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前沿量子科技研究中心 Center for Quantum Frontiers of Research & Technology

When Quantum Boundary Meets The Non-signaling Boundary

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Outline

- Bell Nonlocality
- The Boundary of Non-signaling Polytope
- Our Results
- Summary

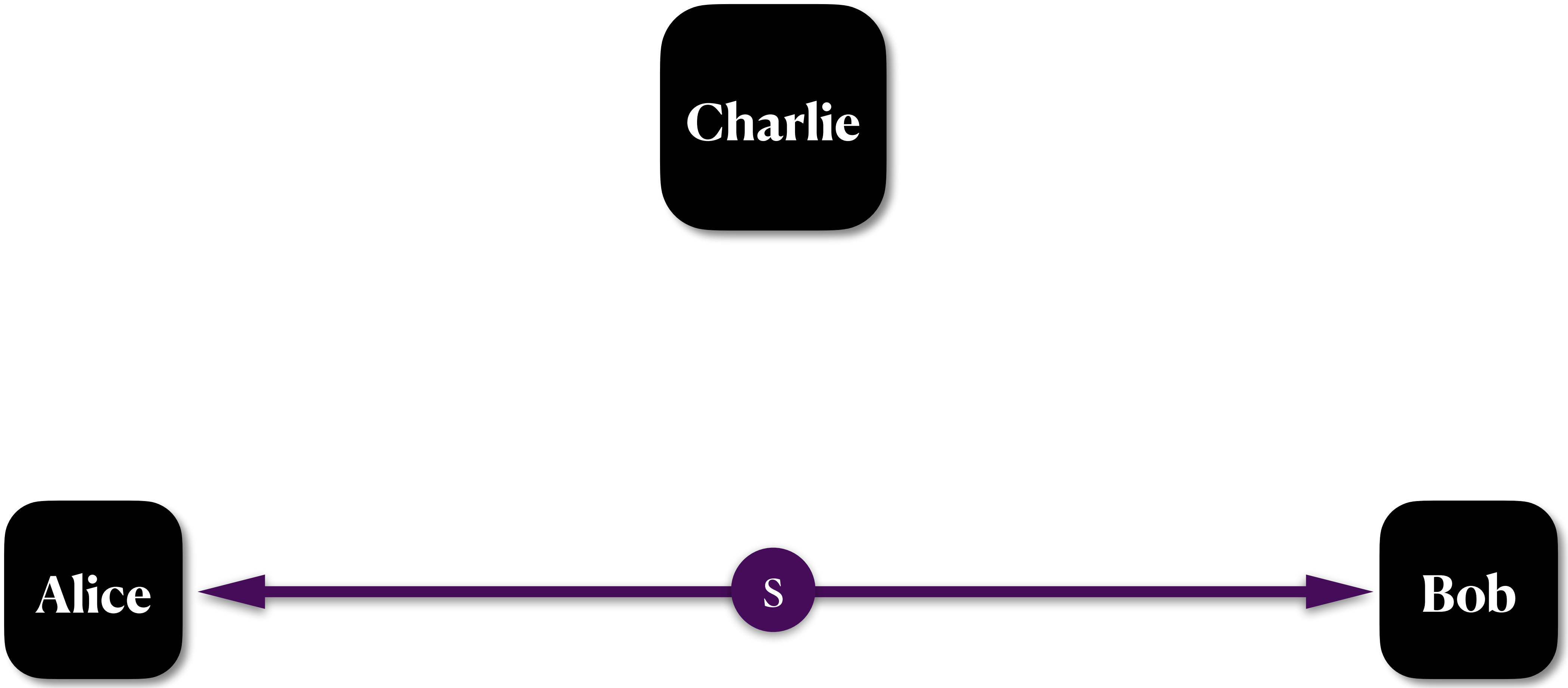
Bell Nonlocality

Bell Nonlocality

The nonlocal game: Clauser-Horne-Shimony-Holt (CHSH) scenario

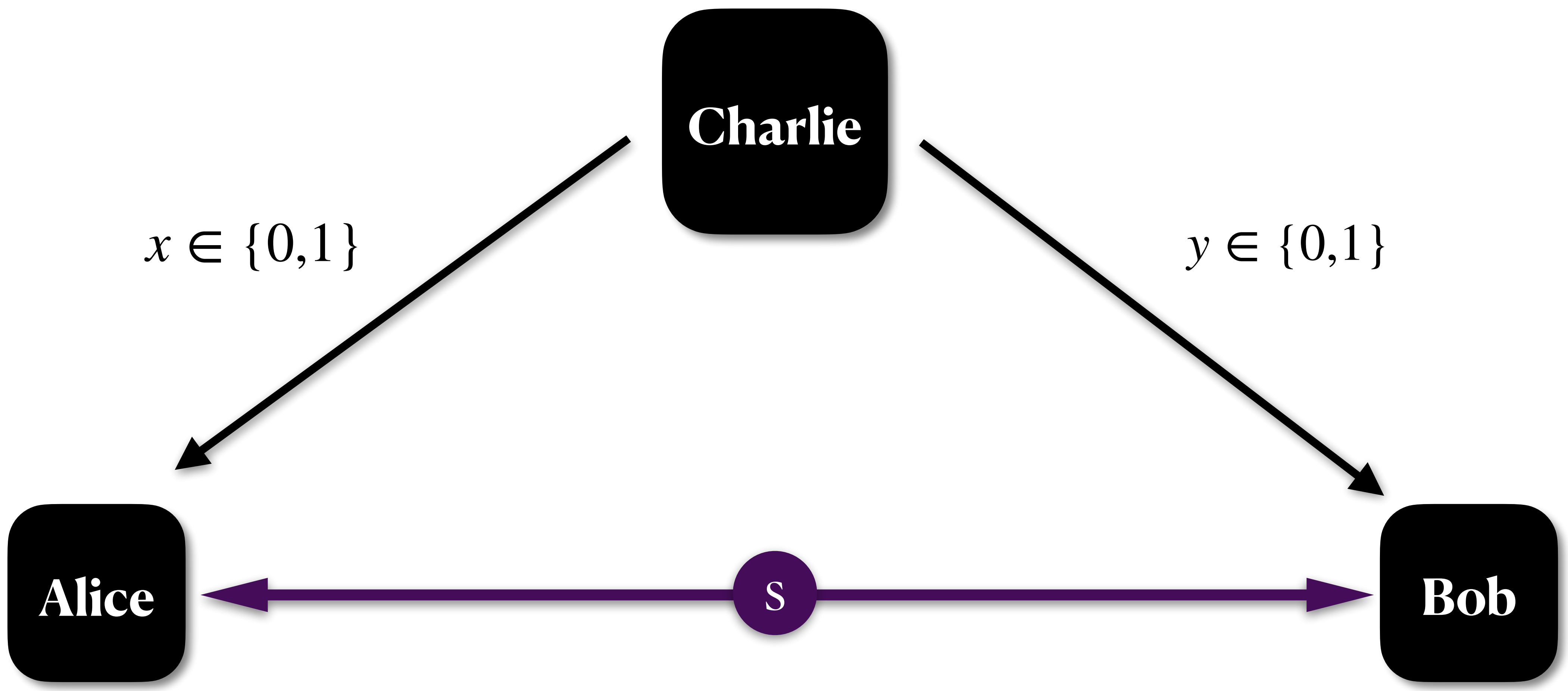
Bell Nonlocality

The nonlocal game: **Clauser-Horne-Shimony-Holt (CHSH) scenario**



Bell Nonlocality

The nonlocal game: Clauser-Horne-Shimony-Holt (CHSH) scenario



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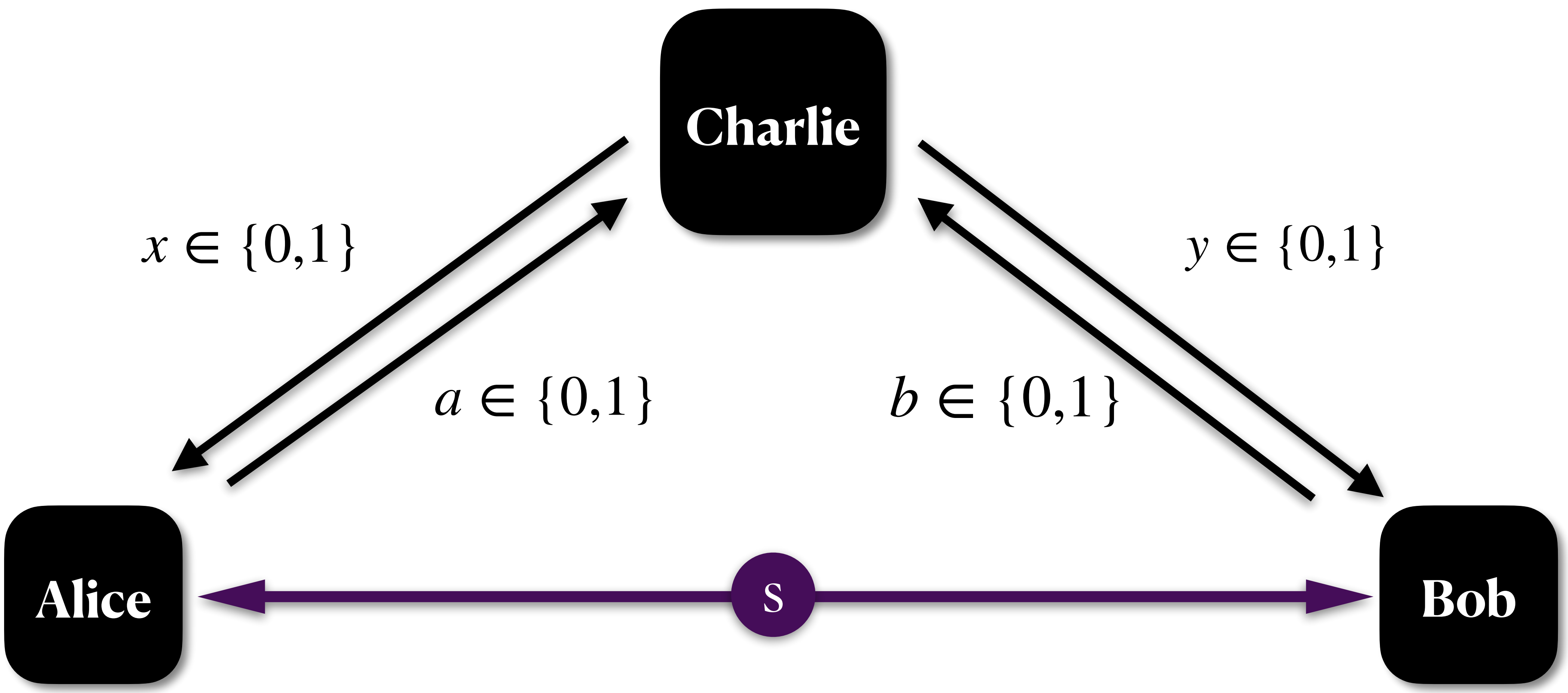

Bell Nonlocality

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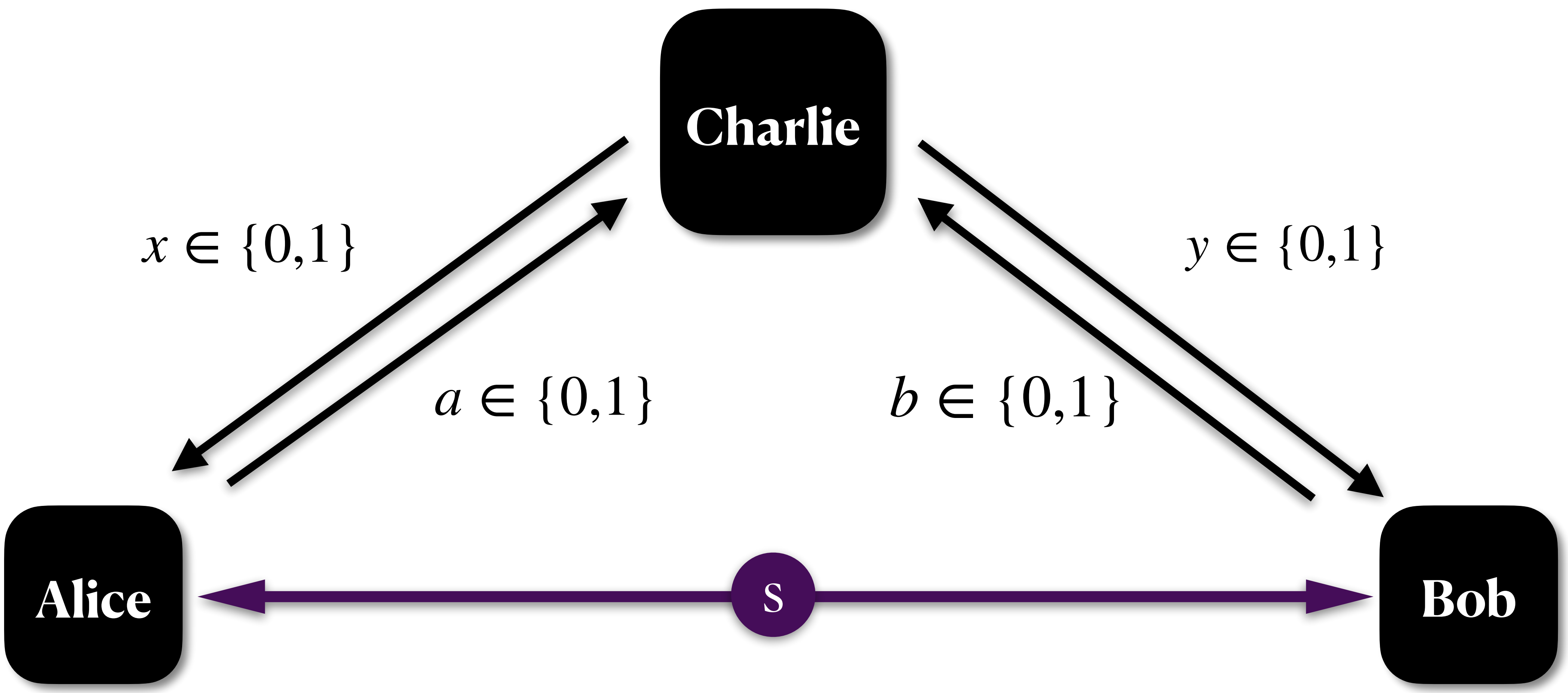

Goal: $a \oplus b = x \cdot y$

Bell Nonlocality

The nonlocal game: Clauser-Horne-Shimony-Holt (CHSH) scenario

Round	a	b	x	y
1	0	1	1	0
2	1	1	0	0
3	1	0	0	1
4	0	1	0	0
...	**...**	**...**	**...**	**...**

Bell Nonlocality

The nonlocal game: Clauser-Horne-Shimony-Holt (CHSH) scenario

Round	a	b	x	y
1	0	1	1	0
2	1	1	0	0
3	1	0	0	1
4	0	1	0	0
...	...	...	...	...

Probability vector : $\overrightarrow{P} = \{P(a, b \,|\, x, y)\}_{a,b,x,y} = \big(P(00\,|\,00) \quad \cdots \quad P(11\,|\,11)\big)$

Bell Nonlocality

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Bell Nonlocality

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Round	a	b	x	y
1	0	1	1	0
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...	...	...	...	...

Probability vector : $\overrightarrow{P} = \{P(a, b \mid x, y)\}_{a,b,x,y} = \big(P(00 \mid 00) \quad \cdots \quad P(11 \mid 11)\big)$

Bell Function : $\overrightarrow{\beta} = (1 \quad -1 \quad -1 \quad \cdots \quad -1 \quad 1)$

Bell Value: $I = \overrightarrow{\beta} \cdot \overrightarrow{P}$

Bell Nonlocality

Various sets of correlations

Bell Nonlocality

Various sets of correlations

- Local hidden-variable models: Local Set: L

$$P_L(a,b\,|\,x,y) = \sum_{\lambda} P_\lambda \; P(a\,|\,x,\lambda)P(b\,|\,y,\lambda)$$

Bell Nonlocality

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- Quantum mechanics (Born's rule): Quantum Set: Q

$$P_Q(a,b|x,y) = \mathrm{tr}[(M_{a|x} \otimes M_{b|y})\; \rho]$$

Bell Nonlocality

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$$P_Q(a,b\,|\,x,y) = \mathrm{tr}[(M_{a|x} \otimes M_{b|y}) \; \rho] \;\; \rightarrow I = \overrightarrow{\beta} \cdot \overrightarrow{P}_Q \leq 2\sqrt{2}$$

Bell Nonlocality

Various sets of correlations

- Local hidden-variable models: Local Set: L

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- The non-signaling conditions: Non-signaling Set: NS

$$\sum_b P(a,b|x,y) = P(a|x,y) = P(a|x)$$

$$\sum_a P(a,b|x,y) = P(b|x,y) = P(b|y)$$

Bell Nonlocality

Various sets of correlations

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- The non-signaling conditions: Non-signaling Set: NS

$$\sum_b P(a,b|x,y) = P(a|x,y) = P(a|x)$$

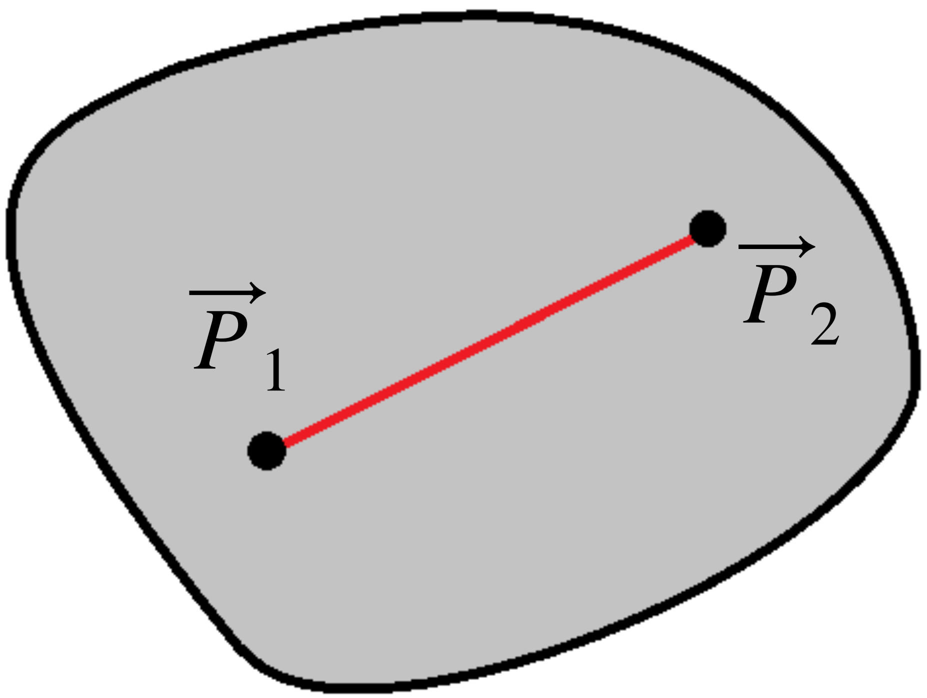
$$\rightarrow I = \vec{\beta} \cdot \vec{P}_{NS} \quad \leq 4$$

$$\sum_a P(a,b|x,y) = P(b|x,y) = P(b|y)$$

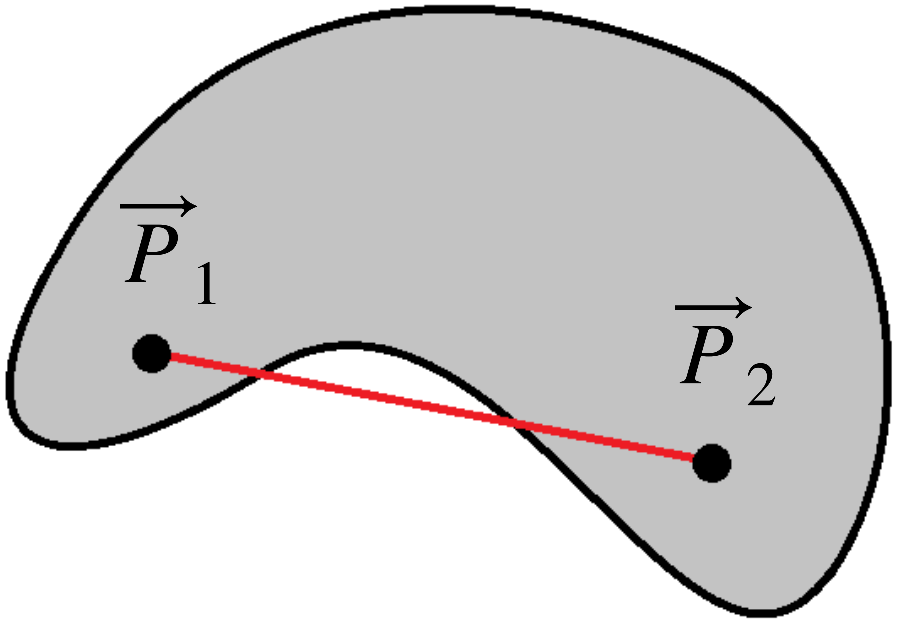
The Boundary of Non-signaling Polytope

The Boundary of Non-signaling Polytope

Convex set

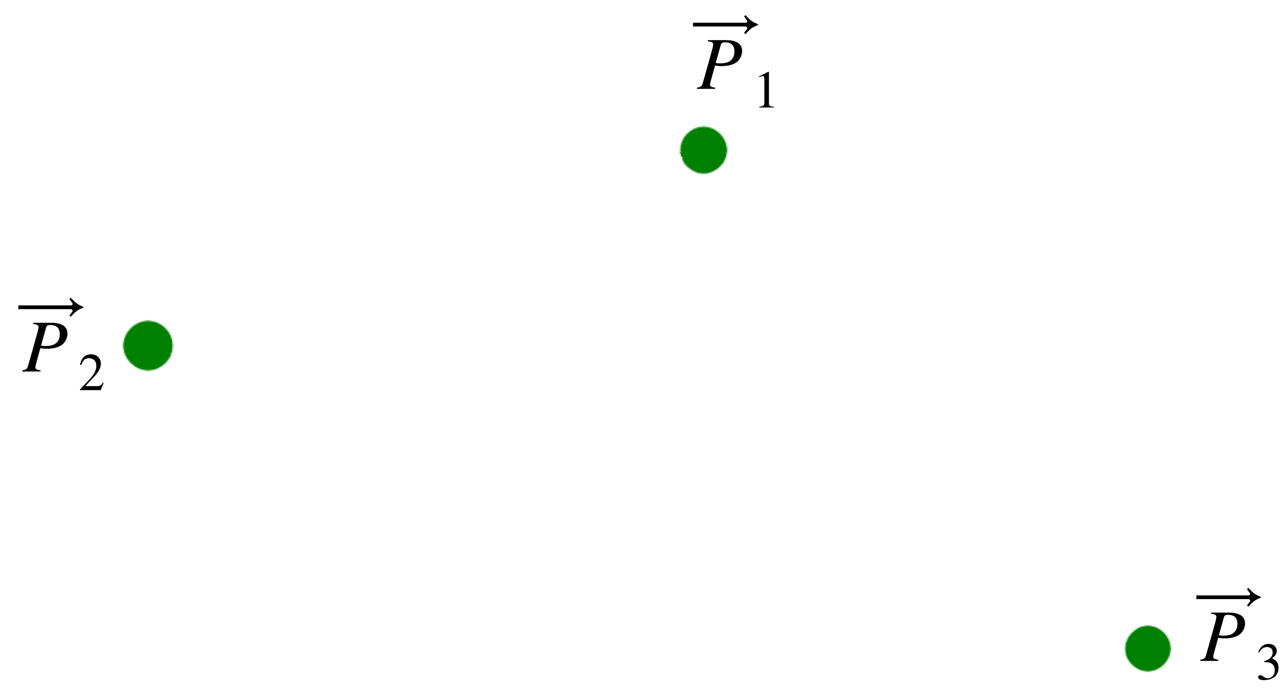


Non-convex set



The Boundary of Non-signaling Polytope

Polytope



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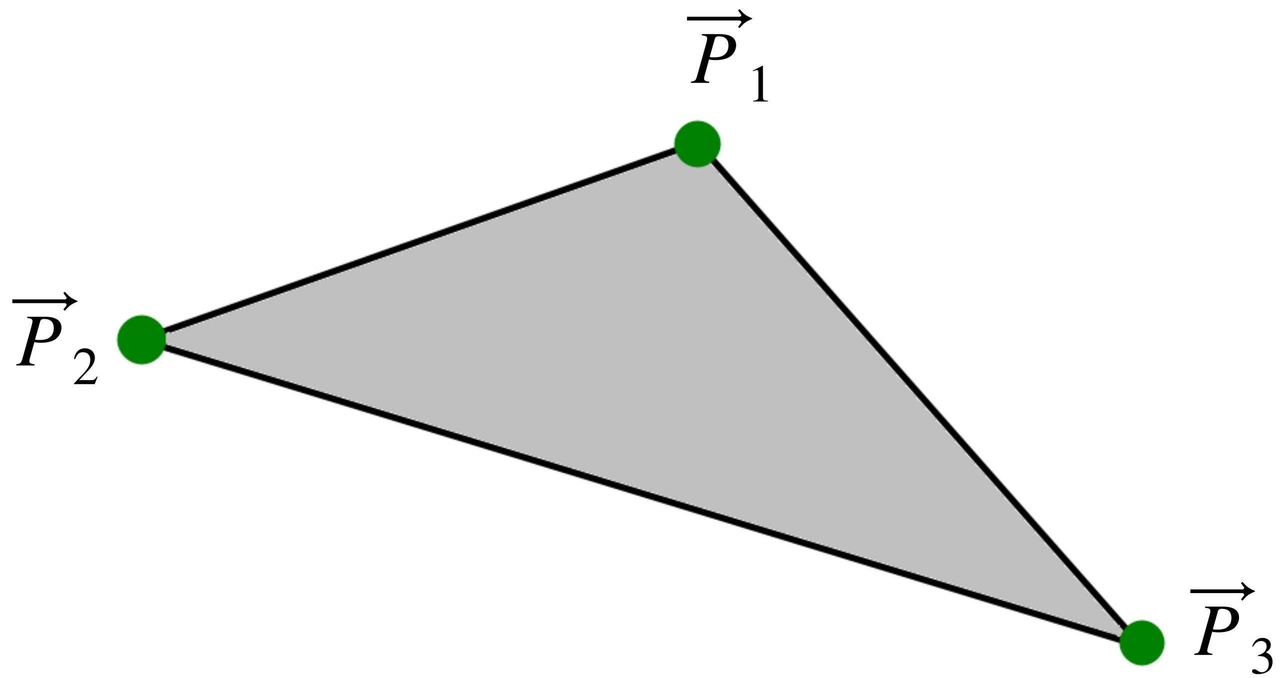
Polytope



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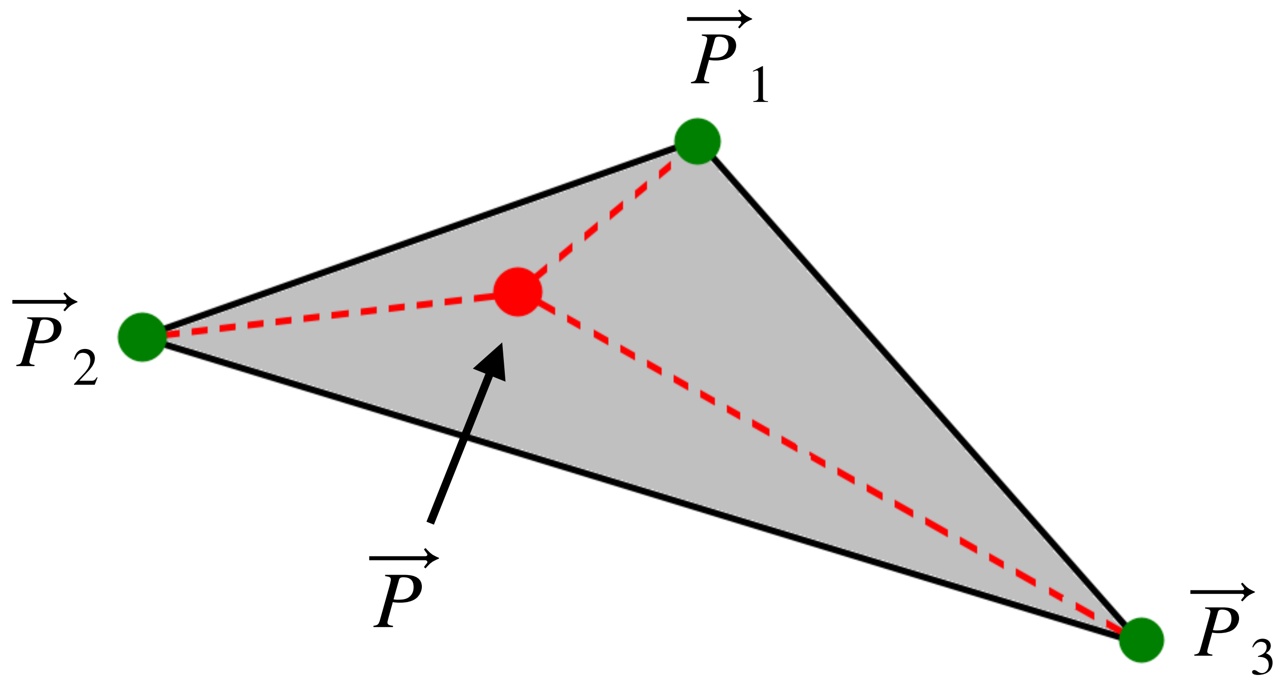
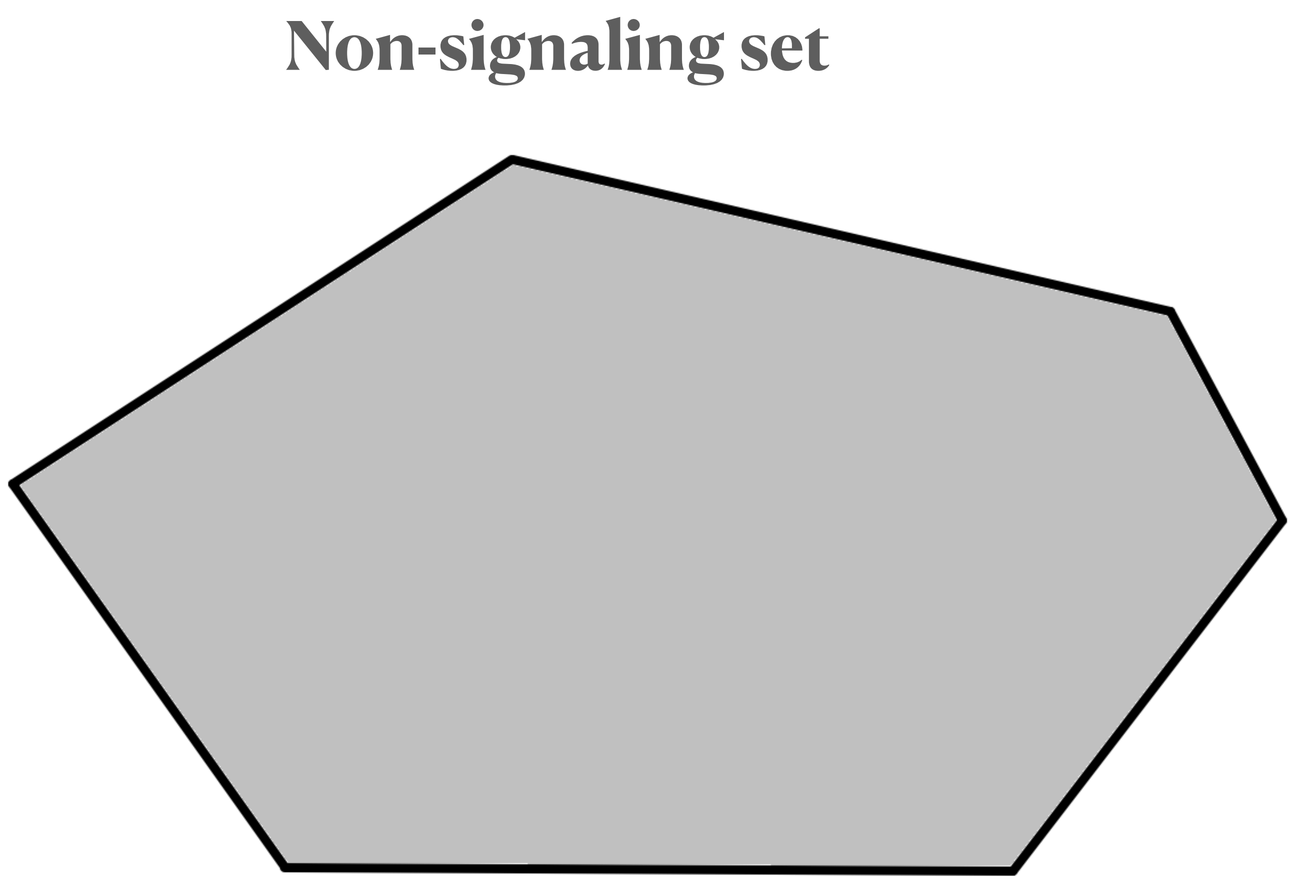
Polytope



- $\overrightarrow{P} = \sum_{i=1}^{3} c_i \overrightarrow{P}_i \ , \ \sum_{i=1}^{3} c_i = 1 \ , \ c_i \geq 0 \ \forall i$

The Boundary of Non-signaling Polytope

Sets of correlations: $L \subsetneq Q \subsetneq NS$



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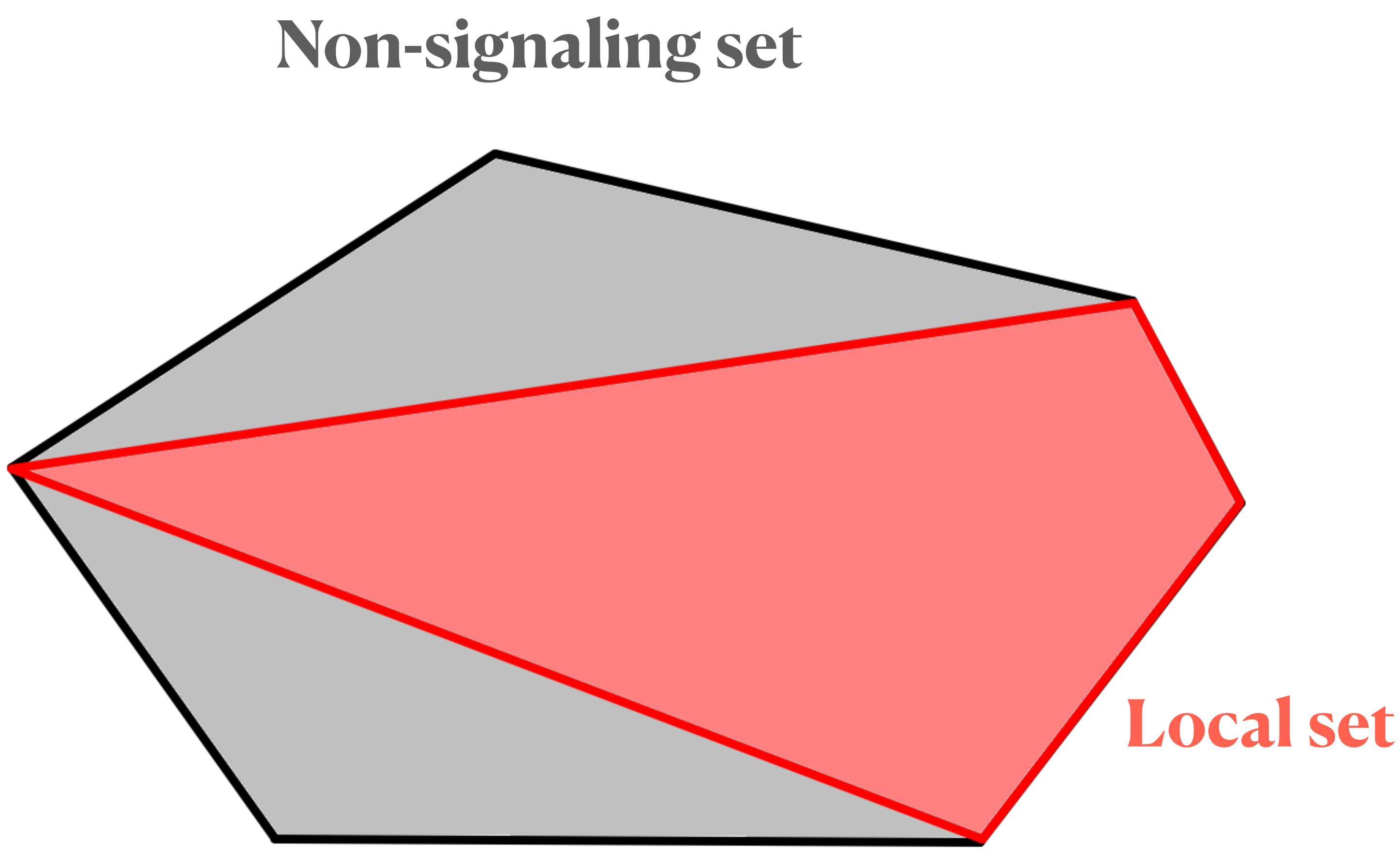

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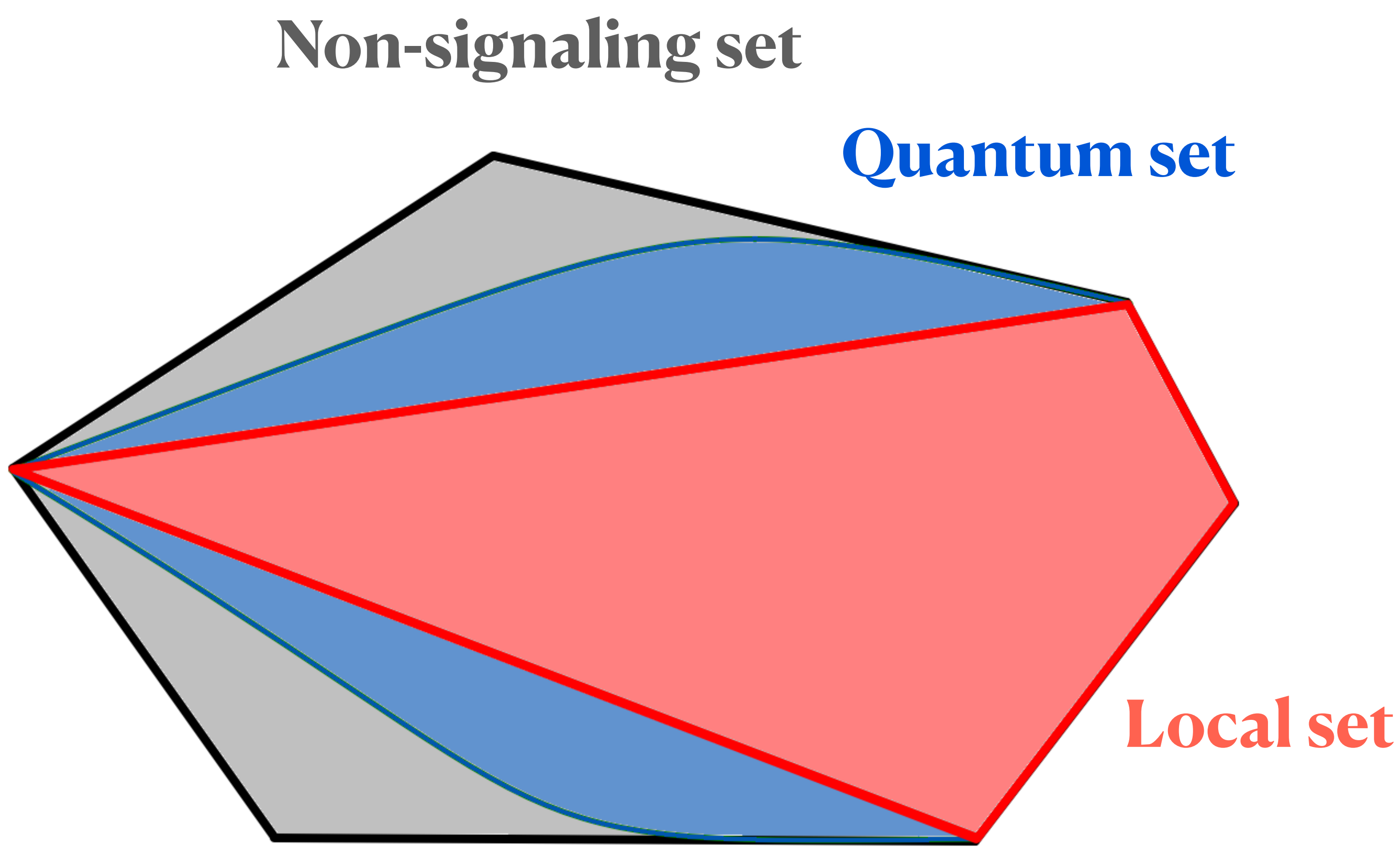

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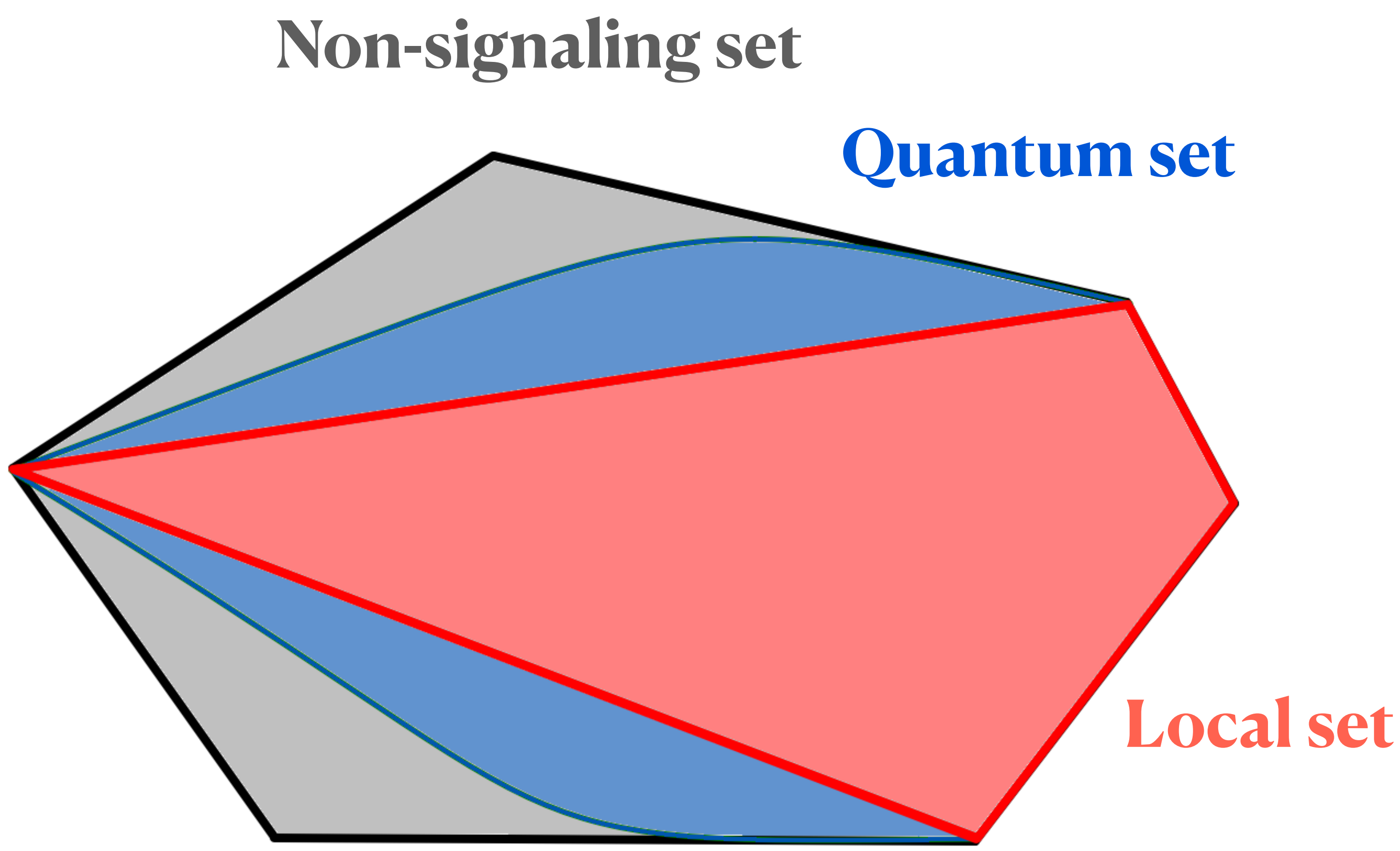

The inequality constraints of NS polytope:

$$P(a, b \mid x, y) \geq 0 \ \forall a, b, x, y$$

The Boundary of Non-signaling Polytope

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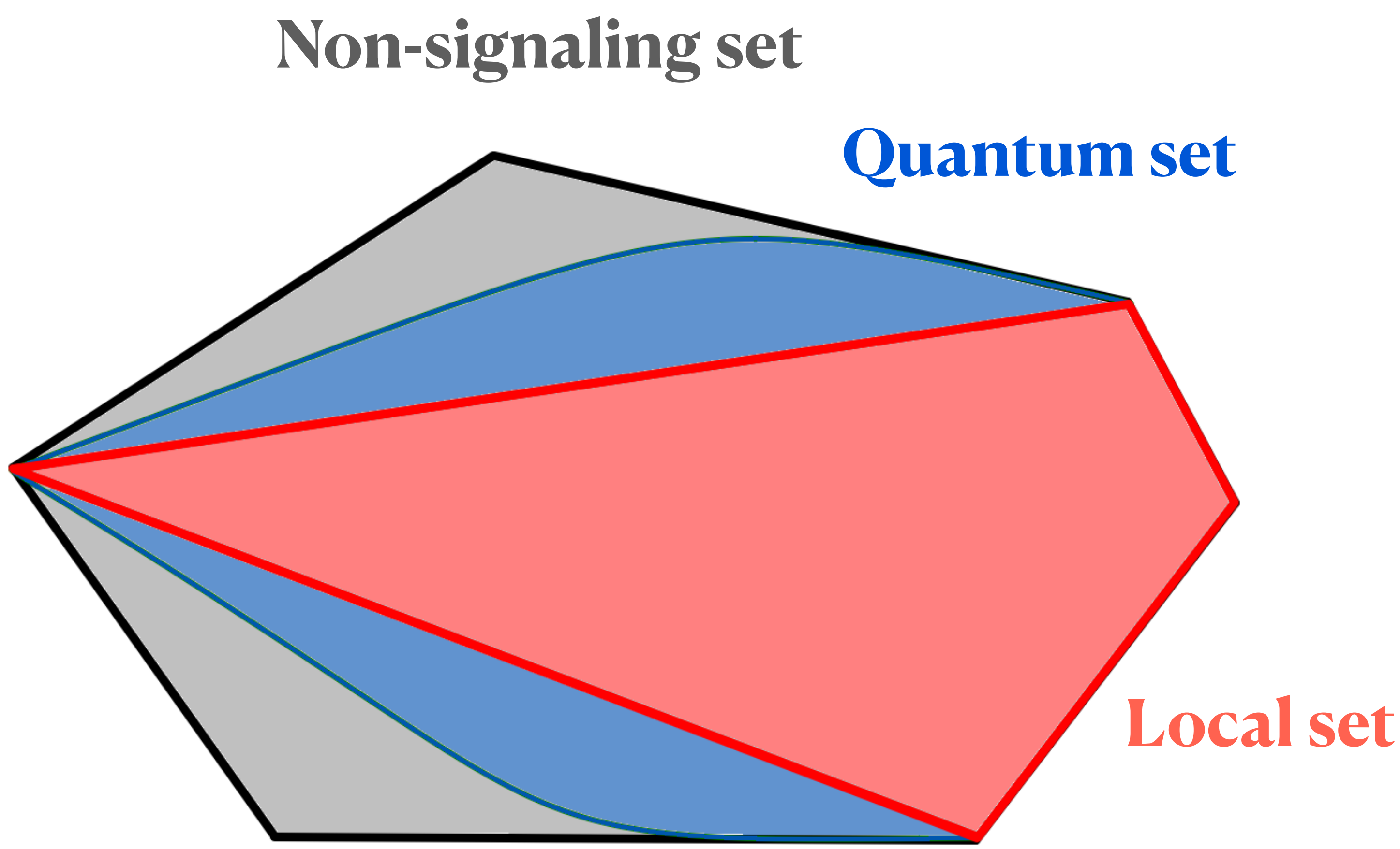


The inequality constraints of NS polytope:

$$P(a, b \,|\, x, y) \geq 0 \ \forall a, b, x, y$$

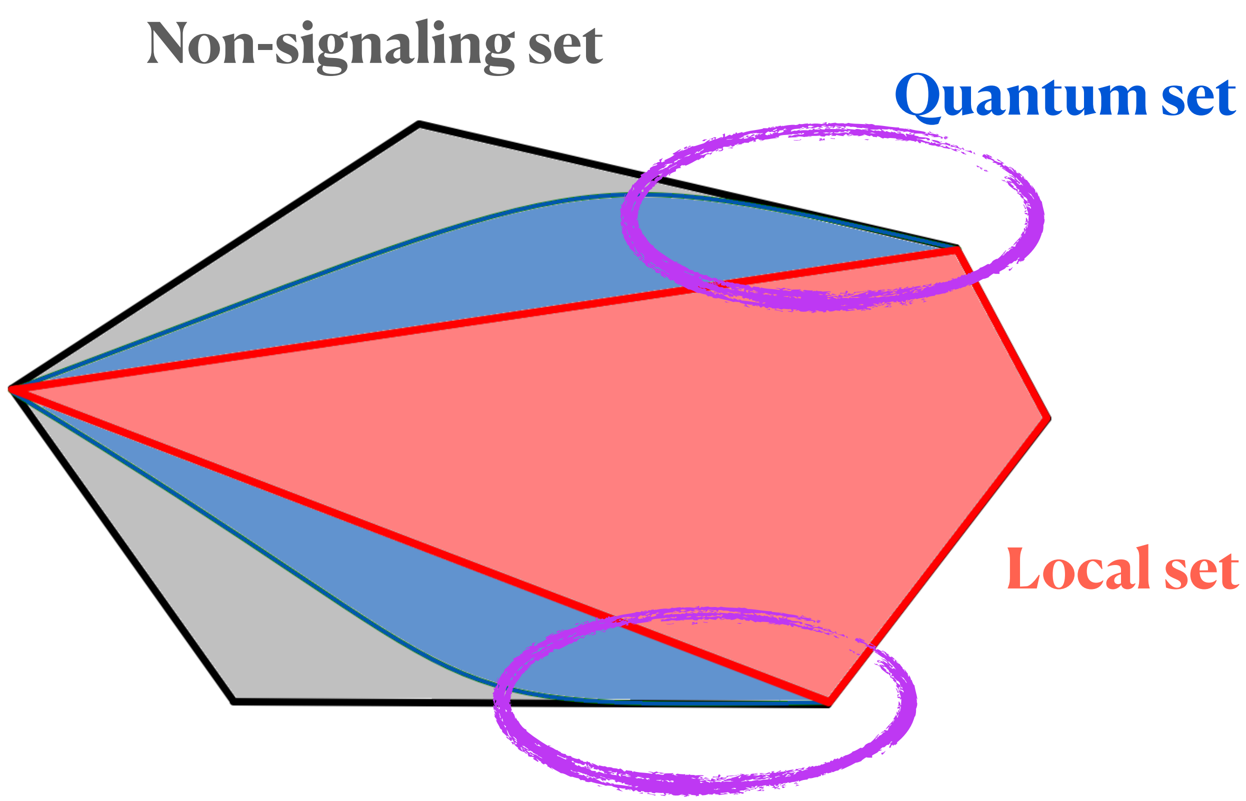
On the boundary of NS polytope:

$$P(a', b' \,|\, x', y') = 0$$

for some a', b', x', y'

The Boundary of Non-signaling Polytope

When quantum boundary meets the non-signaling boundary



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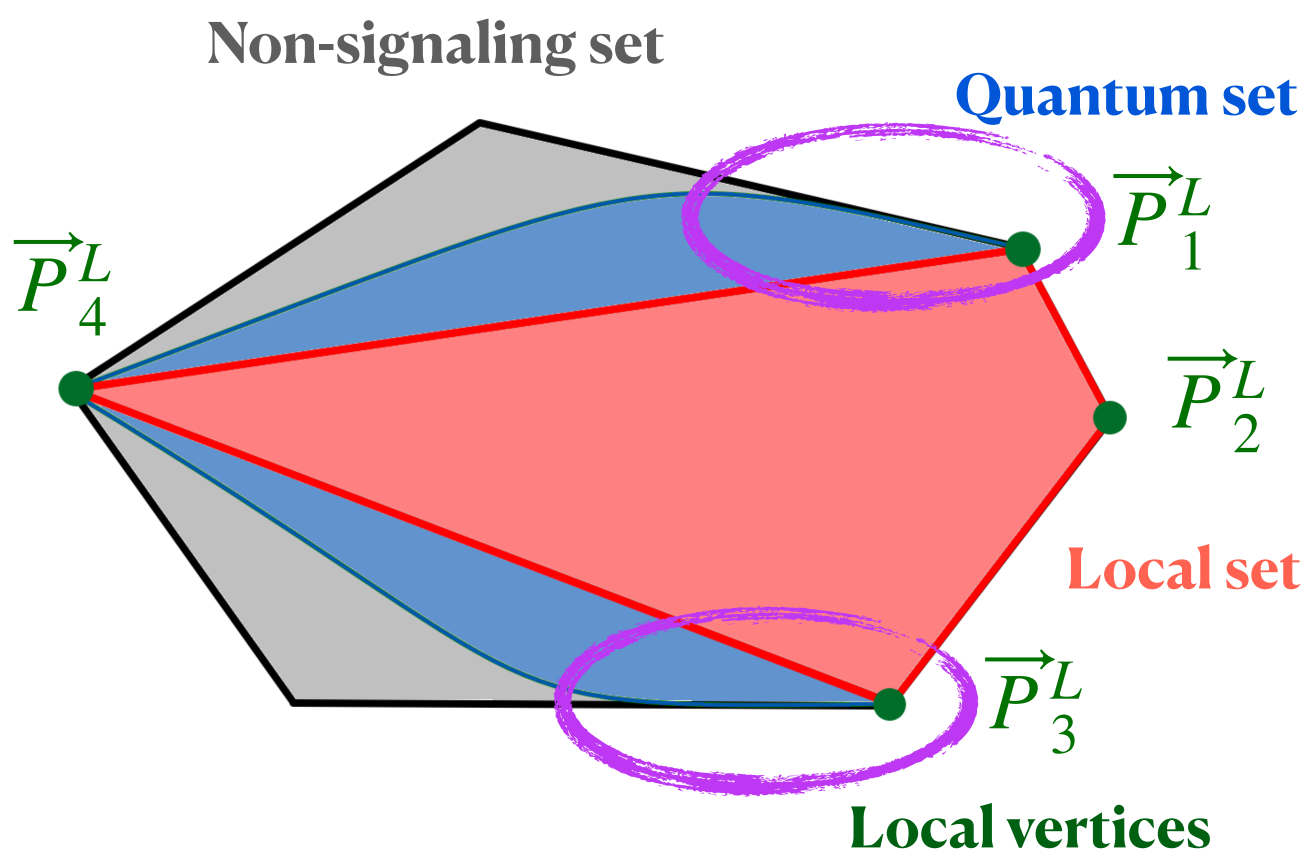

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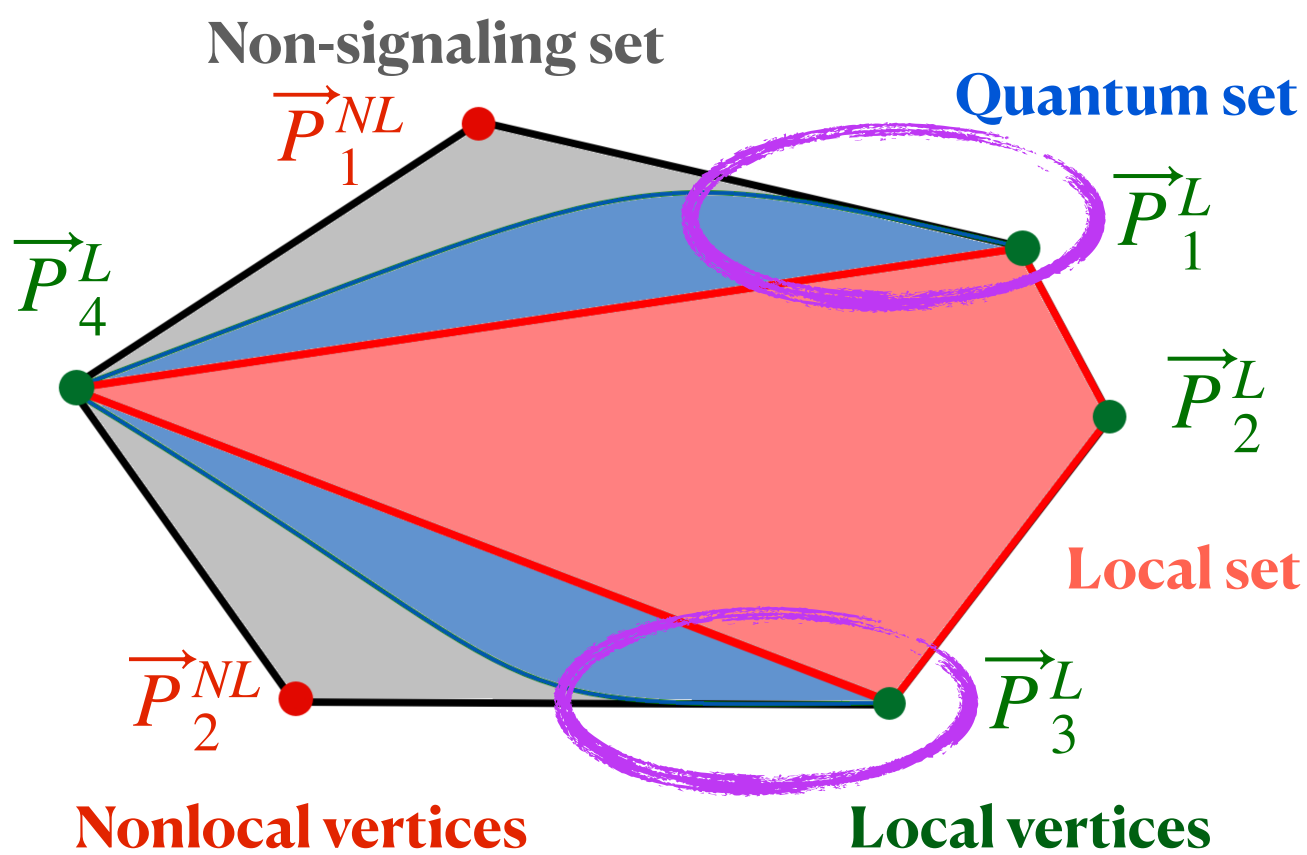

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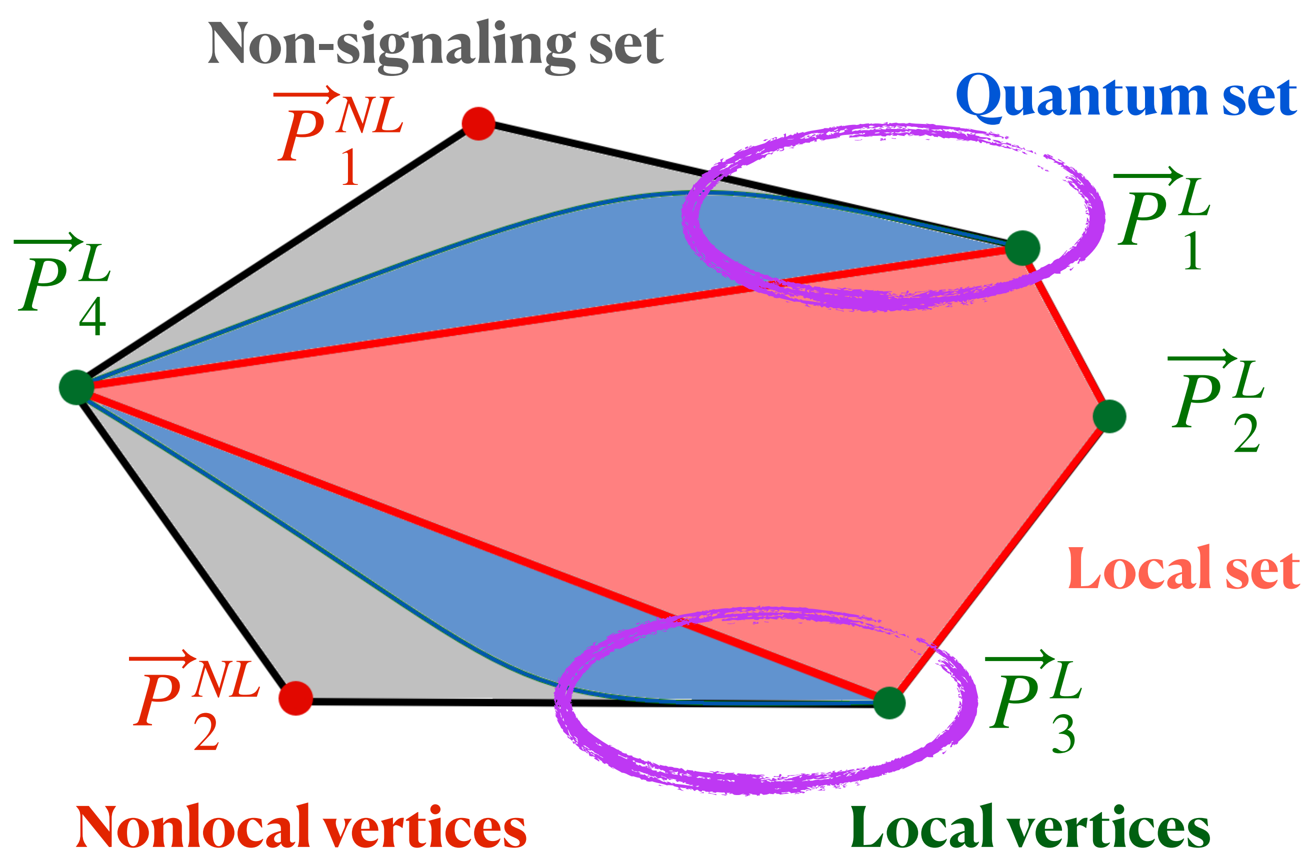


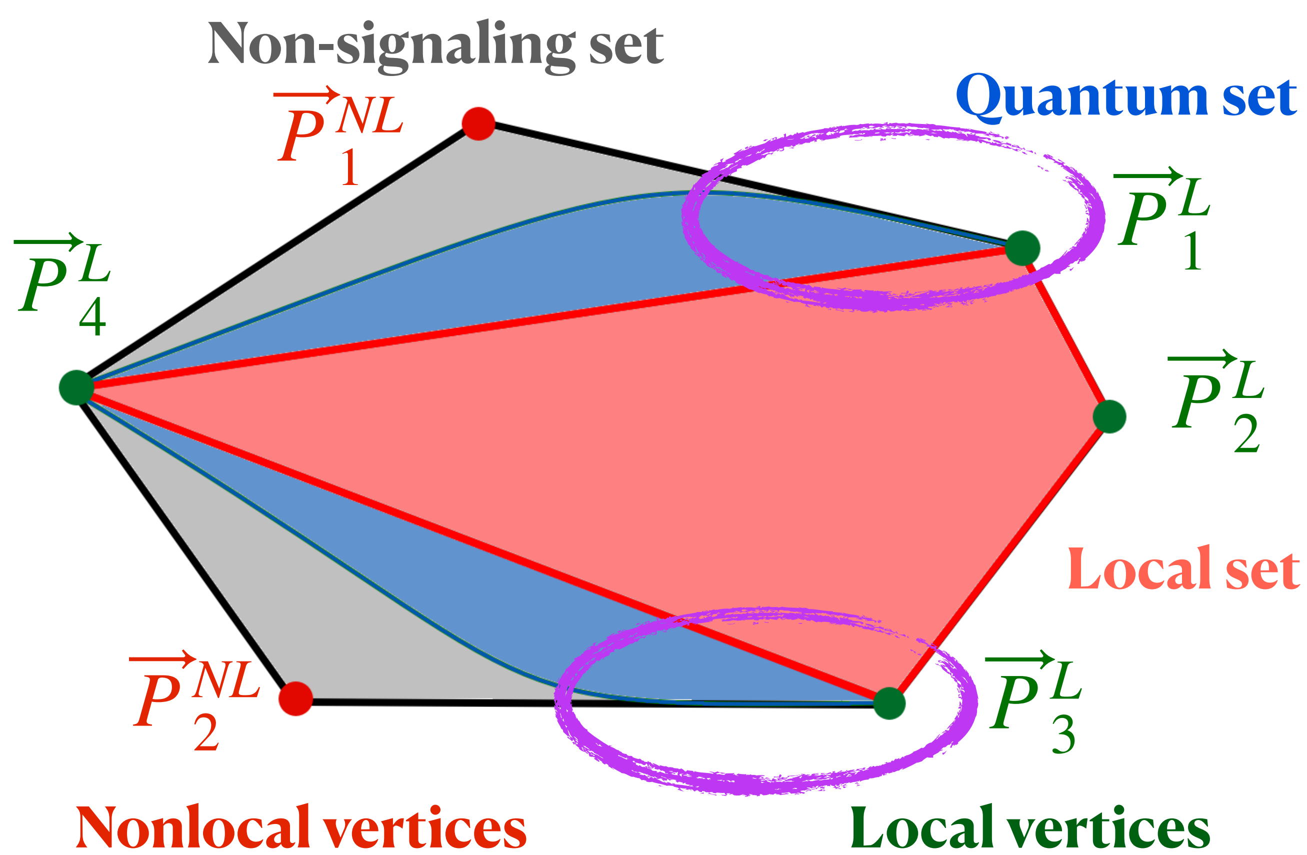
- Probability inside the non-signaling set:

$$\vec{P} = \sum_{i=1}^{8} c_i \vec{P}_i^{NL} + \sum_{j=1}^{16} d_j \vec{P}_j^{L}$$

$$\sum_{i=1}^{8} c_i + \sum_{j=1}^{16} d_j = 1, \;\; c_i \geq 0 \; \forall i, \;\; d_j \geq 0 \; \forall j$$

The Boundary of Non-signaling Polytope

When quantum boundary meets the non-signaling boundary



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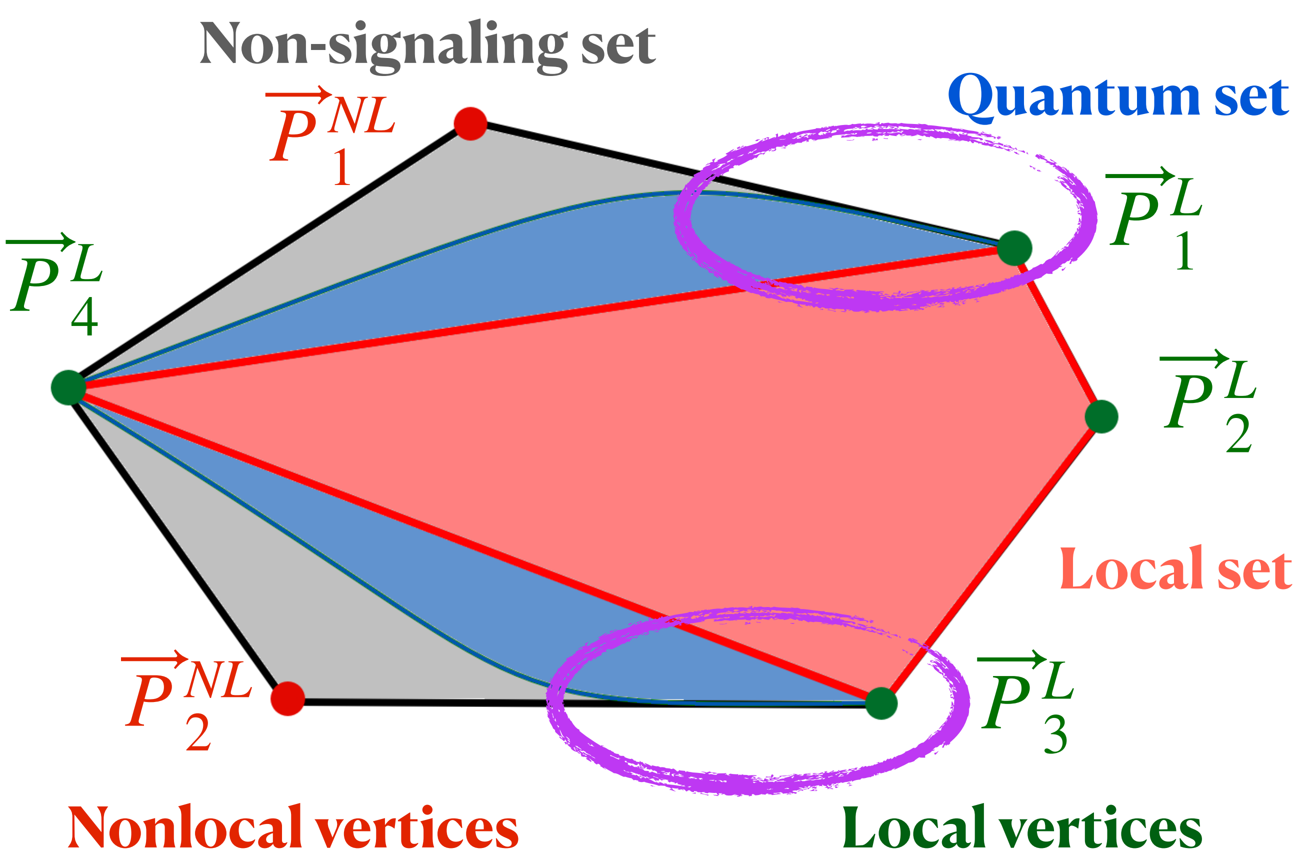
$\vec{P}_1^{NL} \rightarrow$

		$x=0$		$x=1$	
		0	1	0	1
$y=0$	0	0	$\frac{1}{2}$	0	$\frac{1}{2}$
	1	$\frac{1}{2}$	0	$\frac{1}{2}$	0
$y=1$	0	0	$\frac{1}{2}$	$\frac{1}{2}$	0
	1	$\frac{1}{2}$	0	0	$\frac{1}{2}$

The Boundary of Non-signaling Polytope

When quantum boundary meets the non-signaling boundary



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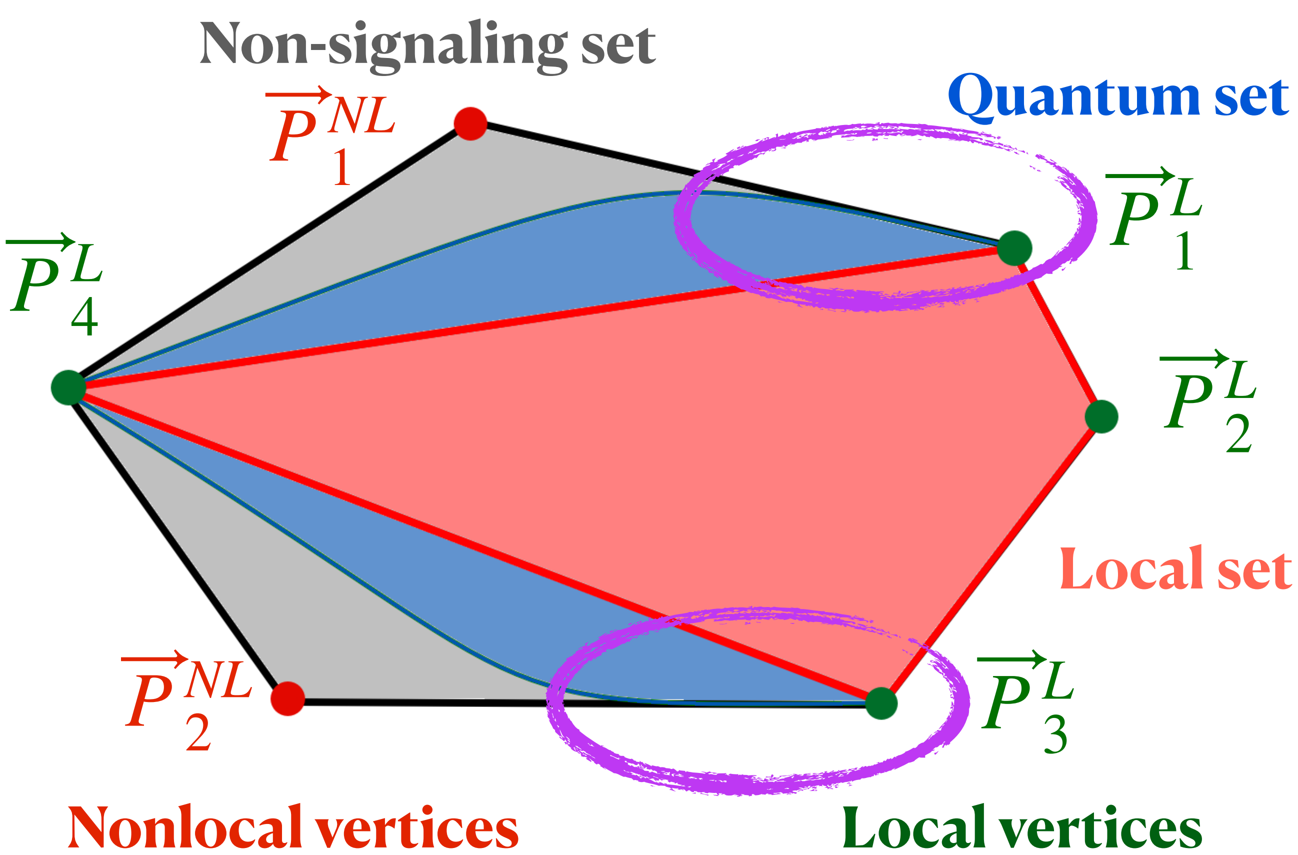
$y=1$	0	0	$\frac{1}{2}$	$\frac{1}{2}$	0
	1	$\frac{1}{2}$	0	0	$\frac{1}{2}$

$P(0,1 \,|\, 1,1) = 0$

The Boundary of Non-signaling Polytope

When quantum boundary meets the non-signaling boundary



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$\vec{P}^{NL}_1 \rightarrow$

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		0	1	0	1
$y=0$	0	0	~~$\frac{1}{2}$~~	0	~~$\frac{1}{2}$~~
	1	~~$\frac{1}{2}$~~	0	~~$\frac{1}{2}$~~	0
$y=1$	0	0	~~$\frac{1}{2}$~~	~~$\frac{1}{2}$~~	0
	1	~~$\frac{1}{2}$~~	0	0	~~$\frac{1}{2}$~~

$P(0,1\,|\,1,1) = 0$

Our Results

Quantum Set

- The maximal number of zeros is three.
- For three zeros cases, there are two feasible classes:

1.

		$x=0$		$x=1$	
		0	1	0	1
$y=0$	0	0			
	1				0
$y=1$	0				
	1		0		

Quantum Set

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- For three zeros cases, there are two feasible classes:

1.

		$x=0$		$x=1$	
		0	1	0	1
$y=0$	0	0			
	1				0
$y=1$	0				
	1		0		

2.

		$x=0$		$x=1$	
		0	1	0	1
$y=0$	0	0			
	1		0		
$y=1$	0				0
	1				

Quantum Set

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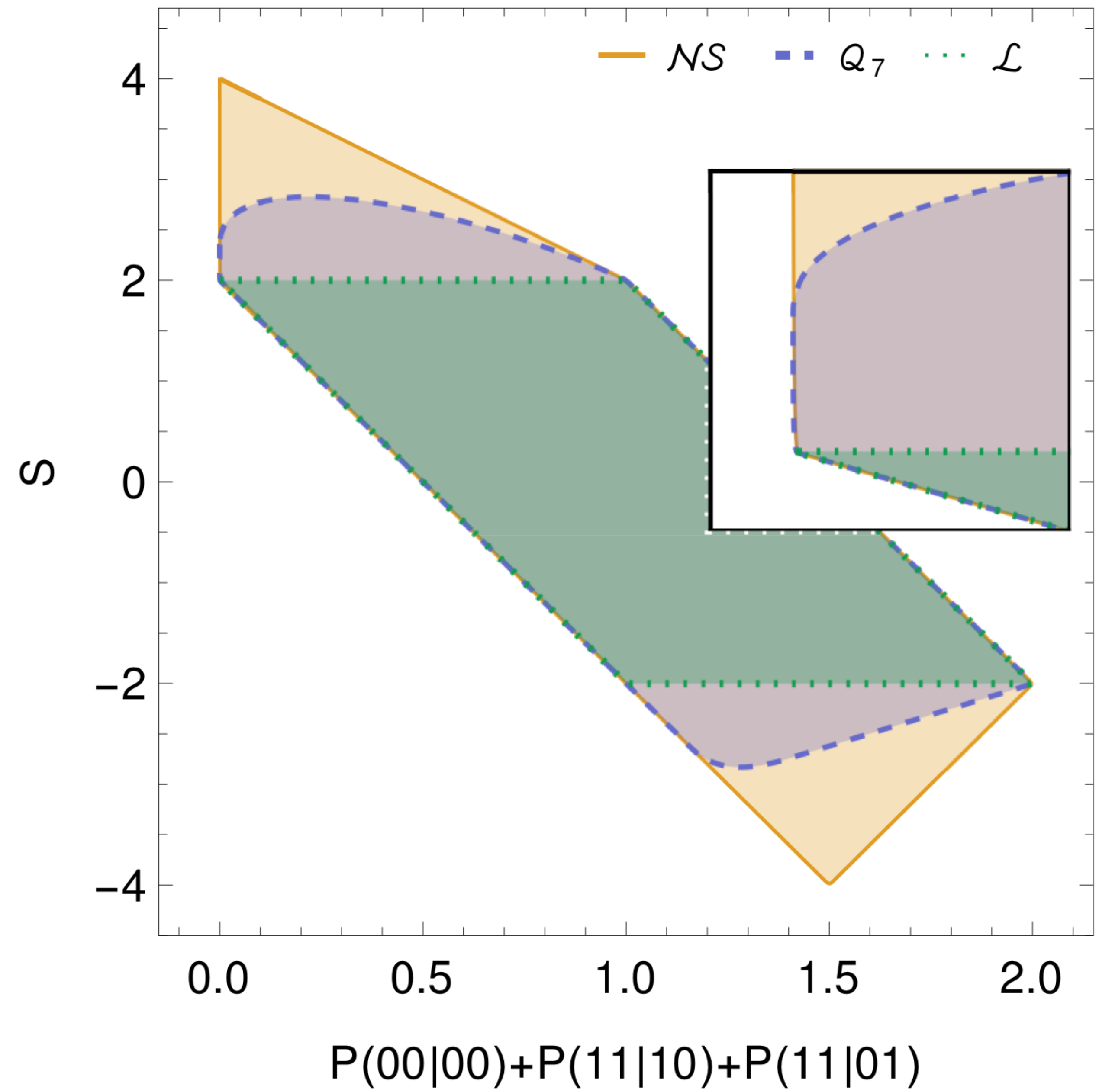
- For three zeros cases, there are two feasible classes:

1.

		$x=0$, 0	$x=0$, 1	$x=1$, 0	$x=1$, 1
$y=0$	0	0			
	1				0
$y=1$	0				
	1		0		

2.

		$x=0$, 0	$x=0$, 1	$x=1$, 0	$x=1$, 1
$y=0$	0	0			
	1		0		
$y=1$	0				0
	1				



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		$x=0$		$x=1$	
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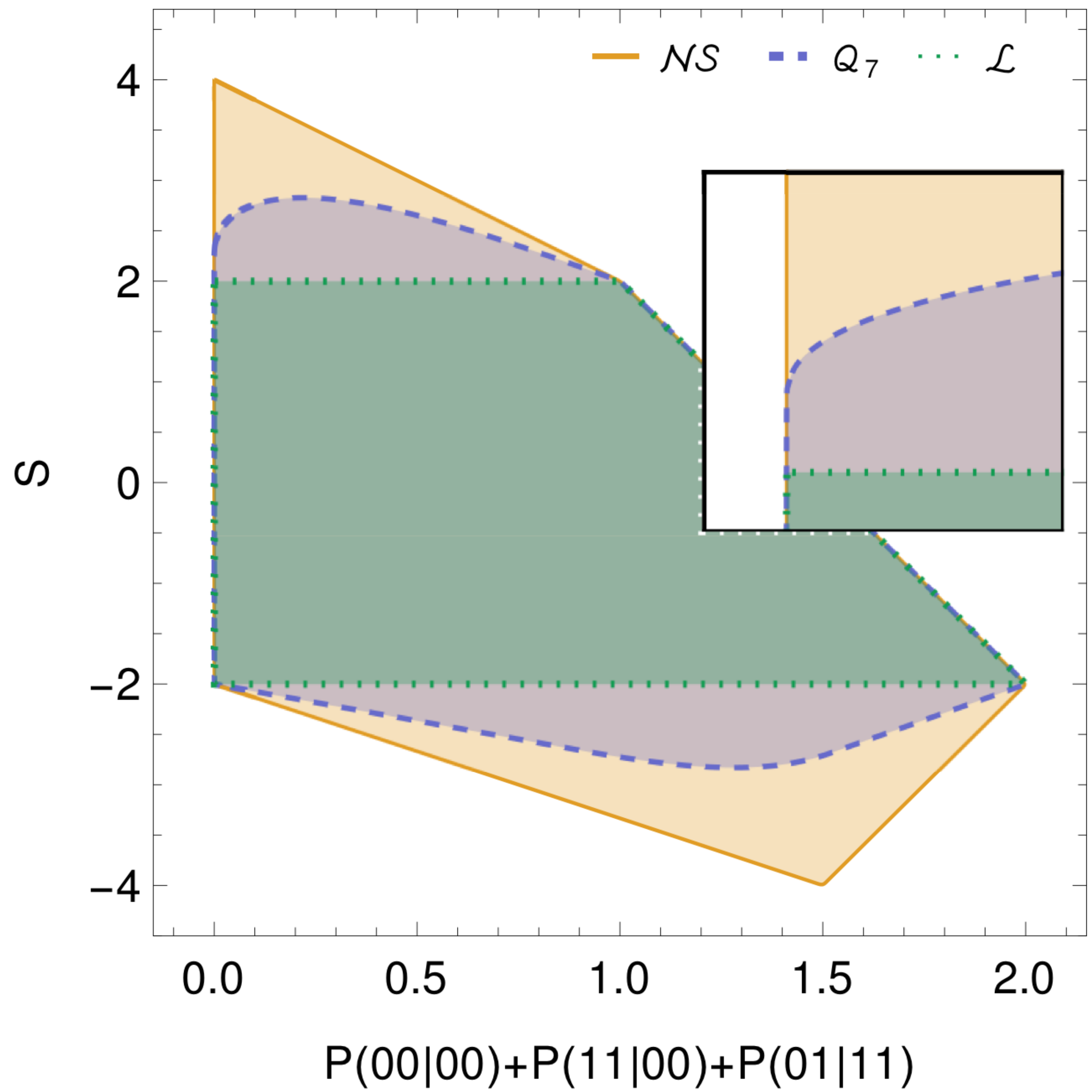
$y=0$	0	0			
	1				0
$y=1$	0				
	1		0		

2.

		$x=0$		$x=1$	
		0	1	0	1
$y=0$	0	0			
	1		0		
$y=1$	0				0
	1				



Quantum Set

<table>
<tr><th># of zeros</th><th>Feasible</th><th>Infeasible</th></tr>
<tr><td>3</td><td>
<table>
<tr><th></th><th></th><th colspan="2">$x = 0$</th><th colspan="2">$x = 1$</th></tr>
<tr><th></th><th></th><th>0</th><th>1</th><th>0</th><th>1</th></tr>
<tr><td rowspan="2">$y = 0$</td><td>0</td><td>0</td><td></td><td></td><td></td></tr>
<tr><td>1</td><td></td><td></td><td></td><td>0</td></tr>
<tr><td rowspan="2">$y = 1$</td><td>0</td><td></td><td></td><td></td><td></td></tr>
<tr><td>1</td><td></td><td>0</td><td></td><td></td></tr>
</table>
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<tr><th></th><th></th><th colspan="2">$x = 0$</th><th colspan="2">$x = 1$</th></tr>
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<tr><td rowspan="2">$y = 0$</td><td>0</td><td>0</td><td></td><td></td><td></td></tr>
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<tr><td rowspan="2">$y = 1$</td><td>0</td><td></td><td></td><td></td><td>0</td></tr>
<tr><td>1</td><td></td><td></td><td></td><td></td></tr>
</table>
</td><td>others</td></tr>
<tr><td>2</td><td>others</td><td>
<table>
<tr><th></th><th></th><th colspan="2">$x = 0$</th><th colspan="2">$x = 1$</th></tr>
<tr><th></th><th></th><th>0</th><th>1</th><th>0</th><th>1</th></tr>
<tr><td rowspan="2">$y = 0$</td><td>0</td><td>0</td><td></td><td></td><td></td></tr>
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<tr><td rowspan="2">$y = 1$</td><td>0</td><td></td><td></td><td></td><td>0</td></tr>
<tr><td>1</td><td></td><td></td><td></td><td></td></tr>
</table>
</td></tr>
<tr><td>1</td><td>all</td><td>none</td></tr>
</table>

Maximally Entangled States Set

Why do we care about finite-dimensional maximally entangled states set?

- **Hardy's nonlocality argument:**

$$P(0,0\,|\,0,0) = 0\,,\ P(1,1\,|\,0,1) = 0$$

$$P(1,1\,|\,1,0) = 0\,,\ P(1,1\,|\,1,1) = q$$

  - Local model: $q = 0$

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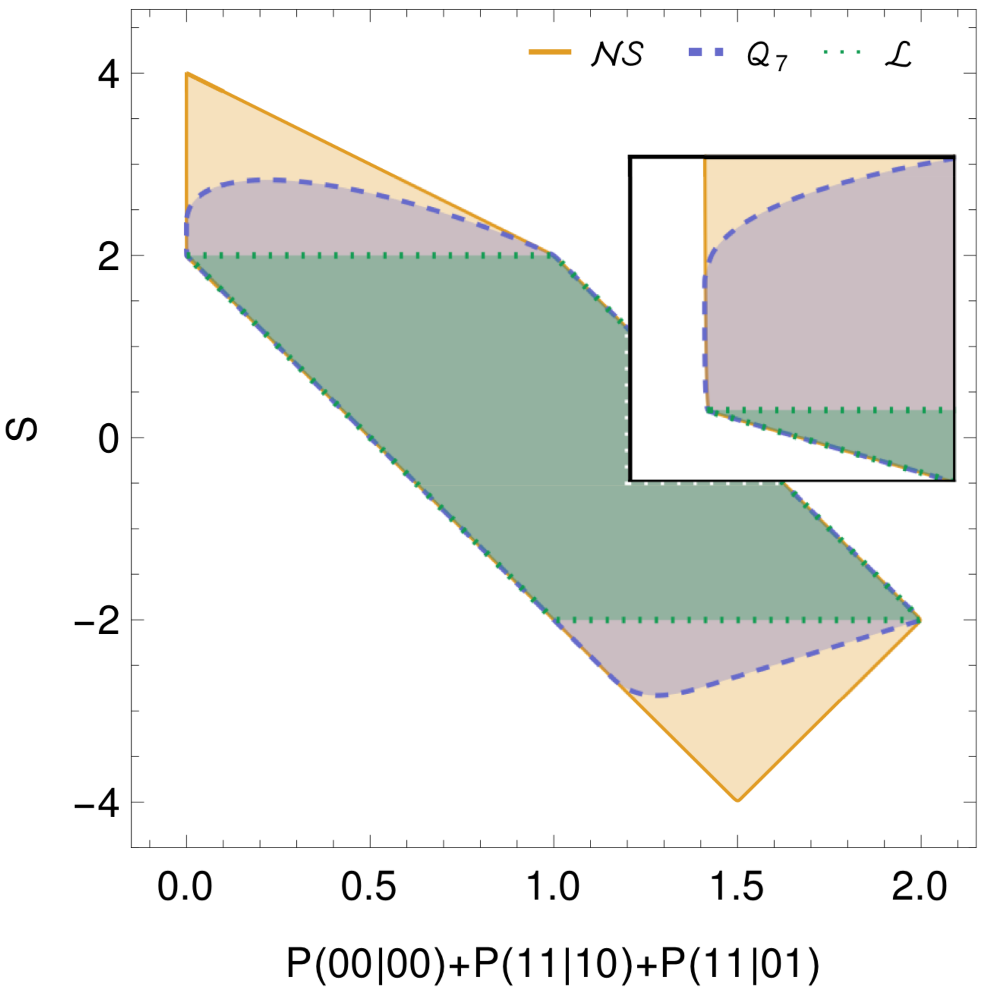
  - Local model: $q = 0$
  - Quantum model: $q \geq 0$



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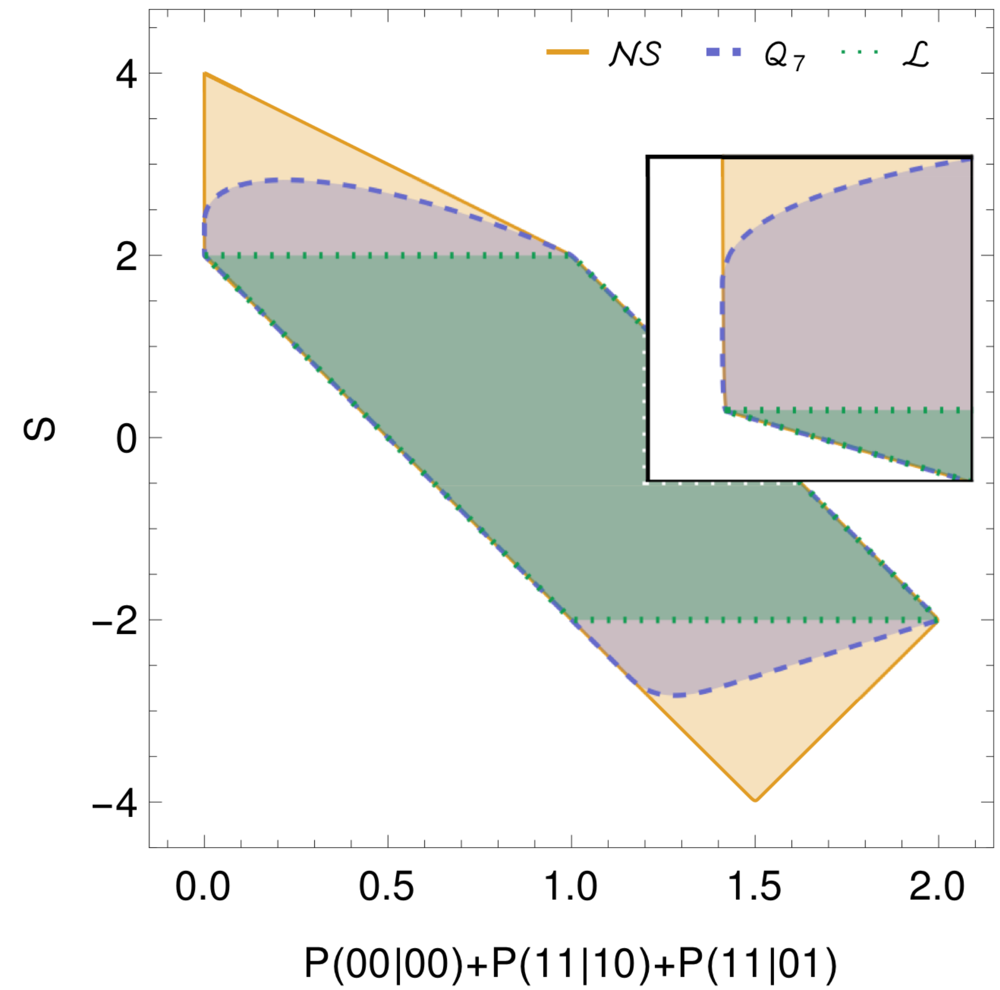
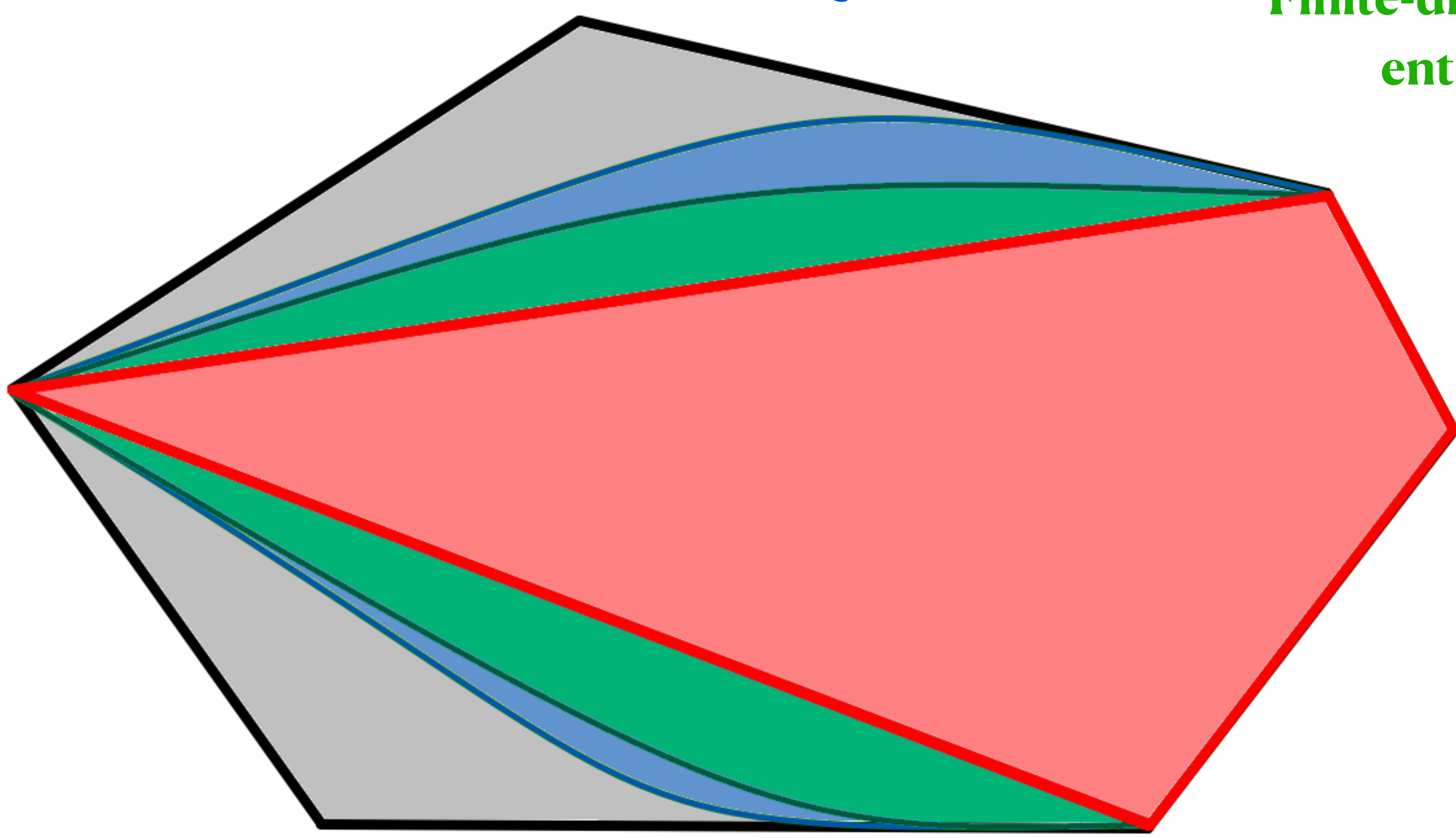
  - Local model: $q = 0$
  - Quantum model: $q \geq 0$
  - Finite-dimensional maximally entangled states: $q = 0$



$$P_{MES}(a,b\,|\,x,y) = \mathrm{tr}[(M_{a|x}\otimes M_{b|y})\,|\Psi_d\rangle\langle\Psi_d\,|\,],\quad |\Psi_d\rangle = \frac{1}{\sqrt{d}}\sum_{i=1}^{d}|ii\rangle$$

Maximally Entangled States Set

Why do we care about finite-dimensional maximally entangled states set?



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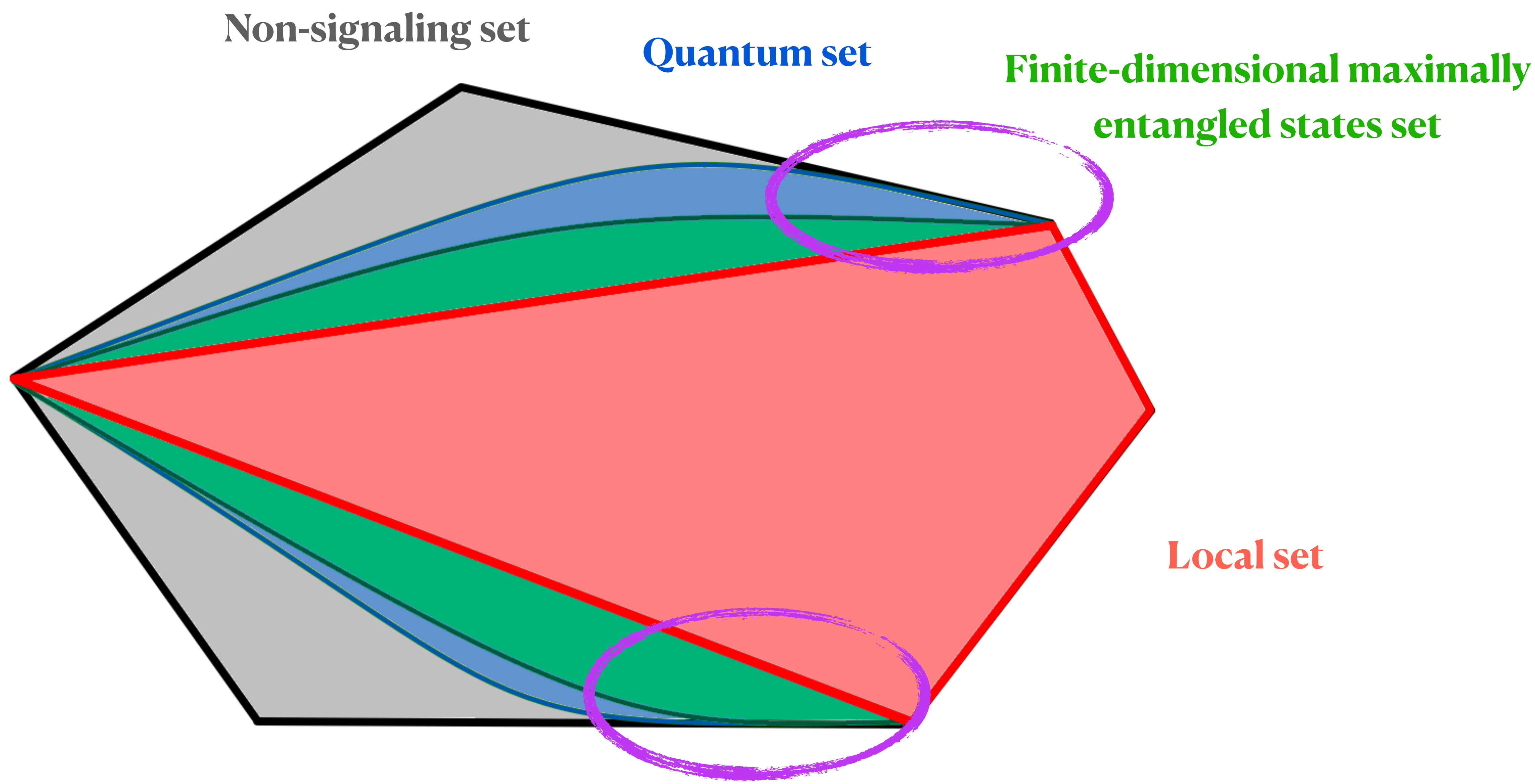


Maximally Entangled States Set

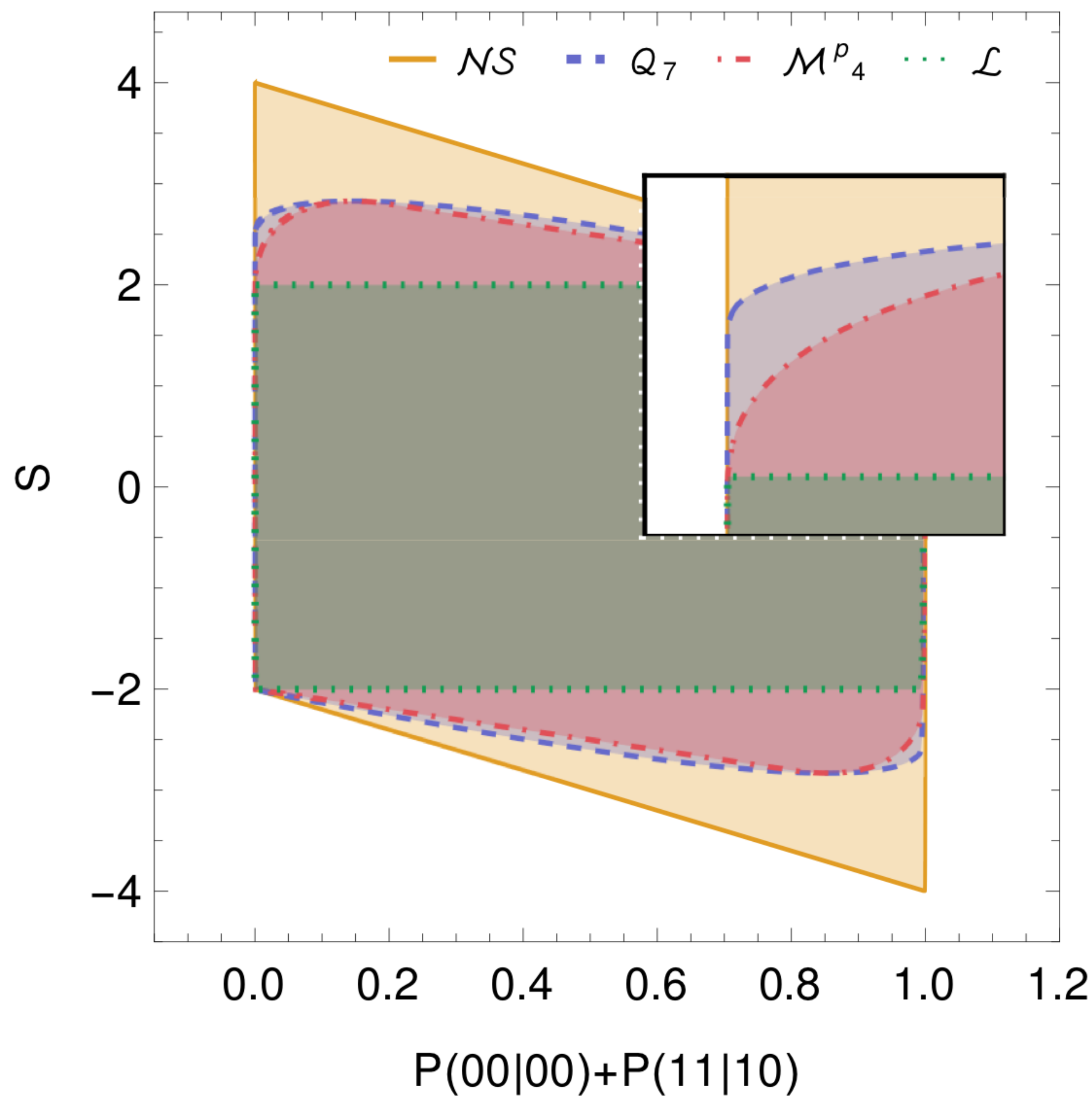
- Three zeros classes are not feasible for maximally entangled states set.

1.

		$x=0$, 0	$x=0$, 1	$x=1$, 0	$x=1$, 1
$y=0$	0	0			
	1				0
$y=1$	0				
	1				

2.

		$x=0$, 0	$x=0$, 1	$x=1$, 0	$x=1$, 1
$y=0$	0	0			
	1				
$y=1$	0				0
	1				



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	1				
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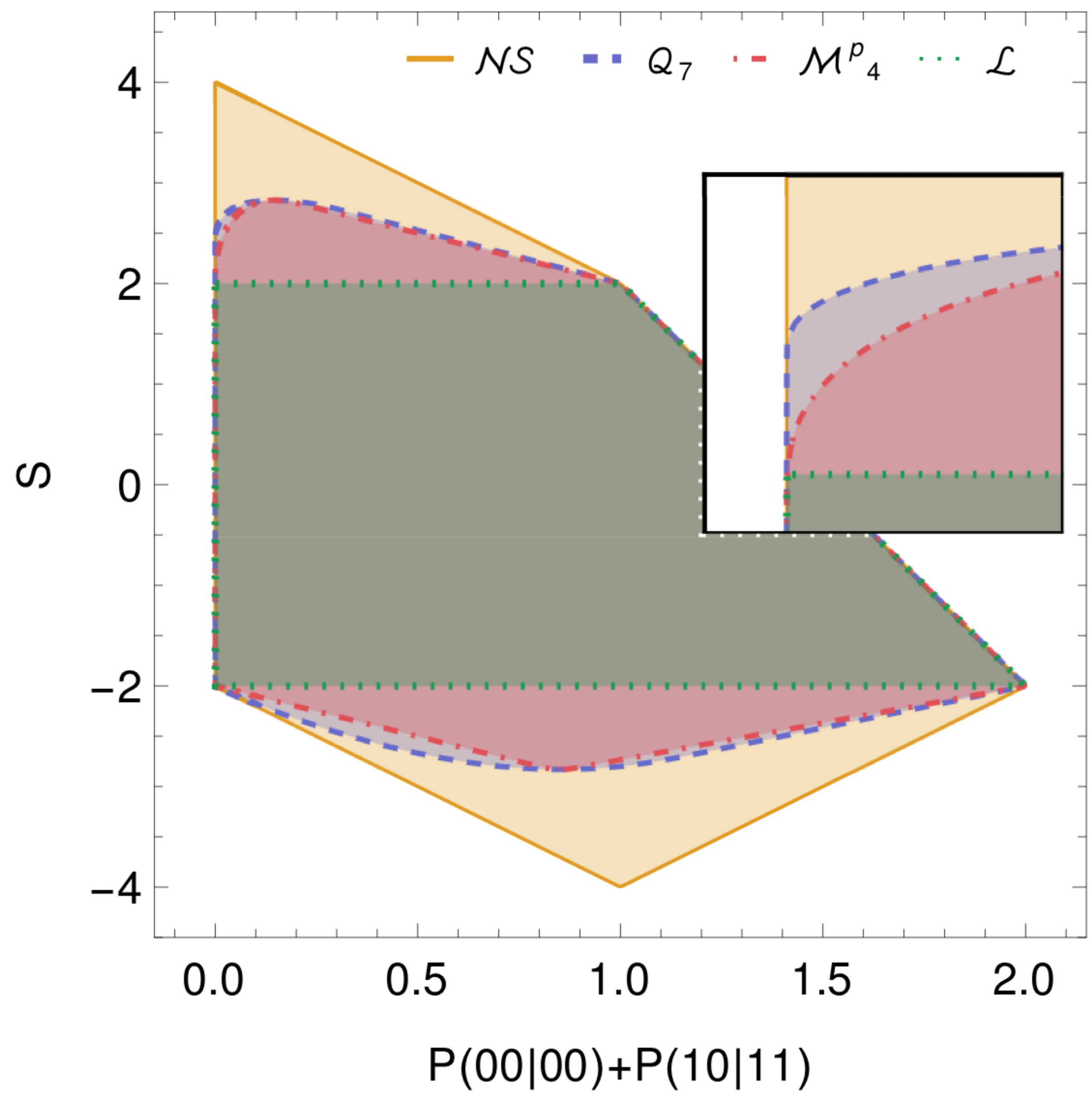
	1				



Maximally Entangled States Set

<table>
<tr><th># of zeros</th><th>Feasible</th><th>Infeasible</th></tr>
<tr><td>3</td><td>none</td><td>all</td></tr>
<tr><td>2</td><td>
<table>
<tr><th></th><th></th><th colspan="2">$x = 0$</th><th colspan="2">$x = 1$</th></tr>
<tr><th></th><th></th><th>0</th><th>1</th><th>0</th><th>1</th></tr>
<tr><td rowspan="2">$y = 0$</td><td>0</td><td>0</td><td></td><td></td><td></td></tr>
<tr><td>1</td><td></td><td>0</td><td></td><td></td></tr>
<tr><td rowspan="2">$y = 1$</td><td>0</td><td></td><td></td><td></td><td></td></tr>
<tr><td>1</td><td></td><td></td><td></td><td></td></tr>
</table>
</td><td>others</td></tr>
<tr><td>1</td><td>all</td><td>none</td></tr>
</table>

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    - Trying other Bell Scenarios.

Thank you for your attention!