Rotating wave approximation and its causality consequences

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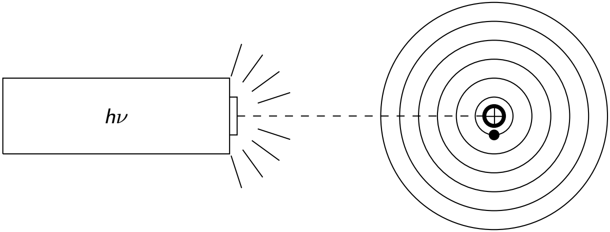
Work done in collaboration with Eduardo Martín-Martínez

Approximations in light-matter interaction



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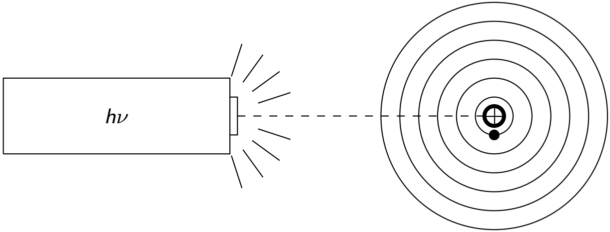

- Full model: Quantum EM field ($\hat{A}_\mu$) and minimal interaction ($\hat{p} \cdot \hat{A}$),
- Dipole approximation: Quantum EM field ($\hat{A}_\mu$) and dipole interaction ($\hat{r} \cdot \hat{E}$),

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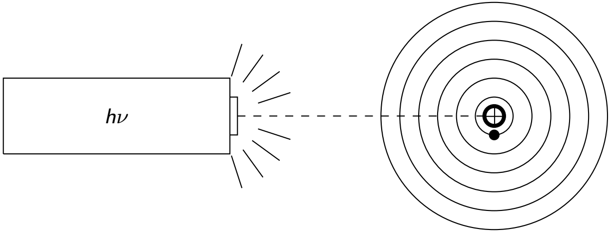

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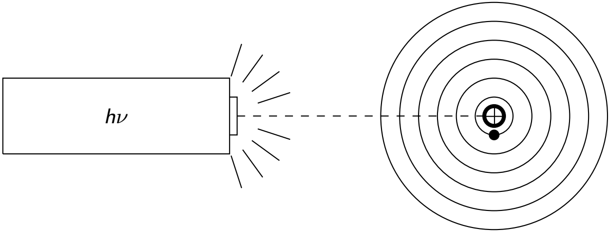
Approximations in light-matter interaction



- Full model: Quantum EM field ($\hat{A}_\mu$) and minimal interaction ($\hat{p} \cdot \hat{A}$),
- Dipole approximation: Quantum EM field ($\hat{A}_\mu$) and dipole interaction ($\hat{r} \cdot \hat{E}$),
- Unruh DeWitt: Quantum scalar field ($\hat{\phi}$) and monopole coupling ($\hat{\sigma}_x \, \hat{\phi}$).
- Further approximations: Rotating wave approximation (RWA) and single mode approximation (SMA).

RWA

UdW: (Note: $\omega = |\boldsymbol{k}|$ and Ω is the detector energy gap.)

$$\begin{aligned}
\hat{H}_{\mathrm{I}}^{\mathrm{UdW}} &= \chi(t)\hat{\sigma}_x(t)\int \mathrm{d}^3\boldsymbol{x}\, F(\boldsymbol{x})\hat{\phi}(\boldsymbol{x},t) \\
&= \chi(t)\left(\hat{\sigma}^+ e^{\mathrm{i}\Omega t} + \hat{\sigma}^- e^{-\mathrm{i}\Omega t}\right)\int \mathrm{d}^3\boldsymbol{x}\, F(\boldsymbol{x})\int \frac{\mathrm{d}^3\boldsymbol{k}}{(2\pi)^{3/2}\sqrt{2\omega}}\left(\hat{a}_{\boldsymbol{k}} e^{-\mathrm{i}\omega t+\mathrm{i}\boldsymbol{k}\cdot\boldsymbol{x}} + \hat{a}^\dagger_{\boldsymbol{k}} e^{\mathrm{i}\omega t-\mathrm{i}\boldsymbol{k}\cdot\boldsymbol{x}}\right) \\
&= \chi(t)\int \mathrm{d}^3\boldsymbol{x}\, F(\boldsymbol{x})\int \frac{\mathrm{d}^3\boldsymbol{k}}{(2\pi)^{3/2}\sqrt{2\omega}}\Big(\hat{a}_{\boldsymbol{k}}\hat{\sigma}^+ e^{-\mathrm{i}(\omega-\Omega)t+\mathrm{i}\boldsymbol{k}\cdot\boldsymbol{x}} + \hat{a}^\dagger_{\boldsymbol{k}}\hat{\sigma}^- e^{\mathrm{i}(\omega-\Omega)t-\mathrm{i}\boldsymbol{k}\cdot\boldsymbol{x}} \\
&\quad + \hat{a}_{\boldsymbol{k}}\hat{\sigma}^- e^{-\mathrm{i}(\omega+\Omega)t+\mathrm{i}\boldsymbol{k}\cdot\boldsymbol{x}} + \hat{a}^\dagger_{\boldsymbol{k}}\hat{\sigma}^+ e^{\mathrm{i}(\omega+\Omega)t-\mathrm{i}\boldsymbol{k}\cdot\boldsymbol{x}}\Big),
\end{aligned}$$

RWA:

$$\hat{H}_{\mathrm{I}}^{\mathrm{RWA}} = \chi(t)\int \mathrm{d}^3\boldsymbol{x}\, F(\boldsymbol{x})\int \frac{\mathrm{d}^3\boldsymbol{k}}{(2\pi)^{3/2}\sqrt{2\omega}}\left(\hat{a}_{\boldsymbol{k}}\hat{\sigma}^+ e^{-\mathrm{i}(\omega-\Omega)t+\mathrm{i}\boldsymbol{k}\cdot\boldsymbol{x}} + \hat{a}^\dagger_{\boldsymbol{k}}\hat{\sigma}^- e^{\mathrm{i}(\omega-\Omega)t-\mathrm{i}\boldsymbol{k}\cdot\boldsymbol{x}}\right).$$

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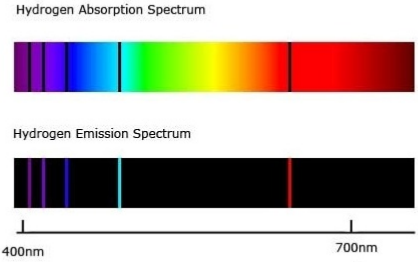
$$= \chi(t)\left(\hat{\sigma}^+ e^{\mathrm{i}\Omega t} + \hat{\sigma}^- e^{-\mathrm{i}\Omega t}\right)\int \mathrm{d}^3\boldsymbol{x}\, F(\boldsymbol{x})\int \frac{\mathrm{d}^3\boldsymbol{k}}{(2\pi)^{3/2}\sqrt{2\omega}}\left(\hat{a}_{\boldsymbol{k}} e^{-\mathrm{i}\omega t + \mathrm{i}\boldsymbol{k}\cdot\boldsymbol{x}} + \hat{a}^\dagger_{\boldsymbol{k}} e^{\mathrm{i}\omega t - \mathrm{i}\boldsymbol{k}\cdot\boldsymbol{x}}\right)$$

$$= \chi(t)\int \mathrm{d}^3\boldsymbol{x}\, F(\boldsymbol{x})\int \frac{\mathrm{d}^3\boldsymbol{k}}{(2\pi)^{3/2}\sqrt{2\omega}}\Big(\hat{a}_{\boldsymbol{k}}\hat{\sigma}^+ e^{-\mathrm{i}(\omega-\Omega)t + \mathrm{i}\boldsymbol{k}\cdot\boldsymbol{x}} + \hat{a}^\dagger_{\boldsymbol{k}}\hat{\sigma}^- e^{\mathrm{i}(\omega-\Omega)t - \mathrm{i}\boldsymbol{k}\cdot\boldsymbol{x}}$$

$$+ \hat{a}_{\boldsymbol{k}}\hat{\sigma}^- e^{-\mathrm{i}(\omega+\Omega)t + \mathrm{i}\boldsymbol{k}\cdot\boldsymbol{x}} + \hat{a}^\dagger_{\boldsymbol{k}}\hat{\sigma}^+ e^{\mathrm{i}(\omega+\Omega)t - \mathrm{i}\boldsymbol{k}\cdot\boldsymbol{x}}\Big),$$

RWA: "Excitation preserving"

$$\hat{H}_{\mathrm{I}}^{\mathrm{RWA}} = \chi(t)\int \mathrm{d}^3\boldsymbol{x}\, F(\boldsymbol{x})\int \frac{\mathrm{d}^3\boldsymbol{k}}{(2\pi)^{3/2}\sqrt{2\omega}}\left(\hat{a}_{\boldsymbol{k}}\hat{\sigma}^+ e^{-\mathrm{i}(\omega-\Omega)t + \mathrm{i}\boldsymbol{k}\cdot\boldsymbol{x}} + \hat{a}^\dagger_{\boldsymbol{k}}\hat{\sigma}^- e^{\mathrm{i}(\omega-\Omega)t - \mathrm{i}\boldsymbol{k}\cdot\boldsymbol{x}}\right).$$



Lore: RWA is good for long times.

UdW probability of vacuum qubit excitation:

$$P\left(|-z\rangle \to |+z\rangle\right) = \int \frac{\mathrm{d}^3\boldsymbol{k}}{(2\pi)^3 2\omega} \left|F(\boldsymbol{k})\right|^2 \int_{-\infty}^{\infty} \mathrm{d}t_1\, \chi\left(\frac{t_1}{T}\right) e^{-\mathrm{i}(\omega+\Omega)t_1} \underbrace{\int_{-\infty}^{\infty} \mathrm{d}t_1'\, \chi\left(\frac{t_1'}{T}\right) e^{\mathrm{i}(\omega+\Omega)t_1'}}_{\to\delta(\omega+\Omega)},$$

UdW probability of vacuum qubit emission:

$$P\left(|+z\rangle \to |-z\rangle\right) = \int \frac{\mathrm{d}^3\boldsymbol{k}}{(2\pi)^3 2\omega} \left|F(\boldsymbol{k})\right|^2 \overbrace{\int_{-\infty}^{\infty} \mathrm{d}t_1\, \chi\left(\frac{t_1}{T}\right) e^{-\mathrm{i}(\omega-\Omega)t_1}}^{\to\delta(\omega-\Omega)} \int_{-\infty}^{\infty} \mathrm{d}t_1'\, \chi\left(\frac{t_1'}{T}\right) e^{\mathrm{i}(\omega-\Omega)t_1'},$$

RWA claim: The RWA transition probabilities become exact when $T\Omega \gg 1$.

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UdW probability of vacuum qubit excitation:

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We will present:

- RWA acausal energy density plots and asymptotic behaviour.
- Exact description of interaction Hamiltonian non-locality.
- A stronger criterion for RWA applicability.
- How results change within a cavity.

Is RWA good? $\hat{H}_{\mathrm{I}}^{\mathrm{RWA}}$ non-locality.

- $\hat{H}_{\mathrm{I}}^{\mathrm{RWA}}$ cannot be written with local operators (Clerk and Sipe 1998).

$$\hat{H}_{\mathrm{I}}^{\mathrm{RWA}} = \chi(t)\int \mathrm{d}^3\boldsymbol{x} F(\boldsymbol{x}) \int \frac{\mathrm{d}^3\boldsymbol{k}}{(2\pi)^{3/2}\sqrt{2\omega}}\left(\hat{a}_{\boldsymbol{k}}\hat{\sigma}^+ e^{-\mathrm{i}(\omega-\Omega)t+\mathrm{i}\boldsymbol{k}\cdot\boldsymbol{x}} + \hat{a}_{\boldsymbol{k}}^\dagger\hat{\sigma}^- e^{\mathrm{i}(\omega-\Omega)t-\mathrm{i}\boldsymbol{k}\cdot\boldsymbol{x}}\right),$$
$$\neq \chi(t)\int \mathrm{d}^3\boldsymbol{x}\, F(\boldsymbol{x})\left(\lambda\hat{\phi}(\boldsymbol{x},t) + \mu\hat{\pi}(\boldsymbol{x},t)\right)$$

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$$\hat{a}_{\boldsymbol{k}} = \frac{1}{(2\pi)^{\frac{3}{2}}} \int \mathrm{d}^3\boldsymbol{x} \left(\sqrt{\frac{\omega}{2}}\hat{\phi}(\boldsymbol{x},t) + \frac{\mathrm{i}}{\sqrt{2\omega}}\hat{\pi}(\boldsymbol{x},t) \right) e^{\mathrm{i}\omega t-\mathrm{i}\boldsymbol{k}\cdot\boldsymbol{x}},$$
$$\hat{a}_{\boldsymbol{k}}^\dagger = \frac{1}{(2\pi)^{\frac{3}{2}}} \int \mathrm{d}^3\boldsymbol{x} \left(\sqrt{\frac{\omega}{2}}\hat{\phi}(\boldsymbol{x},t) - \frac{\mathrm{i}}{\sqrt{2\omega}}\hat{\pi}(\boldsymbol{x},t) \right) e^{-\mathrm{i}\omega t+\mathrm{i}\boldsymbol{k}\cdot\boldsymbol{x}}.$$

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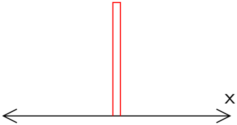
$$\hat{a}^{\dagger}_{\boldsymbol{k}} = \frac{1}{(2\pi)^{\frac{3}{2}}}\int \mathrm{d}^3\boldsymbol{x}\left(\sqrt{\frac{\omega}{2}}\hat{\phi}(\boldsymbol{x},t) - \frac{\mathrm{i}}{\sqrt{2\omega}}\hat{\pi}(\boldsymbol{x},t)\right)e^{-\mathrm{i}\omega t+\mathrm{i}\boldsymbol{k}\cdot\boldsymbol{x}}.$$

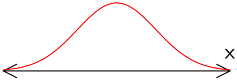
$$\int \mathrm{d}^3\boldsymbol{x}\, F(\boldsymbol{x})\hat{\sigma}_x(t)\hat{\phi}(\boldsymbol{x},t) \to \frac{1}{2}\left(\hat{\sigma}^{+}e^{\mathrm{i}\Omega t} + \hat{\sigma}^{-}e^{-\mathrm{i}\Omega t}\right)\int \mathrm{d}^3\boldsymbol{x}\, F(\boldsymbol{x})\hat{\phi}(\boldsymbol{x},t)$$
$$- \frac{\mathrm{i}}{(2\pi)^2}\left(\hat{\sigma}^{+}e^{\mathrm{i}\Omega t} - \hat{\sigma}^{-}e^{-\mathrm{i}\Omega t}\right)\int \mathrm{d}^3\boldsymbol{x}\, F(\boldsymbol{x})\int \mathrm{d}^3\boldsymbol{y}\,\frac{\hat{\pi}(\boldsymbol{y},t)}{|\boldsymbol{x}-\boldsymbol{y}|^2},$$

$\hat{H}_I^{RWA}$ non-locality.

UdW



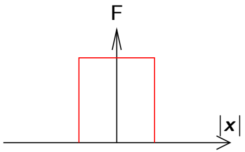
RWA

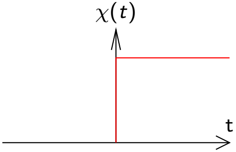


RWA causality effects on field observables

Now consider

$$F(\boldsymbol{x}) = \Theta(R - |\boldsymbol{x}|),$$
$$\chi(t) = \Theta(t).$$





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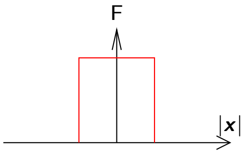
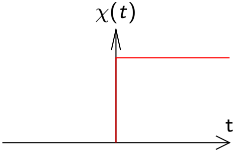


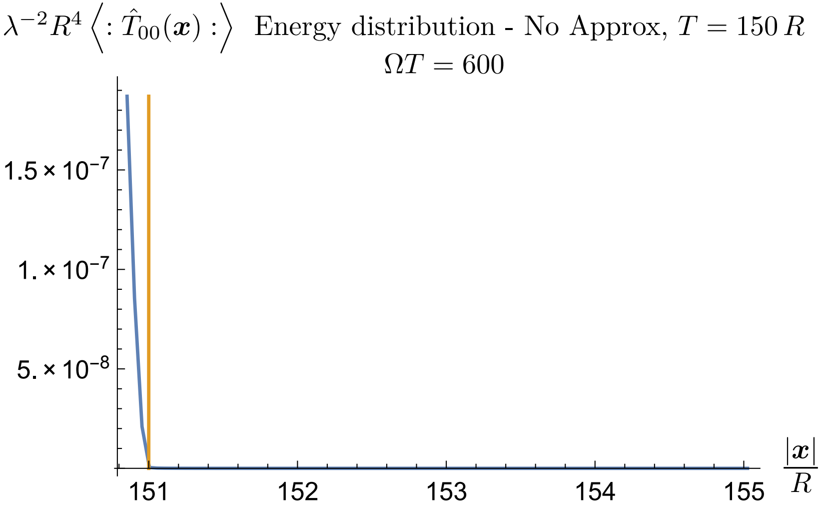


$$\hat{U}\left|0\right\rangle\left|+z\right\rangle = \left(\mathbb{I} - \mathrm{i}\int_{-\infty}^{T}\mathrm{d}t_1\,\hat{H}_{\mathrm{I}}(t_1) - \int_{-\infty}^{T}\mathrm{d}t_1\int_{-\infty}^{t_1}\mathrm{d}t_2\hat{H}_{\mathrm{I}}(t_1)\hat{H}_{\mathrm{I}}(t_2)\right)\left|0\right\rangle\left|+z\right\rangle,$$

$$\hat{H}_{\mathrm{I}}^{\mathrm{UdW}} = \chi(t)\left(\hat{\sigma}^{+}e^{\mathrm{i}\Omega t} + \hat{\sigma}^{-}e^{-\mathrm{i}\Omega t}\right)\int\mathrm{d}^3\boldsymbol{x}\,F(\boldsymbol{x})\int\frac{\mathrm{d}^3\boldsymbol{k}}{(2\pi)^{3/2}\sqrt{2\omega}}\left(\hat{a}_{\boldsymbol{k}}e^{-\mathrm{i}\omega t+\mathrm{i}\boldsymbol{k}\cdot\boldsymbol{x}} + \hat{a}_{\boldsymbol{k}}^{\dagger}e^{\mathrm{i}\omega t-\mathrm{i}\boldsymbol{k}\cdot\boldsymbol{x}}\right)$$

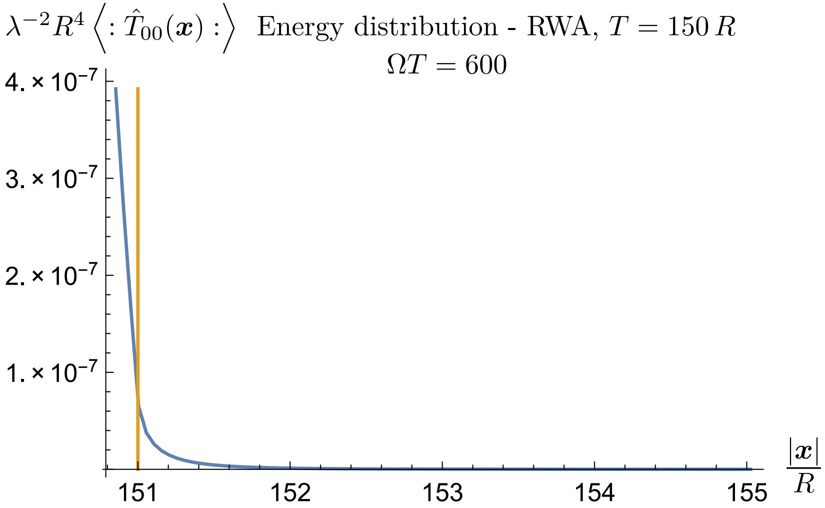
$$\hat{H}_{\mathrm{I}}^{\mathrm{RWA}} = \chi(t)\int\mathrm{d}^3\boldsymbol{x}\,F(\boldsymbol{x})\int\frac{\mathrm{d}^3\boldsymbol{k}}{(2\pi)^{3/2}\sqrt{2\omega}}\left(\hat{a}_{\boldsymbol{k}}\hat{\sigma}^{+}e^{-\mathrm{i}(\omega-\Omega)t+\mathrm{i}\boldsymbol{k}\cdot\boldsymbol{x}} + \hat{a}_{\boldsymbol{k}}^{\dagger}\hat{\sigma}^{-}e^{\mathrm{i}(\omega-\Omega)t-\mathrm{i}\boldsymbol{k}\cdot\boldsymbol{x}}\right).$$

Perturbative plots - Full model



$\Omega = 4R^{-1}$. Causal boundary/light cone surface at $|\boldsymbol{x}|/R = 150 + 1$.

Perturbative plots - RWA



$\Omega = 4R^{-1}$. Causal boundary/light cone surface at $|\boldsymbol{x}|/R = 150 + 1$. Note: No improvement with time.

Should RWA be used for field observable expectations?

$$\left\langle :\hat{\phi}^2(\boldsymbol{x},t):\right\rangle_{\text{Full}} = \frac{\lambda^2}{4(2\pi)^6}\left(2\left|M_e^1\right|^2 - M_e^2 - M_e^{2*}\right),$$

$$\left\langle :\hat{\phi}^2(\boldsymbol{x},t):\right\rangle_{\text{RWA}} = \frac{\lambda^2}{4(2\pi)^6}\left(2\left|M_e^1\right|^2\right),$$

$$M_e^1(\boldsymbol{x},t) := \int \frac{\mathrm{d}^3\boldsymbol{k}}{\omega}\tilde{F}(\boldsymbol{k})e^{\mathrm{i}\omega t - \mathrm{i}\boldsymbol{k}\cdot\boldsymbol{x}}\int_{-t}^{t}\mathrm{d}t_1\,\chi(t_1)e^{-\mathrm{i}(\omega-\Omega)t_1},$$

$$M_e^2(\boldsymbol{x},t) = 2\int\frac{\mathrm{d}^3\boldsymbol{k}\mathrm{d}^3\boldsymbol{k}'}{\omega\omega'}\tilde{F}(\boldsymbol{k})\tilde{F}(\boldsymbol{k}')e^{\mathrm{i}\omega t-\mathrm{i}\boldsymbol{k}\cdot\boldsymbol{x}}e^{\mathrm{i}\omega' t-\mathrm{i}\boldsymbol{k}'\cdot\boldsymbol{x}}$$

$$\int_{-t}^{t}\mathrm{d}t_1\int_{-t}^{t_1}\mathrm{d}t_2\,\chi(t_1)\chi(t_2)e^{-\mathrm{i}(\omega+\Omega)t_1-\mathrm{i}(\omega'-\Omega)t_2}.$$

Note, M_e^2 requires 2nd order Dyson expansion.

$$M_e^2(\boldsymbol{x},t) = 2\int \frac{\mathrm{d}^3\boldsymbol{k}\mathrm{d}^3\boldsymbol{k}'}{\omega\omega'}\tilde{F}(\boldsymbol{k})\tilde{F}(\boldsymbol{k}')e^{\mathrm{i}\omega t-\mathrm{i}\boldsymbol{k}\cdot\boldsymbol{x}}e^{\mathrm{i}\omega' t-\mathrm{i}\boldsymbol{k}'\cdot\boldsymbol{x}}$$

$$\int_{-t}^{t}\mathrm{d}t_1\int_{-t}^{t_1}\mathrm{d}t_2\,\chi(t_1)\chi(t_2)e^{-\mathrm{i}(\omega+\Omega)t_1-\mathrm{i}(\omega'-\Omega)t_2},$$

i.e. Light cone is at $|\boldsymbol{x}| = 2t$.

$$= \frac{2}{(2\pi)^4\,|\boldsymbol{x}|^2}\int \mathrm{d}\omega\,\mathrm{d}\omega'\,\tilde{F}(\omega)\tilde{F}(\omega')$$

$$\frac{1}{-\mathrm{i}(\omega'-\Omega)}\left[\frac{e^{\mathrm{i}(\omega+\omega')|\boldsymbol{x}|}-e^{\mathrm{i}(\omega+\omega')(2t+|\boldsymbol{x}|)}}{-\mathrm{i}(\omega+\omega')}-\frac{e^{-2\mathrm{i}\Omega t+\mathrm{i}\omega|\boldsymbol{x}|+\mathrm{i}\omega'(2t+|\boldsymbol{x}|)}-e^{\mathrm{i}(\omega+\omega')(2t+|\boldsymbol{x}|)}}{-\mathrm{i}(\omega+\Omega)}\right]$$

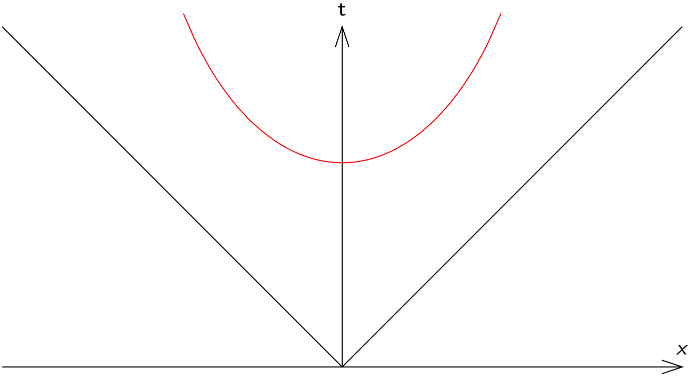
$$+\frac{\mathrm{i}}{-\mathrm{i}(\omega'-\Omega)}\left[\frac{e^{\mathrm{i}(\omega-\omega')|\boldsymbol{x}|}-e^{\mathrm{i}\omega(2t+|\boldsymbol{x}|)+\mathrm{i}\omega'(2t-|\boldsymbol{x}|)}}{-\mathrm{i}(\omega+\omega')}-\frac{e^{-2\mathrm{i}\Omega t+\mathrm{i}\omega|\boldsymbol{x}|+\mathrm{i}\omega'(2t-|\boldsymbol{x}|)}-e^{\mathrm{i}\omega(2t+|\boldsymbol{x}|)+\mathrm{i}\omega'(2t-|\boldsymbol{x}|)}}{-\mathrm{i}(\omega+\Omega)}\right]$$

$$+\frac{\mathrm{i}}{-\mathrm{i}(\omega'-\Omega)}\left[\frac{e^{-\mathrm{i}(\omega-\omega')|\boldsymbol{x}|}-e^{\mathrm{i}\omega(2t-|\boldsymbol{x}|)+\mathrm{i}\omega'(2t+|\boldsymbol{x}|)}}{-\mathrm{i}(\omega+\omega')}-\frac{e^{-2\mathrm{i}\Omega t-\mathrm{i}\omega|\boldsymbol{x}|+\mathrm{i}\omega'(2t+|\boldsymbol{x}|)}-e^{\mathrm{i}\omega(2t-|\boldsymbol{x}|)+\mathrm{i}\omega'(2t+|\boldsymbol{x}|)}}{-\mathrm{i}(\omega+\Omega)}\right]$$

$$-\frac{1}{-\mathrm{i}(\omega'-\Omega)}\left[\frac{e^{-\mathrm{i}(\omega+\omega')|\boldsymbol{x}|}-e^{\mathrm{i}(\omega+\omega')(2t-|\boldsymbol{x}|)}}{-\mathrm{i}(\omega+\omega')}-\frac{e^{-2\mathrm{i}\Omega t-\mathrm{i}\omega|\boldsymbol{x}|+\mathrm{i}\omega'(2t-|\boldsymbol{x}|)}-e^{\mathrm{i}(\omega+\omega')(2t-|\boldsymbol{x}|)}}{-\mathrm{i}(\omega+\Omega)}\right].$$

RWA$\rightarrow$UdW as $\Omega t \rightarrow \infty$, provided $|\boldsymbol{x}| \ll 2t$.

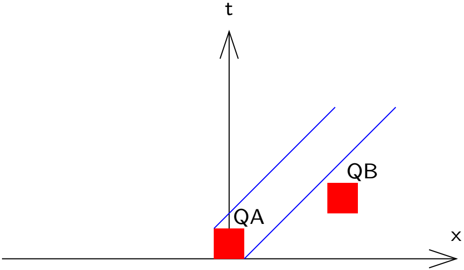
Where can we use RWA?



Communications setup

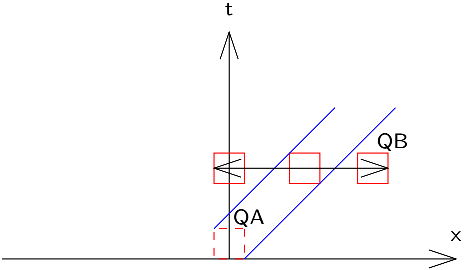
Now consider 2 qubits

$$F_j(\boldsymbol{x}) = \Theta(R_j - |\boldsymbol{x}|),$$
$$\chi_j(t) = \Theta(t - t_j^i)\Theta(t_j^f - t).$$



Martín-Martínez (2015), PRD 92,104019

Communications setup



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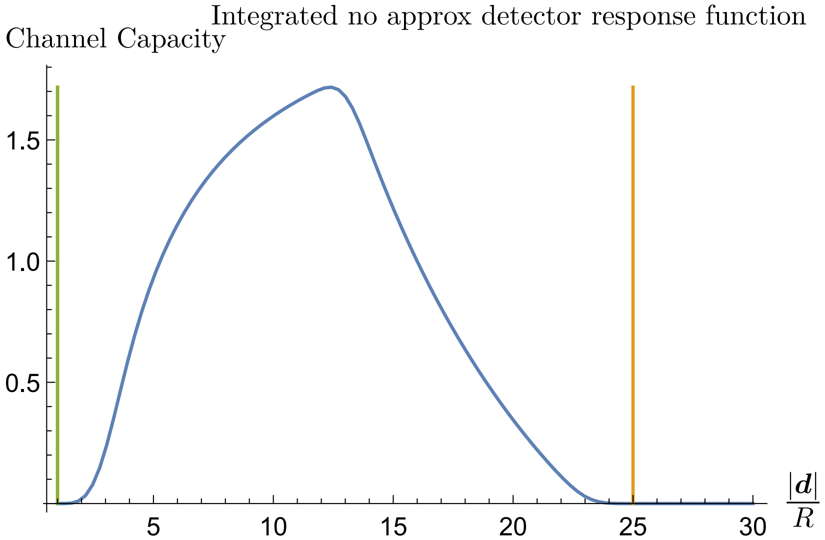
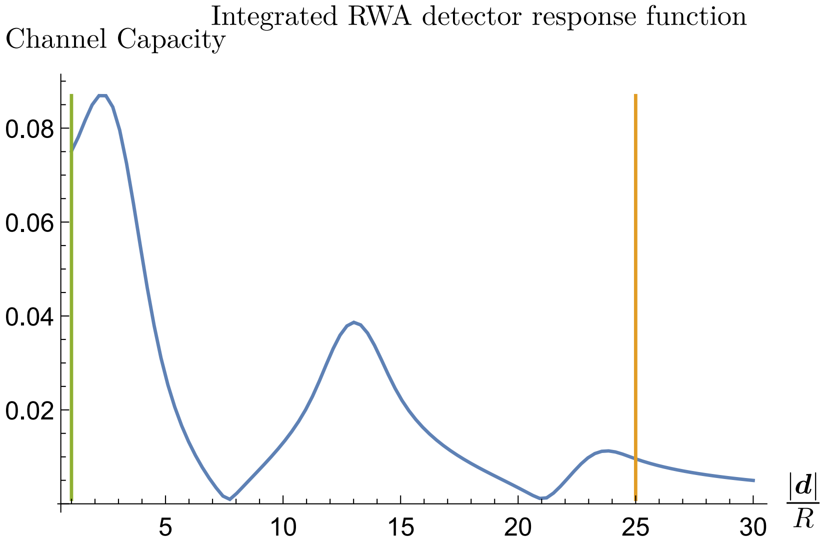


$R_A = R_B = 1$, $\Omega_A = \Omega_B = 1$. $\chi_A = 1$ for $t \in (0, 10)$, $\chi_B = 1$ for $t \in (13, 23)$.



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Asymptotics

- $\chi(t) = \Theta(t)$, i.e. sudden switching, $F(\boldsymbol{x}) = \Theta(R - |\boldsymbol{x}|)$, i.e. solid sphere, hard cutoff.

Asymptotics

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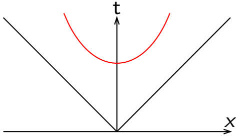
- Causality violation does not improve with increasing $t\Omega$.
- RWA does not always work for long times!



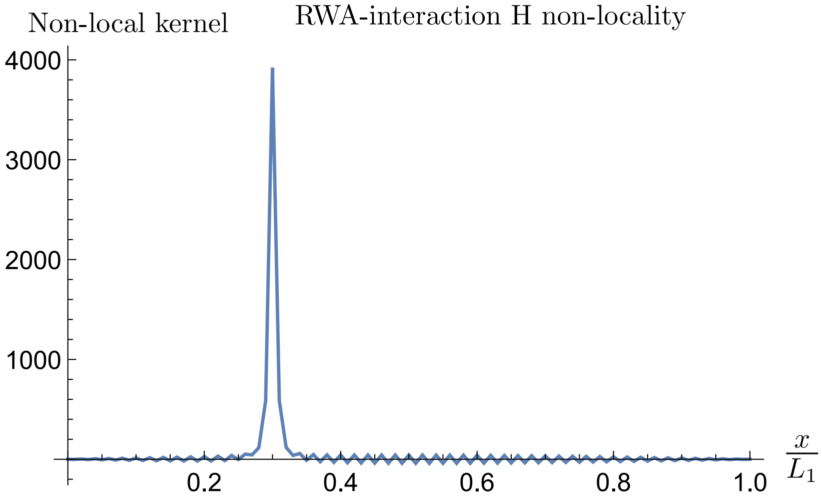
What about cavities?

$$\int \mathrm{d}^3\boldsymbol{x}\, F(\boldsymbol{x})\hat{\sigma}_x(t)\hat{\phi}(\boldsymbol{x},t) \to \frac{1}{2}\left(\hat{\sigma}^+ e^{\mathrm{i}\Omega t} + \hat{\sigma}^- e^{-\mathrm{i}\Omega t}\right)\int \mathrm{d}^3\boldsymbol{x}\, F(\boldsymbol{x})\hat{\phi}(\boldsymbol{x},t)$$
$$- \frac{\mathrm{i}}{(2\pi)^2}\left(\hat{\sigma}^+ e^{\mathrm{i}\Omega t} - \hat{\sigma}^- e^{-\mathrm{i}\Omega t}\right)\int \mathrm{d}^3\boldsymbol{x}\, F(\boldsymbol{x})\int \mathrm{d}^3\boldsymbol{y}\,\frac{\hat{\pi}(\boldsymbol{y},t)}{|\boldsymbol{x}-\boldsymbol{y}|^2}.$$

Consider a cavity $[0,L_1]\times[0,L_2]\times[0,L_3]$

$$\int_0^{L_i} \mathrm{d}^3\boldsymbol{x}\, F(\boldsymbol{x})\hat{\sigma}_x(t)\hat{\phi}(\boldsymbol{x},t) \to \frac{1}{2}\left(\hat{\sigma}^+ e^{\mathrm{i}\Omega t} + \hat{\sigma}^- e^{-\mathrm{i}\Omega t}\right)\int_0^{L_i} \mathrm{d}^3\boldsymbol{x}\, F(\boldsymbol{x})\hat{\phi}(\boldsymbol{x},t)$$
$$+ \frac{4\mathrm{i}}{\pi^3}\left(\hat{\sigma}^+ e^{\mathrm{i}\Omega t} - \hat{\sigma}^- e^{-\mathrm{i}\Omega t}\right)\int_0^{L_i} \mathrm{d}^3\boldsymbol{x}\, F(\boldsymbol{x})\int_0^{L_i} \mathrm{d}^3\boldsymbol{y}\,\hat{\pi}(\boldsymbol{y},t)$$
$$\times \underbrace{\sum_{\boldsymbol{m}=1}^{\infty}\frac{\boldsymbol{\Delta k}}{\omega_{\boldsymbol{m}}}\prod_{i=1}^{3}\sin\left(\frac{\pi m_i x_i}{L_i}\right)\sin\left(\frac{\pi m_i y_i}{L_i}\right)}_{\text{Non-local kernel}}.$$

Cavity $\hat{H}_{\text{int}}$ non-locality



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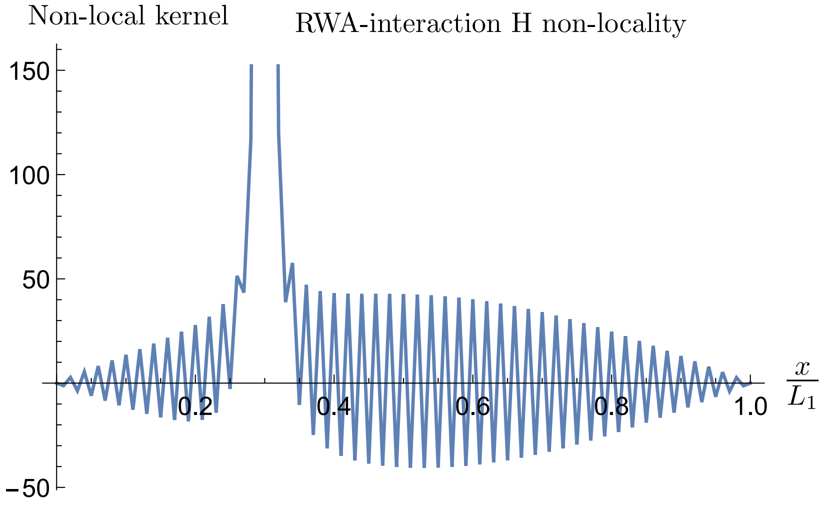


Long-times in cavities

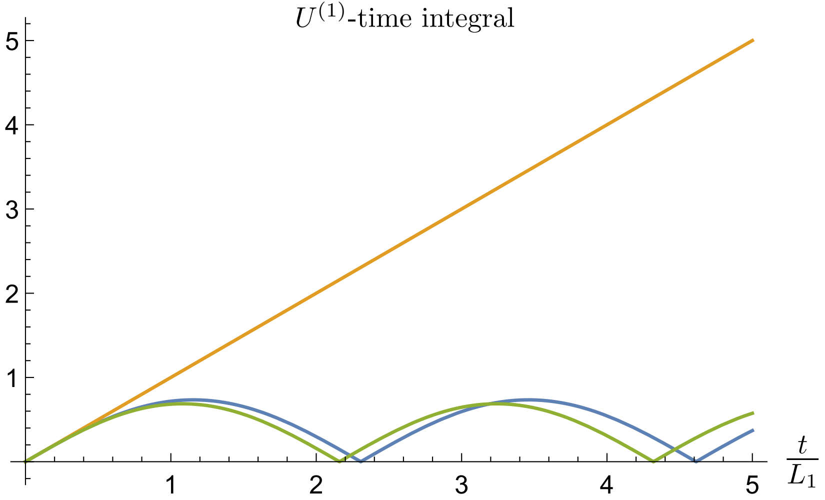
$$M_e^1 = \sum_{m\neq 0} \frac{\boldsymbol{\Delta k}}{\omega} \frac{\mathcal{G}(\boldsymbol{k})}{\mathrm{i}} e^{\mathrm{i}(\omega t - \boldsymbol{k}\cdot\boldsymbol{x})} \int_{-\infty}^{\infty} \mathrm{d}t_1\, \chi(t_1) e^{-\mathrm{i}(\omega-\Omega)t_1},$$

$$M_e^2 = \sum_{m,m'\neq 0} \frac{\boldsymbol{\Delta k}^2}{\omega\omega'} (-1)\mathcal{G}(\boldsymbol{k})\mathcal{G}(\boldsymbol{k}') e^{\mathrm{i}(\omega t - \boldsymbol{k}\cdot\boldsymbol{x}) + \mathrm{i}(\omega' t - \boldsymbol{k}'\cdot\boldsymbol{x})}$$

$$\times \int_{-\infty}^{\infty} \mathrm{d}t_1 \int_{-\infty}^{t_1} \mathrm{d}t_2\, \chi(t_1)\chi(t_2) \left(e^{-\mathrm{i}(\omega+\Omega)t_1} e^{-\mathrm{i}(\omega'-\Omega)t_2} + e^{-\mathrm{i}(\omega'+\Omega)t_1} e^{-\mathrm{i}(\omega-\Omega)t_2} \right),$$

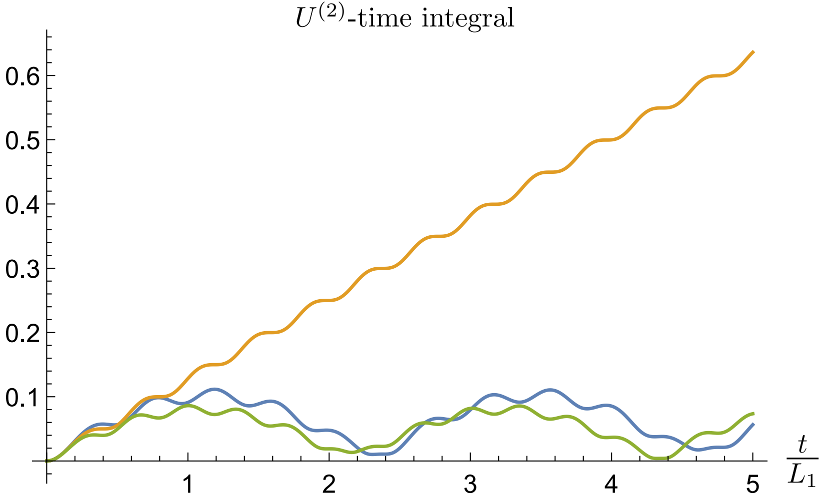
$$\left\langle : \hat{\phi}^2(\boldsymbol{x}, t) : \right\rangle_{\text{Full}} = \frac{\lambda^2}{4(2\pi)^6} \sum_{i\in\{e,g\}} \hat{\Pi}_i \left[2\left| M_i^1 \right|^2 - M_i^2 - \left(M_i^2\right)^* \right]$$

1st order time integral



t^2 contribution to $\left\langle :\hat{\phi}^2(\boldsymbol{x},t): \right\rangle$

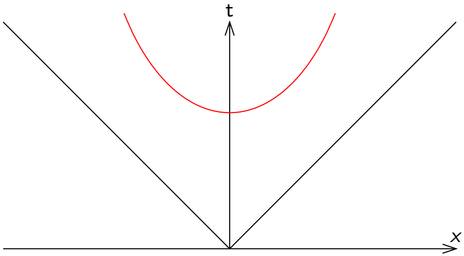
2nd order time integral



t contribution to $\left\langle : \hat{\phi}^2(\boldsymbol{x}, t) : \right\rangle$

Conclusion

- The RWA always gives polynomial causality violations.
- RWA should not be used to model detectors measuring correlations (non-locality/causality violations taint prediction)
- Inside the lightcone, far from light surface and qubit the RWA works best, for field and qubit observables.
- Cavities also suffer polynomial non-localities.



arXiv: 1905.02235