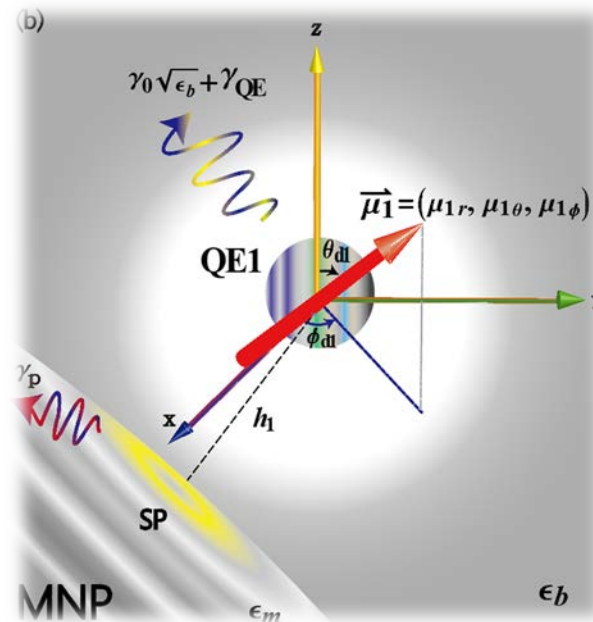


points of quantum emitters coupled to nanoparticle surface plasmons

Po-Chen Kuo, Neill Lambert, Adam Miranowicz ,
Hong-Bin Chen, Guang-Yin Chen, Yueh-Nan Chen,
and Franco Nori



Outline

- ◆ **Exceptional point (EP) and Diaboloic-point (DP)**
- ◆ **Localized surface plasmon (LSP)**
- ◆ **Quantum emitter (QE) coupled to LSP**
 - ◆ **Single-QE & a nanoparticle (NP)**
 - ◆ **Multi-QE & a nanoparticle (NP)**
- ◆ **Detection of EP by power spectrum**
- ◆ **Conclusion**

◆ EP and DP

◆ Single-dot Coupled to MNP

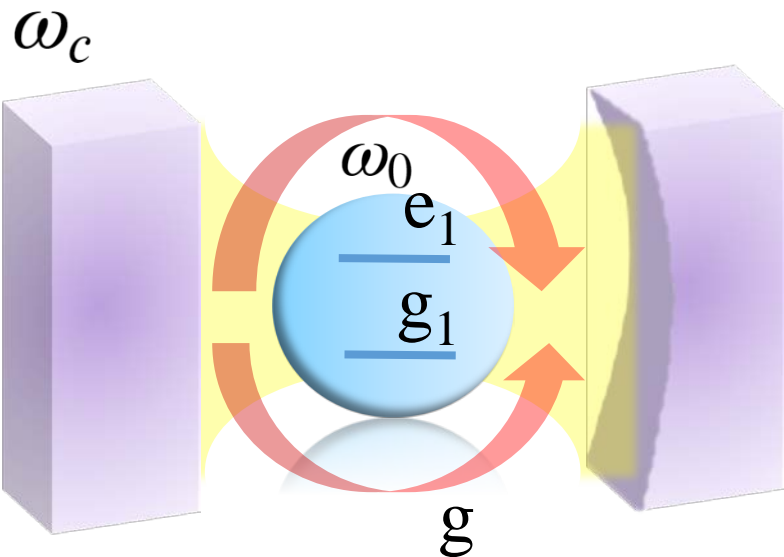
◆ Exceptional point

◆ QD couples to MNW

◆ Double-dots Coupled to MNP

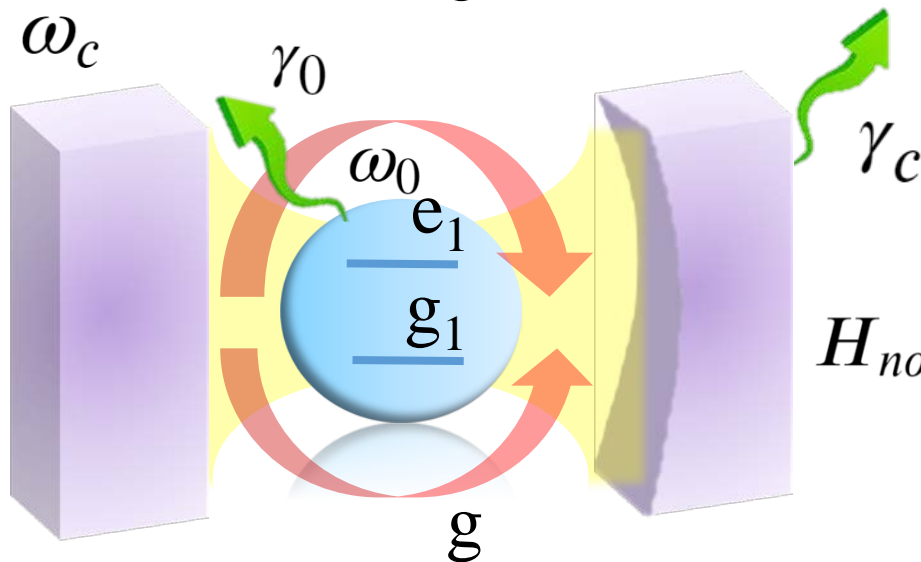
◆ Conclusion

Hermitian and Non-Hermitian system



Hermitian

$$H = \begin{pmatrix} \omega_0 & g \\ g & \omega_c \end{pmatrix}$$



Non-Hermitian

$$H_{non} = \begin{pmatrix} \omega_0 - i\frac{\gamma_0}{2} & g \\ g & \omega_c - i\frac{\gamma_c}{2} \end{pmatrix}$$

◆ surface plasmons

◆ Single-dot Coupled to MNP

◆ Exceptional point

◆ QD couples to MNW

◆ Double-dots Coupled to MNP

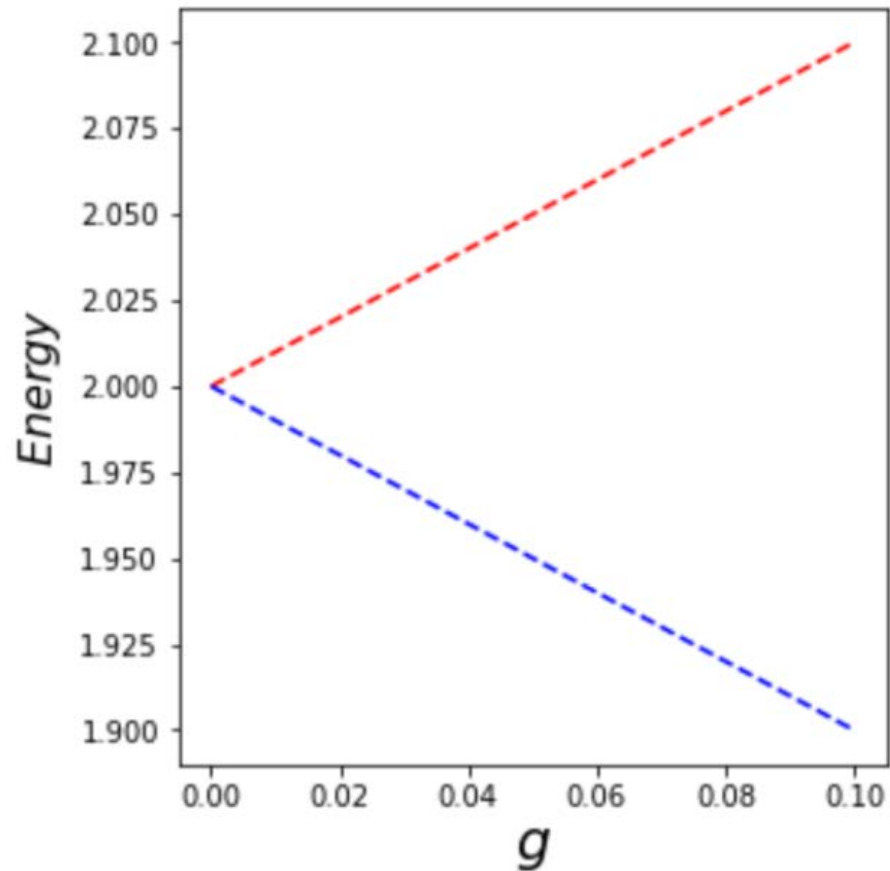
◆ Conclusion

Exceptional point (EP)

Hermitian

$$H = \begin{pmatrix} \omega_0 & g \\ g & \omega_c \end{pmatrix}$$

$$\omega_0 = 2eV \quad \omega_c = 2eV$$



◆ surface plasmons

◆ Single-dot Coupled to MNP

◆ Exceptional point

◆ QD couples to MNW

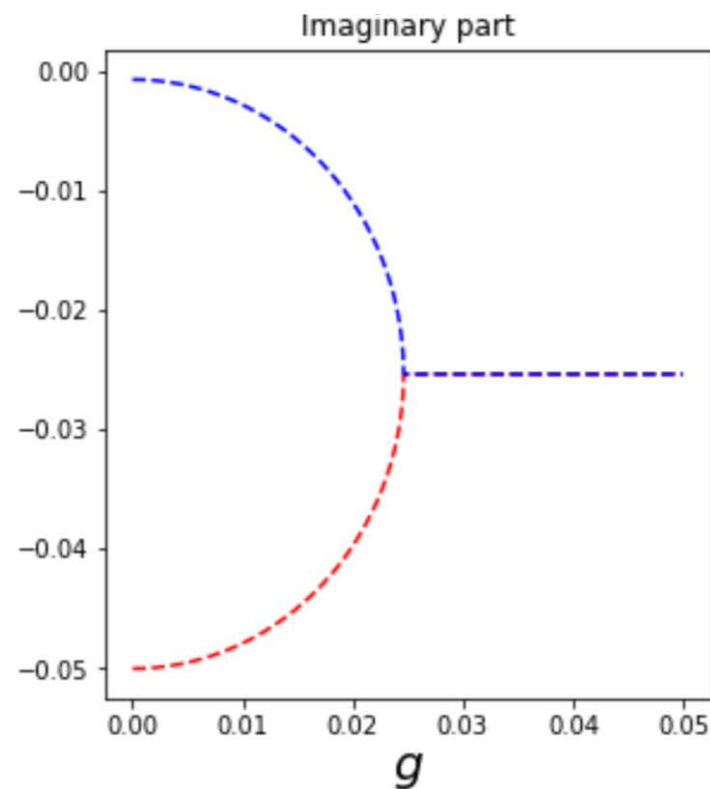
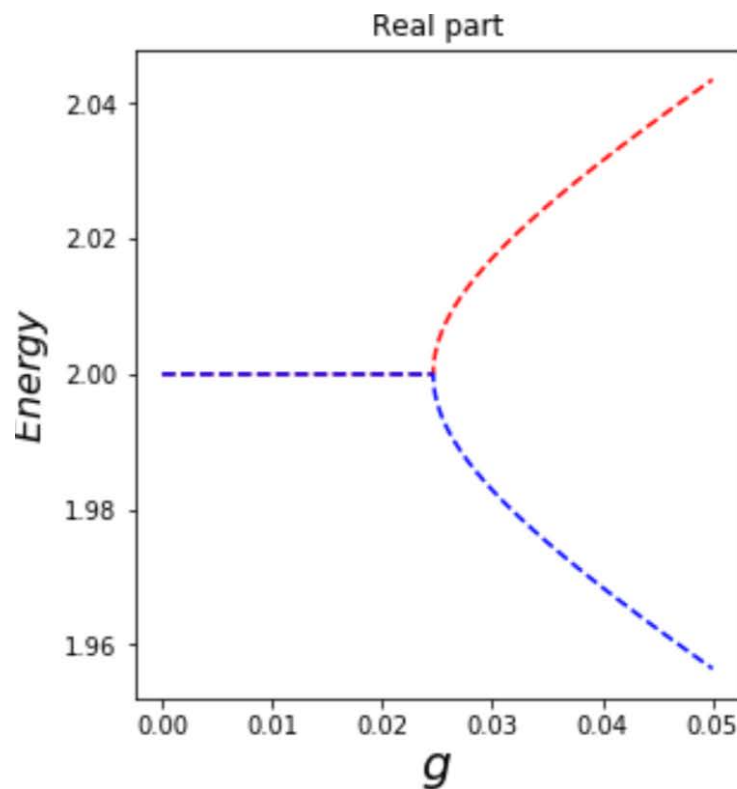
◆ Double-dots Coupled to MNP

◆ Conclusion

Exceptional point (EP)

Non-Hermitian

$$H_{non} = \begin{pmatrix} \omega_0 - i\frac{\gamma_0}{2} & g \\ g & \omega_c - i\frac{\gamma_c}{2} \end{pmatrix}$$



◆ surface plasmons

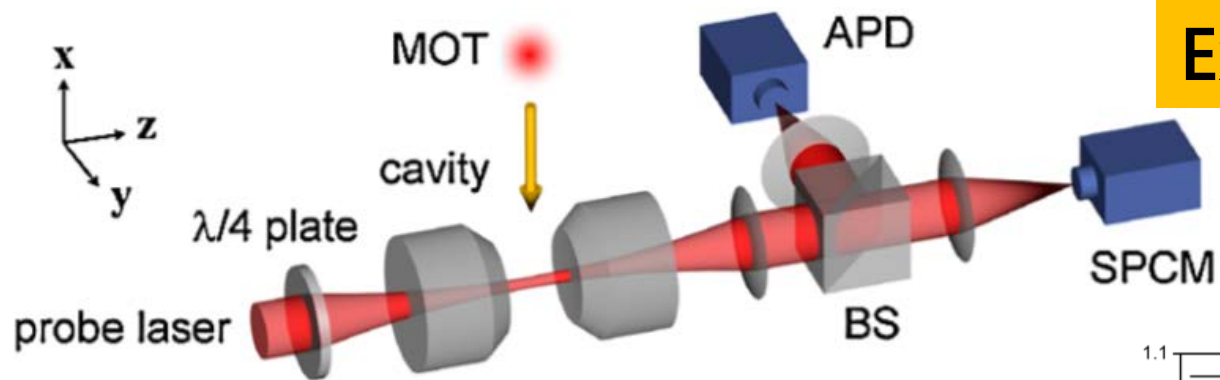
◆ Single-dot Coupled to MNP

◆ Exceptional point

◆ QD couples to MNW

◆ Double-dots Coupled to MNP

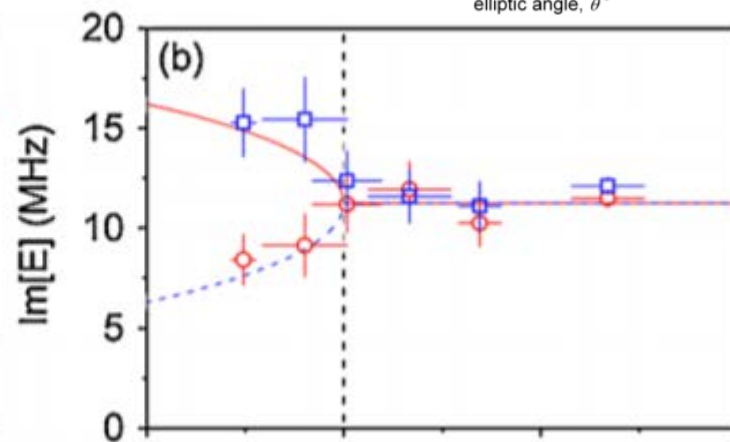
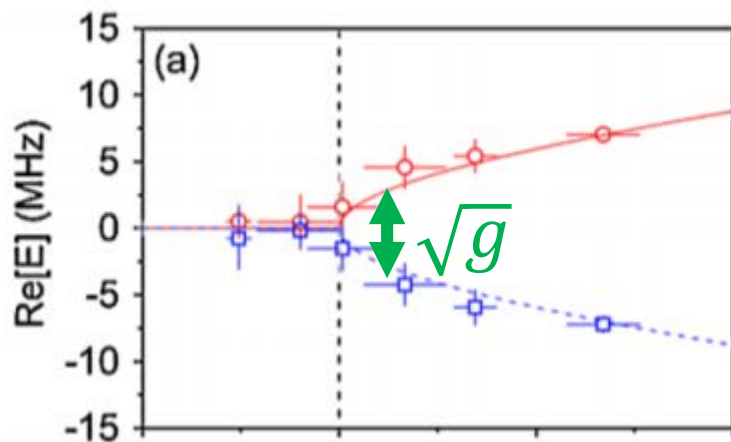
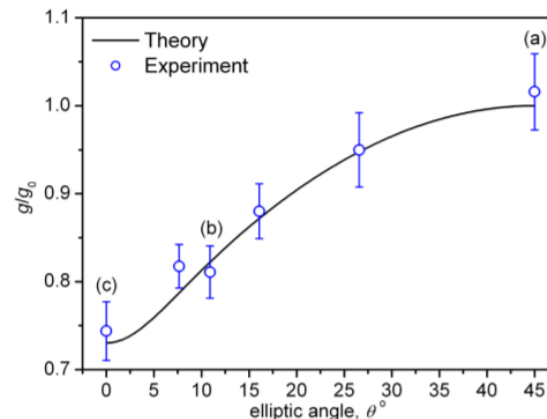
◆ Conclusion



Exceptional point

Sensing

$$H' = \hbar \begin{bmatrix} \omega_a - i\gamma_a & g \\ g & \omega_c - i\gamma_c \end{bmatrix}$$



◆ surface plasmons

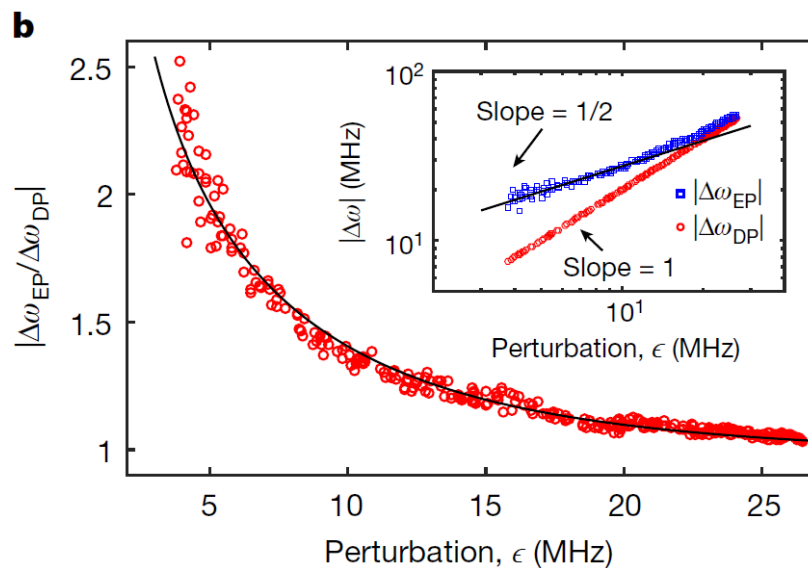
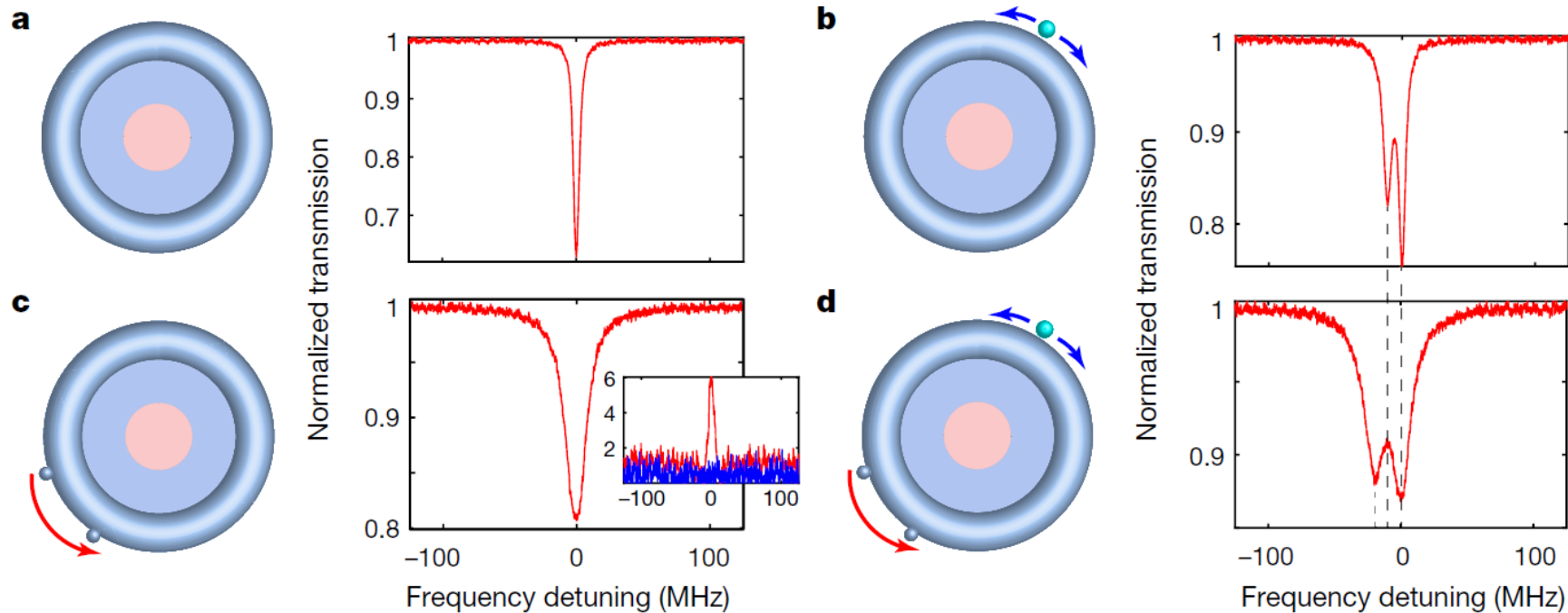
◆ Single-dot Coupled to MNP

◆ Exceptional point

◆ QD couples to MNW

◆ Double-dots Coupled to MNP

◆ Conclusion





◆ surface plasmons

◆ Single-dot Coupled to MNP

◆ Exceptional point

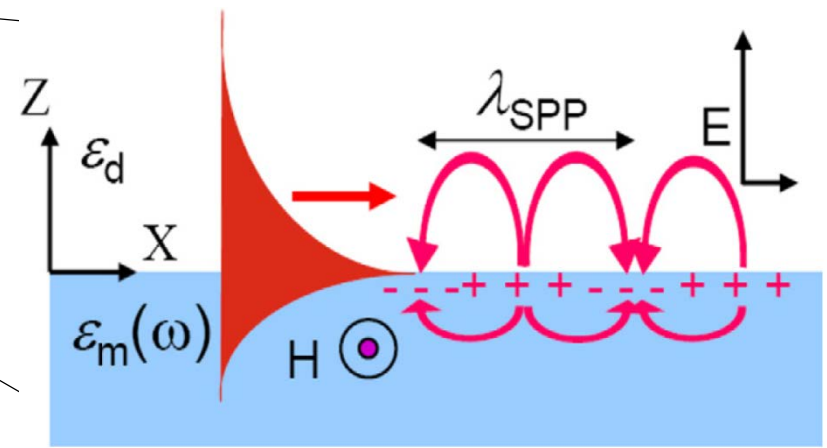
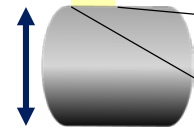
◆ QD couples to MNW

◆ Double-dots Coupled to MNP

◆ Conclusion

surface plasmons (SP)

← wavelength →



surface plasmon polaritons (SPP)

→ {
 confinement of the electromagnetic field
 Enhancement of light matter interaction

◆ surface plasmons

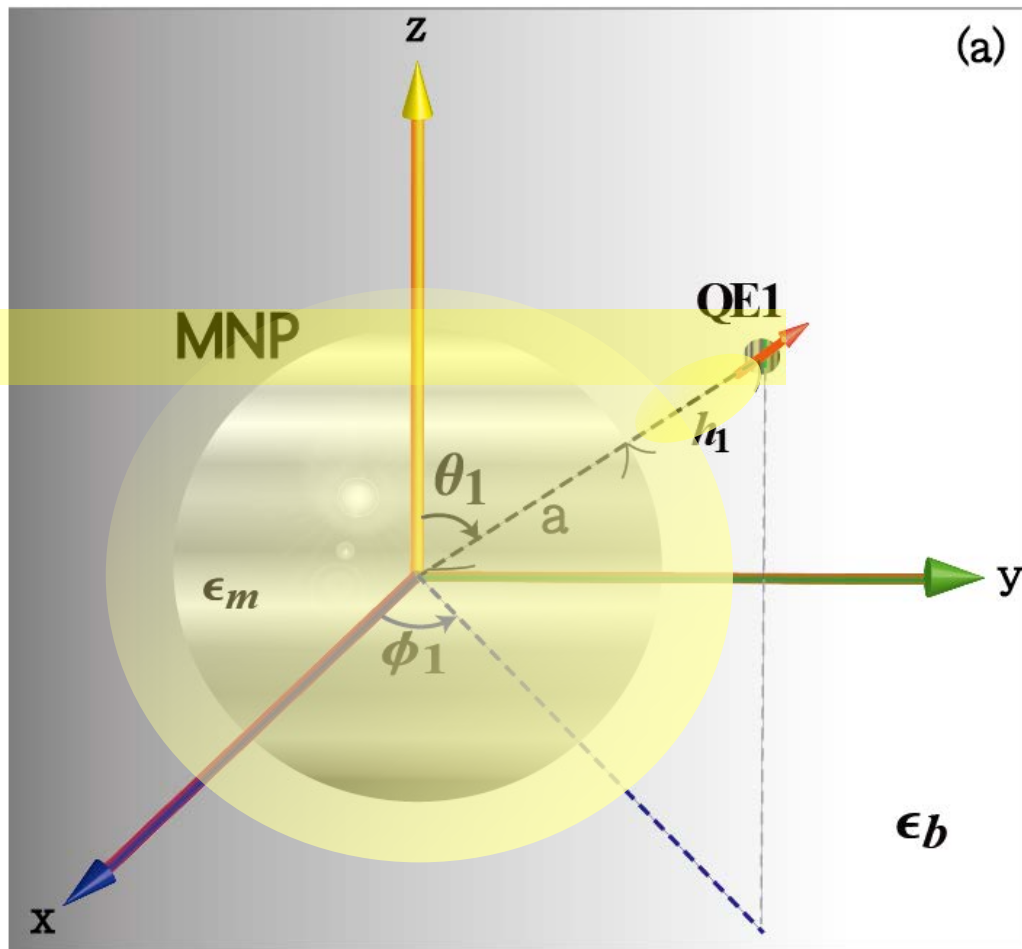
◆ Single-dot Coupled to MNP

◆ Exceptional point

◆ QD couples to MNW

◆ Double-dots Coupled to MNP

◆ Conclusion



$$\begin{aligned} & \hbar \left[\omega_0 - i \left(\frac{\gamma_{QE}}{2} \right) \right] \sigma_{e_1, e_1} \\ & + \int d^3 \vec{r} \int_0^\infty d\omega \hbar \omega \hat{f}^\dagger(\vec{r}, \omega) \hat{f}(\vec{r}, \omega) \\ & - \int_0^\infty d\omega \left[\vec{\mu}_1 \cdot \left(\hat{E}(\vec{r}, \omega) \sigma_+^{(1)} + H.c. \right) \right] \end{aligned}$$

◆ surface plasmons

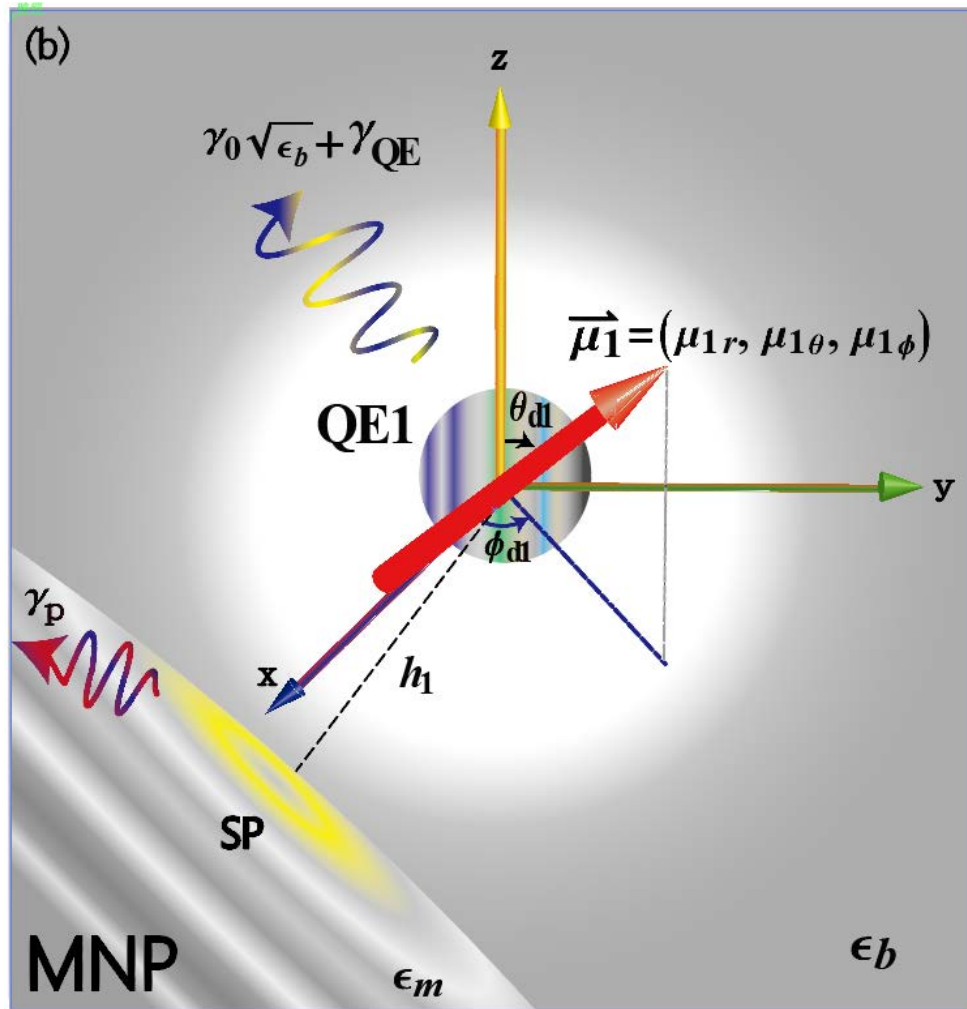
◆ Single-dot Coupled to MNP

◆ Exceptional point

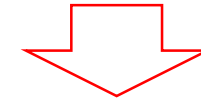
◆ QD couples to MNW

◆ Double-dots Coupled to MNP

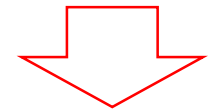
◆ Conclusion



$$-\int_0^{\infty} d\omega \left[\vec{\mu}_1 \cdot \left(\hat{E}(\vec{r}, \omega) \sigma_+^{(1)} + H.c. \right) \right]$$



$$\hat{E}(\vec{r}, \omega) = i \sqrt{\frac{\hbar}{\pi \epsilon_0}} \frac{\omega^2}{c^2} \int d^3 \vec{r}_1 \sqrt{\epsilon^I(\vec{r}_1, \omega)} \hat{G}(\vec{r}, \vec{r}_1, \omega) \hat{f}(\vec{r}_1, \omega)$$



$$\nabla \times \nabla \times \hat{G}(\vec{r}, \vec{r}_1, \omega) - \frac{\omega^2}{c^2} \epsilon(\vec{r}, \omega) \hat{G}(\vec{r}, \vec{r}_1, \omega) = \mathbf{I} \delta(\vec{r}, \vec{r}_1)$$

$$\tilde{G}_{es(r,R)}^{(11)} = \begin{pmatrix} G_{es}^{(rr)} & G_{es}^{(r\theta)} & G_{es}^{(r\phi)} \\ G_{es}^{(\theta r)} & G_{es}^{(\theta\theta)} & G_{es}^{(\theta\phi)} \\ G_{es}^{(\phi r)} & G_{es}^{(\phi\theta)} & G_{es}^{(\phi\phi)} \end{pmatrix}$$

◆ surface plasmons

◆ Single-dot Coupled to MNP

◆ Exceptional point

◆ QD couples to MNW

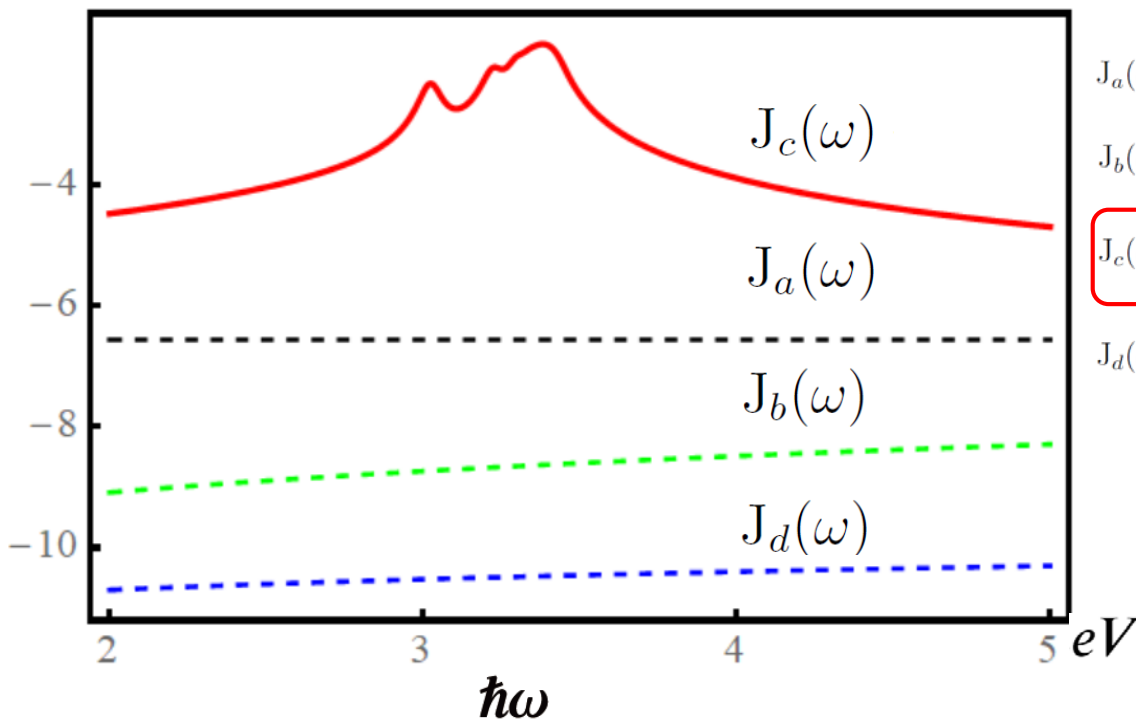
◆ Double-dots Coupled to MNP

◆ Conclusion

$$H \cdot \psi \Rightarrow \dot{C}_1(t) = - \int_0^t dt_1 \int_0^\infty d\omega \mathbf{J}(\omega) e^{i(\omega_0 - \omega)(t - t_1)} C_1(t_1)$$

$$J(\omega) \approx J_a(\omega) + J_b(\omega) + \mathbf{J}_c(\omega) + J_d(\omega)$$

$\log(\hbar J)$



$$J_a(\omega) = \frac{\gamma_0 \sqrt{\epsilon_b}}{2\pi}$$

$$J_b(\omega) = \frac{\gamma_0 \epsilon_b^{3/2} \omega^2 (a + h_1)^2 (\mu_{1\theta}^2 + \mu_{1\phi}^2)}{8\pi c^2 \mu_1^2}$$

$$\mathbf{J}_c(\omega) = \sum_{n=0}^{\infty} \frac{g_{nr}^2(\mu_1) + g_{n\theta}^2(\mu_1) + g_{n\phi}^2(\mu_1)}{\pi} \frac{\gamma_p/2}{(\omega - \omega_n)^2 + (\gamma_p/2)^2}$$

$$J_d(\omega) = \sum_{\eta=0}^{\infty} \frac{g_{\eta\theta}^2(\mu_1) + g_{\eta\phi}^2(\mu_1)}{\pi} \frac{\gamma_p}{\omega^2 + \gamma_p^2}$$

◆ surface plasmons

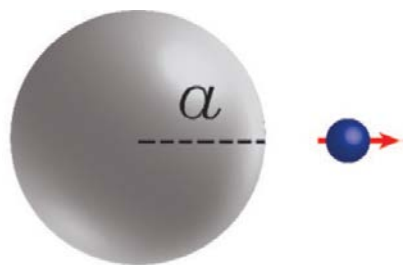
◆ Single-dot Coupled to MNP

◆ Exceptional point

◆ QD couples to MNW

◆ Double-dots Coupled to MNP

◆ Conclusion

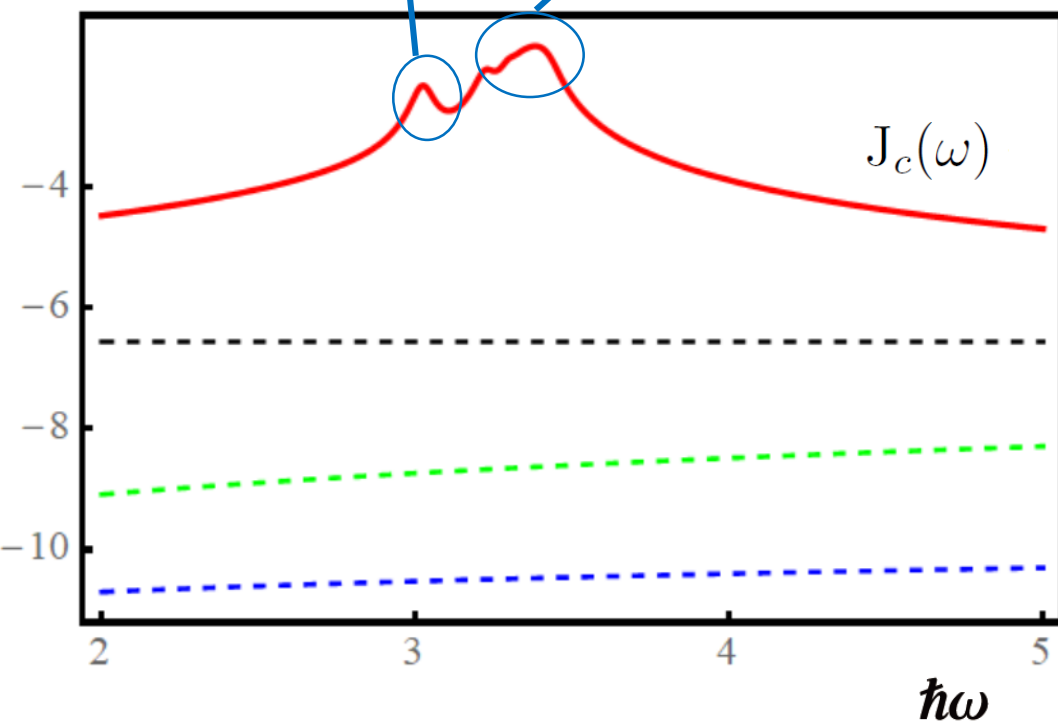


$$J(\omega) \approx \sum_{n=0}^{\infty} \frac{g_n^2}{\pi} \frac{\gamma_p/2}{(\omega - \omega_n)^2 + (\gamma_p/2)^2}$$

$$\omega_n = \omega_p / \sqrt{\epsilon_{\infty} + \epsilon_d(n+1)/n}$$

$$\omega_d = \omega_1 \quad \omega_M = \frac{\sum_{n=2}^{\infty} \omega_n g_n^2}{\sum_{n=2}^{\infty} g_n^2}$$

$\log(\hbar J)$



◆ surface plasmons

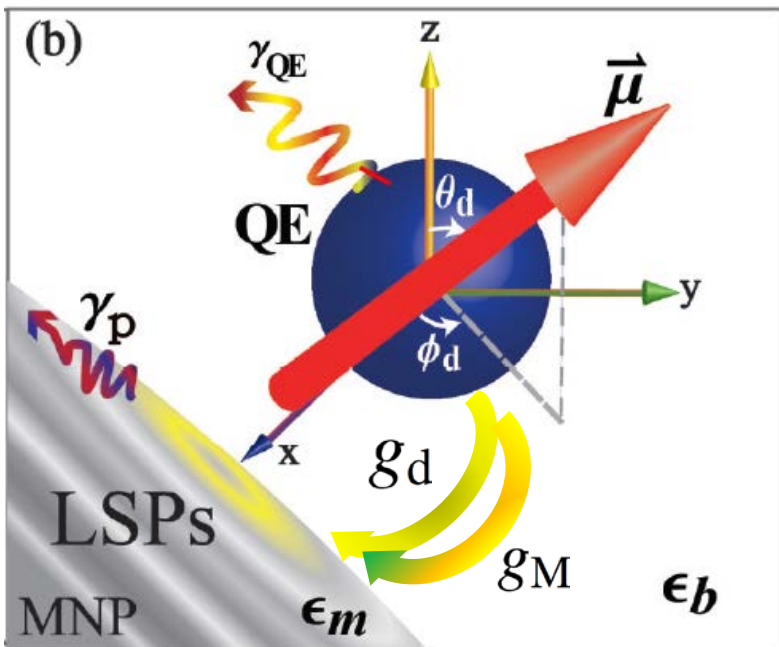
◆ Single-dot Coupled to MNP

◆ Exceptional point

◆ QD couples to MNW

◆ Double-dots Coupled to MNP

◆ Conclusion



$$\hat{H}_{3 \times 3} = \begin{bmatrix} \omega_0 - i\frac{\gamma_{QE}}{2} & g_d & g_M \\ g_d & \omega_d - i\frac{\gamma_p}{2} & 0 \\ g_M & 0 & \omega_M - i\frac{\gamma_p}{2} \end{bmatrix}$$

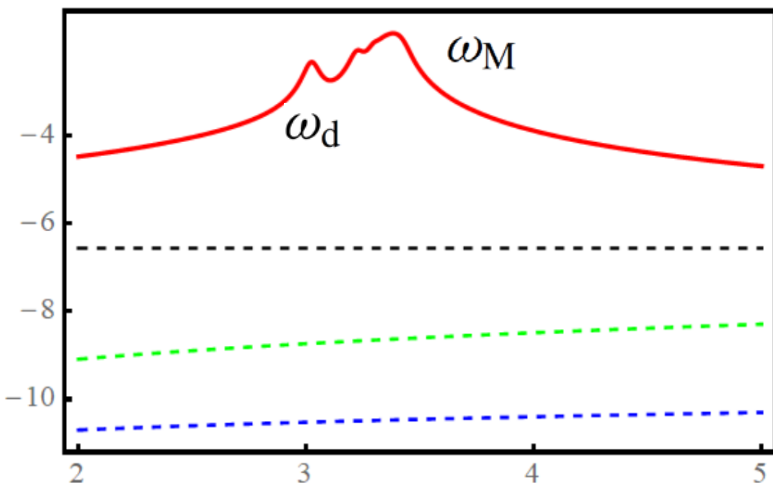
$$g_d = g_1$$

$$g_M^2 = \sum_{n=2}^{\infty} g_n^2$$

$$g_n^2 = \sum_{\alpha=r,\theta,\phi} g_{n\alpha}^2$$

$$g_{nr}^2 = \mu_r^2 (n+1)^2 f_n(\omega_n)$$

$$g_{n\theta(\phi)}^2 = \mu_{\theta(\phi)}^2 \sum_{m=0}^n \mathcal{D}_{nm} [mP_n^m(0)]^2 f_n(\omega_n)$$



◆ surface plasmons

◆ Single-dot Coupled to MNP

◆ Exceptional point

◆ QD couples to MNW

◆ Double-dots Coupled to MNP

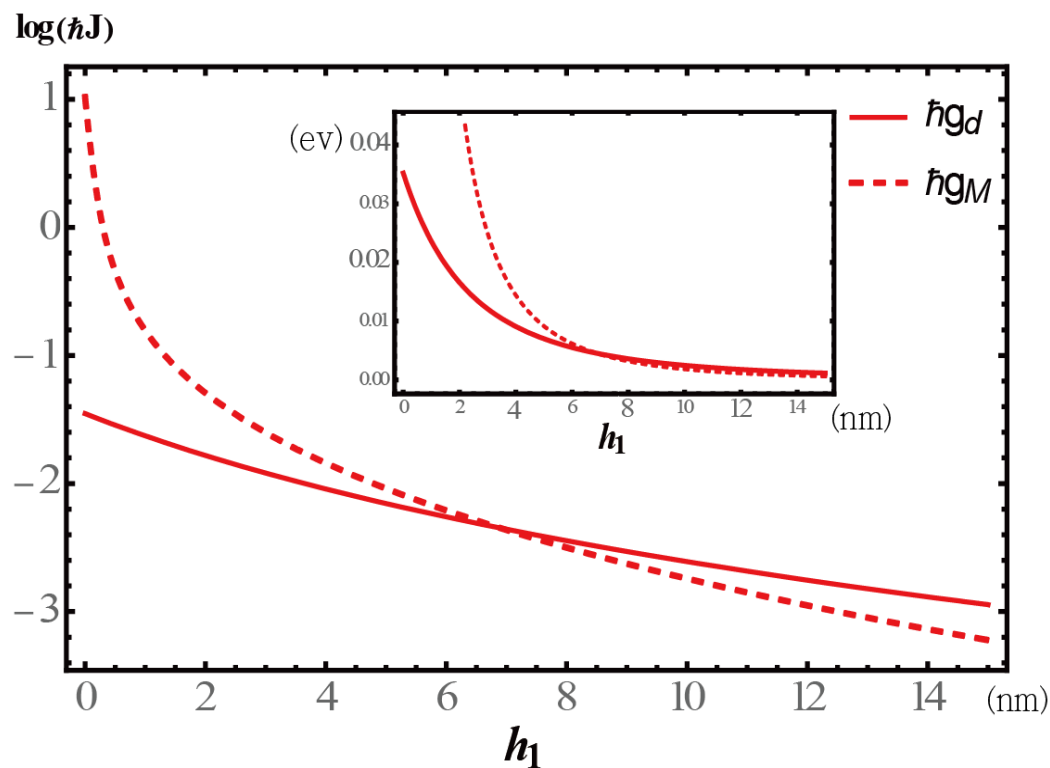
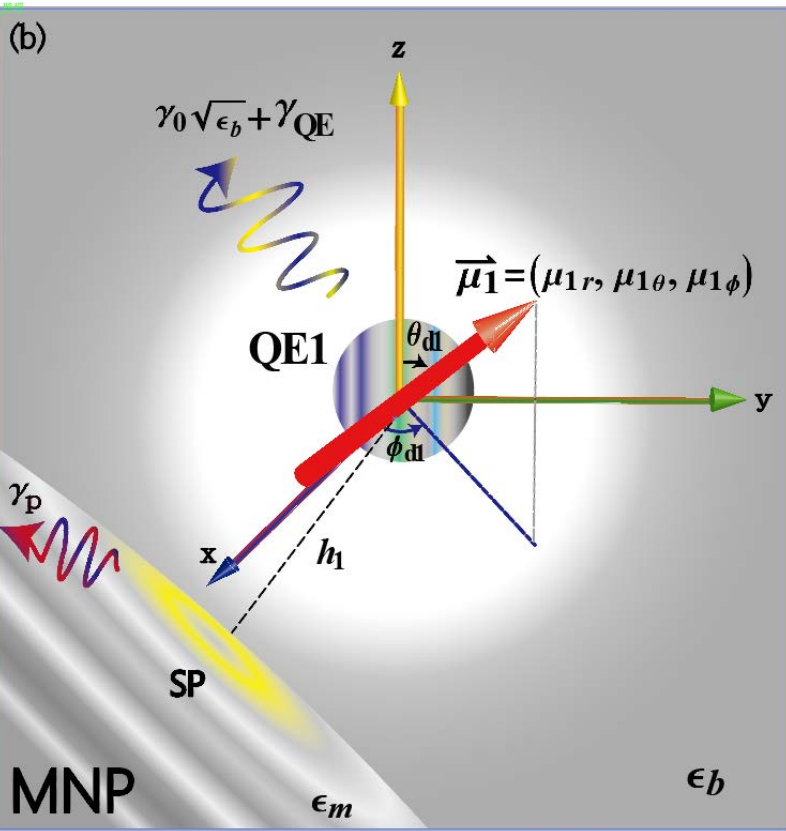
◆ Conclusion

$$g_n^2 = \sum_{\alpha=r,\theta,\phi} g_{n\alpha}^2$$

$$g_{nr}^2 = \mu_r^2 (n+1)^2 f_n(\omega_n)$$

$$g_{n\theta(\phi)}^2 = \mu_{\theta(\phi)}^2 \sum_{m=0}^n \mathcal{D}_{nm} [mP_n^m(0)]^2 f_n(\omega_n)$$

$$f_n(\omega_n) = \frac{a^{2n+1}}{(a+h)^{2n+4}} \left(1 + \frac{1}{2n}\right) \frac{\omega_p}{4\pi\epsilon_0\hbar} \left(\frac{\omega_n}{\omega_p}\right)^3$$



◆ surface plasmons

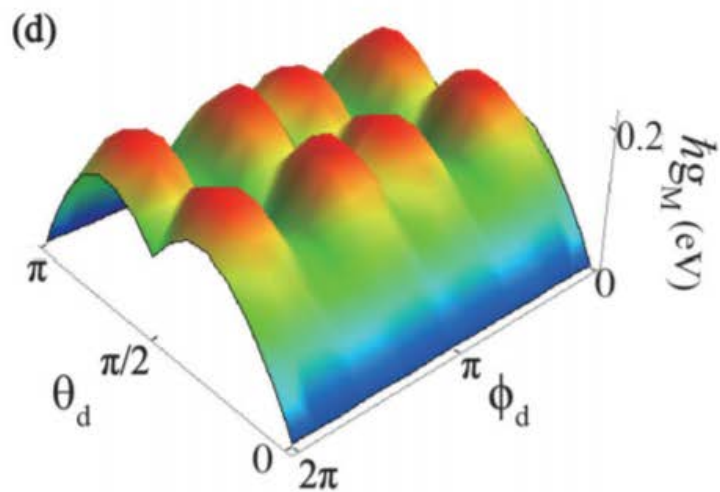
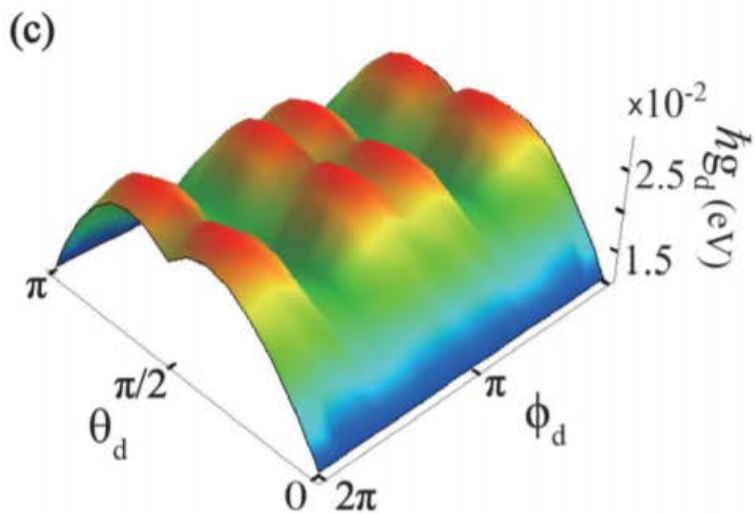
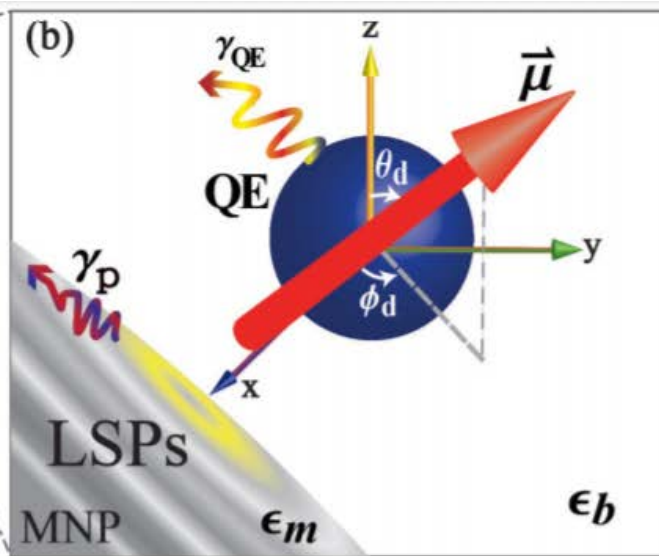
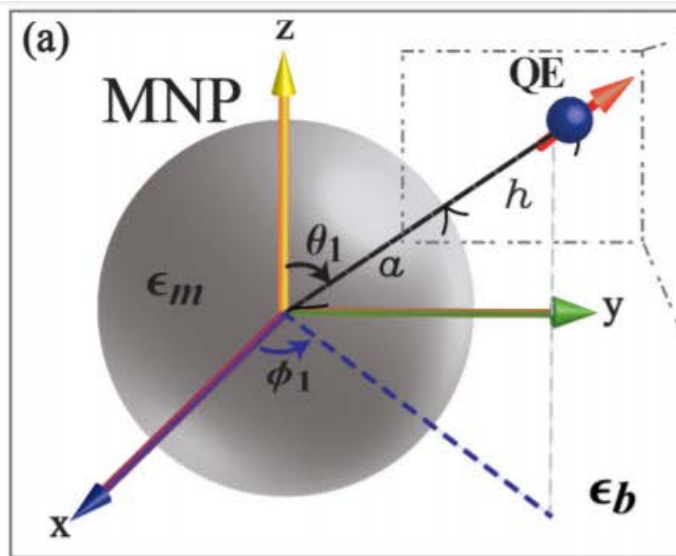
◆ Single-dot Coupled to MNP

◆ Exceptional point

◆ QD couples to MNW

◆ Double-dots Coupled to MNP

◆ Conclusion



◆ surface plasmons

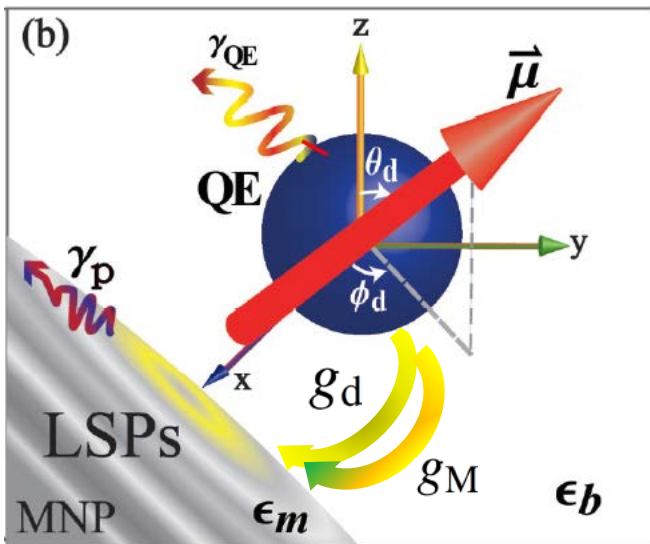
◆ Single-dot Coupled to MNP

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◆ Double-dots Coupled to MNP

◆ Conclusion



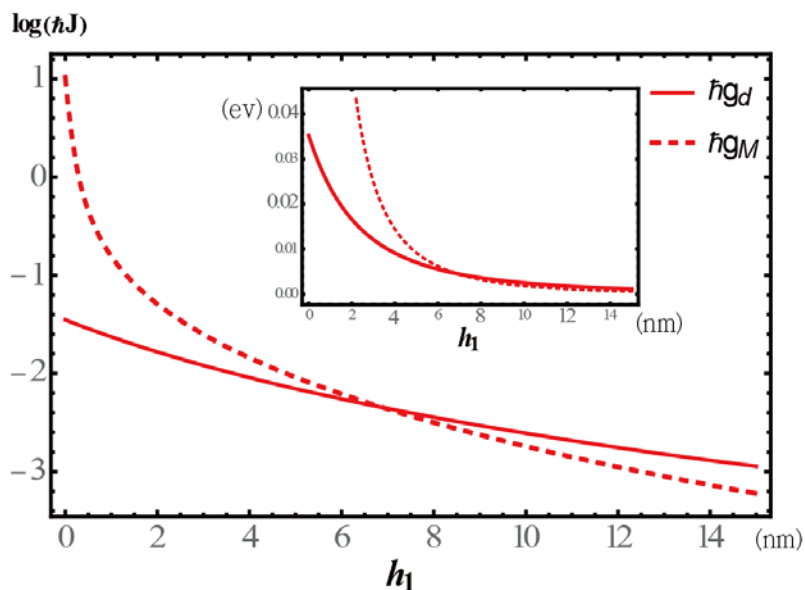
$$\hat{H}_{3 \times 3} = \begin{bmatrix} \omega_0 - i\frac{\gamma_{QE}}{2} & g_d & g_M \\ g_d & \omega_d - i\frac{\gamma_p}{2} & 0 \\ \textcircled{g_M} & 0 & \omega_M - i\frac{\gamma_p}{2} \end{bmatrix}$$



$$\hat{H}_{2 \times 2} = \begin{bmatrix} \omega_0 - i\frac{\gamma_{QE}}{2} & \textcircled{g_M} \\ g_M & \omega_M - i\frac{\gamma_p}{2} \end{bmatrix}$$



$$g_M = (\gamma_p - \gamma_{QE})/4$$



◆ surface plasmons

◆ Single-dot Coupled to MNP

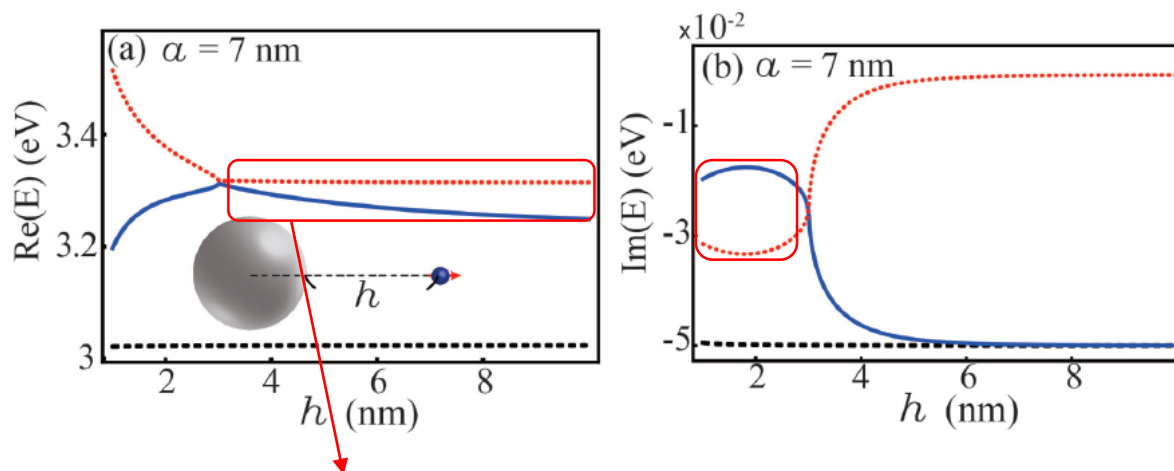
◆ Exceptional point

◆ QD couples to MNW

◆ Double-dots Coupled to MNP

◆ Conclusion

$$\hat{H}_{3 \times 3} = \begin{bmatrix} \omega_0 - i\frac{\gamma_{QE}}{2} & g_d & g_M \\ g_d & \omega_d - i\frac{\gamma_p}{2} & 0 \\ g_M & 0 & \omega_M - i\frac{\gamma_p}{2} \end{bmatrix}$$



$$\Delta E = \frac{-\sqrt{3}i[u^2 + p(\omega_d, g_d)^{2/3} - 48g_d^2]}{12p(\omega_d, g_d)^{1/3}}$$

$$p(\omega_d, g_d) = 144g_d^2(2\omega_{d\Delta} + i\gamma_\Delta) + iu^3 + 12q(\omega_d, g_d)$$

$$q(\omega_d, g_d) = -96g_d^4(\gamma_\Delta^2 - 10i\gamma_\Delta\omega_{d\Delta} + 2\omega_{d\Delta}^2) - 3\gamma_\Delta g_d^2 u^3 - 768g_d^6,$$

$u = \gamma_\Delta + 4i\omega_{d\Delta}$, $\omega_{d\Delta} = \omega_d - \omega_0$, and $\gamma_\Delta = \gamma_p - \gamma_{QE}$

◆ surface plasmons


◆ Single-dot Coupled to MNP

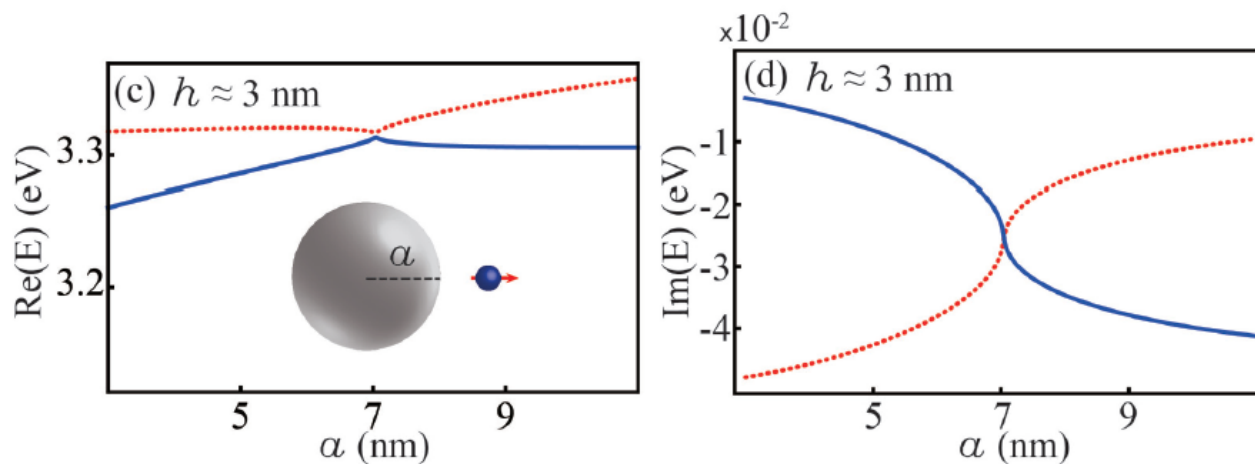
◆ Exceptional point

◆ QD couples to MNW

◆ Double-dots Coupled to MNP

◆ Conclusion

$$\hat{H}_{3 \times 3} = \begin{bmatrix} \omega_0 - i\frac{\gamma_{QE}}{2} & g_d & g_M \\ g_d & \omega_d - i\frac{\gamma_P}{2} & 0 \\ g_M & 0 & \omega_M - i\frac{\gamma_P}{2} \end{bmatrix}$$




◆ surface plasmons

◆ Single-dot Coupled to MNP

◆ Exceptional point

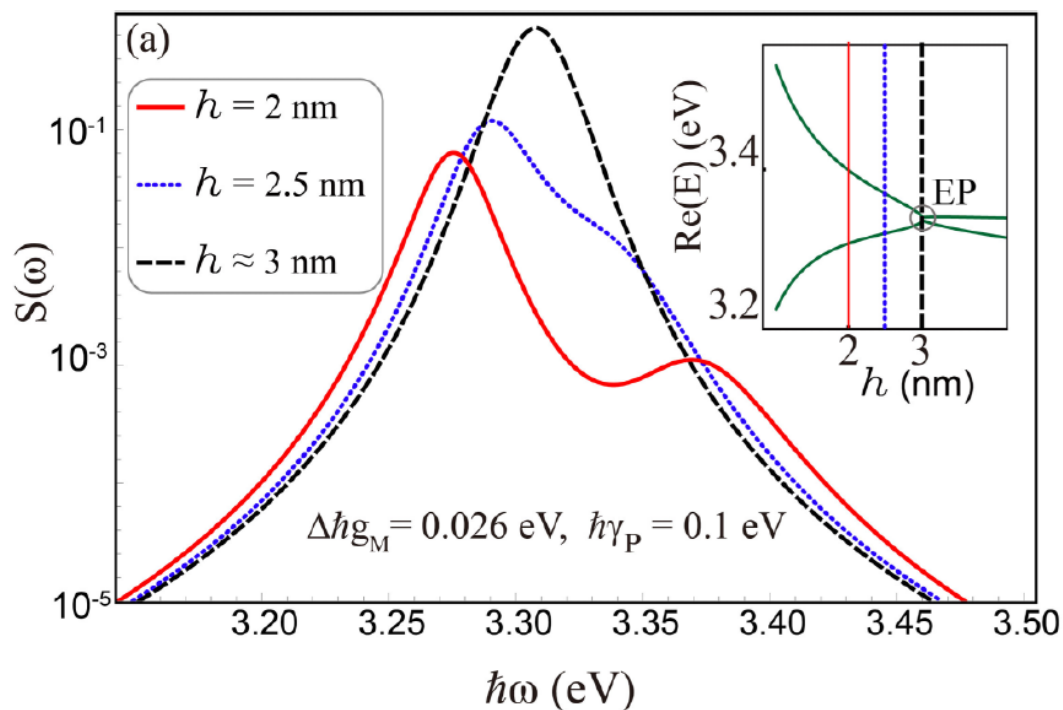
◆ QD couples to MNW

◆ Double-dots Coupled to MNP

◆ Conclusion

$$\hat{H}_{3 \times 3} = \begin{bmatrix} \omega_0 - i\frac{\gamma_{QE}}{2} & g_d & g_M \\ g_d & \omega_d - i\frac{\gamma_P}{2} & 0 \\ g_M & 0 & \omega_M - i\frac{\gamma_P}{2} \end{bmatrix}$$

$$S(\omega) = \frac{1}{\pi} \text{Re} \int_0^\infty d\tau \langle \hat{\sigma}_+^{(1)}(0) \hat{\sigma}_-^{(1)}(\tau) \rangle e^{i\omega\tau}$$



◆ surface plasmons

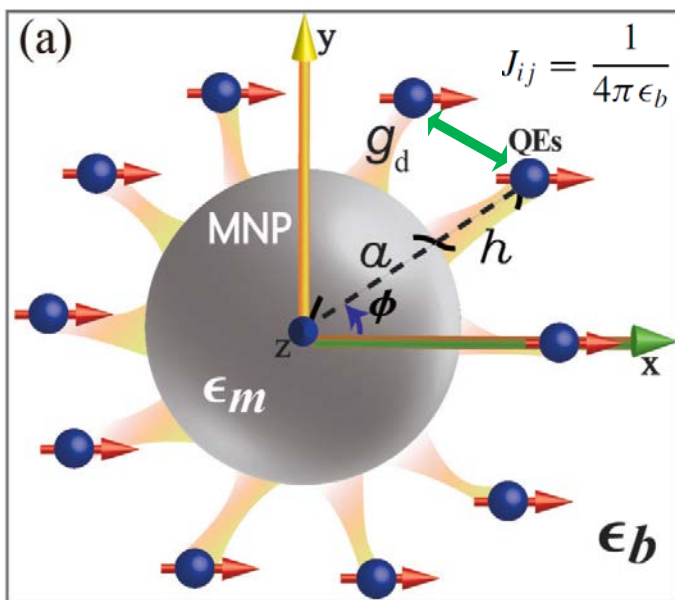
◆ Single-dot Coupled to MNP

◆ Exceptional point

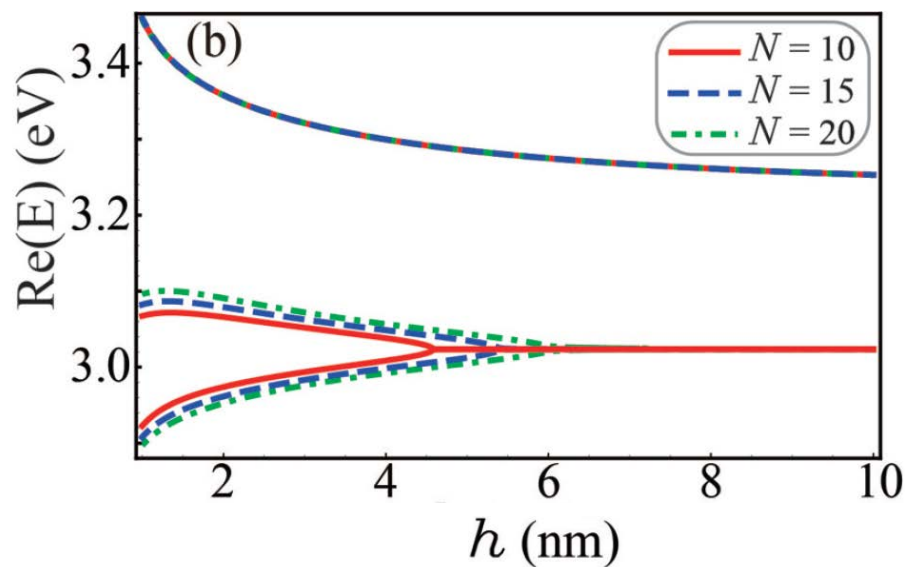
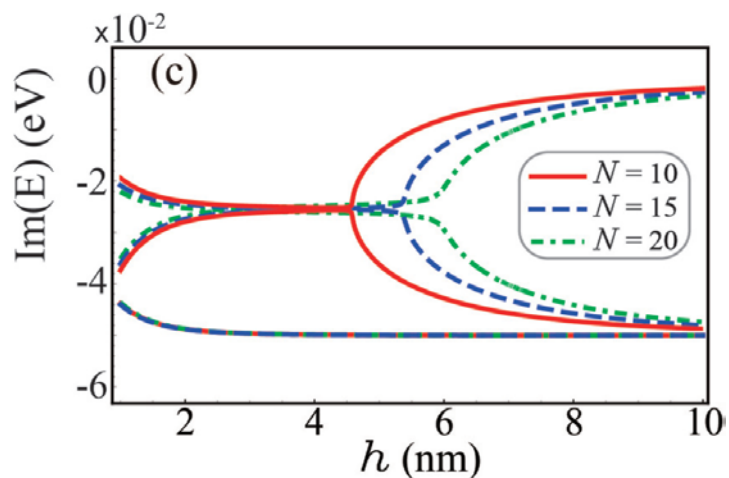
◆ QD couples to MNW

◆ Double-dots Coupled to MNP

◆ Conclusion



$$H_{3 \times 3} = \begin{bmatrix} \omega_0 - i\frac{\gamma_{QE}}{2} & g_d & g_M \\ g_d & \omega_d - i\frac{\gamma_p}{2} & 0 \\ g_M & 0 & \omega_M - i\frac{\gamma_p}{2} \end{bmatrix}$$



◆ surface plasmons

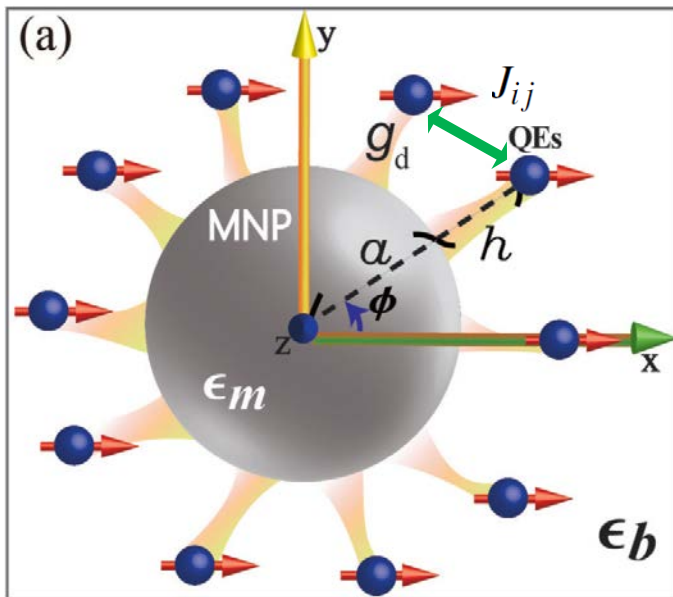
◆ Single-dot Coupled to MNP

◆ Exceptional point

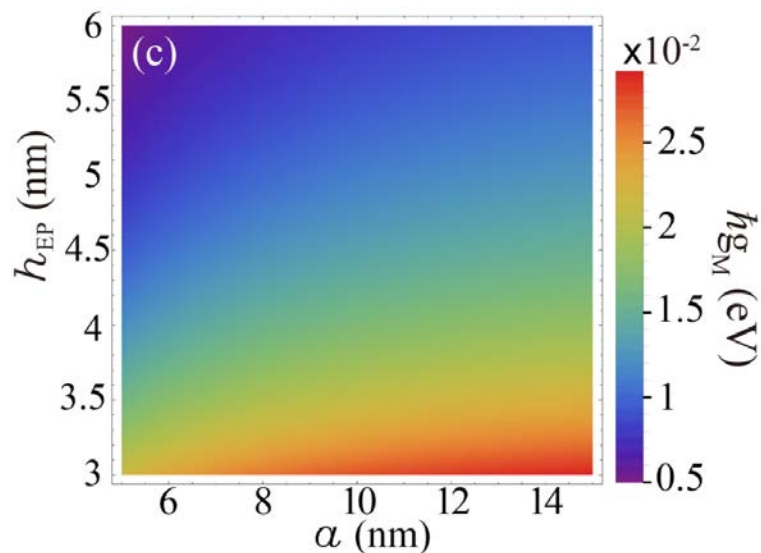
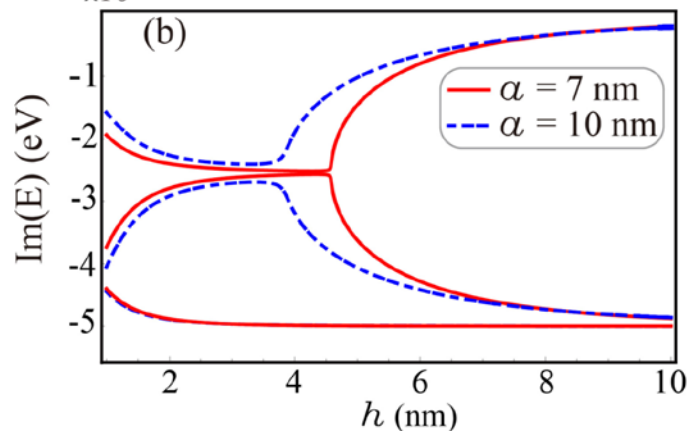
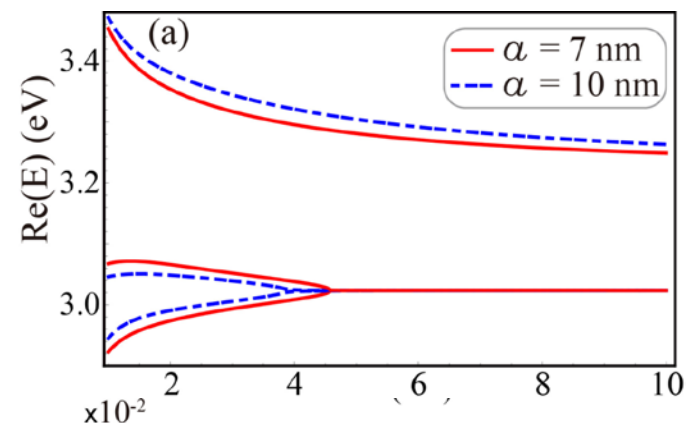
◆ QD couples to MNW

◆ Double-dots Coupled to MNP

◆ Conclusion



$$H_{3 \times 3} = \begin{bmatrix} \omega_0 - i\frac{\gamma_{QE}}{2} + \delta_J & \sqrt{N}g_d & g_M \\ \sqrt{N}g_d & \omega_d - i\frac{\gamma_P}{2} & 0 \\ g_M & 0 & \omega_M - i\frac{\gamma_P}{2} \end{bmatrix}$$



◆ surface plasmons

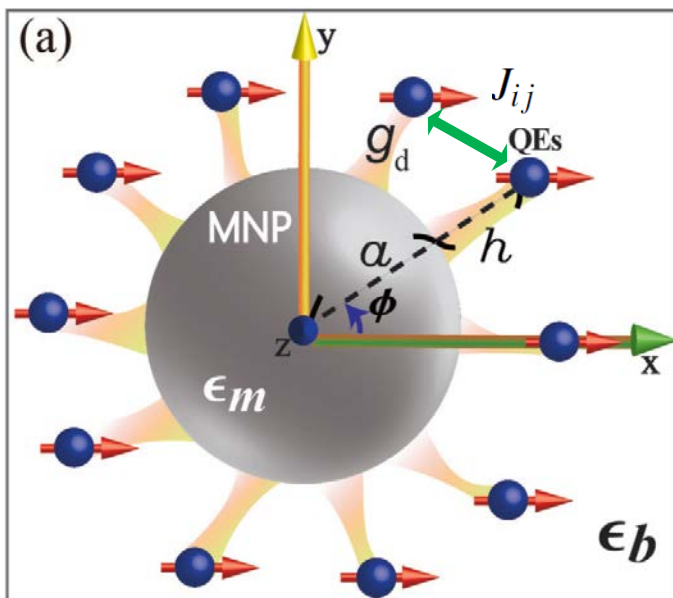
◆ Single-dot Coupled to MNP

◆ Exceptional point

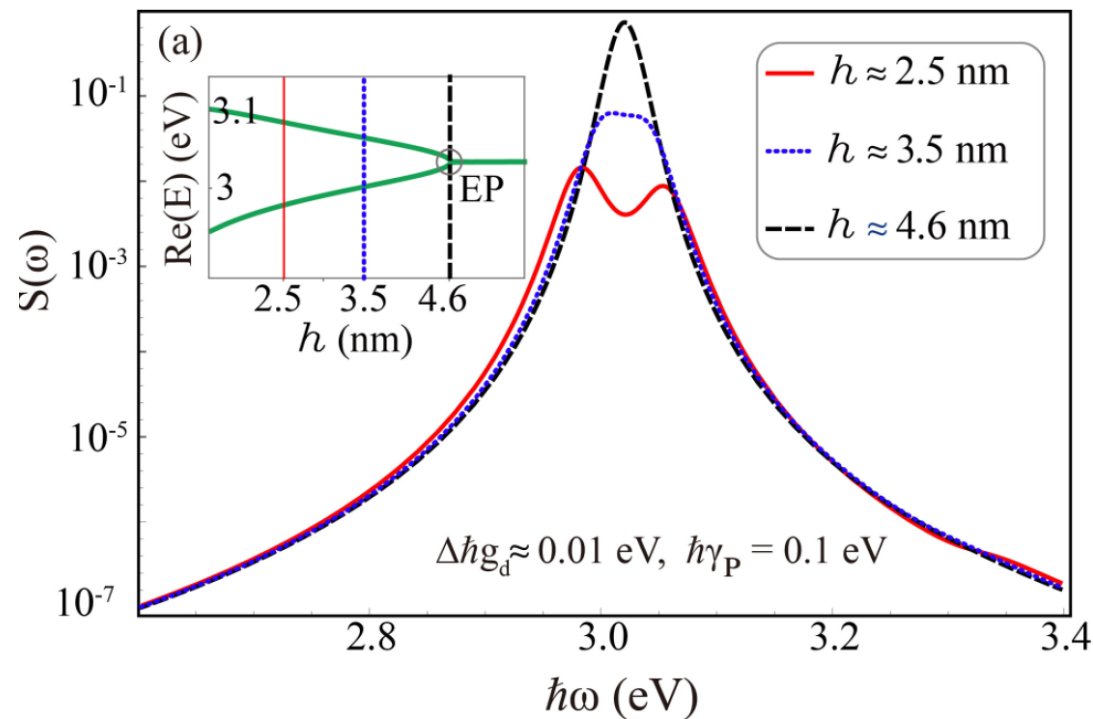
◆ QD couples to MNW

◆ Double-dots Coupled to MNP

◆ Conclusion



$$H_{3 \times 3} = \begin{bmatrix} \omega_0 - i\frac{\gamma_{QE}}{2} + \delta_J & \sqrt{N}g_d & g_M \\ \sqrt{N}g_d & \omega_d - i\frac{\gamma_P}{2} & 0 \\ g_M & 0 & \omega_M - i\frac{\gamma_P}{2} \end{bmatrix}$$



$$S(\omega) = \frac{1}{\pi} \text{Re} \int_0^\infty d\tau \langle \hat{\sigma}_+^{(c)}(0) \hat{\sigma}_-^{(c)}(\tau) \rangle e^{i\omega\tau}$$

◆ surface plasmons

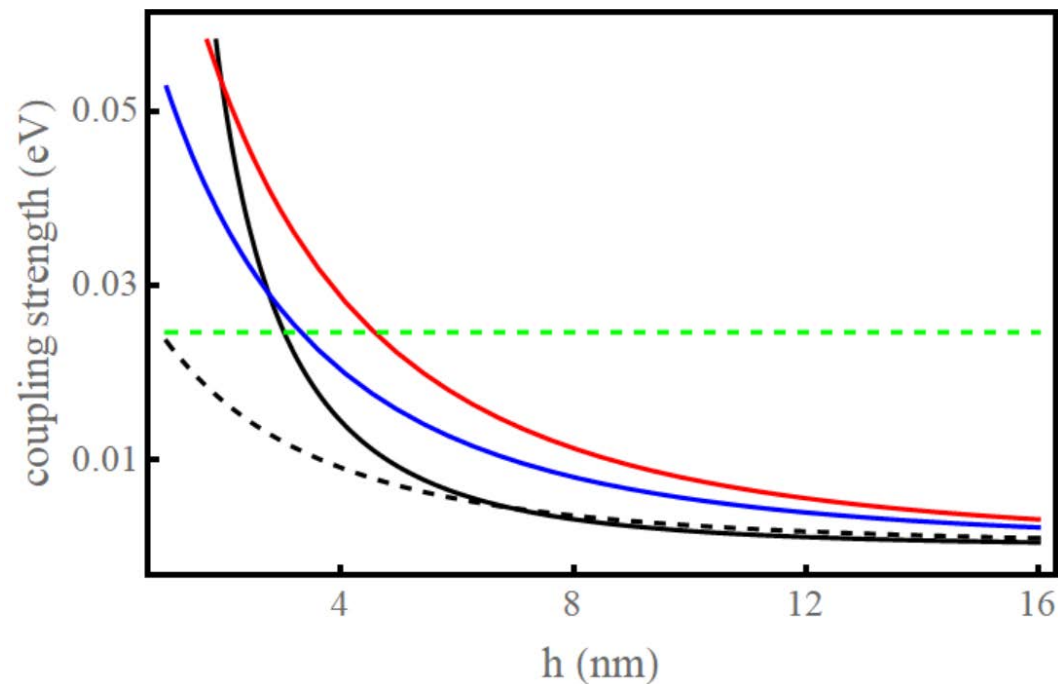
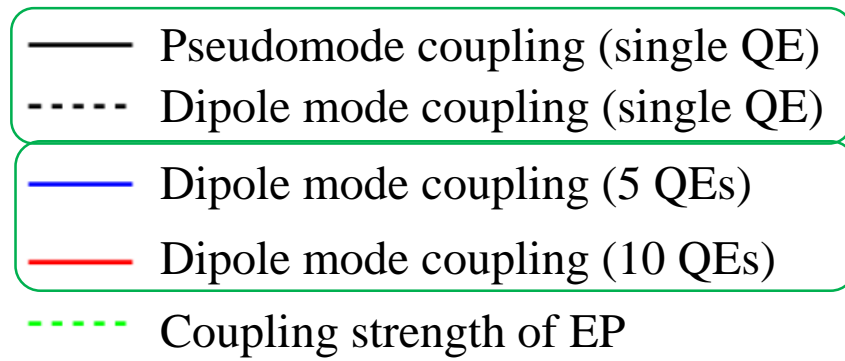
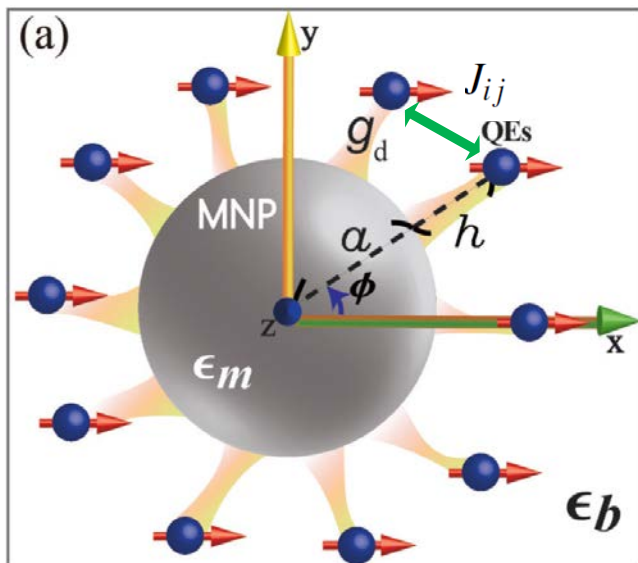
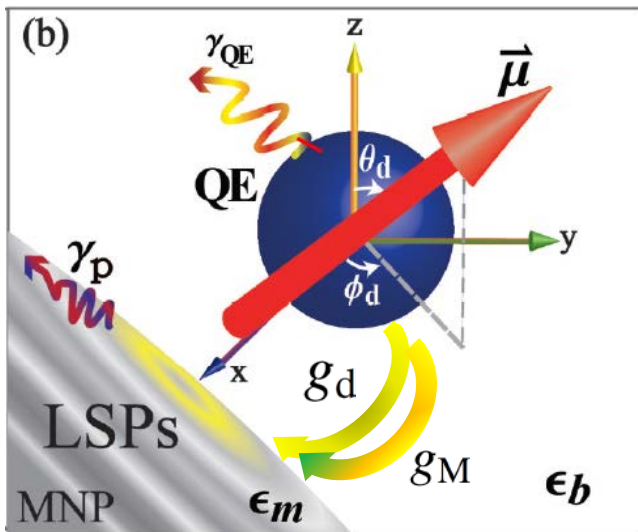
◆ QD couples to MNW

◆ Single-dot Coupled to MNP

◆ Double-dots Coupled to MNP

◆ Exceptional point

◆ Conclusion



$$\gamma_0 \sqrt{\epsilon_b} + \gamma_{QE}$$

**Thanks for
listening !!**

$$\vec{\mu}_1 = (\mu_{1r}, \mu_{1\theta}, \mu_{1\phi})$$

QE1

θ_{dl}

ϕ_{dl}

y

γ_P

x

h_1

SP

MNP

ϵ_m

ϵ_b

