Exploring the Origin of Quantum Advantages in Quantum Technologies

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IAMS

中央研究院 ACADEMIA SINICA 1928


學習、研究與人生
一個探險、探索與欣賞的過程
studying, research & life: adventure, exploration & appreciation

Significant Advancements in AMO Physics

躬逢其盛！
那個激動人心的年代！
The exciting ages !

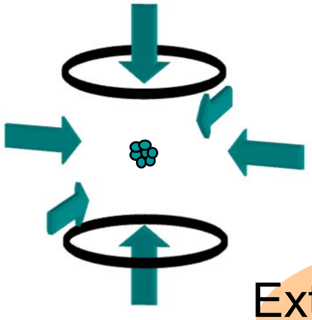

Norman Ramsey

20th ICAP, Innsbruck, Austria, July 2006

Norman Ramsey:
"I wish I were 29 instead of 92 years old such that I could participate in the exciting advancement of atomic physics".

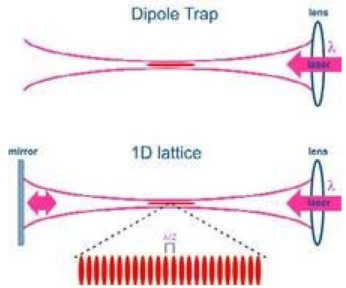
And now the ages of quantum science and technology!

Atom (& Molecules) & Photon Manipulation



Laser cooling
& trapping

Optical trap/
Optical lattices



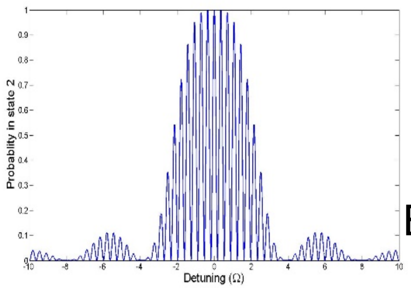
External degree of freedom

Environment

Atoms (molecules)

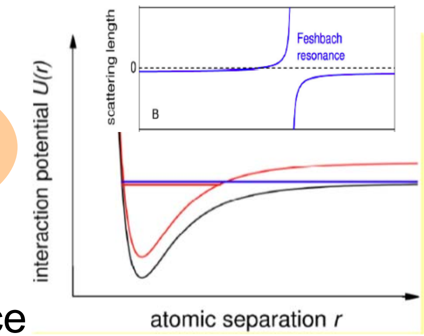
Internal degree of freedom

Atom-atom Interaction



Electromagnetic excitations

Magnetic-tuned
Feshbach resonance



τ<100 *atto sec.*

Ultra-short

Lasers (photons)

Ultra-intense

P>1*PW(10^{15}W)*

Precision~10^{-19}

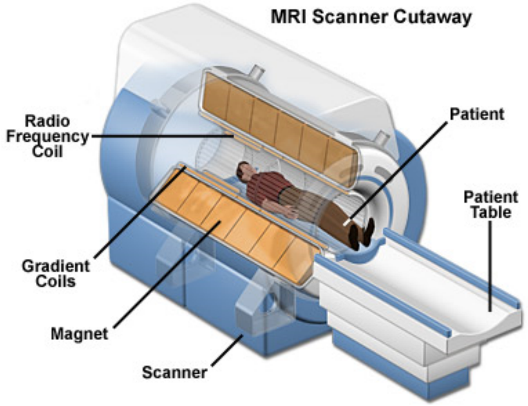
Ultra-stable

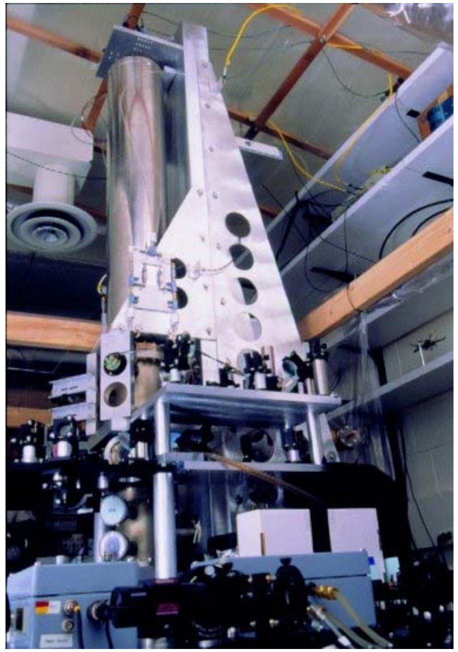
Non-classical

single photon
Photon pairs
Squeezed state

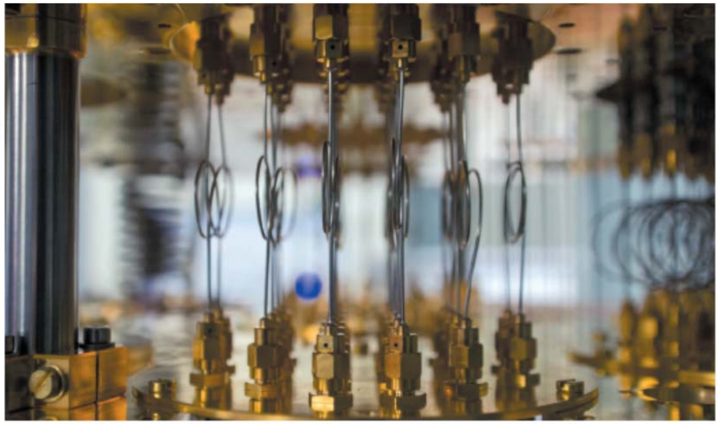
Quantum 1.0 in Modern Life

- Material design (e.g. quantum-well diode laser, transistor) based on Schrodinger equation.





The Dizzy News on Quantum Technologies

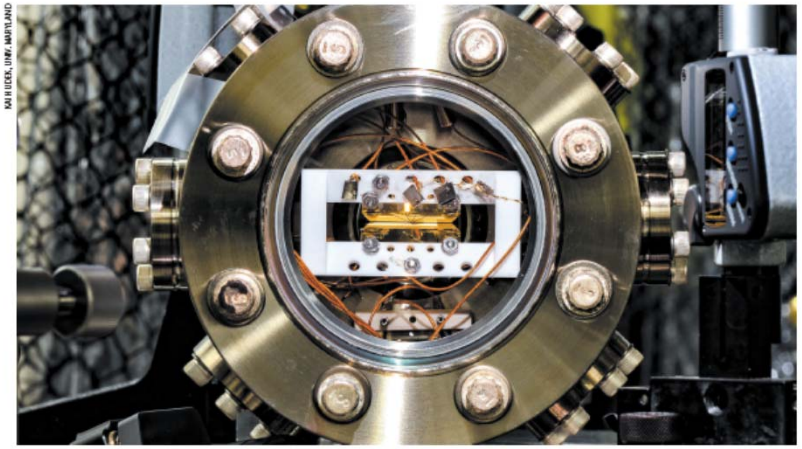


Superconducting lines for IBM's quantum computer.

QUANTUM COMPUTING

Nature 543,159 (2017)

Quantum cloud goes commercial

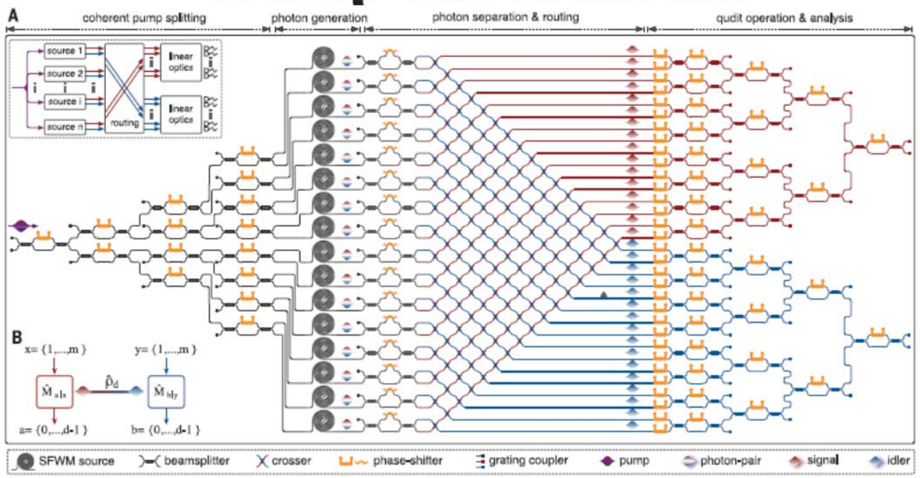


A front runner in the pursuit of quantum computing uses single ions trapped in a vacuum.

PHYSICS

Nature 541,9 (2017)

Quantum computers ready to leap out of the lab



Multidimensional quantum entanglement with large-scale integrated optics

Science 360, 285(2018)

Quantum 2.0 : Emphasize information technology

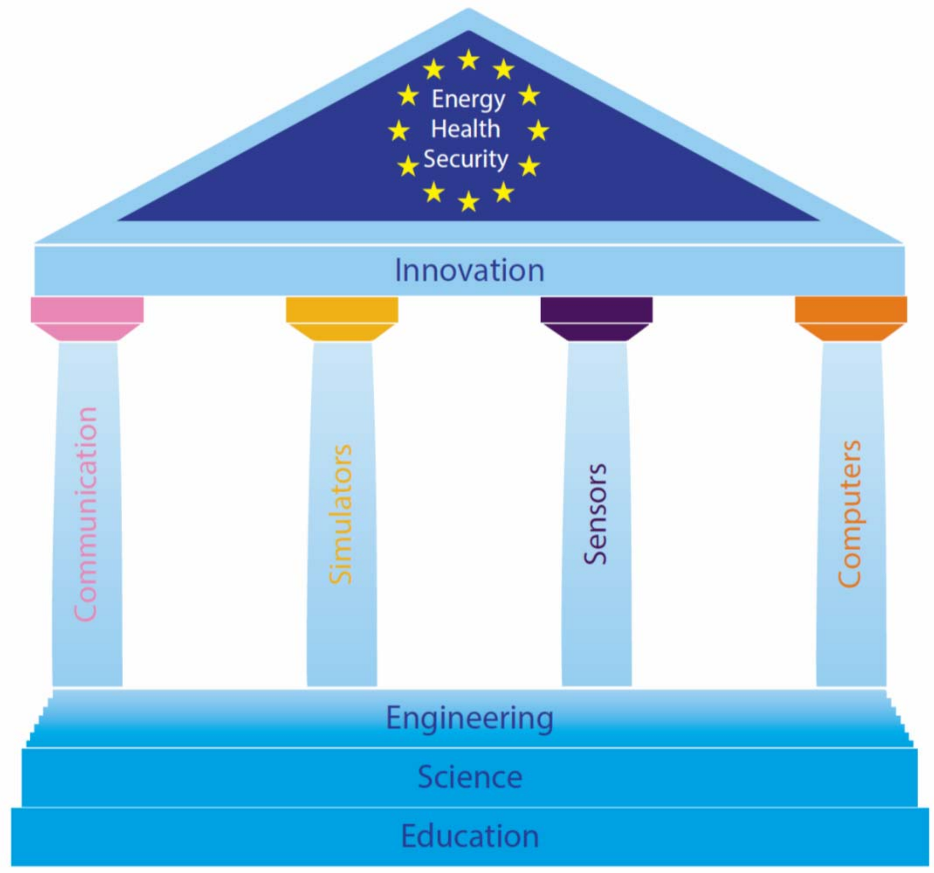
Key Technology of the 21st century

EU
QT Flagship initiative (2016)
Budge: 1 Billion Euro / 10 yr
10 Startup companies (IDQ, CQC, …)

China:
Satellite-Ground Quant. Comm.
City wide Q.N. (Jinan): 100 Billion RMB,

US
Nat'l Quantum Initiative Act (Dec. 2018)
DoE $625M / 5 yr for 2~5 QIS RC's
QC by IBM, Google, Intel, Microsoft,
Honeywell, IonQ, ColdQuanta, Rigetti…

CA: D-Wave, Xanadu
…



Elements of a European programme in quantum technologies.

Quantum 2.0: Turning Quantum Weirdness into Use

Wave-particle duality

Probability description

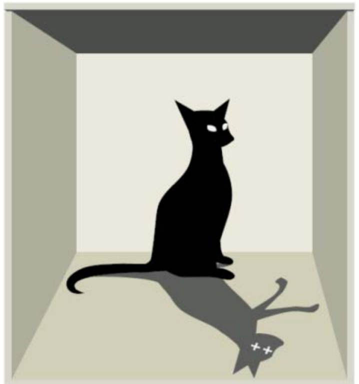
Coherent superposition

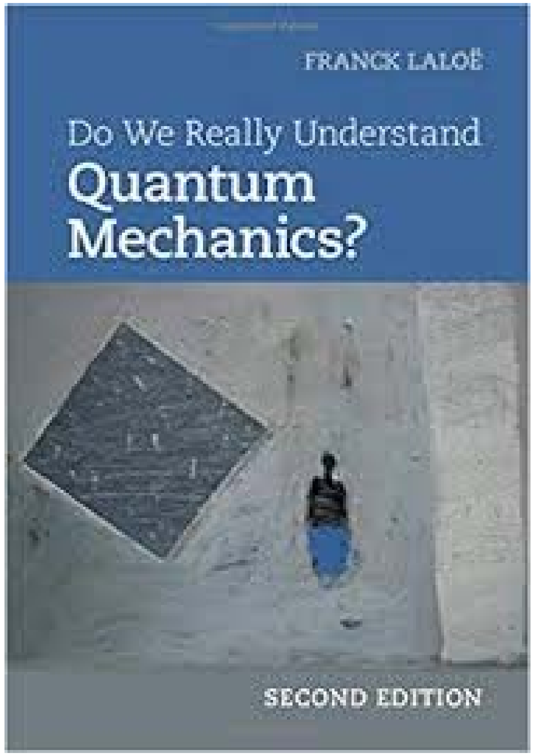
Uncertainty relation

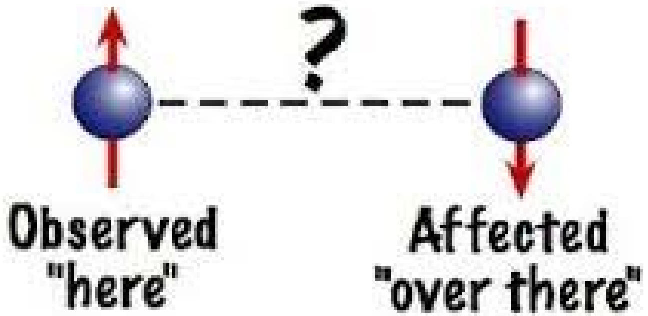
Quantum entanglement

Nonlocality

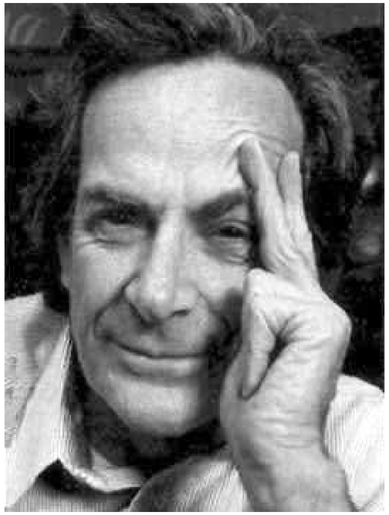
Non-classical correlation…







Quantum Weirdness: we all have these feelings



Richard P. Feynman (1982)

We have always had a great deal of difficulty in understanding the world view that quantum mechanics represents…

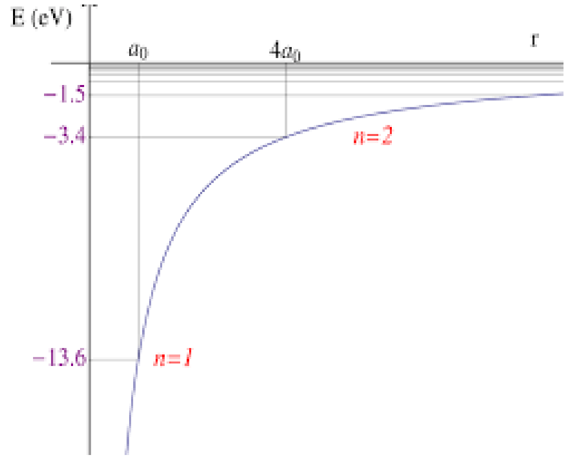
…Okay, I still get nervous with it…

It has not yet become obvious to me that there is no real problem. I cannot define the real problem, therefore I suspect there's no real problem, but I'm not sure there's no real problem.

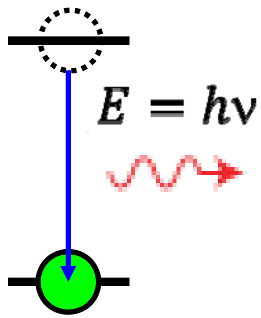
Quantum 1.0:
Shut up and calculate !

Quantum 2.0:
Shut up and just use it !

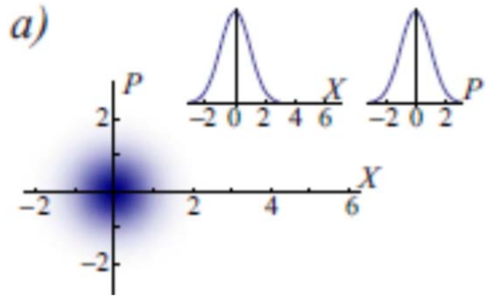
Different meaning of “Quantum” in different places



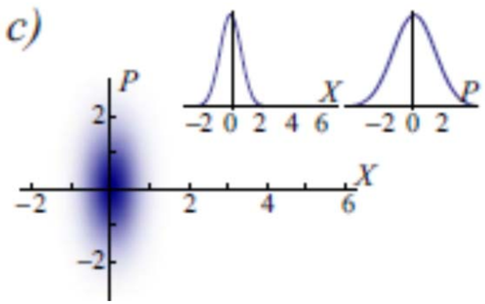
Hydrogen atom:
quantized energy levels
Atomic clock



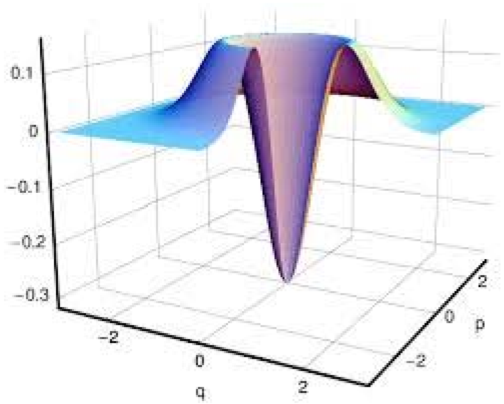
Quanta: basic unit
Photon, electron,
atom…



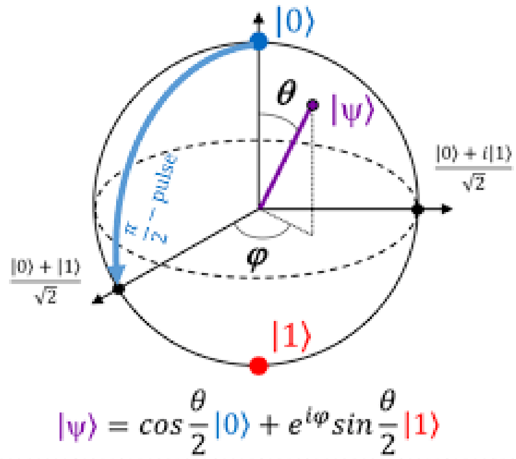
Vacuum state
Quantum fluctuation
Uncertainty relation
Commutation relation



Squeezed state

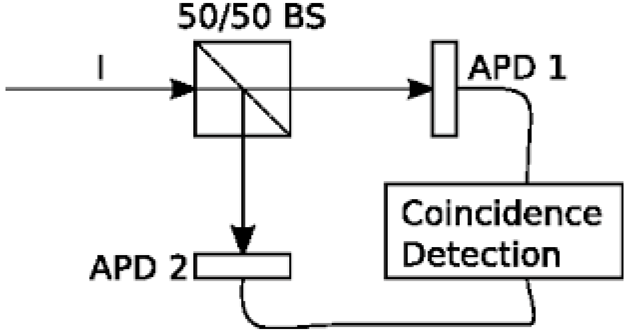


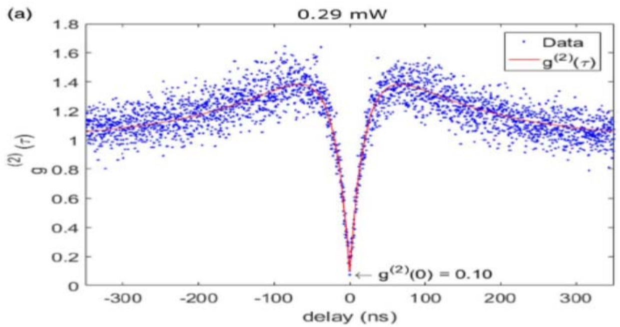
Negativity in Winger
function, n=1 Fock state



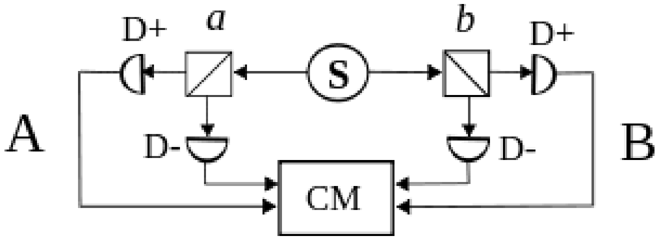
Bloch sphere: 2-level system
Coherent superposition, qubit
Phase/ off-diagonal coherence

Second-order correlation function,
$g^{(2)}(\tau=0)<1$ for single photon



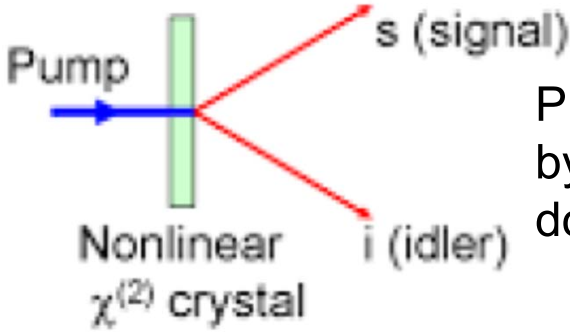


CHSH Bell inequality
Nonlocality, entanglement
QKD, foundation of QM

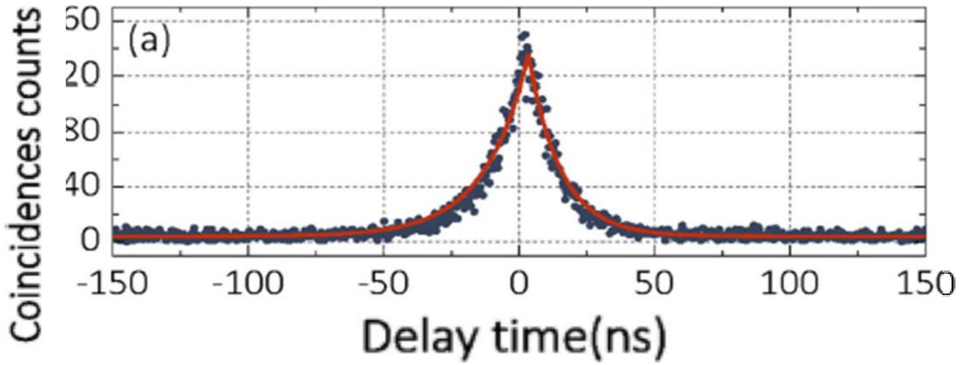


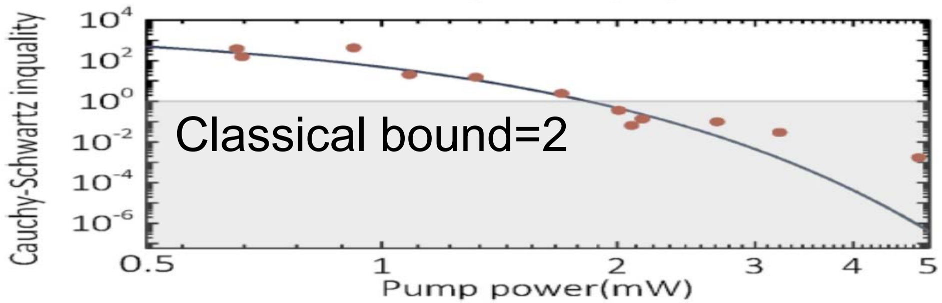
$$E=\frac{N_{++}-N_{\pm-}-N_{-+}+N_{--}}{N_{++}+N_{\pm-}+N_{-+}+N_{--}}$$

a,b: polarization angle



Photon-pair generation
by spontaneous parametric
down-conversion (SPDC)

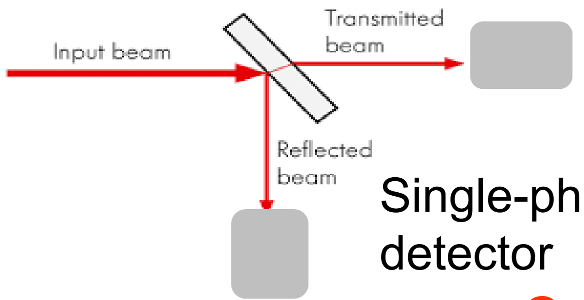




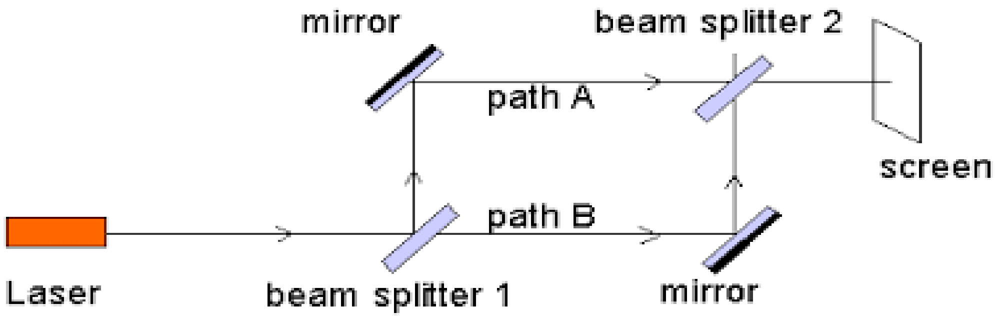
S=E(a,b)-E(a,b')+E(a',b)+E(a',b'),
|S|≤2, local realism,
$S = 2\sqrt{2}$ for (a,a',b,b')=($0,45^0,22.5^0,67.5^0$),
Quantum Mechanics

Meet many "Quantum" by a piece of glass !

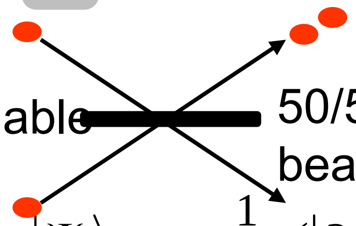
1. The single photon "quanta" by $g^{(2)}$ measurement.
2. Intrinsic randomness !
3. Path-superposition state!
4. Quantum vacuum injected by the open port.
5. Wave-particle duality: photon interfere with itself
6. Hong-Ou-Mandel two-photon interference.
7. Even more quantum (projective measurement/wavefunction collapse, uncertainty relation) in addition with three pieces of polarizer.



Single-photon detector

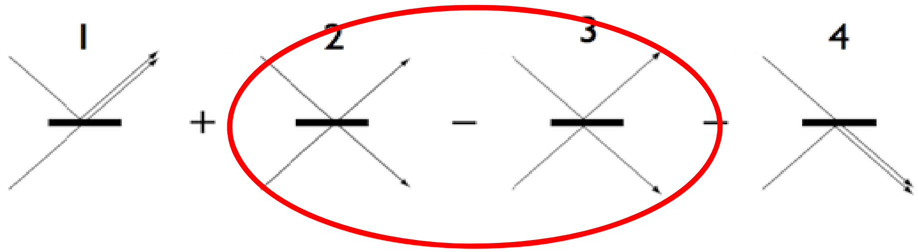


two indistinguishable photons

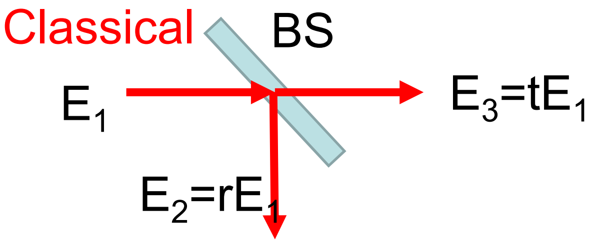


50/50 beam splitter

$$|\Psi\rangle_{out} = \frac{1}{\sqrt{2}}(|2,0\rangle - |0,2\rangle)$$



Notes on Lossless Beam Splitter in Quantum Optics



Analog from classical to quantum case violates the commutation relation !

$$\hat{a}_2 = r\hat{a}_1,\ \hat{a}_3 = t\hat{a}_1 \Rightarrow [\hat{a}_2, \hat{a}_3^{+}] = rt^{*} \neq 0$$

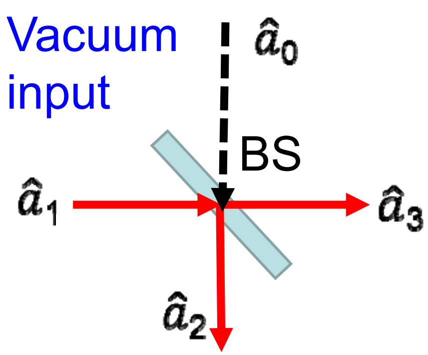
Why so? Need to consider the vacuum input port !

$$a_2 = ra_1 + ta_0; a_3 = ta_1 + ra_0$$

$$\Rightarrow [a_2, a_2^{+}] = [ra_1 + ta_0, r^{*}a_1^{+} + t^{*}a_0^{+}] = |r|^2[a_1, a_1^{+}] + |t|^2[a_0, a_0^{+}] + rt^{*}[a_1, a_0^{+}] + tr^{*}[a_0, a_1^{+}] = 1$$

$$if\ (|r|^2 + |t|^2) = 1$$

Quantum BS



Commutation relations

$$[\hat{a}_i, \hat{a}_j] = 0; [\hat{a}_i^{+}, \hat{a}_j^{+}] = 0;$$
$$[\hat{a}_i, \hat{a}_j^{+}] = \delta_{ij}$$

$$\begin{bmatrix} \hat{a}_2 \\ \hat{a}_3 \end{bmatrix} = \begin{bmatrix} t & r \\ t & r \end{bmatrix} \begin{bmatrix} \hat{a}_0 \\ \hat{a}_1 \end{bmatrix}$$

$$B^{+}B = 1 \Rightarrow \begin{pmatrix} t^{*} & r^{*} \\ r^{*} & t^{*} \end{pmatrix} \begin{pmatrix} t & r \\ r & t \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \Rightarrow |t|^2 + |r|^2 = 1; tr^{*} + rt^{*} = 0$$

$$t = |t|e^{i\delta_t}; r = |r|e^{i\delta_r} \Rightarrow e^{i(\delta_t - \delta_r)}[1 + e^{2i(\delta_r - \delta_t)}] = 0 \Rightarrow \delta_r - \delta_t = \pi/2$$

$$choose\ , \delta_t = 0 \Rightarrow \delta_r = \pi/2,\ for\ 50/50\ BS \Rightarrow B = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & i \\ i & 1 \end{pmatrix}$$

General case

$$B = \begin{bmatrix} |t| & i|r| \\ i|r| & |t| \end{bmatrix}$$

What should we ask behind these fancy news?

- What problem do you want to solve ?
- Why quantum can help?
  Where does the quantum advantage come from?
- How much quantum can help ?
- What's the requirement to gain quantum advantage?
- How far away from gaining those advantages?
  Where do we still need to improve?
- Does it helps to know more on foundation of QM ?
- What insight do you learn ?
- …

Where does quantum advantage come from?

- Case study

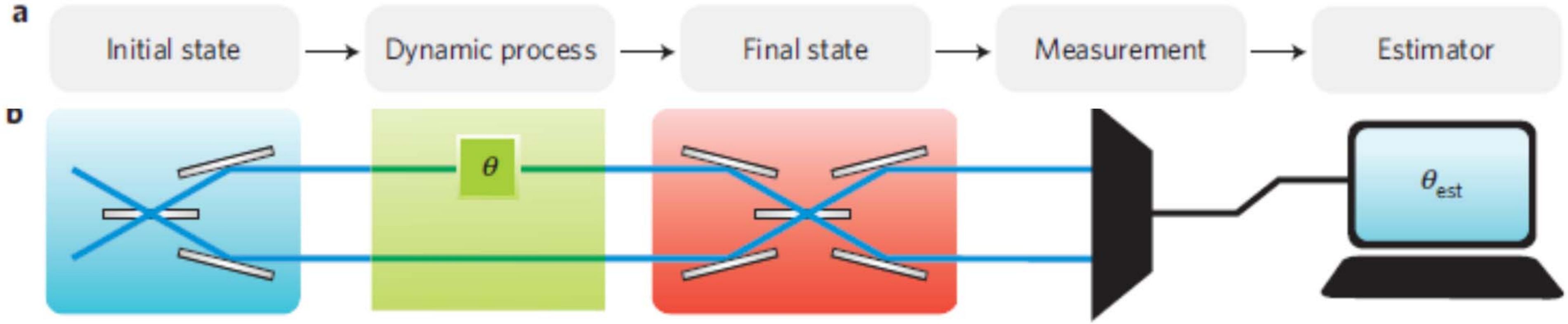
1. Many-body GHZ (or NOON) entangled state & squeezed state in quantum metrology.

2. "Interaction-free" measurement

3. SPDC photon pairs, Quantum correlation, Quantum-enhanced Radar & Imaging.

4. Quantum repeater/ Twin-field QKD (no time)

General Concepts of Quantum Metrology



Fisher information $I(\theta) = N\int [\frac{\partial}{\partial\theta}\log\Lambda(x;\theta)]^2 \Lambda(x;\theta)dx = N\int \frac{1}{\Lambda}[\frac{\partial\Lambda}{\partial\theta}]^2 dx$

Probability distribution of detected photons

parameter to be determined

Measurement outcomes

Cramer-Rao bound

$$(\Delta\theta)^2 = \int dx\Lambda(x)[\hat{\theta}(x) - \theta]^2 \geq \frac{1}{I(\theta)}$$

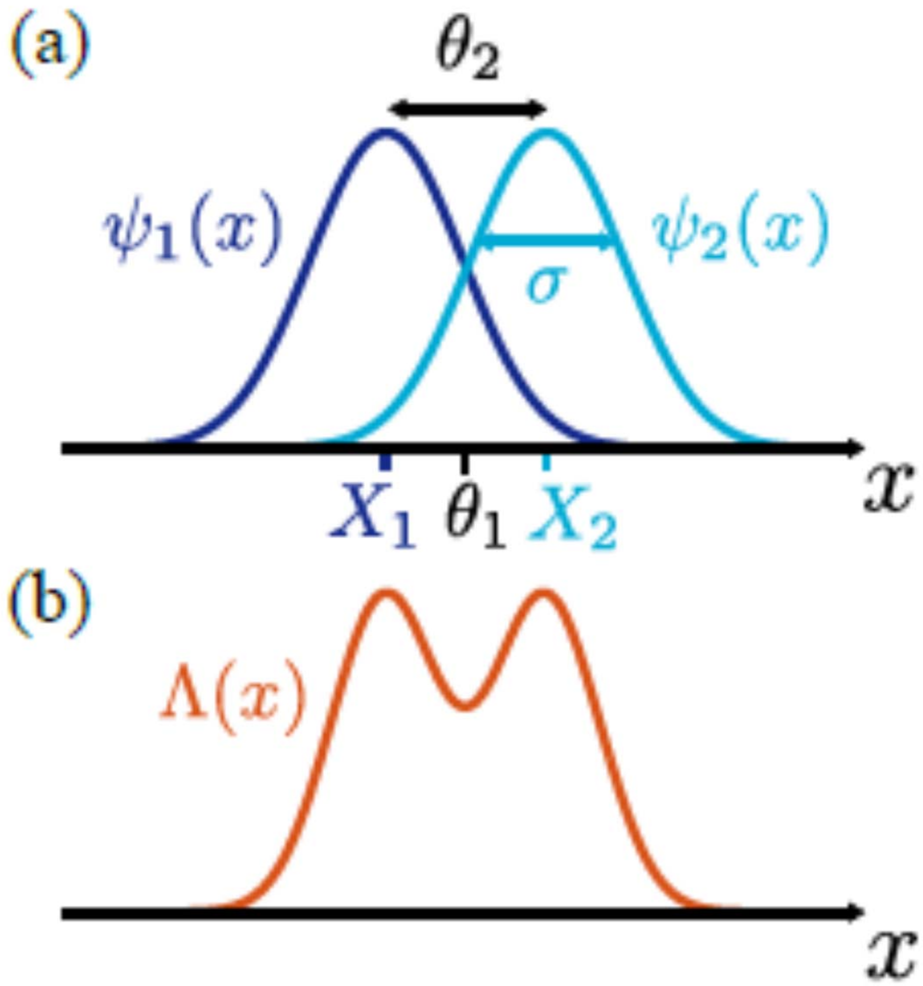
Can be generalized to multiple unknowns and quantum version

C.W. Helstrom (1969)
M Tsang, CM Caves (2011)

Notes: Uncertainy relation, SQL, LIGO

Nat. Phys. 7, 406(2011) , Rev. Mod. Phys. 90,035005(2018), J. Statis Phys, 1,231(1969)

An example: Rayleigh's Criterion



Mathematical Notes

- Cramer-Rao bound: inverse of the Fisher information is a lower bound on the variance of unbiased estimator θ.

Unbiased: $\int (\hat{\theta}(x)-\theta) f(\theta;x)dx = 0$

It holds independent of θ:

$$0=\frac{\partial}{\partial\theta}(\int(\hat{\theta}(x)-\theta)f(x;\theta)dx=\int(\hat{\theta}(x)-\theta)\frac{\partial f}{\partial\theta}dx-\int f(x;\theta)dx \Rightarrow \int(\hat{\theta}(x)-\theta)\frac{\partial f}{\partial\theta}dx=1$$

Because $\int f dx = 1$ And using $\frac{\partial f}{\partial\theta} = f\frac{\partial \log f}{\partial\theta}$

We have $\int(\hat{\theta}(x)-\theta) f\frac{\partial \log f}{\partial\theta}dx = 1 = \int[(\hat{\theta}(x)-\theta)\sqrt{f}][\sqrt{f}\frac{\partial \log f}{\partial\theta}]dx$

Using Cauchy-Schwarz inequality & the integral is actually the matrix product over all data points

$$1=\{\int[(\hat{\theta}(x)-\theta)\sqrt{f}][\sqrt{f}\frac{\partial \log f}{\partial\theta}]dx\}^2 \le [\int(\hat{\theta}(x)-\theta)^2 f dx][\int(\frac{\partial \log f}{\partial\theta})^2 f dx]$$

Thus $Var[\hat{\theta}] \ge \frac{1}{I(\theta)}$

Notes on Quantum Fisher Information

2. Quantum Fisher information matrix

2.1. Definition

Consider a vector of parameters $\vec{x} = (x_0, x_1, ..., x_a, ...)^{\mathrm{T}}$ with x_a the ath parameter. $\vec{x}$ is encoded in the density matrix $\rho = \rho(\vec{x})$. In the entire paper we denote the QFIM as $\mathcal{F}$, and an entry of $\mathcal{F}$ is defined as [1, 2]

$$\mathcal{F}_{ab} := \frac{1}{2}\mathrm{Tr}\left(\rho\{L_a, L_b\}\right), \tag{1}$$

where $\{\cdot,\cdot\}$ represents the anti-commutation and L_a (L_b) is the symmetric logarithmic derivative (SLD) for the parameter x_a (x_b), which is determined by the equation[6]

$$\partial_a \rho = \frac{1}{2}\left(\rho L_a + L_a \rho\right). \tag{2}$$

The SLD operator is a Hermitian operator and the expected value $\mathrm{Tr}(\rho L_a) = 0$. Utilizing the equation above, $\mathcal{F}_{ab}$ can also be expressed by [24]

$$\mathcal{F}_{ab} = \mathrm{Tr}\left(L_b \partial_a \rho\right) = -\mathrm{Tr}(\rho \partial_a L_b). \tag{3}$$

Based on equation (1), the diagonal entry of QFIM is

$$\mathcal{F}_{aa} = \mathrm{Tr}\left(\rho L_a^2\right), \tag{4}$$

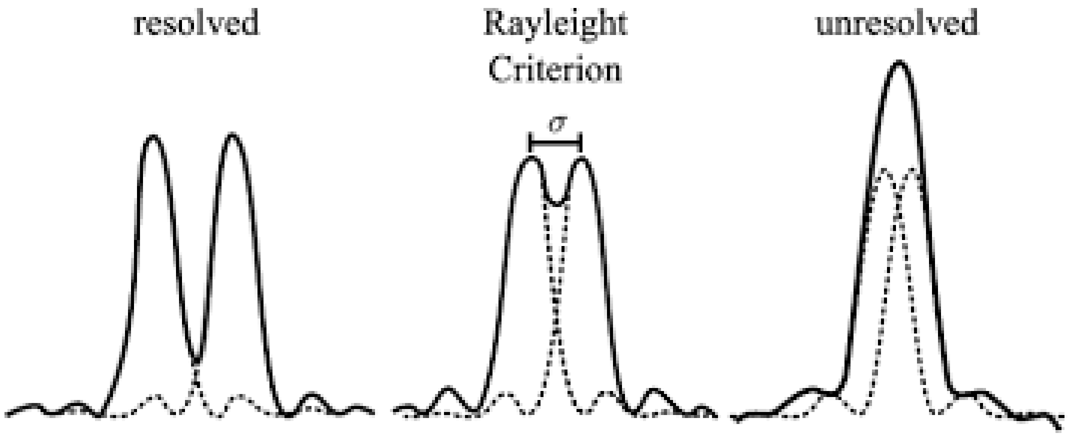
which is exactly the QFI for parameter x_a.

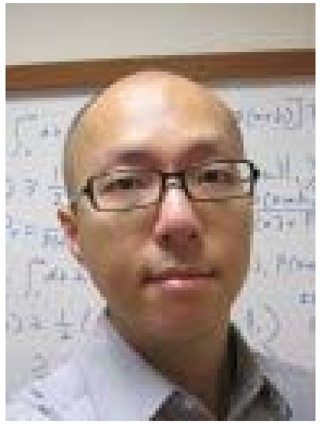
Note: C N Yang's joke

For a review on Quantum Fisher information, see J. Phys. A: 53, 023001(2020)

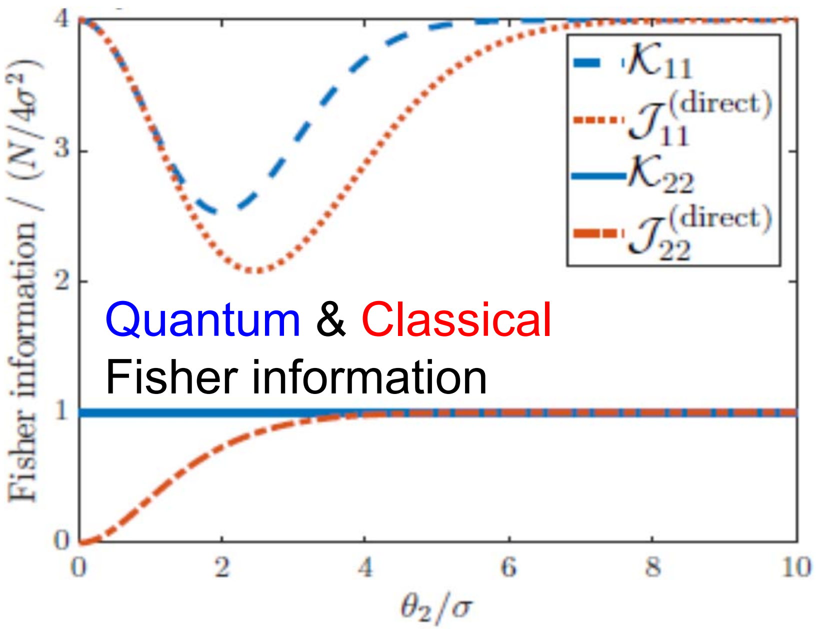
Surprise by Q. Fisher Analysis: Beat Rayleigh's Curse

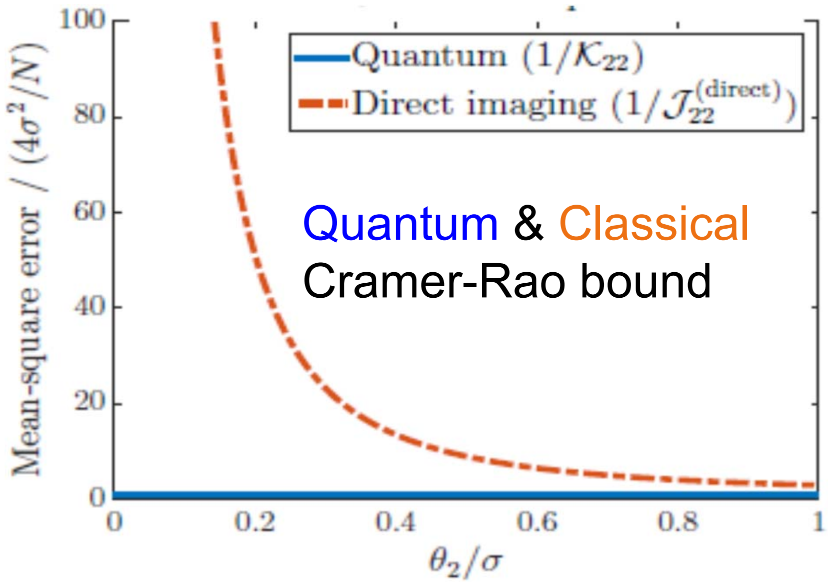
Mankei Tsang (2015) : The quantum Fisher information maintains a constant value for any separation of two incoherent sources if optimal measurement is performed.





Mankei Tsang（曾文祺）





Optica 2, 646(2015); PRX 6, 031033(2016)

Proof-of-Principle Experiment:

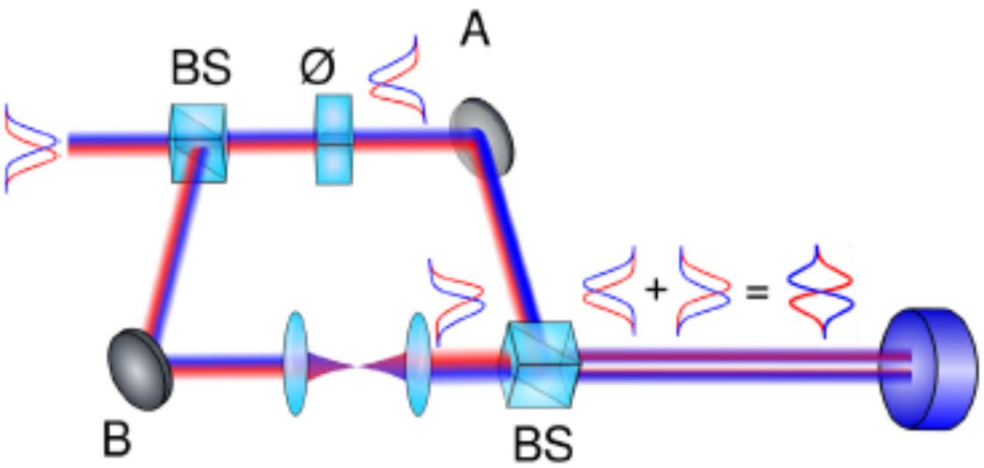
$$E_s(\rho) := \frac{E(\rho)+E(-\rho)}{2} = \frac{S}{2}\left[\psi\left(\rho+\frac{\mathbf{d}}{2}\right)+\psi\left(\rho-\frac{\mathbf{d}}{2}\right)\right]$$

$$E_a(\rho) := \frac{E(\rho)-E(-\rho)}{2} = \frac{D}{2}\left[\psi\left(\rho+\frac{\mathbf{d}}{2}\right)-\psi\left(\rho-\frac{\mathbf{d}}{2}\right)\right].$$

$$I_s(\rho_s) = \frac{|S|^2}{4}\left[\left|\psi\left(\rho_s+\frac{\mathbf{d}}{2}\right)\right|^2+\left|\psi\left(\rho_s-\frac{\mathbf{d}}{2}\right)\right|^2+2I_{\text{int}}(\rho_s,\mathbf{d})\right],$$

$$I_a(\rho_a) = \frac{|D|^2}{4}\left[\left|\psi\left(\rho_a+\frac{\mathbf{d}}{2}\right)\right|^2+\left|\psi\left(\rho_a-\frac{\mathbf{d}}{2}\right)\right|^2-2I_{\text{int}}(\rho_a,\mathbf{d})\right],$$

$$I_{\text{int}}(\rho,\mathbf{d}) = \mathrm{Re}\,\psi^*\left(\rho+\frac{\mathbf{d}}{2}\right)\psi\left(\rho-\frac{\mathbf{d}}{2}\right)$$



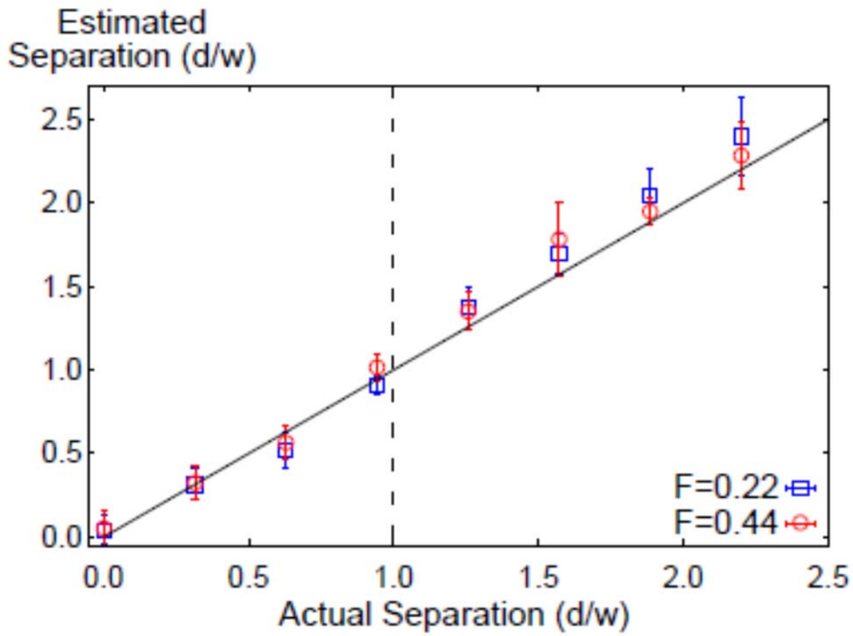
Opt Exp 24, 254222 & 24,268580(2016)

$$\mathbb{E}[N_s|S] = \frac{|S|^2}{2}\int d\rho_s I_s(\rho_s) = \frac{|S|^2}{2}[1+\delta(d)],$$

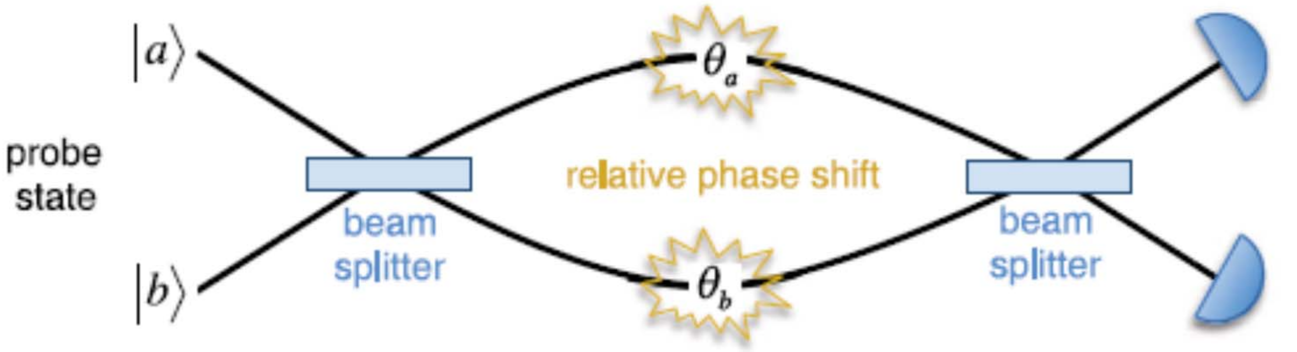
$$\mathbb{E}[N_a|D] = \frac{|D|^2}{2}\int d\rho_a I_a(\rho_a) = \frac{|D|^2}{2}[1-\delta(d)],$$

Measure the ratio of Ns and Na, one can determine δ(d) and thus d.

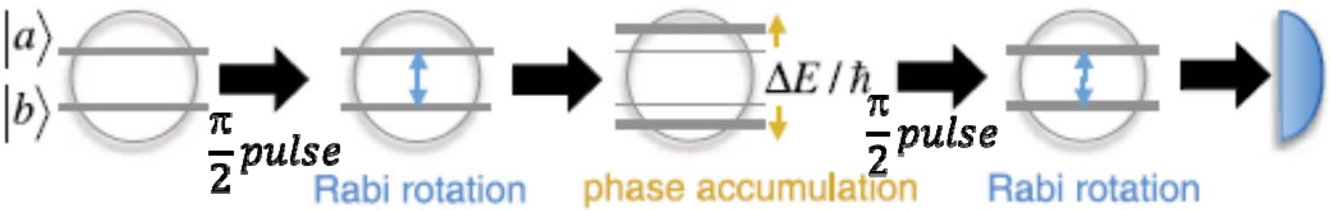
The antisymmetric part has no Rayleigh's curse.



Interferometric Phase Measurement



Mach-Zhender Interferometer



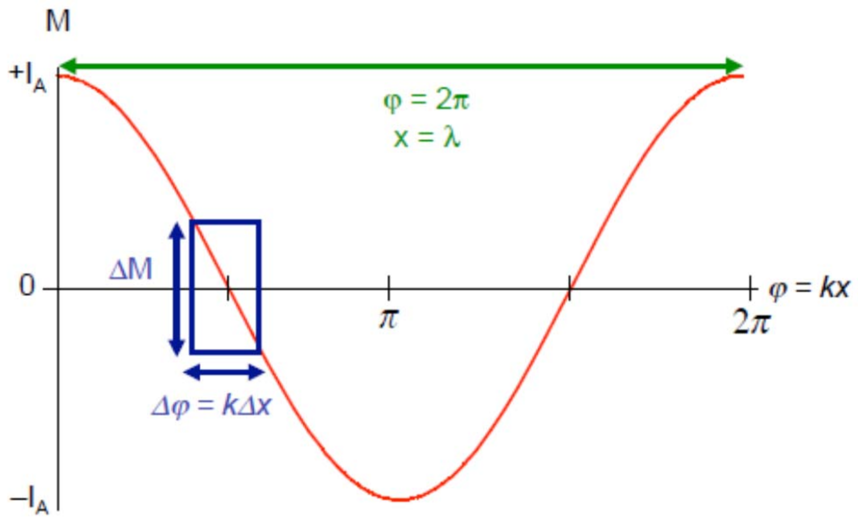
Ramsey Interferometer

Measure the population difference

$$\hat{M} = |a\rangle\langle a| - |b\rangle\langle b|$$

Mean $\quad \langle \hat{M} \rangle = \cos\theta$

Variance $\quad \Delta^2 \hat{M} = \sin^2\theta$



L. Pezze et. al. Rev. Mod. Phys. 90,035005(2018)

Notes

$$|\psi_{out}\rangle = \underbrace{\frac{1}{\sqrt{2}}\begin{bmatrix}1 & i\\ i & 1\end{bmatrix}}_{\text{50/50 beam splitter}} \underbrace{\begin{bmatrix}e^{i\theta/2} & 0\\ 0 & e^{-i\theta/2}\end{bmatrix}}_{\text{Phase shift to Be measured}} \underbrace{\frac{1}{\sqrt{2}}\begin{bmatrix}1 & i\\ i & 1\end{bmatrix}}_{\text{50/50 beam splitter}} \underbrace{\begin{bmatrix}1\\ 0\end{bmatrix}}_{\text{Input state}} = i\begin{bmatrix}\sin(\frac{\theta}{2})\\ \cos(\frac{\theta}{2})\end{bmatrix}$$

Different textbook might has a different expression due to the phase choice

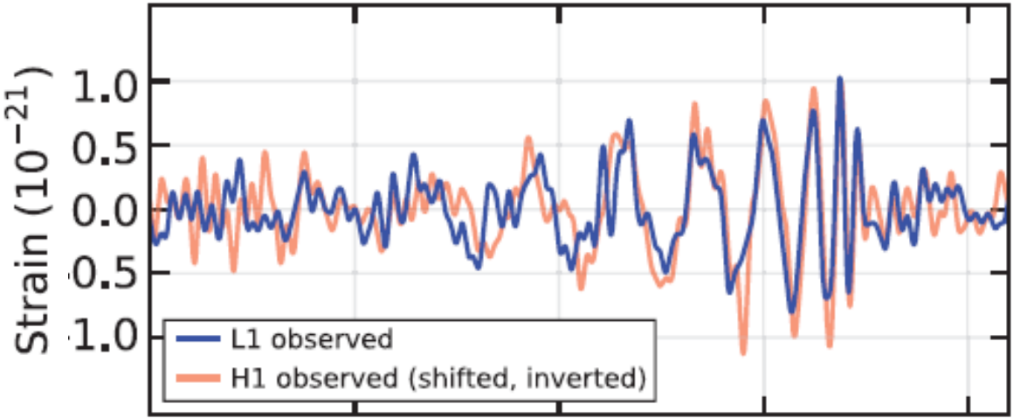
$$\langle\psi_{out}|\hat{M}|\psi_{out}\rangle = sin^2\left(\frac{\theta}{2}\right) - cos^2\left(\frac{\theta}{2}\right) = -\cos(\theta)$$

$$\Delta^2 M = \langle\psi_{out}|\hat{M}^2|\psi_{out}\rangle - (\langle\psi_{out}|\hat{M}|\psi_{out}\rangle)^2 = 1 - cos^2(\theta) = sin^2(\theta)$$

Laser Interferometer Gravitational-Wave Observatory (LIGO) & Atomic Clock

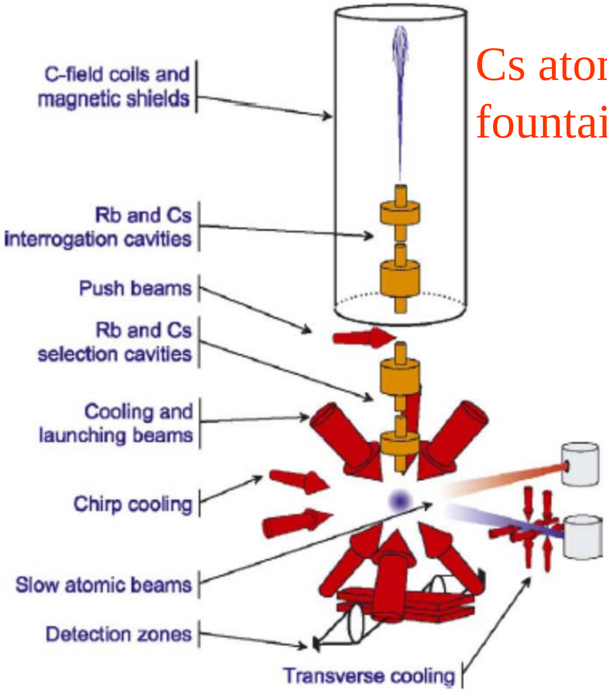
L=4km, sensitivity $\Delta L \sim 10^{-18}$ m, $\Delta L/L < 10^{-21}$

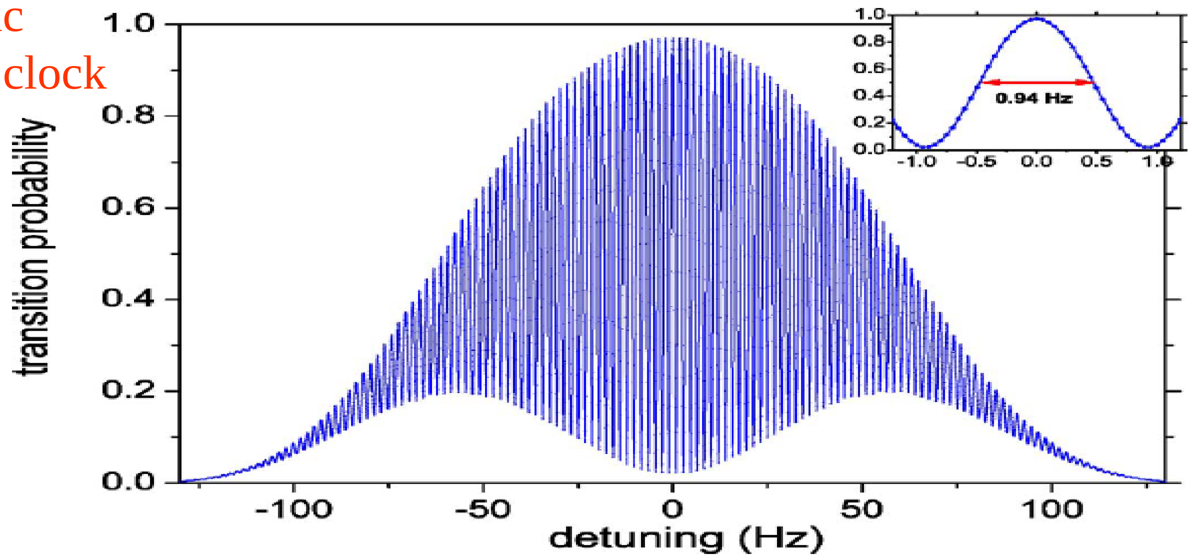
Binary Black Hole Merger



B.P. Abbott et al. PRL 116, 061102(2016)

Cs atomic fountain clock

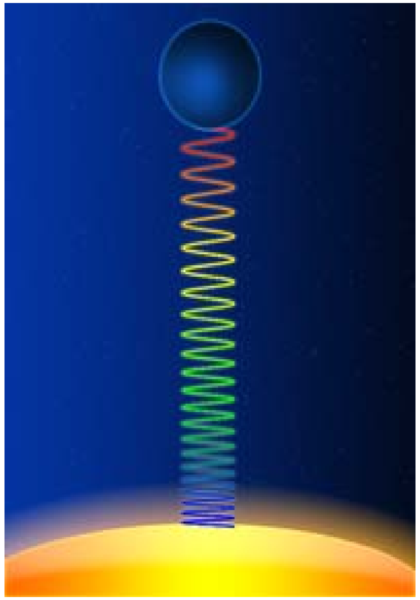




S. Bize et al. C.R. Physique, 5, 829(2004)

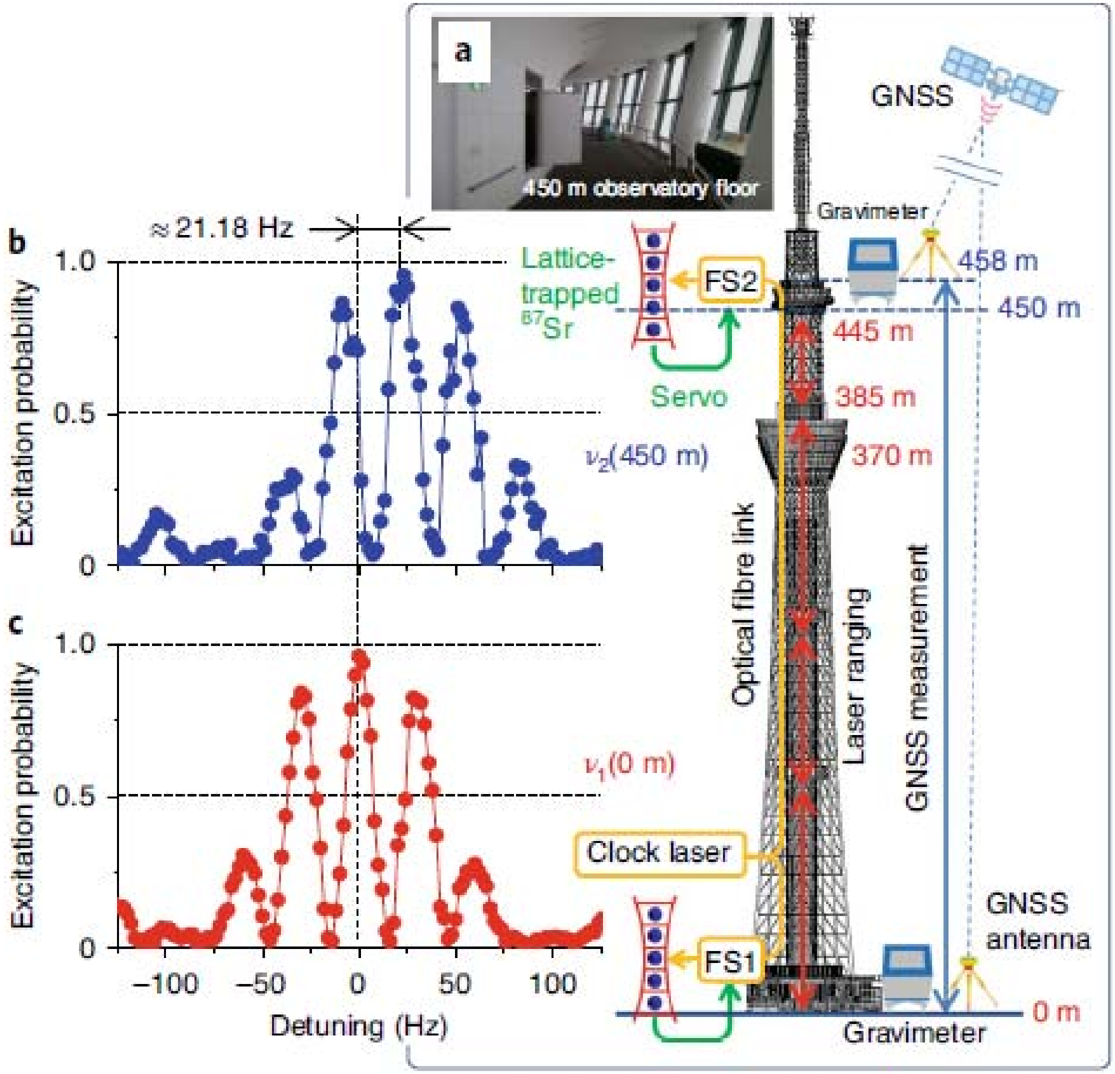
Gravitational Redshift Test by Atomic Clock

Gravitational red shift
/ time dilation



$$\frac{\delta f}{f} = \frac{g\Delta h}{c^2}$$
$$\approx 10^{-16},$$
for Δh=1m

Clock precision: $\frac{\delta f}{f} < 10^{-18}$



Redshift test to 10^{-5} precision: Katori's group, Nat. Photo. 14, 414(2020)

Note: me & Ni

How Quantum Helps Metrology?

- The particle (quantum) nature of photon and atom results in a phase uncertainty. For N uncorrelated particles, it is called the SQL.

$$\Delta^2\hat{M} = \sin^2\theta \times \mathrm{N}$$
$$\langle\hat{M}\rangle = \cos\theta \times \mathrm{N}$$

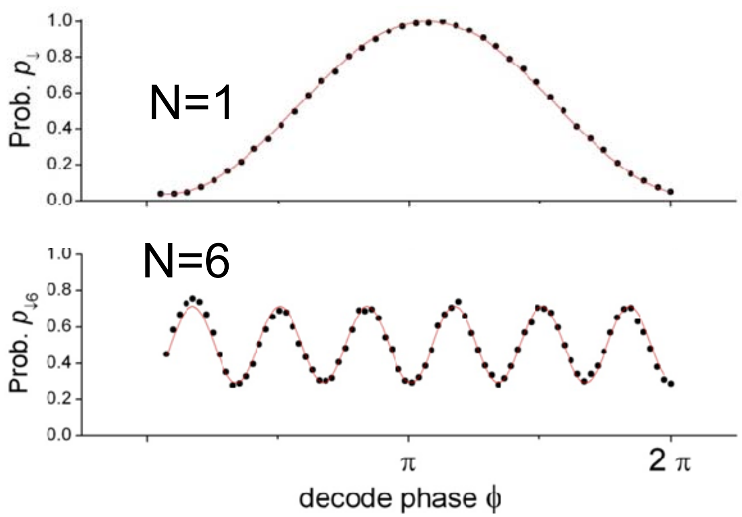
$$\boxed{\Delta\theta = \Delta\hat{M}/|d\langle\hat{M}\rangle/d\theta| = 1/\sqrt{N}.}$$

Standard quantum (shot-noise) limit

- Many-body entanglement can reduce uncertainty.

$$|GHZ\rangle(t) = (|a\rangle^{\otimes N} + e^{-iN\theta}|b\rangle^{\otimes N})/\sqrt{2}$$

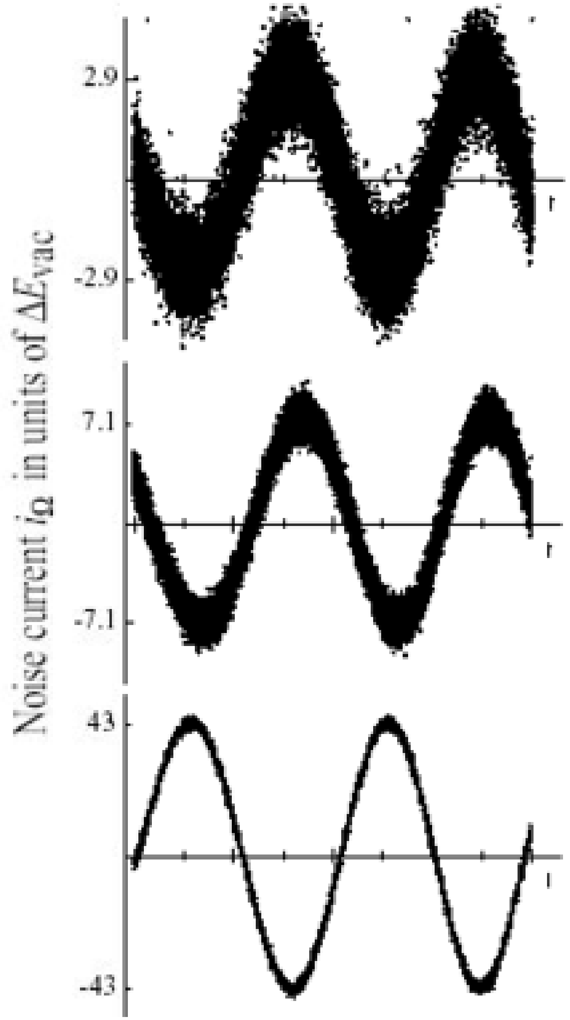
GHZ/NOON state for atom/photon



$$\langle M\rangle = \cos(N\theta)$$
$$(\Delta M)^2 = \sin^2(N\theta).$$

$$\boxed{\Delta\theta = \frac{1}{N}}$$

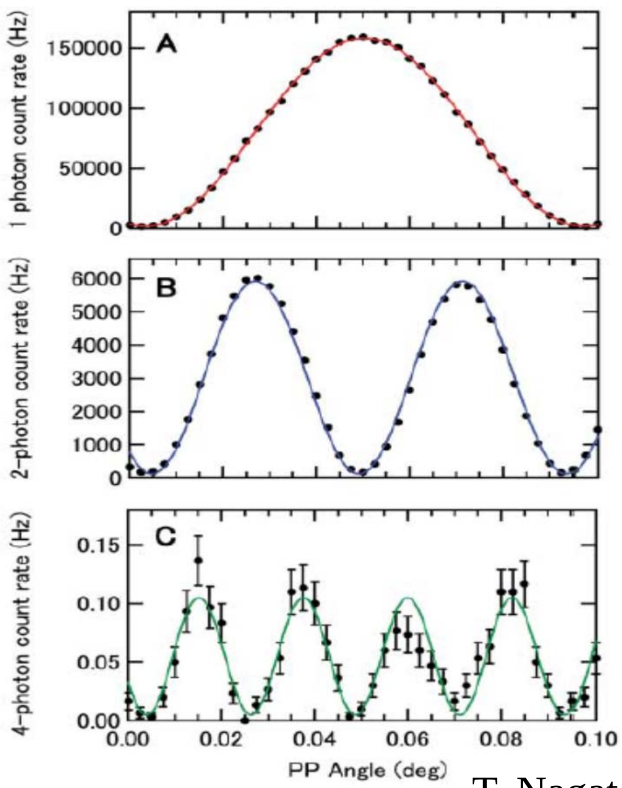
Heisenberg limit



Leibfried *et al.*, Nature 438, 639 (2005)

Fragile Many-Body Entangled States

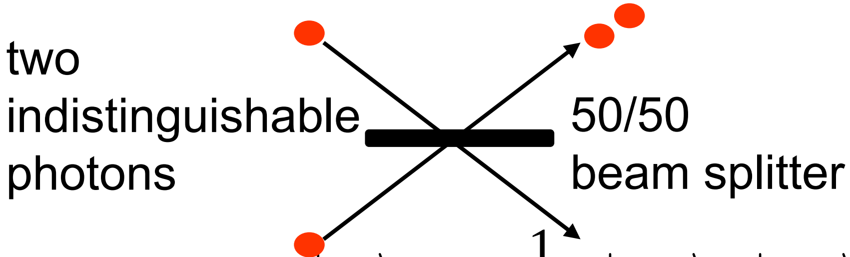
- Difficult to generate the GHZ (or NOON) state, usually require high nonlinearity, post-selection, or photon-number-resolving detector with high quantum efficiency.
- Loss and decoherence can easily destroy the entanglement.
- Super-resolution doesn't necessarily imply super-sensitivity, only if the interference contrast is high.



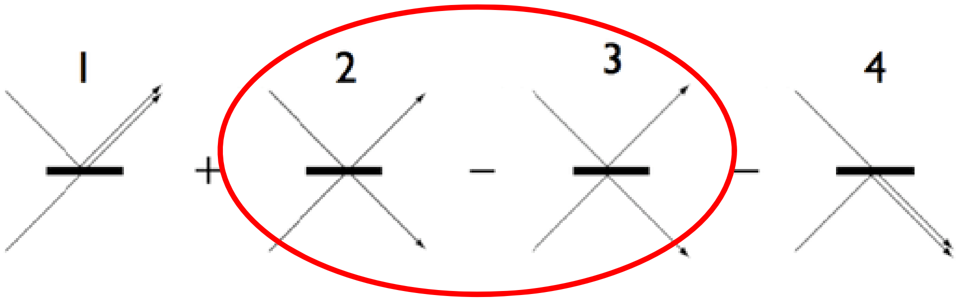
T. Nagata et al. Science 316, 726(2007)

Generation of NOON state

N=2 NOON state, the simplest case



$$\left|\Psi\right\rangle_{out} = \frac{1}{\sqrt{2}}\left(\left|2,0\right\rangle - \left|0,2\right\rangle\right)$$

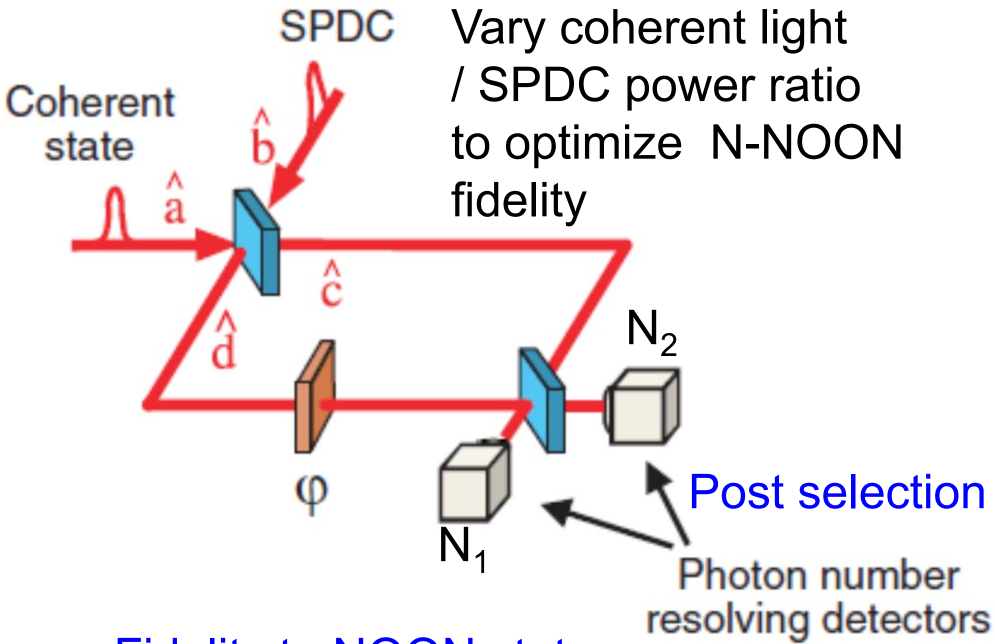


Cancelled

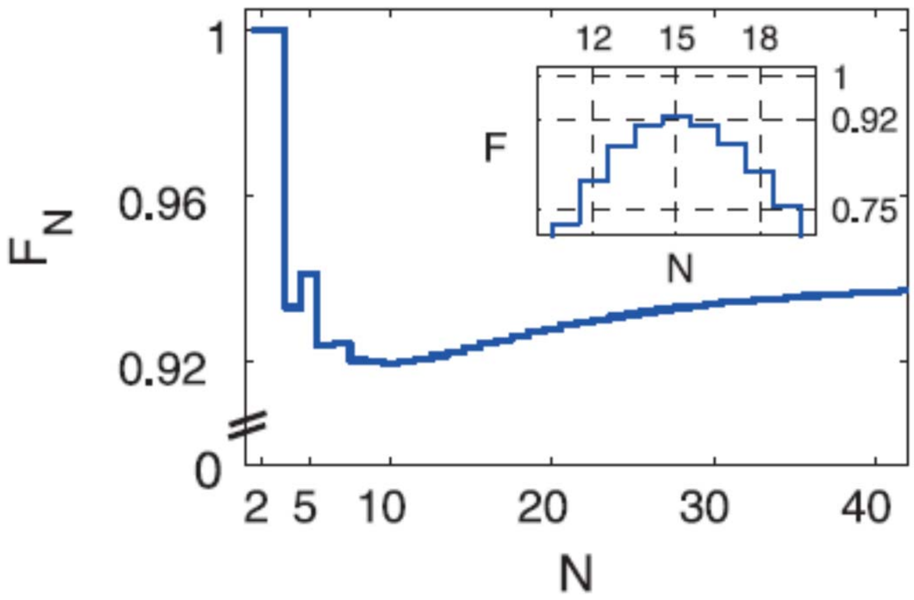
For high-NOON state generation, see e.g.
PRA 65, 052104(2002),
Nature, 429, 158&161 (2004)

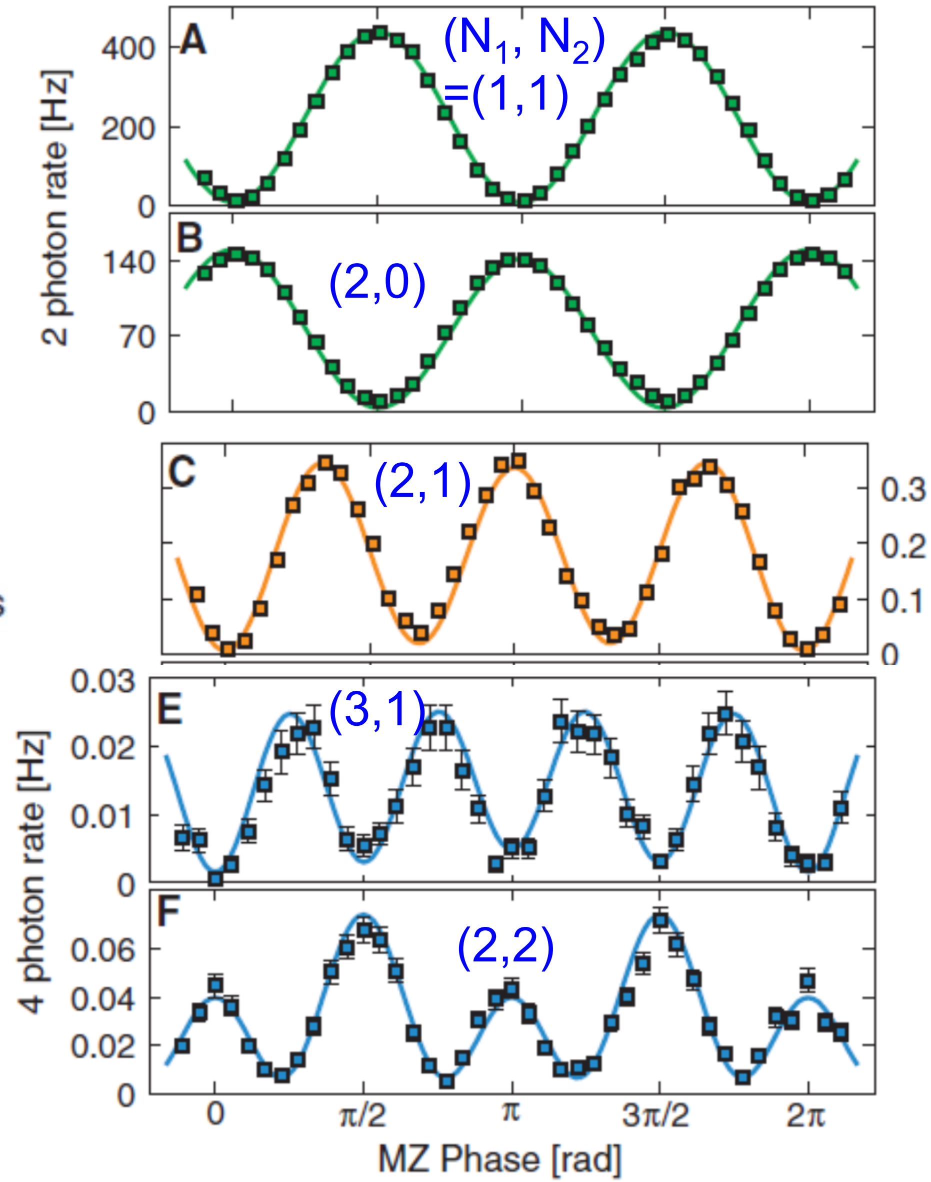
Generation of High-NOON State

Vary coherent light / SPDC power ratio to optimize N-NOON fidelity



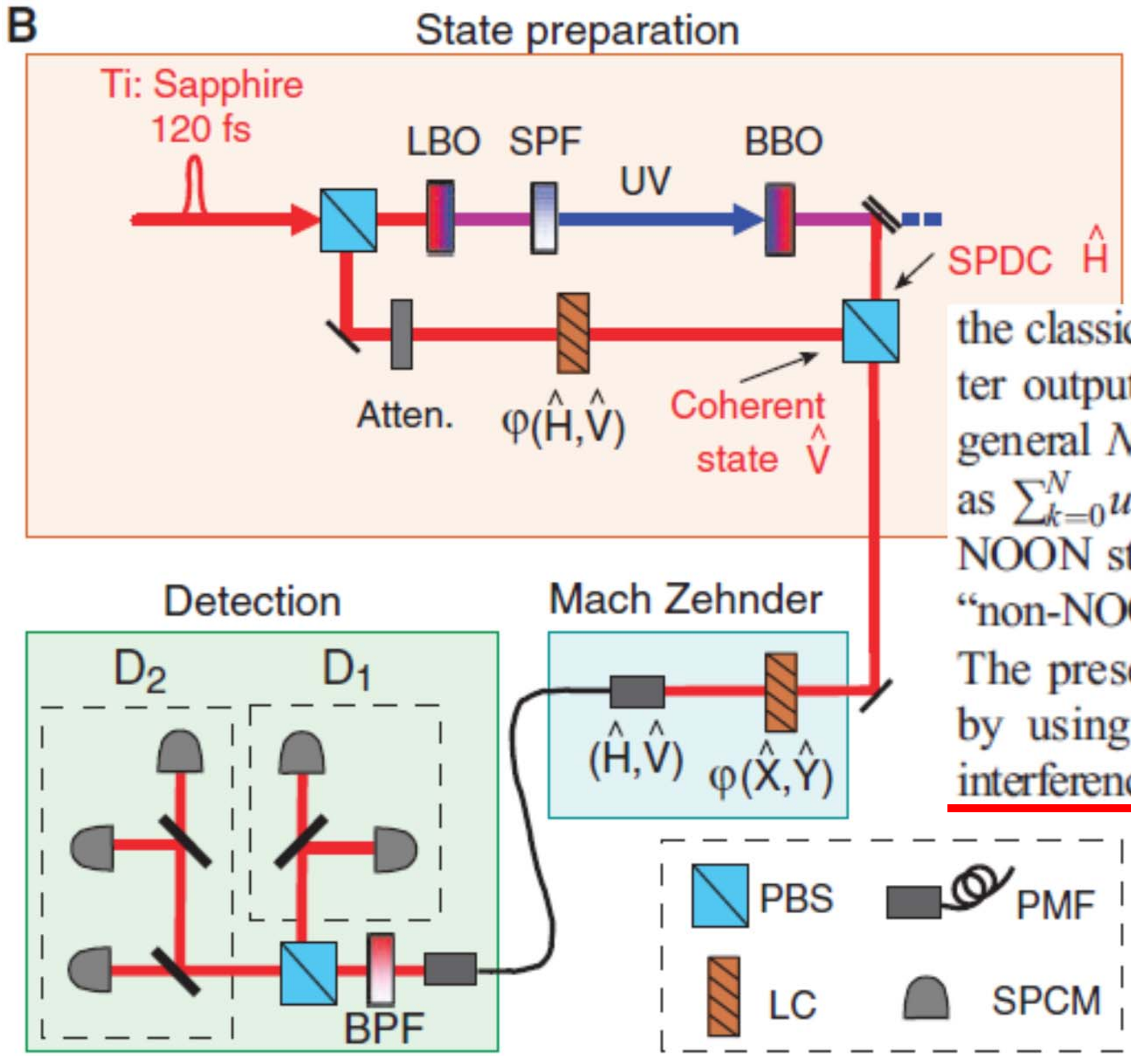
Fidelity to NOON state





PRA 76, 031806(2007); Science 328, 879(2010)

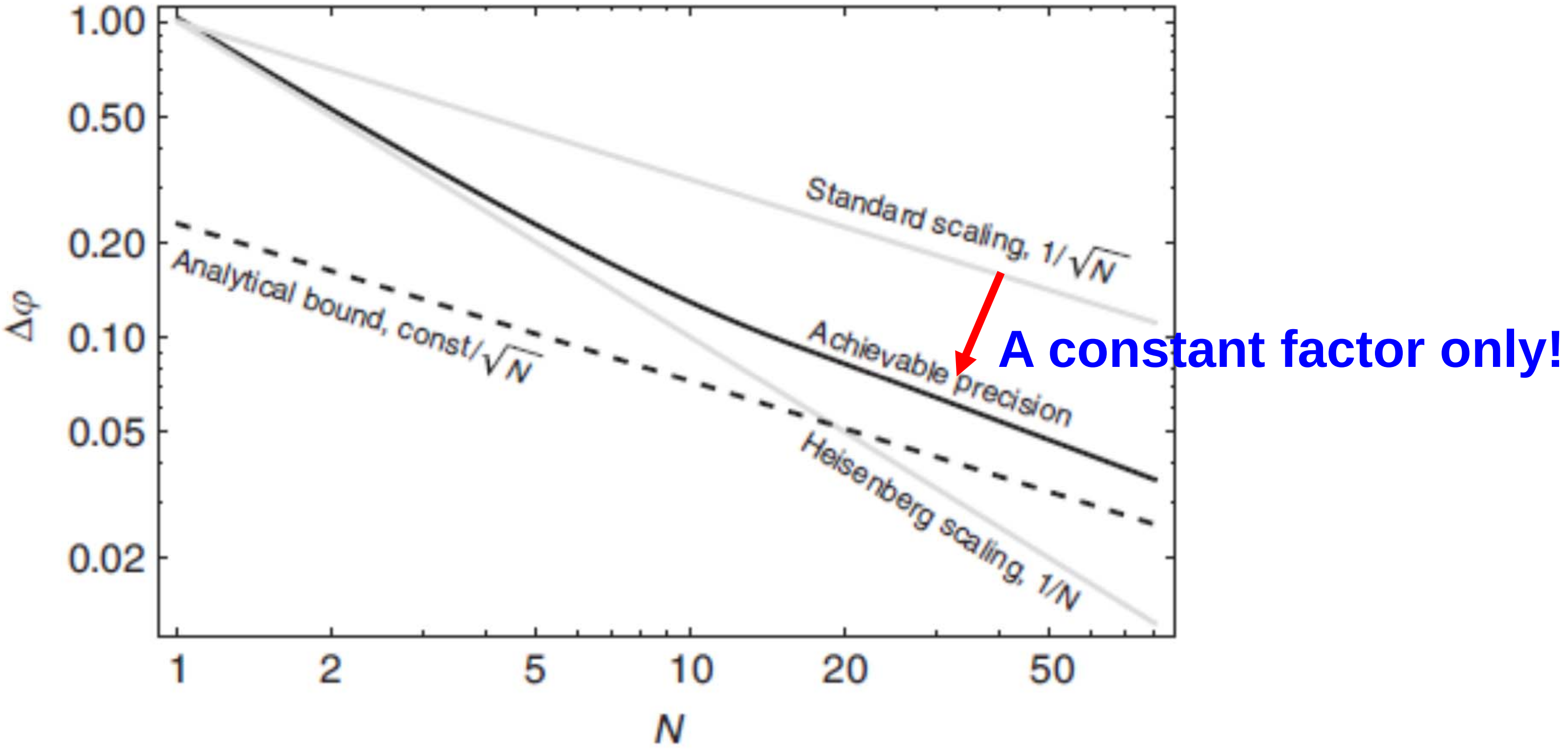
Notes on High-NOON State Generation



the classical resources. The state at the beamsplitter output, $|\psi_{out}\rangle_{c,d}$, is highly path-entangled. A general N-photon two-mode state can be written as $\sum_{k=0}^{N} u_k |k\rangle_c |N-k\rangle_d$. The creation of an ideal NOON state would require elimination of all the "non-NOON" components (i.e., $u_1, \ldots, u_{N-1} = 0$). The present scheme does this almost perfectly by using the naturally emerging multiphoton interference (Fig. 1A). The fidelity of the output

PRA 76, 031806(2007); Science 328, 879(2010)

Theoretical Study on Quantum Advantage

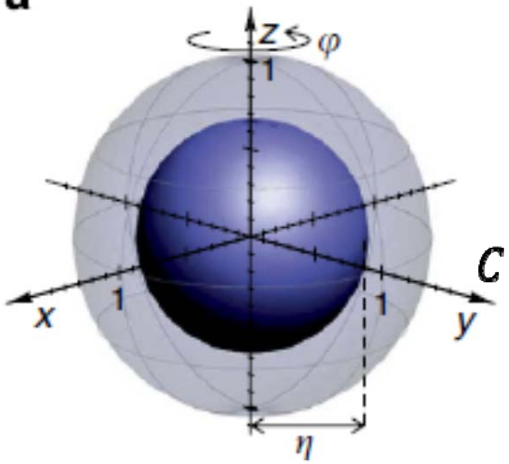


- Decoherence degrades the 1/N scaling to $C/\sqrt{N}$ at large N with a constant improvement factor *C*.
- Theoretical study based on quantum Fisher information/Cramer-Rao bound. It is important before doing experiment.

New J Phys, 15, 073005(2013); Nat. Commun3:1063(2012)

Decoherence Models & Precision Bounds

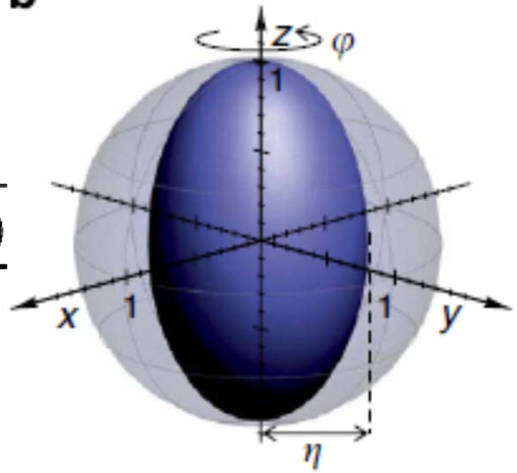
a



Depolarization:

$$C=\sqrt{\frac{(1-\eta)(1+2\eta)}{2\eta^2}}$$

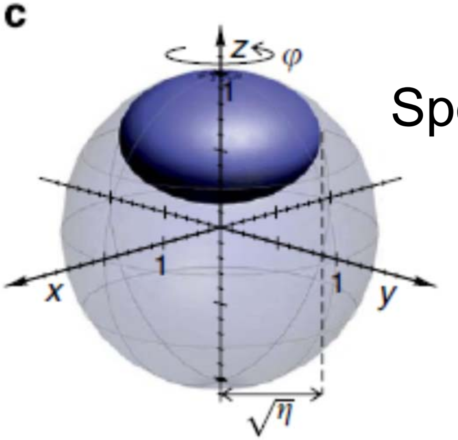
b



Dephasing:

$$C=\sqrt{\frac{1-\eta^2}{\eta}}$$

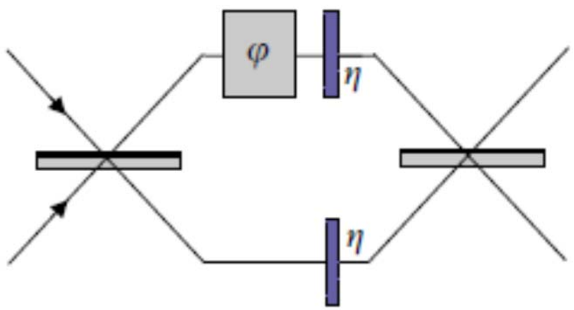
c



Spontaneous decay:

$$C=\frac{1}{2}\sqrt{\frac{1-\eta}{\eta}}$$

d



Photon loss:

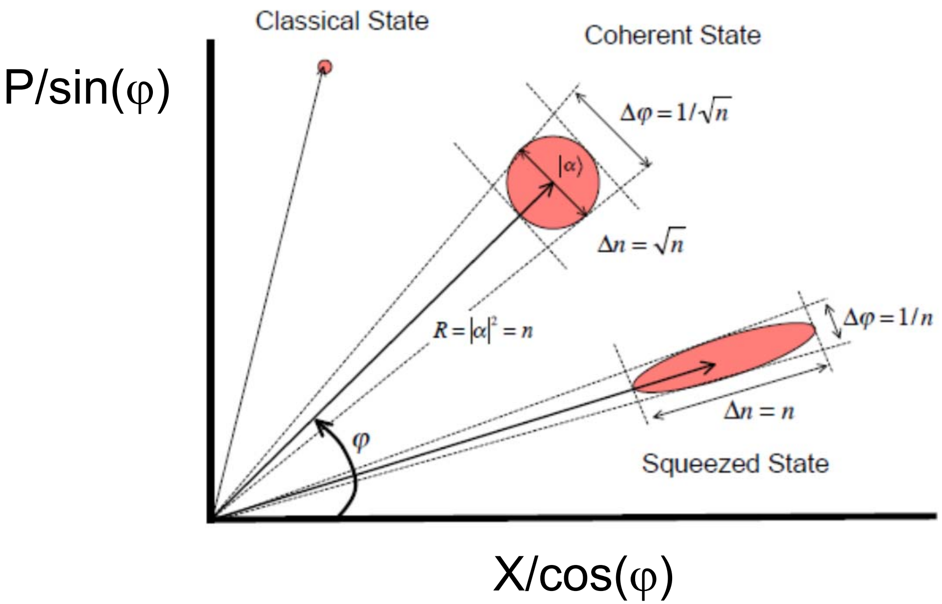
$$C=\sqrt{\frac{1-\eta}{\eta}}$$

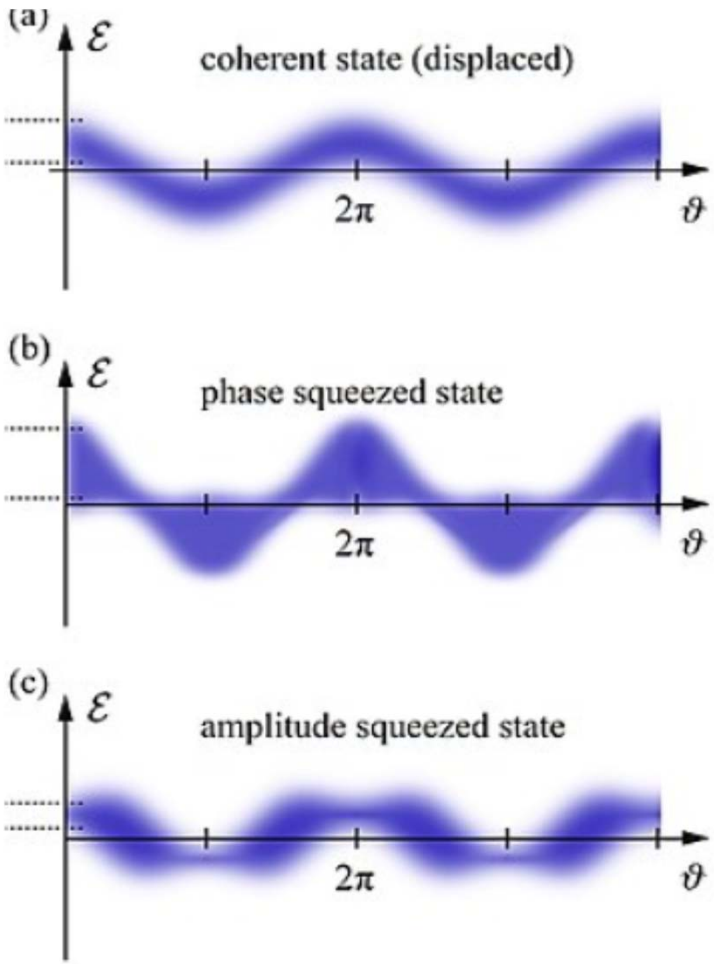
- The better η (close to 1), the better the improvement factor.
  $\Rightarrow$ Pursuit for perfect !

New J Phys, 15, 073005(2013); Nat. Commun3:1063(2012)

Squeezed State for Photons

- More reliable way to go below SQL is to use the squeezed state.
- High squeezing requires high nonlinearity,
  good mode matching, low crystal loss,
  high detection efficiency, phase stability…etc.
- Advanced LIGO used the squeezed light
  with a 3 dB improvement in sensitivity.





Generation of Squeezed Light

$$|\alpha, R\rangle = \hat{S}(R)|\alpha\rangle$$

Squeezed state

Coherent state

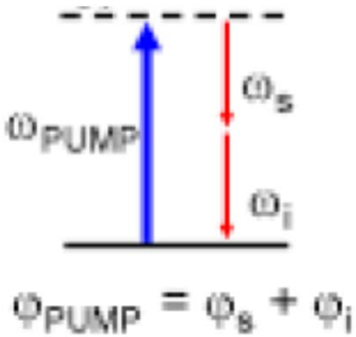
$$\hat{S}(R) = \exp[R(\hat{a}^2 - \hat{a}^{+2})/2]$$

Squeezing operator

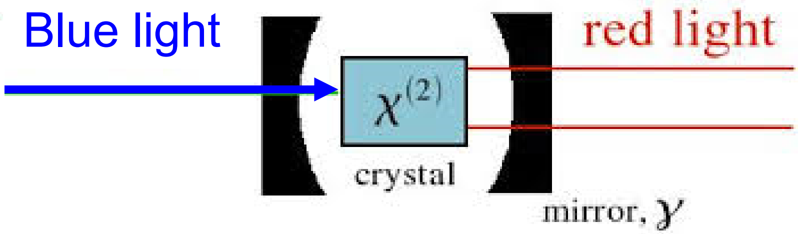
$$(\Delta E_Q)^2 = [E^{(1)}]^2 e^{2R}; \; (\Delta E_P)^2 = [E^{(1)}]^2 e^{-2R};$$

$$(\Delta E_Q)(\Delta E_P) = [E^{(1)}]^2$$

This offer such a Hamiltonian



Spontaneous Parametric Down-Conversion (SPDC)



- Frequency-degenerate Optical parametric oscillator (OPO) operating below the lasing threshold $\Rightarrow$ Squeezed light
- Strong nonlinear atom-photon coupling (active role)is a key for squeezing & thus the quantum advantage!

Ref. Grynberg, Aspect, and Fabre: Intro. to Quantum Optics, Complement 7A

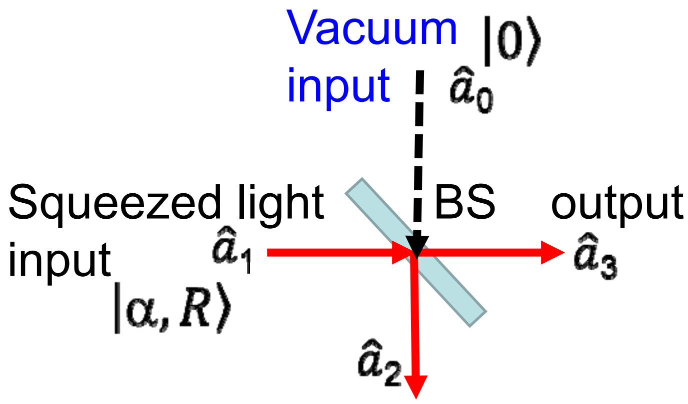
Note: Theo vs Exp author

Squeezed State with Loss (Beam Splitter Model)

$$\hat{a}_3 = t\hat{a}_1 + r\hat{a}_0$$

Field quadrature $\hat{E}_Q = E^{(1)}(\hat{a} + \hat{a}^+)$

$$\hat{E}_{Q3} = t\hat{E}_{Q1} + r\hat{E}_{Q0}$$



Input state $|\psi\rangle = |\alpha, R\rangle_1 \otimes |0\rangle_0$

$$\hat{E}_{Q3}|\psi\rangle = t\hat{E}_{Q1}|\alpha, R\rangle_1 \otimes |0\rangle_0 + E^{(1)} r|\alpha, R\rangle_1 \otimes |1\rangle_0$$

Mean $\langle\psi|\hat{E}_{Q3}|\psi\rangle = t\langle\alpha, R|\hat{E}_{Q1}|\alpha, R\rangle$

$$\langle\psi|\hat{E}_{Q3}^2|\psi\rangle = |t|^2\langle\alpha, R|\hat{E}_{Q1}^2|\alpha, R\rangle + |r|^2 (E^{(1)})^2$$

Variance $(\Delta\hat{E}_{Q3})^2 = \langle\psi|\hat{E}_{Q3}^2|\psi\rangle - \left(\langle\psi|\hat{E}_{Q3}|\psi\rangle\right)^2 = |t|^2\left(\Delta\hat{E}_{Q1}\right)^2 + |r|^2\left(E^{(1)}\right)^2$

$$= \left(E^{(1)}\right)^2 (|t|^2 e^{-2R} + |r|^2)$$

- Fluctuation from vacuum port adds to the overall output fluctuation.
- The photon loss (passive role) degrades the degree of squeezing !

Ref. Grynberg, Aspect, and Fabre: Intro. to Quantum Optics, Complement 7A

QST: Pursuit for Perfect for Squeezed Light

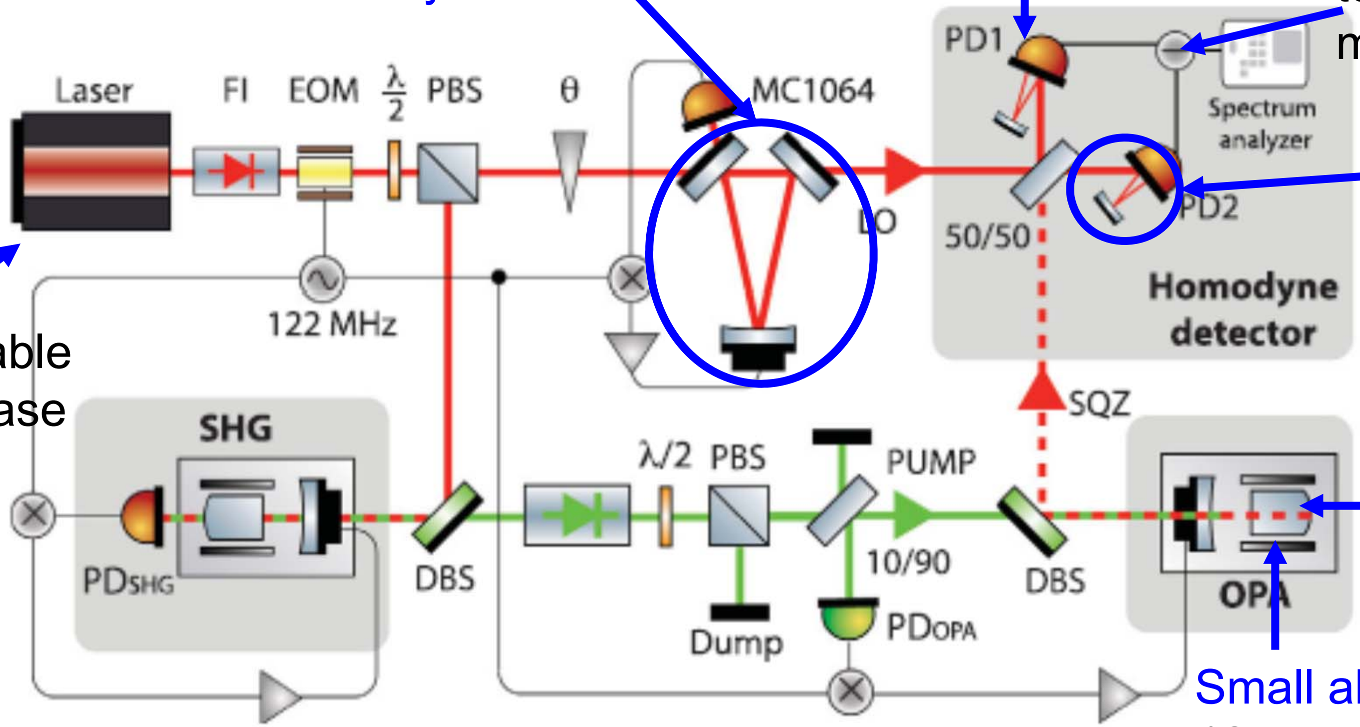
Mode-cleaning cavity for a good spatial mode to achieve a high interference visibility 99.6%

Quantum efficiency >99.5%

Balanced detection to reduce common mode noise 80dB

Recycle the residual reflection to reduce loss

Very stable Low phase noise



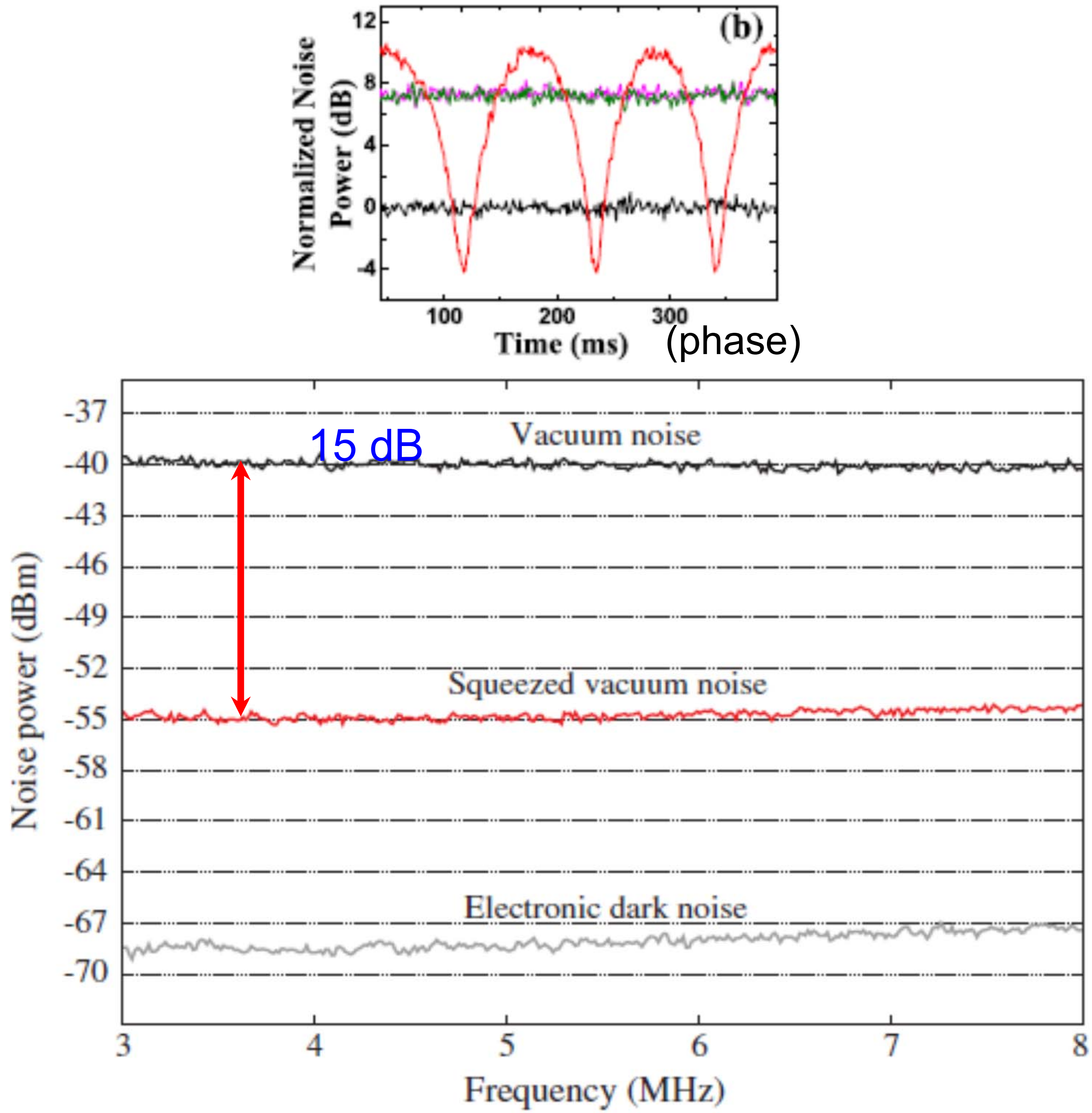
Overall 2% loss

high specs on AR and HR coatings

Small absorption 12 ppm/cm for PPKTP crystal

- A total optical loss ~ 2.5 % with a phase noise of 1.7 mrad
- Require absolute high specs !

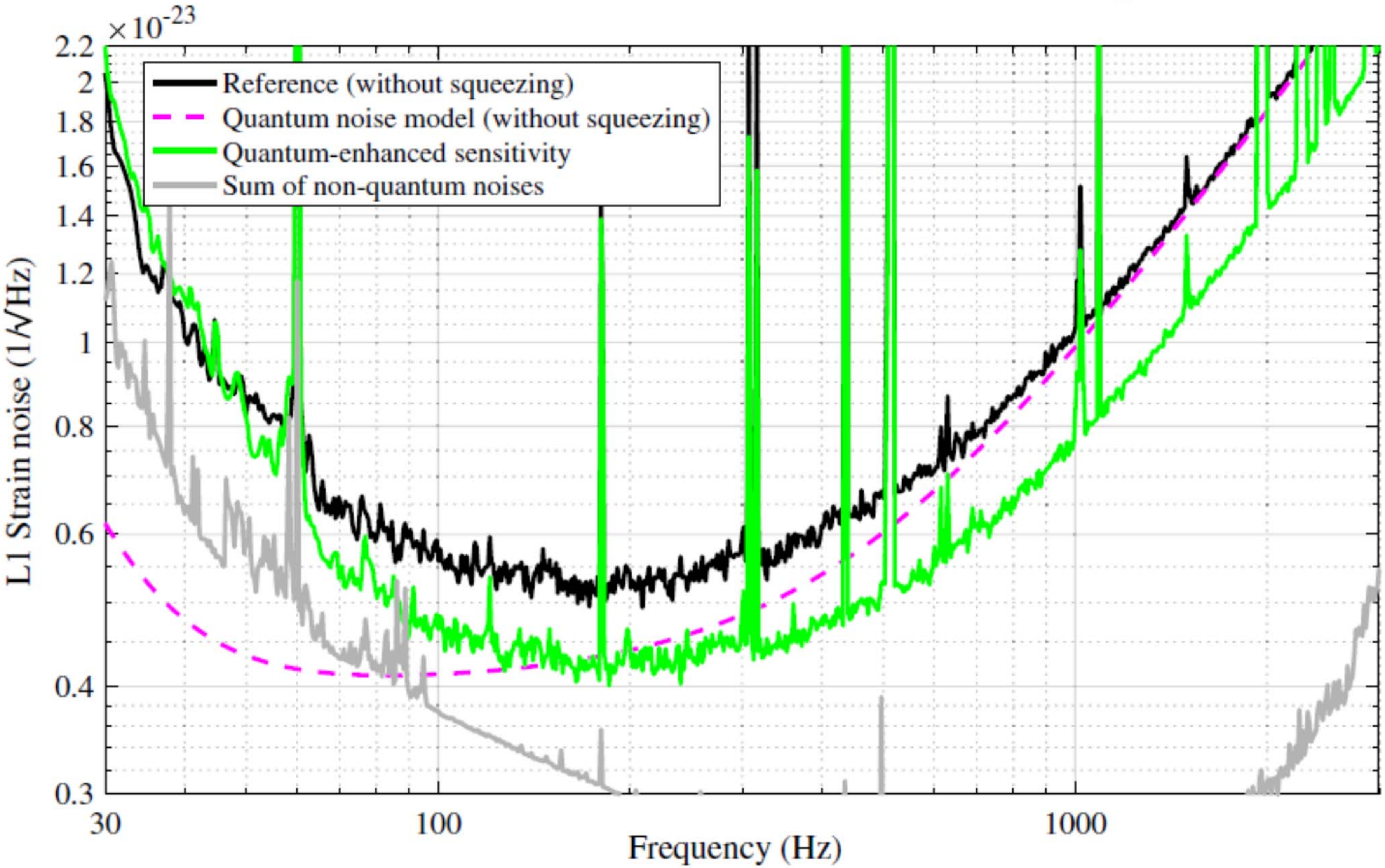
H. Vahlbruch et al. PRL 117, 110801(2016)



(b)
Normalized Noise Power (dB)
12
8
4
0
-4
100
200
300
Time (ms)
(phase)
Vacuum noise
15 dB
Squeezed vacuum noise
Electronic dark noise
Noise power (dBm)
-37
-40
-43
-46
-49
-52
-55
-58
-61
-64
-67
-70
3
4
5
6
7
8
Frequency (MHz)

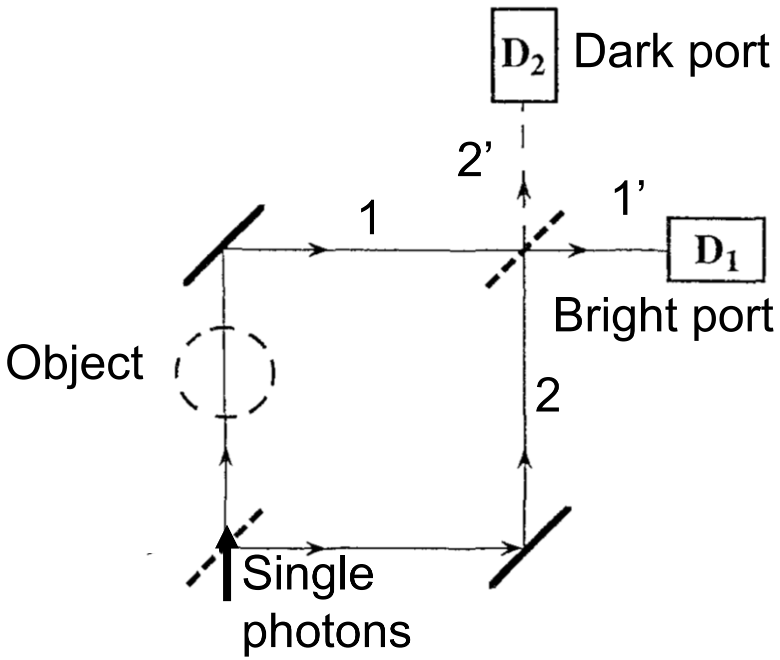
PHYSICAL REVIEW LETTERS **123**, 231107 (2019)

Quantum-Enhanced Advanced LIGO Detectors in the Era of Gravitational-Wave Astronomy



"Interaction-Free" Measurement ?

- An arrangement to show the weird nonlocality of the wavefunction。



No object → No click on D_2

With object → some clicks on D_2
and those photons go through arm 2,
without interaction with the object!

⇒ "Interaction-free" measurement?
(Knowing the existence of object without interacting with it?)
What's "wrong" ?

$$|in\rangle \rightarrow \frac{1}{\sqrt{2}}(|1\rangle + i|2\rangle) \rightarrow \frac{1}{\sqrt{2}}(|scattered\rangle + i|2\rangle) \rightarrow \frac{1}{\sqrt{2}}|scatt\rangle + \frac{i}{2}(|1'\rangle + i|2'\rangle)$$

(i) no detector clicks, P=1/2

(ii) detector D_1 clicks, P=1/4

(iii) detector D_2 clicks. P=1/4

Define IMF efficiency

$$\eta = \frac{P_{IMF}}{P_{IMF} + P_{abs}} = \frac{1}{3}$$

A. Elitzur, L Vaidman, Foundation of Phys, 23, 987, 1993

Better IFM Efficiency for Variable BS ?

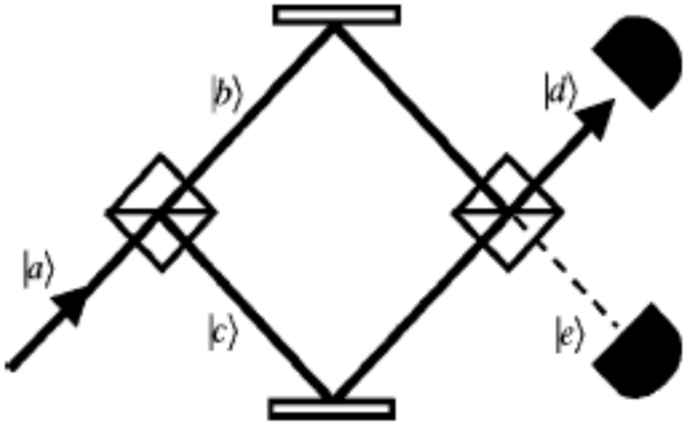
$$|b\rangle = \sqrt{T_1}|a\rangle, |c\rangle = i\sqrt{R_1}|a\rangle,$$

$$|d\rangle = \sqrt{T_2}|c\rangle + i\sqrt{R_2}|b\rangle = i(\sqrt{R_1T_2} + \sqrt{R_2T_1})|a\rangle$$

$$|e\rangle = \sqrt{T_2}|b\rangle + i\sqrt{R_2}|c\rangle = (\sqrt{T_1T_2} - \sqrt{R_1R_2})|a\rangle$$

If $T_2=R_1$ (and thus $T_1=R_2$) => |e>=0

Dark port

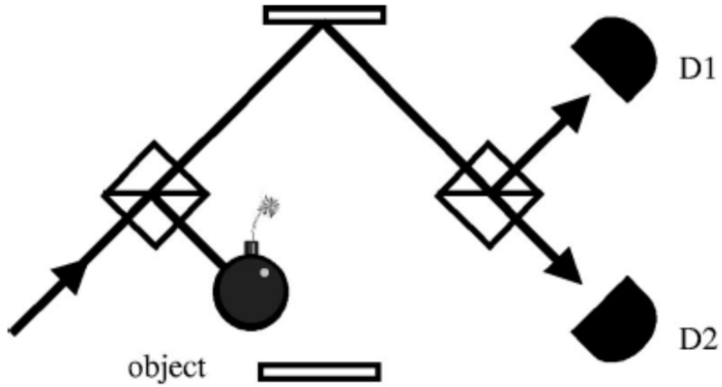


$$P_{abs} = R_1 \qquad P_{IFM} = T_1\, T_2$$

$$\eta = \frac{P_{IFM}}{P_{abs} + P_{IFM}} = \frac{T_1T_2}{R_1 + T_1T_2}$$

$(T_2=R_1)$

$$= \frac{T_1}{1+T_1} = \frac{(1-R_1)}{1+(1-R_1)} = \frac{1-R_1}{2-R_1}$$



⇨ Approaching 0.5 when R_1 approaching 0

Am J Phys, 70, 272, 2002

An Improved IFM Scheme

Choose a transmission for each BS to be $T = \sin^2(\pi/2N)$.

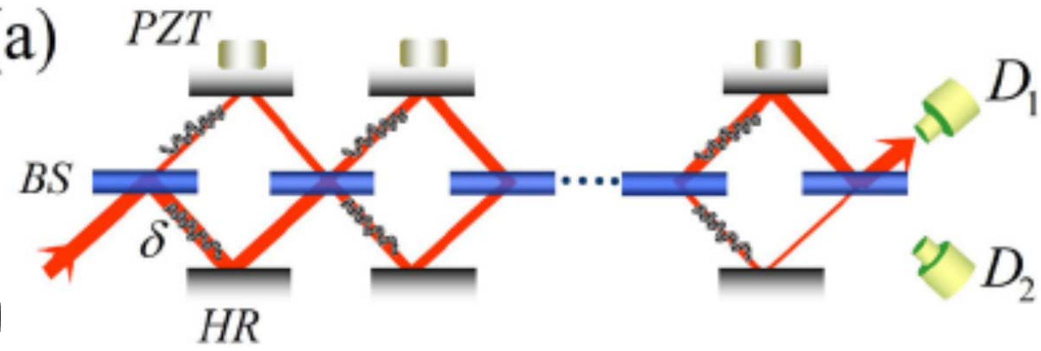
$$U = \begin{bmatrix} \sqrt{T} & i\sqrt{R} \\ i\sqrt{R} & \sqrt{T} \end{bmatrix} = \begin{bmatrix} \cos(\pi/2N) & i\sin(\pi/2N) \\ i\sin(\pi/2N) & \cos(\pi/2N) \end{bmatrix}$$

But

$$\mathcal{R}(\theta)\mathcal{R}(\phi) = \begin{pmatrix} \cos\theta & i\sin\theta \\ i\sin\theta & \cos\theta \end{pmatrix}\begin{pmatrix} \cos\phi & i\sin\phi \\ i\sin\phi & \cos\phi \end{pmatrix} = \begin{pmatrix} \cos(\theta+\phi) & i\sin(\theta+\phi) \\ i\sin(\theta+\phi) & \cos(\theta+\phi) \end{pmatrix} = \mathcal{R}(\theta+\phi).$$

$$\begin{aligned} U^N &= \mathcal{R}^N(\pi/2N) \\ &= \mathcal{R}(N\cdot\pi/2N) \end{aligned} = \begin{pmatrix} \cos(\pi/2) & i\sin(\pi/2) \\ i\sin(\pi/2) & \cos(\pi/2) \end{pmatrix} = \begin{pmatrix} 0 & i \\ i & 0 \end{pmatrix} \quad |\text{out}\rangle = \begin{pmatrix} 0 & i \\ i & 0 \end{pmatrix}\begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} i \\ 0 \end{pmatrix}$$

(a)



$\begin{pmatrix} 0 \\ 1 \end{pmatrix}$

Dark port

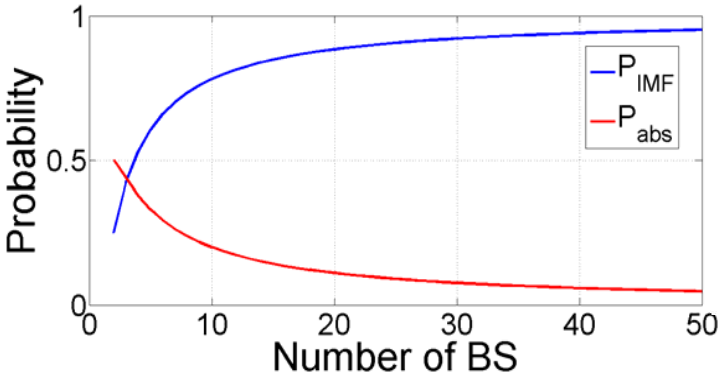
Am J Phys, 70, 272(2002); Las Phys Lett 15, 065211(2018)

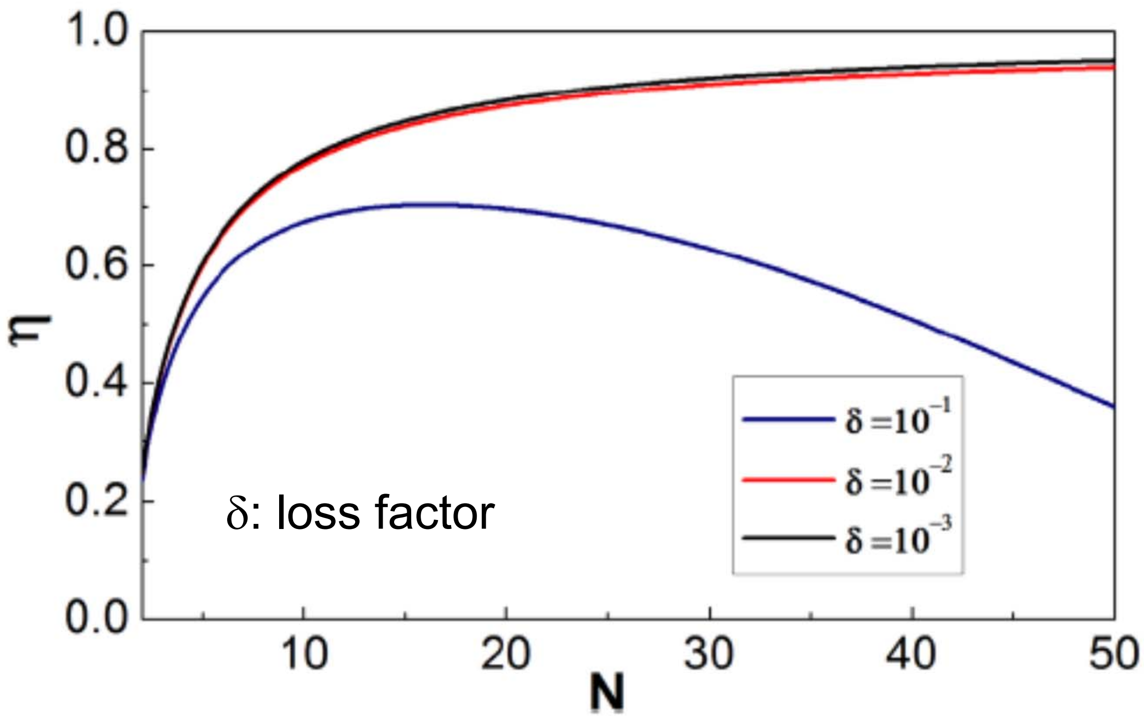
Can IFM Efficiency Approach Unity? ⇒Real IFM !

$$P_{\mathrm{IFM}} = [\cos^2(\pi/2N)]^N.$$

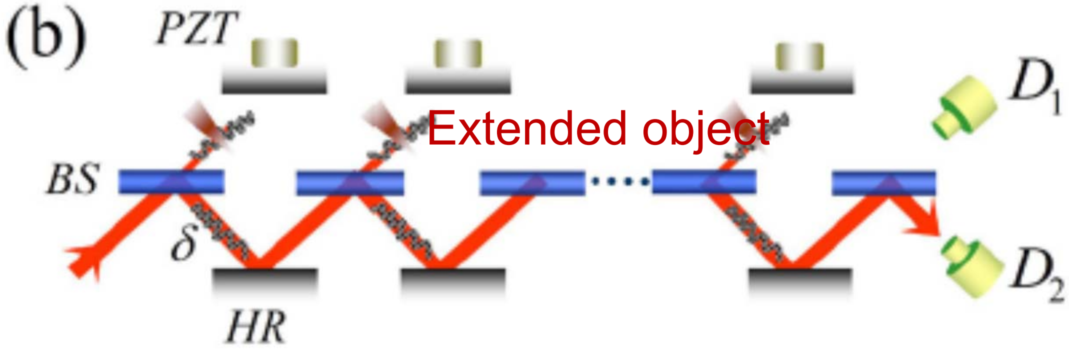
$$P_{\mathrm{abs}} = T + RT + R^2T + \cdots + R^{N-2}T = \sin^2\left(\frac{\pi}{2N}\right)\sum_{k=0}^{N-2}\left[\cos^2\left(\frac{\pi}{2N}\right)\right]^k.$$

$$\eta = \frac{P_{IMF}}{P_{IMF} + P_{abs}}$$



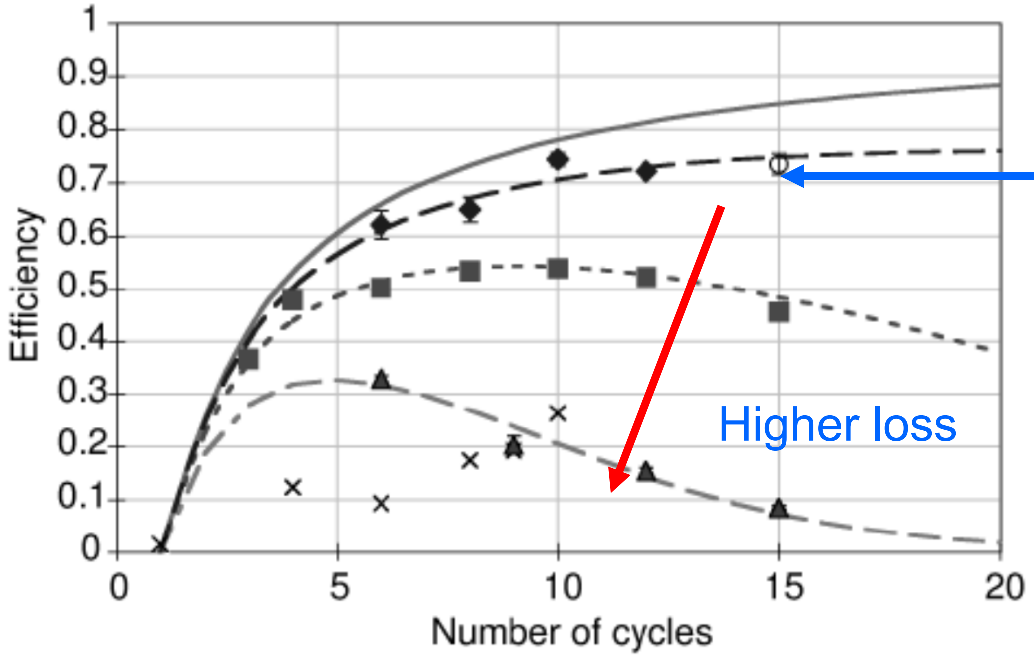


Low photon loss is crucial !

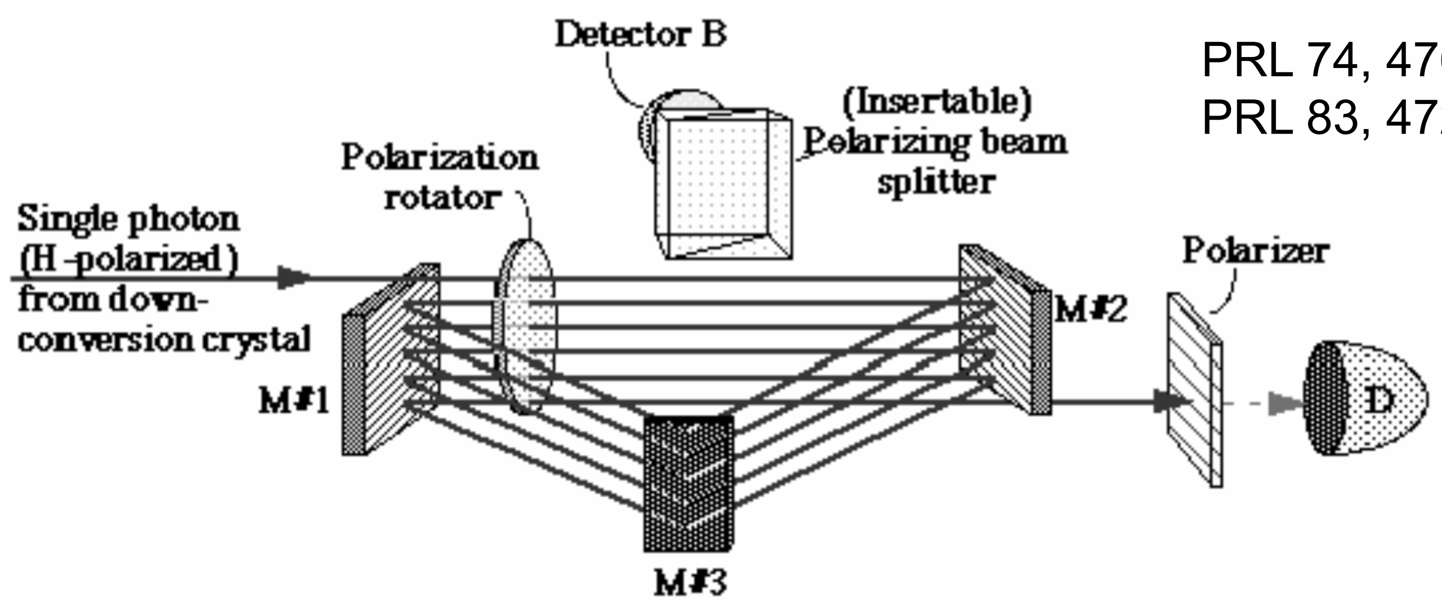


Am J Phys, 70, 272(2002);
Las Phys Lett 15, 065211(2018)

Realization of IFM Experiment



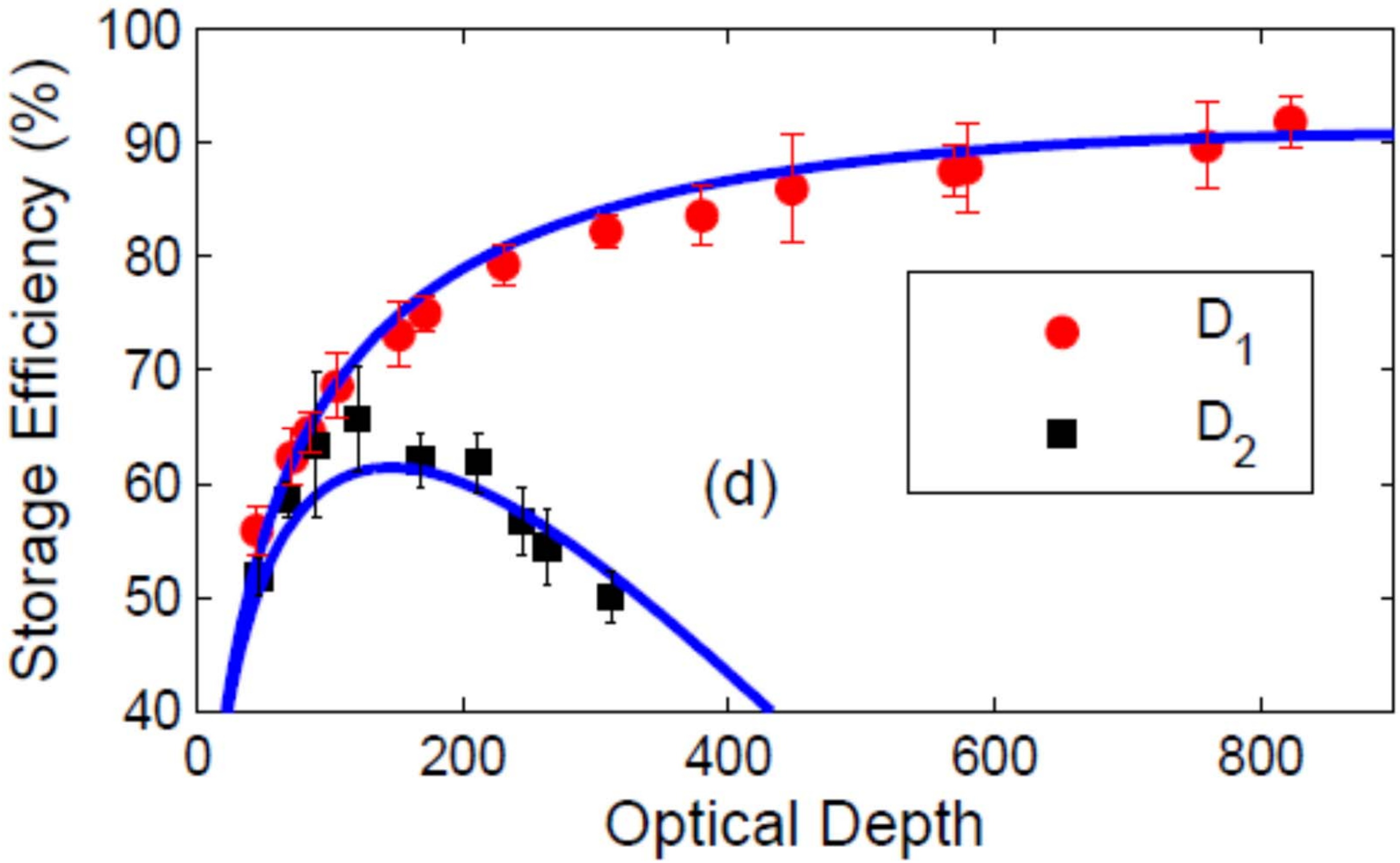
T=97.7%(Pocket cell)
R=99.4%(Recycling mirror)



PRL 74, 4763(1995)
PRL 83, 4725(1999)

Why looks so familiar ?

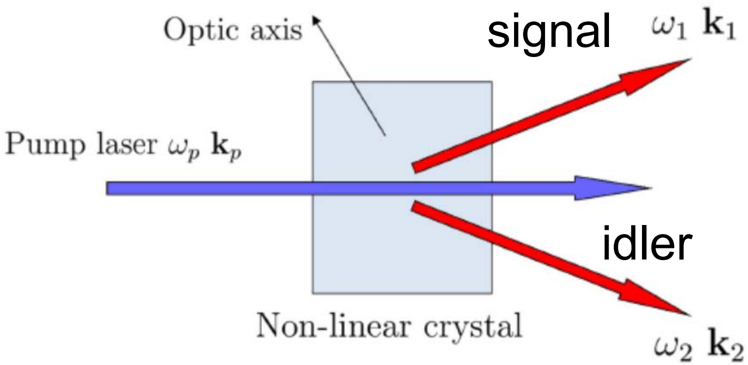
Efficiency of EIT coherent optical memory

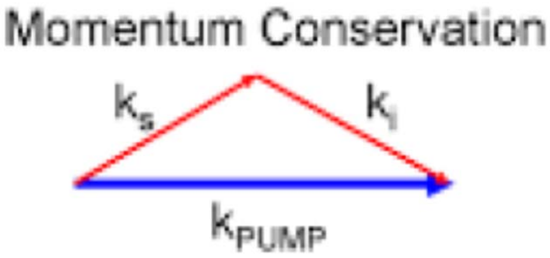


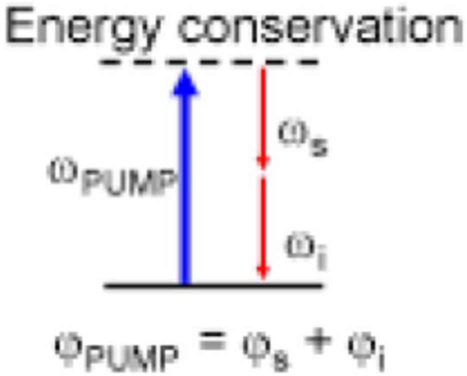
YF Hsiao et al, Phys. Rev. Lett. 120, 183602(2018)

Case Study 3
Photon Pairs by SPDC, Non-classical Correlation, Quantum Illumination (Radar) & Imaging

Spontaneous Parametric Down Conversion (SPDC)







$$P = \chi^1 E + \underbrace{\chi^2 EE + \chi^3 EEE + \ldots}_{\text{Nonlinear response}}$$

$$H = \int_V \frac{1}{2}\vec{E}\cdot(\epsilon_0\vec{E} + \vec{P}) \;\; = \int_V \left[\frac{1}{2}\epsilon_0 E^2(r,t) + X_1(r,t) + X_2(r,t)\right]\mathrm{d}V.$$

$$X_1(r,t) = \frac{1}{2}\chi^1_{i,j}E_iE_j \qquad X_2(r,t) = \frac{1}{3}\chi^2_{i,j,k}E_iE_jE_k,$$ PDC term

$$|\psi\rangle = \hat{S}|0\rangle = \exp\left[-\frac{1}{\mathrm{i}\hbar}\int H_I(t')\mathrm{d}t'\right]|0\rangle$$

$$= \bigotimes_{q,\Omega} \exp[f(q,\Omega)\hat{a}^\dagger_{q,\Omega}\hat{a}^\dagger_{-q,-\Omega} - \text{H.C.}]|0\rangle.$$

Transverse momentum (q)

frequency (Ω)

Interaction strength & phase matching

$$f(q,\Omega) = \chi^{(2)}A_0Le^{\mathrm{i}\Delta z/2}\mathrm{sinc}\left(\frac{\Delta k(q,\Omega)L}{2}\right).$$

$$= re^{\mathrm{i}\theta}$$

r : squeezing parameter

J Opt 19, 094002(2017)

SPDC Photon Statistics

- Two-mode entangled states in photon number (not related to phase), or called multimode twin-beam (TWB) state.

$$|\psi\rangle = \bigotimes_{q,\Omega} |TWB\rangle_{q,\Omega} = \bigotimes_{q,\Omega} \sum_{n} c_{q,\Omega}(n) |n\rangle_{q,\Omega} |n\rangle_{-q,-\Omega} = \bigotimes_{k} \operatorname{sech}(r_k) \sum_{n=0}^{\infty} \tanh^{n}(r_k) \left| n_k^{(s)}, n_k^{(i)} \right\rangle.$$

For a specific mode pairs, mean photon number $\mu = \langle \hat{a}_j^{\dagger} \hat{a}_j \rangle = \sinh^2(r).$

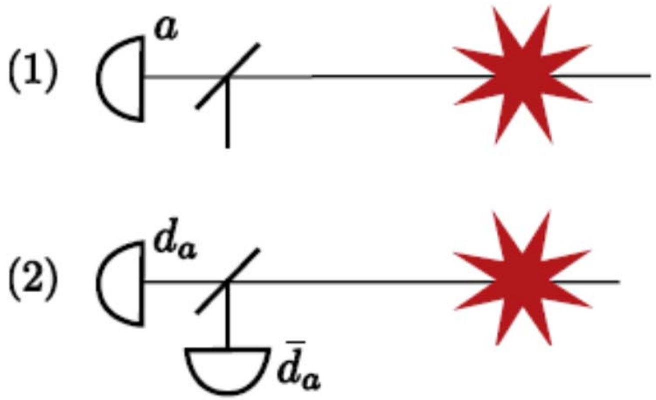
- For each mode in the pairs, SPDC radiation has thermal statistics.

$$\langle (\Delta \hat{n}_1)^2 \rangle = \mu(1 + \mu) = \langle \hat{n}_1 \rangle (1 + \langle \hat{n}_1 \rangle) = \langle (\Delta \hat{n}_2)^2 \rangle$$

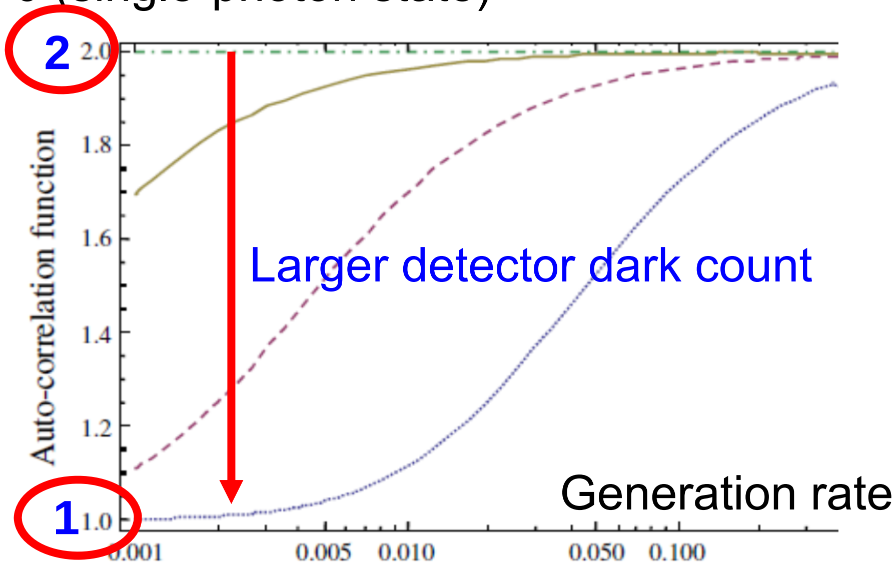
- Auto-correlation function

$g^{(2)}$=1(coherent state), =2(thermal state), =0 (single-photon state)

$$g_a^{(2)} = \frac{\langle a^{\dagger 2} a^2 \rangle}{\langle a^{\dagger} a \rangle^2},$$



Avoid imperfect detectors



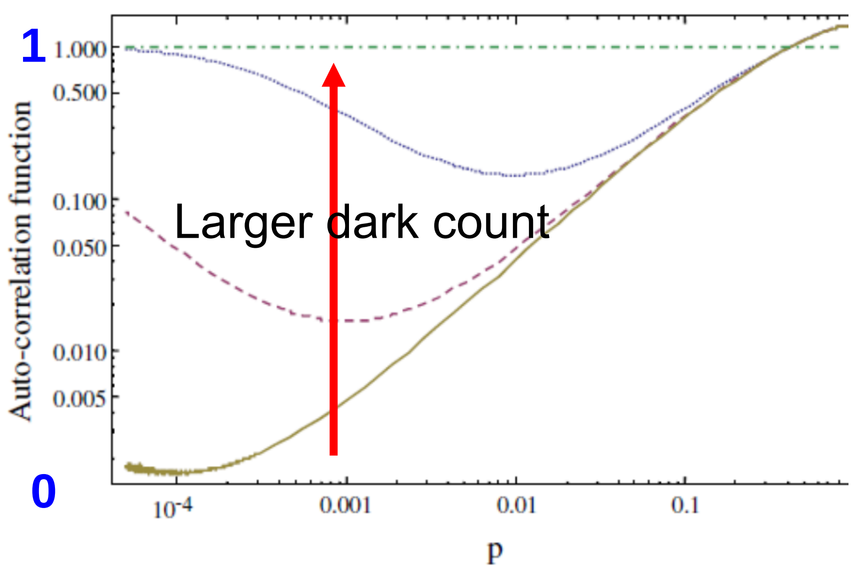
J Phys B 45, 124016(2012),PRA 85, 023829,J Opt 19, 094002(2017)

SPDC Photon Statistics

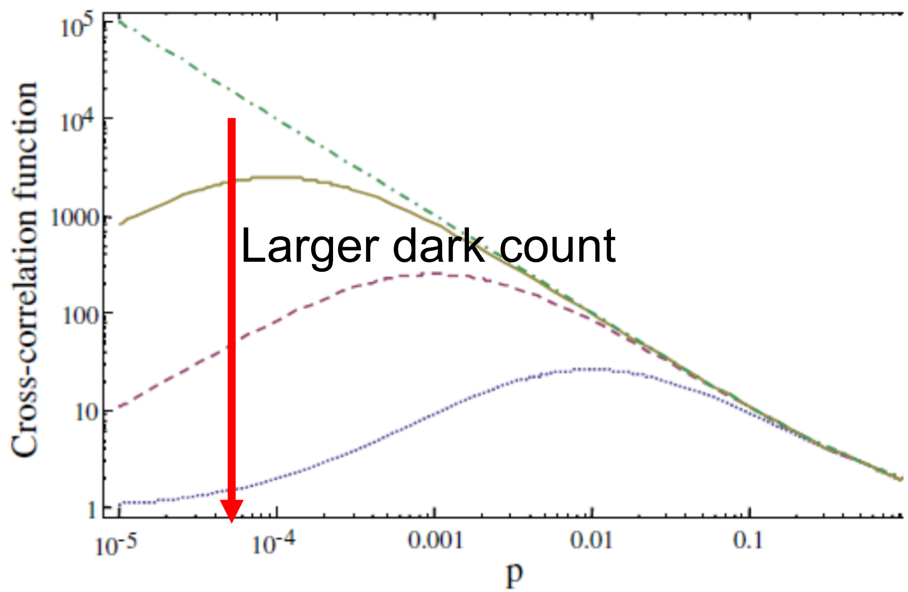
- Auto-correlation conditioned when mode b is detected, showing non-classicality, heralded single photons !
- Second-order Corss-correlation function, the larger value the larger nonclasicality.

$$g_{ab}^{(2)} = \frac{\langle a^\dagger b^\dagger b a\rangle}{\langle a^\dagger a\rangle \langle b^\dagger b\rangle},$$

Conditioned auto-correlation

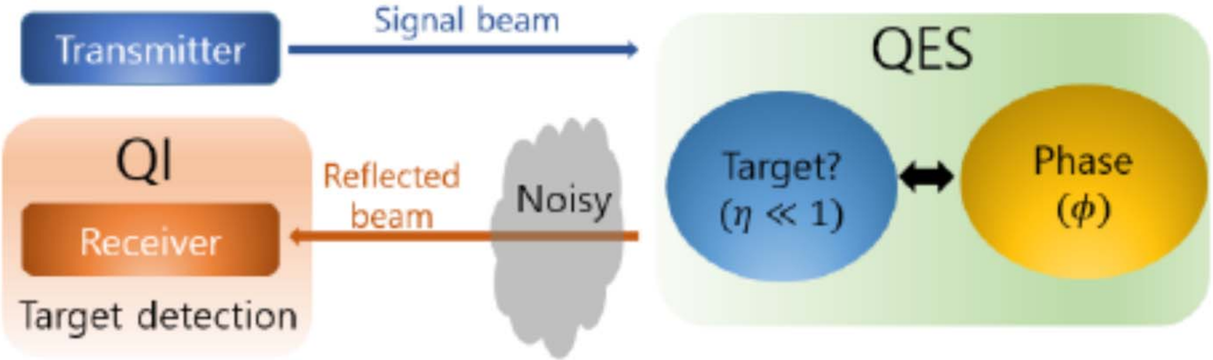


cross-correlation function

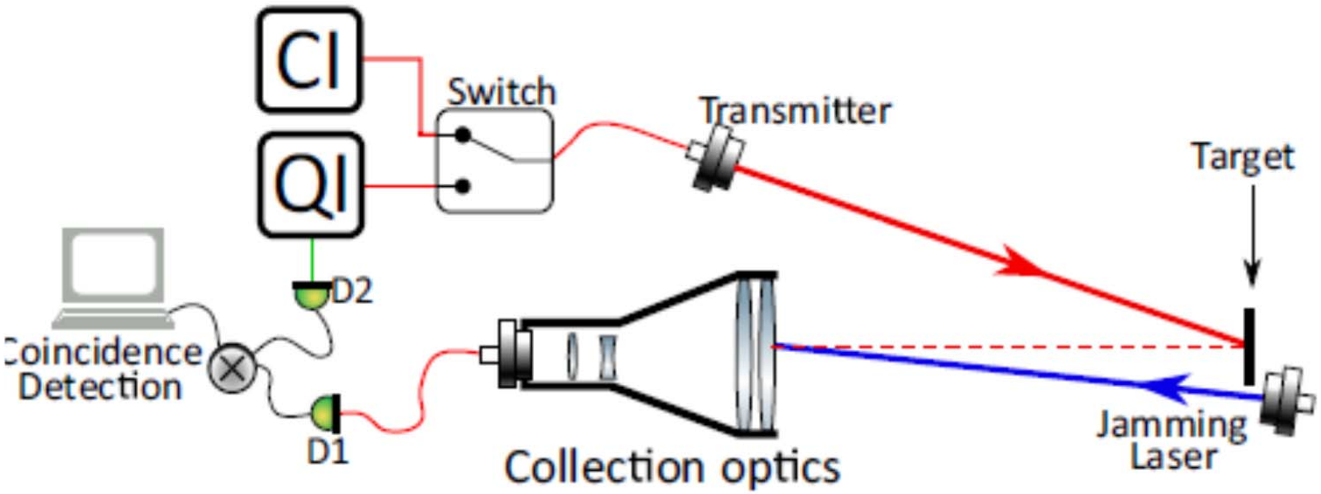


Quantum Illumination with Correlated Photon Pairs

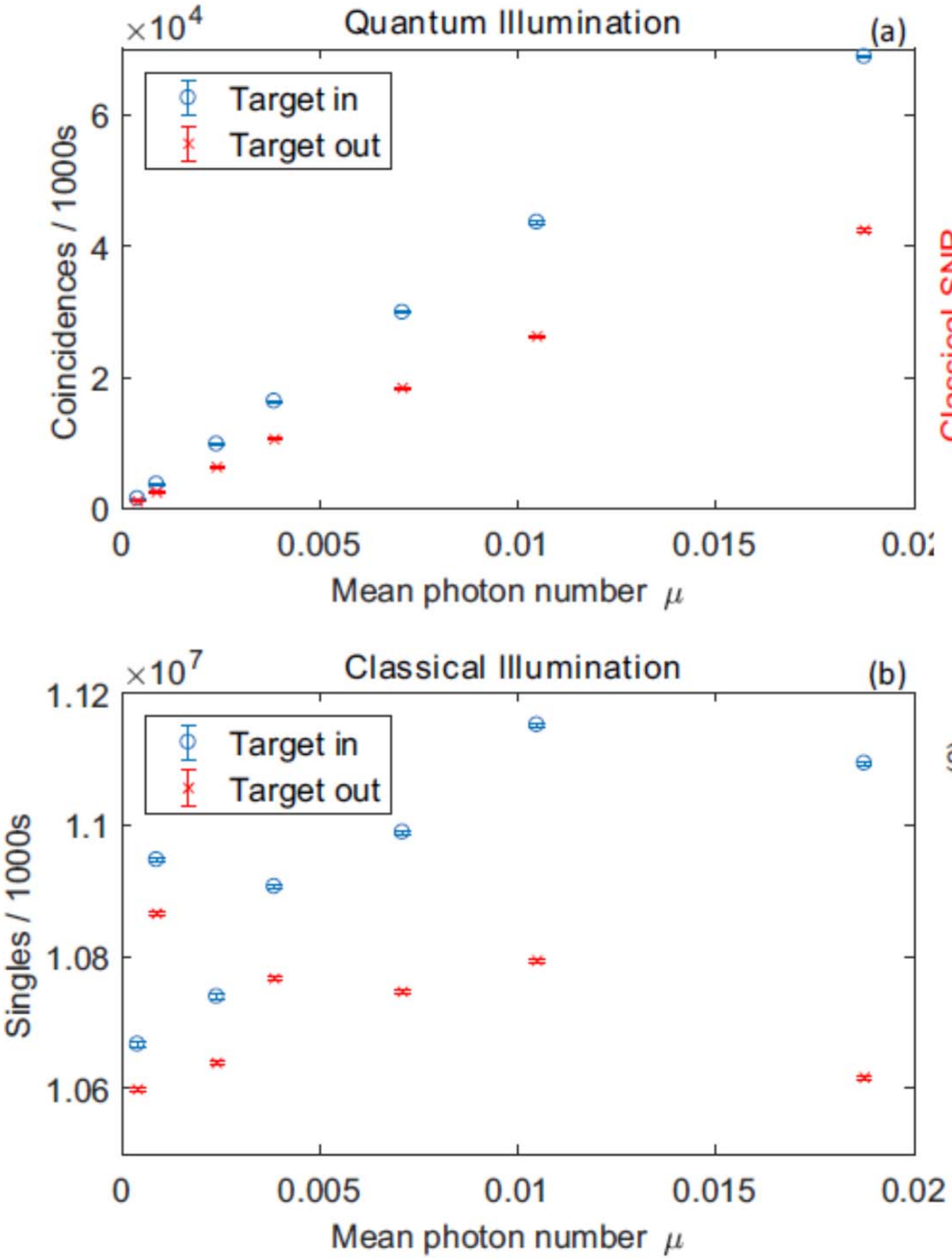
- Can one gain the quantum advantage in the high loss condition?
- What quantum state is optimal? What is optimal measurement scheme? What’s the maximum quantum advantage (6 dB?)

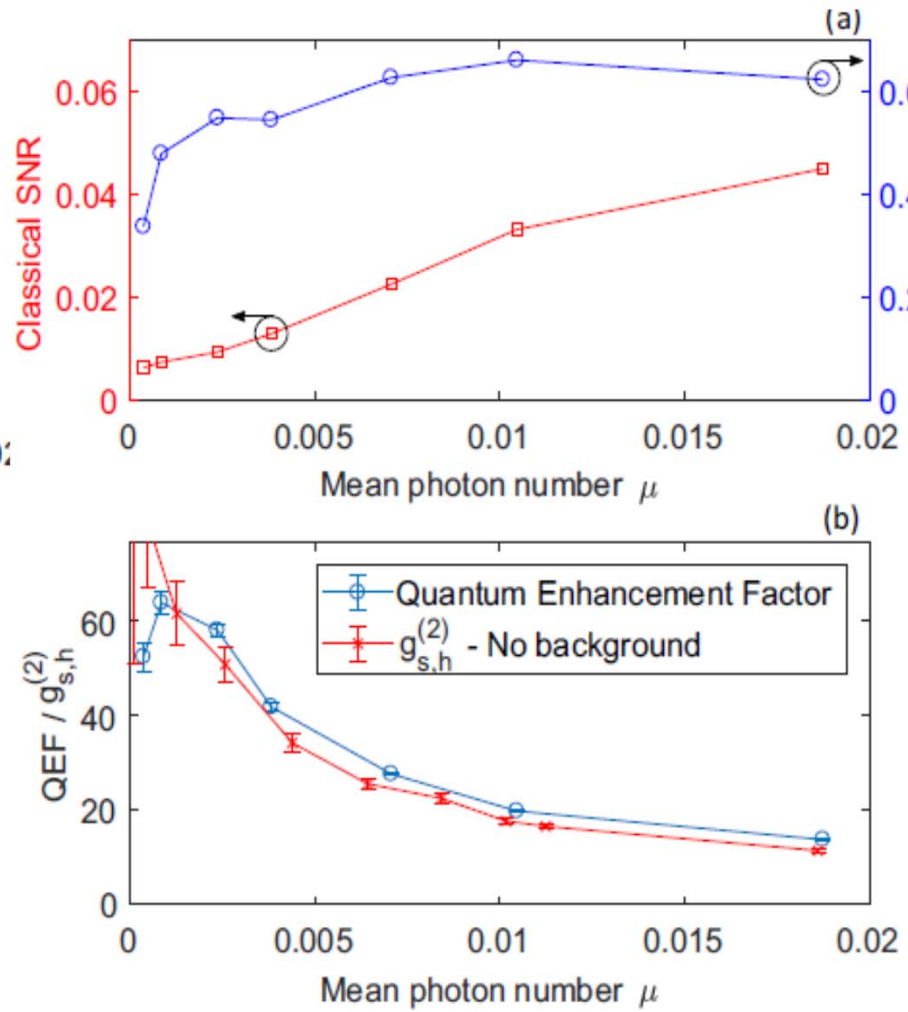


Quantum Radar (LIDAR) ?



For a review on QI, Arxiv: 1910.12277; 2004.09234;PRA 99, 023828(2019)

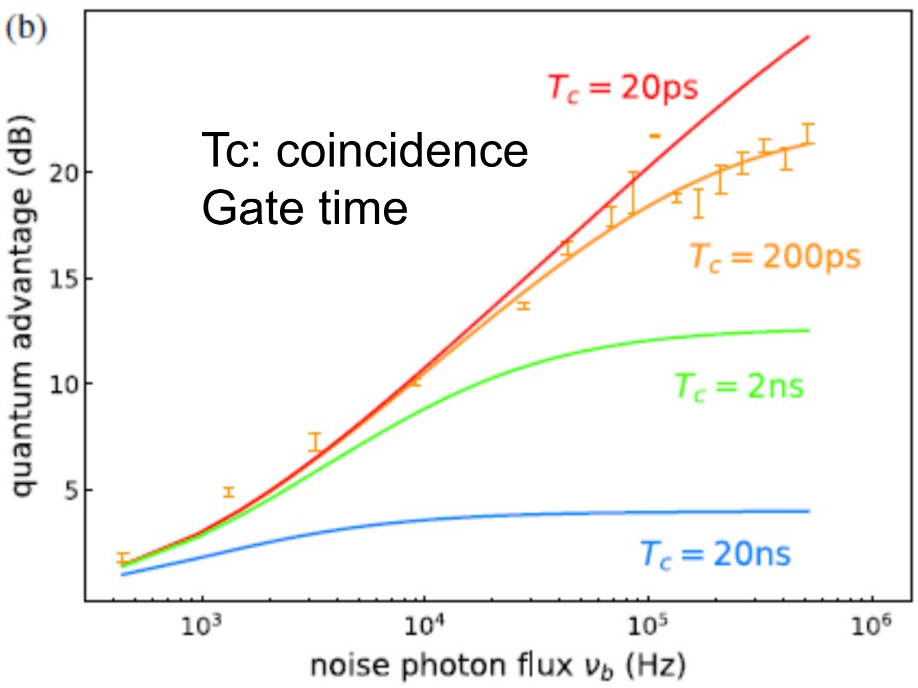


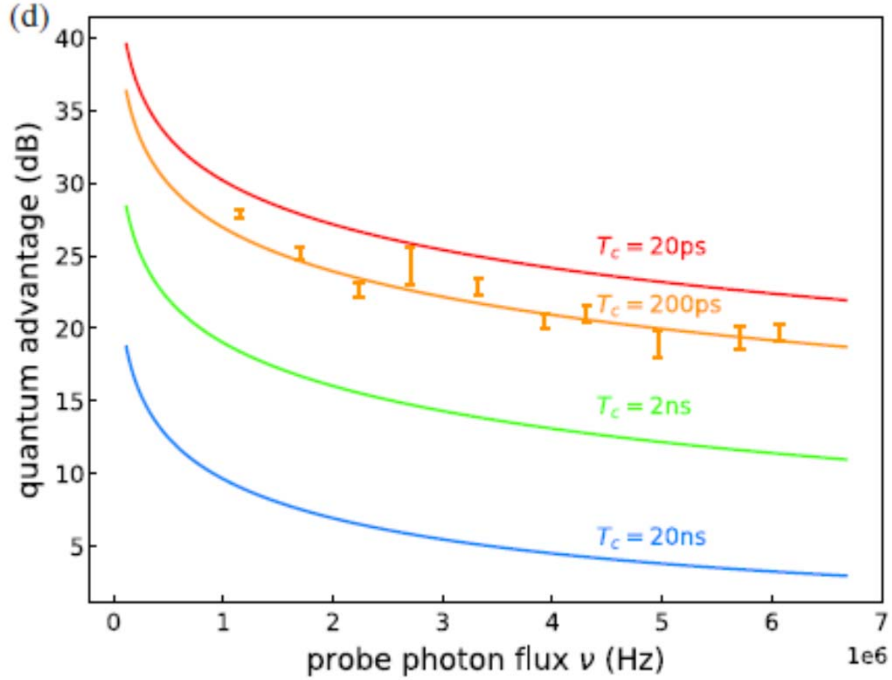


PRA 99, 023828(2019)

A Recent Result

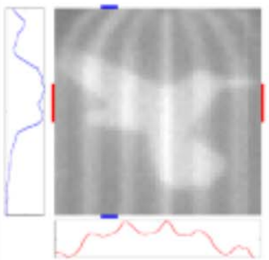
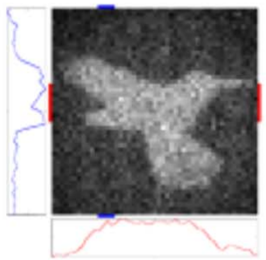
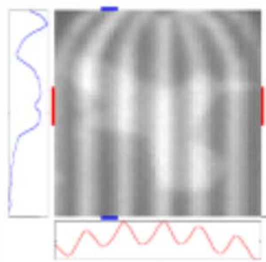
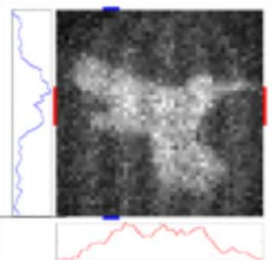
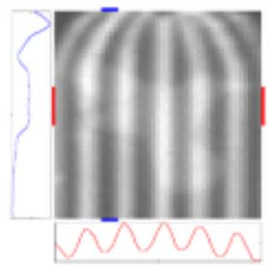
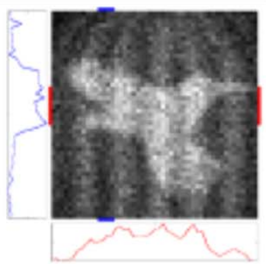
- Using continuous-wave pump laser & superconducting nanowire single photon detector which has a low time jittering (~100 ps).
- Such simple to get ~ 26 dB quantum advantage ?

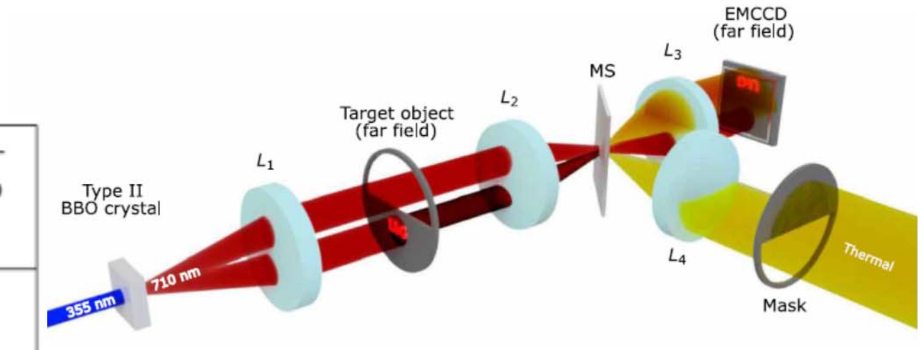




PRA 99, 023828(2019)

Quantum-Enhanced Imaging

Thermal to SPDC illumination ratio	Classically acquired image	Quantum illumination AND-image	Distinguish-ability ratio D_Q/D_C
0.5			1.35
1.5			2.39
2.5			3.50

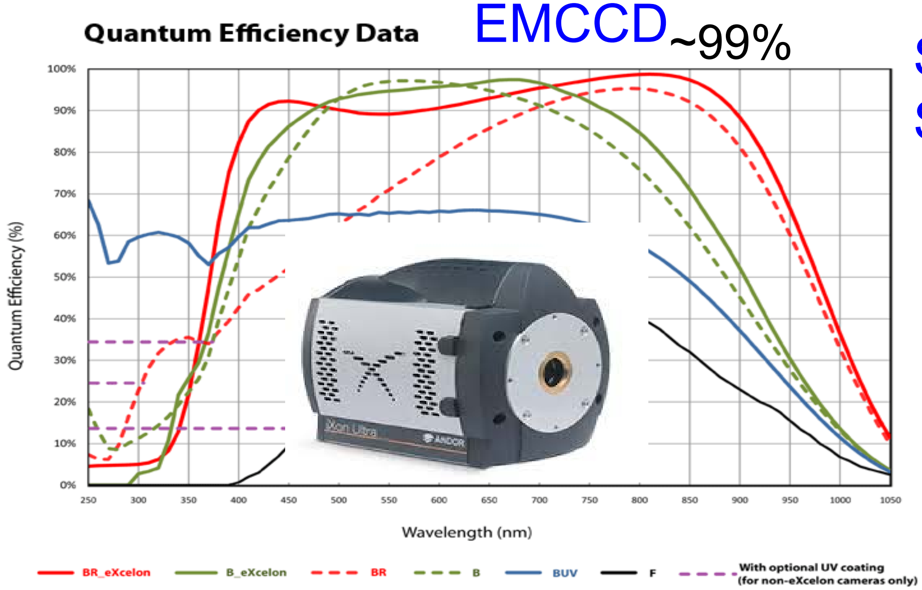


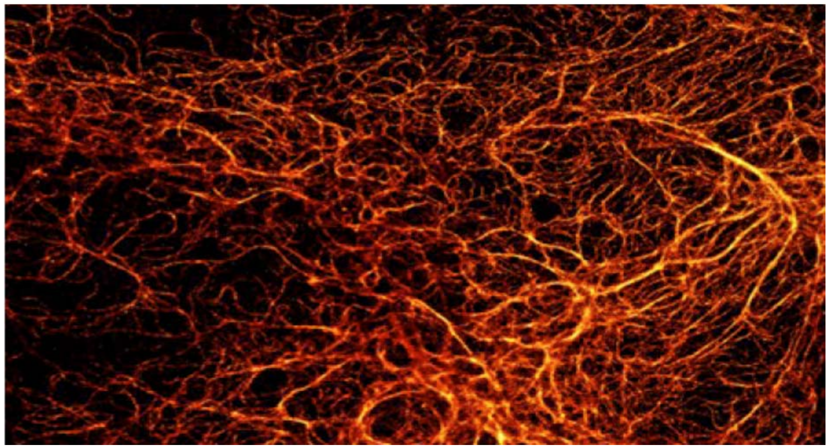
Sci. Avd. 6:eaay2652(2020)

Spirit (Price) of QST: the Pursuit for Perfect

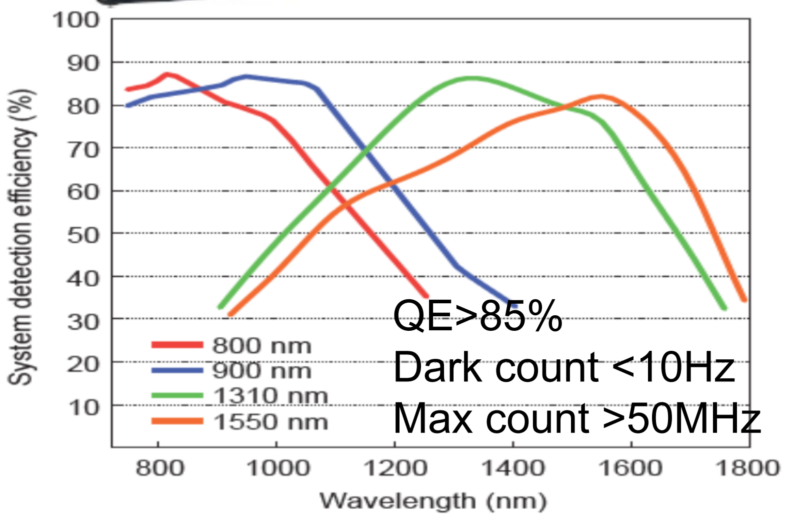
- The absolute high requirements of QST drive us to pursuit for perfect technology/material which benefit all communities.

EMCCD ~99%





Superconducting Nanowire Single-Photon Detector



QE>85%
Dark count <10Hz
Max count >50MHz

Summary on my personal viewpoints

Informational analysis on theoretical quantum advantage should be on the highest priority (even important than physical implementation) !

Quantum advantage:
Sensitivity improvement,
Complexity reduction, security…

from

Non-classicality (quantumness):
Entanglement, squeezing,
Superposition, correlation…

Fight against

Requires active role

Dephasing (decoherence), loss,
noise, spontaneous decay…

Nonlinearity, strong atom-photon
Interaction, post selection,
Feedforward, N-resolving
Capability, precise control...

That's why people
develop…

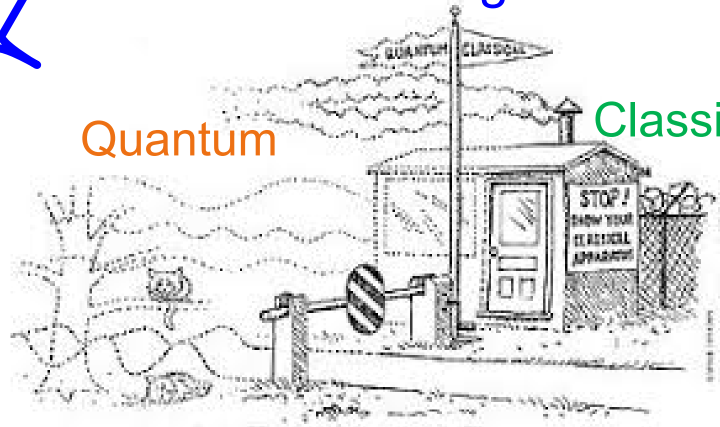
Pursuit for perfect

Can these developments help
our understanding of ...

Topological quantum computing
Topological insulator,
Majorana fermions…

Quantum

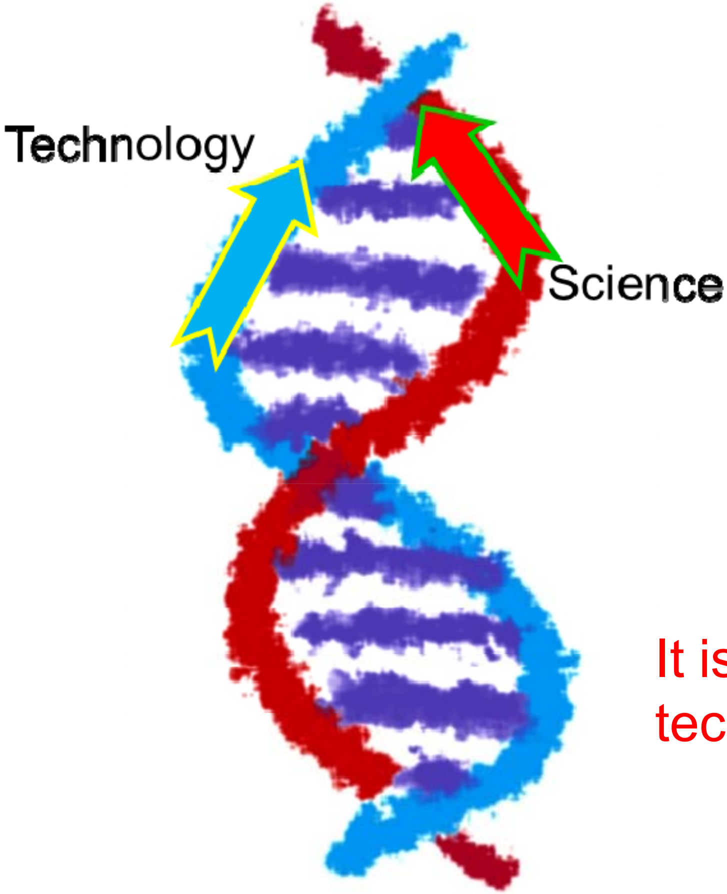
Classical



Summary of my Viewpoints on QST

- Quantum advantages come from noc-lassicality or quantumness.
- Pursuit for perfect, benefit to all research fields.
  [AMO/QST技術領頭羊]
- Theoretical studies on maximum quantum advantage are important before one really do the experiment. [資訊理論很重要!]
- Quantum Fisher information and Cramer-Rao bound analysis is important.[要讀書!]
- QST deepen our understanding on the fundamental side, e.g.
  [科學與技術相輔相成]
  physical limit of quantum devices,
  structure of many-body entanglement,
  topological quantum devices less affected by decoherence,
  boundary between classical and quantum world.
- Although quantum computer may not make big fortune soon, the supporting equipment/device does [研究人口一多就有錢賺].
- …

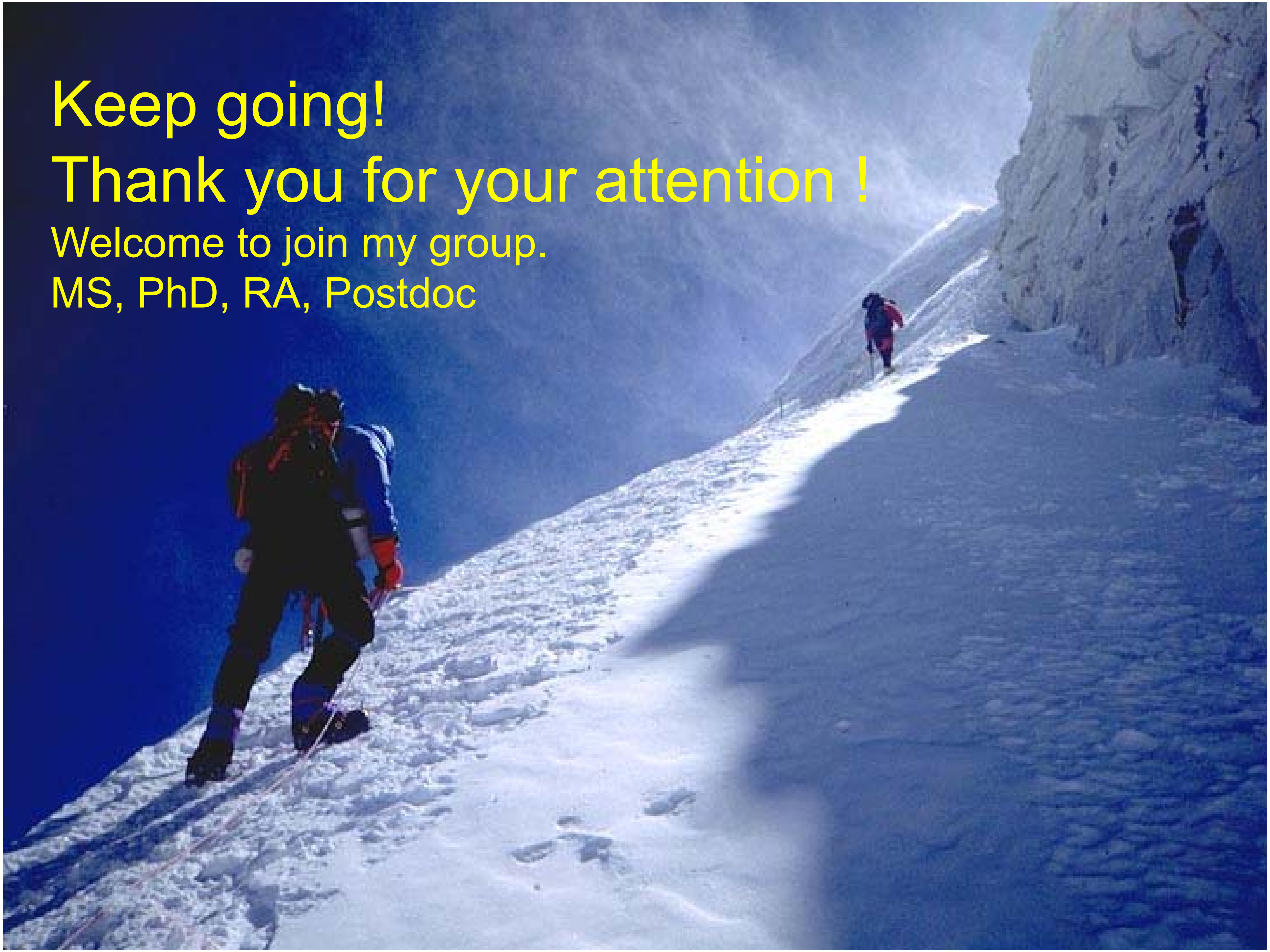
Double Helix of Science & Technology



Better understanding of science helps technology moving forward

Better technology helps to explore new science

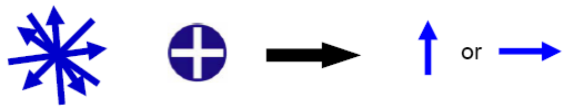
It is a tradition of AMO physics to develop new technologies and explore physics at new regime !


Keep going!
Thank you for your attention !
Welcome to join my group.
MS, PhD, RA, Postdoc

Quantum Cryptography: Positive use of “negative” QM laws

1. Use of single photons and the irreducible randomness.

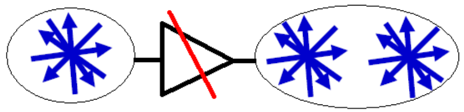
2. Observation disturbs a system:



guaranteed by the Uncertainty principle:
(for any non-commute operation)

$$\Delta x \Delta p \geq \hbar / 2$$

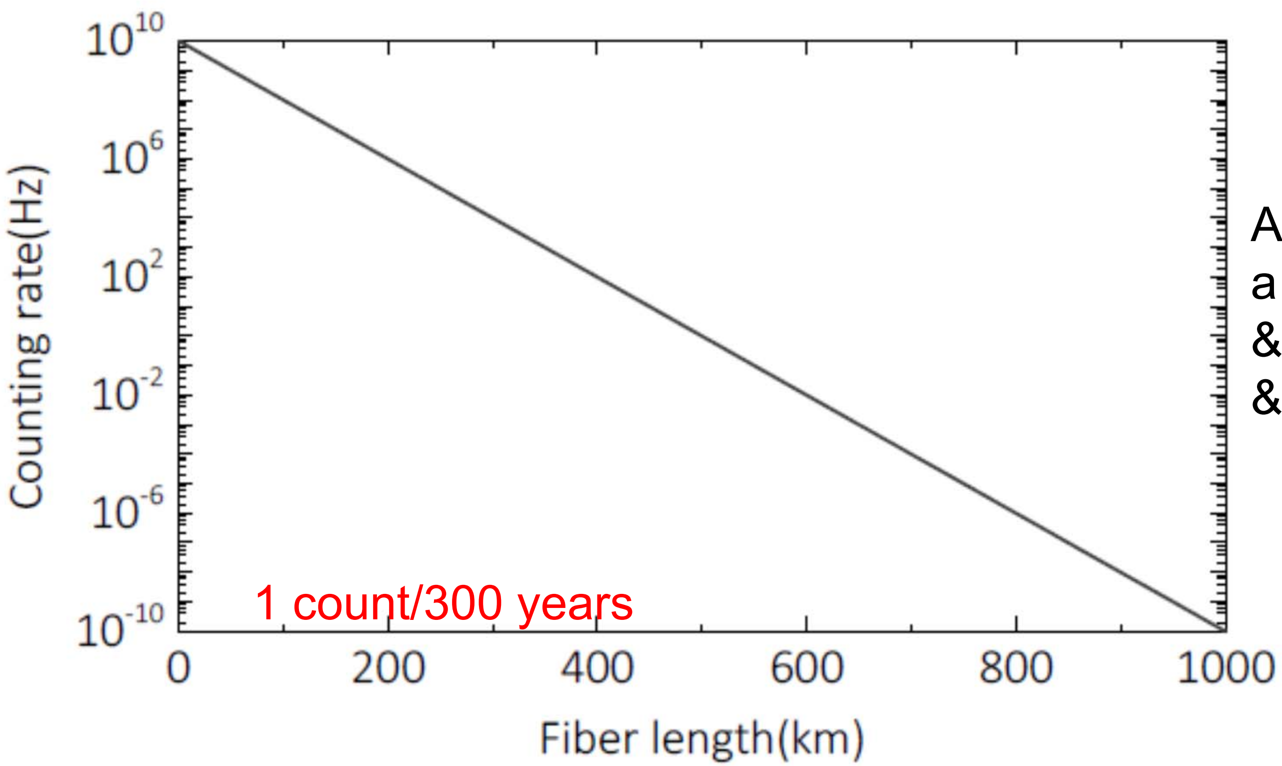
3. Linearity prohibits duplication:
(No cloning theorem)



- **Absolute security is guaranteed by the fundamental physical laws.**

Issue for Long-Distance Quantum Communication

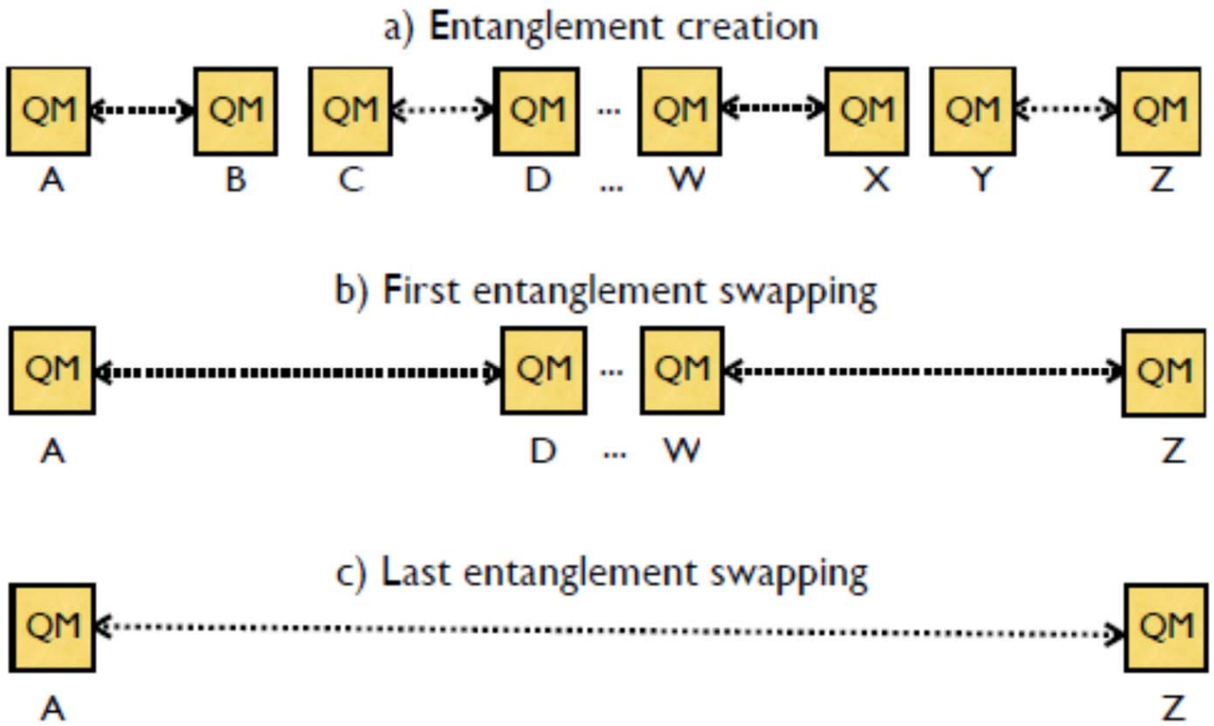
- Photon loss and decoherence prohibit the fiber-based long-distance quantum communication due to the use of single photons and no-cloning theorem (can't use amplifier or repeater), which is the same reason for its security.



Assuming
a10 GHz photo rate
& 100% detection efficiency
& 0.2 dB/km fiber loss

Quantum Repeaters

- Utilize entanglement swapping and quantum memory.
- Quantum memory allows the wait-until-success strategy.
- For sufficient high memory and detection efficiency, the entanglement distribution rate can outperform the direct transmission of light.



Rev. Mod. Phys. 83, 33(2011)

Trends: Always Making Better Quantum Devices 1/5

The quest for a perfect single-photon source Nat. Photon 13, 731 & 770(2019)

Counting near-infrared single-photons with 95% efficiency

Opt. Express 16, 3032(2008)

High quantum-efficiency photon-number-resolving detector for photonic on-chip information processing

Opt. Express 21, 22657(2013)

Near-Unity Coupling Efficiency of a Quantum Emitter to a Photonic Crystal Waveguide

PRL 113, 093603(2014)

Highly efficient frequency conversion with bandwidth compression of quantum light

Nat. Com 8:14288(2017)

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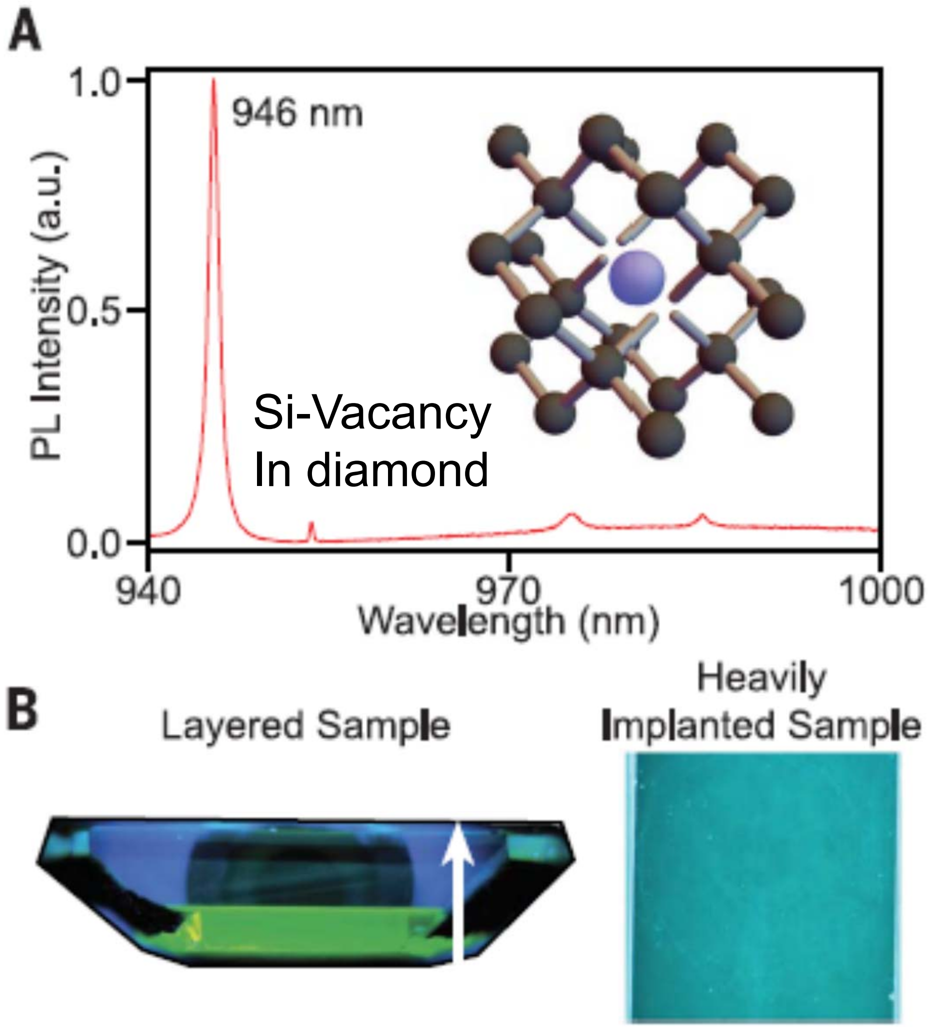
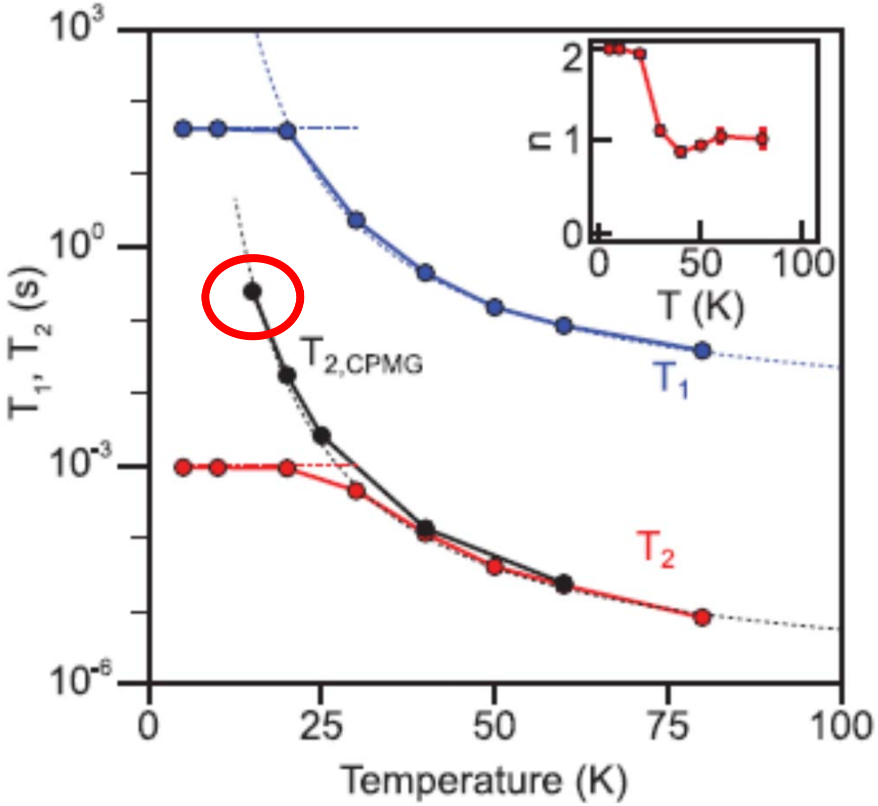
Trends: Solid-State Atom-like (Defect) System 2/5

- Finding better defect system behaves more like real atom.
- Good to fabricate desired pattern and combine with waveguide etc.



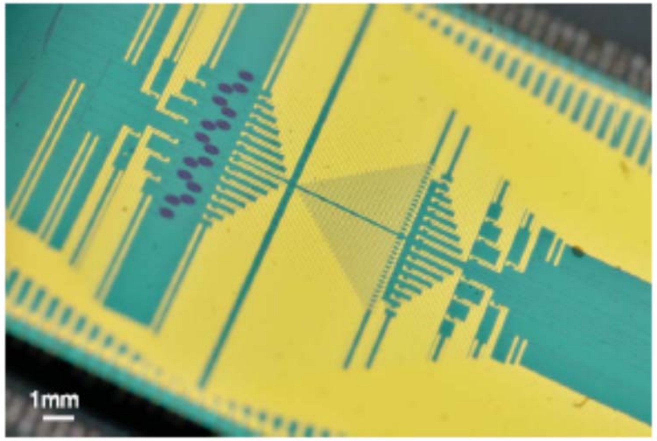
Coherence time ~0.2 sec



B Rose et al. Science, 361,60(2018)

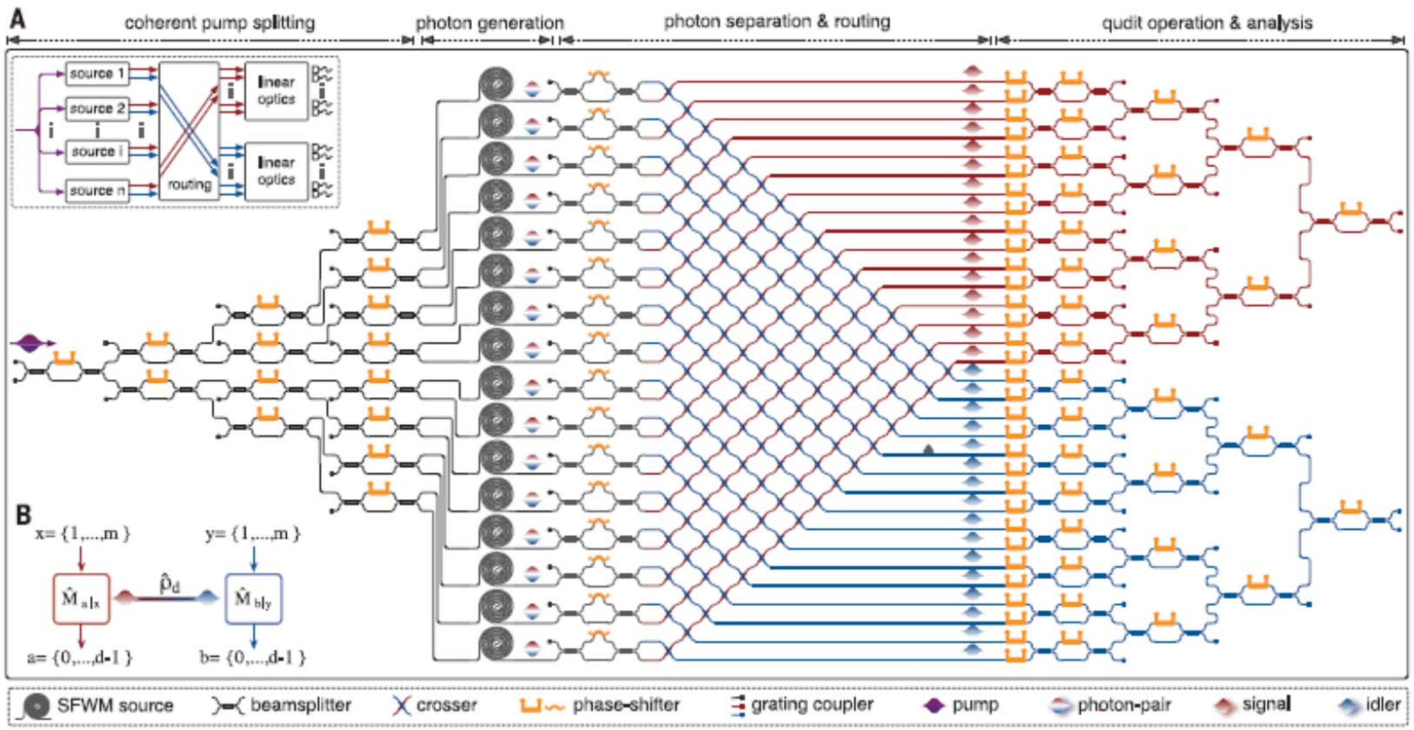
Trends: Chip-size Quantum Optical System 3/5

C

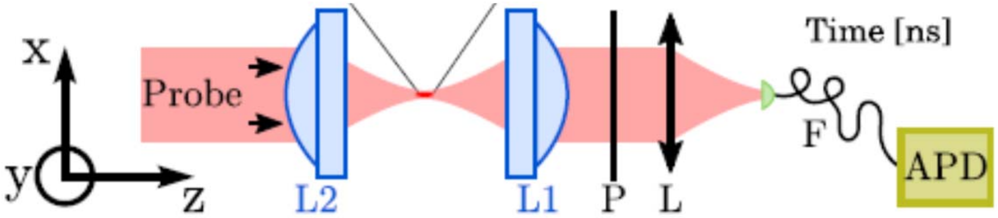


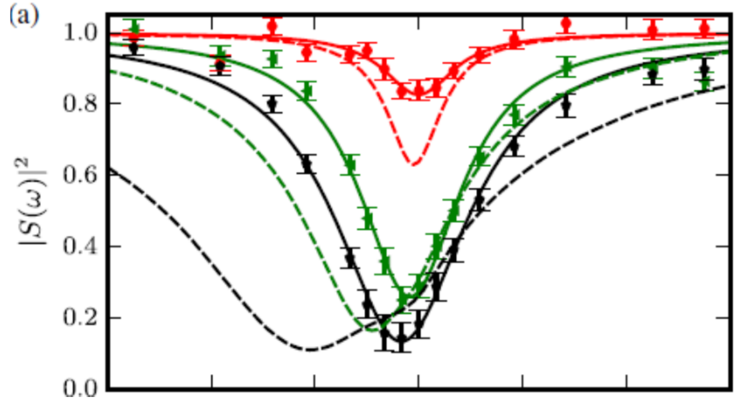
Multidimensional quantum entanglement with large-scale integrated optics

J Wang et al. Science 360, 285(2018)



Trends: Utilize Many-body Cooperative State 4/5



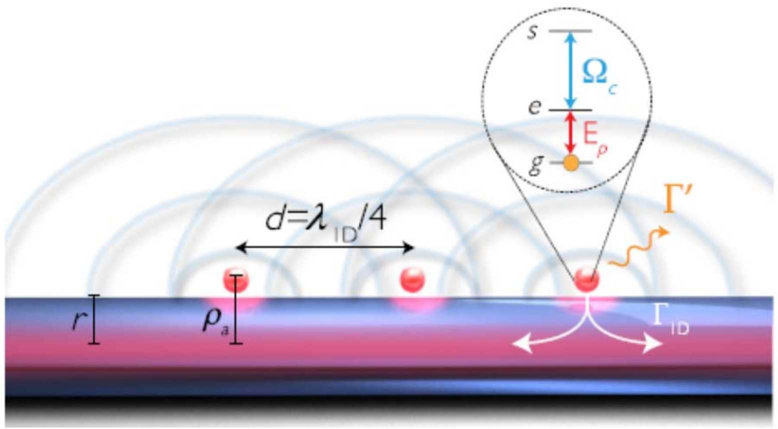


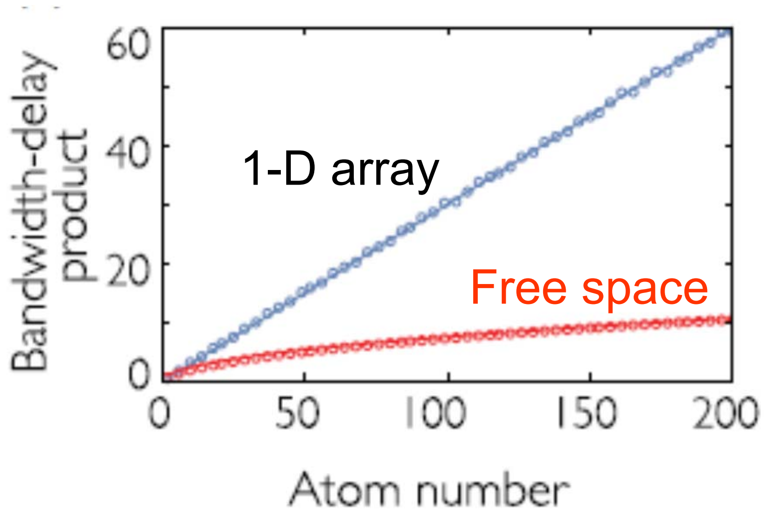
the microscopic model shows that a quantitative understanding of the light-induced interactions even in a relatively simple situation is still a challenge.

PRL, 116, 233601(2016)

Exponential Improvement in Photon Storage Fidelities Using Subradiance and "Selective Radiance" in Atomic Arrays

PRX, 7, 031024(2017)

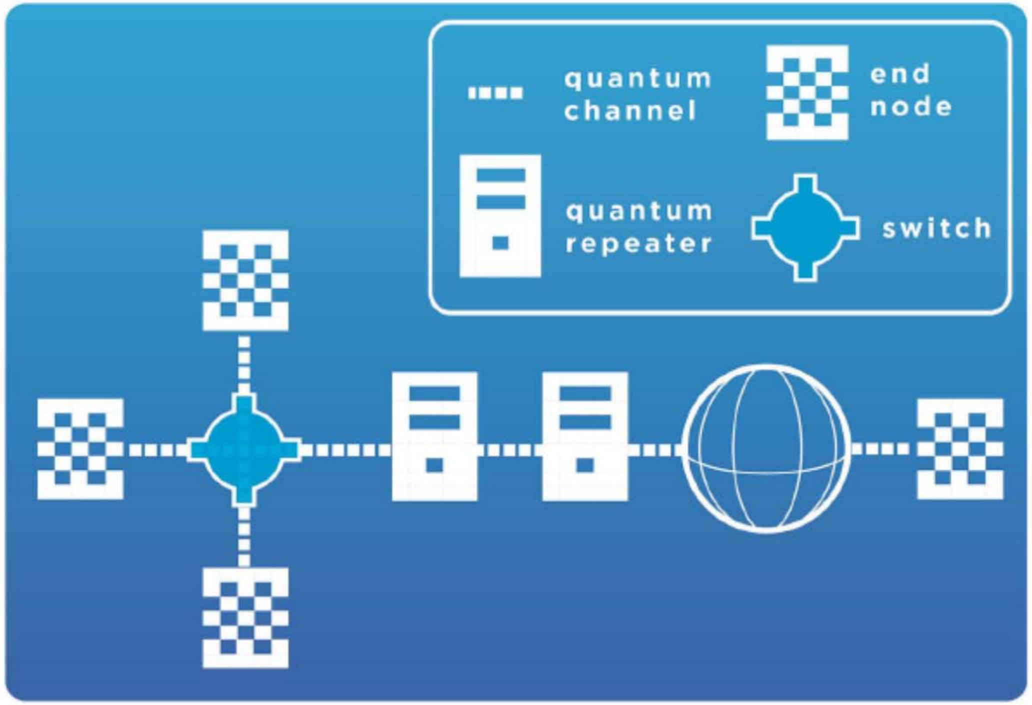


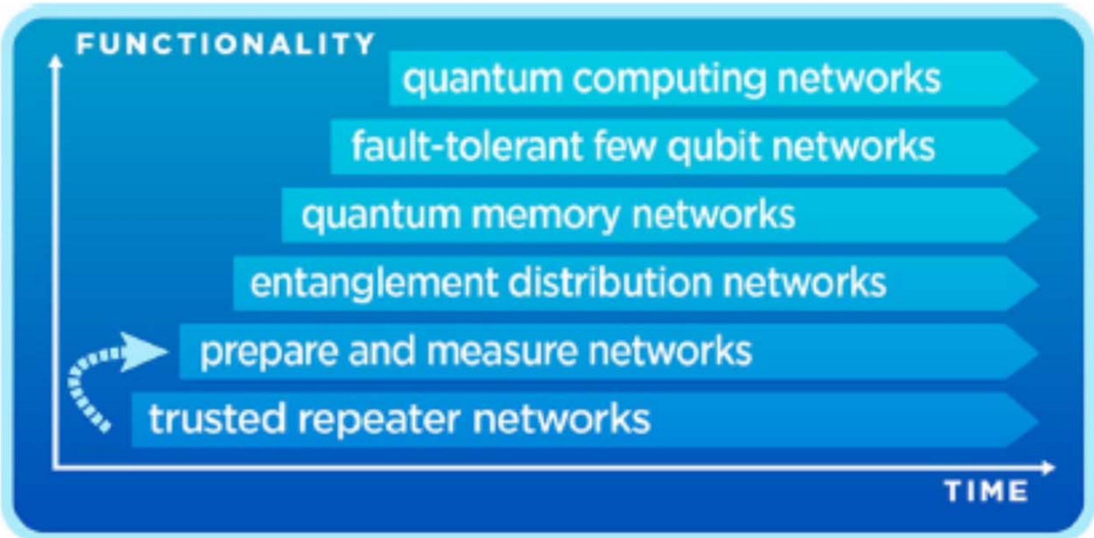


Large-scale Quantum Network 5/5

Quantum internet: A vision for the road ahead

S Wehner at al. Science, 362, 303(2018)





The real conclusions in my mind

這個研究讓我體會到了一些更一般的道理。多年前當我們一開始進行這方面的研究時就已估計推進光學密度至 1000 以上是可能的，經多年的堅持與努力的確也達成。做研究必須有一定信念與堅持。而當一個物理量推至一極致，你也成為一定程度的先驅者，會有一些迷惑處也會有一些有趣的新發現，畢竟這是在探索一個前人未到達過的新疆域。另外，要去實現一些量子資訊科學的應用，在物理上的限制必須深入研究，惟有瞭解問題的所在才可能想出辦法解決它。而量子資訊科技，對物理系統的要求極高，往往必須將相關的物理與技術推至極限，才有實際應用的可能，這也呼應我前面已提到的一點:不管其應用前景如何，量子資訊科學的發展終將推進人類對微觀世界更深刻瞭解與操控能力。

中研院週報1512期 2015/4/6