2008 Summer workshop on nanoscale materials

CTS, NCKU

Spin dipole induced by the spin-Hall effect in the diffusion region

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Outline:

1. Introduction charge and spin dipoles around a scatterer
2. Spin-Hall effect
3. Spin dipole induced by one single impurity
4. Spin accumulation in a semi-infinite system
5. Summary

Charge and spin dipoles:

The presence of a point scatterer leads to a dipole field about the scatterer in a conducting current.

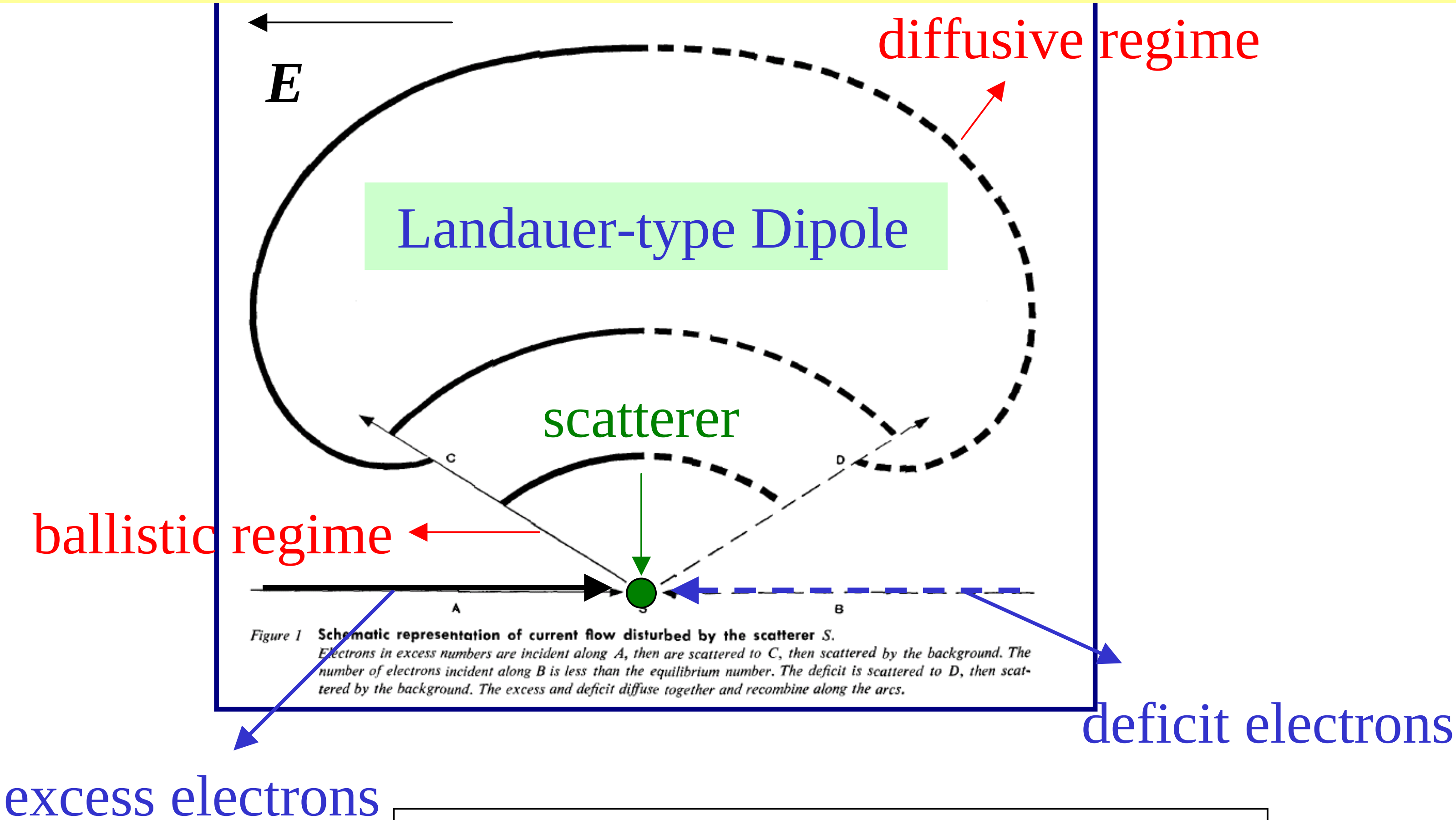
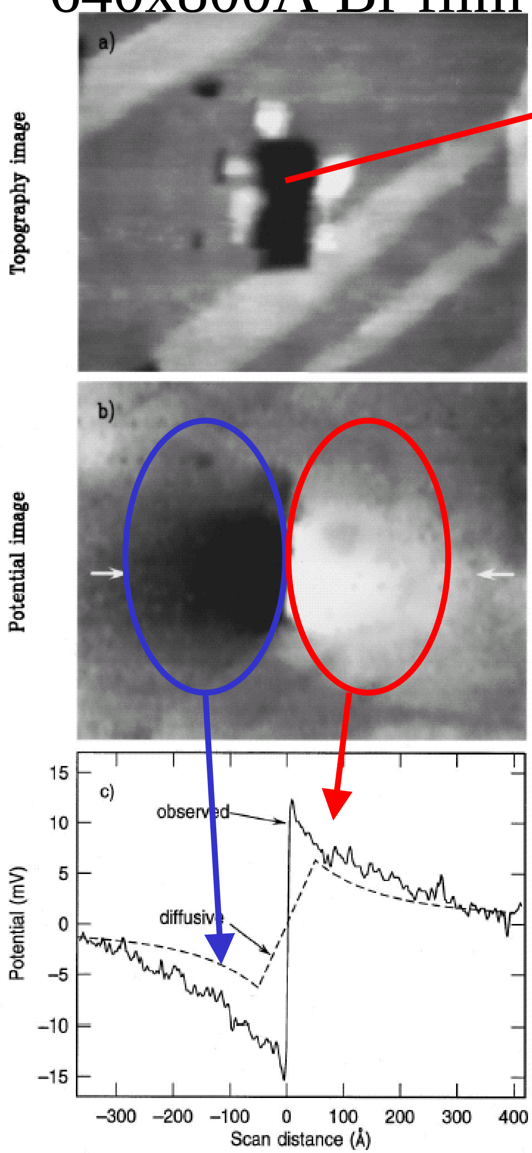


Figure 1 **Schematic representation of current flow disturbed by the scatterer *S*.** *Electrons in excess numbers are incident along A, then are scattered to C, then scattered by the background. The number of electrons incident along B is less than the equilibrium number. The deficit is scattered to D, then scattered by the background. The excess and deficit diffuse together and recombine along the arcs.*

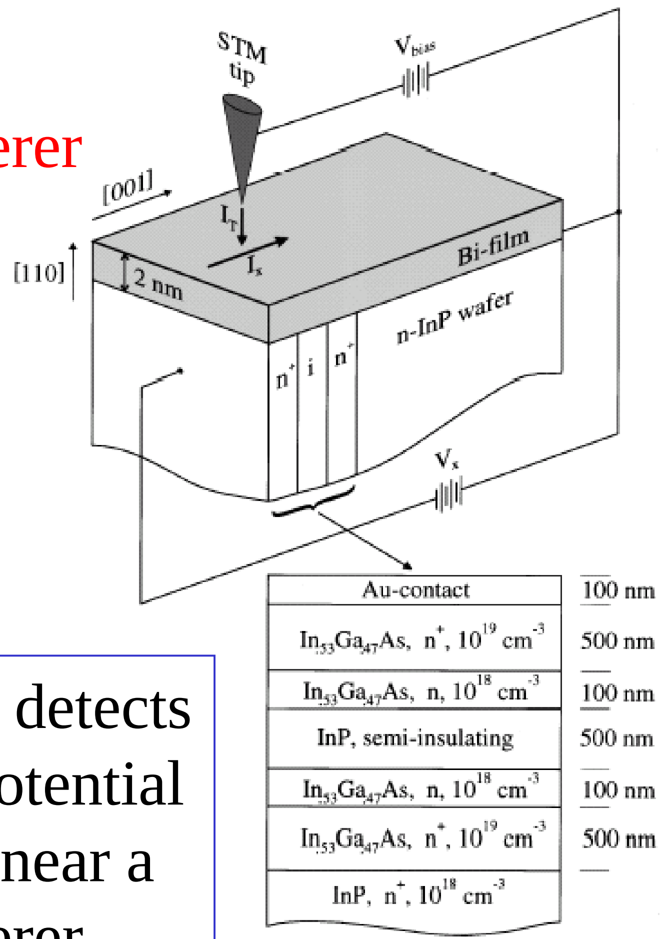
Landauer, IBM J. Res. Dev. 1, 223 (1957)

Experimental measurement of the charge dipole

640x800Å Bi-film



scatterer

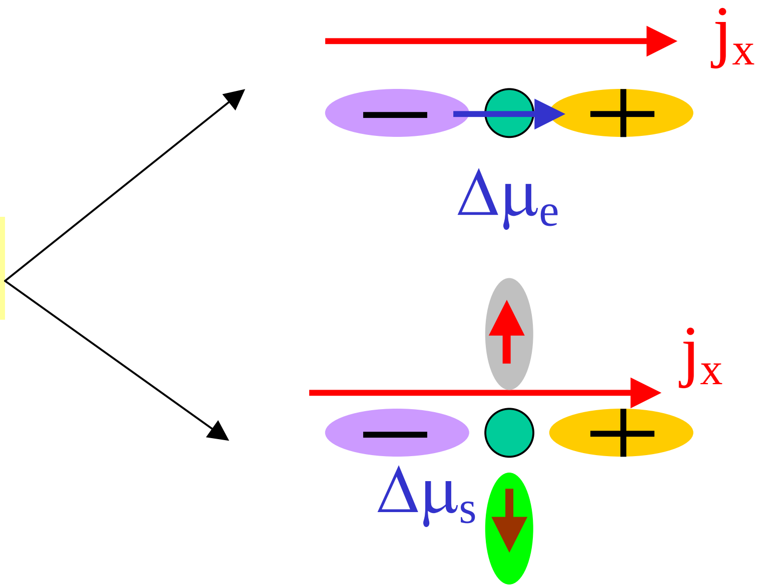


STM detects the potential drop near a scatterer.

BG Briner, RM Feenstra *et al.* *PRB* **54**, 5283(R) (1996)

Spin dipoles (spin cloud) induced via SOI

Electric current j_x



Charge dipole!

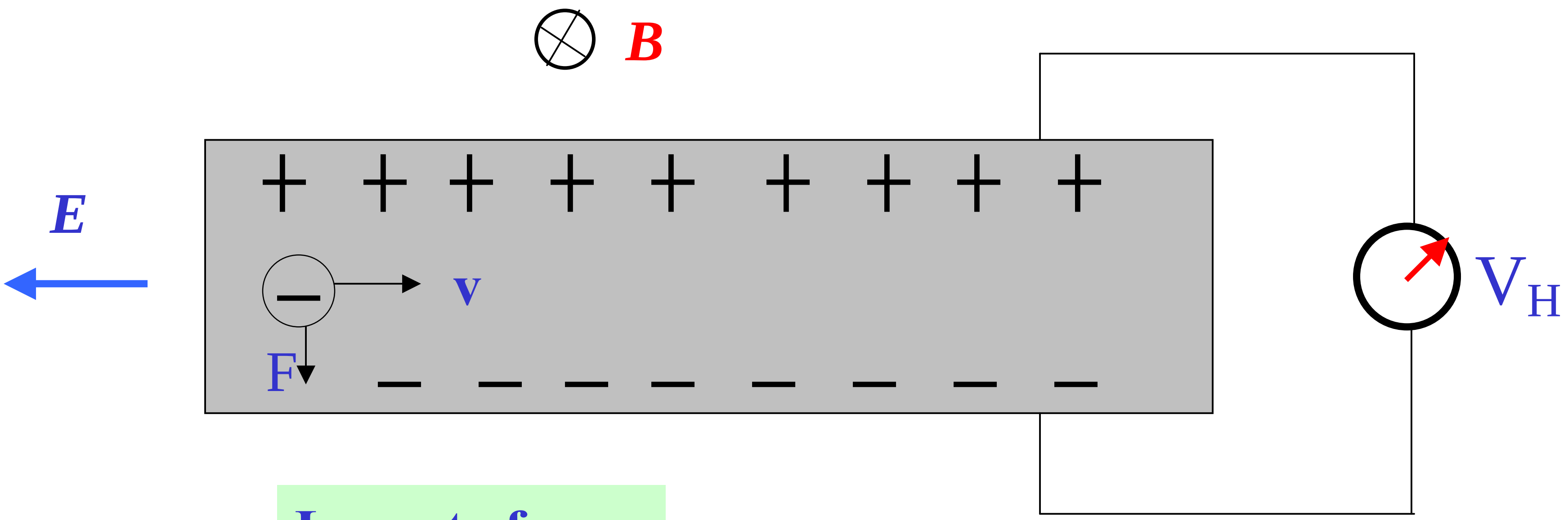
Spin dipole ?

a. Electric current induces nonequilibrium charge dipole around a scatterer.

b. Our concern here is: Does unpolarized electric current induce nonequilibrium spin dipole around a normal scatterer in a SOI semiconductor ?

Spin-Hall effect

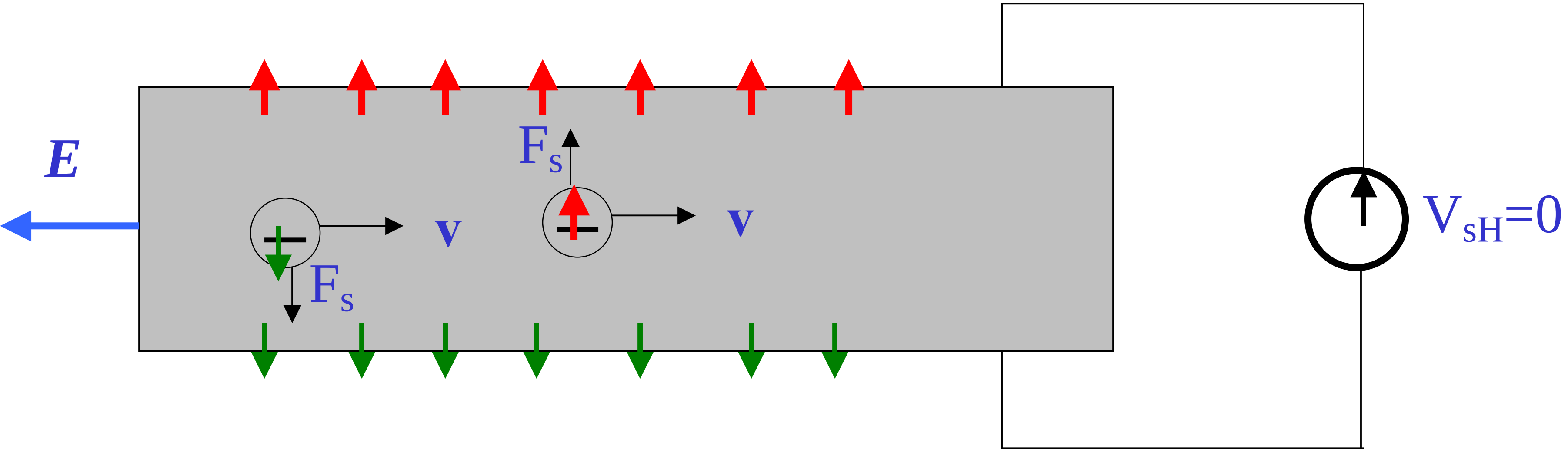
Conventional Hall Effect for charges:
A driving electric field ***E*** is applied in the hall-bar accompanying a external magnetic field ***B*** perpendicular to the sample.



Lorentz force

Spin-Hall effect:

In response to an influx of electric current, spin is found to accumulate at the lateral edges of the sample. No magnetic field is needed but SOI is crucial.



Case of unpolarized electric current

Microscopic origins of SHE and experimental status: “ *Intrinsic SHE* ” and “ *extrinsic SHE* ”.

(1) The “Extrinsic SHE” arises from SOI impurities.

Experimental result in n-doped system

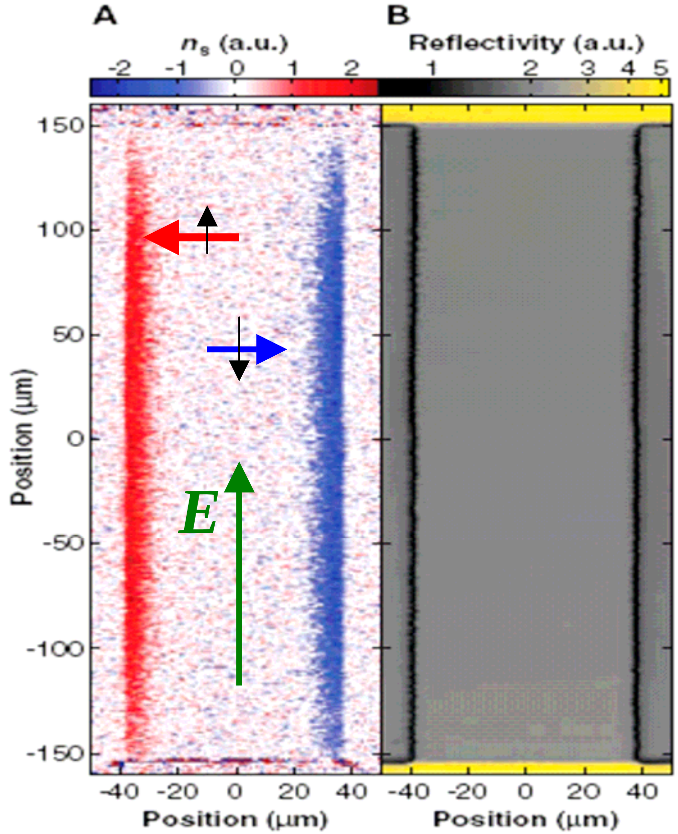
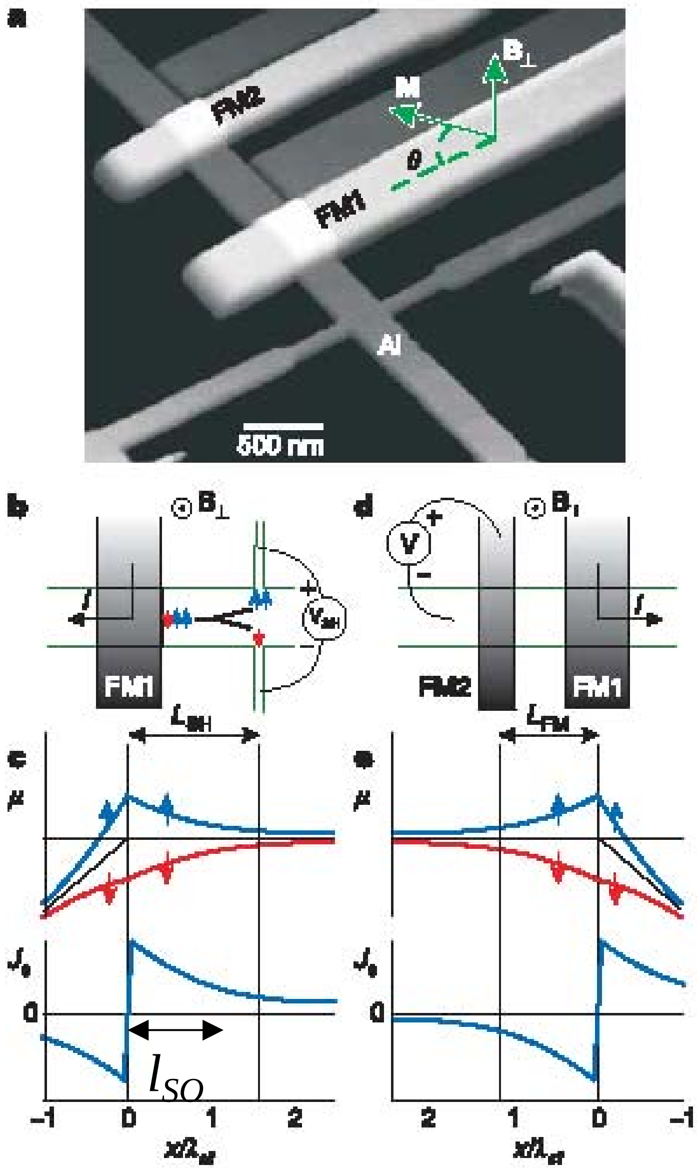


Fig. 2. (A and B) Two-dimensional images of spin density n_s and reflectivity R, respectively, for the unstrained GaAs sample measured at T = 30 K and E = 10 mV μm^{-1}.

Extrinsic spin-Hall effect experiment (Science, **306**, 1910 (2004) by Awschalom *et al*.)

(2) The “Intrinsic SHE” arises from the band structure effect.

Experimental result in p-doped system

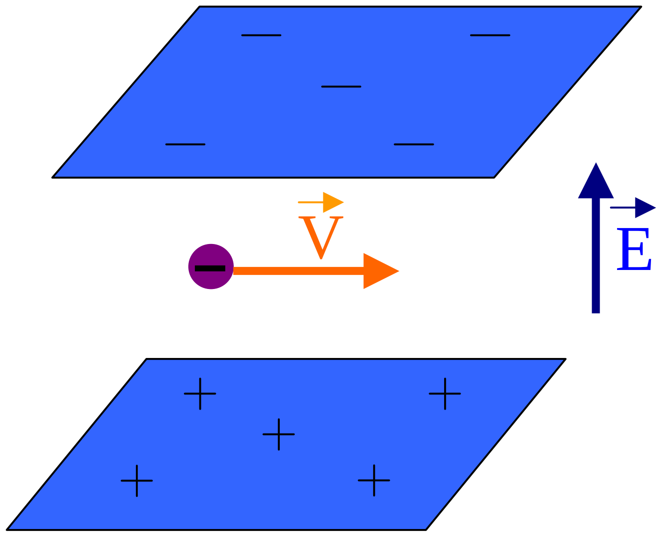


The spin-up and spin-down charges move toward the opposite lateral edges due to the SOI of **Al**.

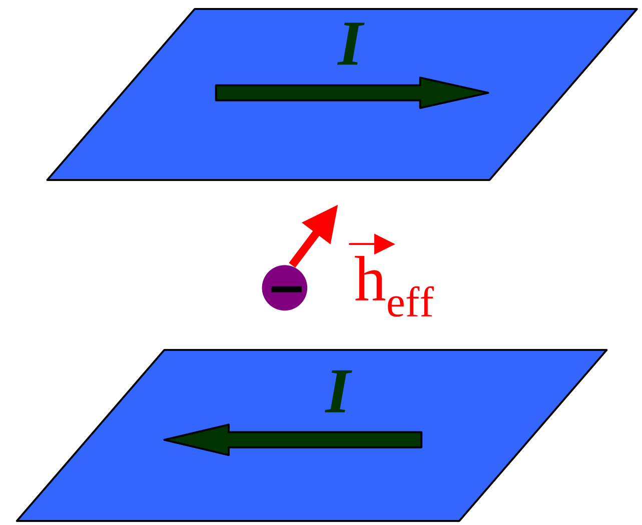
S. O.Valenzuela and M. Tinkham, *Nature* **442**, 176 (2006)

An electron moves between two charged plane

Effective magnetic field induced by the effective current *I*.



In Lab. frame



In the rest frame of an electron

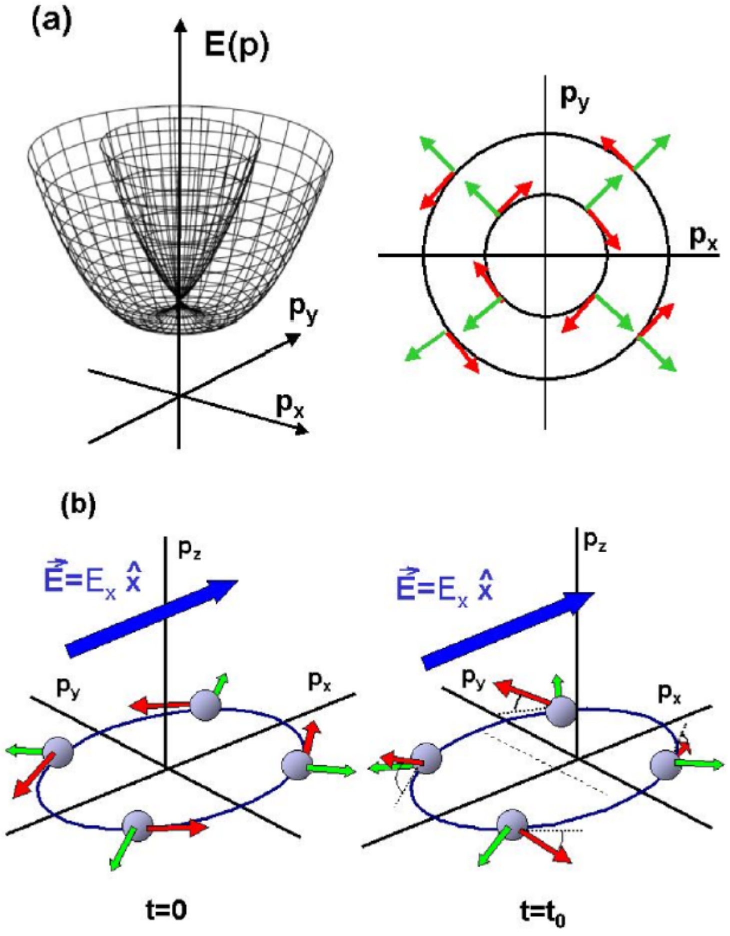
The SOI Hamiltonian is given by

$$H = -\vec{\mu}\cdot\vec{h}_{eff} \propto \vec{\sigma}\cdot\left(-\vec{V}\times\vec{E}\right) = \vec{\sigma}\cdot\vec{h}_{eff}$$

$$H_{Rashba} \equiv \alpha\left(\vec{p}\times\hat{z}\right)\cdot\vec{\sigma}$$

where α **is called the Rashba constant.**

Intrinsic spin-Hall effect in ballistic regime:



Green arrows: wavevector

Red arrows:
effective magnetic field

Rashba SOI

J. Sinova, *et al* PRL <u>92</u>, 126603 (2004)

However, there is no *z-polarized* spin polarization induced in the diffusion limit for Rashba SOI.

Intrinsic spin-Hall effect in the diffusion region

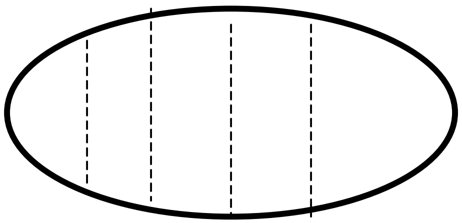
Diffusion region: spin relaxation length l_{SO} >> mean free path l_{mean}

on e can write down the diffusion propagator:

$$D^{il}(r) = -\delta^{il} D\nabla^2 + 4\tau\varepsilon^{iln}\overline{[h^n_{P_F} V^m_F]}\nabla_m - 2\varepsilon^{ilm}B^m - 4\tau\overline{h^2_{P_F}\left(\delta^{il} - n^i_k n^l_k\right)}$$

$$+4\tau^2 \overline{h^3_p \frac{\partial n^i_p}{\partial \mathbf{p}}} \bullet \vec{\nabla}$$

$$\equiv \delta^{il} D\nabla^2 + R^{ilm}\nabla_m - 2\varepsilon^{ilm}B^m - \Gamma^{il} + M^{i0}$$



vE

$\Gamma = \dfrac{1}{2\tau}$: scattering rate

R^{ilm} : spin precession term due to spatial variation of spin density

Γ^{il} : DP relaxation

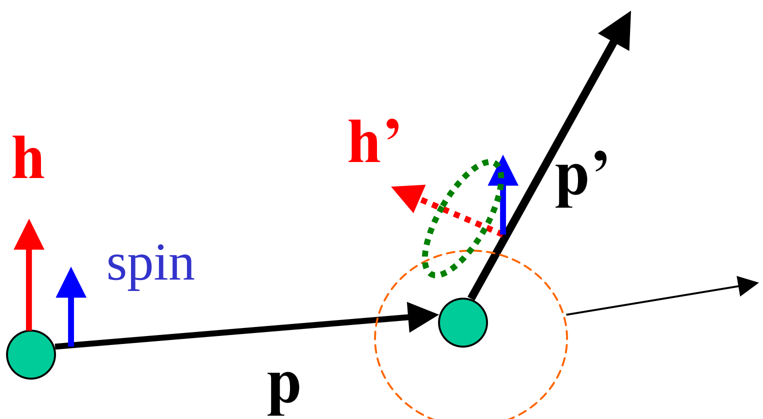
M^{i0} : spin-charge coupling

A.G. Mal'shukov, L.Y. Wang, C.S. Chu, PRL **95**, 146601 (2005)

Spin precession due to the SOI field :

D'yakonov Perel' (DP) spin relaxation:

Spin relaxation due to spin precession between collisions with no spin-flipping impurities. A change in momentum changes the precession axis of the spin.



Scattering from a normal scatterer : no spin-flipping

Spin dipole induced by a single impurity

Ballistic regime: $l_{mean} >> l_{so}$

Diffusive regime: $l_{mean} << l_{so}$

$$H = \frac{p^2}{2m^*} + \vec{h} \bullet \vec{\sigma}$$

The nth component spin polarization is

$$S_n(r) = \frac{-e}{m^*} \iint dr' \frac{d\omega}{2\pi} \frac{dn_F(\omega)}{d\omega} \left\langle Tr\left[\sigma^n G^r(\mathbf{r}, \mathbf{r}', \omega) \mathbf{v} \bullet \mathbf{E} G^a(\mathbf{r}', \mathbf{r}, \omega)\right]\right\rangle$$

Velocity operator:

$$v^j = \frac{k^j}{m^*} + \frac{\partial \mathbf{h_k} \bullet \boldsymbol{\sigma}}{\partial k^j}$$

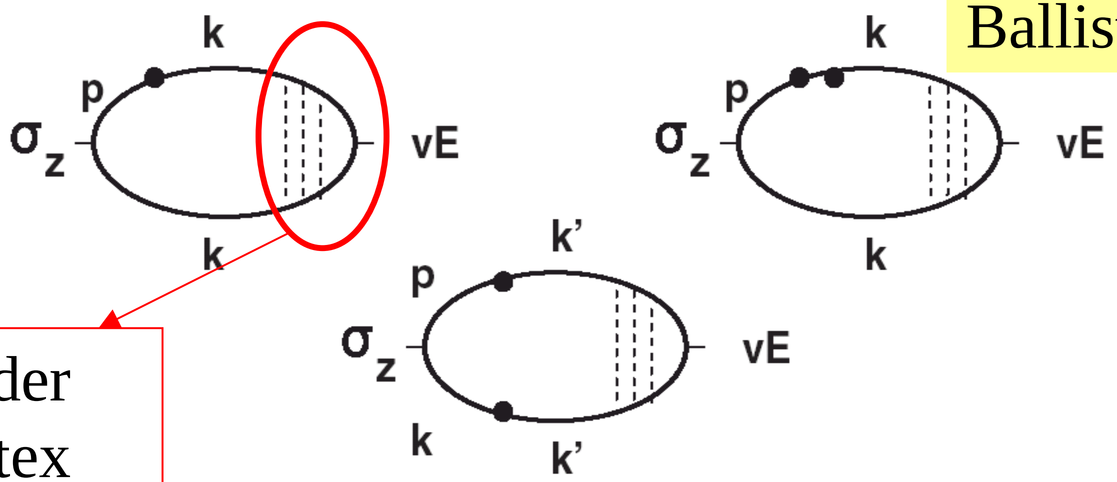
The effective SOI field is given by

$$h_k^x = \alpha k_y, h_k^y = -\alpha k_x$$

for Rashba SOI.

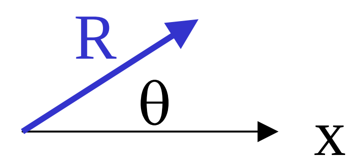
In the ballistic regime (Rashba SOI case)

Ballistic regime: $l_{\text{mean}} >> l_{so}$



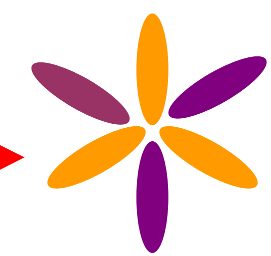
ladder
vertex

R= r-r$_i$



$$\sigma_z(r) = -\frac{m^* v_d}{2\pi^2 R L_{so}} \sin\left(\frac{2R}{L_{so}}\right) \sin\theta \quad \text{(dipole)}$$

$$+\frac{m^* v_d \sigma_t}{2\pi^2 R^2} \sin^2\left(\frac{2R}{L_{so}}\right) \sin 3\theta \left(\sigma_{tot} + \sqrt{\frac{8\pi}{k_F}} \operatorname{Re}\left[f(\pi) e^{i2k_F R}\right]\right)$$



Spin accumulation does occur regardless of vanishing spin current in the bulk (Rashba SOI case).

(A.G. Mal'shukov and C.S. Chu, *PRL* **97**, 76601 (2006))

The ***target impurity*** is at $\boldsymbol{r}_i$ with a scattering potential $V_{tg}(\boldsymbol{r}-\boldsymbol{r}_i)$ and the Green's functions can be expanded up to 2nd order of scattering potential of the target impurity:

$$G^{r(a)}(\boldsymbol{r},\boldsymbol{r}')=G^{r(a)0}(\boldsymbol{r},\boldsymbol{r}')+\int d\boldsymbol{s}^2 G^{r(a)0}(\boldsymbol{r},\boldsymbol{s})V_{tg}(\boldsymbol{s}-\boldsymbol{r}_i)G^{r(a)0}(\boldsymbol{s},\boldsymbol{r}')$$

$$+\int d\boldsymbol{s}^2 d\boldsymbol{s}'^2\, G^{r(a)0}(\boldsymbol{r},\boldsymbol{s})V_{tg}(\boldsymbol{s}-\boldsymbol{r}_i)G^{r(a)0}(\boldsymbol{s},\boldsymbol{s}')V_{tg}(\boldsymbol{s}'-\boldsymbol{r}_i)G^{r(a)0}(\boldsymbol{s}',\boldsymbol{r}')$$

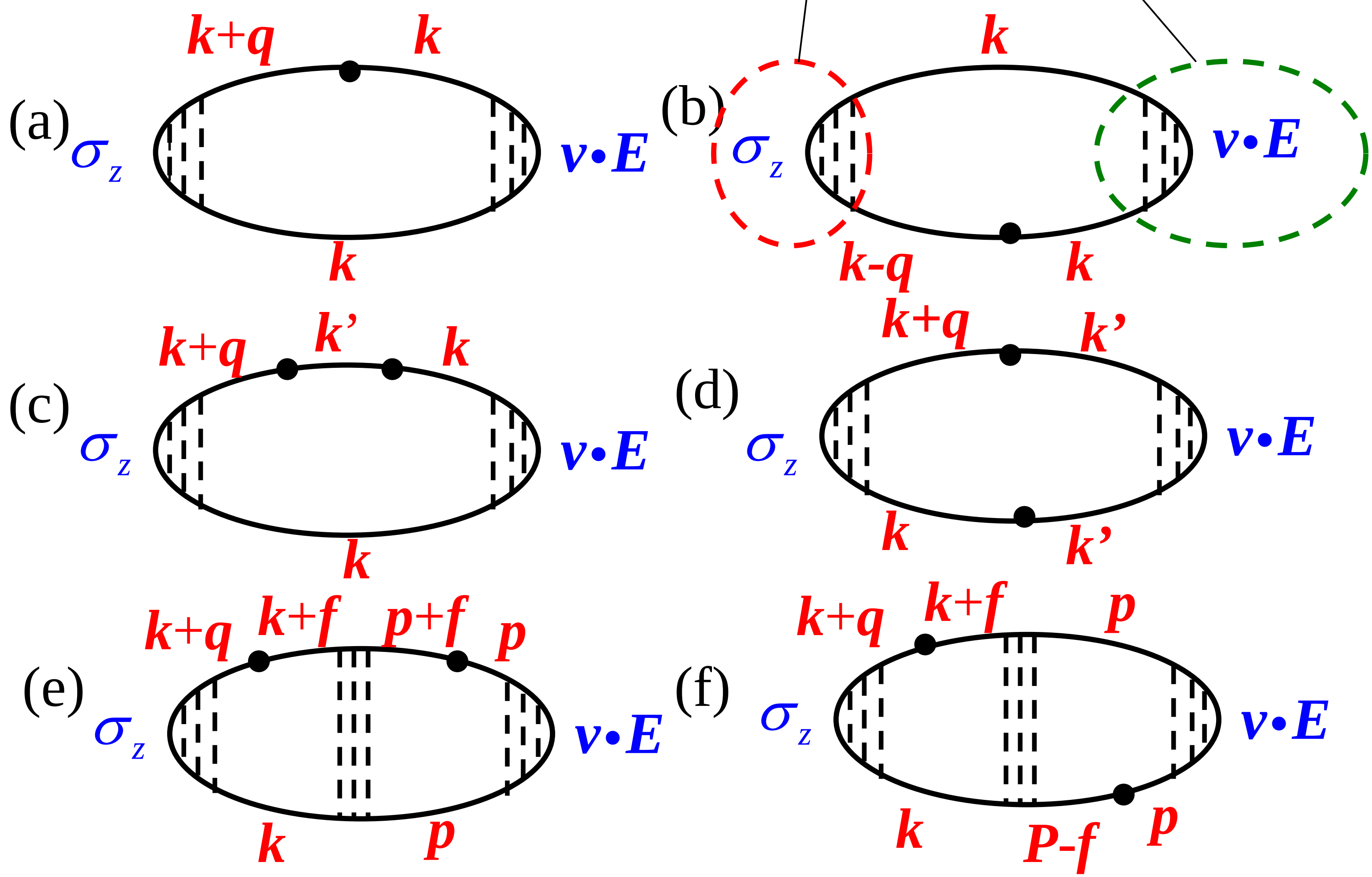
Calculating the background impurities averaging:

$$G_k^{r(a)}=\int d^2(\boldsymbol{r}-\boldsymbol{r}')e^{i\boldsymbol{k}(\boldsymbol{r}-\boldsymbol{r}')}\overline{G^{r(a)0}(\boldsymbol{r},\boldsymbol{r}')}=\frac{1}{E_F-E_k-\boldsymbol{h}_{\boldsymbol{k}}\bullet\boldsymbol{\sigma}+(-)i\Gamma}$$

Assuming the semiclassical approximation $E_F\tau\gg 1$ is valid, the above Green's functions become the building block for ladder perturbation series.

In the diffusive regime (Rashba SOI case)

$$S_z(q) = \frac{1}{2\pi}\sum_{p,k} Tr\left[G^a_{p,k}\Sigma_z(q)G^r_{k+q,p}T(\mathbf{p})\right]$$



The main contributions come from (a)-(d).

Calculation of spin polarization $S_z(\boldsymbol{q})$

The vertex $\Sigma_z(\boldsymbol{q})$ can be represented by a diffusion propagator $D^{zb}(\boldsymbol{q})$:

$$\Sigma_z(\boldsymbol{q}) = \sum_b D^{zb}(\boldsymbol{q})\,\tau^{b=0,x,y,z}, \qquad \tau^0 = 1, \tau^j = \sigma^j$$

The source $I^n(\boldsymbol{q})$ presents the spin-polarized particle emitted from a target impurity.

$$S_z(\boldsymbol{q}) = \sum_{n=x,y,z} D^{zn}(\boldsymbol{q})\, I^n(\boldsymbol{q}), \qquad \boldsymbol{q} \ll l_{mean}^{-1} \ll k_F$$

One can obtain the spin polarization $S_z(\boldsymbol{q})$ by calculating the source terms $I^n(\boldsymbol{q})$ and matrix elements $D^{zn}(\boldsymbol{q})$.

Source terms I_1 and I_2 are the first and second order with respect to the scattering potential V_{tg}

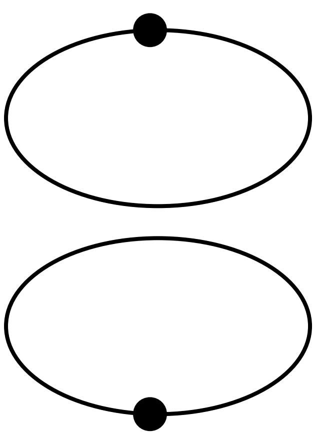
$$I^n(\boldsymbol{q}) = I_1^n(\boldsymbol{q}) + I_2^n(\boldsymbol{q})$$

$$I_1^n(q) = \frac{eV_{tg}}{2\pi m^*} e^{iq\bullet r_i} \sum_p Tr[G_p^r G_p^a \left(\sigma^n G_{p+q}^r + G_{p-q}^a \sigma^n\right)]\boldsymbol{p}\bullet\boldsymbol{E}$$

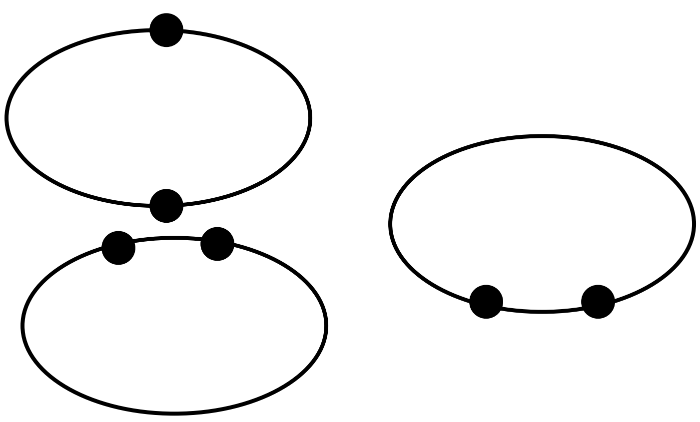
$$I_2^n(q) = \frac{eV_{tg}^2}{2\pi m^*} e^{iq\bullet r_i} \sum_p Tr[G_p^r G_p^a \left(G_k^a \sigma^n G_{k+q}^r - \gamma\sigma^n G_{p+q}^r + \gamma G_{p-q}^a \sigma^n\right)]\boldsymbol{p}\bullet\boldsymbol{E}$$

$$\gamma = i\,\mathrm{Im}\left[\sum_{k'} G_{k'}^a\right] = i\pi N_0$$

$I_1^n(\boldsymbol{q})$



$I_2^n(\boldsymbol{q})$



Symmetric properties for source $I^n(\boldsymbol{q})$

$$I^n(\boldsymbol{q}) = \frac{e}{2\pi m^*}\sum_{p,k} Tr\left[G^r_{p,k}\sigma^n G^a_{k+q,p}\right]\boldsymbol{p}\bullet\boldsymbol{E}$$

$$\left(h_x^{Rashba}, h_y^{Rashba}\right) = \left(\alpha k_y, -\alpha k_x\right)$$

---

(1) $p_{x,y} \to -p_{x,y}, \sigma^i \to \sigma^z\sigma^i\sigma^z$ (for Rashba SOI)

$$I^{x,y}\left(q_x, q_y\right) = I^{x,y}\left(-q_x, -q_y\right); I^z\left(q_x, q_y\right) = -I^z\left(-q_x, -q_y\right)$$

(2) $p_y \to -p_y, \sigma^i \to \sigma^y\sigma^i\sigma^y$ (for Rashba SOI)

$$I^y\left(q_x, q_y\right) = I^y\left(q_x, -q_y\right); I^{x,z}\left(q_x, q_y\right) = -I^{x,z}\left(q_x, -q_y\right)$$

in final, we have the relations:

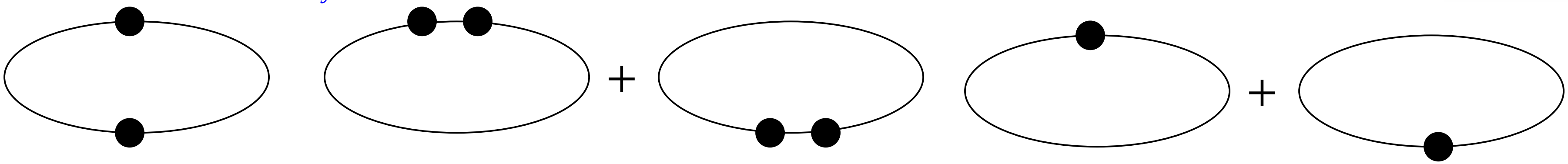
$$I^z \sim q_y$$

$$I^x \sim c_1 + q_x q_y \Rightarrow c_1 = 0$$

$$I^y \sim c_2 + q_y^2$$

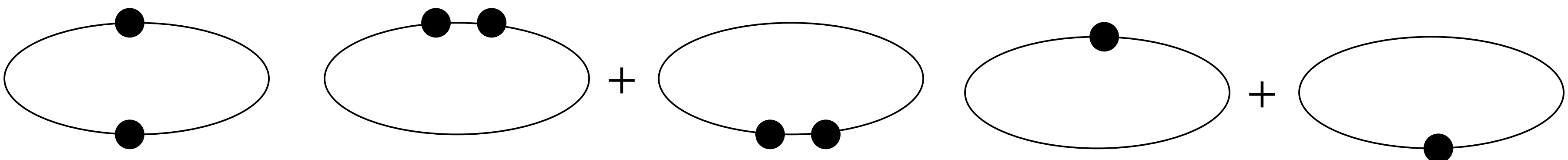
Brief summary:

For q=0 cases: σ_y



$-v_d m^* N_0 \alpha h_{k_F}^2 \dfrac{\Gamma'}{\Gamma^3}$ $\qquad$ $+2v_d m^* N_0 \alpha h_{k_F}^2 \dfrac{\Gamma'}{\Gamma^3}$ $\qquad$ 0

For small q cases: σ_z



$-iq_y v_d \mathrm{N}_0 h_{k_F}^2 \dfrac{\Gamma'}{2\Gamma^3}$ $\qquad$ $2iq_y v_d \mathrm{N}_0 h_{k_F}^2 \dfrac{\Gamma'}{2\Gamma^3}$ $\qquad$ 0

Source terms:

$$I^x = 0;$$

$$I^y = v_d m^* N_0 \alpha h_{k_F}^2 \frac{\Gamma'}{\Gamma^3};$$ ← constant

$$I^z = iq_y v_d \mathrm{N}_0 h_{k_F}^2 \frac{\Gamma'}{2\Gamma^3}.$$ ← proportional to q_y

$\left(v_d = eE\tau / m^*\right)$

The matrix element $D^{zn}(\boldsymbol{q})$ satisfies the spin-diffusion equation:

$$\sum_l \left(-\delta^{il} D q^2 - \Gamma^{il} + i \sum_m R^{ilm} q_m \right) D^{lj}(q) = -\frac{\delta^{ij}}{\tau} = -2\Gamma\delta^{ij}$$

PRL 95, 146601 (2005)

DP relaxation term: $\Gamma^{il} = 4\tau \left\langle \delta^{il} h_{k_F}^2 - h_{k_F}^i h_{k_F}^l \right\rangle$

precession term: $R^{ilm} = 4\tau \sum_p \varepsilon^{ilp} \left\langle h_k^p v_F^m \right\rangle$

$$D^{zz} = \frac{1}{2h_{k_F}^2 \tau^2} \frac{\tilde{q}^2 + 1}{(\tilde{q}^2 + 2)(\tilde{q}^2 + 1) - 4\tilde{q}^2}; D^{yy} = \frac{1}{2h_{k_F}^2 \tau^2} \frac{\tilde{q}^2 + 2}{(\tilde{q}^2 + 2)(\tilde{q}^2 + 1) - 4\tilde{q}^2}$$

$$-D^{zy} = D^{yz} = \frac{1}{2h_{k_F}^2 \tau^2} \frac{2i\tilde{q}_y}{(\tilde{q}^2 + 2)(\tilde{q}^2 + 1) - 4\tilde{q}^2}. \qquad (2\tilde{q} = q l_{so})$$

Spin polarization around a target impurity

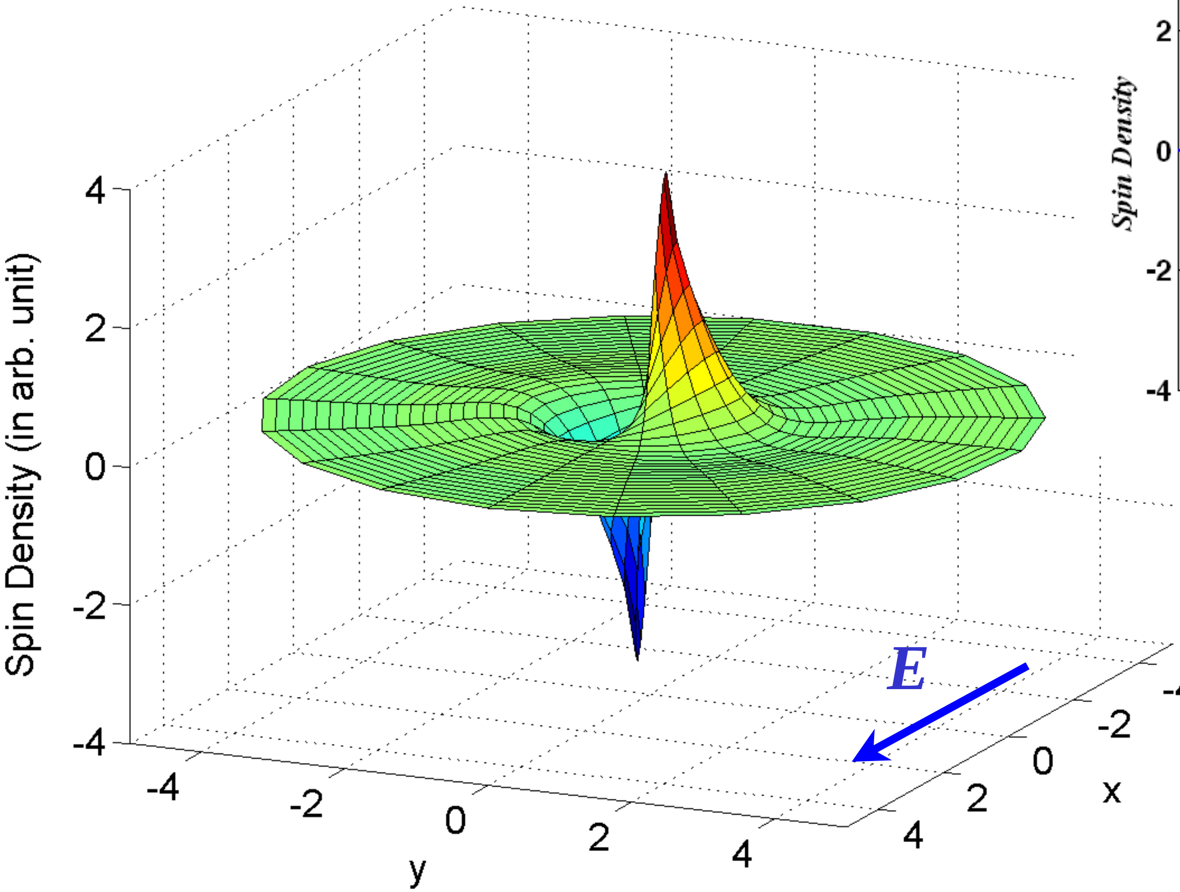
$$\begin{cases} S_z = -2v_d \dfrac{m^*\alpha}{\hbar} N_0 \dfrac{\Gamma'}{\Gamma} \dfrac{i\tilde{q}_y\left(\tilde{q}^2+3\right)}{\left(\tilde{q}^2+2\right)\left(\tilde{q}^2+1\right)-4\tilde{q}^2} \\ S_y = 2v_d \dfrac{m^*\alpha}{\hbar} N_0 \dfrac{\Gamma'}{\Gamma} \dfrac{\left(3\tilde{q}^2+2\right)}{\left(\tilde{q}^2+2\right)\left(\tilde{q}^2+1\right)-4\tilde{q}^2} \end{cases}$$

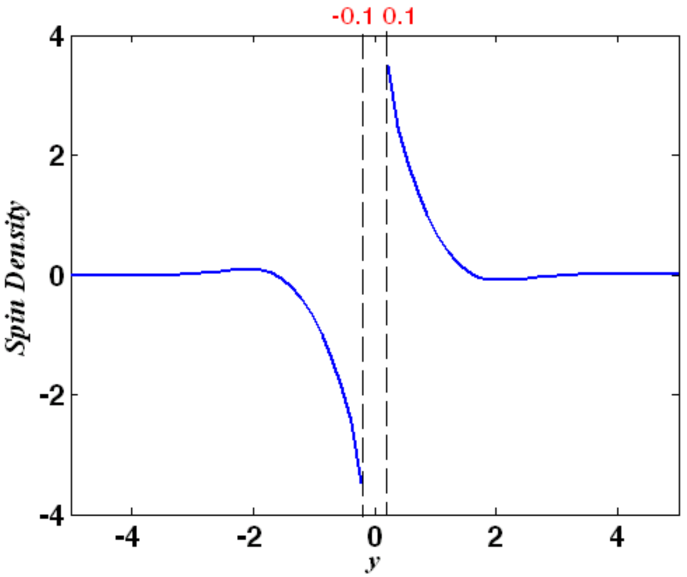
In bulk system: $q \rightarrow 0$ and $\Gamma' = \Gamma/n_i$ (target impurity is the same as the background impurities)

$$\begin{cases} S_z = 0 \\ S_y = 2v_d \dfrac{m^*\alpha}{\hbar} N_0 \end{cases}$$

← Rashba case

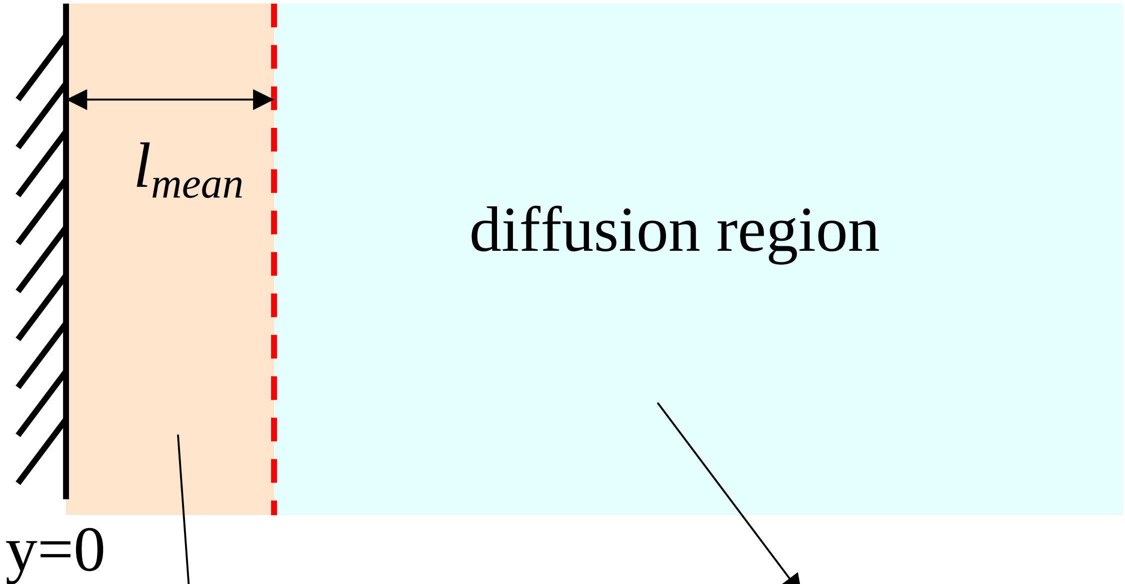
Spin polarization S_z induced by a single impurity





PRB **75**, 085315(2007)

Spin accumulation in a semi-infinite system



Spin diffusion equation

Boltzmann equation involving spin

$$v\nabla_y \vec{g}_{\vec{k}} + 2\left(\vec{g}_{\vec{k}} \times \overrightarrow{h_k}\right) + eE_x \frac{\partial g^0_{\vec{k}}}{\partial k_x} = \frac{1}{\tau}\left(\vec{S}(y) - \vec{g}_{\vec{k}}\right)$$

distribution function

$$f_{\vec{k}}\left(\vec{r}\right) = \vec{\sigma} \bullet \vec{g}_{\vec{k}}\left(\vec{r}\right)$$

The ith component averaging spin density: $S_{av}^{i}(y)=\sum_{\vec{k}}\vec{g}_{\vec{k}}$

Spin difffusion equation:

$$\begin{cases} \dfrac{\partial^2 S_{av}^{z}}{\partial y^2}-4m^*\alpha\dfrac{\partial S_{av}^{y}}{\partial y}-8m^{*2}\alpha^2 S_{av}^{z}=0 \\ \dfrac{\partial^2 S_{av}^{y}}{\partial y^2}+4m^*\alpha\dfrac{\partial S_{av}^{z}}{\partial y}-4m^{*2}\alpha^2 S_{av}^{y}=\dfrac{S^{bulk}}{4m^{*2}\alpha^2} \end{cases}$$

boundary conditions:

$$\begin{cases} -D\left.\dfrac{\partial S_{av}^{z}(y)}{\partial y}\right|_{y=0}+2Dm^*\alpha\left(S_{av}^{y}(0)-S^{bulk}\right)=0 \\ -D\left.\dfrac{\partial S_{av}^{y}(y)}{\partial y}\right|_{y=0}-2Dm^*\alpha S_{av}^{z}(0)=0 \end{cases}$$

Finally, there is no spin accumulation near the boundary:

$$S_{av}^{y}=S^{bulk}=2\tau eE_x N_0\alpha, \qquad S_{av}^{z}=0$$

Summary:

1. Spin dipoles can be induced around impurities for the vanishing bulk spin-Hall current system.
2. The spin accumulation from the impurity averaging is exactly cancel by the spin contribution of hard-wall boundary. It is consistent with previous works.

Thank you!

Extrinsic spin-Hall effect

1. Spin-orbit interaction (SOI):

From Dirac equation:

$$-\frac{\hbar^2}{4m_0^2c^2}\vec{\sigma}\cdot\left[\vec{k}\times\vec{\nabla}V\right]$$

→

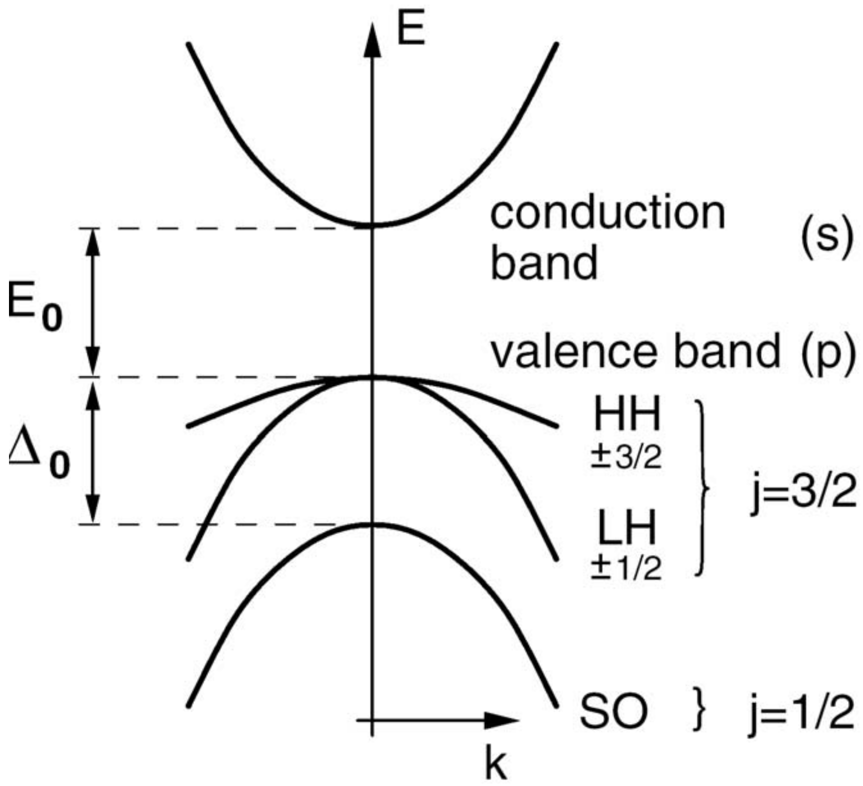
$$\lambda\,\vec{\sigma}\cdot\left(\vec{k}\times\vec{\nabla}V\right)$$ in vacuum

We have **$\lambda = -3.7\times10^{-6}$ Å^2** **in vacuum.**

In semiconductor: GaAs: **$\lambda = 5.3$ Å^2**

The SOI can be enhanced greatly in the semiconductor.

Band structure in the semiconductor:



Using the k⋅p to calculate and we have

In semiconductor:

$$\lambda = \frac{\hbar^2}{6m^* E_0}\left[1 - \frac{E_0^2}{(E_0 + \Delta_0)^2}\right]$$

In vacuum:

$$\lambda = -\frac{\hbar^2}{4m_0^2 c^2} = -\frac{\hbar^2}{2m_0(2m_0 c^2)}$$

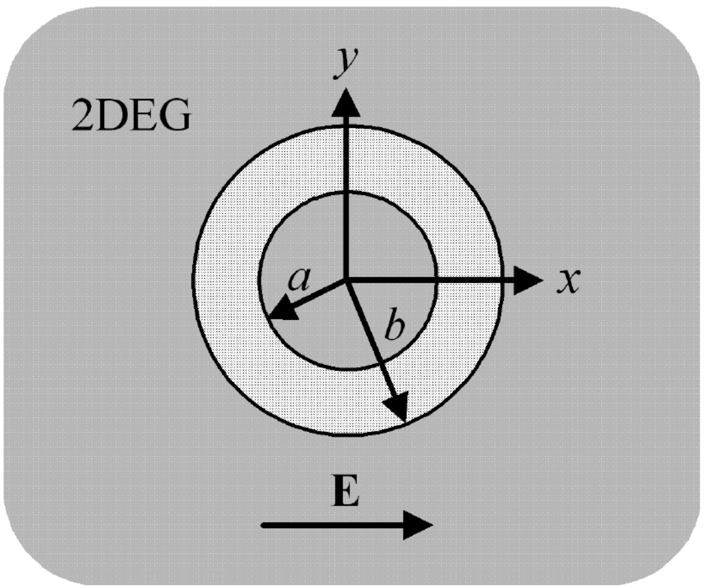
the ratio between the energy gap in vacuum and in semiconductor

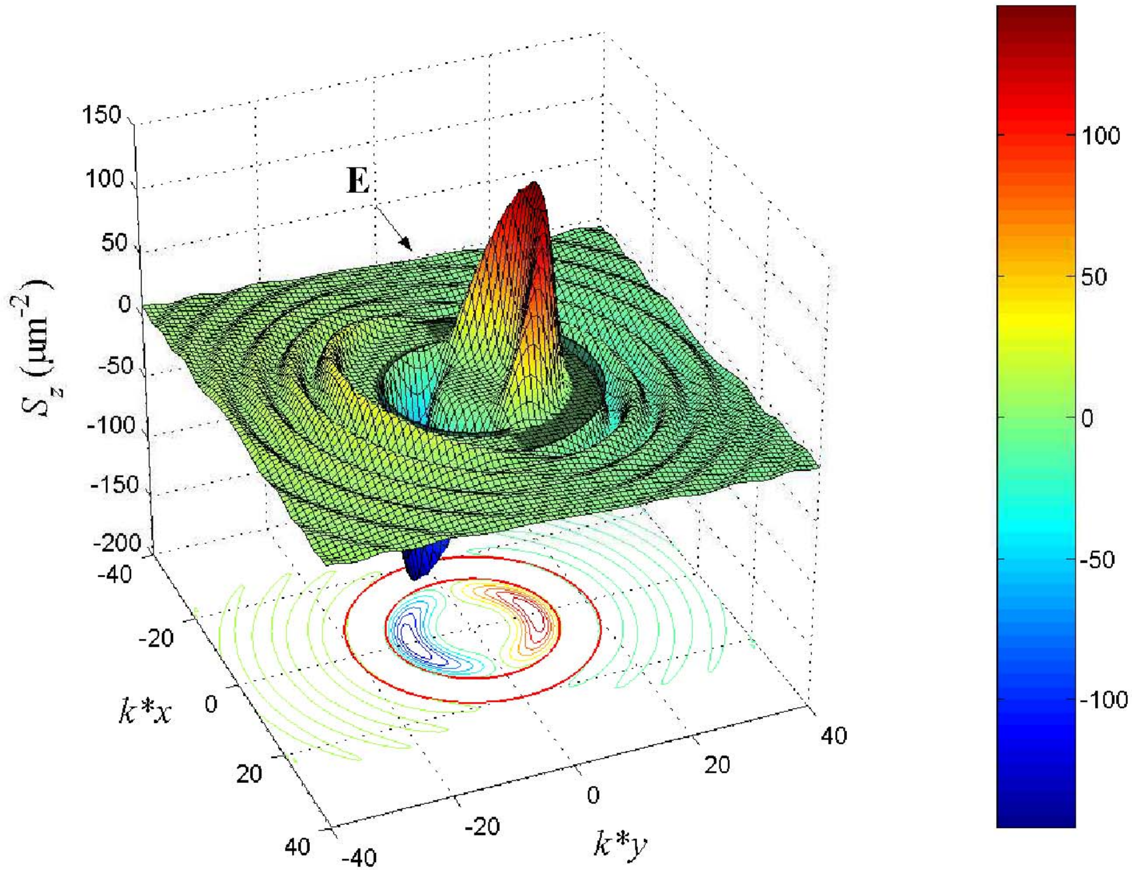
$$\frac{m^* c^2}{E_0} \sim 10^6$$

such the SOI is not small in the semiconductor.

A ring-shaped potential pattern is embedded in a 2DEG.

Chen KY, Chu, and Mal'shukov , PRB (2007).





$$H_{\mathrm{SO}} = \lambda\ \vec{\sigma}\cdot(\vec{k}\times\vec{\nabla}V)$$

$$V(\rho) = V_O\left[\theta(\rho-a)-\theta(\rho-b)\right]$$