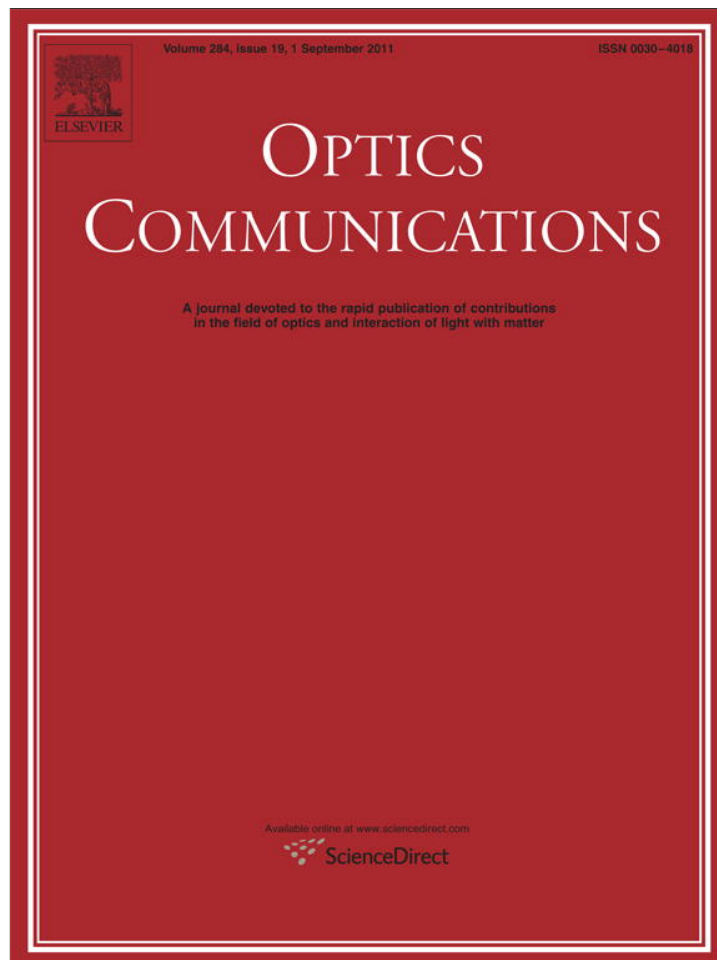


Provided for non-commercial research and education use.
Not for reproduction, distribution or commercial use.



This article appeared in a journal published by Elsevier. The attached copy is furnished to the author for internal non-commercial research and education use, including for instruction at the authors institution and sharing with colleagues.

Other uses, including reproduction and distribution, or selling or licensing copies, or posting to personal, institutional or third party websites are prohibited.

In most cases authors are permitted to post their version of the article (e.g. in Word or Tex form) to their personal website or institutional repository. Authors requiring further information regarding Elsevier's archiving and manuscript policies are encouraged to visit:

<http://www.elsevier.com/copyright>



Contents lists available at ScienceDirect

Optics Communications

journal homepage: www.elsevier.com/locate/optcom

Retardation effects in quantum dot systems coupled via one-dimensional waveguides

Hong-Bin Chen, Guang-Yin Chen^{*}, Yueh-Nan Chen^{**}

Department of Physics and National Center for Theoretical Sciences, National Cheng-Kung University, Tainan 701, Taiwan

ARTICLE INFO

Article history:

Received 11 February 2011
Received in revised form 26 April 2011
Accepted 28 April 2011
Available online 14 May 2011

Keywords:

Quantum dot
Retardation
Radiative decay
Superradiance
Non-Markovian
Entanglement

ABSTRACT

We investigate the retardation effect on the radiative decay and entanglement of two quantum dots. The retardation effect is found to be very weak if the dots are coupled to free-space vacuum reservoir. To enhance the effect, we propose to embed the dots inside a one-dimensional waveguide. It is found that populations and entanglement can saturate to non-vanishing values with appropriate conditions. Furthermore, entanglement sudden-rise and sudden-fall are also observed due to this non-Markovian retardation.

© 2011 Elsevier B.V. All rights reserved.

1. Introduction

Retardation is one of the classic problems in quantum theory of radiation. Fermi first considered in 1930s [1] the excitation transfer between two separated two-level atoms with one being in its excited state and the other one being in its ground state. In accordance with Einstein causality, he showed that the photon emitted by the excited atom would be reabsorbed by the other atom after a retardation time due to the causal propagation. This model has been further examined in the following decades. Some of the results [2] are consistent with Fermi's conclusion. However, some argued that Fermi's conclusion is a result of approximations [3]. In contrast to Einstein causality, Hegerfeldt proposed that the excitation would go from the excited atom to the other non-excited atom instantaneously without any time-delay [4]. Several authors attributed the absence of the retardation time to the employment of the rotating-wave approximation (RWA) [5].

Although the concept of retardation has been introduced since 1930s, due to the difficulties such as manipulating a single atom and the precision measurement of photons, there is no convincing experimental results until the progress in the technology of high-Q cavities [6,7]. Therefore, one of the main purposes of this work is to propose a promising way for experimental realizations by using the newly-developed technologies in solid state physics to enhance the phenomenon of retardation.

In solid state systems, an exciton in a quantum dot (QD) can be regarded as a two-level system. Radiative properties of QD excitons, such as superradiance [8] and Purcell effect [9], have attracted great attention during the past two decades. With the advances of technologies, it is now possible to observe the Purcell effect by embedding the QDs inside the photonic crystals [10]. In addition, a well-designed one-dimensional photonic crystal waveguide with 50 cm-long decay length of the photons was fabricated [11]. Even more, an order of magnitude lower in propagation loss has also been demonstrated [12].

Based on these developments in fabrication technologies, we consider in this work the retardation between two QDs embedded inside a one-dimensional waveguide. To preserve the causal propagation time, RWA is not employed in this work. As will be seen in the followings, the retardation effect in free space is usually too weak to be observed. The advantage of the photons confined in one dimension is to enhance the visibility of the retardation effect. In addition, it is well-known that the QDs can never be made perfectly identical during the fabrication process. Therefore, we will also show that how this difference destroys the photon trapping state. Finally, entanglement sudden-rise and sudden-fall will also be demonstrated.

2. Model and formulation

Let us consider the system which consists of two QDs with exciton resonant frequencies $\hbar\omega_1$ and $\hbar\omega_2$, respectively. The free Hamiltonian describing the two QDs and the photon field is written as

$$\hat{H}_0 = \hbar\omega_1 |e, g\rangle\langle e, g| + \hbar\omega_2 |g, e\rangle\langle g, e| + \hbar(\omega_1 + \omega_2) |e, e\rangle\langle e, e| + \sum_{\mathbf{k}} \hbar\omega_{\mathbf{k}} \hat{a}_{\mathbf{k}}^\dagger \hat{a}_{\mathbf{k}}, \quad (1)$$

^{*} Corresponding author. Tel: +886 6 2757575x65082x15; fax: +886 6 2373981.

^{**} Corresponding author. Tel.: +886 6 2757575x65268; fax: +886 6 2373981.

E-mail addresses: guangyin.py93g@nctu.edu.tw (G.-Y. Chen), yuehnan@mail.ncku.edu.tw (Y.-N. Chen).

where $|e, g\rangle$ ($|g, e\rangle$) denotes that there is one exciton in QD 1 (2), and $|e, e\rangle$ represents both the QDs contain one exciton. Here, $\hat{a}_{\mathbf{k}}$ is the photon annihilation operator with wave vector \mathbf{k} and energy $\hbar\omega_{\mathbf{k}}$. The interaction between the QDs and the photon field can be written as [5]

$$\hat{H}_I = -\sum_{j=1}^2 \sum_{\mathbf{k}} \left(\hbar g_{j,\mathbf{k}} \hat{\sigma}_j^+ \hat{a}_{\mathbf{k}} + \hbar \tilde{g}_{j,\mathbf{k}} \hat{\sigma}_j^- \hat{a}_{\mathbf{k}} + \hbar \tilde{g}_{j,\mathbf{k}}^* \hat{\sigma}_j^+ \hat{a}_{\mathbf{k}}^\dagger + \hbar g_{j,\mathbf{k}}^* \hat{\sigma}_j^- \hat{a}_{\mathbf{k}}^\dagger \right) \quad (2)$$

with

$$\begin{cases} g_{j,\mathbf{k}} = \frac{E_{0,\mathbf{k}}(\vec{\epsilon}_{\mathbf{k}} \cdot \vec{p}) e^{i\mathbf{k} \cdot \mathbf{r}_j}}{\hbar} \\ \tilde{g}_{j,\mathbf{k}} = \frac{E_{0,\mathbf{k}}(\vec{\epsilon}_{\mathbf{k}} \cdot \vec{p}^*) e^{i\mathbf{k} \cdot \mathbf{r}_j}}{\hbar} \end{cases} \quad (3)$$

Here, $\hat{\sigma}_j^\pm = |e\rangle_j \langle g|$ is the atomic operator of the j -th QD with the position at \mathbf{r}_j , $E_{0,\mathbf{k}} = \sqrt{\frac{2\pi\hbar\omega_{\mathbf{k}}}{V}}$ is the amplitude of radiation fields, V is the quantization volume, and $\vec{\epsilon}_{\mathbf{k}}$ is the polarization vector. Note that we also assume the dipole moment \vec{p} for the two QDs are identical. In Eq. (2), the second term lowers the excited atom and annihilates a photon simultaneously, and the third term represents an inverse procedure. These two terms are dropped while performing the so-called rotating-wave approximation (RWA). However, they are in fact related to the retardation effect [5]. Hence, instead of performing the RWA, we keep these two terms throughout.

The state vector of the system in the Schrödinger picture can be expressed as

$$\begin{aligned} |\Psi(t)\rangle &= e^{-i\omega_1 t} b_1(t) |e, g, 0\rangle + e^{-i\omega_2 t} b_2(t) |g, e, 0\rangle \\ &+ \sum_{\mathbf{k}} e^{-i\omega_{\mathbf{k}} t} b_{\mathbf{k}}(t) |g, g, 1_{\mathbf{k}}\rangle \\ &+ \sum_{\mathbf{k}} e^{-i(\omega_1 + \omega_2 + \omega_{\mathbf{k}}) t} \tilde{b}_{\mathbf{k}}(t) |e, e, 1_{\mathbf{k}}\rangle, \end{aligned} \quad (4)$$

where the state $|e, g, 0\rangle$ ($|g, e, 0\rangle$) represents that there is one exciton in QD1 (2) with no photon, while $|g, g, 1_{\mathbf{k}}\rangle$ means the exciton recombines and a photon with wavevector \mathbf{k} is emitted. In Eq. (4), we have assumed that there is only one excitation in QD1 initially.

Together with the Schrödinger's equation, one obtains four equations for the coefficients:

$$\frac{\partial b_1(t)}{\partial t} = i \sum_{\mathbf{k}} \left[g_{1,\mathbf{k}} e^{-i(\omega_{\mathbf{k}} - \omega_1)t} b_{\mathbf{k}}(t) + \tilde{g}_{2,\mathbf{k}} e^{-i(\omega_{\mathbf{k}} + \omega_2)t} \tilde{b}_{\mathbf{k}}(t) \right], \quad (5)$$

$$\frac{\partial b_2(t)}{\partial t} = i \sum_{\mathbf{k}} \left[\tilde{g}_{1,\mathbf{k}} e^{-i(\omega_{\mathbf{k}} + \omega_1)t} \tilde{b}_{\mathbf{k}}(t) + g_{2,\mathbf{k}} e^{-i(\omega_{\mathbf{k}} - \omega_2)t} b_{\mathbf{k}}(t) \right], \quad (6)$$

$$\frac{\partial b_{\mathbf{k}}(t)}{\partial t} = i g_{1,\mathbf{k}}^* e^{i(\omega_{\mathbf{k}} - \omega_1)t} b_1(t) + i g_{2,\mathbf{k}}^* e^{i(\omega_{\mathbf{k}} - \omega_2)t} b_2(t), \quad (7)$$

$$\frac{\partial \tilde{b}_{\mathbf{k}}(t)}{\partial t} = i \tilde{g}_{1,\mathbf{k}}^* e^{i(\omega_{\mathbf{k}} + \omega_1)t} \tilde{b}_2(t) + i \tilde{g}_{2,\mathbf{k}}^* e^{i(\omega_{\mathbf{k}} + \omega_2)t} \tilde{b}_1(t). \quad (8)$$

With the initial conditions, $b_1(0) = 1, b_2(0) = b_{\mathbf{k}}(0) = \tilde{b}_{\mathbf{k}}(0) = 0$, the Laplace transformations of Eqs. (5) and (6) can be expressed as

$$B_1(s) = \frac{s + i(\omega_2 - \omega_1) + \beta_1}{[s + i(\omega_2 - \omega_1) + \beta_1](s + \beta_1) - f_1^2(s)} \quad (9)$$

$$B_2(s) = \frac{-f_2(s)}{[s - i(\omega_2 - \omega_1) + \beta_2](s + \beta_2) - f_2^2(s)},$$

where

$$\beta_1 = \sum_{\mathbf{k}} \left[\frac{|\tilde{g}_{1,\mathbf{k}}|^2}{s + i(\omega_{\mathbf{k}} + \omega_2)} + \frac{|g_{2,\mathbf{k}}|^2}{s + i(\omega_{\mathbf{k}} - \omega_1)} \right], \quad (10)$$

$$\beta_2 = \sum_{\mathbf{k}} \left[\frac{|\tilde{g}_{2,\mathbf{k}}|^2}{s + i(\omega_{\mathbf{k}} + \omega_1)} + \frac{|g_{1,\mathbf{k}}|^2}{s + i(\omega_{\mathbf{k}} - \omega_2)} \right],$$

and

$$f_1(s) = \sum_{\mathbf{k}} \left[\frac{g_{1,\mathbf{k}} g_{2,\mathbf{k}}^*}{s + i(\omega_{\mathbf{k}} - \omega_1)} + \frac{\tilde{g}_{1,\mathbf{k}}^* \tilde{g}_{2,\mathbf{k}}}{s + i(\omega_{\mathbf{k}} + \omega_2)} \right], \quad (11)$$

$$f_2(s) = \sum_{\mathbf{k}} \left[\frac{g_{1,\mathbf{k}} g_{2,\mathbf{k}}^*}{s + i(\omega_{\mathbf{k}} - \omega_2)} + \frac{\tilde{g}_{1,\mathbf{k}}^* \tilde{g}_{2,\mathbf{k}}}{s + i(\omega_{\mathbf{k}} + \omega_1)} \right].$$

The inverse Laplace transformation can then be written as

$$b_j(t) = \frac{1}{2\pi i} \int_{\epsilon - i\infty}^{\epsilon + i\infty} B_j(s) e^{st} ds = C_{j+}(t) + C_{j-}(t), j = 1, 2, \quad (12)$$

where

$$\begin{aligned} C_{1\pm}(t) &= \frac{1}{4\pi i} \int_{-\infty}^{\infty} e^{izt} \left[z + (\omega_2 - \omega_1) - i\tilde{\beta}_1 \right. \\ &\quad \left. \pm \sqrt{\left(iA_1 e^{\frac{i(\omega_1 - z)r}{v}} \right)^2 + (\omega_2 - \omega_1) \left[z + (\omega_2 - \omega_1) - i\tilde{\beta}_1 \right]} \right]^{-1} dz, \end{aligned} \quad (13)$$

$$C_{2\pm}(t) = \frac{1}{4\pi i} \int_{-\infty}^{\infty} e^{izt} \left[-iA_2 e^{\frac{i(\omega_2 - z)r}{v}} \pm \sqrt{\left[z - (\omega_2 - \omega_1) - i\tilde{\beta}_2 \right] \left(z - i\tilde{\beta}_2 \right)} \right]^{-1} dz, \quad (14)$$

with $\tilde{\beta}_j = 2p^2 \omega_j^3 / 3\hbar v^3$. Here, r denotes the inter-dot distance and v is the velocity of the photon in medium. For photons in free space, the function A_j is written as

$$A_j = -3\tilde{\beta}_j \cdot \left[i \frac{1 - P_2(\cos \theta \vec{p})}{3 \cdot \frac{\omega_j r}{v}} + \frac{P_2(\cos \theta \vec{p})}{\left(\frac{\omega_j r}{v} \right)^2} + i \frac{P_2(\cos \theta \vec{p})}{\left(\frac{\omega_j r}{v} \right)^3} \right],$$

where $\theta \vec{p}$ is the angle between \vec{p} and \vec{z} and P_2 is the Legendre function.

If the following condition is satisfied:

$$\forall z \in \Re, \left| \frac{\sqrt{\left(iA_1 e^{\frac{i(\omega_1 - z)r}{v}} \right)^2 + (\omega_2 - \omega_1) \left[z + (\omega_2 - \omega_1) - i\tilde{\beta}_1 \right]}}{z + (\omega_2 - \omega_1) - i\tilde{\beta}_1} \right| < 1. \quad (15)$$

Eq. (13) can be expanded into a power series

$$\begin{aligned} C_{1\pm}(t) &= \int_{-\infty}^{\infty} \sum_{n=0}^{\infty} (\mp 1)^n e^{izt} \\ &\quad \times \left[\left(iA_1 e^{\frac{i(\omega_1 - z)r}{v}} \right)^2 + (\omega_2 - \omega_1) \left(z + (\omega_2 - \omega_1) - i\tilde{\beta}_1 \right) \right]^{\frac{n}{2}} \\ &\quad \times \left[z + (\omega_2 - \omega_1) - i\tilde{\beta}_1 \right]^{-(n+1)} dz. \end{aligned} \quad (16)$$

After adding $C_{1\pm}(t)$ together, all those terms with odd n 's are canceled out. Let $n = 2k$, one obtains

$$b_1(t) = \frac{1}{2\pi i} \sum_{k=0}^{\infty} \sum_{j=0}^k \binom{k}{j} (\omega_2 - \omega_1)^{k-j} \left(-A_1^2 e^{\frac{2\omega_1 t}{v}}\right)^j \times \int_{-\infty}^{\infty} \frac{e^{iz\left(t - \frac{2jr}{v}\right)}}{\left[z + (\omega_2 - \omega_1) - i\tilde{\beta}_1\right]^{k+j+1}} dz. \quad (17)$$

The integration in Eq. (17) can be expressed as

$$\int_{-\infty}^{\infty} \frac{e^{i\alpha z}}{(z - z_0)^n} dz = \frac{2\pi i (i\alpha)^{n-1} e^{i\alpha z_0}}{(n-1)!} H(\alpha), \quad (18)$$

where $H(\alpha)$ is the Heaviside or unitstep function. This leads to the final result:

$$b_1(t) = \sum_{k=0}^{\infty} \sum_{j=0}^k \frac{k!}{j!(k-j)!(k+j)!} \left[i\left(t - \frac{2jr}{v}\right)\right]^{k+j} \times (\omega_2 - \omega_1)^{k-j} \left(-A_1^2 e^{\frac{2\omega_1 t}{v}}\right)^j \times e^{[-\tilde{\beta}_1 - i(\omega_2 - \omega_1)]\left(t - \frac{2jr}{v}\right)} H\left(t - \frac{2jr}{v}\right). \quad (19)$$

Similarly, if the following condition is fulfilled:

$$\forall z \in \Re, \left| \frac{iA_2 e^{\frac{i(\omega_2 - z)r}{v}}}{\sqrt{\left[z - (\omega_2 - \omega_1) - i\tilde{\beta}_2\right]\left(z - i\tilde{\beta}_2\right)}} \right| < 1, \quad (20)$$

one can add $C_{2\pm}(t)$ and cancel all even- n terms out. Finally, one can obtain the desired result:

$$b_2(t) = \sum_{k=0}^{\infty} \sum_{j=0}^{\infty} -(i)^k \left(A_2 e^{\frac{\omega_2 t}{v}}\right)^{2j+1} \left[t - \frac{(2j+1)r}{v}\right]^{2j+k+1} \times \frac{(j+k)!}{(2j+k+1)!j!k!} e^{-\tilde{\beta}_2\left(t - \frac{(2j+1)r}{v}\right)} \times \left[e^{i(\omega_2 - \omega_1)\left(t - \frac{(2j+1)r}{v}\right)} (\omega_1 - \omega_2)^k + (\omega_2 - \omega_1)^k \right] \times H\left(t - \frac{(2j+1)r}{v}\right). \quad (21)$$

Eqs. (19) and (21) give the probability amplitudes containing all the retardation time. In the following chapter, we will numerically sketch the time evolutions of the populations.

3. Results and discussions

We first consider that two QDs are placed in free space. To demonstrate the retardation effect obviously, we choose some unrealistic parameters and numerically depict the time evolution of excitation populations in Fig. 1. The solid (dashed) curve represents the result of $\omega_1 = \omega_2 = 2.1\tilde{\beta}_1$ ($2.5\tilde{\beta}_1$). As seen, the population of QD1 first decays exponentially, and the population of QD2 becomes non-zero at $t = r/v$ since it starts to receive the photon emitted from QD1. Then, the population of QD1 revives at $t = 2r/v$ since it may receive the photon from QD2. The wavy curves reveal the fact that a photon is passed back and forth between two QDs. One also finds that, if the energy spacing $\hbar\omega$ is increased, the population decays more rapidly (dashed curve) and the retardation effect dies away eventually.

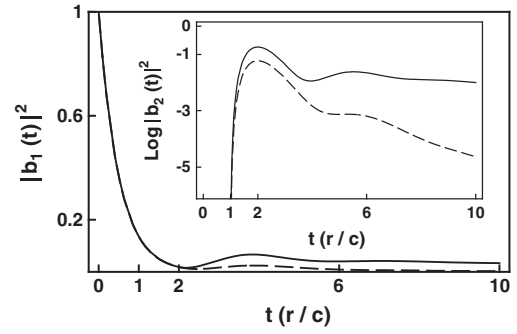


Fig. 1. Occupation probabilities of two QDs with unrealistic parameters: $\omega_1 = \omega_2 = 2.1\tilde{\beta}_1$ (solid) and $\omega_1 = \omega_2 = 2.5\tilde{\beta}_1$ (dashed). The wavy curves reveal that a photon is passed back and forth between two QDs.

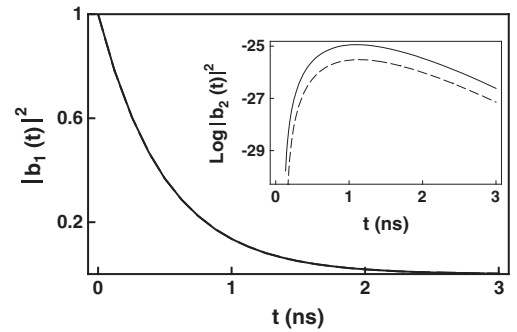


Fig. 2. Occupation probabilities of two QDs with realistic parameters: $\hbar\omega_1 = \hbar\omega_2 = 1.89 \text{ eV}$, $\tilde{\beta}_1 = \tilde{\beta}_2 = 1 \text{ GHz}$, $r = 3 \text{ cm}$ (solid) or 4 cm (dashed), $v = 10^8 \text{ m/s}$. For this realistic setting, QD2 can hardly receive the photon emitted from QD1. Hence, QD1 can not be aware of the existence of QD2, either.

Fig. 2 shows the numerical results for the realistic parameters of QD excitons: $\hbar\omega_1 = \hbar\omega_2 = 1.89 \text{ eV}$, $r = 3 \text{ cm}$ (solid) or 4 cm (dashed), and $\tilde{\beta}_1 = \tilde{\beta}_2 = 1 \text{ GHz}$. As seen from the inset, it is almost impossible to see the effect of retardation for such a small population.

Let us now consider that the QDs are embedded inside a one-dimensional photonic crystal waveguide as shown in Fig. 3. This means the summation of \mathbf{k} in above equations is restricted in one dimension. In plotting Fig. 4(a), some realistic parameters are chosen: $\hbar\omega_1 = \hbar\omega_2 = 1.89 \text{ eV} = \hbar\omega$, $r = 6.55 \text{ cm}$ (solid), 13.1 cm (dashed), and 19.65 cm (dotted), and $\tilde{\beta}_1 = \tilde{\beta}_2 = 1 \text{ GHz}$. We can see that the retardation effect is enhanced considerably. As the distance is increased, the reviving of QD2 is delayed due to the longer traveling time of the photon. Moreover, the probabilities of the two QDs saturate to non-

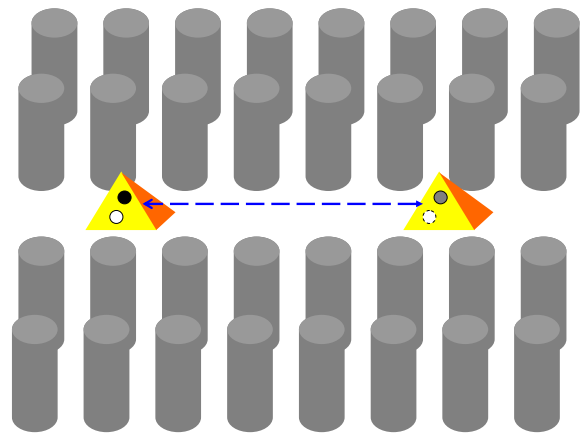


Fig. 3. Schematic view of two QDs embedded inside a one-dimensional waveguide.

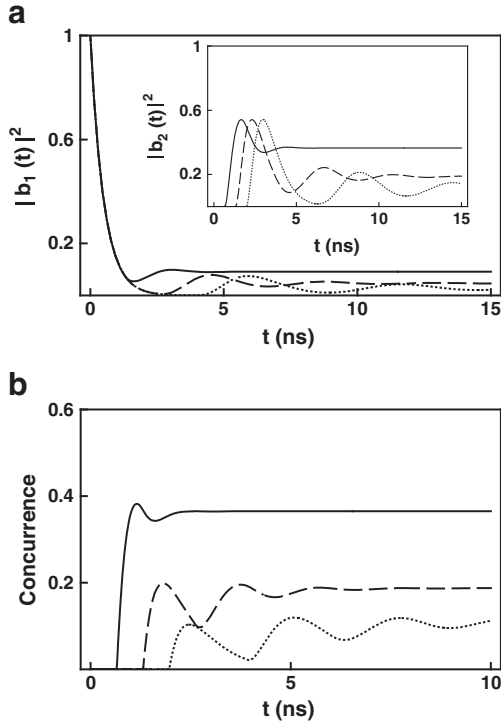


Fig. 4. (a) Occupation probabilities of two QDs in a one-dimensional waveguide. The parameters are chosen as: $\hbar\omega_1 = \hbar\omega_2 = 1.89$ eV, $\tilde{\beta}_1 = \tilde{\beta}_2 = 1$ GHz, $r = 6.55$ cm (solid), 13.1 cm (dashed) or 19.65 cm (dotted), $v = 10^8$ m/s. As $\omega r/v$ is a multiple of π , the curves saturate to non-zero values instead of decay to zero gradually. (b) The corresponding entanglement as a function of time.

zero values due to our special selection here: $\omega r/v$ is a multiple of π . To understand this, one recalls that, without retardation, the probability amplitudes of the dots are [13]

$$\begin{cases} b_1(t) = e^{-2\tilde{\beta}_1 t} (1 + e^{2\tilde{\beta}_1 t}) / 2, \\ b_2(t) = e^{-i\omega r/v - 2\tilde{\beta}_1 t} (-1 + e^{2\tilde{\beta}_1 t}) / 2. \end{cases} \quad (22)$$

As seen from Eq. (22), there is always 50% chance for the two dots to evolve into the trapping state: $|eg\rangle + e^{-i\omega r/v}|ge\rangle$. This means the retardation tends to suppress the formation of this trapping state as the inter-dot distance r is increased as shown in Fig. 4(a). Since the double dot system can also be viewed as two qubits, we also plot in Fig. 4(b) the concurrence as a function of time for different inter-dot distances. One clearly observes that the entanglement also reveals the features of sudden-rise and photon-trapping.

A natural question to ask is that what happens if the inter-dot distance r does not satisfy the condition of $\omega r/cv = n\pi$? or What if the energy spacings of two dots are not identical ($\hbar\omega_1 \neq \hbar\omega_2$)? To answer the questions, the inter-dot distance of the dashed curve in Fig. 5 is shifted by 5×10^{-6} cm, compared to that of the black curve ($r = 6.55$ cm). While for the dotted curve, the energy difference between two dots is 6.5×10^{-7} eV, compared to the zero-energy difference of the black curve. As seen from Fig. 5, non-zero populations and concurrence no longer exist in long time limit for both variations, meaning that the trapping state is easily destroyed in real situations.

It is also interesting to consider different initial conditions. If some other initial conditions ($b_1(0) \neq 1$ and $b_2(0) \neq 0$) are used, the new excitation populations are the linear combinations of Eqs. (19) and (21):

$$\begin{cases} \tilde{b}_1(t) = b_1(0)b_1(t) + b_2(0)b_2(t), \\ \tilde{b}_2(t) = b_2(0)b_1(t) + b_1(0)b_2(t). \end{cases} \quad (23)$$

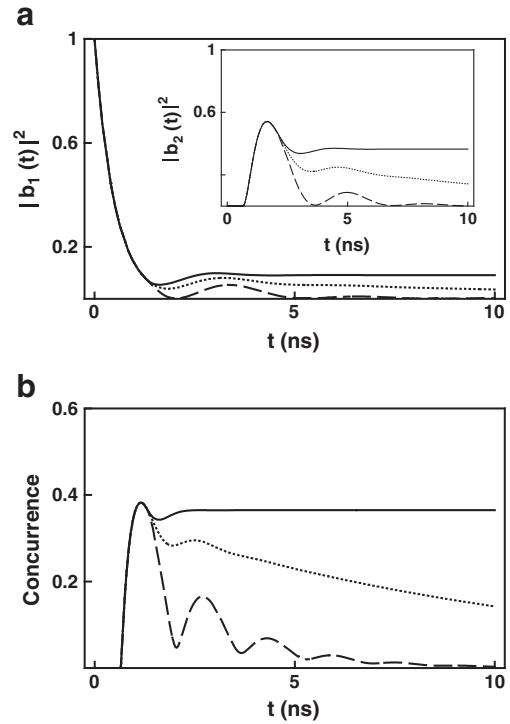


Fig. 5. (a) Populations and (b) entanglement of two QDs for non-saturating conditions. The parameters for the black curve are $\hbar\omega_1 = \hbar\omega_2 = 1.89$ eV, $\tilde{\beta}_1 = \tilde{\beta}_2 = 1$ GHz, $r = 6.55$ cm, and $v = 10^8$ m/s. If the separation distance between two QDs is shifted by 5×10^{-6} cm (dashed) or the energy spacings of both QDs are shifted by 6.5×10^{-7} eV (dotted), the saturation condition is broken and the populations and entanglement die away eventually.

In plotting Fig. 6, the following initial conditions are employed: $b_1(0) = -b_2(0) = 1/\sqrt{2}$. As seen, QD1 initially decays with an exponential way as if it is placed in an isolated environment. As the retardation effect comes in, it starts to show the collective decay (superradiant) behavior.

In addition to the feature of sudden rise, there is also the feature of sudden fall if the conditions: $b_1(0) = b_2(0) = 1/\sqrt{2}$ are chosen as shown in Fig. 7. To understand this, we explicitly write down the terms in early time:

$$\tilde{b}_1(t) = e^{-\tilde{\beta}_1 t} H(t) + \frac{1}{2!} \left[A_1 \left(t - \frac{2r}{v} \right) e^{i\frac{\omega r}{v}} \right]^2 e^{-\tilde{\beta}_1 \left(t - \frac{2r}{v} \right)} H \left(t - \frac{2r}{v} \right) \quad (24)$$

$$+ \frac{1}{4!} \left[A_1 \left(t - \frac{4r}{v} \right) e^{i\frac{\omega r}{v}} \right]^4 e^{-\tilde{\beta}_1 \left(t - \frac{4r}{v} \right)} H \left(t - \frac{4r}{v} \right) + \dots,$$

$$\tilde{b}_2(t) = - \left[A_2 \left(t - \frac{r}{v} \right) e^{i\frac{\omega r}{v}} \right] e^{-\tilde{\beta}_1 \left(t - \frac{r}{v} \right)} H \left(t - \frac{r}{v} \right) \quad (25)$$

$$- \frac{2}{3!} \left[A_2 \left(t - \frac{3r}{v} \right) e^{i\frac{\omega r}{v}} \right]^3 e^{-\tilde{\beta}_1 \left(t - \frac{3r}{v} \right)} H \left(t - \frac{3r}{v} \right) + \dots,$$

where we have assumed $\tilde{\beta}_1 = \tilde{\beta}_2$. First of all, if $\omega r/v$ is a multiple of π , then all terms add constructively, leading to the saturating behavior. Second, if $b_1(0) = 1/\sqrt{2}$, while $b_2(0) = \pm 1/\sqrt{2}$, then the population of the QD1 in the time period $0 < t < \frac{2r}{v}$ can be expressed as

$$\tilde{b}_1(t) = \frac{1}{\sqrt{2}} e^{-\tilde{\beta}_1 t} \mp 2 \left[A_2 \left(t - \frac{r}{v} \right) e^{i\frac{\omega r}{v}} \right] e^{-\tilde{\beta}_1 \left(t - \frac{r}{v} \right)} \cdot H \left(t - \frac{r}{v} \right). \quad (26)$$

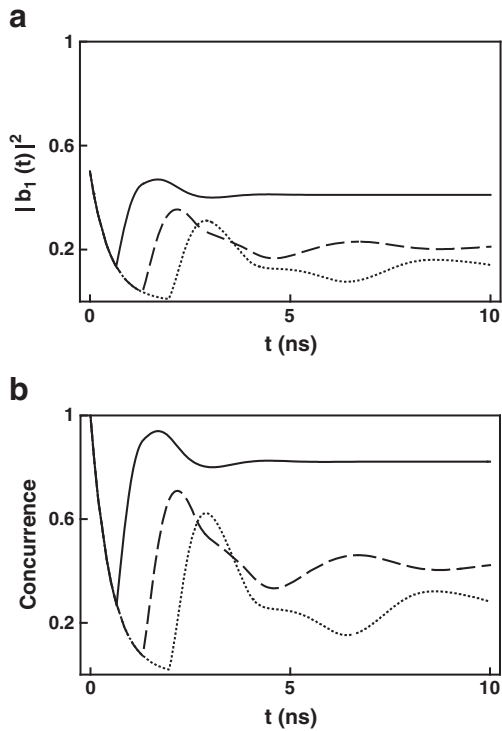


Fig. 6. (a) Populations and (b) entanglement of two QDs with the initial conditions: $b_1(0) = 1/\sqrt{2}$, $b_2(0) = -1/\sqrt{2}$. The parameters are chosen as: $\hbar\omega_1 = \hbar\omega_2 = 1.89$ eV, $\beta_1 = \beta_2 = 1$ GHz, $r = 6.55$ cm (solid), 13.1 cm (dashed), 19.65 cm (dotted), $v = 10^8$ m/s.

The $-$ and $+$ signs in Eq. (26) give the features of sudden fall and sudden rise, respectively.

A few remarks about entanglement issue presented in this work should be addressed here. Ever since the phenomenon of entanglement sudden death pointed out by Yu and Eberly [14], great attention has been focused on this issue [15]. In our work, entanglement sudden death [16] does not occur since we consider only one excitation initially. Therefore, the entanglement sudden rise and sudden fall found in this work are actually due to the non-Markovian (retardation) effect. In addition, one notes that the entanglement trapping phenomenon can also occur in other environments. For example, if two qubits are placed in photonic band-gap materials, entanglement trapping can occur and entanglement sudden death is thus prevented [17].

A few remarks on the experimental realizations should be addressed here. Recently, the progress of the high-Q photonic crystal waveguide cavities [18] provides promising properties such as strong coupling, one-dimensional propagation, and low leakage. Thus, if the QDs are placed inside such a one-dimensional waveguide, the retardation effect can be enhanced considerably. Another Alternative is that the QDs can be coupled to the metal nanowire surface plasmons [13,19] since the surface plasmon propagates quasi-one-dimensionally on the surface of the metal nanowire. Due to the strong coupling between QDs and surface plasmons [13,19], this also provides a practical way for experimental realizations.

4. Summary

In this work, we investigate the retardation effect between two QDs. If the QDs are placed in free space, the retardation effect is too weak to be observed. However, if the propagating photons are confined in one-dimensional space, the retardation effect can be enhanced considerably. For equal energy-level spacings and $\omega r/v = n\pi$, the populations and entanglement of the dots saturate to non-vanishing values. The features

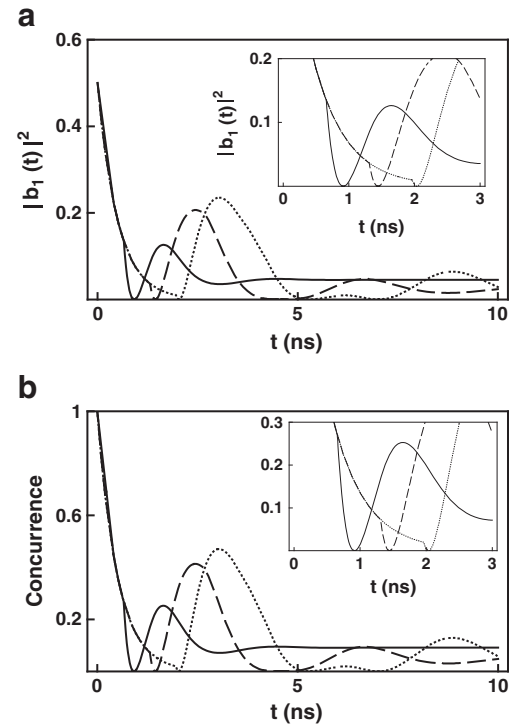


Fig. 7. (a) Populations and (b) entanglement of two QDs with the initial conditions: $b_1(0) = b_2(0) = 1/\sqrt{2}$. The parameters are chosen as: $\hbar\omega_1 = \hbar\omega_2 = 1.89$ eV, $\beta_1 = \beta_2 = 1$ GHz, $r = 6.55$ cm (solid), 13.1 cm (dashed), 19.65 cm (dotted), $v = 10^8$ m/s. The insets are the zoom-in plots to demonstrate more clearly the sudden-fall.

of sudden-rise and sudden-fall are pointed out and can be explained by non-Markovian retardation effect.

Acknowledgments

This work is supported partially by the National Science Council, Taiwan under the grant number 98-2112-M-006-002-MY3.

References

- [1] E. Fermi, Rev. Mod. Phys. 4 (1932) 87.
- [2] B. Feretti, R.E. Peierls, Nature 160 (1947) 531; J. Hamilton, Proc. Phys. Soc. A 62 (1949) 12; W. Heitler, S.T. Ma, Proc. R. Irish Acad. 52 (1949) 123.
- [3] M.I. Shirokov, Yad. Fiz. 4 (1966); Sov. J. Nucl. Phys. 774 (1967).
- [4] G.C. Hegerfeldt, Phys. Rev. Lett. 72 (1994) 596.
- [5] P.W. Milonni, P.L. Knight, Phys. Rev. A 10 (1974) 1096; P.W. Milonni, P.L. Knight, Phys. Rev. A 11 (1975) 1090; P.W. Milonni, D.F.V. James, H. Fearn, Phys. Rev. A 52 (1995) 1525.
- [6] D. Meschede, W. Jhe, E.A. Hinds, Phys. Rev. A 41 (1990) 1587; U. Dorner, P. Zoller, Phys. Rev. A 66 (2002) 023816; A. Beige, J. Pachos, H. Walther, Phys. Rev. A 66 (2002) 063801.
- [7] J. Eschner, Raab Ch., F. Schmidt-Kaler, R. Blatt, Nature 413 (2001) 495; F. Dubin, D. Rotter, M. Mukherjee, C. Russo, J. Eschner, R. Blatt, Phys. Rev. Lett. 98 (2007) 183003.
- [8] E. Hanamura, Phys. Rev. B 38 (1988) 1228; Y.N. Chen, D.S. Chuu, T. Brandes, Phys. Rev. Lett. 90 (2003) 166802.
- [9] J.M. Gerard, B. Sermage, B. Gayral, B. Legrand, E. Costard, V. Thierry-Mieg, Phys. Rev. Lett. 81 (1998) 1110; G.S. Solomon, M. Pelton, Y. Yamamoto, Phys. Rev. Lett. 86 (2001) 3903; J.P. Reithmaier, G. Sek, A. Löffler, C. Hofmann, S. Kuhn, S. Reitzenstein, L.V. Keldysh, V.D. Kulakovskii, T.L. Reinecke, A. Forchel, Nature 432 (2004) 197.
- [10] S. Noda, M. Fujita, T. Asano, Nature Photonics 1 (2007) 449 and the references therein.
- [11] C.J.M. Smith, H. Benisty, S. Olivier, M. Rattier, C. Weisbuch, T.F. Krauss, Appl. Phys. Lett. 77 (2000) 2813.
- [12] M.V. Kotlyar, T. Karle, M.D. Settle, L. O'Faolain, T.F. Krauss, Appl. Phys. Lett. 84 (2004) 3588.
- [13] G.Y. Chen, Y.N. Chen, D.S. Chuu, Opt. Lett. 33 (2008) 2212; Y.N. Chen, G.Y. Chen, D.S. Chuu, T. Brandes, Phys. Rev. A 79 (2009) 033815.

- [14] T. Yu, J.H. Eberly, Phys. Rev. Lett. 93 (2004) 140404;
T. Yu, J.H. Eberly, Phys. Rev. Lett. 97 (2006) 140403.
- [15] T. Yu, J.H. Eberly, Science 323 (2009) 598 and the references therein.
- [16] Z. Ficek, R. Tanaś, Phys. Rev. A 77 (2008) 054301.
- [17] B. Bellomo, R. LoFranco, S. Maniscalco, G. Compagno, Phys. Rev. A 78 (2008) 060302 (R).
- [18] Y. Akahane, T. Asano, B.S. Song, S. Noda, Nature 425 (2003) 944;
B.S. Song, S. Noda, T. Asano, Y. Akahane, Nature Material 4 (2005) 207;
F.S.F. Brossard, X.L. Xu, D.A. Williams, M. Hadjipanayi, M. Hugues, M. Hopkinson, X. Wang, R.A. Taylor, Appl. Phys. Lett. 97 (2010) 111101;
R. Bose, D. Sridharan, G.S. Solomon, E. Waks, Opt. Express 19 (2011) 5398.
- [19] A.V. Akimov, A. Mukherjee, C.L. Yu, D.E. Chang, A.S. Zibrov, P.R. Hemmer, H. Park, M.D. Lukin, Nature 450 (2007) 402;
D.E. Chang, A.S. Sørensen, P.R. Hemmer, M.D. Lukin, Phys. Rev. Lett. 97 (2006) 053002;
D.E. Chang, A.S. Sørensen, E.A. Demler, M.D. Lukin, Nature Physics 3 (2007) 807.