Transitivity of entanglement

Gelo Noel M. Tabia

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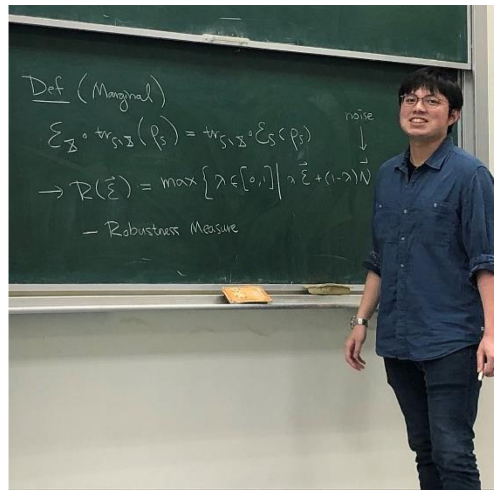
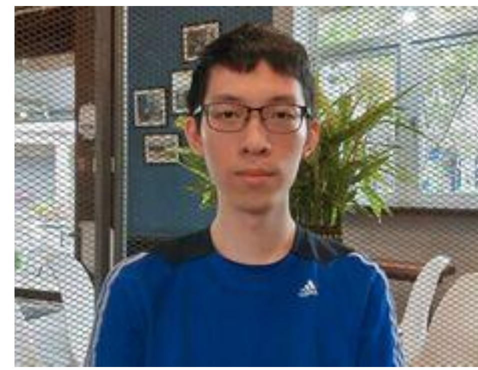
Collaborators





Yeong-Cherng Liang
NCKU

Chung-Yun Hsieh
ICFO

Yu-Chun Yin
NCKU

Overview

What is transitivity of entanglement?

How do we certify it?

Do we have some physical picture for it?

Can we characterize states that display it?

Motivation

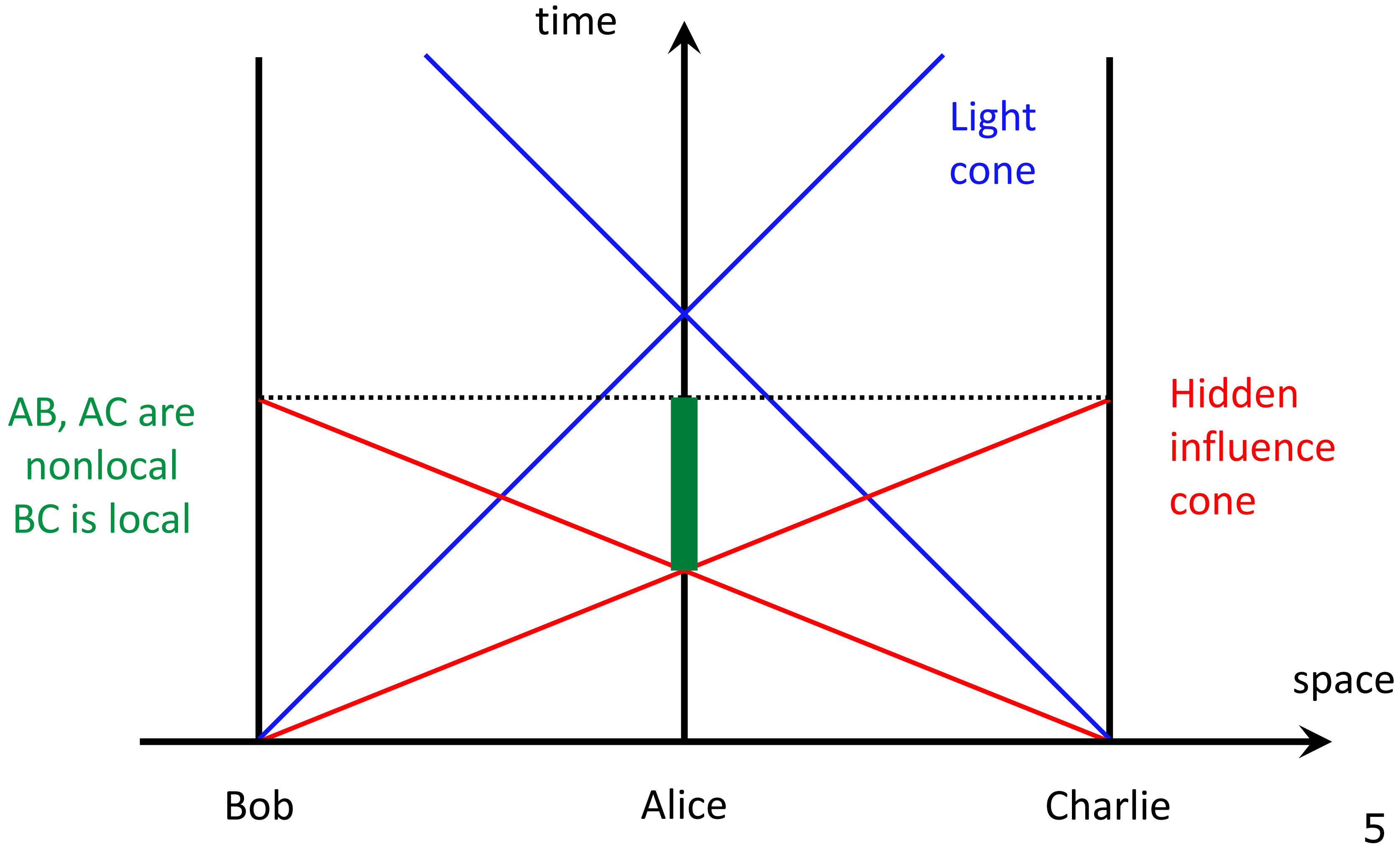
There are quantum correlations that cannot be explained by a local hidden variables.

But maybe Bell-nonlocal correlations come from a hidden superluminal influence between particles.

One way to rule it out is to show transitivity of nonlocality.

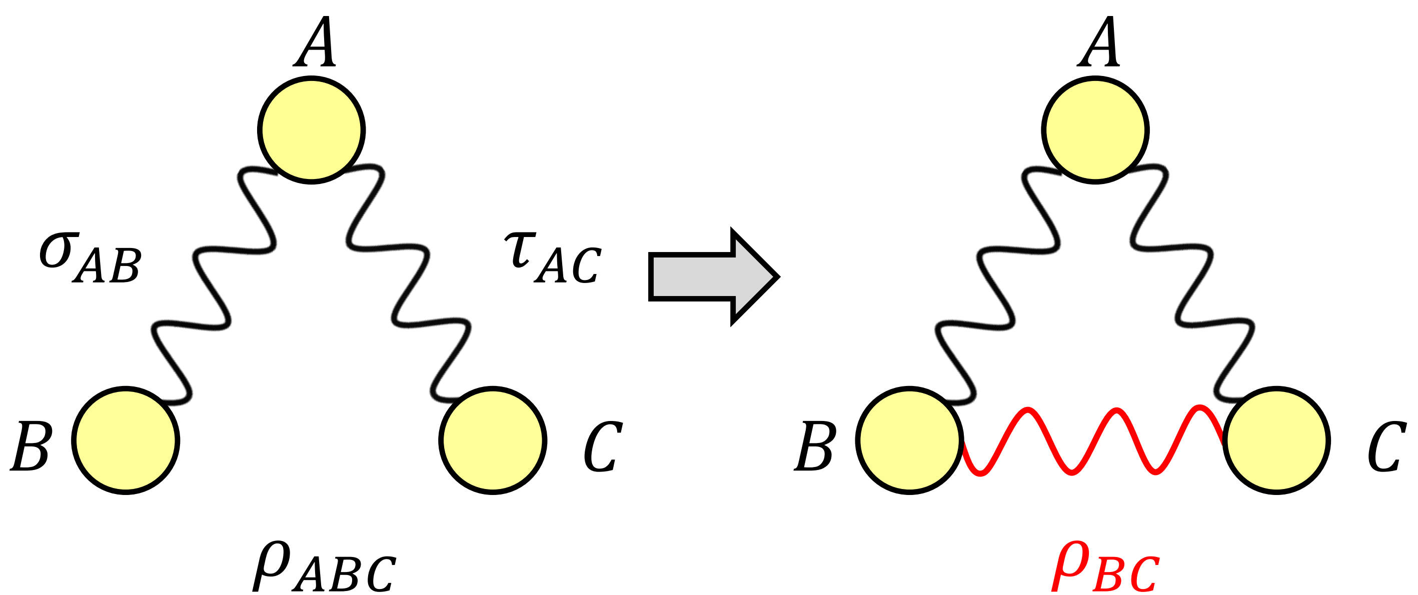
Coretti *et al.*, PRL 107, 100402 (2011)

Superluminal influence



Entanglement transitivity

We say two compatible entangled states σ_{AB} and τ_{AB} exhibit transitivity if for all ρ_{ABC} with $\rho_{AB} = \sigma$, $\rho_{AC} = \tau$, ρ_{BC} is entangled.



Certifying transitivity

We can check if 2-qubit entangled σ_{AB}, τ_{AC} display transitivity with a semidefinite program (SDP):

$$
\begin{aligned}
\max_{\rho_{ABC}} \ & \lambda \\
\text{subject to } \ & \rho_{ABC} \geq 0, \\
& \rho_{AB} = \sigma, \\
& \rho_{AC} = \tau, \\
& \rho_{BC}^{T_B} \geq \lambda I.
\end{aligned}
$$

Dual problem

The dual SDP says it is equivalent to a certain 2-local Hamiltonian problem:

$$\zeta_{ABC} := H_{AB} \otimes I_C$$
$$+ H_{AC} \otimes I_B - \eta_{BC}^{T_B} \otimes I_A$$

$$E_{AB} := \mathrm{tr}(\sigma_{AB} H_{AB})$$
$$E_{AC} := \mathrm{tr}(\tau_{AC} H_{AC})$$

ρ_{ABC} is a zero-energy ground state of ζ_{ABC}

$$\begin{aligned}
\min_{H_{AB}, H_{AC}, \eta_{BC}} \quad & E_{AB} + E_{AC} \\
\text{subject to} \quad & \eta_{ABC} \geq 0, \\
& \mathrm{tr}(\eta_{ABC}) = 1, \\
& H_{AB} = H_{AB}^{\dagger}, \\
& H_{AC} = H_{AC}^{\dagger}, \\
& \zeta_{ABC} \geq 0.
\end{aligned}$$

Example: W-state

Let $\sigma_{AB} = \tau_{AC} = \frac{2}{3}|\Psi^+\rangle\langle\Psi^+| + \frac{1}{3}|00\rangle\langle 00|$

Transitivity SDP has $\lambda_\star = \frac{1-\sqrt{5}}{6}$ for state

$$|W\rangle_{ABC} = \frac{1}{\sqrt{3}}(|100\rangle + |010\rangle + |001\rangle)$$

The dual SDP gives 2-local Hamiltonian

$$\zeta_{ABC} = \zeta_0 + \sum_\alpha J_\alpha^x\,(XX + YY)_\alpha + \sum_\alpha J_\alpha^z\,(ZZ)_\alpha + \sum_i h_i Z_i$$

$$\alpha \in \{AB, AC, BC\}, \qquad i \in \{A, B, C\}$$

Example: W-state

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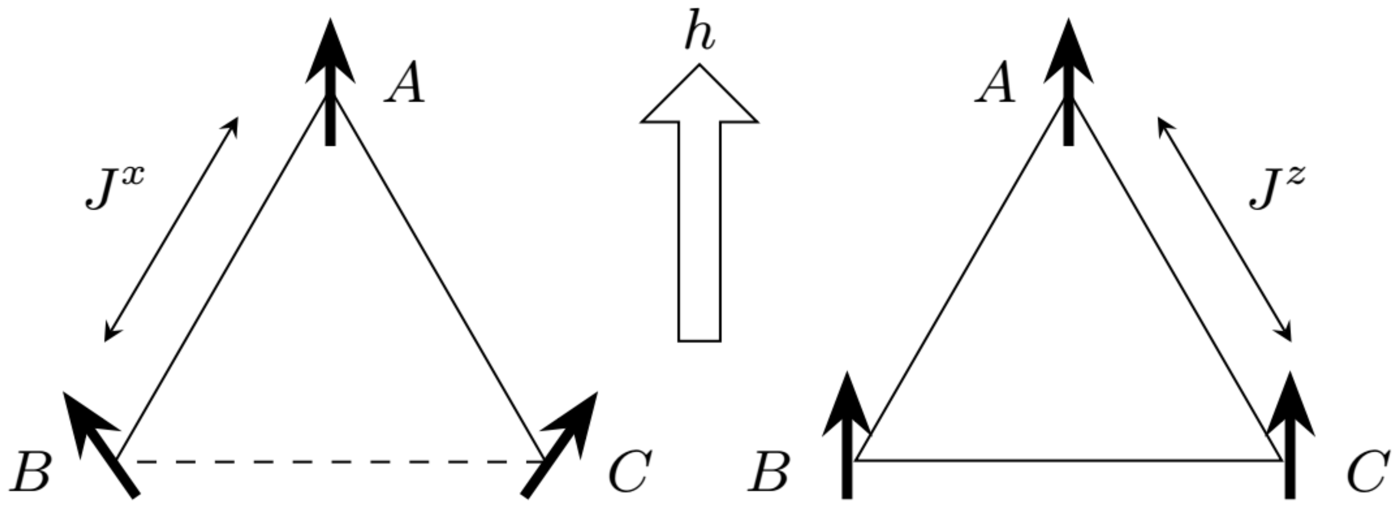


Generalized Heisenberg XXZ model

Symmetric extension

We say ρ_{ABC} is a symmetric extension (SE) of σ_{AB} with respect to A if

$$\mathrm{tr}_C(\rho_{ABC}) = \mathrm{tr}_B(\rho_{ABC}) = \sigma.$$

Transitivity for $\sigma = \tau$ means the bipartite marginals for every SE of σ is entangled

When does a 2-qubit σ_{AB} have a pure and unique SE (e.g., $|W\rangle$)?

Quantum operations

Quantum operations are represented by completely positive (CP) linear maps.

Choi operator: $J(\Lambda) = (\mathrm{id}_A \otimes \Lambda_B)(\Phi_d)$

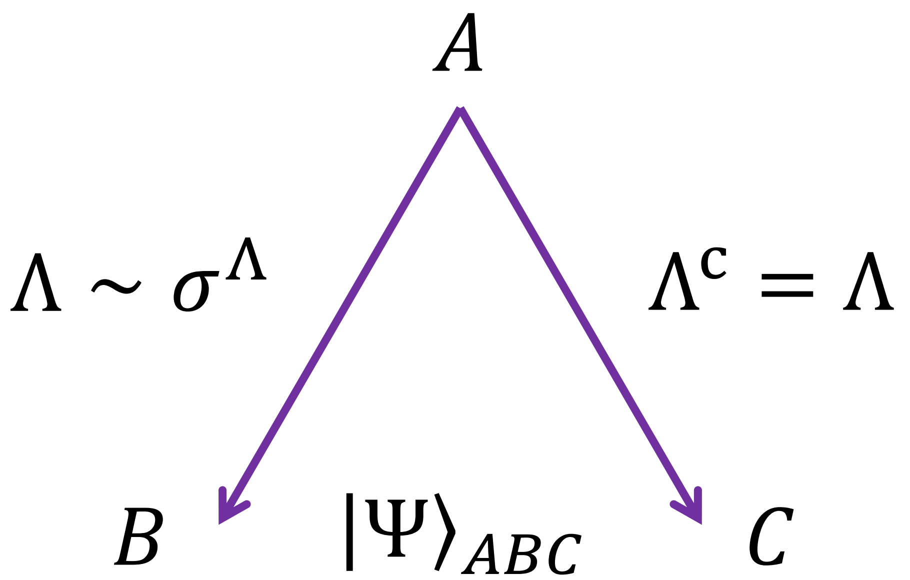
Stinespring isometry: joint operation on SE then discard E: $\Lambda(\rho) = \mathrm{tr}_E\left(V_{SE}\rho_S V_{SE}^{\dagger}\right)$

Complementary operation Λ^{c} of Λ:

$$\Lambda^{\mathrm{c}}(\rho) = \mathrm{tr}_S\left(V_{SE}\rho_S V_{SE}^{\dagger}\right)$$

Self-complementary

Two-qubit σ_{AB} has a pure unique SE iff it describes a self-complementary operation.



$$|\Psi\rangle_{ABC} = a_0|000\rangle + a_1 e^{it}|011\rangle + \frac{b}{\sqrt{2}}(|110\rangle + |101\rangle)$$

Conclusion

We introduced entanglement transitivity and provided a way to certify it.

It is equivalent to finding a 2-local 3-body Hamiltonian of a particular ground state.

Two-qubit entangled σ_{AB} with unique SE $|\Psi\rangle_{ABC}$ shows transitivity if $\mathrm{tr}_A(|\Psi\rangle\langle\Psi|)$ is NPT.

Nonlocality example

A $d \otimes d$ entangled state σ_{AB} that is useful for teleportation is k-copy Bell-nonlocal

W-state marginal $\sigma_{AB} = \frac{2}{3}|\Psi^+\rangle\langle\Psi^+| + \frac{1}{3}|00\rangle\langle 00|$ violates Khot-Vishnoi inequality for $k \geq 28$

Unique SE of $\sigma_{AB}^{\otimes k}$ is $|W\rangle^{\otimes k}$

Thus, k-copy W-state exhibits (state) nonlocality transitivity

Cavalcanti, *et al.*, PRA 87, 042104 (2013)