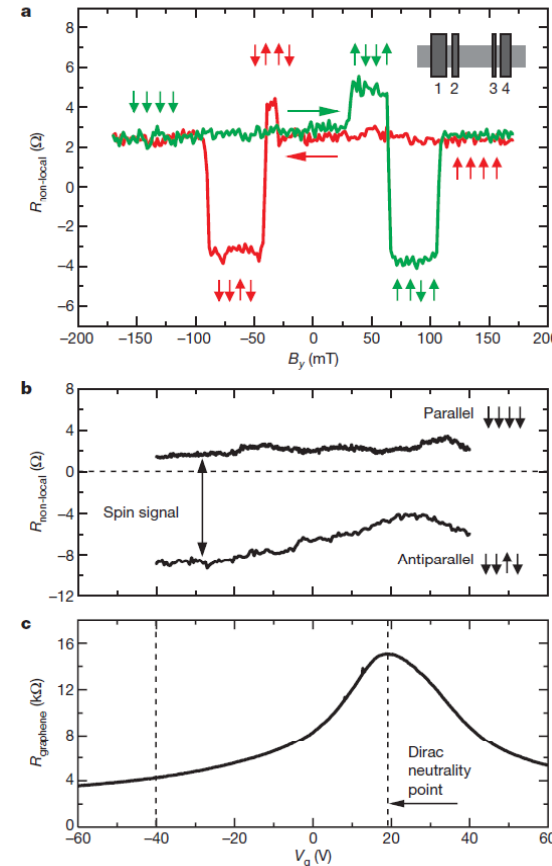
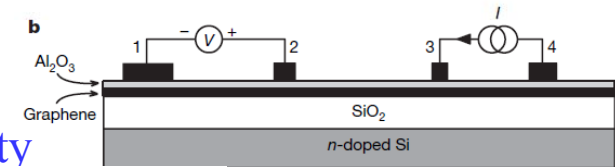
Josephson effect in graphene



[Heersche, Nature 446 (2007) 56]



Ferromagnetism on
graphene by proximity

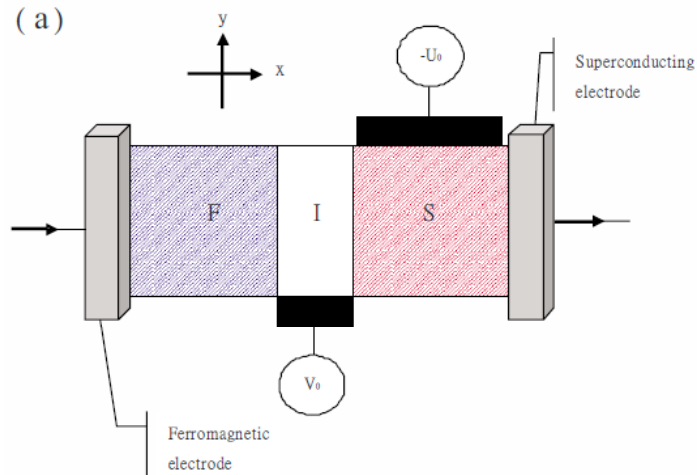


Spin transport at
room temperature
in graphene

[Tombros, Nature
448 (2007) 571]

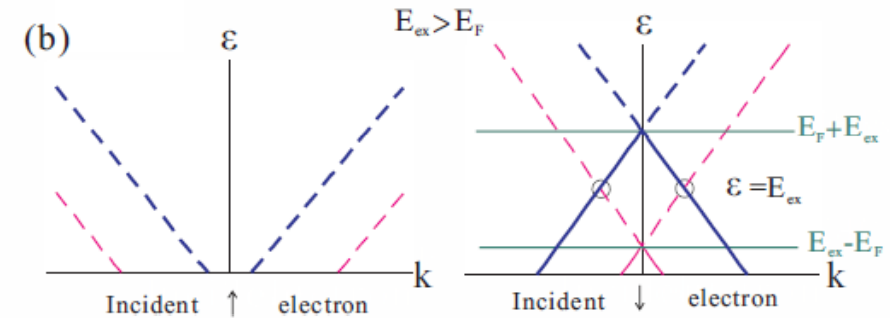
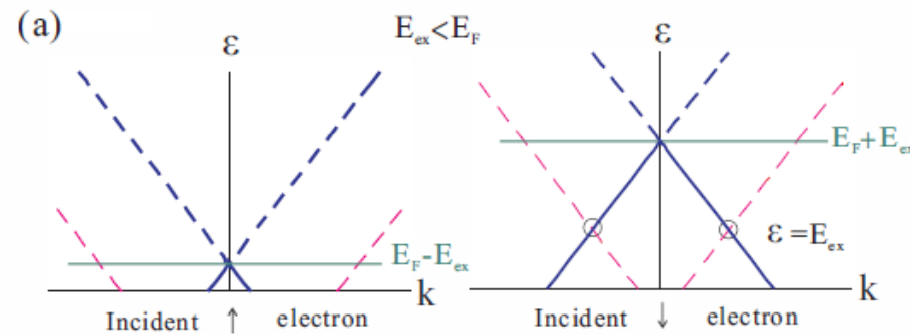
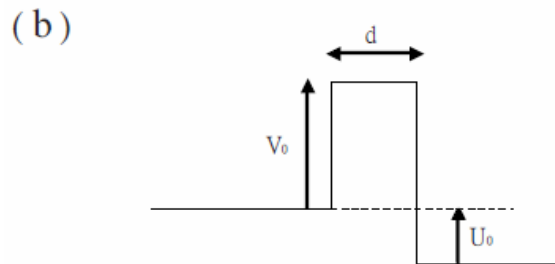
Ferromagnetic metal-insulator-superconductor (FIS) junction

[Hsu, Guo, PRB 81 (2010) 045412]

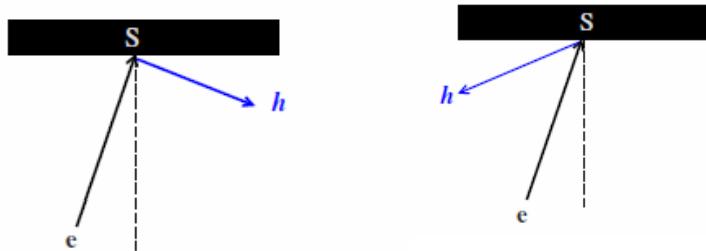


$$\begin{pmatrix} H_{\pm} - E_F - \rho_{\sigma} h & \Delta \\ \Delta^* & E_F - H_{\pm} - \rho_{\sigma} h \end{pmatrix} \begin{pmatrix} u_{\sigma}^{\pm} \\ v_{\sigma}^{\pm} \end{pmatrix} = \varepsilon \begin{pmatrix} u_{\sigma}^{\pm} \\ v_{\sigma}^{\pm} \end{pmatrix}$$

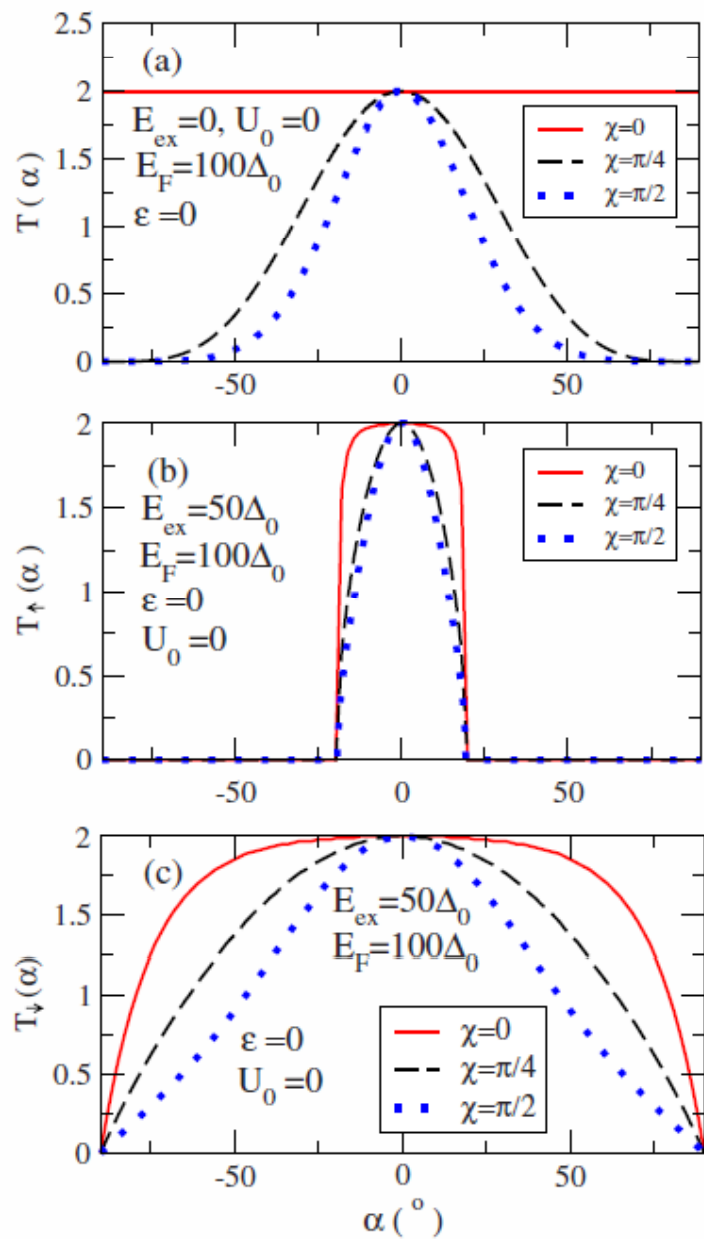
$$H_{\pm} = -i\hbar v_F (\sigma_x \partial_x \pm \sigma_y \partial_y) + U. \quad (\text{spin-polarized Dirac-BdG equation})$$



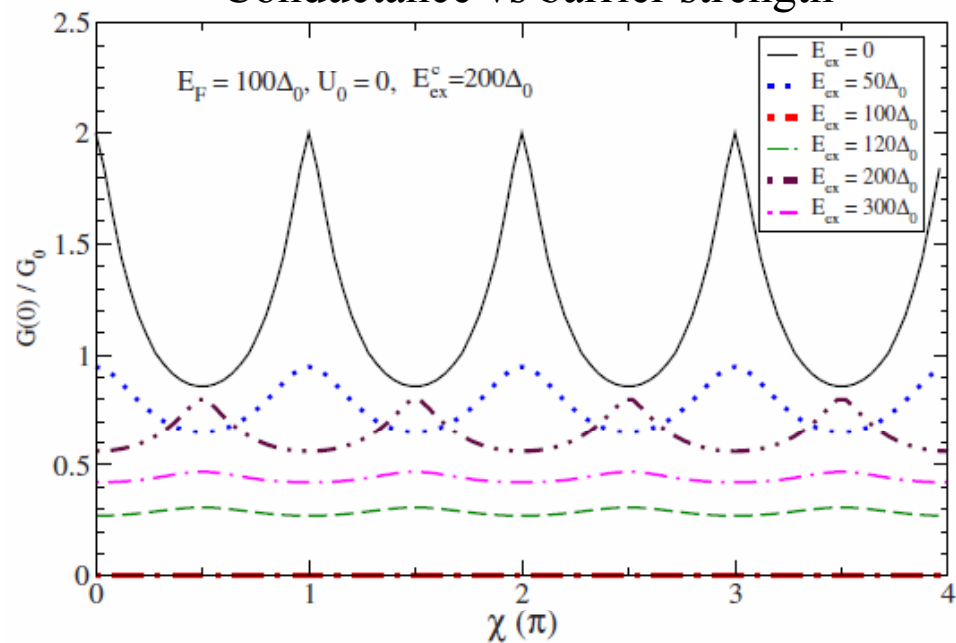
(a) modified specular Andreev reflection (b) modified retro-Andreev reflection



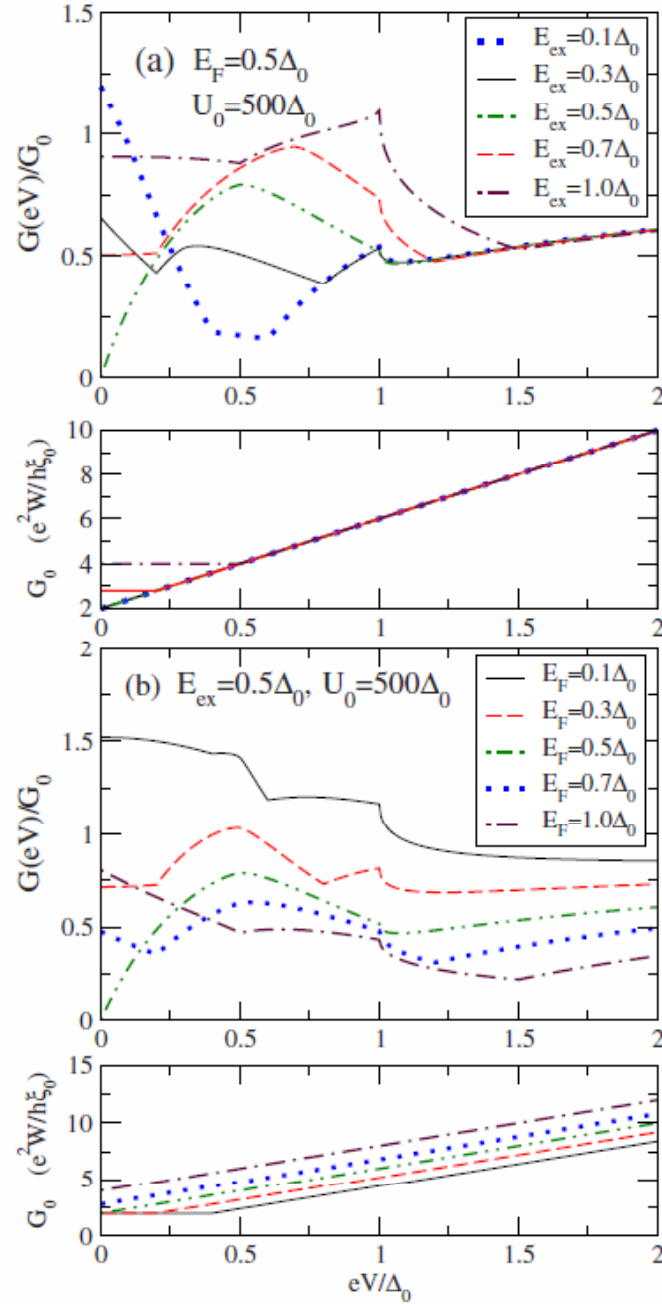
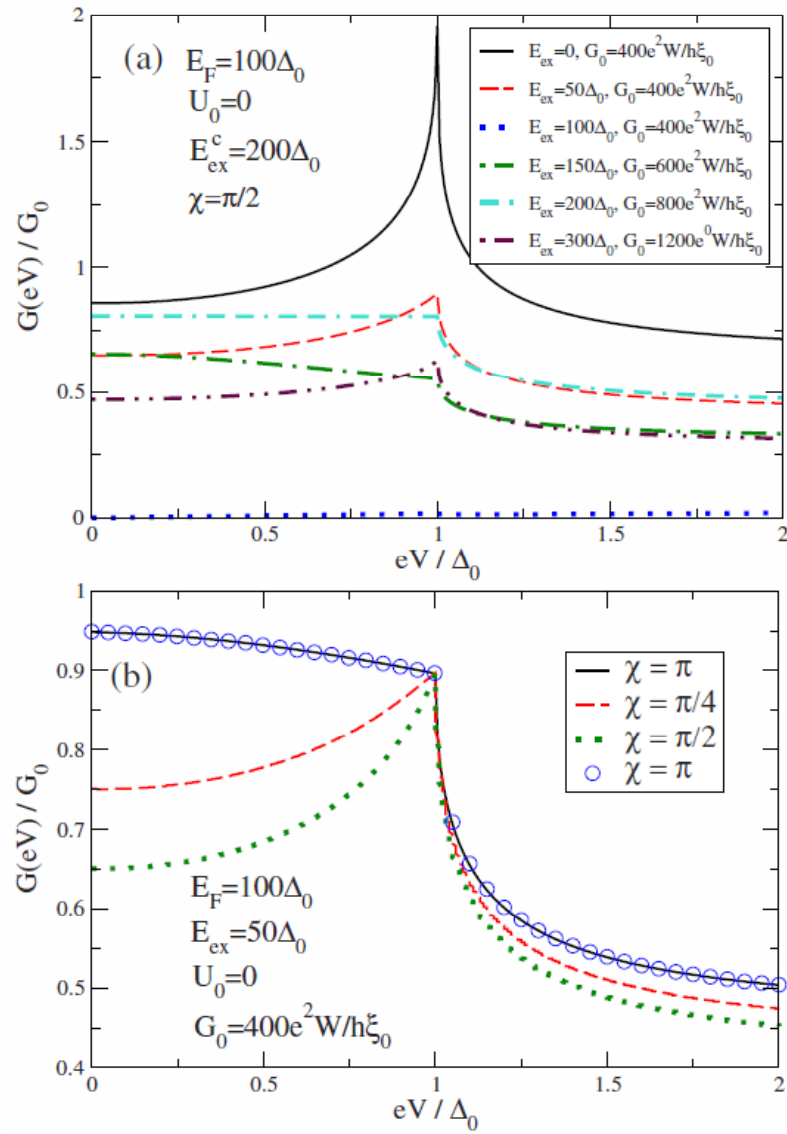
Transmission vs incident angle



Conductance vs barrier strength

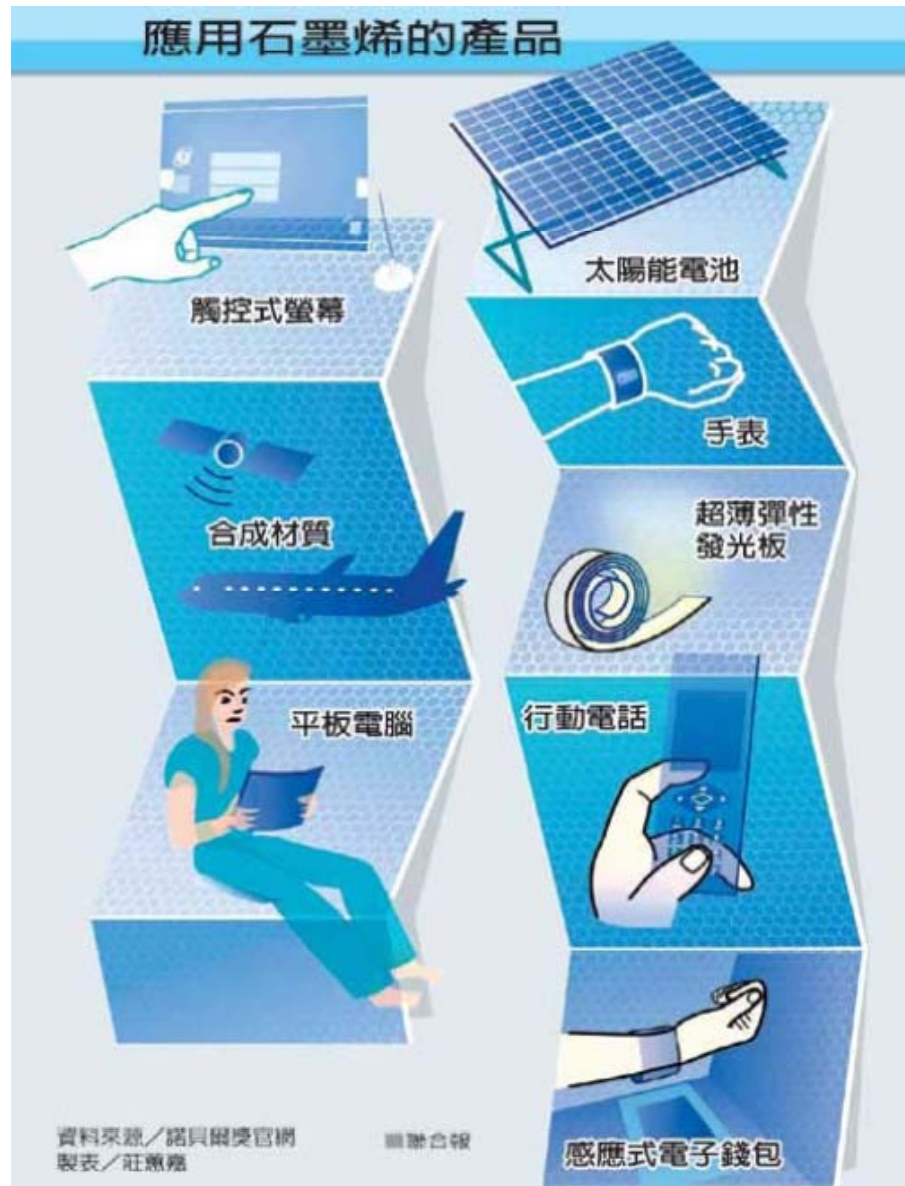


Conductance vs bias at different E_{ex}



Conductance vs bias
at different E_{ex}

IV. Possible applications of graphene



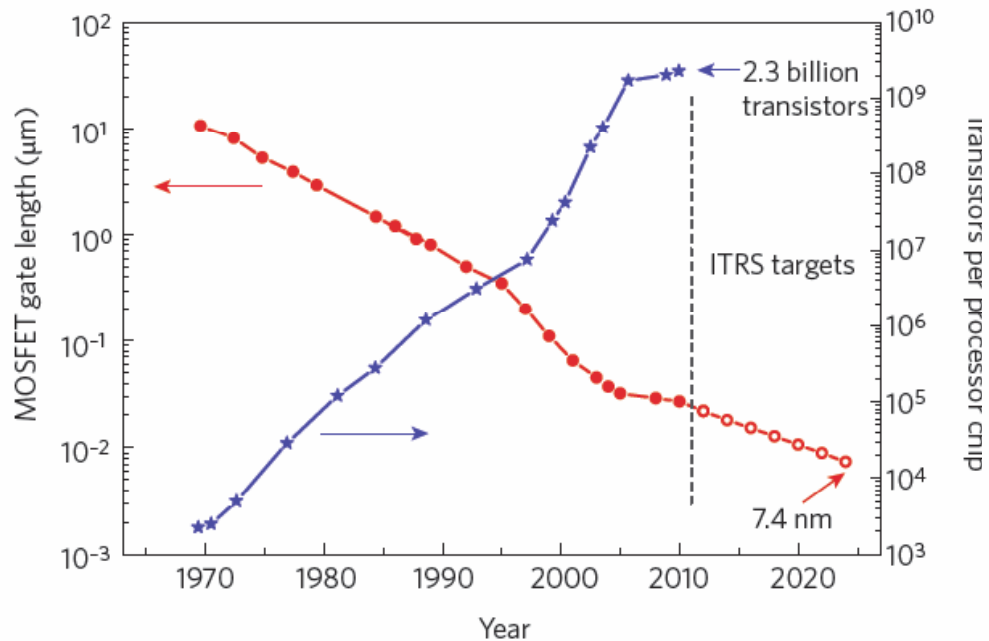
Possible graphen products

[from official Nobel Web Site]

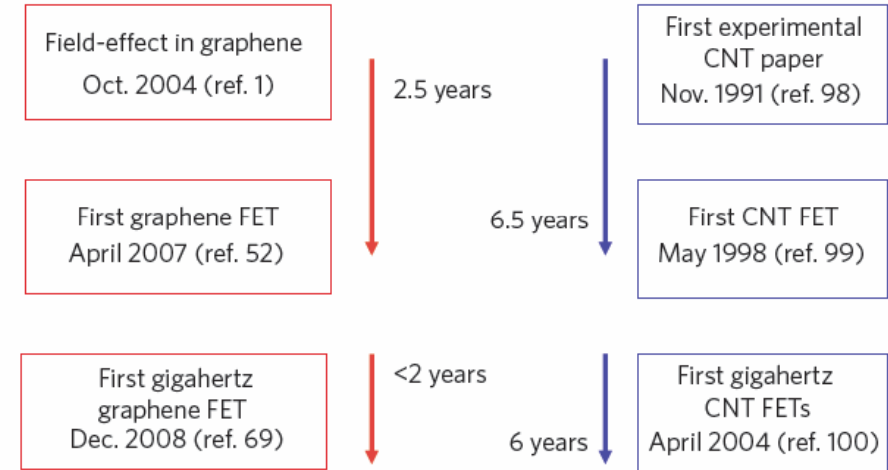
Dream worlds

1. Utrasmall and fast nanoelectronic devices

Graphene transistors [Schwierz, et al., NatNano 5 (2010) 487]



Structure and evolution of graphene

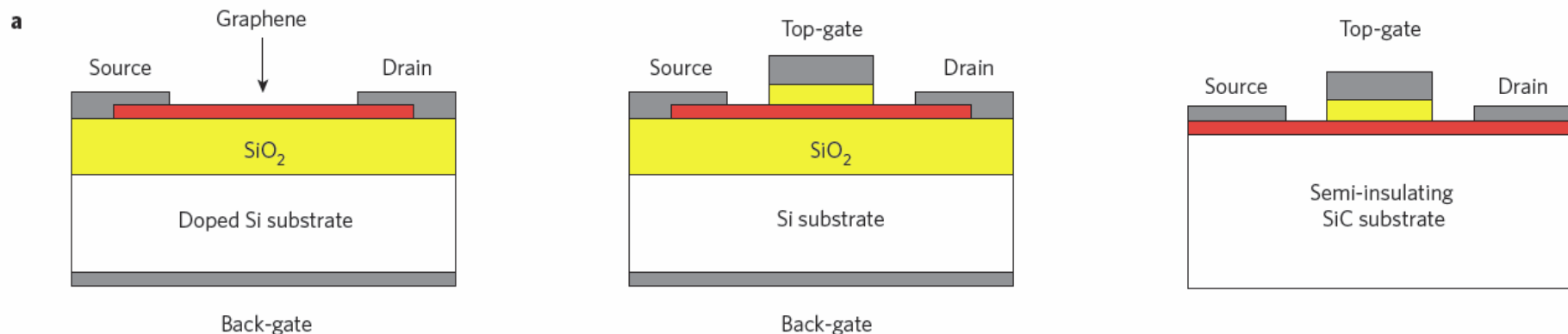


Record cut-off frequencies:

Graphene FET 100 GHz (ref. 73)

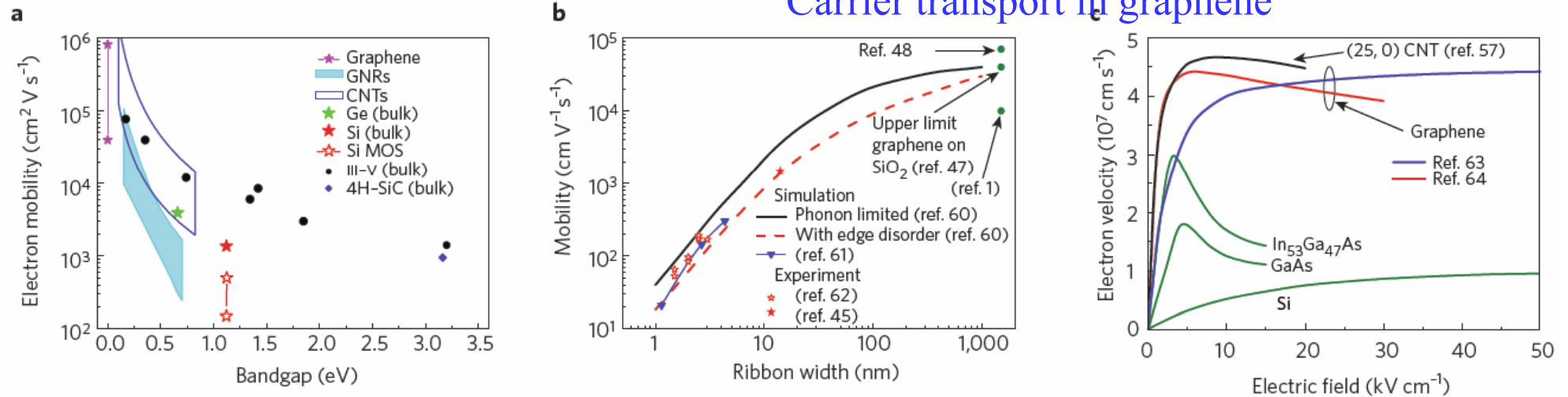
CNT FET 80 GHz (ref. 78)

Trends in digital electronics

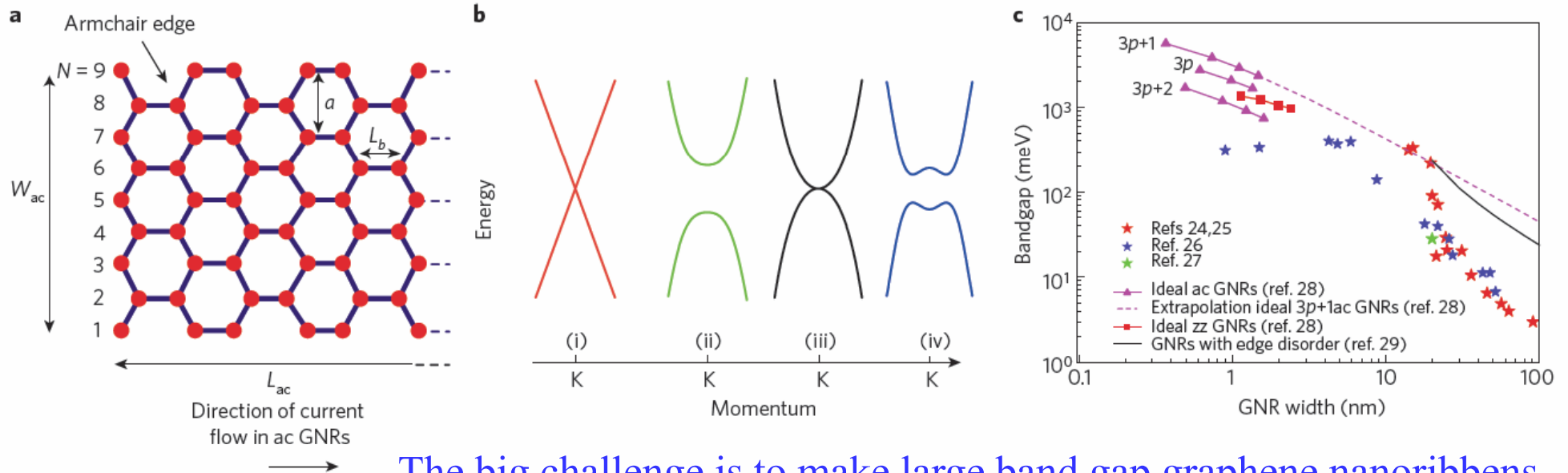


[Schwierz, et al., NatNano 5 (2010) 487]

Carrier transport in graphene



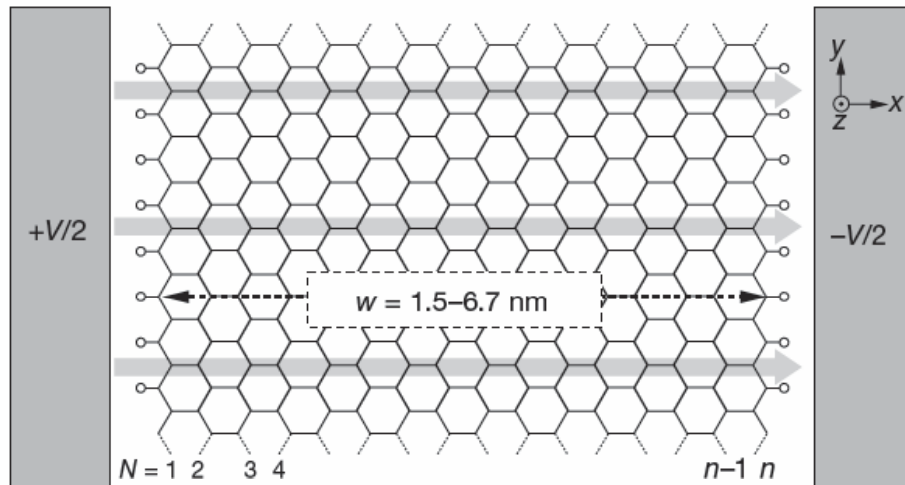
Properties of graphene and graphene nanoribbons



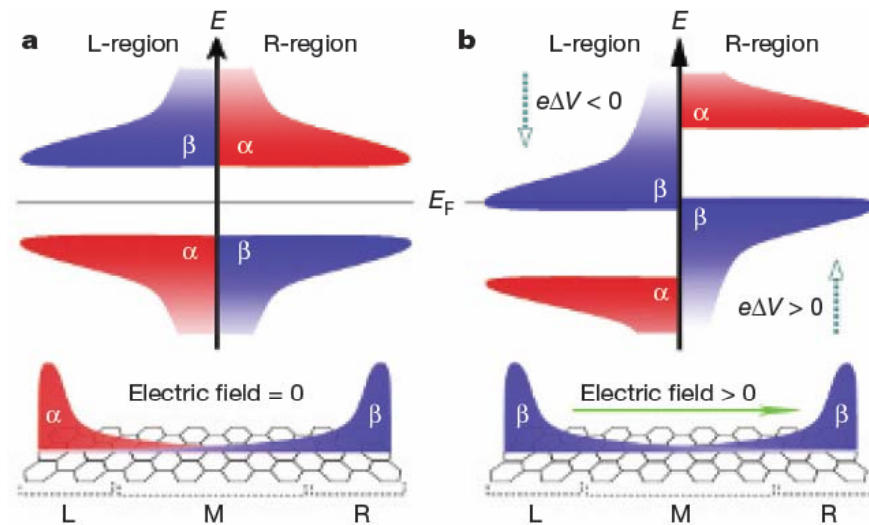
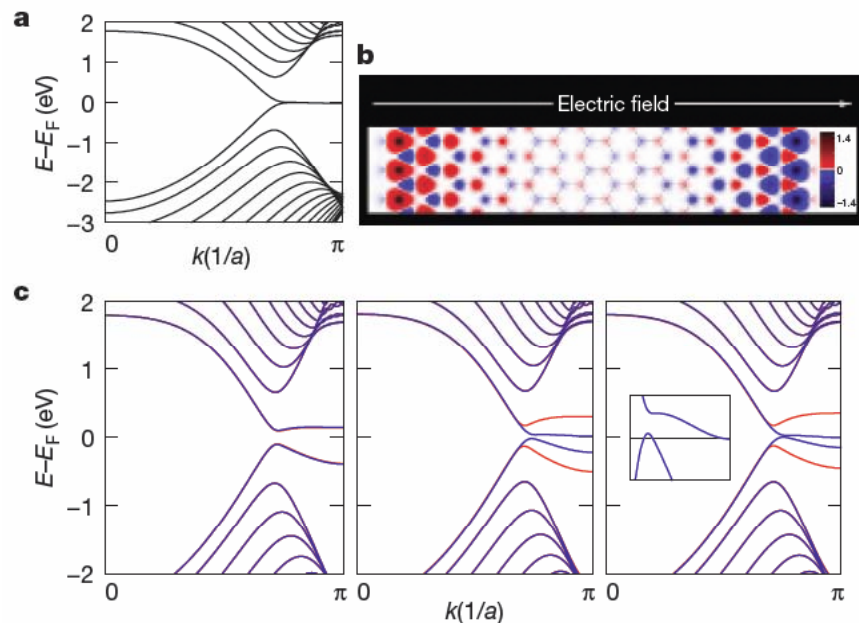
The big challenge is to make large band gap graphene nanoribbons.

2. Graphene-based spintronics and magneto-electronics

Half-metallic graphene nanoribbons



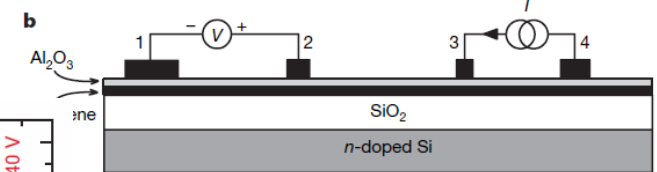
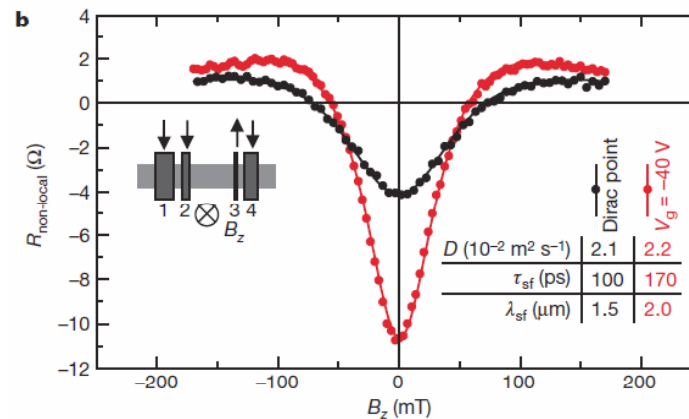
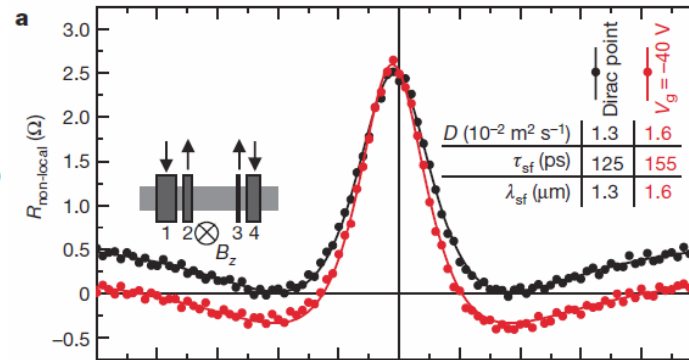
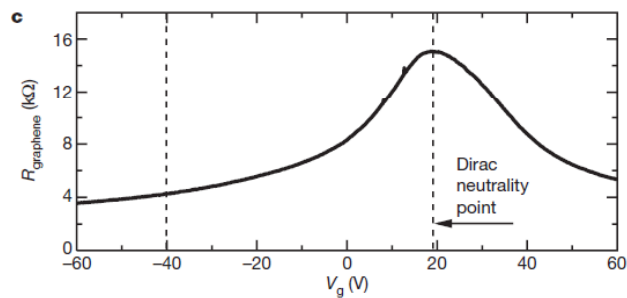
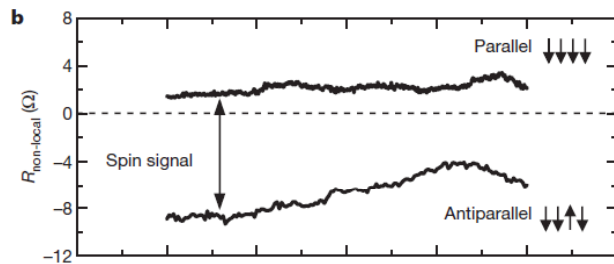
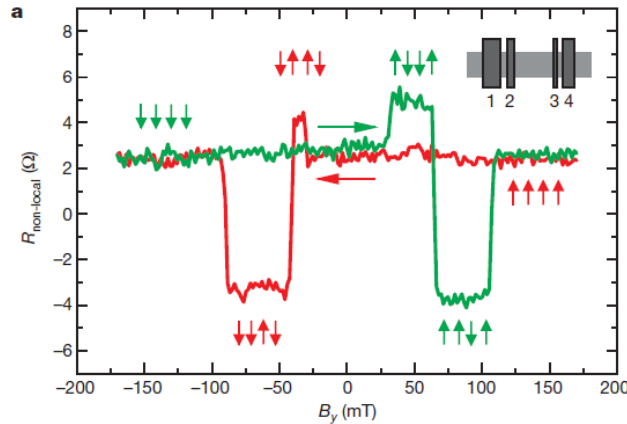
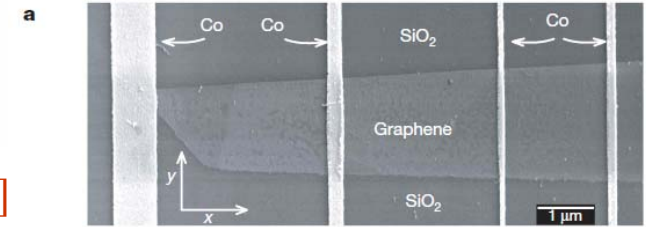
[Son, Cohen, Louie, Nature 444 (2006) 347]



Electronic spin transport and spin precession in single graphene layers at room temperature

Nikolaos Tombros¹, Csaba Jozsa¹, Mihaita Popinciuc², Harry T. Jonkman² & Bart J. van Wees¹

[Tombros, Nature 448 (2007) 571]



Four-terminal spin valve device

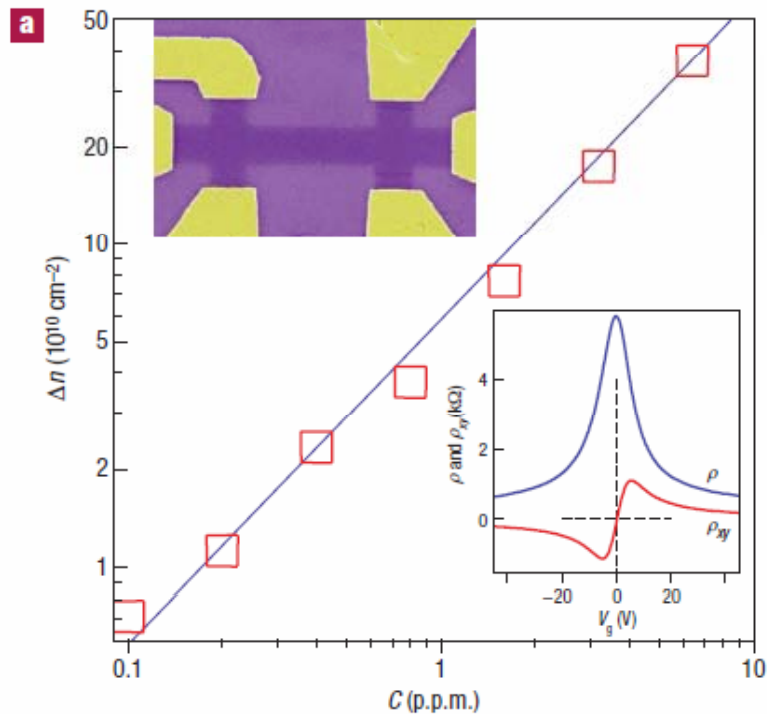
Hanle spin precession

Spin transport at room temperature in graphene

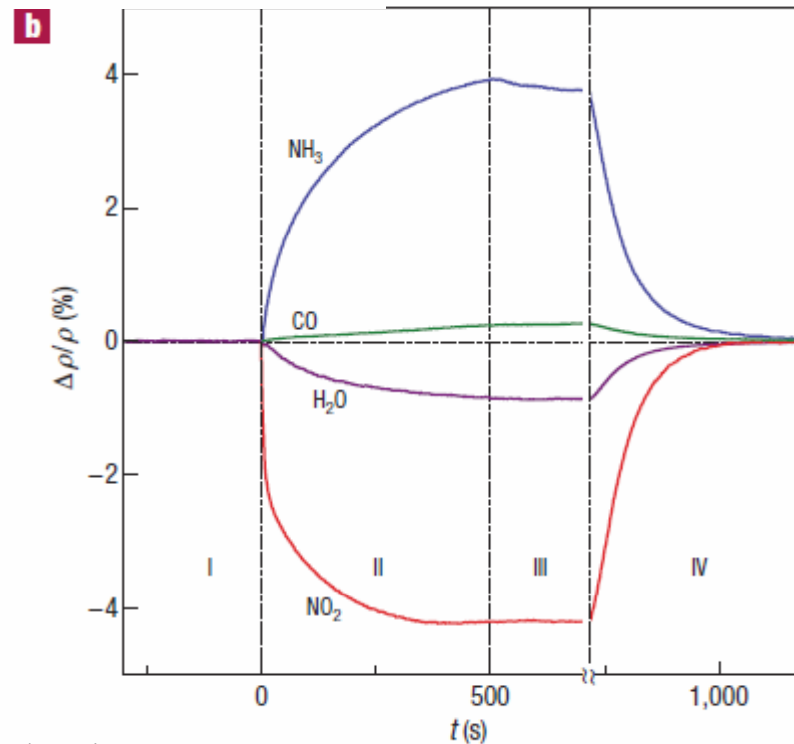
3. Molecule sensors and graphene biodevices

Detection of individual gas molecules
adsorbed on graphene [NatMat 6 (2007) 652]

F. SCHEDIN¹, A. K. GEIM¹, S. V. MOROZOV², E. W. HILL¹, P. BLAKE¹, M. I. KATSNELSON³
AND K. S. NOVOSELOV^{1*}



Sensitivity of graphene to chemical doping



Acknowledgements:

Discussions and Collaborations:

Ya-Fen Hsu (許雅芬) & Tsung-Wei Chen (陳宗緯) (NTU)
Wei-Li Lee (李偉立) (Academia Sinica)

Financial Support:

National Science Council

Thank you for your attention