QUANTUM SUPERPOSITION OF CAUSAL ORDERS

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The University of Hong Kong

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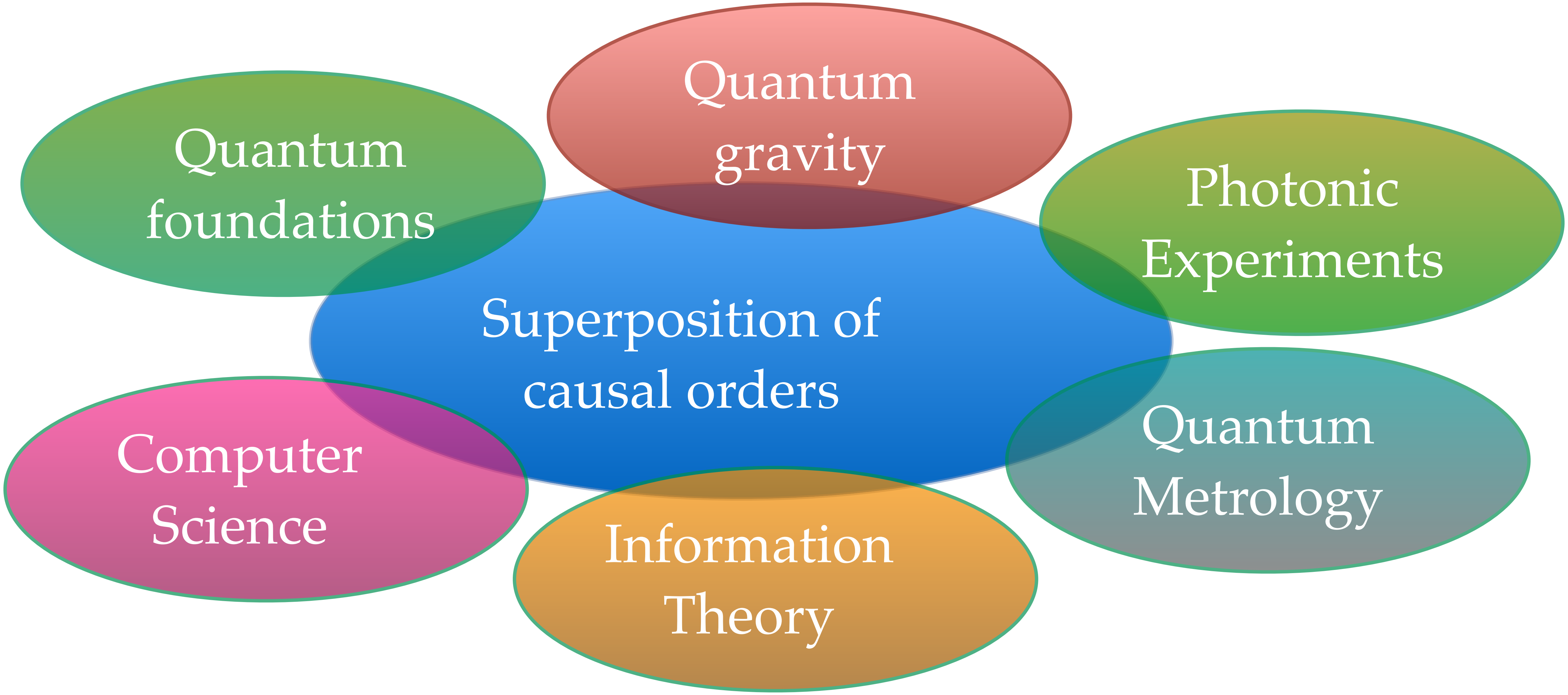
SUPERPOSITION OF CAUSAL ORDERS



FROM GRAVITY TO QUANTUM INFORMATION

General relativity is
a deterministic theory with non-fixed causal structure.
Quantum theory is
a probabilistic theory with fixed causal structure.
In this paper we build a framework
for probabilistic theories with non-fixed causal structure.
This combines the radical elements
of general relativity and quantum theory.

Journal of Physics A: Mathematical and Theoretical

Towards quantum gravity: a framework for probabilistic theories with non-fixed causal structure

Lucien Hardy

Published 7 March 2007 • 2007 IOP Publishing Ltd
Journal of Physics A: Mathematical and Theoretical, Volume 40, Number 12

Springer Link

Quantum Reality, Relativistic Causality, and Closing the Epistemic Circle pp 379-401 | Cite as

Quantum Gravity Computers: On the Theory of Computation with Indefinite Causal Structure

Authors Authors and affiliations

Lucien Hardy

SUPERPOSITION OF CAUSAL STRUCTURES

Beyond Quantum Computers

G. Chiribella, G. M. D'Ariano, P. Perinotti, B. Valiron

The manuscript poses and addresses a new very fundamental issue in Quantum Computer Science, which is going to have a radical impact on the way we currently conceive quantum computation. We show that there exists a new kind of "higher-order" quantum computation, potentially much more powerful than the usual quantum processing, which is feasible, but cannot be realized by a usual quantum circuit. In order to implement this new kind of computations one needs to change the rules of quantum circuits, also considering circuits with the geometry of the connections that can be itself in a quantum superposition. The new kind of computation poses also fundamental problems for unexplored aspects of quantum mechanics in a non-fixed causal framework, which go far beyond computer-science problems, and may be of relevance in quantum gravity.

arXiv:0912.0195

Quantum computations without definite causal structure

Giulio Chiribella, Giacomo Mauro D'Ariano, Paolo Perinotti, and Benoit Valiron
Phys. Rev. A **88**, 022318 – Published 14 August 2013

We show that quantum theory allows for transformations of black boxes that cannot be realized by inserting the input black boxes within a circuit in a predefined causal order. The simplest example of such a transformation is the *classical switch of black boxes*, where two input black boxes are arranged in two different orders conditionally on the value of a classical bit. The quantum version of this transformation—the *quantum switch*—produces an output circuit where the order of the connections is controlled by a quantum bit, which becomes entangled with the circuit structure. Simulating these transformations in a circuit with fixed causal structure requires either postselection or an extra query to the input black boxes.

Phys. Rev. A 88, 022318 (2013)

THE QUANTUM SWITCH

An hypothetical machine that combines two black boxes in a coherent superposition of alternative orders.

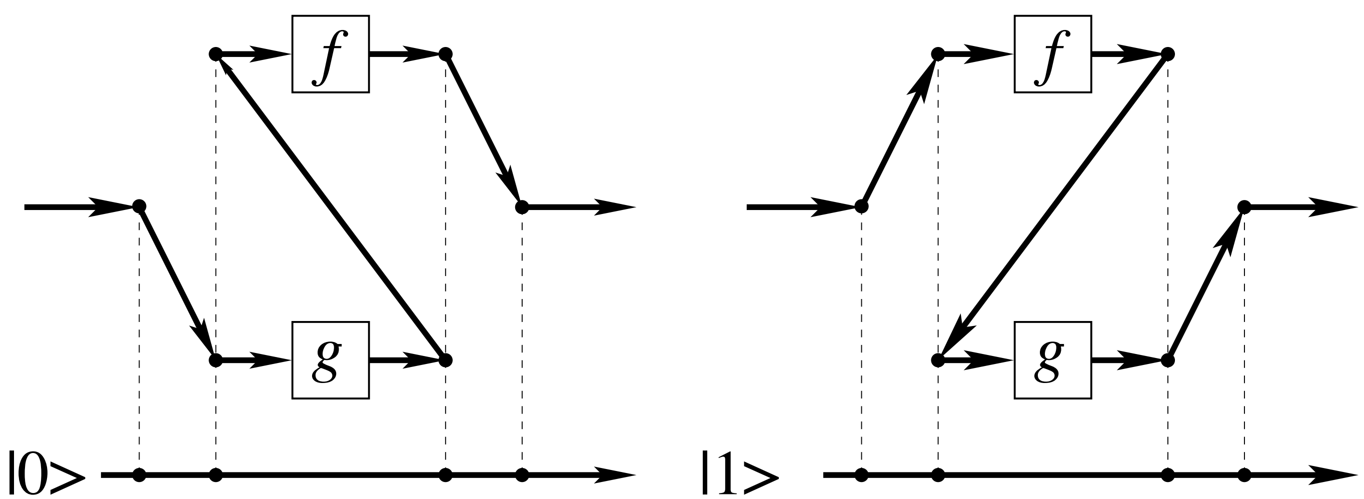


FIG. 1: Quantum machine with classical control over movable wires.

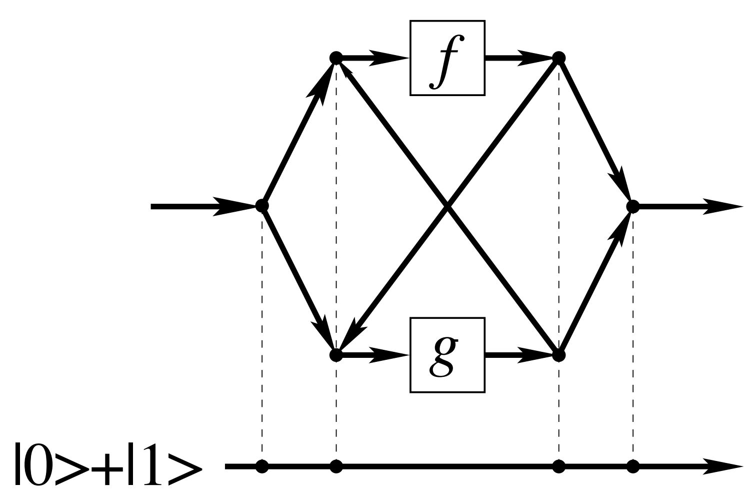


FIG. 2: Quantum machine with quantum control over movable wires.

figures from arXiv:0912.0195

INFORMATION-THEORETIC ADVANTAGES

Open Access | Published: 02 October 2012

Quantum correlations with no causal order

Ognyan Oreshkov, Fabio Costa & Časlav Brukner

Nature Communications 3, Article number: 1092 (2012) | Cite this article

Rapid Communication

Perfect discrimination of no-signalling channels via quantum superposition of causal structures

Giulio Chiribella
Phys. Rev. A **86**, 040301(R) – Published 10 October 2012

Editors' Suggestion

Computational Advantage from Quantum-Controlled Ordering of Gates

Mateus Araújo, Fabio Costa, and Časlav Brukner
Phys. Rev. Lett. **113**, 250402 – Published 18 December 2014

Enhanced Communication with the Assistance of Indefinite Causal Order

Daniel Ebler, Sina Salek, and Giulio Chiribella
Phys. Rev. Lett. **120**, 120502 – Published 22 March 2018

Quantum Metrology with Indefinite Causal Order

Xiaobin Zhao, Yuxiang Yang, and Giulio Chiribella
Phys. Rev. Lett. **124**, 190503 – Published 14 May 2020

Quantum Refrigeration with Indefinite Causal Order

David Felce and Vlatko Vedral
Phys. Rev. Lett. **125**, 070603 – Published 11 August 2020

EXPERIMENTS

Experimental superposition of orders of quantum gates

Lorenzo M. Procopio, Amir Moqanaki, Mateus Araújo, Fabio Costa, Irati Alonso Calafell, Emma G. Dowd, Deny R. Hamel, Lee A. Rozema, Časlav Brukner & Philip Walther

Nature Communications **6**, Article number: 7913 (2015) | Cite this article

Experimental verification of an indefinite causal order

Giulia Rubino[1,*], Lee A. Rozema[1], Adrien Feix[1,2], Mateus Araújo[1,2], Jonas M. Zeuner[1], Lorenzo ...
+ See all authors and affiliations

Science Advances 24 Mar 2017:
Vol. 3, no. 3, e1602589
DOI: 10.1126/sciadv.1602589

Editors' Suggestion

Indefinite Causal Order in a Quantum Switch

K. Goswami, C. Giarmatzi, M. Kewming, F. Costa, C. Branciard, J. Romero, and A. G. White
Phys. Rev. Lett. **121**, 090503 – Published 31 August 2018

Experimental Quantum Switching for Exponentially Superior Quantum Communication Complexity

Kejin Wei, Nora Tischler, Si-Ran Zhao, Yu-Huai Li, Juan Miguel Arrazola, Yang Liu, Weijun Zhang, Hao Li, Lixing You, Zhen Wang, Yu-Ao Chen, Barry C. Sanders, Qiang Zhang, Geoff J. Pryde, Feihu Xu, and Jian-Wei Pan
Phys. Rev. Lett. **122**, 120504 – Published 28 March 2019

Experimental Transmission of Quantum Information Using a Superposition of Causal Orders

Yu Guo, Xiao-Min Hu, Zhi-Bo Hou, Huan Cao, Jin-Ming Cui, Bi-Heng Liu, Yun-Feng Huang, Chuan-Feng Li, Guang-Can Guo, and Giulio Chiribella
Phys. Rev. Lett. **124**, 030502 – Published 24 January 2020

PLAN OF THE LECTURE

- Theoretical framework: quantum supermaps
  - -quantum causal networks
  - -definite vs indefinite causal order
  - -the quantum SWITCH
  - -general higher-order maps

- Applications of the quantum SWITCH

- Physical realizations

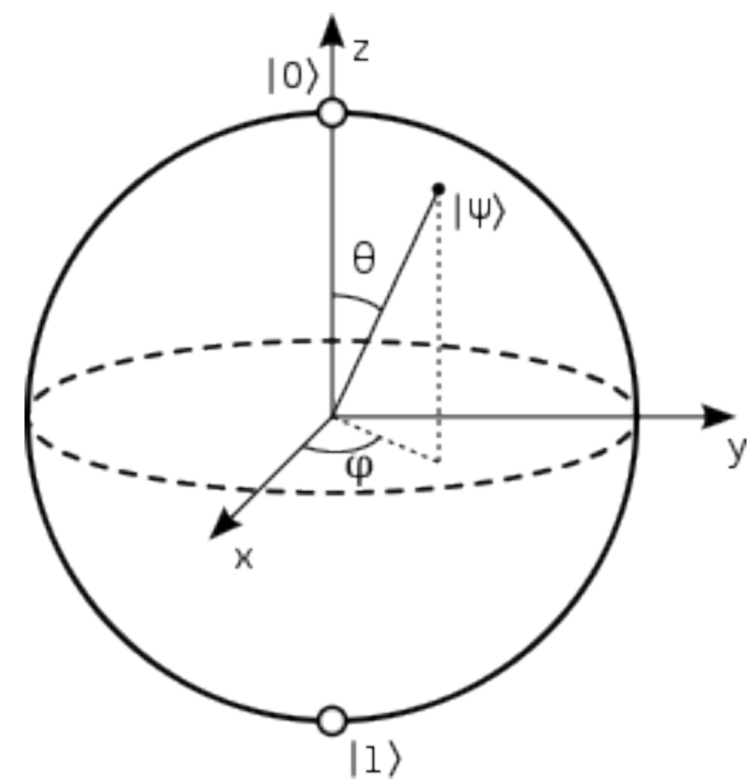
THEORETICAL FRAMEWORK: QUANTUM SUPERMAPS

To see a World in a Grain of Sand
And a Heaven in a Wild Flower
Hold Infinity in the palm of your hand
And Eternity in an hour.

William Blake, ca. 1803

FORGET EVERYTHING, EXCEPT QUANTUM STATES

Promise: there exist quantum systems.



Quantum system $\longrightarrow$ Hilbert space $\mathscr{H} = \mathbb{C}^d$

Quantum states = density matrices

$$\rho \in L(\mathscr{H}), \quad \boxed{\langle\psi|\rho|\psi\rangle \geq 0 \quad \forall|\psi\rangle \in \mathscr{H},} \quad \mathrm{Tr}[\rho] = 1$$

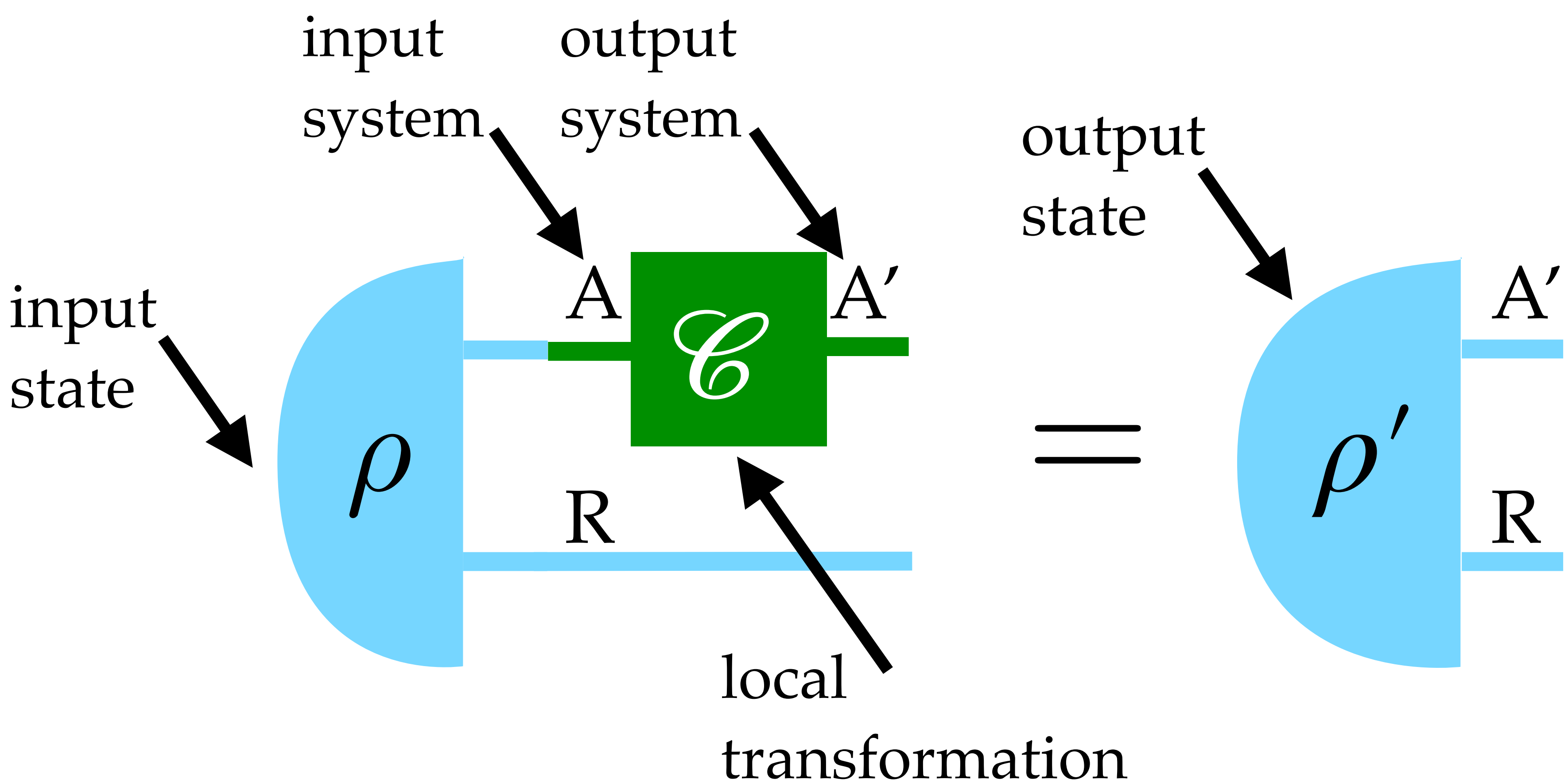
$$\rho \geq 0$$

Question:

What is the most general way
to transform
quantum states into quantum states?

ADMISSIBLE MAPS

Admissible map: must be **linear*** and **send states into states**, even when acting locally on one part of a composite system



*why linear?

see **Chiribella, D'Ariano, and Perinotti, in Chiribella and Spekkens, eds. arXiv:1506.00398, p. 11**

EXAMPLE AND NON-EXAMPLE

- Example: unitary map $\mathcal{U}(\rho) := U\rho U^\dagger, \quad U^\dagger U = UU^\dagger = I$

- Non-example: transpose map $\Theta(\rho) := \rho^T \quad \forall\rho$

Apply it to $|\Phi^+\rangle := \sum_k |k\rangle \otimes |k\rangle/\sqrt{d}$,

get

$$(\Theta \otimes \mathcal{I}_R)(|\Phi^+\rangle\langle\Phi^+|) = \sum_{k,l} \Theta(|k\rangle\langle l|) \otimes |k\rangle\langle l|/d$$

$$= \sum_{k,l} |l\rangle\langle k| \otimes |k\rangle\langle l|/d$$

$$= \mathtt{SWAP}/d \ngeq 0$$

CHARACTERIZATION OF THE ADMISSIBLE MAPS

Every admissible map has a *Kraus representation*

$$\mathscr{C}(\rho) = \boxed{\sum_i C_i \, \rho \, C_i^\dagger} \quad \text{with} \quad \boxed{\sum_i C_i^\dagger C_i = I}$$

completely positive

trace-preserving

Admissible maps = completely positive, trace-preserving maps

=: quantum channels

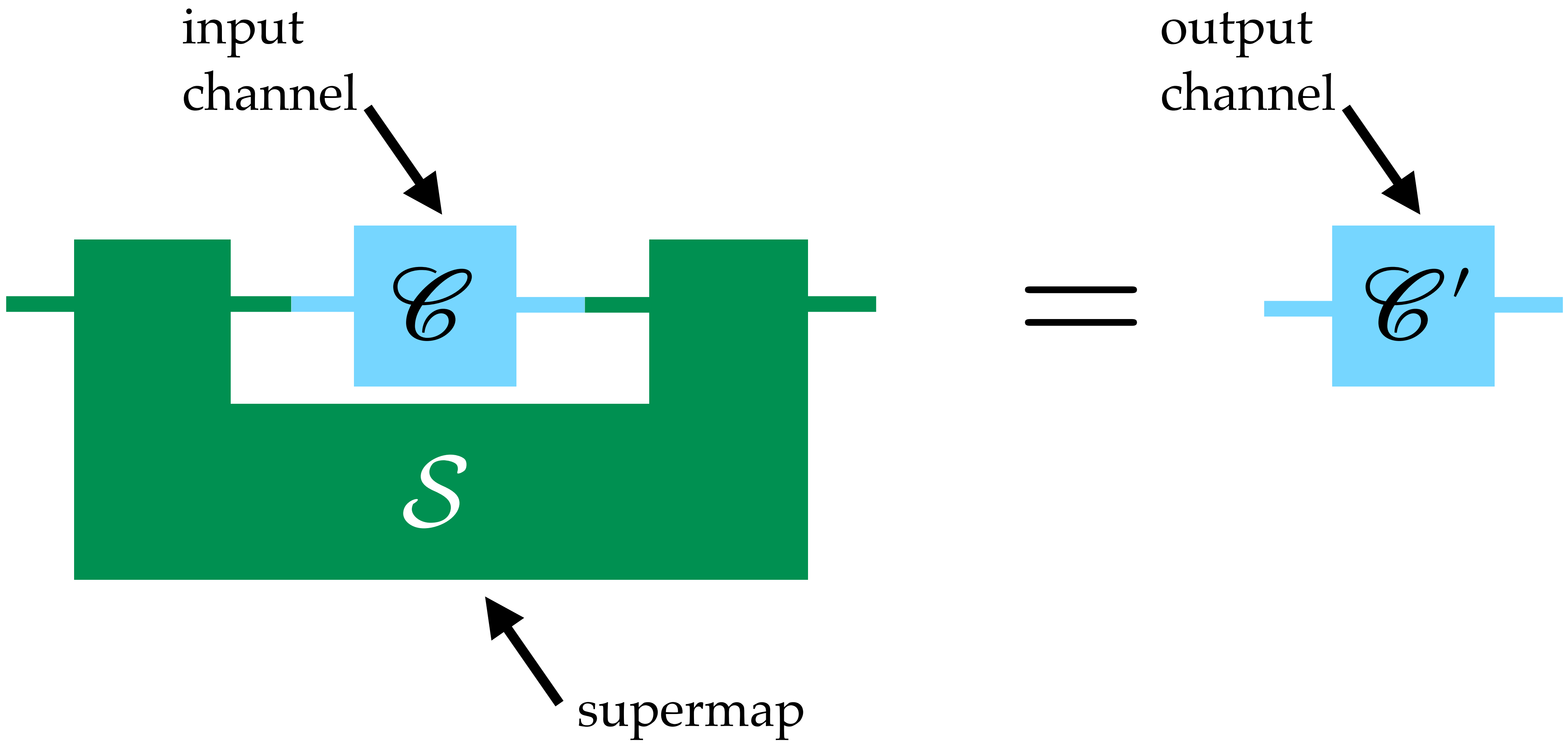
cf. Heinosaari and Ziman, Cambridge University Press (2011).

Next Level:

What is the most general way
to transform
quantum channels into quantum channels?

SUPERMAPS

Supermaps = transformations of quantum channels



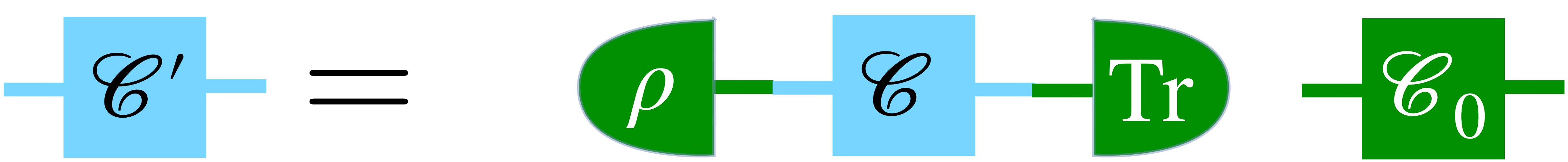
ADMISSIBLE SUPERMAPS

Admissible supermap: must be **linear***
and **send channels into channels**,
even when acting locally on one part of
a bipartite channel

EXAMPLES

- Encoding-decoding $\mathscr{S}(\mathscr{C}) := \mathscr{D} \circ \mathscr{C} \circ \mathscr{E}$,
  with $\mathscr{E}$ and $\mathscr{D}$ fixed quantum channels

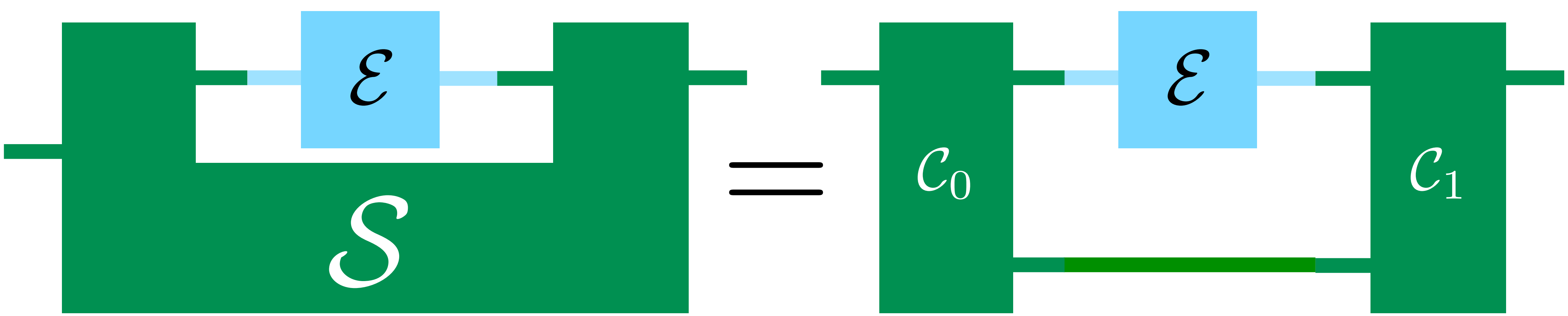
- Replacement $\mathscr{S}(\mathscr{C}) := \mathrm{Tr}[\mathscr{C}(\rho)]\,\mathscr{C}_0$,
  with ρ fixed state and $\mathscr{C}_0$ fixed channel



CHARACTERIZATION
OF THE ADMISSIBLE SUPERMAPS

Theorem

Every admissible supermap can be realized by a network of channels with memory:



Chiribella, D'Ariano, and Perinotti, EPL 83, 30004 (2008)

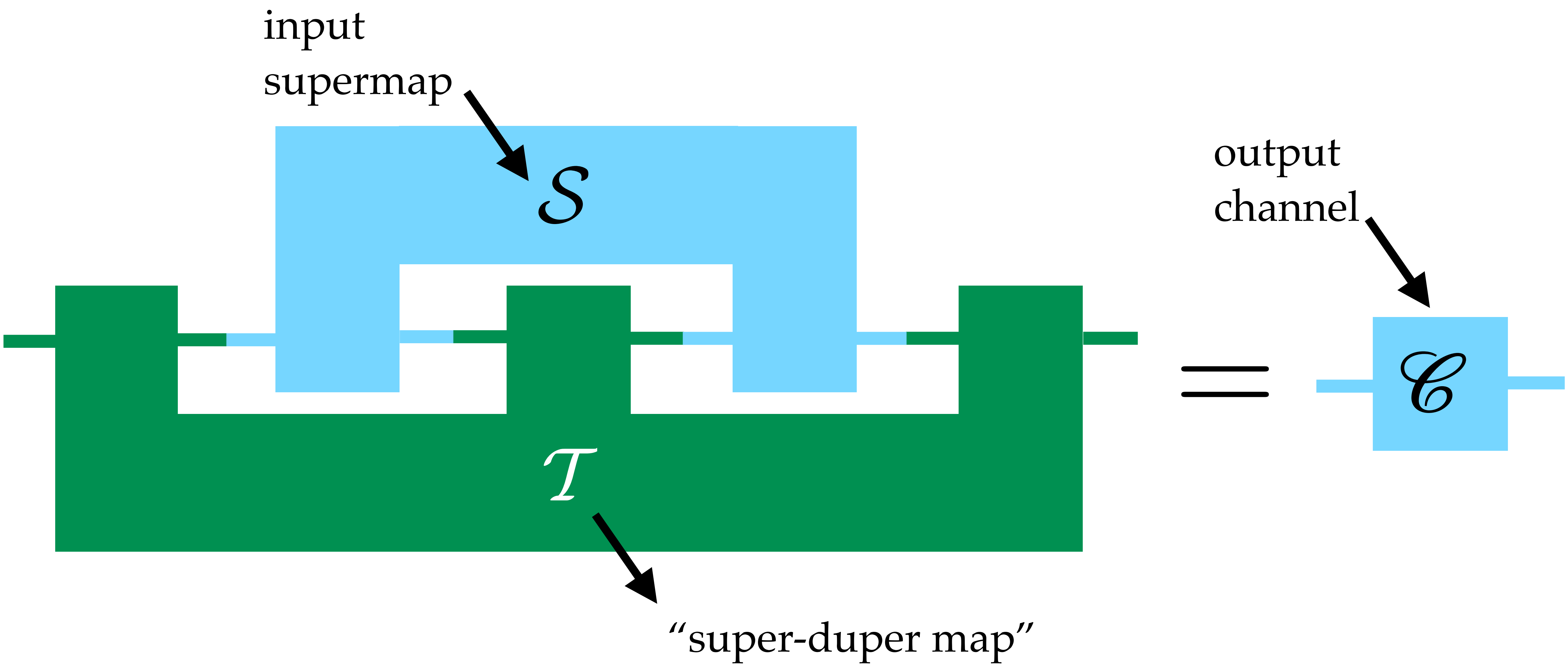
Next Next Level:

What is the most general way

to transform

admissible supermaps into admissible channels?

HIGHER-ORDER SUPERMAPS

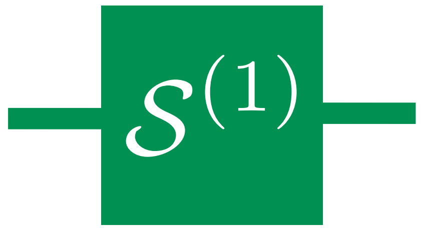


Admissible "super-duper map":

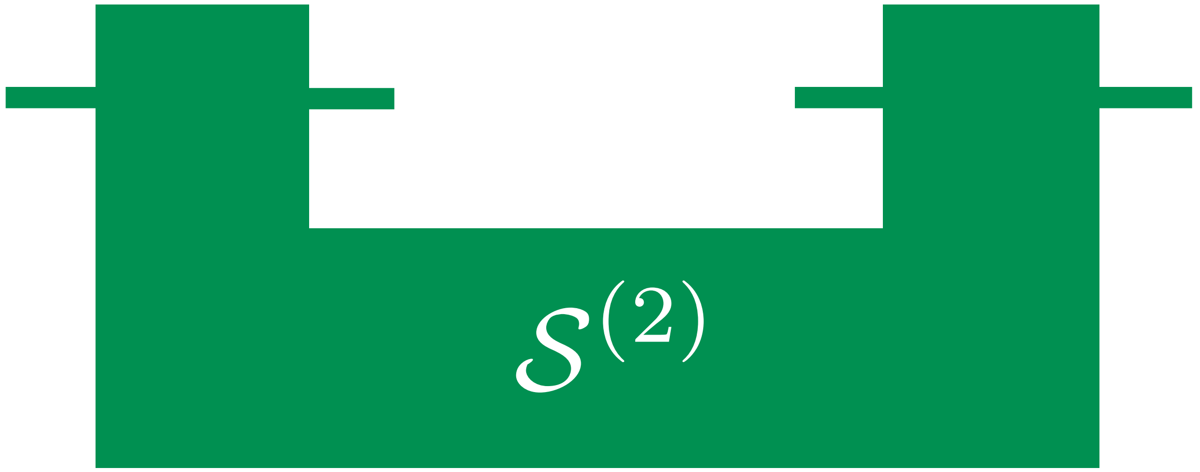
must be **linear*** and **send admissible supermaps into channels**, even when acting locally on one part of a bipartite supermap

ADMISSIBLE N-MAPS

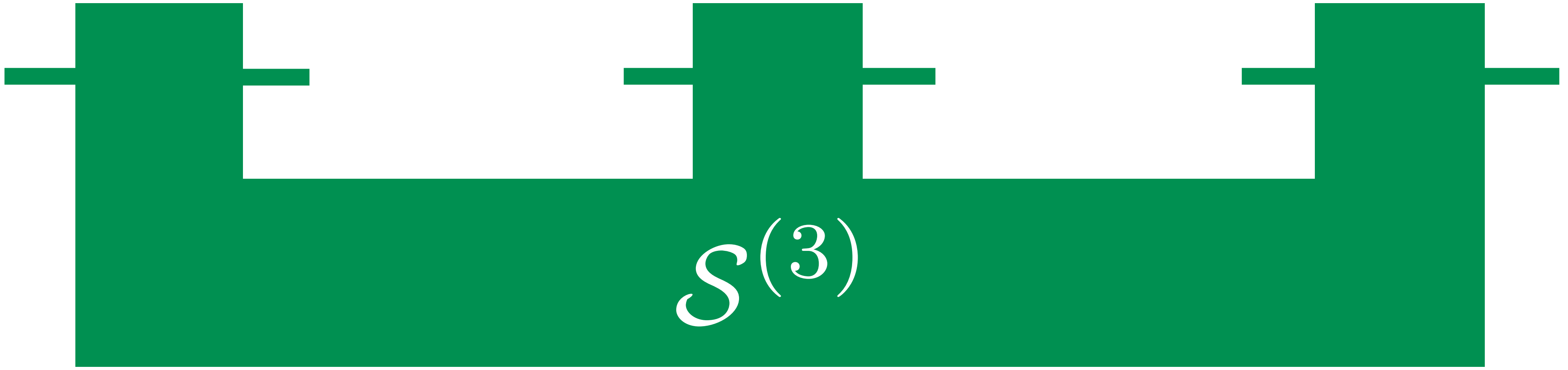
N=1 quantum channel



N=2



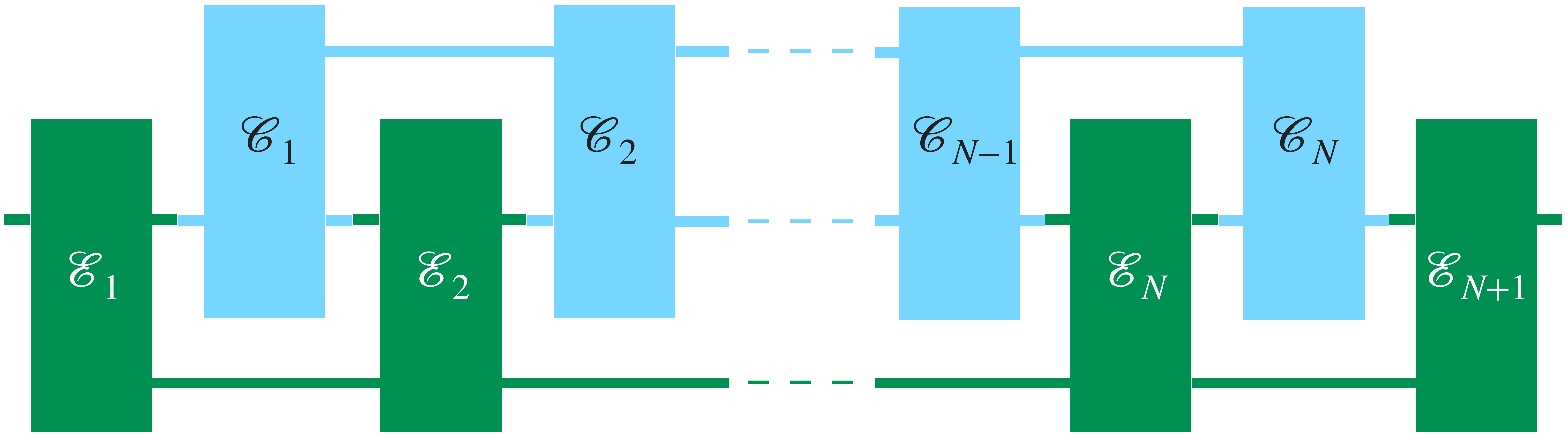
N=3



REALIZATION OF ADMISSIBLE N-MAPS

Theorem

Any admissible N-map can be realized by a sequential network of quantum channels with memory:



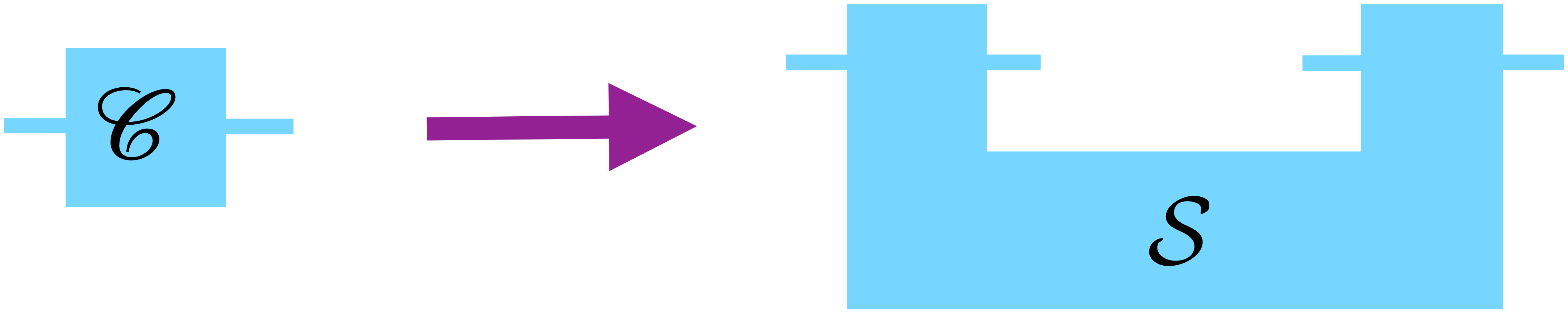
Chiribella, D'Ariano, and Perinotti, Phys. Rev. A 80, 022339 (2009)

Getting to the weird levels:

What is the most general way
to transform
admissible supermaps into admissible supermaps?

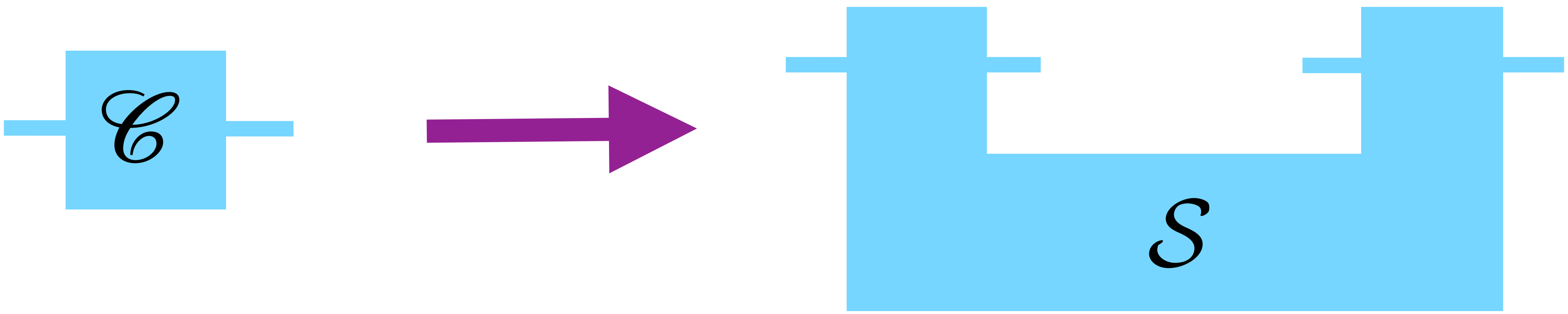
THE EASIEST EXAMPLE

Question: what is the most general way to transform a quantum channel into a supermap?

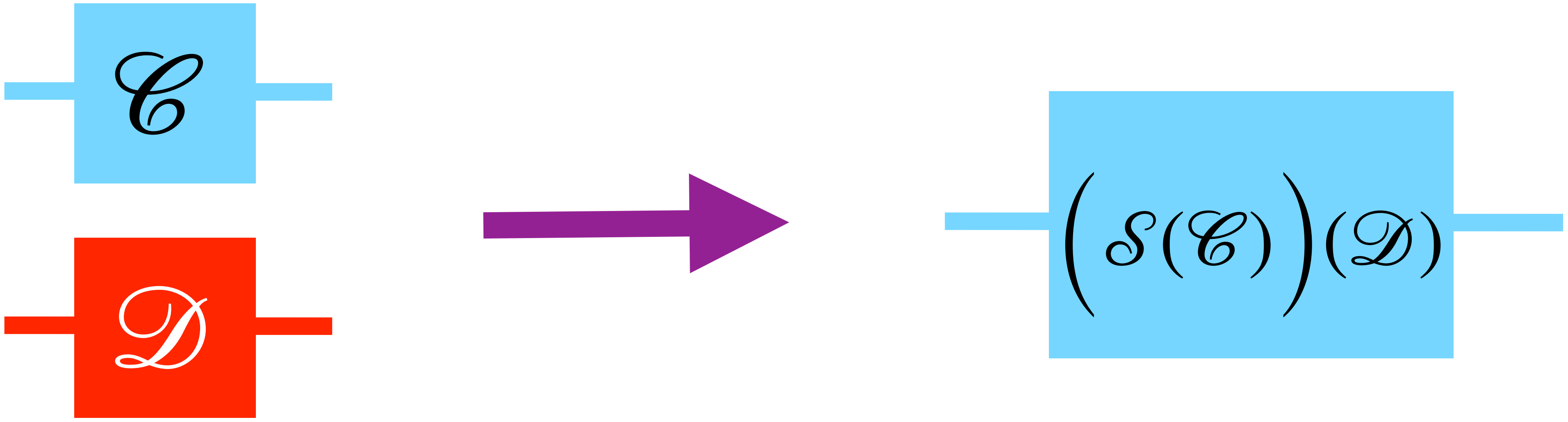


EQUIVALENT FORMULATION

Transforming a channel into a supermap



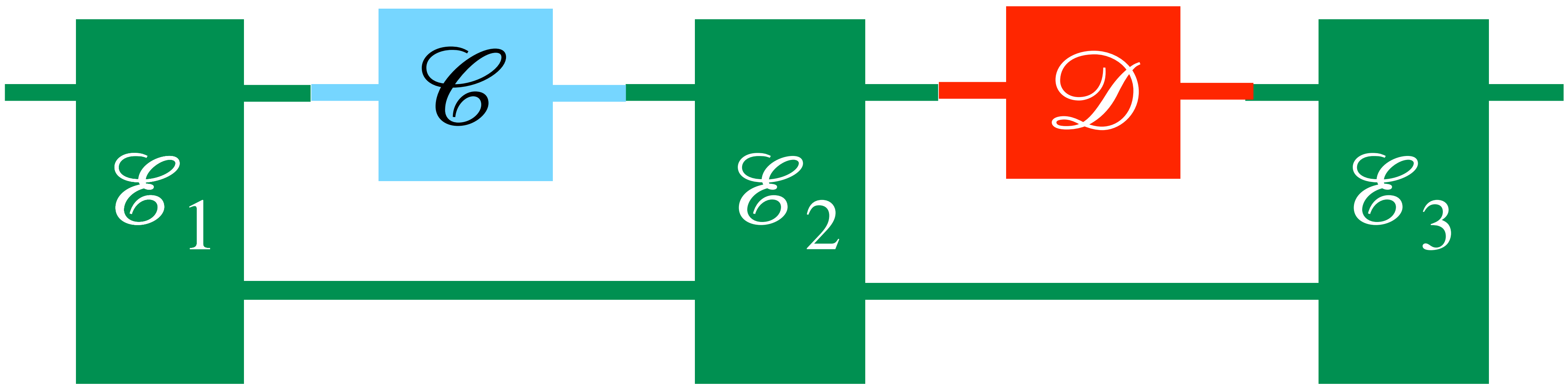
is equivalent to
transforming a *pair* of channels into a channel



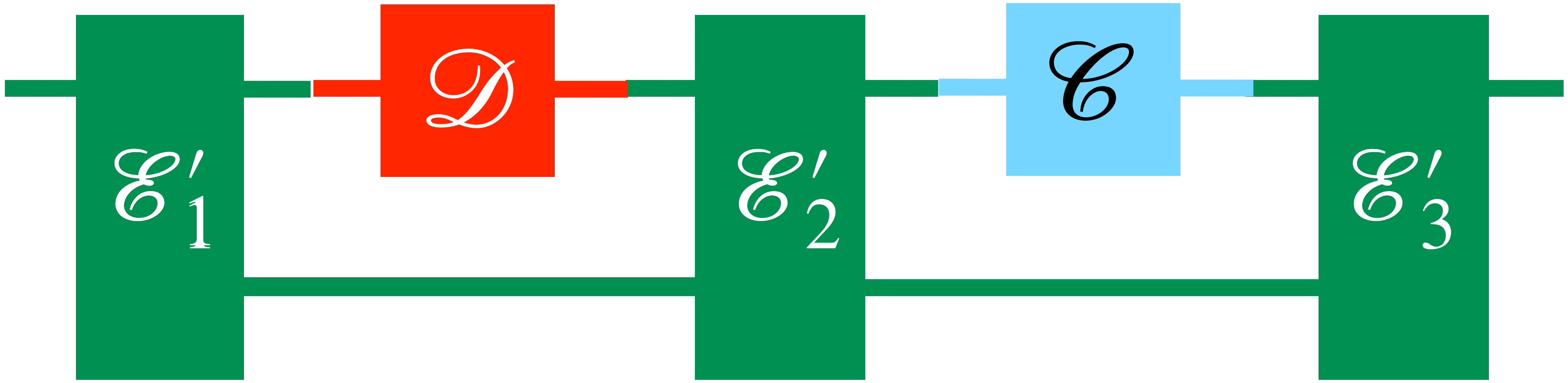
TWO COMPLEMENTARY ORDERS

There are two alternative causal networks.

- First realization: place $\mathscr{C}$ before $\mathscr{D}$

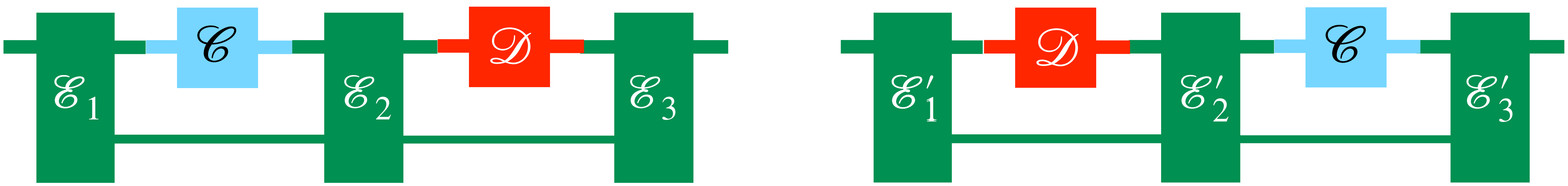


- Second realization: place $\mathscr{D}$ before $\mathscr{C}$



MIXTURE VS SUPERPOSITION OF CAUSAL STRUCTURES

Two complementary choices of causal networks:



We could choose randomly between these two supermaps.

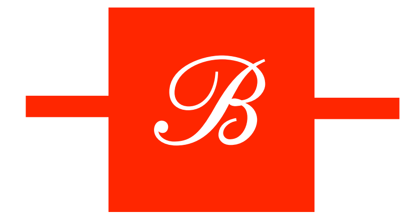
But we can also choose coherently,
depending on the state of a control qubit.
This gives us a coherent superposition of causal structures.

THE SIMPLIFIED QUANTUM SWITCH

THE SIMPLIFIED QUANTUM SWITCH

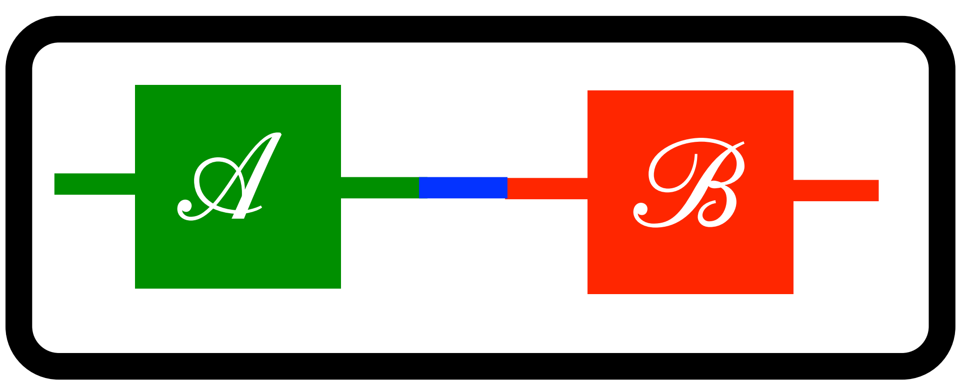
The *simplified* quantum SWITCH is the supermap that

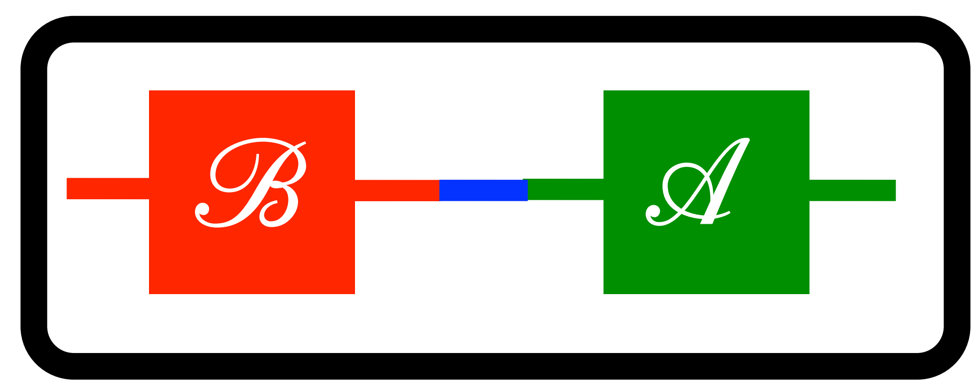
takes as input the two channels 

and 

with equal inputs/outputs

and *connects* them in a coherent superposition of the following configurations:



and


Chiribella, D'Ariano, Perinotti, Valiron, arXiv:0912.0195; Phys. Rev. A 88, 022318 (2013) [hereafter CDPV 2009/2013]

MATHEMATICAL DEFINITION

- Input channels:

$$\mathscr{A}(\rho) = \sum_i A_i \rho A_i^\dagger \qquad \text{and} \qquad \mathscr{B}(\rho) = \sum_j B_j \rho B_j^\dagger$$

- Output of simplified quantum SWITCH:

$$\mathcal{S}(\mathscr{A}, \mathscr{B})(\rho) = \sum_{i,j} S_{ij} \rho S_{ij}^\dagger$$

$$S_{ij} := A_i B_j \otimes |0\rangle\langle 0| + B_j A_i \otimes |1\rangle\langle 1|$$

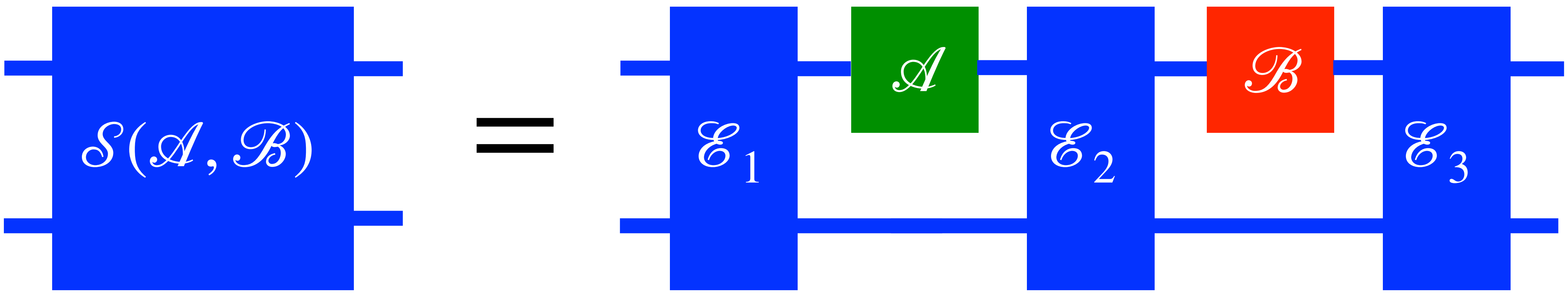
states of the control qubit

INCOMPATIBILITY
WITH
FIXED CAUSAL ORDER

INCOMPATIBILITY WITH FIXED CAUSAL ORDER

Theorem (CDPV 2009/2013)

It is impossible to find quantum channels $\mathscr{E}_1$, $\mathscr{E}_2$, and $\mathscr{E}_3$ such that



for all unitary 

and 

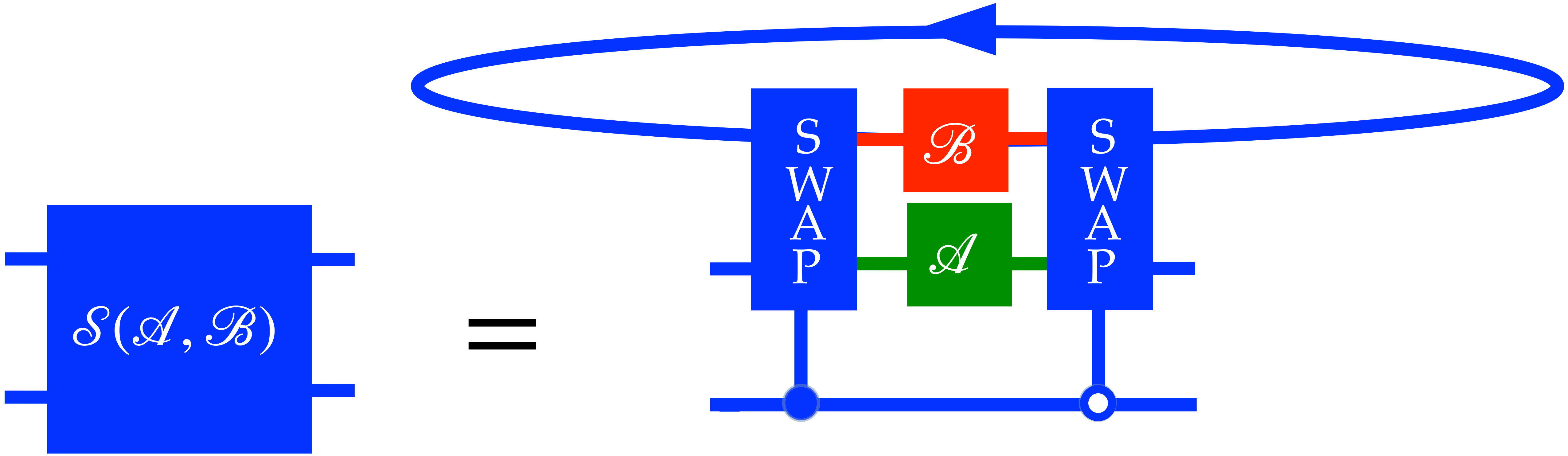
(same holds with $\mathscr{A}$ and $\mathscr{B}$ in the opposite order,
and for classical mixtures of the two orders)

QUANTUM SWITCH AND TIME LOOPS

If a network of channels implements the quantum SWITCH (in the sense of the previous theorem), then it must contain a loop.

The converse also holds:
If we have access to a network with a loop,
then we use it to construct a circuit that implements the quantum SWITCH deterministically.

REALIZATION OF THE SWITCH IN A CIRCUIT WITH LOOP [CDPV 2009/2013]



- **True time loop:** maybe in exotic quantum gravity scenarios
- **Simulated time loop:** with conclusive teleportation

THE FULL QUANTUM SWITCH

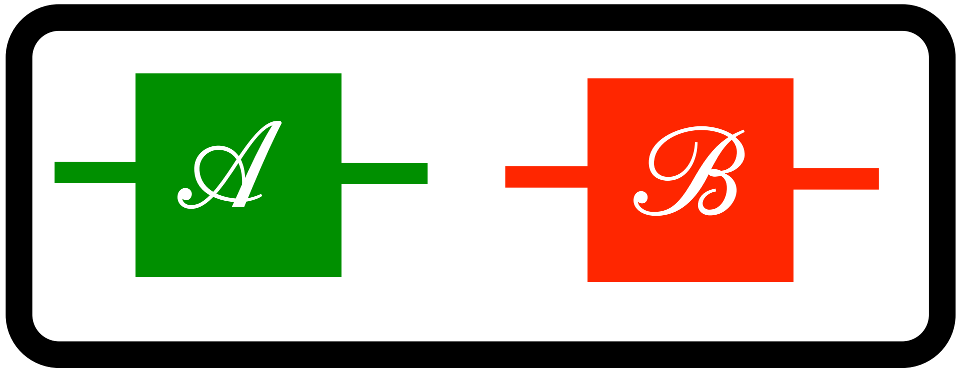
THE FULL QUANTUM SWITCH [CDPV 2009/2013]

The *full* quantum SWITCH is the supermap that

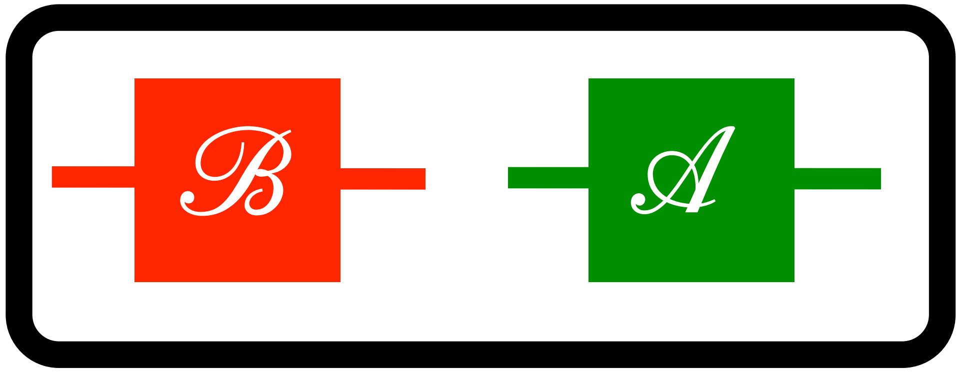
takes as input the two channels 

and 

and *places them* in a coherent superposition of the following configurations:



and



MATHEMATICAL DEFINITION

- Output channel of simplified quantum SWITCH:

$$\mathcal{S}(\mathcal{A}, \mathcal{B})(\rho) = \sum_{i,j} S_{ij} \rho S_{ij}^\dagger$$

$$S_{ij} := A_i \otimes B_j \otimes |0\rangle\langle 0| + B_j \otimes A_i \otimes |1\rangle\langle 1|$$

acting in the
1st time slot

acting in the
2nd time slot

EXAMPLE

- Switching a channel with the identity

$$\mathcal{S}(\mathcal{A}, \mathcal{I})(\rho) = \sum_{i,j} S_i \rho S_i^\dagger$$

$$S_i := A_i \otimes I \otimes |0\rangle\langle 0| + I \otimes A_i \otimes |1\rangle\langle 1|$$

acting in the 1st time slot

acting in the 2nd time slot

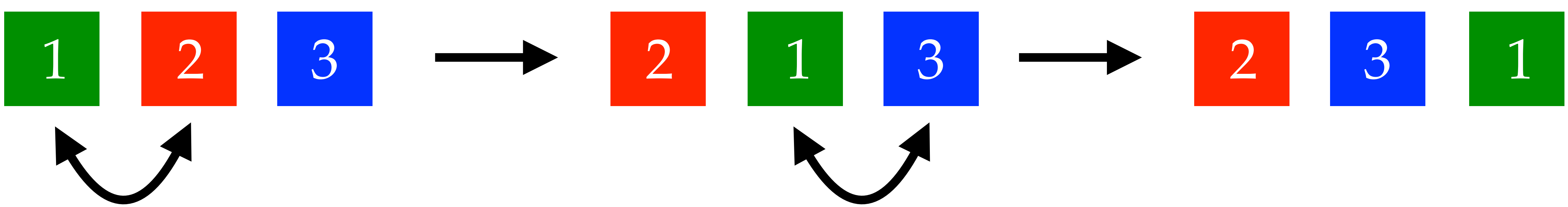
Switch of $\mathcal{A}$ with identity = time-delocalized $\mathcal{A}$

SWITCHING
MORE THAN
TWO CHANNELS

FROM 2 TO N

Fact: every permutation of N objects can be decomposed into a sequence of transpositions (i.e. “switches” of 2 objects).

Example: cyclic permutation (1,2,3) $\longrightarrow$ (2,3,1)



By combining $\Theta(N \log N)$ quantum switches of two-channels, we can coherently control arbitrary permutations of N channels.

Colnaghi, D’Ariano, Perinotti, and Facchini, Phys. Lett. A 376 (2012).
Facchini and Simon Perdrix, in International Conference on Theory and Applications of Models of Computation, p. 324, Springer (2015).

QUANTUM SWITCH OF N CHANNELS

$$\mathcal{S}(\mathscr{C}_1, \mathscr{C}_2, \ldots, \mathscr{C}_N)(\rho) = \sum_{i_1, i_2, \ldots, i_N} S_{i_1, i_2, \ldots, i_N} \, \rho \, S^{\dagger}_{i_1, i_2, \ldots, i_N}$$

$$S_{i_1, \ldots, i_N} := \sum_{\pi \in \mathsf{S}_N} C^{\pi(1)}_{i_{\pi(1)}} \otimes C^{\pi(2)}_{i_{\pi(2)}} \otimes \cdots \otimes C^{\pi(N)}_{i_{\pi(N)}} \otimes |\pi\rangle\langle\pi|$$

acting in the 1st time slot

acting in the 2nd time slot

acting in the Nth time slot

$$\left\{ C^n_{i_n} \right\} = \text{Kraus operators of } \mathscr{C}_n$$

ALL
POSSIBLE
SUPERMAPS

RECURSIVE DEFINITION

Types of maps (GC, slide from Tainan Workshop 2015)

- Maps of type 0 (quantum states)

- If x and y are allowed types, then (x,y) is an allowed type

Admissible (x,y) maps: all linear maps transforming maps of type x into maps of type y, even when acting locally.

Most general processes compatible with quantum mechanics!

Explicitly characterized in:

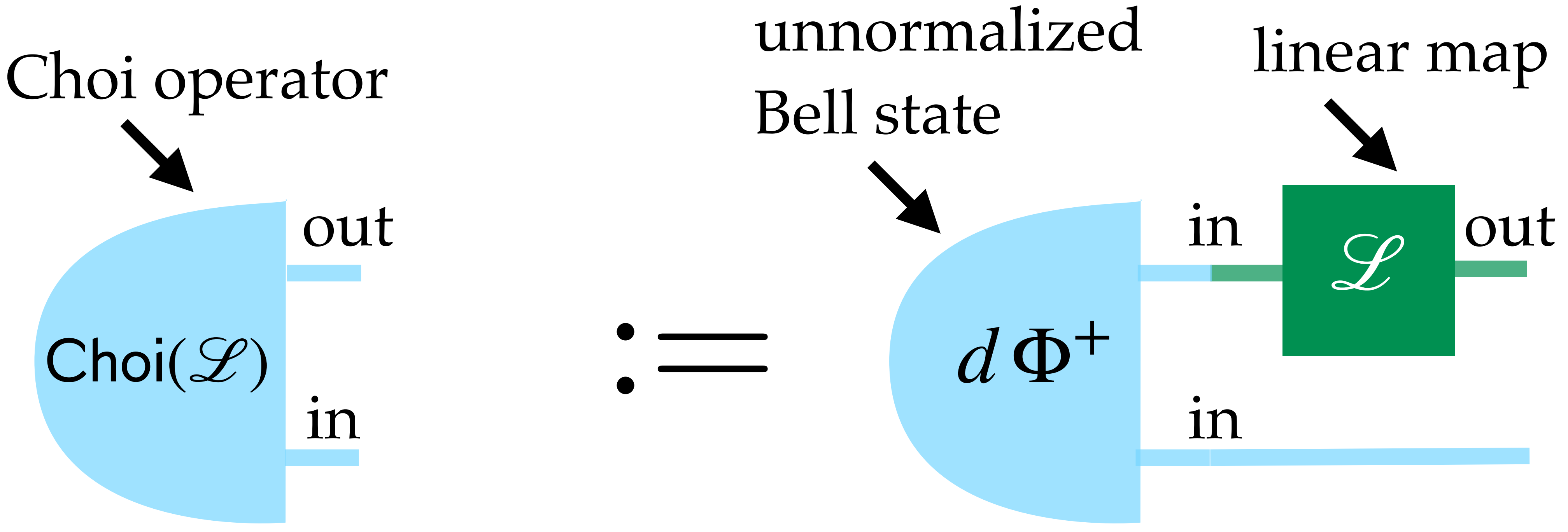
Bisio and Perinotti, Proceedings of the Royal Society A, 475, 20180706 (2019).

CHOI OPERATOR REPRESENTATION

CHOI OPERATORS

For a linear map $\mathscr{L} : \rho \mapsto \mathscr{L}(\rho)$
the Choi operator is

$$\mathsf{Choi}(\mathscr{L}) := \sum_{i,j} \mathscr{L}(|i\rangle\langle j|) \otimes |i\rangle\langle j|$$

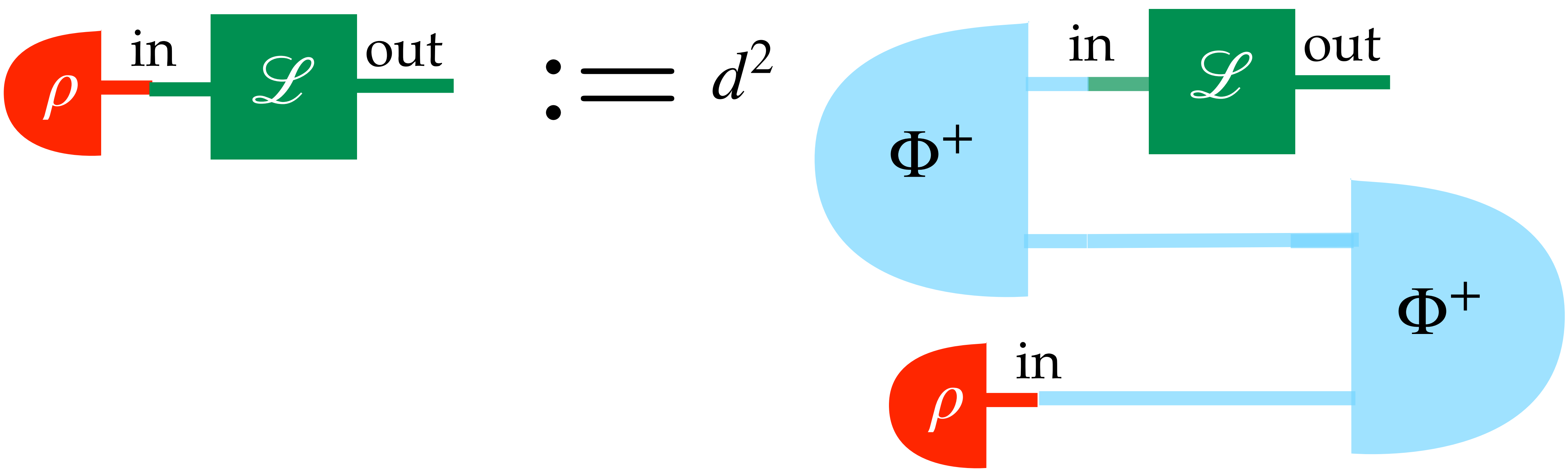


Choi, Lin. Alg. Appl. 10, 285 (1976)

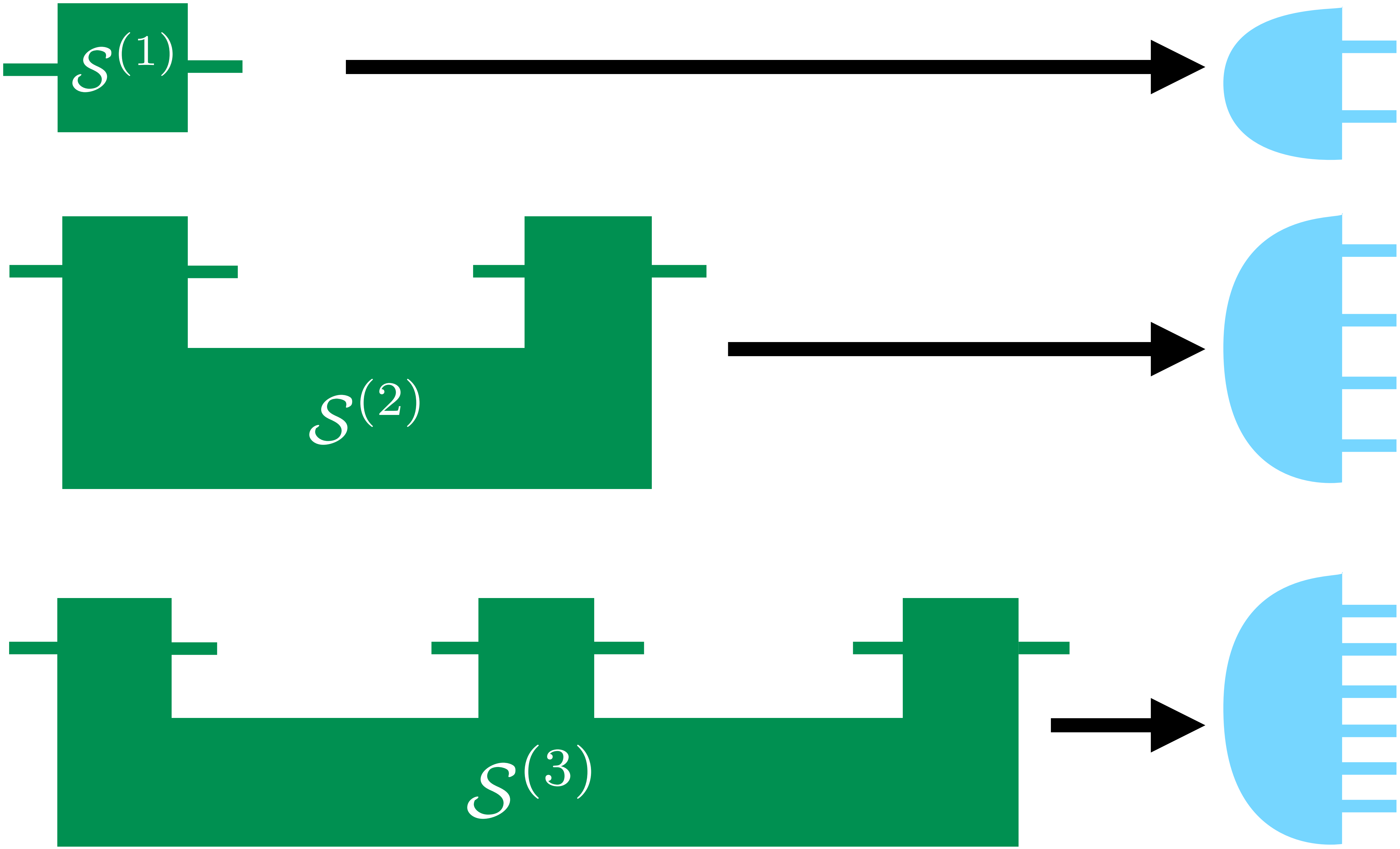
ONE-TO-ONE CORRESPONDENCE

The correspondence $\mathscr{L} \mapsto \mathsf{Choi}(\mathscr{L})$ is one-to-one:

$$\mathscr{L}(\rho) = \mathrm{Tr}_{\mathrm{in}}\left[\left(I_{\mathrm{out}} \otimes \rho^T\right)\ \mathsf{Choi}(\mathscr{L})\right]$$



CHOI REPRESENTATION OF SUPERMAPS

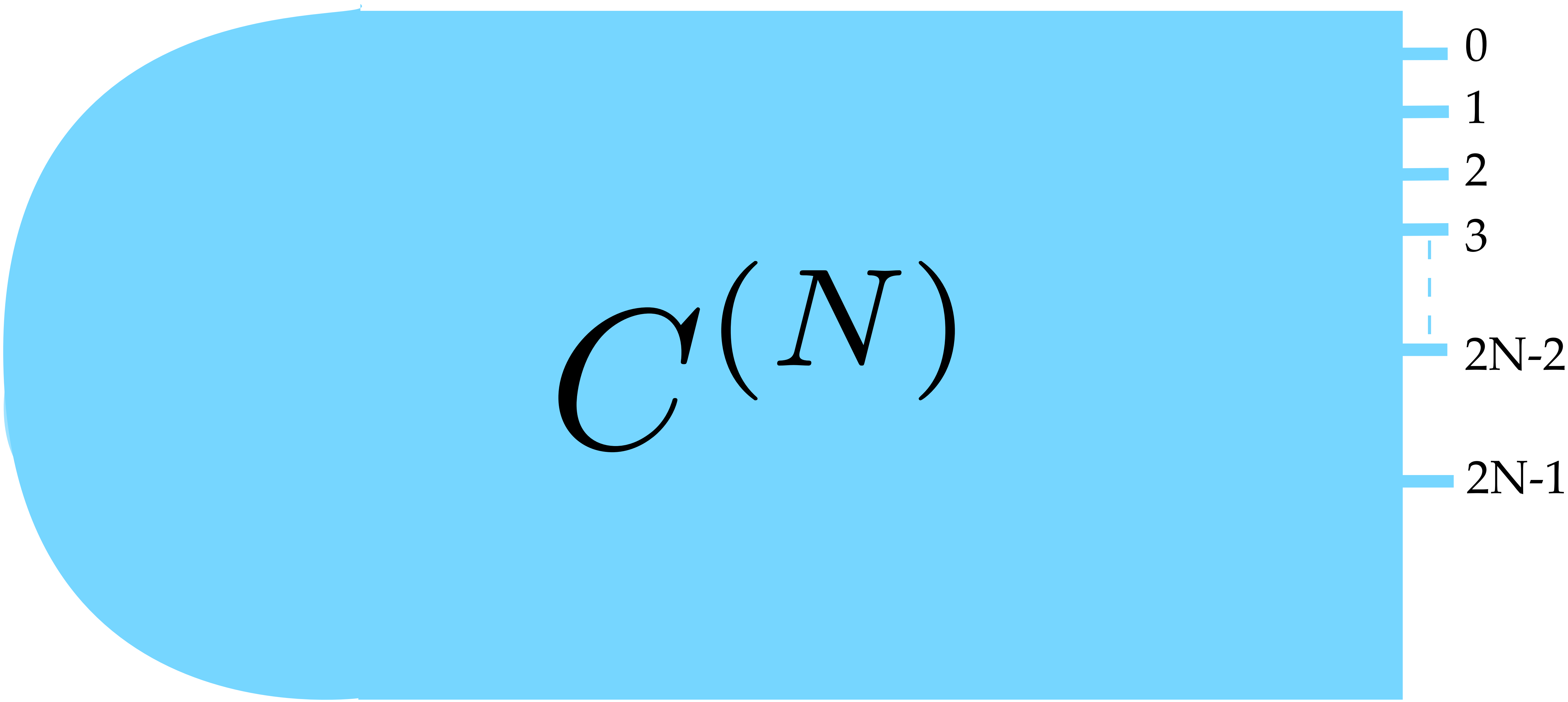


Chiribella, D'Ariano, and Perinotti, PRA 80 022339 (2009)

EXAMPLE: QUANTUM COMBS

Quantum causal network = sequence of quantum channels

Quantum comb = Choi operator of the causal network



THE MATHEMATICAL FORM OF THE CAUSAL STRUCTURE

Characterization of quantum combs:

$$C^{(N)} = \text{quantum comb} \iff \begin{cases} C^{(N)} \geq 0 \\ \mathsf{Tr}_{2N-1}\left[C^{(N)}\right] = I_{2N-2} \otimes C^{(N-1)} \\ \vdots \\ \mathsf{Tr}_3\left[C^{(2)}\right] = I_2 \otimes C^{(1)} \\ \mathsf{Tr}_1\left[C^{(1)}\right] = I_0 \end{cases}$$

Gutoski and Watrous, Proc. STOC (2007)

Chiribella, D'Ariano, and Perinotti, PRL 101, 060401 (2008)

OPTIMIZING QUANTUM NETWORKS

Optimizing over quantum networks is a semidefinite program.

Chiribella, NJP 14 125008 (2012);
Chiribella and Ebler, NJP 18 093053 (2016)

Applications:

- quantum metrology / tomography / channel discrimination
- quantum cryptography / game theory
- quantum interactive proof systems
- quantum machine learning

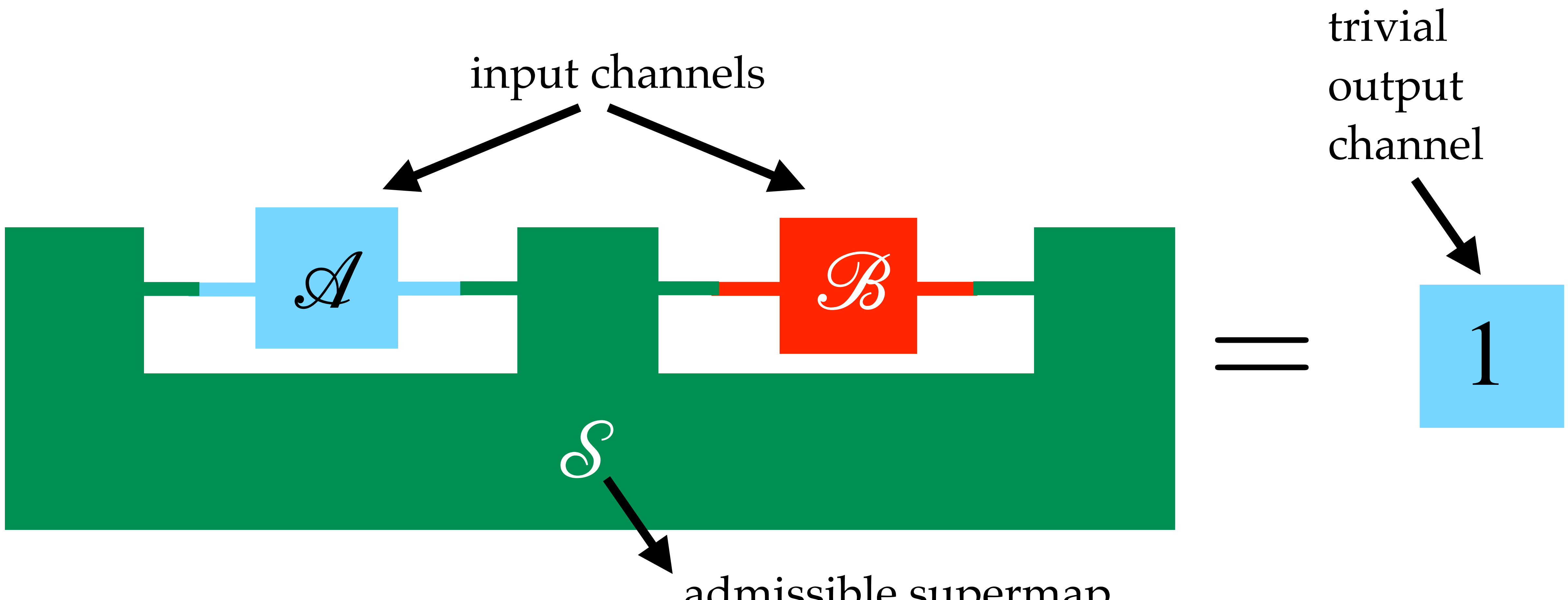
…

RELATION WITH PROCESS MATRICES

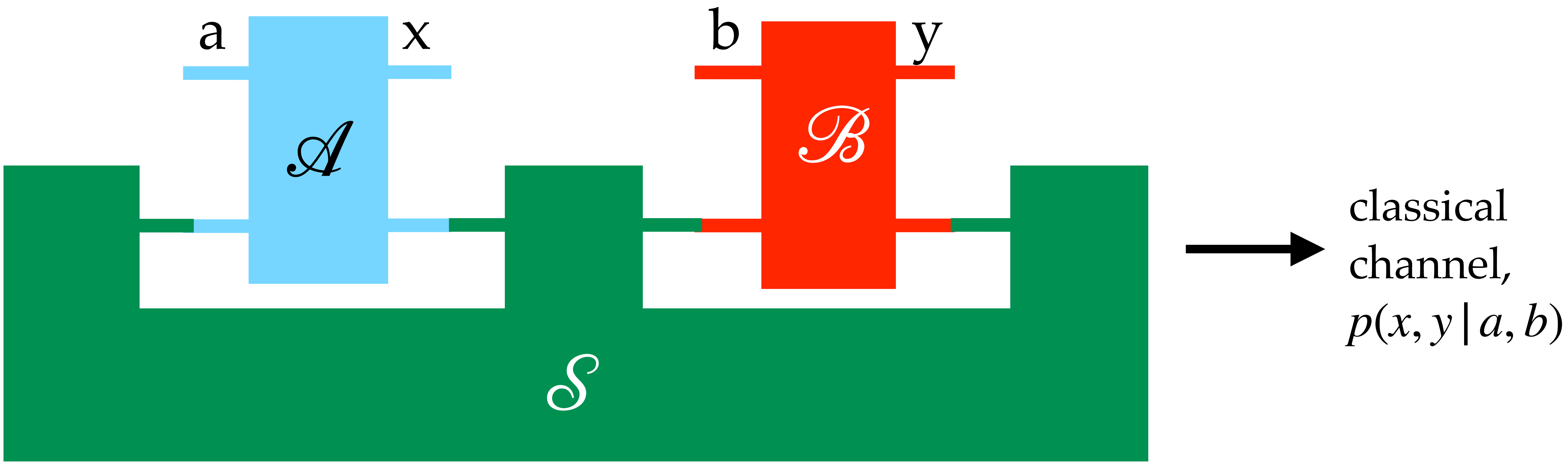
PROCESS MATRICES

Oreshkov, Costa, Brukner, Nat. Commun. 3, 1 (2012)
popular framework for studying indefinite causal order

Process matrices = Choi operators of admissible supermaps
from N channels to the *trivial channel*
(the number 1)



CAUSAL INEQUALITIES



Causal inequalities = Bell-like inequalities that certify that $\mathscr{S}$ is incompatible with a definite causal order.

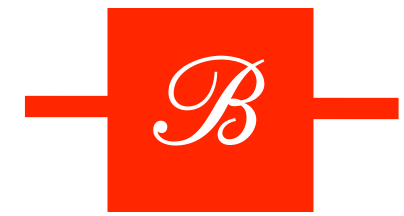
Fact: they are violated by some quantum supermaps,
but *not by the quantum SWITCH*

Oreshkov, Costa, Brukner, Nat. Commun. 3, 1 (2012)

APPLICATIONS
OF
THE QUANTUM SWITCH

QUERY COMPLEXITY AND COMMUNICATION COMPLEXITY

REDUCING QUERY COMPLEXITY

- Testing properties of processes  and 

e.g. ***discover if operators commute or anti-commute***

probability of correct answer = 1 with the quantum SWITCH

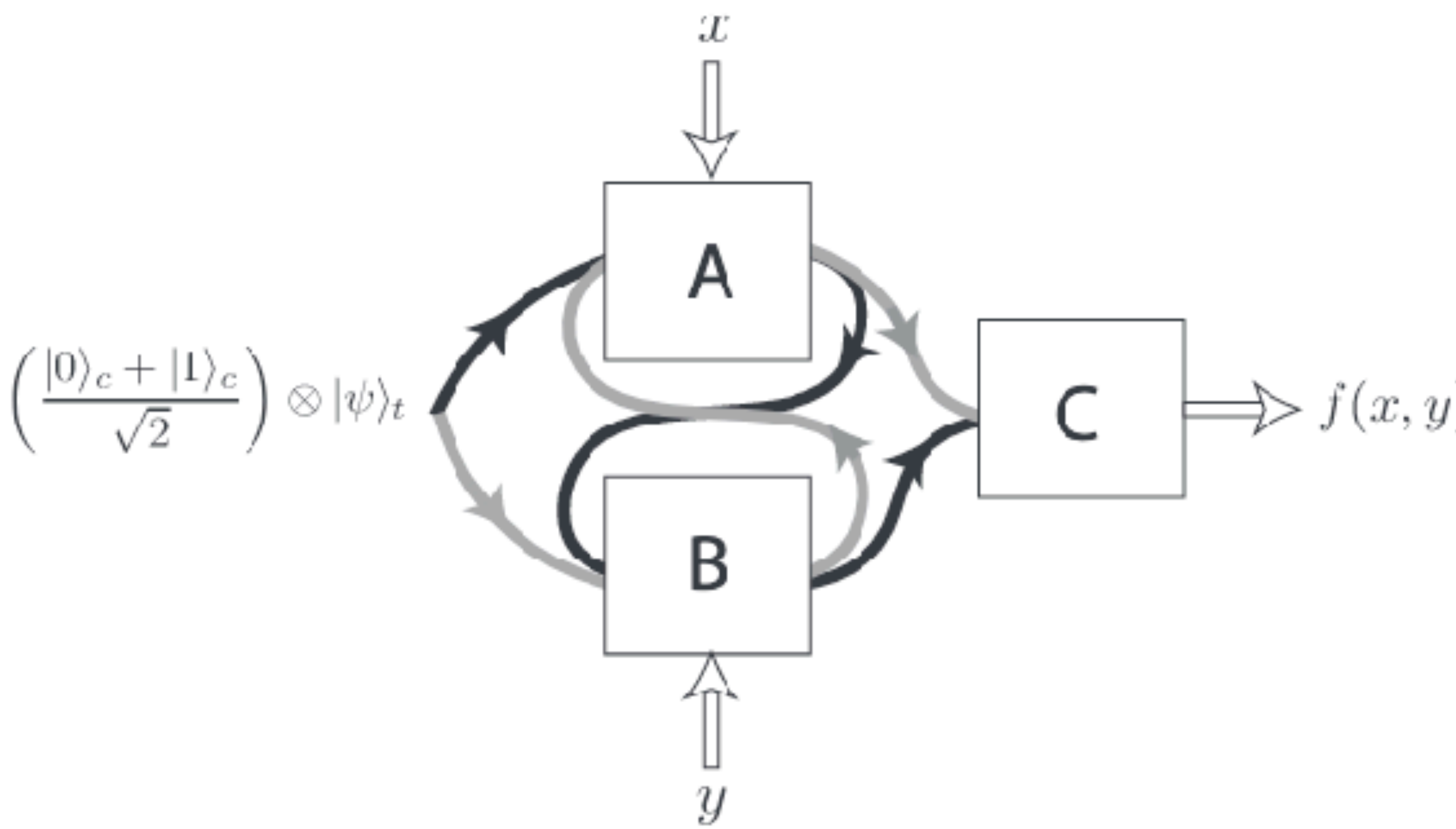
< 1 for every testing strategy where $\mathscr{A}$ and $\mathscr{B}$ are connected in a definite order.

Chiribella, PRA 86, 040301(R) (2012)

Extension to N channels:
Araujo, Costa, and Brukner, PRL 113, 250402 (2014)

REDUCING COMMUNICATION COMPLEXITY

The ability to distinguish between commuting and anticommuting channels is a primitive that can be used to ***reduce the amount of communication needed by 3 distant parties to compute a desired function.***



- Causally ordered: $O(N)$ bits
- SWITCH: $O(\log N)$ bits

$N =$ size of the input strings

Guerin, Feix, Araujo, and Brukner, PRL 117, 100502 (2016)

BEATING THE HEISENBERG LIMIT IN QUANTUM METROLOGY

Zhao and Chiribella, PRL 124, 190503 (2020)

BACK TO THE HARMONIC OSCILLATOR

Consider harmonic oscillator with position and momentum operators X and P, respectively, and Hamiltonian $H = (X^2 + P^2)/2$

Canonical commutation relation $[X, P] = i\,I$
or equivalently, $D_x D_p = e^{ixp}\, D_p D_x$
with $D_x = \exp[-ixP]$ and $D_p = \exp[ipX]$

Phase space area: $A = x\,p$

MEASURING THE PRODUCT OF TWO AVERAGE DISPLACEMENTS

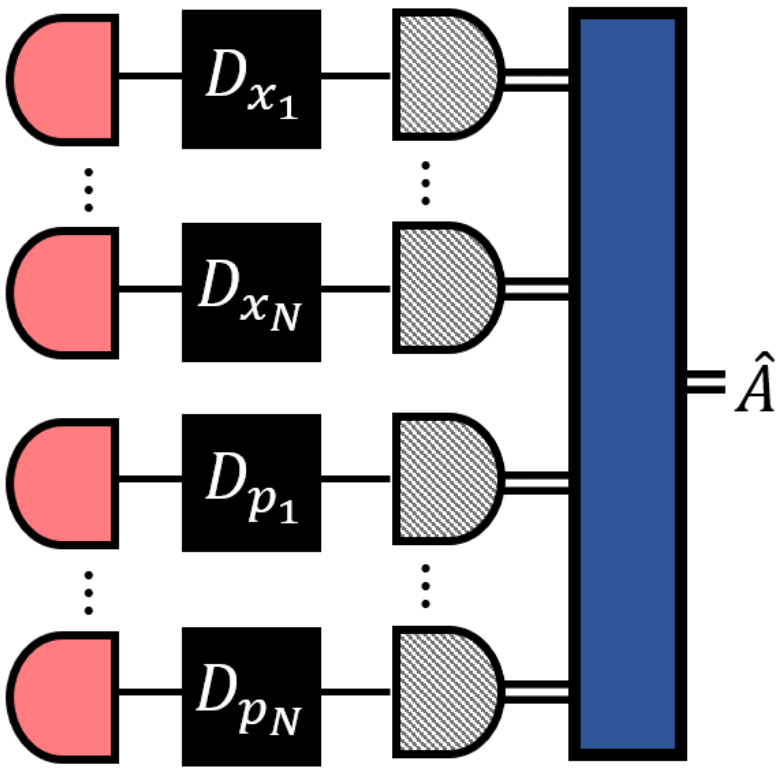
Settings: A harmonic oscillator is subject to N displacements, either of its position or of its momentum

$$D_{x_j} = \exp[-ix_j P] \qquad j \in \{1,\ldots,N\}$$
$$D_{p_k} = \exp[ip_k X] \qquad k \in \{1,\ldots,N\}$$

Task: estimate the product $A = \overline{x}\,\overline{p}$ of the average displacements $\overline{x} = \frac{1}{N}\sum_j x_j$ and $\overline{p} = \frac{1}{N}\sum_k p_k$

CAUSALLY ORDERED STRATEGIES (1)

Strategy 1: estimate each displacement independently



For a single displacement z,
the *Root Mean Square Error (RMSE)* is

$$\Delta z = \frac{1}{\sqrt{8\nu E}} \quad \text{with}$$

ν = number of repetitions of the experiment
$E = \langle X^2 + P^2 \rangle / 2$ = average energy of the probe

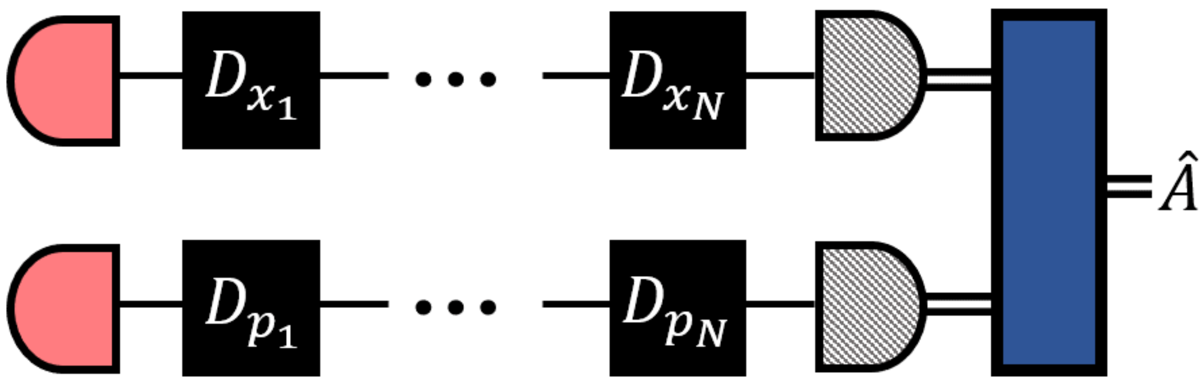
The averages and their product are computed classically.

RMSE for the product: $\Delta A = O\left(\frac{1}{\sqrt{\nu N}}\right)$

(standard quantum limit w.r.t. N)

CAUSALLY ORDERED STRATEGIES (2)

Strategy 2: directly estimate the two average displacements



For an average displacement $\bar{z}$, the *RMSE* is $\Delta\bar{z} = \dfrac{1}{\sqrt{8\nu E}\,N}$

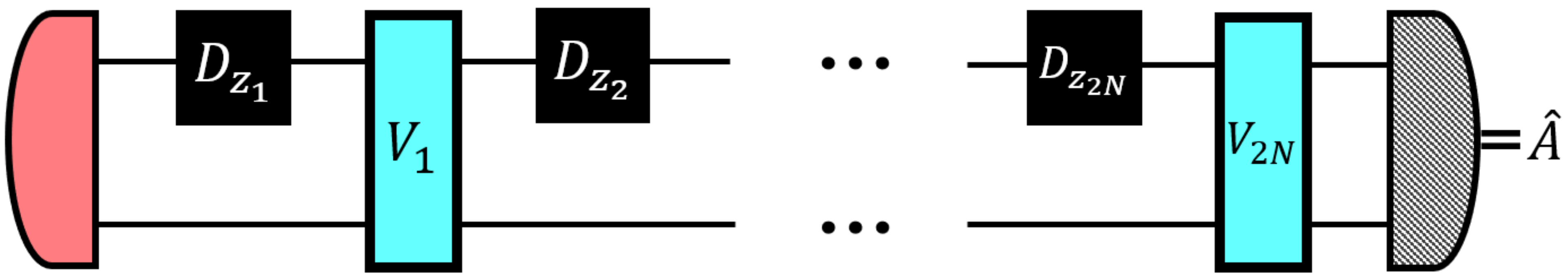
The product of $\bar{x}$ and $\bar{p}$ is computed classically.

RMSE for the product: $\Delta A = O\left(\dfrac{1}{\sqrt{\nu}N}\right)$

(Heisenberg limit w.r.t. N)

CAUSALLY ORDERED STRATEGIES (3)

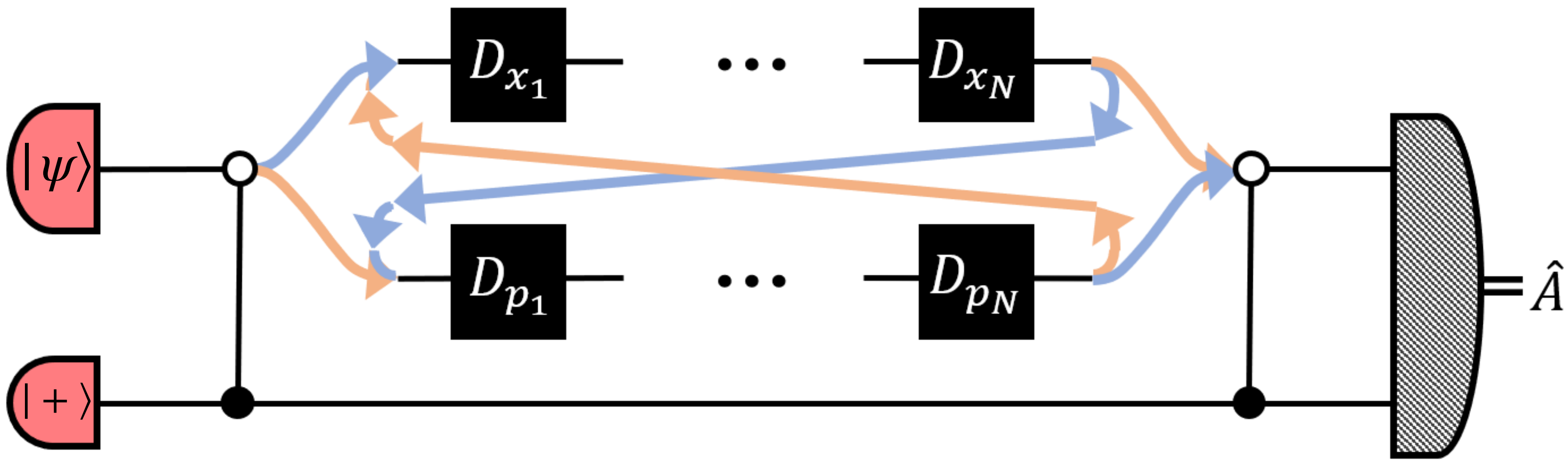
Most general causally ordered strategy



General bound on RMSE: $$\Delta A_{\text{fixed}} = \Omega\left(\frac{1}{\sqrt{\nu}N}\right)$$

No causally ordered scheme can beat Heisenberg limit w.r.t. N

SWITCH-ENHANCED ESTIMATION



Final state of the probe and control qubit:

$$\frac{1}{\sqrt{2}} D_{N\overline{p}} D_{N\overline{x}} |\psi\rangle \otimes |0\rangle + \frac{1}{\sqrt{2}} D_{N\overline{x}} D_{N\overline{p}} |\psi\rangle \otimes |1\rangle$$

$$= \; D_{N\overline{p}} D_{N\overline{x}} |\psi\rangle \otimes \frac{|0\rangle + e^{iN^2 A}|1\rangle}{\sqrt{2}}$$

ADVANTAGE OF THE QUANTUM SWITCH

For small A, the quantum SWITCH yields

$$\Delta A_{\text{switch}} = \frac{1}{\sqrt{\nu}N^2}$$

to be contrasted with the bound $\Delta A_{\text{fixed}} = \Omega\left(\frac{1}{\sqrt{\nu}N}\right)$ for general strategies with definite causal order.

In general, the quantum SWITCH enables estimation of the phase $\phi = \sum_{i,j} x_i p_j \mod 2\pi$ with error $\Delta\phi = \frac{1}{\sqrt{\nu}}$

whereas causally-ordered strategies have error $\Delta\phi = \Omega\left(\frac{N}{\sqrt{\nu}}\right)$

TOWARDS IMPLEMENTATIONS

- Applications to tests of canonical commutation relations and modification thereof.

- Experimental challenge: to implement *quantum SWITCH on a harmonic oscillator.*
  e.g.
  -vibrational modes of molecule, with path as control
  -axial modes of ion traps, with spin as control.

QUANTUM SHANNON THEORY ON QUANTUM SPACETIMES

(CLASSICAL) SHANNON THEORY:

The carriers of information are classical:
classical states, classical channels, classical spacetime

Claude E. Shannon

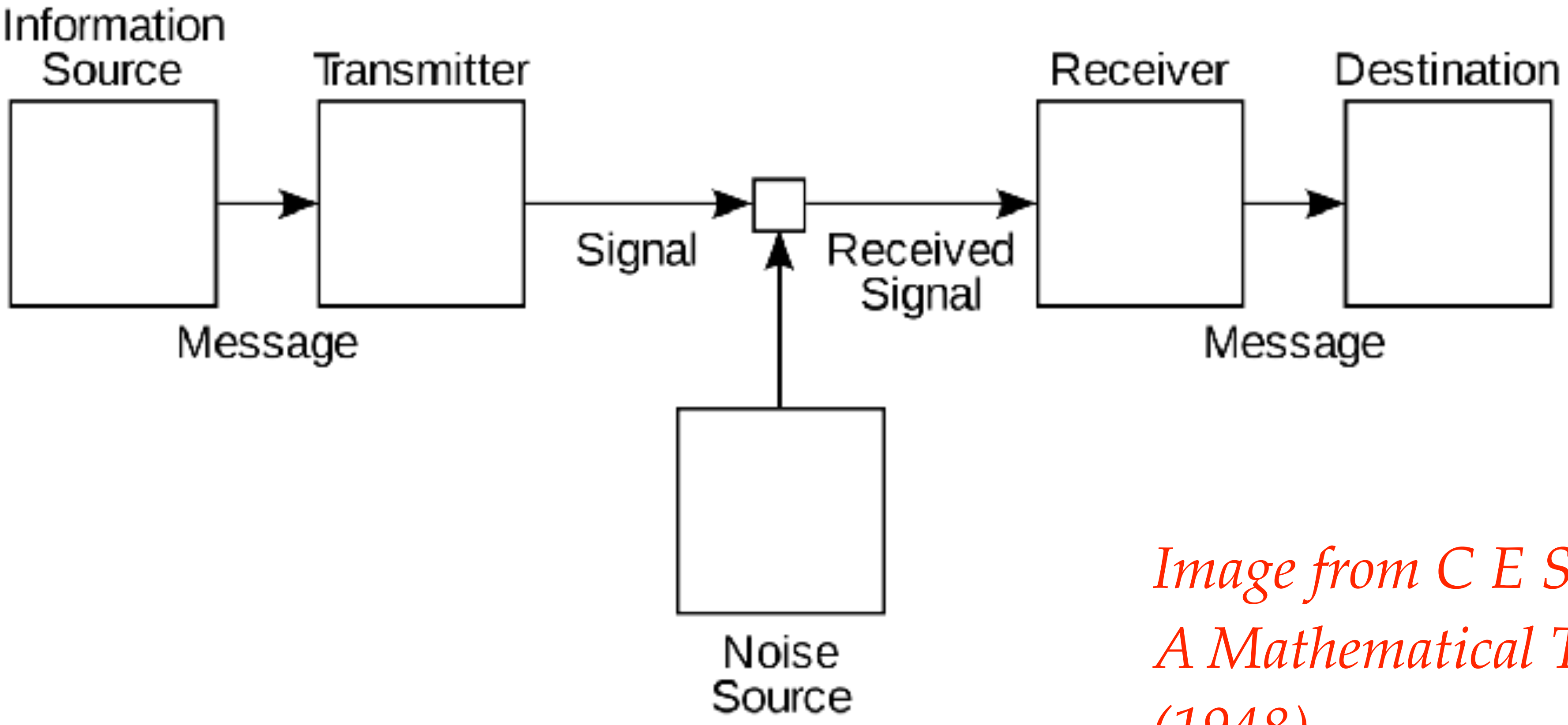


Image from C E Shannon, A Mathematical Theory of Communication (1948)

QUANTUM SHANNON THEORY

Allows the state of the information carriers and the channels be quantum.

Messages can be quantum:
not just strings of bits, like 0010110111,
but also quantum superpositions, like

$$|\Psi\rangle = \frac{|0010110111\rangle + |1010100011\rangle}{\sqrt{2}}$$

Still, spacetime is classical
and
the configuration of the communication channels is fixed.

Alexander Holevo

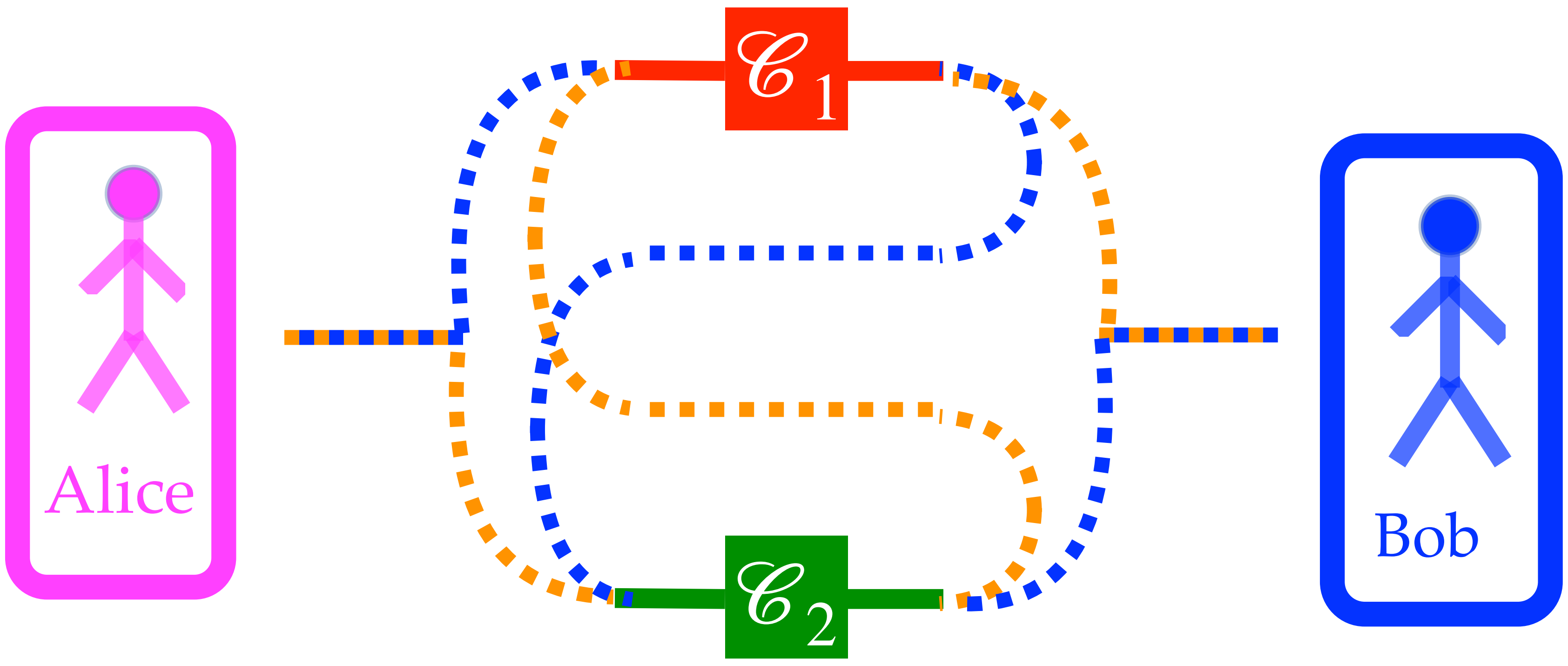
Benjamin Schumacher

Charles Bennett

Gilles Brassard

QUANTUM SHANNON THEORY ASSISTED BY QUANTUM SPACETIMES

Suppose that Alice and Bob are embedded in a superposition of spacetimes, so that the communication devices between them are placed in an indefinite order.



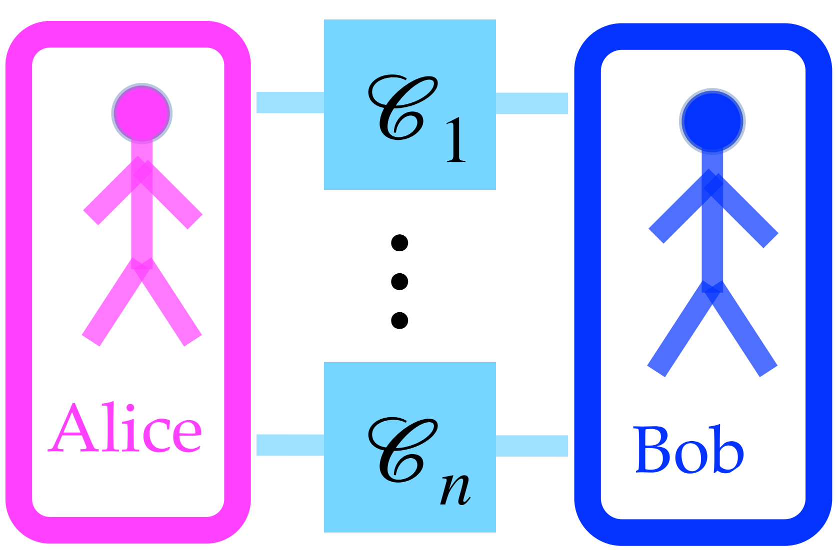
How is the exchange of information affected?

PLACEMENT OF THE CHANNELS

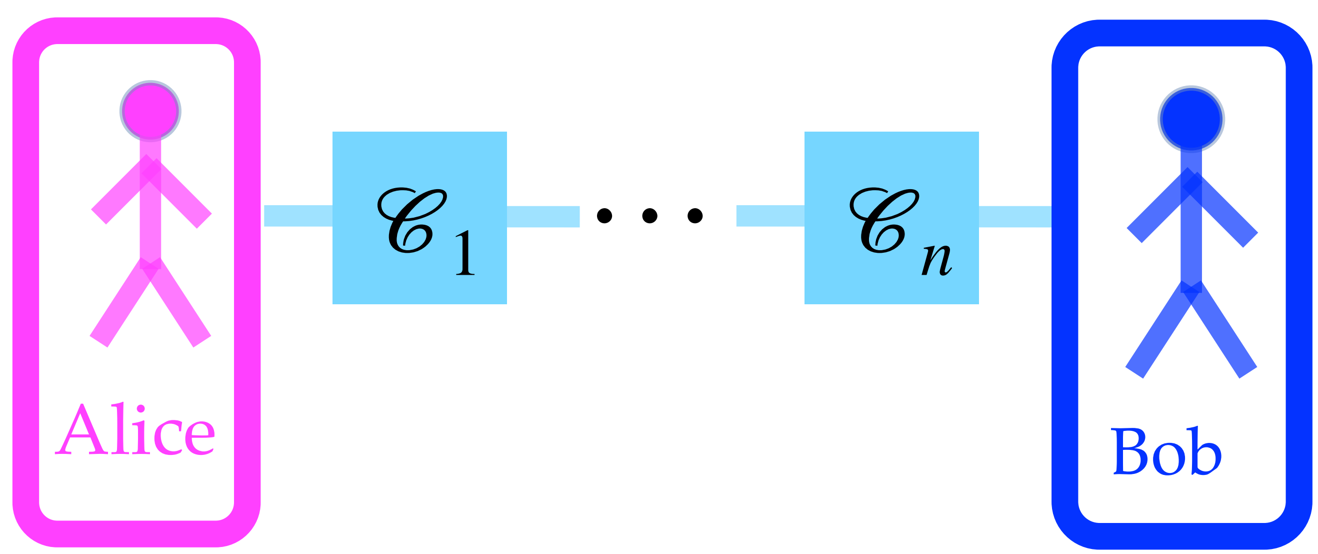
Suppose that the communication between a sender (Alice) and a receiver (Bob) uses n communication devices, corresponding to channels $\mathscr{C}_1, \mathscr{C}_2, \ldots, \mathscr{C}_n$

The structure of spacetime determines
how the devices are placed between Alice and Bob.

e.g. parallel placement



vs sequential placement



Idea: adding superposition of orders to the allowed placements.

SHANNON THEORETIC ADVANTAGES

- Model 1: parallel and sequential placement only
- Model 2: parallel, sequential, **and superposition of orders**

Model 2 outperforms Model 1 in several situations:

-classical data transmission:

Ebler, Salek, Chiribella, Phys. Rev. Lett. 120, 120502 (2018)

Goswami, Romero, White arXiv:1807.07383.

-quantum data transmission:

Salek, Ebler, Chiribella, arXiv:1809.06655

Chiribella et al, arXiv:1810.10457

CLASSICAL COMMUNICATION WITH COMPLETELY DEPOLARIZING CHANNELS

Depolarizing channel: $$\mathscr{A}(\rho) = \frac{I}{d} \qquad \forall \rho$$

- Without superposition of orders, no communication is possible.

- With superposition of orders, classical communication becomes possible at a rate of 0.0488 bits per channel(s) use.

Non-zero classical capacity eperimentally verified with more than 34.8 standard deviations (see Hefei experiment, later)

D. Ebler, S. Salek, G. Chiribella, Phys. Rev. Lett. 120, 120502 (2018)

PERFECT QUANTUM COMMUNICATION WITH ENTANGLEMENT-BREAKING CHANNEL

XY-channel: $$\mathscr{A}(\rho) = \frac{1}{2}(X\rho X + Y\rho Y)$$

- Without superposition of orders, every quantum superposition is decohered to a classical mixture.
  Quantum communication impossible!

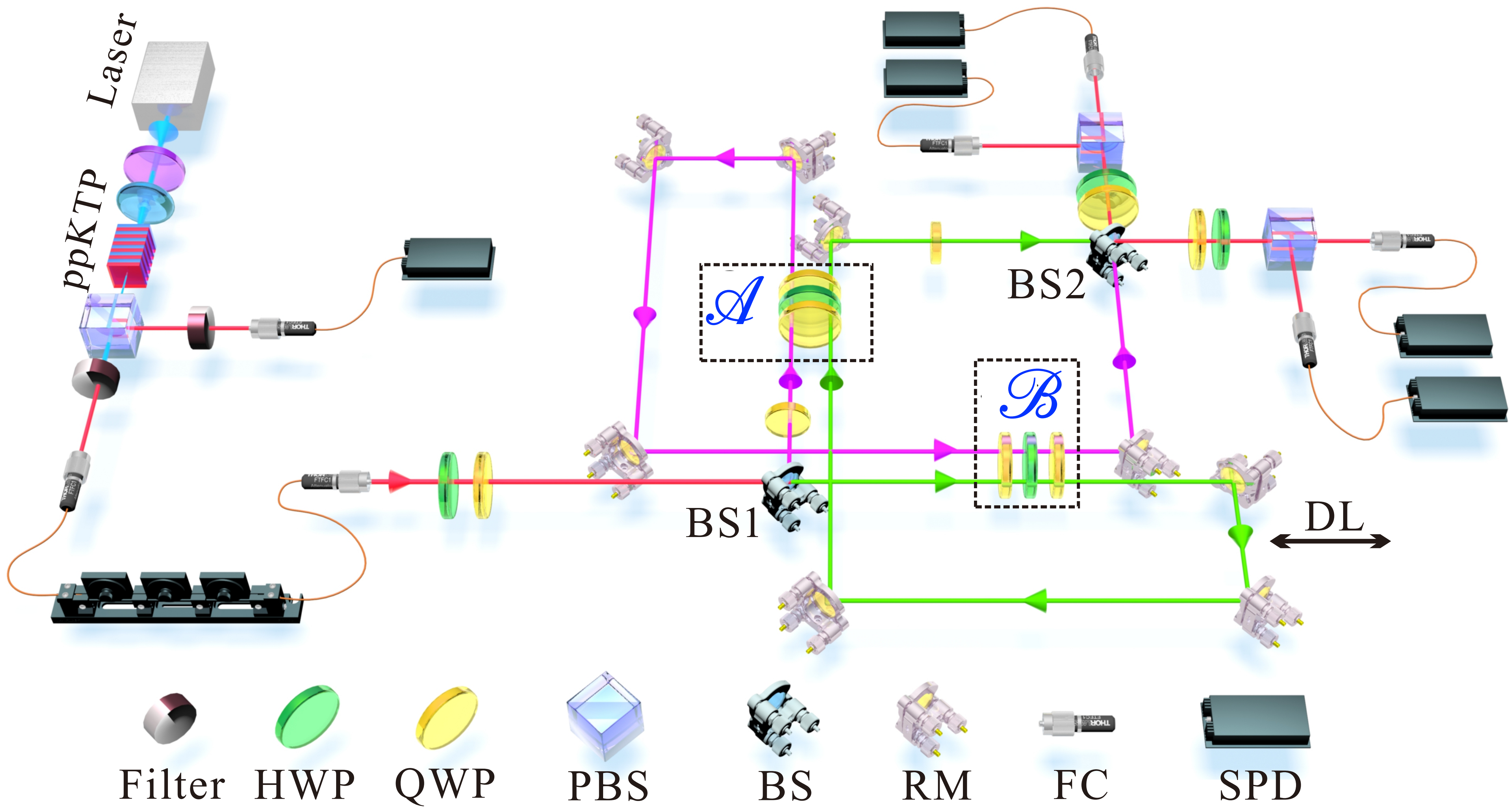
- With superposition of orders, the resulting channel is

$$\mathscr{C}(\rho) = \frac{1}{2}\rho \otimes |+\rangle\langle+| + \frac{1}{2}Z\rho Z \otimes |-\rangle\langle-|$$

Noiseless quantum communication possible!

Experimentally verified with fidelity higher than 98%

HEFEI EXPERIMENT



Y Guo, X-M Hu, Z-B Hou, H Cao, J-M Cui, B-H Liu, Y-F Huang, C-F Li, G-C Guo, G Chiribella, Phys. Rev. Lett. 124, 030502 (2020).

RELATED RESEARCH

The extension of quantum Shannon theory with superposition of orders has revived interest in other forms of superpositions:

-superpositions of communication channels

Gisin, Linden, Massar, Popescu, PRA 72, 012338 (2005).
Abbott, Wechs, Horsman, Mhalla, Branciard, arXiv:1810.09826
Chiribella and Kristjánsson, Proc. Royal Soc. A 475, 20180903 (2019)

-superpositions of encoding/decoding operations

Guerin, Rubino, Brukner PRA 99, 062317 (2019)

Much debated question: how much of the advantages of the quantum SWITCH is specific to the superposition of orders as opposed to being generic to all kinds of superpositions?

Some answers: Kristjánsson *et al*, New J. Phys. 22 073014 (2020)
Rubino *et al*, https://arxiv.org/abs/2007.05005

PHYSICAL REALIZATION / SIMULATION OF THE QUANTUM SWITCH

DIFFERENT LEVELS OF REALIZATION / SIMULATION

The quantum SWITCH is an abstract supermap, defined irrespectively of its physical realization.

Existing ways to "implement it" fall into 3 basic categories:

- implementations with closed time loops (real or simulated)
  CDP arXiv:0912.0195 and Phys. Rev. A 88, 022318 (2013),

- gravitational implementations with superpositions of masses
  Zych *et al*, Nat. Comm. 10, 3772 (2019)

- table-top implementations with known physics

REALIZATION OR SIMULATION?

All the 3 types of implementation produce the *same output* of the quantum SWITCH.
e.g. they produce the unitary evolution

$$S(U,V) := UV \otimes |0\rangle\langle 0| + VU \otimes |1\rangle\langle 1|$$

However, the physical mechanism that produces $S(U,V)$ from U and V is radically different.

Much debated question:
which mechanisms count as genuine physical realizations?

CONCLUSIONS

(1) Quantum supermaps: a way to explore extensions of quantum mechanics.

(2) The quantum SWITCH:
indefinite causal order and relation with time loops

(3) Information-processing advantages:
query complexity, communication complexity,
quantum metrology, quantum Shannon theory,
quantum thermodynamics…
Open problems: *proving* computational speedups,
getting closer to applications.

(4) Physical Realizations
Open problem: better understanding of realizations,
closer relation with quantum gravity?