Consequences of preserving reversibility in quantum superchannels

Young Researchers Forum on Quantum Information Science

August 28th, 2020

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arXiv:2003.05682

Introduction of quantum superchannels

Quantum channel

Quantum superchannel with one slot

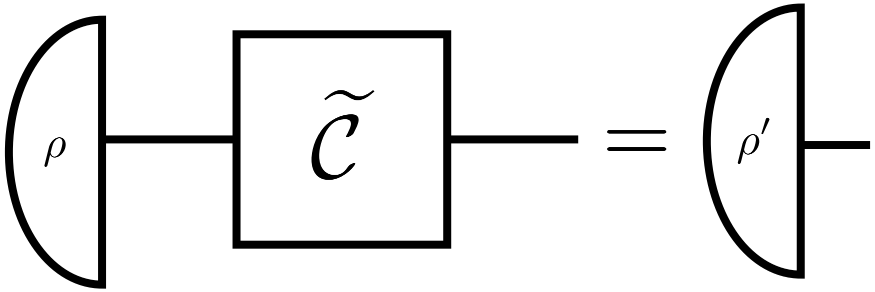
Quantum superchannel with two slots

Quantum switch

Pure superchannel

Quantum channel

- Deterministic linear transformation between **quantum states**
- CPTP map
- Quantum channel $\tilde{\mathcal{C}} : \mathcal{L}(\mathcal{H}_I) \to \mathcal{L}(\mathcal{H}_O)$
- Map representation $\tilde{\mathcal{C}}(\rho) = \rho'$
- Choi operator representation $\mathrm{C} * \rho = \rho'$ ($\mathrm{C} \coloneqq \Sigma_{ij} |i\rangle\langle j| \otimes \tilde{\mathcal{C}}(|i\rangle\langle j|)$)
- Circuit figure



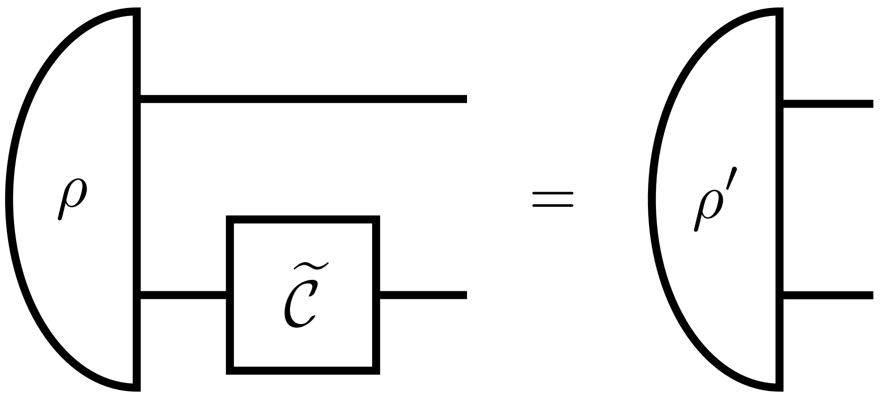
Quantum channel

- Note complete positivity of quantum channels

$$\left(\tilde{\mathcal{I}} \otimes \tilde{\mathcal{C}}\right)(\rho) = \rho'$$

($\tilde{\mathcal{I}}$: Identity map)

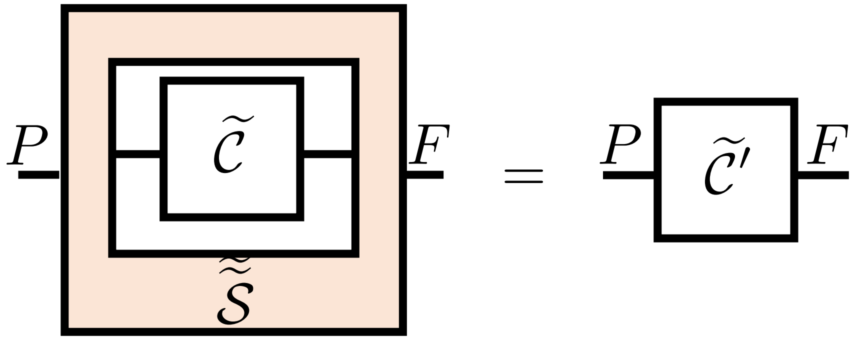
$$\mathrm{C} * \rho = \rho'$$

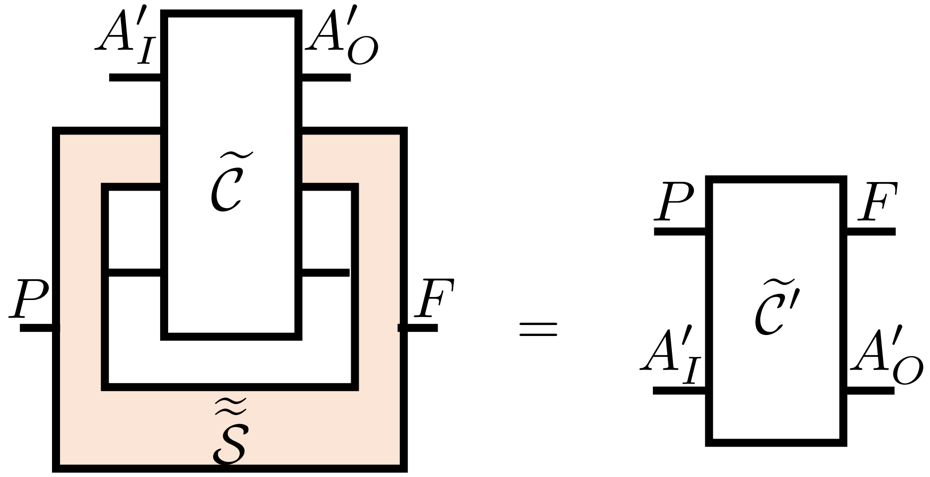


Quantum superchannel with one slot

- Deterministic linear transformation between **quantum channels**
- Quantum superchannel $\tilde{\tilde{S}} : [\mathcal{L}(A_I) \to \mathcal{L}(A_O)] \to [\mathcal{L}(P) \to \mathcal{L}(F)]$

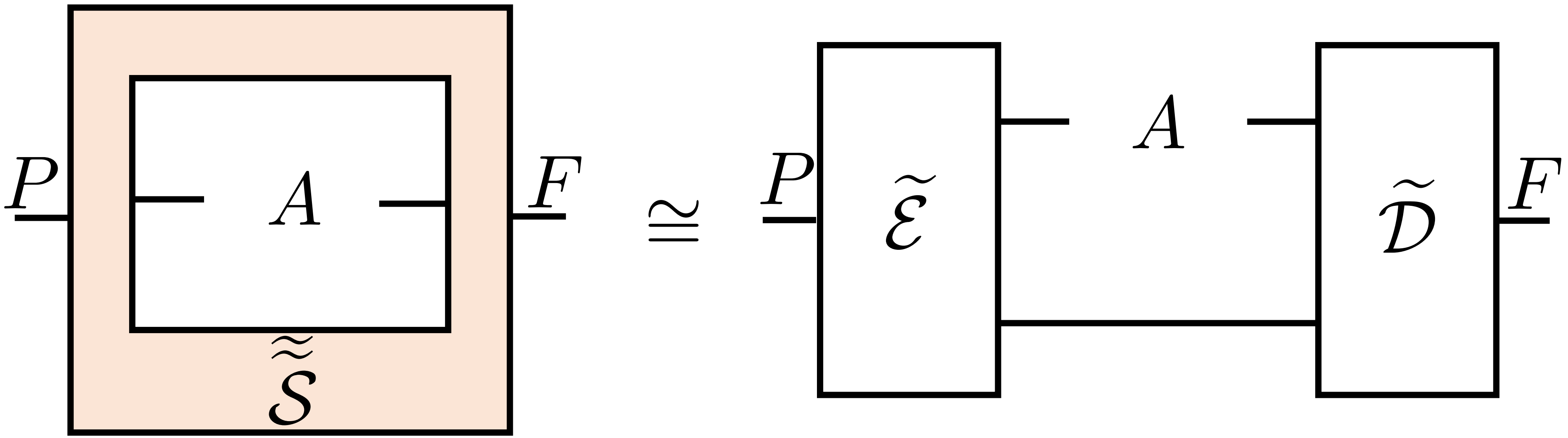
- Circuit-like figure





Quantum superchannel with one slot

- Can be realized in a quantum circuit



Quantum superchannel with two slots

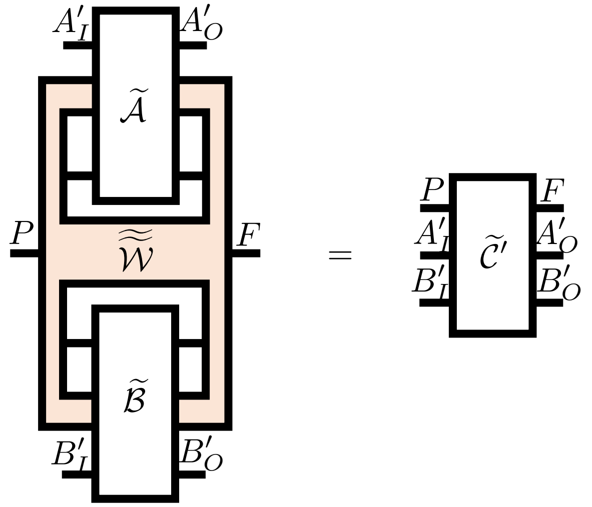
- Input: a product of two quantum channels

  → Output: a quantum channel

- Quantum superchannel

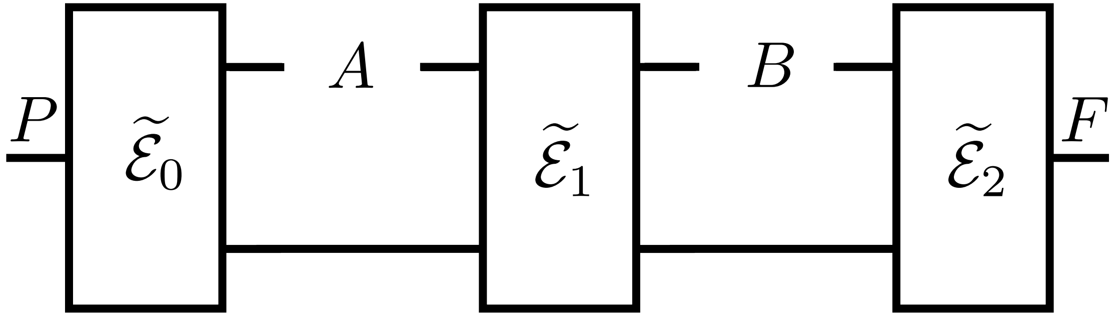
  $\widetilde{\widetilde{\mathcal{W}}} : [\mathcal{L}(A_I) \to \mathcal{L}(A_O)] \otimes [\mathcal{L}(B_I) \to \mathcal{L}(B_O)] \to [\mathcal{L}(P) \to \mathcal{L}(F)]$

- $\widetilde{\widetilde{\mathcal{W}}}\left(\tilde{\mathcal{A}} \otimes \widetilde{\mathcal{B}}\right) = \tilde{\mathcal{C}}'$
- $\mathrm{W} * (A \otimes B) = C'$



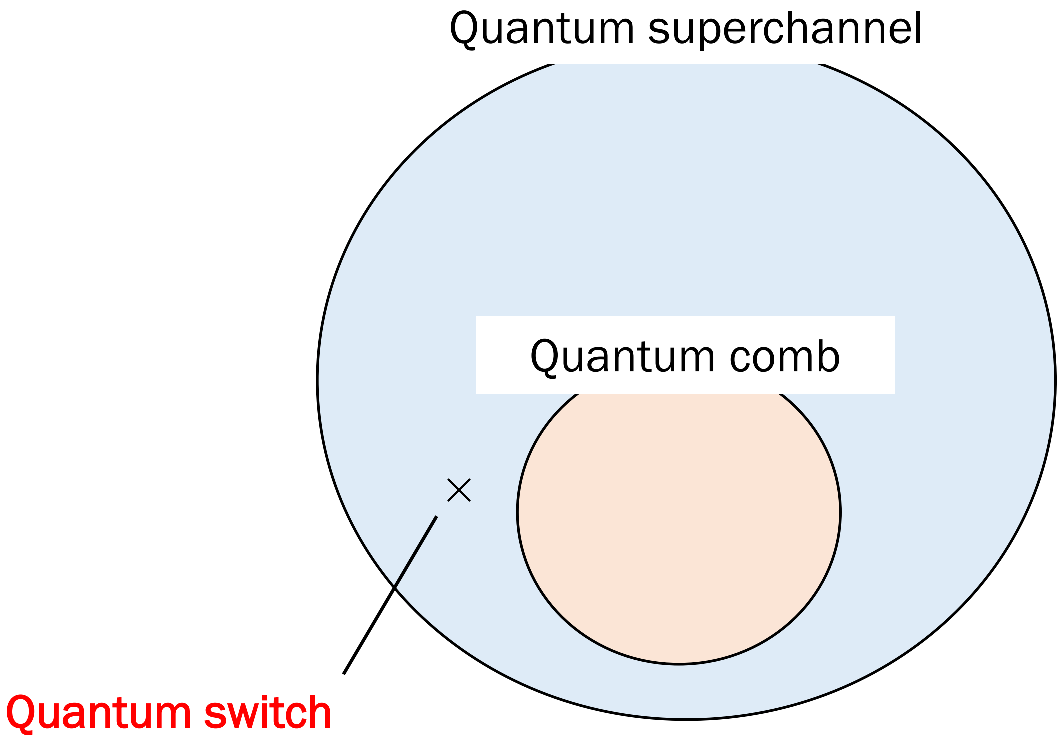
Quantum comb

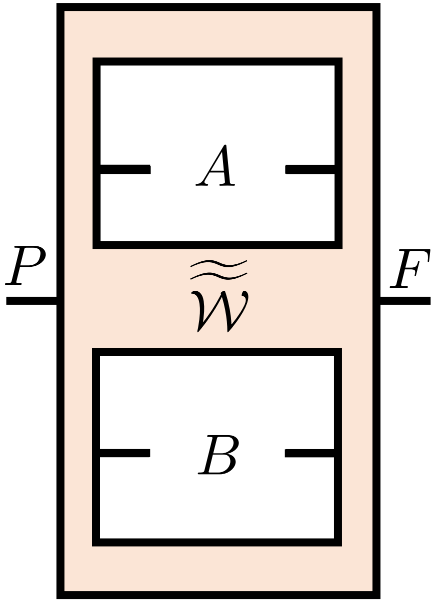
- Quantum superchannel with causal order $A \prec B$ between slots
- Quantum comb can be realized in a quantum circuit.



Quantum superchannel with two slots

- Question: $\widetilde{\widetilde{\widetilde{\mathcal{W}}}}$ can be realized in a quantum circuit?
- Answer: NOT generally





Quantum switch

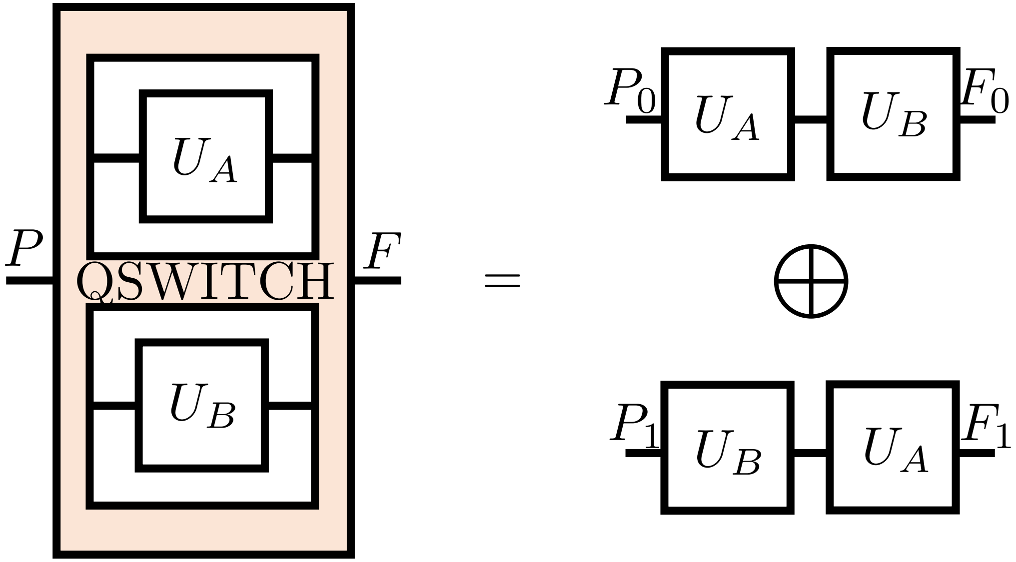
- Quantum switch $W_{\mathrm{QS}} \in \mathcal{L}(P \otimes A_I \otimes A_O \otimes B_I \otimes B_O \otimes F)$

- (Choi vector) $|U\rangle\rangle \coloneqq \sum_i |i\rangle \otimes U|i\rangle = (I \otimes U)|I\rangle\rangle$
- (Choi vector) $\mathcal{E}_U(\rho) = U\rho U^\dagger,\ E_U = |U\rangle\rangle\langle\langle U|$
- Action of quantum switch on unitary transformations
  $W_{\mathrm{QS}} * (|U_A\rangle\rangle\langle\langle U_A| \otimes |U_B\rangle\rangle\langle\langle U_B|) = |U_G(U_A, U_B)\rangle\rangle\langle\langle U_G(U_A, U_B)|$
  $\big(U_G(U_A, U_B)\big)\big((\alpha|0\rangle^{P_c} + \beta|1\rangle^{P_c}) \otimes |\psi\rangle^{P_t}\big)$
  $= \alpha|0\rangle^{F_c} U_B U_A |\psi\rangle^{F_t} + \beta|1\rangle^{F_c} U_A U_B |\psi\rangle^{F_t}$

Quantum switch

- Action of quantum switch on unitary transformations

$$U_G(U_A, U_B) = |0\rangle^{F_c}\langle 0|^{P_c} \otimes U_B^{\mathcal{H}\to F_t} U_A^{P_t\to\mathcal{H}} + |1\rangle^{F_c}\langle 1|^{P_c} \otimes U_A^{\mathcal{H}\to F_t} U_B^{P_t\to\mathcal{H}}$$
$$= U_B U_A \oplus U_A U_B$$



$P = P_c \otimes P_t = P_0 \oplus P_1$

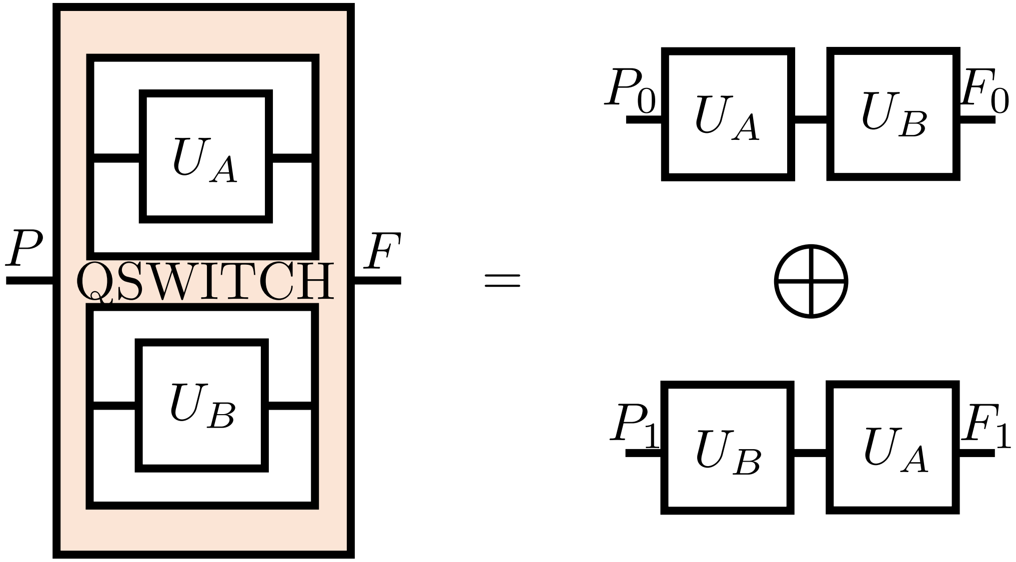
$F = F_c \otimes F_t = F_0 \oplus F_1$

$P_c, F_c \cong \mathbb{C}^2$: control system

$P_t, F_t \cong \mathbb{C}^d$: target system

Quantum switch

- Cannot be realized in a quantum circuit
- Superchannel in “indefinite causal order”



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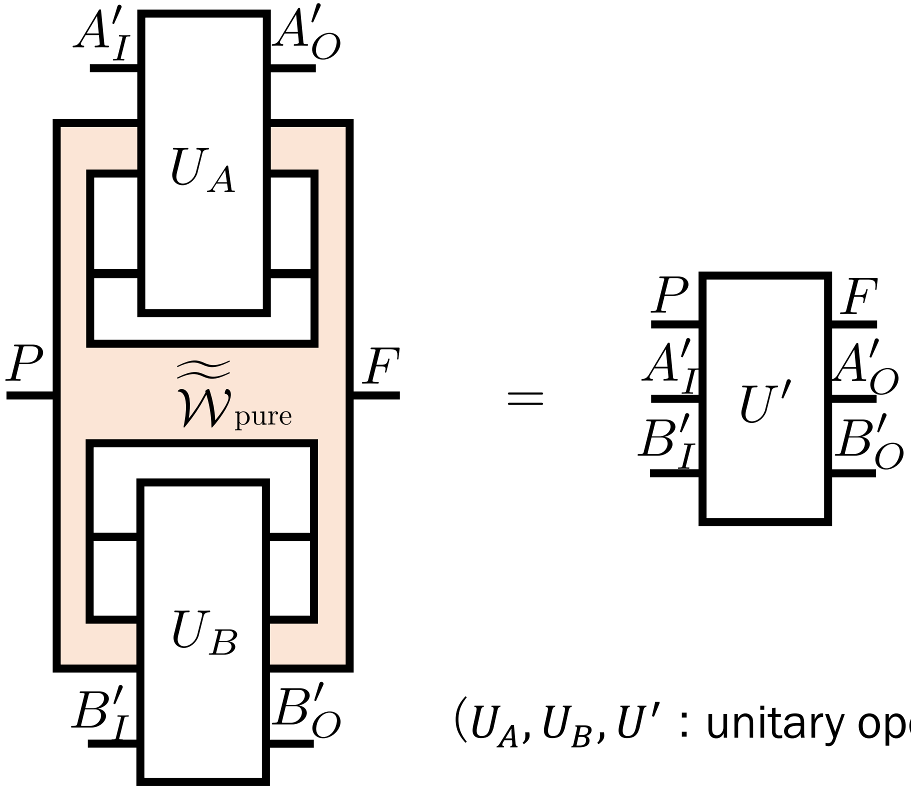
Pure superchannel

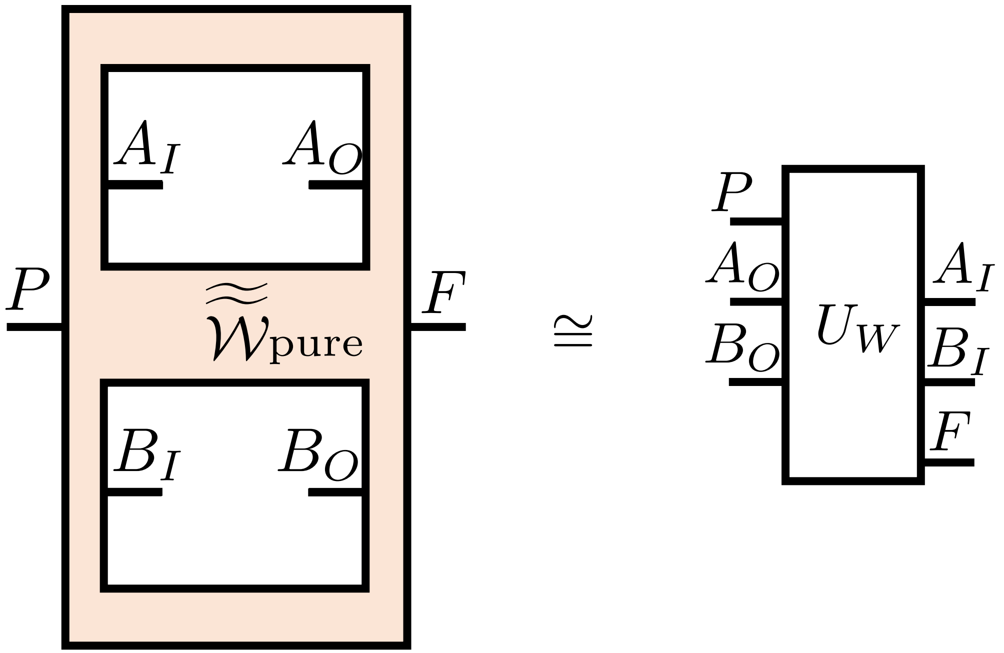
- *Pure* superchannel … transforms unitary channels into unitary channels



(U_A, U_B, U' : unitary operators)

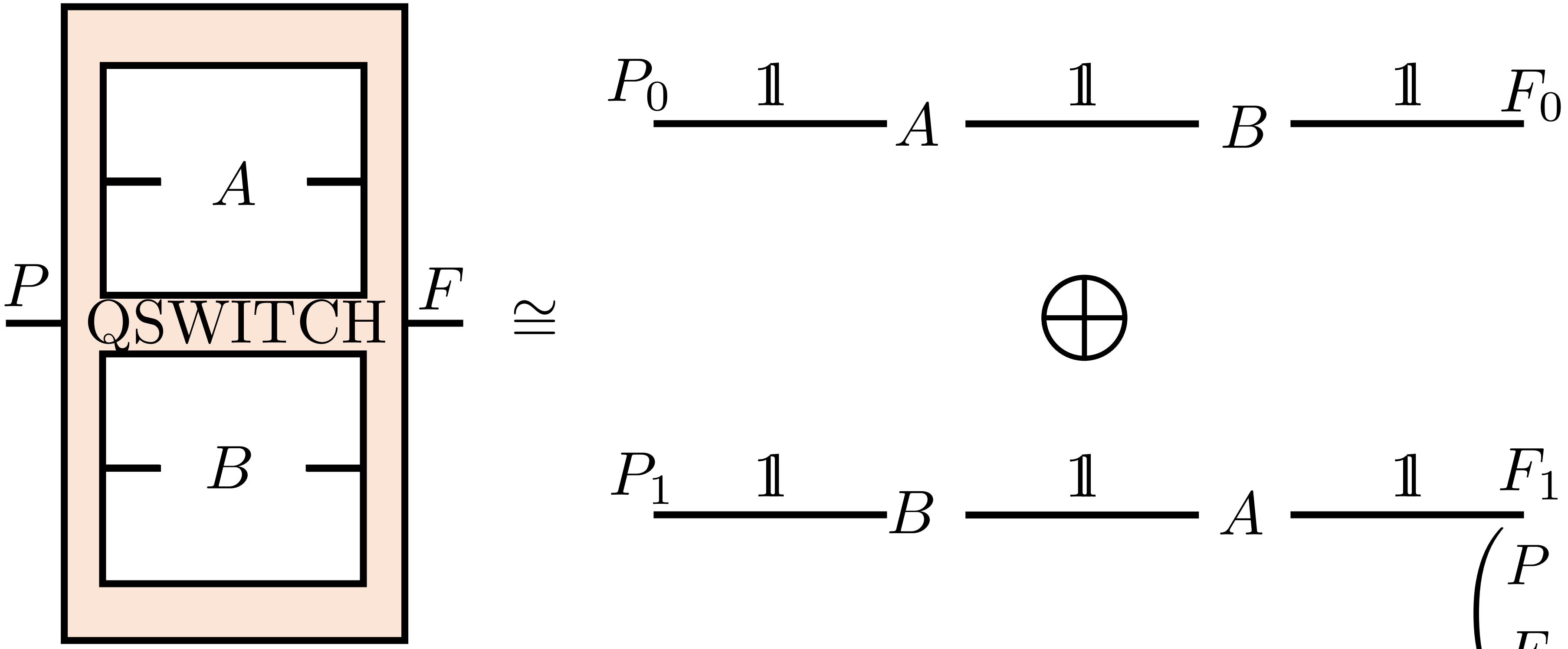
Pure superchannel

- **Theorem.**
  A superchannel W is pure iff $W = |U_W\rangle\rangle\langle\langle U_W|$
  $(U_W : P \otimes A_O \otimes B_O \to A_I \otimes B_I \otimes F, \text{unitary})$



Pure superchannel

- Quantum switch is a pure superchannel
- A direct sum of circuits with different causal orders



$$\begin{pmatrix} P = P_0 \oplus P_1 \\ F = F_0 \oplus F_1 \end{pmatrix}$$

Main results

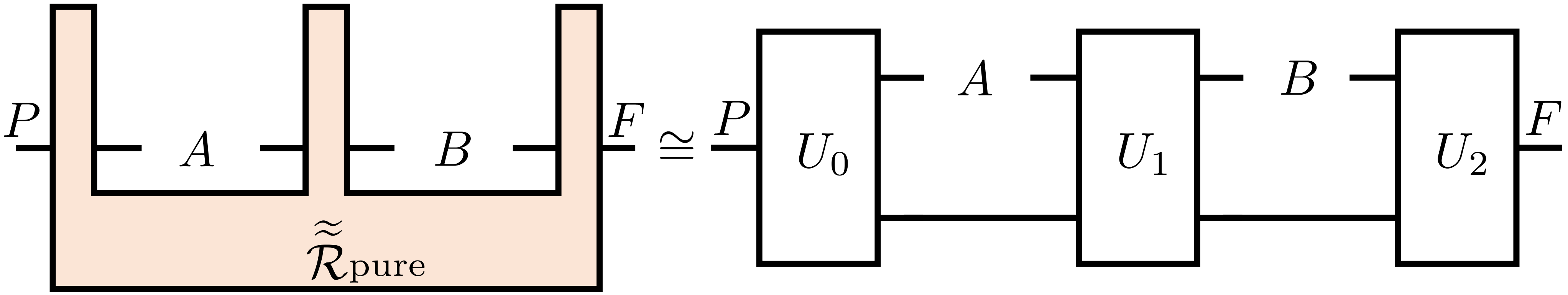
Decomposition of pure comb with N slots

Decomposition of pure superchannel with two slots

Decomposition of pure comb

- **Theorem 1.**

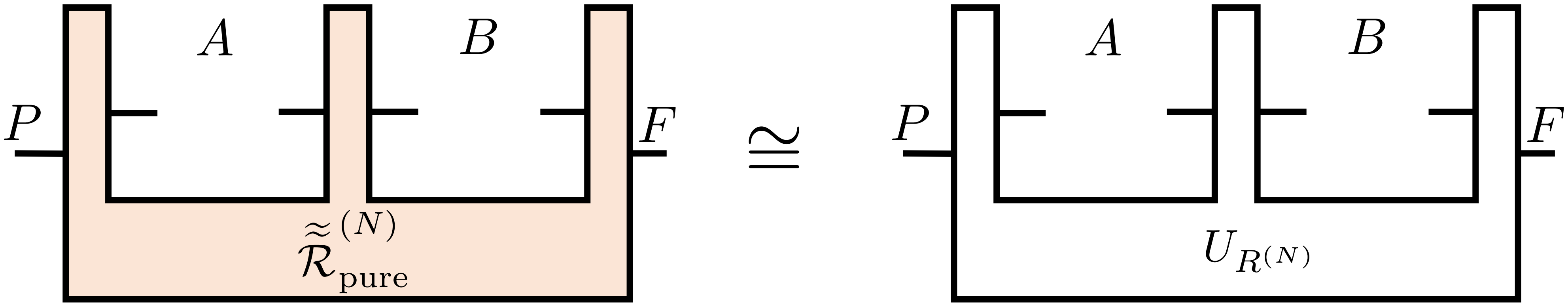
Pure combs with $\boldsymbol{N}$ **slots** can be decomposed into sequences of unitary channels



$(U_0, U_1, U_2$: unitary operators$)$

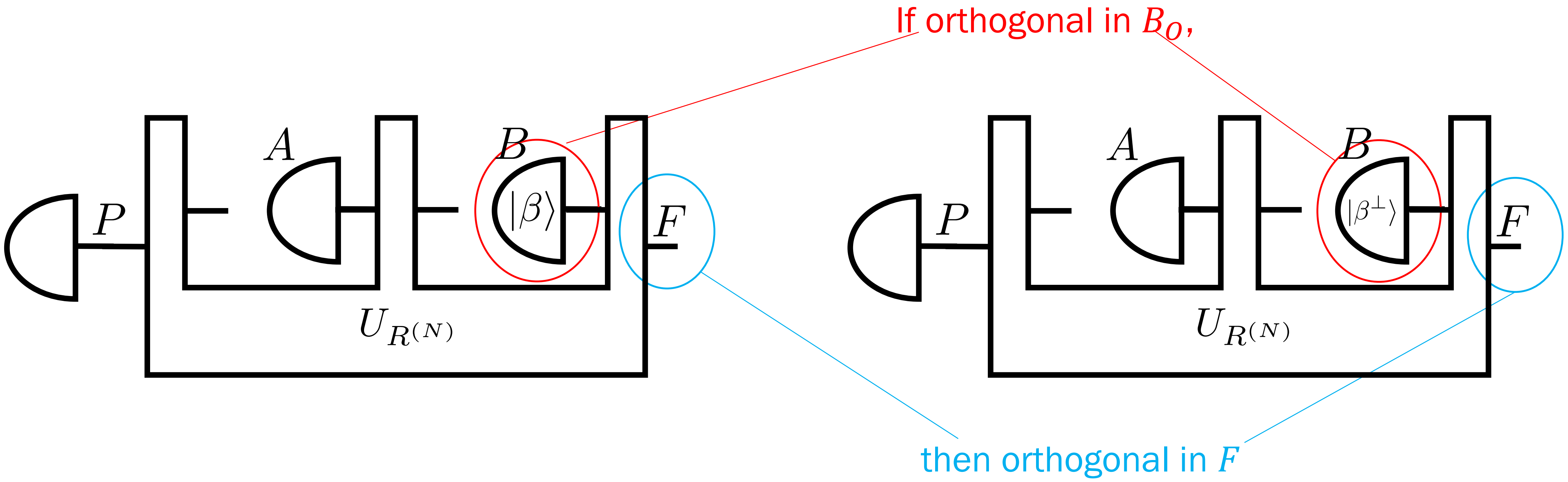
Sketch of the proof of Theorem 1

- Focus on the unitary operators $U_{R^{(N)}}$ representing the pure combs $\tilde{\tilde{\mathcal{R}}}^{(N)}_{\text{pure}}$



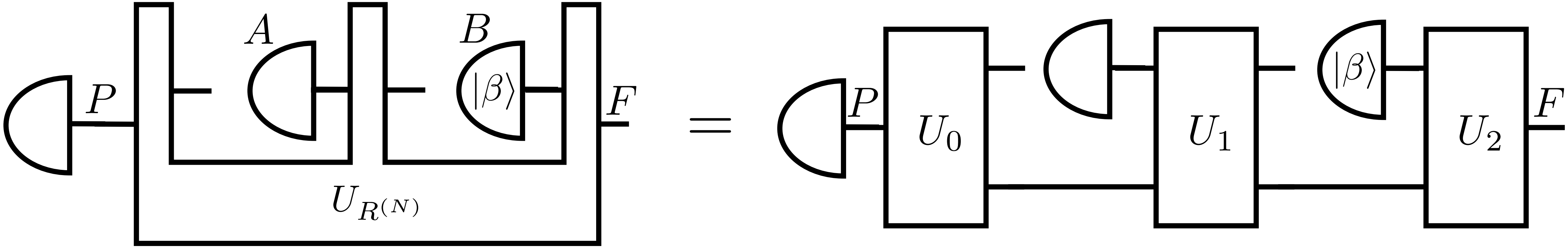
Sketch of the proof of Theorem 1

- Imply properties of $U_{R^{(N)}}$



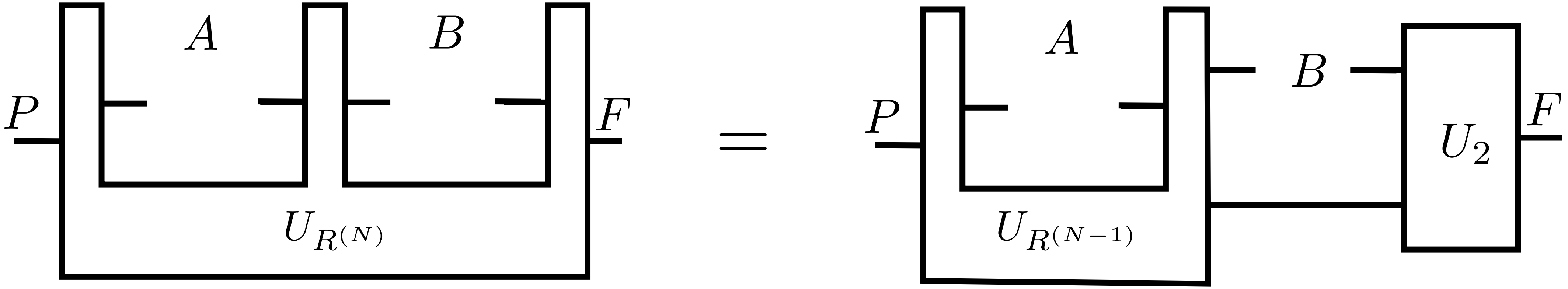
Sketch of the proof of Theorem 1

- Imply properties of $U_{R(N)}$
- If we know the decomposition, this property is trivial



Sketch of the proof of Theorem 1

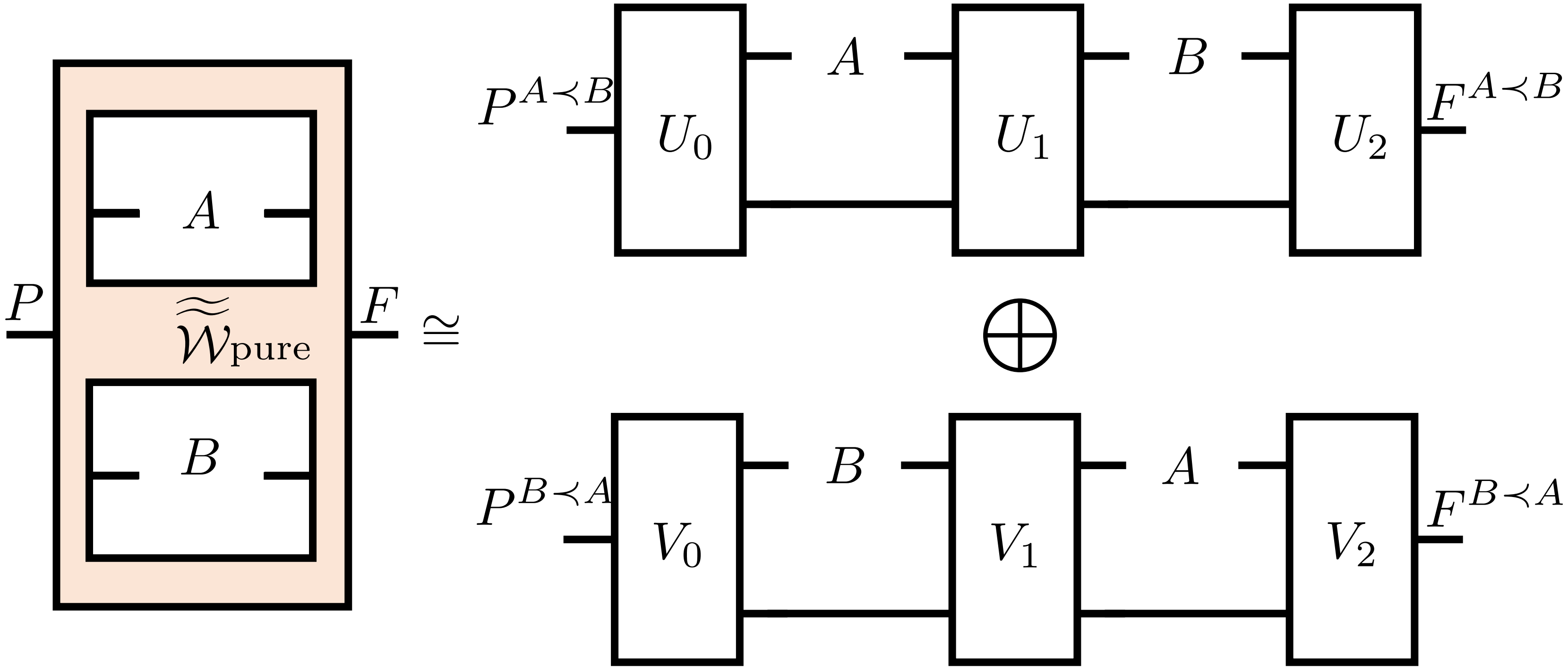
- Decompose $U_{R(N)}$ using the properties inductively



Decomposition of pure superchannel with two slots

- **Theorem 2.**

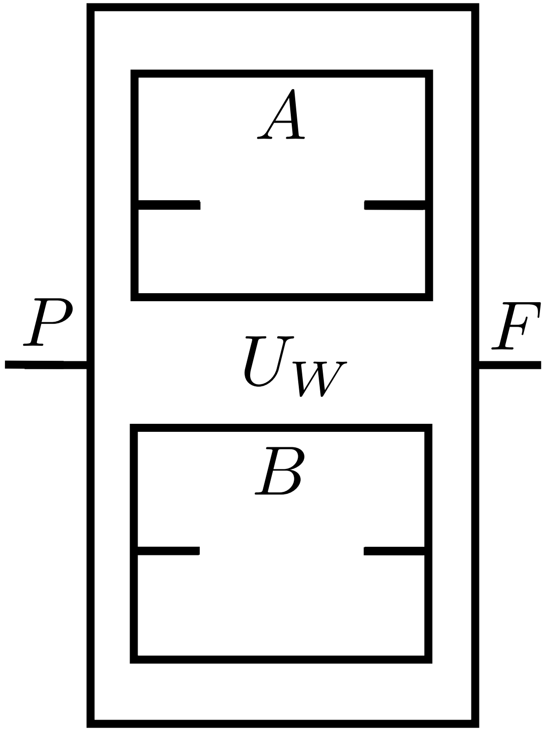
Pure superchannels with **two slots** can be decomposed into direct sum of pure combs, which is a generalized form of quantum switch



Sketch of the proof of Theorem 2

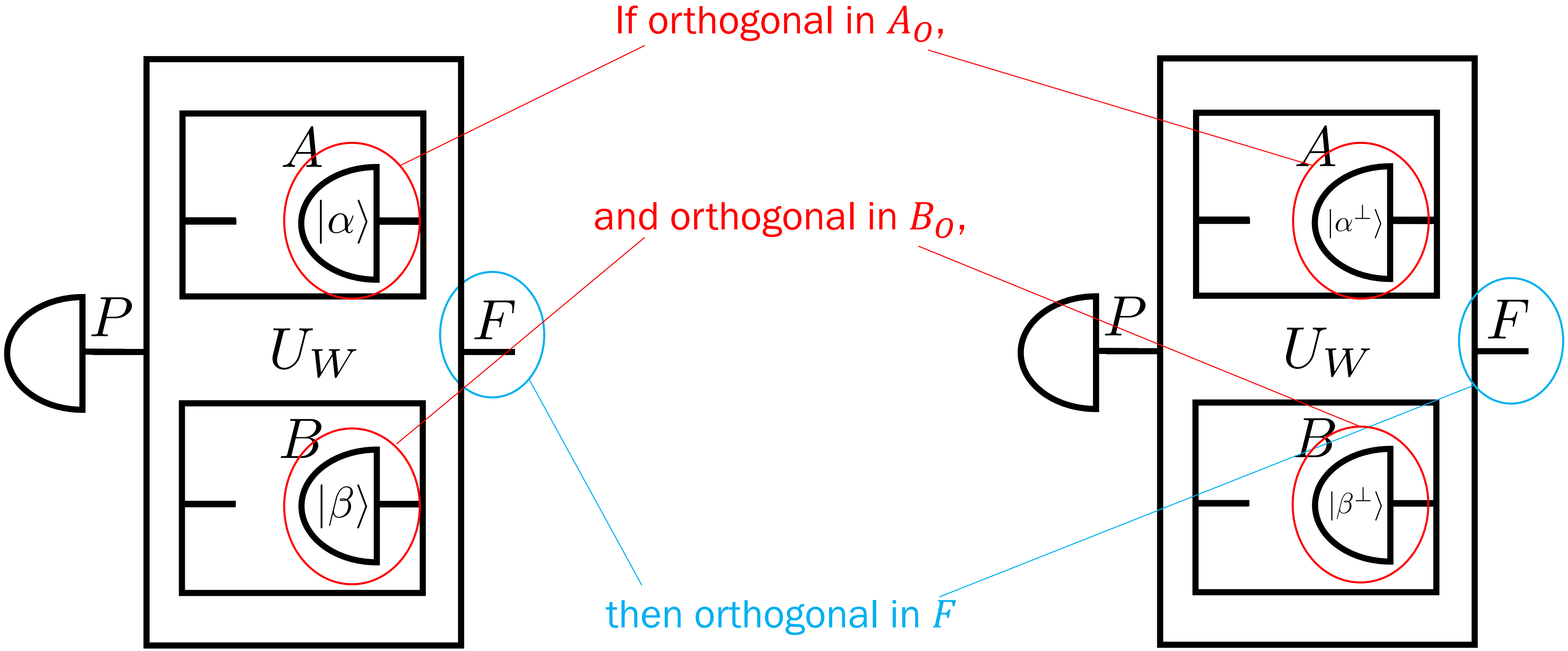
(The flow is similar as that of Theorem 1)

- Focus on the unitary operators U_W representing the pure superchannels $\tilde{\tilde{\tilde{\mathcal{W}}}}_{\mathrm{pure}}$
- Imply **three properties** of U_W
- Decompose U_W using the properties



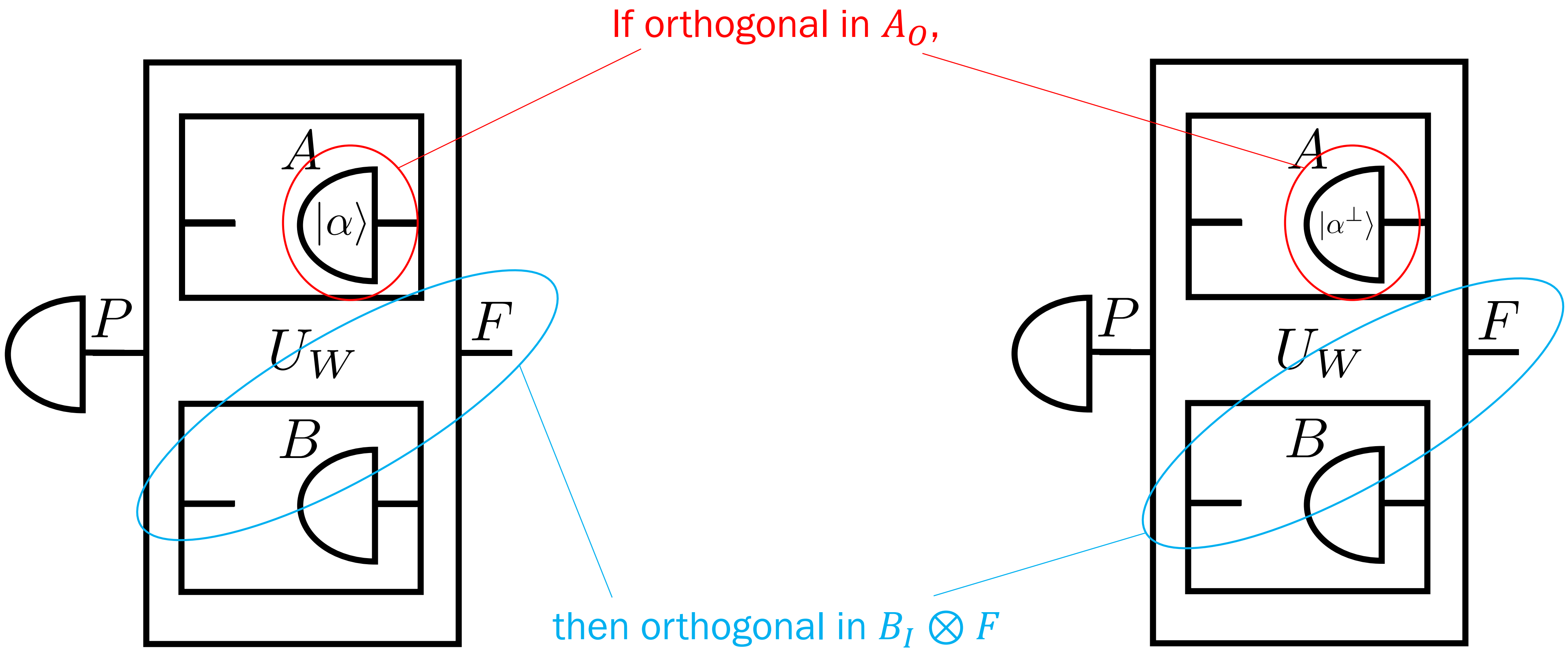
Sketch of the proof of Theorem 2

Property 1



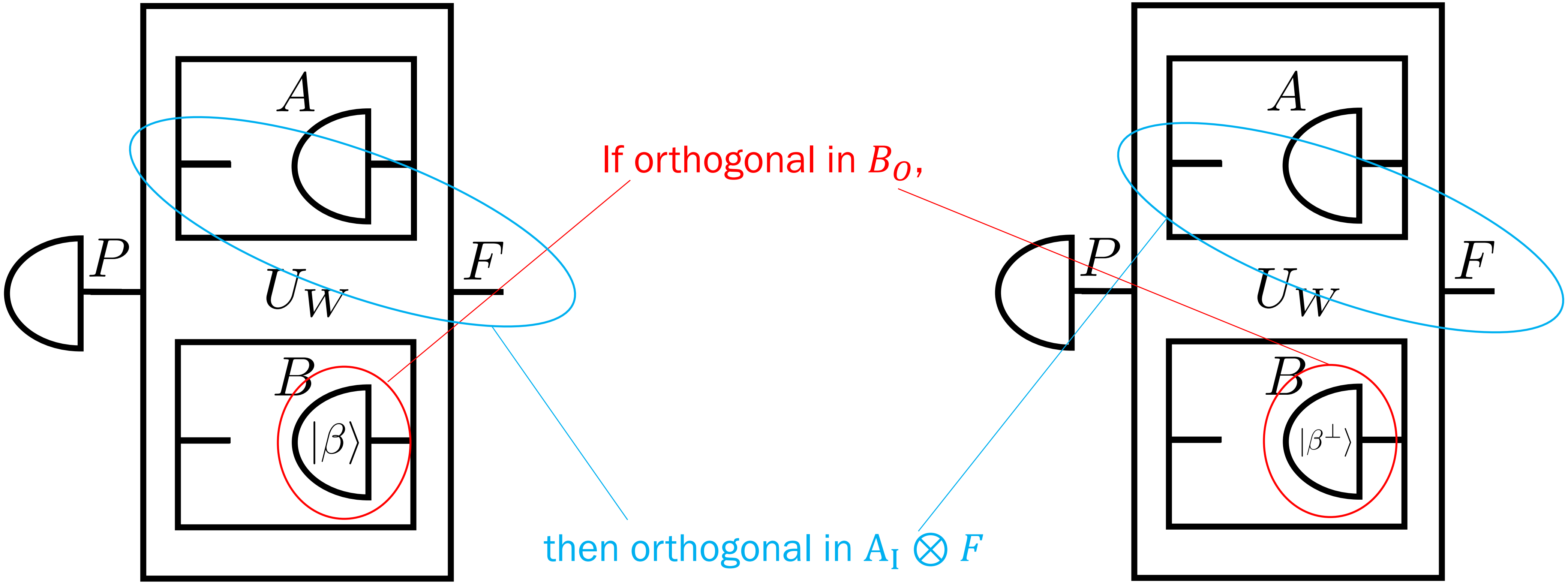
Sketch of the proof of Theorem 2

Property 2



Sketch of the proof of Theorem 2

Property 3



Open questions

- What about pure superchannels with more than two slots?

There are pure superchannels with three slots which are not in the form of direct sum of pure combs.

Conclusion

Conclusion

- Pure superchannels are quantum superchannels which preserve reversibility
- We decompose pure combs with N slots into sequences of unitary channels
- We decompose pure superchannels with two slots into direct sums of pure combs
- It is an open question what form pure superchannels with more than two slots are decomposed

Main references

- https://arxiv.org/abs/0804.0180 quantum supermap
- https://arxiv.org/abs/0712.1325 quantum comb
- https://arxiv.org/abs/0912.0195 quantum switch
- https://arxiv.org/abs/1109.5154 supermap in indefinite causal order
- https://arxiv.org/abs/1105.4464 quantum process, causal inequality
- https://arxiv.org/abs/1506.03776 witness of causal nonseparablity
- https://arxiv.org/abs/1506.05449 causal inequality
- https://arxiv.org/abs/1611.08535 pure superchannel
- https://arxiv.org/abs/1801.07594 realization of quantum switch
- https://arxiv.org/abs/2003.05682 pure superchannel decomposition