On Nonlocality and Incompatibility breaking channel

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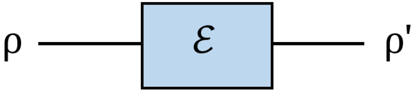
Outline

- Quantum channel, Nonlocality breaking and Incompatibility breaking channel (IBC)
- CHSH nonlocality breaking and 2-IBC
- Nonlocality breaking and 2-IBC in tripartite scenario
- 2-IBC's in bi-locality, n-locality and n-star network scenario

Quantum Channel

- **Quantum Channel:** A *quantum channel* (QC) (CPTP) map $\mathcal{E} : \mathcal{L}(\mathcal{H}^{\mathcal{A}}) \to \mathcal{L}(\mathcal{H}^{\mathcal{B}})$,



$$\rho' = \mathcal{E}(\rho) = \sum_{j=1}^{n} K_j \rho K_j^{\dagger}$$

where K_i satisfies $\sum_j K_j^{\dagger} K_j = \mathbb{I}$.

In Heisenberg picture, the corresponding description is given in terms of dual channel $\mathcal{E}^*$ acting on operators

$$Tr[\mathcal{E}(\rho)A] = Tr[\rho\mathcal{E}^*(A)]; \quad \forall \rho, A \tag{1}$$

Nonlocality and Incompatibility: The Bell operator associated with the CHSH inequality has the form

$$\mathcal{B} = A_1 \otimes (B_1 + B_2) + A_2 \otimes (B_1 - B_2),$$

Then the Bell-CHSH inequality, for any state ρ is

$$Tr[\rho \mathcal{B}(A_1, A_2, B_1, B_2)] \leq 2. \tag{2}$$

The violation of above inequality (2) is sufficient to justify the non-locality of quantum state. However incompatible observables acting on entangled particles enable the nonlocality. Thus, incompatibility is the necessary to lead the violation of CHSH inequality.

M. Wolf et.al., PRL, 103, 230402 (2009).

Nonlocality Breaking Channel

Definition

Any channel $\mathcal{E} : \mathcal{L}(\mathcal{H}) \to \mathcal{L}(\mathcal{H})$ is said to be *NBC* if applying on one side of (arbitrary) bipartite state ρ_{AB}, it produces a state $\rho'_{AB} = (\mathcal{E} \otimes \mathbb{I})(\rho_{AB})$ which satisfies the Bell-CHSH inequality (2).

A unital channel is particularly important as it breaks the non-locality for any state, when it breaks for maximally entangled states.

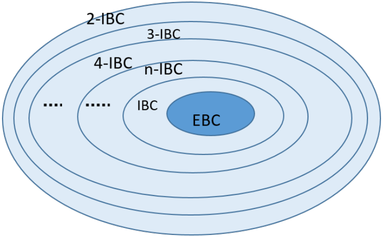
R. Pal and S. Ghosh, J. Phys. A, 48, 155302 (2015).

Incompatibility Breaking Channel

Definition

Any channel $\mathcal{E} : \mathcal{L}(\mathcal{H}) \rightarrow \mathcal{L}(\mathcal{H})$ is said to be *IBC* if after mapping $\mathcal{E}(\mathcal{A})$ is compatible, for incompatible subset $\mathcal{A}$ of given observables.

If a channel $\mathcal{E}$ breaks the incompatibility of every class of n observables, it is said to be $n -$ **IBC**, $2 -$ **IBC**'s is the channel which breaks the incompatibility for pairs.



T. Heinosaari *et.al.*, J. Phys. A, 48, 435301 (2015).

CHSH nonlocality and Incompatibility Breaking Channel

Theorem

If $(\mathcal{E} \otimes \mathbb{I})$ *is* 2*-IBC* , *then* $(\mathcal{E}^* \otimes \mathbb{I})$ *is CHSH non-locality breaking.*

Proof.

Let $(\mathcal{E} \otimes \mathbb{I})$ be 2-*IBC*, then by the application of channel on Alice side incompatible measurements, say A_1 and A_2 becomes compatible, if the measurement on Bob side is B_1 and B_2, then we can write

$$
\begin{aligned}
& Tr[\rho \mathcal{B}(\mathcal{E}(A_1), \mathcal{E}(A_2), B_1, B_2)] \leq 2; \quad \forall \rho, A_1, A_2, B_1, B_2 \\
\Rightarrow \quad & Tr[\rho \mathcal{E}(A_1) \otimes B_1] + Tr[\rho \mathcal{E}(A_2) \otimes B_1] \\
& + Tr[\rho \mathcal{E}(A_1) \otimes B_2] - Tr[\rho \mathcal{E}(A_2) \otimes B_2] \leq 2 \\
\Rightarrow \quad & Tr[\mathcal{E}^*(\rho) A_1 \otimes B_1] + Tr[\mathcal{E}^*(\rho) A_2 \otimes B_1] \\
& + Tr[\mathcal{E}^*(\rho) A_1 \otimes B_2] - Tr[\mathcal{E}^*(\rho) A_2 \otimes B_2] \leq 2
\end{aligned}
$$

$\Rightarrow (\mathcal{E}^* \otimes \mathbb{I})$ is CHSH non-locality breaking. □

CHSH nonlocality and Incompatibility Breaking Channel

Theorem

If $(\mathcal{E} \otimes \mathbb{I})$ *is CHSH non-locality breaking, then* $(\mathcal{E}^* \otimes \mathbb{I})$ *is* 2*-IBC, provided the channel is unital.*

Proof.

Let $(\mathcal{E} \otimes \mathbb{I})$ be CHSH non-locality breaking, then

$$\begin{aligned} & Tr[\mathcal{E}(\rho)\mathcal{B}(A_1, A_2, B_1, B_2)] \leq 2; \quad \forall \rho, A_1, A_2, B_1, B_2 \\ \Rightarrow \quad & Tr[\rho\mathcal{E}^*(A_1) \otimes B_1] + Tr[\rho\mathcal{E}^*(A_1) \otimes B_2] \\ & + Tr[\rho\mathcal{E}^*(A_2) \otimes B_1] - Tr[\rho\mathcal{E}^*(A_2) \otimes B_2] \leq 2 \end{aligned}$$

$\Rightarrow \mathcal{E}^*(A_1)$ and $\mathcal{E}^*(A_2)$ is compatible Hence, $\mathcal{E}^*$ is 2-*IBC*, where $\mathcal{E}^*$ is unital, i.e., $\mathcal{E}^*(\mathbb{I}) = \mathbb{I}$. □

The Svetlichny inequality and the maximal violation The violation of Svetlitchny inequality shows the genuine tripatite non-locality, hereby, we consider the Svetlichly operator S is given by,

$$\begin{aligned} S &= A \otimes \left[(B+B') \otimes C + (B-B') \otimes C'\right] \\ &+ A' \otimes \left[(B-B') \otimes C - (B+B') \otimes C'\right] \end{aligned}$$

where A, A', B, B', C, C' are observables of the form $G = \vec{g}.\vec{\sigma} = \sum_k g_k \sigma_k$, $G \in \left(A, A', B, B', C, C'\right)$ and

Nonlocality breaking channel in tripartite scenario

$g \in \left(\vec{a}, \vec{a'}, \vec{b}, \vec{b'}, \vec{c}, \vec{c'}\right)$,$\sigma_k (k = 1, 2, 3)$ are the Pauli matrices, $\vec{\sigma} = (\sigma_1, \sigma_2, \sigma_3), \vec{g} = (g_1, g_2, g_3)$ is a three dimensional real unit vectors.

For any three qubit state $|\psi\rangle$, admitting bi-LHV model, the mean value of the Svelitchny operator is bounded as,

$$|\langle\psi|S|\psi\rangle| \leq 4 \tag{3}$$

Definition:3 For any three qubit quantum state ρ, the maximum quantum value $\langle S\rangle_\rho$ of the Svetlichny operator S is bounded as

$$|\langle S\rangle|_\rho \leq 4\lambda_1 \tag{4}$$

where $\langle S\rangle_\rho = Tr[S\rho]$ and λ_1 is the maximum singular value of the matrix $M = (M_{j,ik})$, with $M = (M_{ijk} = Tr[\rho(\sigma_i \otimes \sigma_j \otimes \sigma_k)]$, $i, j, k = 1, 2, 3$.

M. Li *et.al.*, PRA 96, 042323 (2017).

Conditions of violation of Svetlichny inequality

Example 1. The generalized GHZ state $|\psi\rangle = \alpha|000\rangle + \beta|111\rangle$, where α and β are real with $\alpha^2 + \beta^2 = 1$; can be stated in the form of Pauli matrices

$$\begin{aligned}
\rho_g &= \frac{1}{8}[I \otimes I \otimes I + I \otimes \sigma_3 \otimes \sigma_3 + \sigma_3 \otimes I \otimes \sigma_3 + \sigma_3 \otimes \sigma_3 \otimes I \\
&+ (\alpha^2 - \beta^2)(\sigma_3 \otimes I \otimes I + I \otimes \sigma_3 \otimes I + I \otimes I \otimes \sigma_3 \\
&+ \sigma_3 \otimes \sigma_3 \otimes \sigma_3) + \alpha\beta(\sigma_1 \otimes \sigma_1 \otimes \sigma_1 - \sigma_1 \otimes \sigma_2 \otimes \sigma_2 \\
&- \sigma_2 \otimes \sigma_1 \otimes \sigma_2 - \sigma_2 \otimes \sigma_2 \otimes \sigma_1)]
\end{aligned}$$

Violation of Svetlichny inequality

From Def(3), we can obtain the matrix M_1 is given by

$$M_1 = \begin{pmatrix} 2\alpha\beta & 0 & 0 & -2\alpha\beta & 0 & 0 & 0 & 0 & 0 \\ 0 & -2\alpha\beta & 0 & 0 & -2\alpha\beta & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \alpha^2 - \beta^2 \end{pmatrix}$$

with two equal singular values $\lambda_1 = 2\alpha\beta\sqrt{2}, \lambda_2 = 2\alpha\beta\sqrt{2}$ and $\lambda_3 = \alpha^2 - \beta^2$ respectively. The maximum of Svelitchny operator for the GGHZ state is given by

$$\langle S \rangle_{\rho_g} = 8\sqrt{2}\alpha\beta$$

Thus the violation of Svetlinchy's inequality for GGHZ state is valid for

$$\alpha\beta > \frac{1}{2\sqrt{2}} \tag{5}$$

The maximum violation $(4\sqrt{2})$ is obtained for GHZ state $(\alpha = \beta = 1/\sqrt{2})$.

Violation of Svetlichny inequality

Example 2. The Maximally Slice state $|\psi\rangle = 1/\sqrt{2}|000\rangle + |11(\alpha|0\rangle + \beta|1\rangle)$, where α and β are real with $\alpha^2 + \beta^2 = 1$; can be stated in the form of Pauli matrices

$$\begin{aligned}
\rho_s &= \frac{1}{8}\Big[I \otimes I \otimes I + \frac{(1+\alpha^2-\beta^2)}{2} I \otimes I \otimes \sigma_3 \\
&+ \frac{(1-\alpha^2+\beta^2)}{2} I \otimes \sigma_3 \otimes \sigma_3 + \frac{(1-\alpha^2+\beta^2)}{2} \sigma_3 \otimes I \otimes \sigma_3 \\
&+ \sigma_3 \otimes \sigma_3 \otimes I + \frac{(1+\alpha^2-\beta^2)}{2} \sigma_3 \otimes \sigma_3 \otimes \sigma_3 \\
&+ \alpha(\sigma_1 \otimes \sigma_1 \otimes I + \sigma_1 \otimes \sigma_1 \otimes \sigma_3 - \sigma_2 \otimes \sigma_2 \otimes I \\
&- \sigma_2 \otimes \sigma_2 \otimes \sigma_3) + \beta(\sigma_1 \otimes \sigma_1 \otimes \sigma_1 - \sigma_1 \otimes \sigma_2 \otimes \sigma_2 \\
&- \sigma_2 \otimes \sigma_1 \otimes \sigma_2 - \sigma_2 \otimes \sigma_2 \otimes \sigma_1) + \alpha\beta(I \otimes I \otimes \sigma_1 \\
&- I \otimes \sigma_3 \otimes \sigma_1 - \sigma_3 \otimes I \otimes \sigma_1 + \sigma_3 \otimes \sigma_3 \otimes \sigma_1)\Big]
\end{aligned}$$

Violation of Svetlichny inequality

From Def(3), we can obtain the matrix M_2 is given by

$$M_2 = \begin{pmatrix} \beta & 0 & \alpha & 0 & -\beta & 0 & 0 & 0 & 0 \\ 0 & -\beta & 0 & -\beta & 0 & -\alpha & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \alpha\beta & 0 & \frac{1+\alpha^2-\beta^2}{2} \end{pmatrix}$$

with $\lambda_1 = \sqrt{\frac{1}{4}\left(\alpha^2 - \beta^2 + 1\right)^2 + (\alpha\beta)^2}$ and two equal singular values $\lambda_2 = \lambda_3 = \sqrt{(\alpha^2 + 2\beta^2)}$ respectively.

$$\langle S \rangle_{\rho_s} = 4\sqrt{(\alpha^2 + 2\beta^2)}$$

Thus the violation of Svetlinchy's inequality for maximally sliced state is valid for

$$\sqrt{(\alpha^2 + 2\beta^2)} > 1 \tag{6}$$

Violation of Svetlichny inequality

We get the maximum violation of Svetlinchy's inequality for maximally sliced state ($\alpha = 0$ and $\beta = 1$) to be $4\sqrt{2}$. Note that for the values of $\alpha = 0$ and $\beta = 1$, maximally sliced state reduces to the GHZ state.

Example 3. Consider the quantum state given by

$$\sigma_A(\rho) = p|GHZ\rangle\langle GHZ| + (1-p)I_2 \otimes \bar{I}$$

$0 \leq p \leq 1$; I_2 stands for 2×2 identity matrix and $\bar{I} = diag(1, 0, 0, 1)$. From Def(3), matrix M_3 is given by

$$M_3 = \begin{pmatrix} p & 0 & 0 & 0 & -p & 0 & 0 & 0 & 0 \\ 0 & -p & 0 & -p & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Nonlocality breaking condition

Maximum quantum value of the state can be obtained as $\langle S \rangle_{\sigma_A} = 4\sqrt{2}p$.

NONLOCALITY BREAKING CONDITION: For any three qubit state ρ_{ABC}, given channel $\mathcal{E}$ is said to be genuine *non-locality breaking* if acting on qubit it gives a state $\rho'_{ABC} = (\mathcal{E} \otimes I \otimes I)(\rho_{ABC})$, which satisfies the Svetlichny inequality, $\langle S \rangle_{\rho'_{ABC}} \leq 4$.

Similarly, the nonlocality breaking condition can be obtained as,

Generalised GHZ state, $\eta \leq \frac{1}{2\sqrt{2}\alpha\beta}$,

Maximally slice state, $\eta \leq \frac{1}{\sqrt{\alpha^2+2\beta^2}}$ and

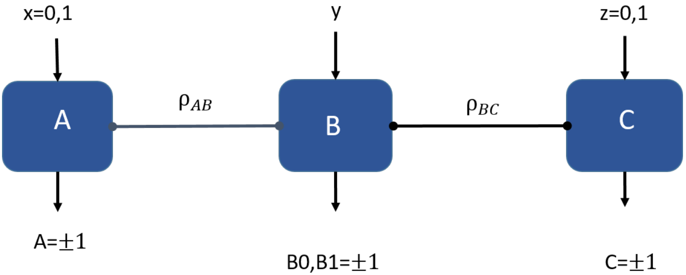
General mixed state, $\eta \leq \frac{1}{\sqrt{2}p}$ respectively.

NBC and IBC

State	Svelitchny violation (Nonlocality witnessing condition)	Nonlocality breaking condition	Incompatibility breaking condition
GGHz state	$\alpha\beta > \frac{1}{2\sqrt{2}}$	$\eta \lesssim \frac{1}{2\sqrt{2}\alpha\beta}$	$\eta \leq \frac{1}{\sqrt{2}}$
Max. Sliced State	$\sqrt{\alpha^2 + 2\beta^2} > 1$	$\eta \lesssim \frac{1}{\sqrt{\alpha^2+2\beta^2}}$	$\eta \lesssim \frac{1}{\sqrt{2}}$
Modified GHZ state	$p > \frac{1}{\sqrt{2}}$	$\eta \lesssim \frac{1}{p\sqrt{2}}$	$\eta \leq \frac{1}{\sqrt{2}}$

Bilocality scenario

No-Signaling and Independence(NSI): Branciard et.al., PRL,2010.



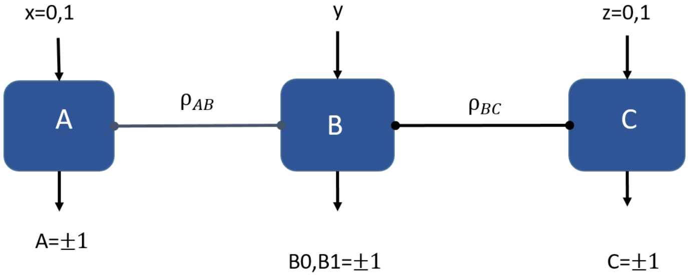
Independent locality(bilocality): $P(\lambda_1, \lambda_2) = P(\lambda_1).P(\lambda_2)$

$$P(ABC|xyz) = \int d\lambda_1 d\lambda_2 q(\lambda_1) q(\lambda_2) P(A|x, \lambda_1) P(B|y, \lambda_1, \lambda_2) P(C|z, \lambda_2)$$

$$\sqrt{|I|} + \sqrt{|J|} < 2$$

where, $I = (A0 + A1)B0(C0 + C1)$; $J = (A0 - A1)B0(C0 - C1)$.

Bilocality scenario



If ρ violates CHSH then $\rho \otimes \rho$ violates bilocality inequality.

($\rho \rightarrow$ all pure entangled state).

N. Gisin, PRA,96,020304(R) (2017).

Hence, 2-IBC $\Leftrightarrow$ non-bilocality breaking. (($\mathcal{E}_A \otimes \mathbb{I}$) and ($\mathbb{I} \otimes \mathcal{E}_C$) are CHSH non-locality breaking then ($\mathcal{E}_A^* \otimes \mathbb{I}$) and ($\mathbb{I} \otimes \mathcal{E}_C^*$) are 2-IBC and vice versa.)

Non-bilocality and Incompatibility Breaking Channel

Definition

A channel $(\mathcal{E}_A \otimes \mathbb{I} \otimes \mathbb{I} \otimes \mathcal{E}_c)$ is said to be non-bilocality breaking if after application of the channel $\rho' \otimes \rho'$ satifies the bilocality inequality.

Theorem

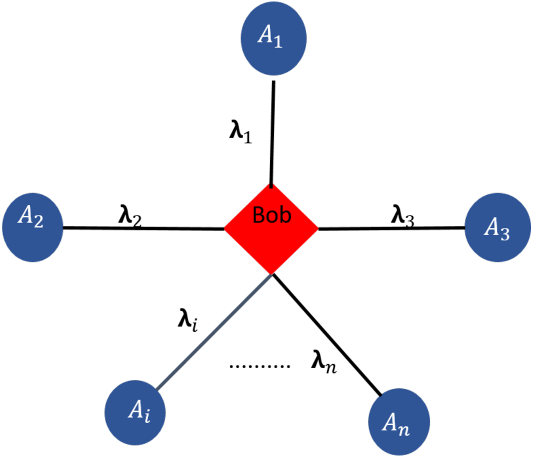
If $(\mathcal{E}_A \otimes \mathbb{I} \otimes \mathbb{I} \otimes \mathcal{E}_c)$ is non-bilocality breaking then $(\mathcal{E}_A^* \otimes \mathbb{I})$ and $(\mathbb{I} \otimes \mathcal{E}_A^*)$ are 2-IBC.

Theorem

If $(\mathcal{E}_A \otimes \mathbb{I})$ and $(\mathbb{I} \otimes \mathcal{E}_A)$ are 2-IBC, then $(\mathcal{E}_A^* \otimes \mathbb{I} \otimes \mathbb{I} \otimes \mathcal{E}_c^*)$ is non-bilocality breaking.

Hence, non-bilocality breaking channels are 2-IBC.

n-star shaped network scenario



Here, we can also show that n-2-IBC are equivalent to non-n-locality breaking channel.

Conclusion

- We show an equivalency relation between CHSH nonlocality and incompatibility breaking channel.
- We extend this study in tripartite scenario using Svetchlichny inequality studied in some well known states.
- We found that, within certain range of channel and state parameter incompatibility assures nonlocality for generalized GHZ and maximally slice state.

Thank You