# DETECTION **LOOPHOLE**

in

# MEASUREMENT DEVICE-INDEPENDENT **ENTANGLEMENT WITNESS**

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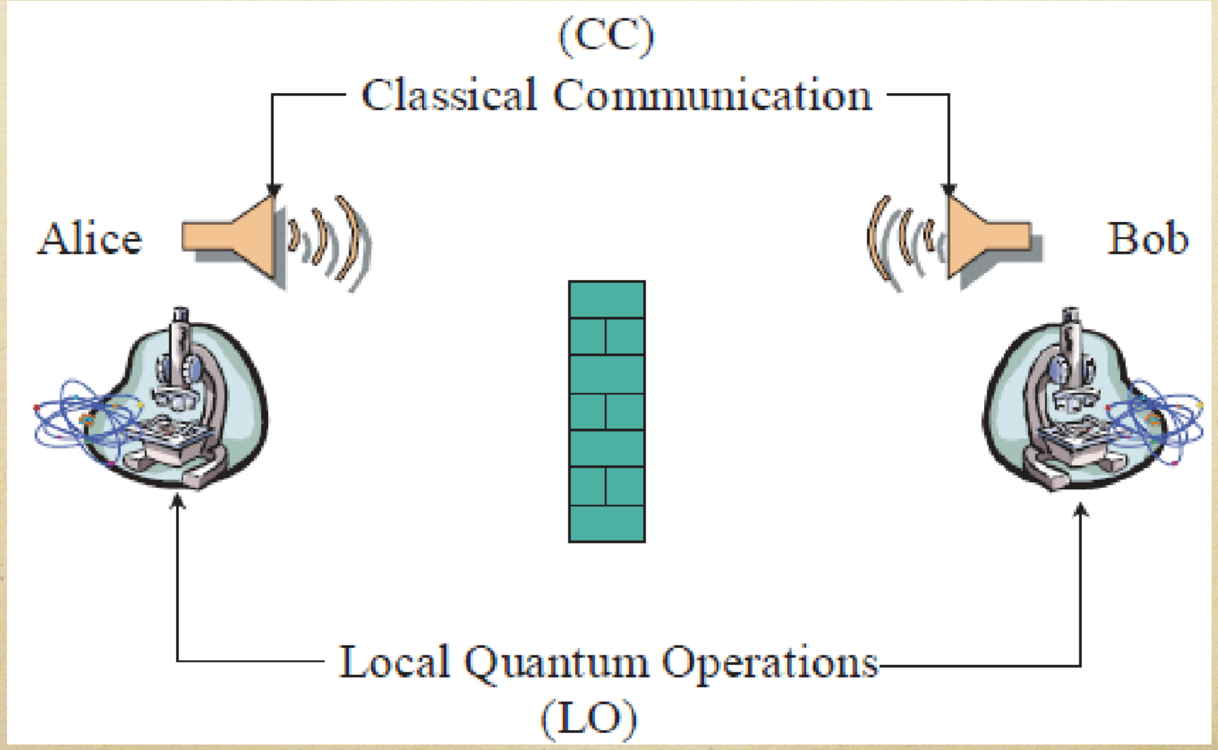
# Entanglement

Definition : $|\Psi_{AB}\rangle \neq |\psi\rangle \otimes |\phi\rangle$ (Pure state)

$$\rho_{AB} \neq \sum_i p_i \rho_i \otimes \sigma_i$$ (Mixed state)

Two spatially separated observers can't prepare entangled state with local operations even with the help of classical communication (LOCC).



# Entanglement Theory

Generation

Characterisation

**Detection**

Quantification

Manipulation

# Entanglement Theory

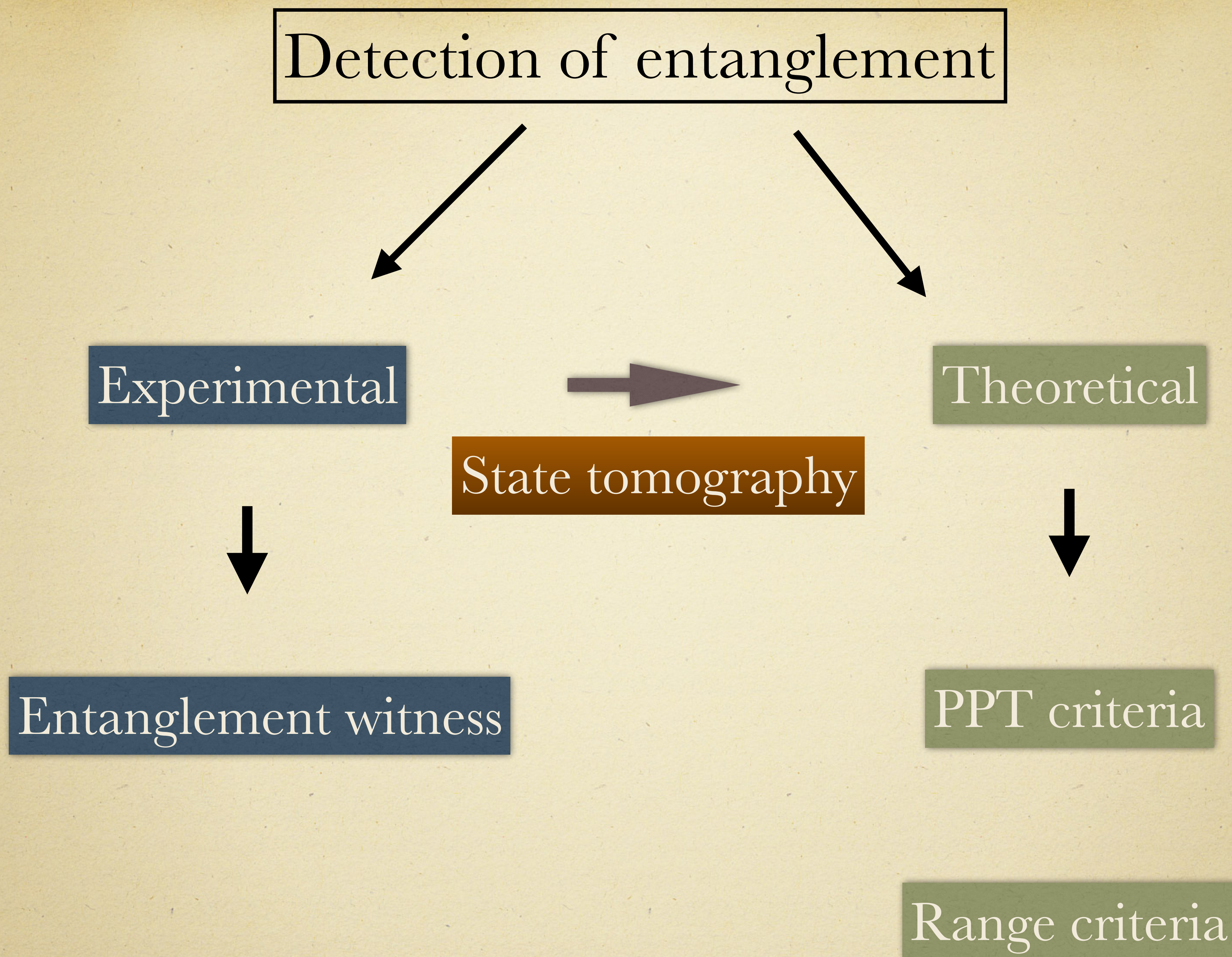
Generation

Characterisation

Detection + Loophole

Quantification

Manipulation


Detection of entanglement
Experimental
Theoretical
State tomography
Entanglement witness
PPT criteria
Range criteria

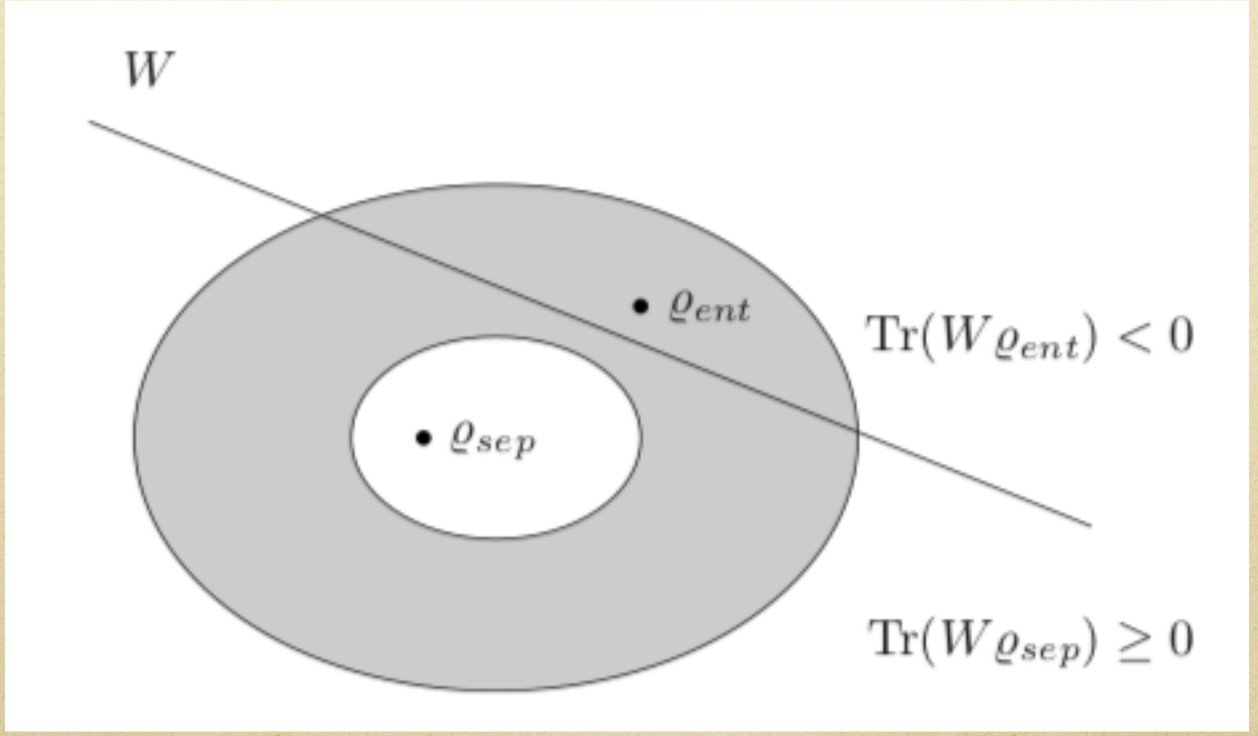
# Entanglement Witness

Definition:

$$\exists \text{ at least one } \rho \notin \mathcal{S}, \text{ s.t. } \operatorname{Tr}(W\rho) < 0$$
$$\text{while } \forall \rho_s \in \mathcal{S}, \operatorname{Tr}(W\rho_s) \geq 0$$

Local decomposition:

$$W = \sum_{i=1}^{k} c_i |e_i\rangle\langle e_i| \otimes |f_i\rangle\langle f_i|.$$



# Maximally entangled state mixed with noise

## Experimental detection of entanglement via witness operators and local measurements

O. Gühne , P. Hyllus , D. Bruss , A. Ekert , M. Lewenstein , C. Macchiavello & A. Sanpera

$$\rho = p|\psi^+\rangle\langle\psi^+| + (1-p)\sigma,$$

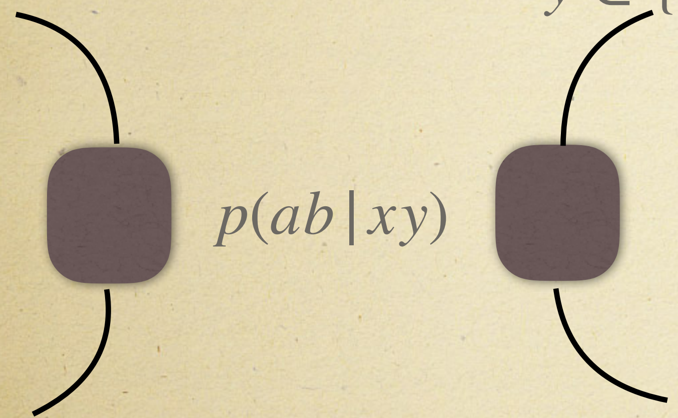
$$||\sigma - \tfrac{1}{4}\mathbb{I}\otimes\mathbb{I}|| \leq d,$$

$$W_0 = |\phi^+\rangle\langle\phi^+|^{T_A} = \frac{1}{4}(\mathbb{I}\otimes\mathbb{I}+\sigma_z\otimes\sigma_z-\sigma_x\otimes\sigma_x-\sigma_y\otimes\sigma_y).$$

# Hierarchy of correlations

## Bell test

$x \in \{0,1\}$ $y \in \{0,1\}$



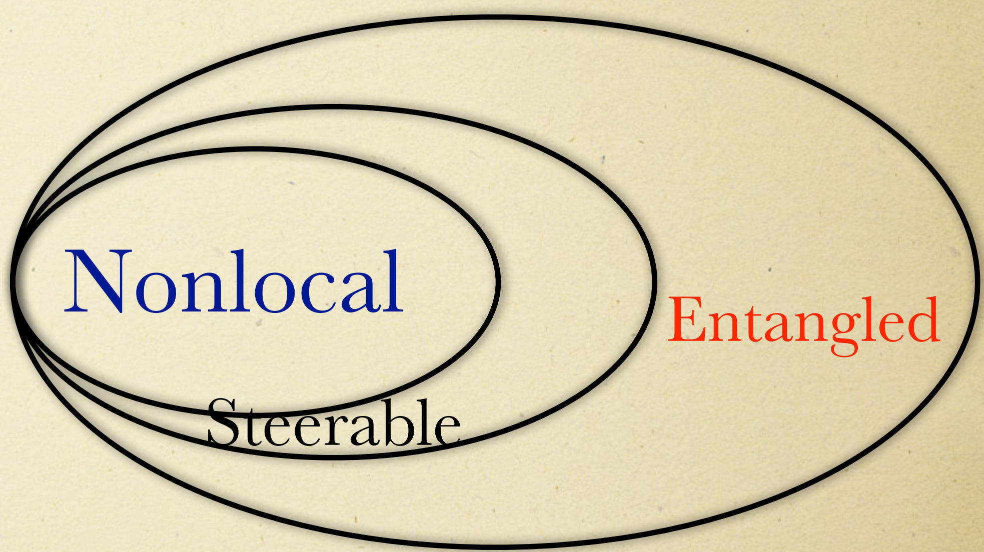
$p(ab|xy)$

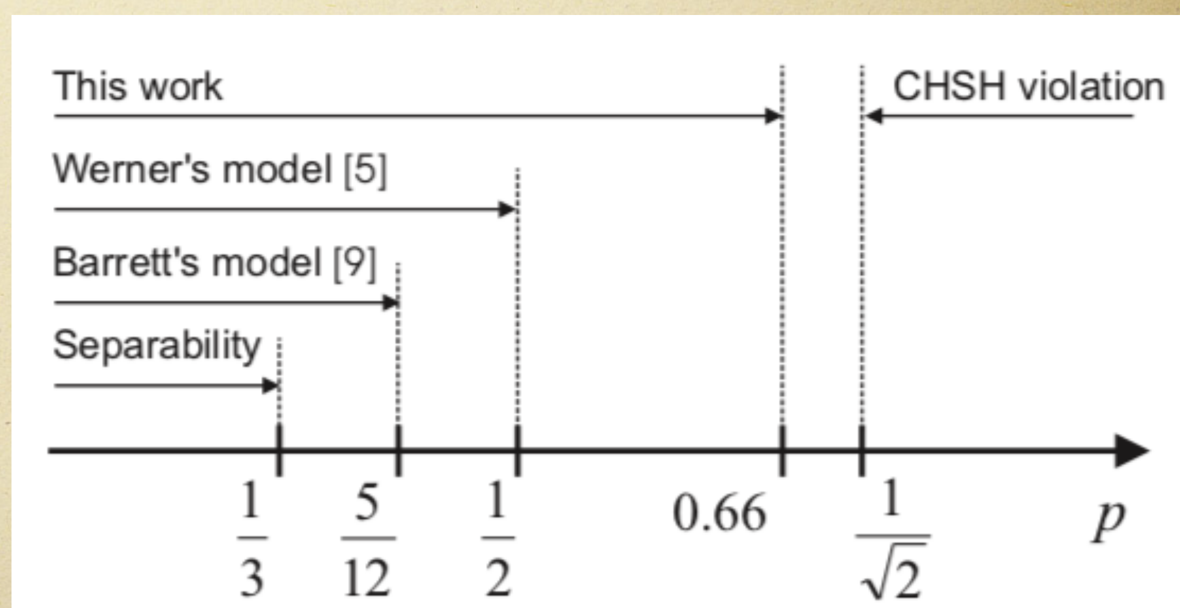
$a \in \{-1,1\}$ $b \in \{-1,1\}$

$$< x, y > = \sum_{ab} ab\ p(ab|xy)$$

$$< x, y > + < x', y > + < x, y' > - < x', y' > \leq 2$$

$$p|\psi^+\rangle\langle\psi^+| + (1-p)I/4$$



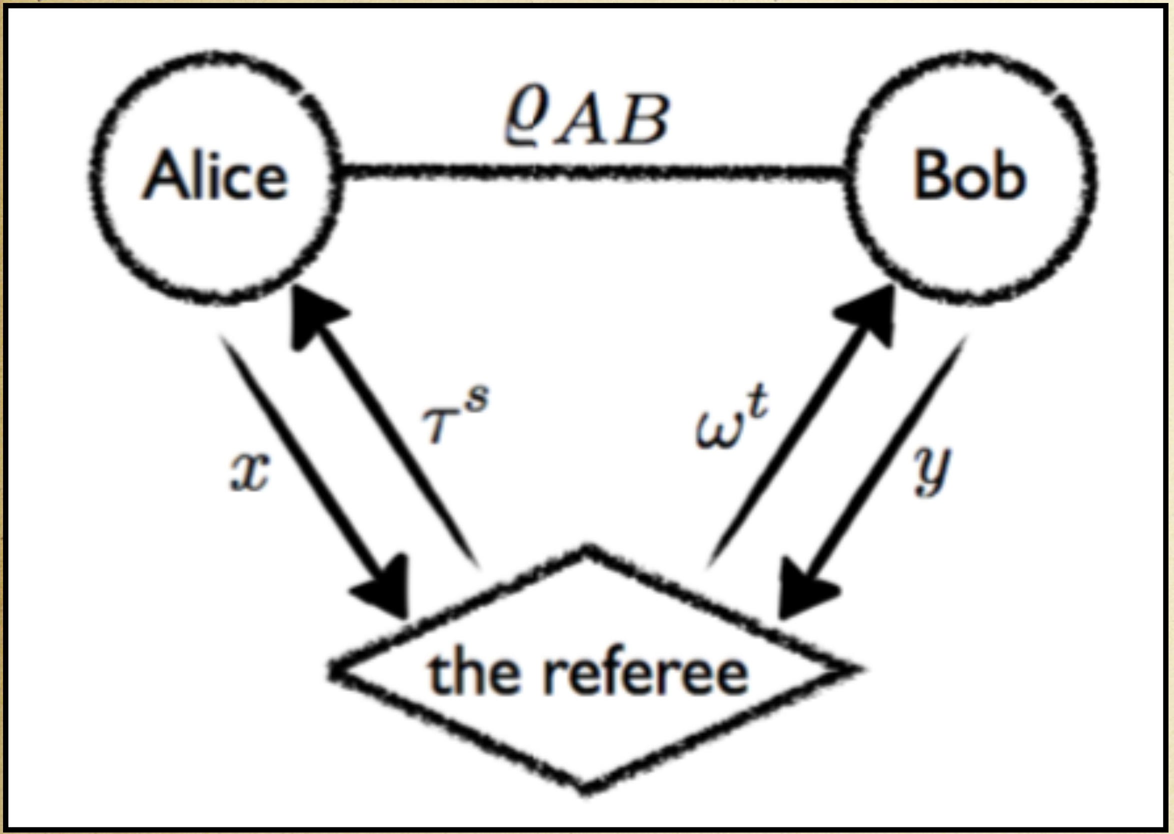



# Witnessing Entanglement in measurement device independent scenario

Selected for a Viewpoint in *Physics*
PRL **108,** 200401 (2012) PHYSICAL REVIEW LETTERS week ending 18 MAY 2012

**All Entangled Quantum States Are Nonlocal**

Francesco Buscemi*




$$\wp^*(\varrho_{AB};\mathsf{G}_{\rm nl}) := \max \sum_{s,t,x,y} p(s)q(t)\wp(s,t,x,y)\mu(x,y|s,t)$$

where $\mu(x,y|s,t)$ is the joint conditional probability distribution computed as

$$\mathrm{Tr}\left[(P^x_{A_0A}\otimes Q^y_{BB_0})(\pi^s_{A_0}\otimes\varrho_{AB}\otimes\pi^t_{B_0})\right],$$

**Corollary 1.** *In any semi-quantum nonlocal game* $\mathsf{G}_{\rm sq}$*, all separable quantum states yield exactly the same payoff* $\wp_{\rm sep}(\mathsf{G}_{\rm sq})$*. Moreover, a quantum state* $\varrho_{AB}$ *is entangled if and only if there exists a semi-quantum nonlocal game* $\mathsf{G}_{\rm sq}$*, for which* $\wp^*(\varrho_{AB};\mathsf{G}_{\rm sq}) > \wp_{\rm sep}(\mathsf{G}_{\rm sq})$.

# Measurement device independent witness from standard Entanglement witness

PRL **110,** 060405 (2013) PHYSICAL REVIEW LETTERS week ending 8 FEBRUARY 2013

**Measurement-Device-Independent Entanglement Witnesses for All Entangled Quantum States**

Cyril Branciard,[1] Denis Rosset,[2] Yeong-Cherng Liang,[2] and Nicolas Gisin[2]

$$W = \sum_{s,t} \beta_{s,t}\, \tau_s^{\top} \otimes \omega_t^{\top}$$

$$I(P) = \sum_{s,t} \beta_{s,t}\, P(1,1|\tau_s,\omega_t)$$

$$P_\rho(1,1|\tau_s,\omega_t) = \mathrm{tr}\big[(|\Phi^+_{AA}\rangle\langle\Phi^+_{AA}| \otimes |\Phi^+_{BB}\rangle\langle\Phi^+_{BB}|) \cdot (\tau_s \otimes \rho_{AB} \otimes \omega_t)\big]$$

$$I(\rho^{w}_{AB}) = \frac{5}{8}\sum_{s=t} P(1,1|\tau_s,\omega_t) - \frac{1}{8}\sum_{s\neq t} P(1,1|\tau_s,\omega_t),$$

where $s$, $t$ takes values 0, 1, 2, and 3, and

$$\tau_s = \sigma_s \frac{\mathbb{I}_2 + \vec{\sigma}\cdot\vec{n}}{2}\sigma_s, \quad \omega_t = \sigma_t \frac{\mathbb{I}_2 + \vec{\sigma}\cdot\vec{n}}{2}\sigma_t,$$

with $\sigma_0 = \mathbb{I}_2$, $\vec{\sigma} = (\sigma_1, \sigma_2, \sigma_3)$ are the usual Pauli matrices, $\vec{n} = \frac{1}{\sqrt{3}}(1,1,1)$.

# Detection loophole

Due to imperfection in local detectors, a separable state may show entangled correlation.

Means non-unity detector efficiency, e.g., loss in detection events.

Goal is to ensure negative measured expectation value of witness is really due to underlying entangled state.

For two qubit states, the detection loophole for witnesses can already be closed with a detection efficiency greater than 2/3.

The purpose of MDI-EW is to ensure entanglement of a shared quantum state, independent of what measurements are being performed.

We investigate whether this type of witness also guarantees entanglement independent of **inefficient detectors.**

Consider 3 cases: lossy events,
dark counts
when both are present

Any EW can be decomposed as

$$W = c_0 \mathbb{I} + \sum_\alpha c_\alpha S_\alpha$$

Lost event efficiency : $\eta_- = \dfrac{\tilde{N}}{\tilde{N} + \epsilon_-}$

$$\langle S_\alpha \rangle_m = \frac{\sum_i (\tilde{n} - \bar{\epsilon}_-)\lambda_i}{\tilde{N} - \epsilon_-}$$

$$\langle W \rangle_m = C_0 + \sum_\alpha C_\alpha \frac{\langle S_\alpha \rangle_t}{\eta_-} = C_0\left(1 - \frac{1}{\eta_-}\right) + \frac{\langle W \rangle_t}{\eta_-}$$

W will detect entanglement when $\langle W \rangle_t < 0$

$$\langle W \rangle_m < C_0\left(1 - \frac{1}{\eta_-}\right)$$

$C_0\left(1 - \frac{1}{\eta_-}\right) \leq \langle W \rangle_m < 0$ is not sufficient to detect entanglement

For Werner state : $\langle W_{\rho_p} \rangle_m < \frac{1}{4}\left(1 - \frac{1}{\eta_-}\right)$

For noisy GHZ state : $\langle W_{\rho_q^{GHZ}} \rangle_m < \frac{3}{8}\left(1 - \frac{1}{\eta_-}\right)$

ideal. Hence, for example, if the lost event efficiency is $\eta_- = \frac{1}{2}$, then the witness operator, $W_{\rho_p}$, can detect an entangled Werner state if $\langle W_{\rho_p} \rangle_m < -\frac{1}{4}$ is satisfied, and $W_{\rho_q^{GHZ}}$ can detect an entangled noisy GHZ state if $\langle W_{\rho_q^{GHZ}} \rangle_m < -\frac{3}{8}$ is satisfied.

Additional event efficiency : $\eta_+ = \dfrac{\tilde{N}}{\tilde{N} + \epsilon_+}$

No separable state can be identified as entangled.

General case : $\langle W \rangle_m < C_0 \left(1 - \dfrac{1}{\eta_- + \frac{1}{\eta_+} - 1}\right)$

$$I(\rho_{AB}) = \sum_{r,s} \beta_{rs} P(1,1|\tau_r,\omega_s).$$

$$I(\rho_p) = \frac{5}{8}\sum_{s=t} P(1,1|\tau_r,\omega_s) - \frac{1}{8}\sum_{s\neq t} P(1,1|\tau_r,\omega_s).$$

Case I : In case of loss

$$I_m(\rho_{AB}) < \frac{\mathrm{tr}(W)}{4}\left(1 - \frac{1}{\Xi_{\eta_-}}\right).$$

$$\sum_{r,s} \beta_{rs} = \mathrm{tr}(W).$$

$$\frac{\mathrm{tr}(W)}{4}\left(1 - \frac{1}{\Xi_{\eta_-}}\right) \leq I_m(\rho'_{AB}) < 0,$$

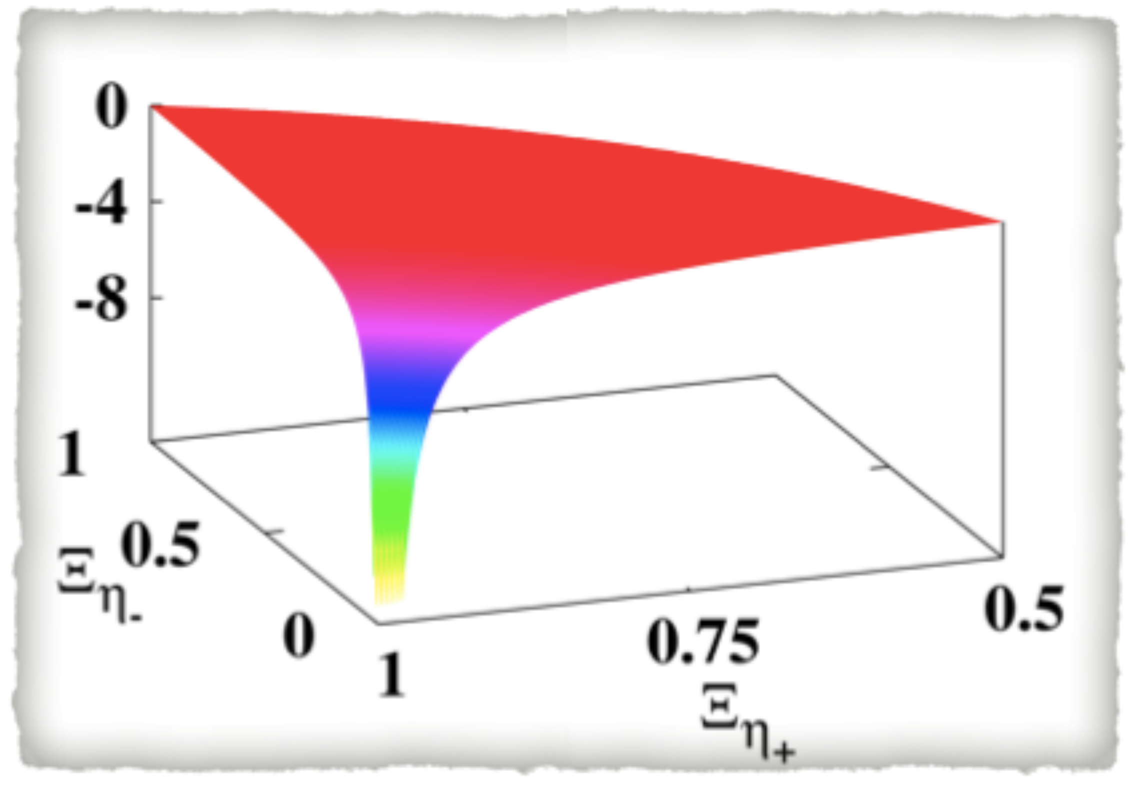
Erroneous conclusion

Case II : In case of dark count, no wrong detection

# Case III : In case of dark count and lost events

$$I_m(\rho_{AB}) < \frac{\mathrm{tr}(W)}{4}\left(1 - \frac{1}{\Xi_{\eta_-} + \frac{1}{\Xi_{\eta_+}} - 1}\right).$$

MDI-EW detect entanglement without any loophole when $\Xi_{\eta_-} + \frac{1}{\Xi_{\eta_+}} = 2.$



# Conclusion

Imperfections in detection devices lead to erroneous Conclusion about the underlying state.

MDI-EW does not show a separable state as entangled When measurements are wrong.

Conditions for detection loophole free test of entanglement via MDI-EW.


THANK YOU!