QFT measurements: Localization, causality and correlations

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Summary

1. Challenges of defining fundamental quantum information concepts in relativistic systems: Localization, Causality, Covariance.

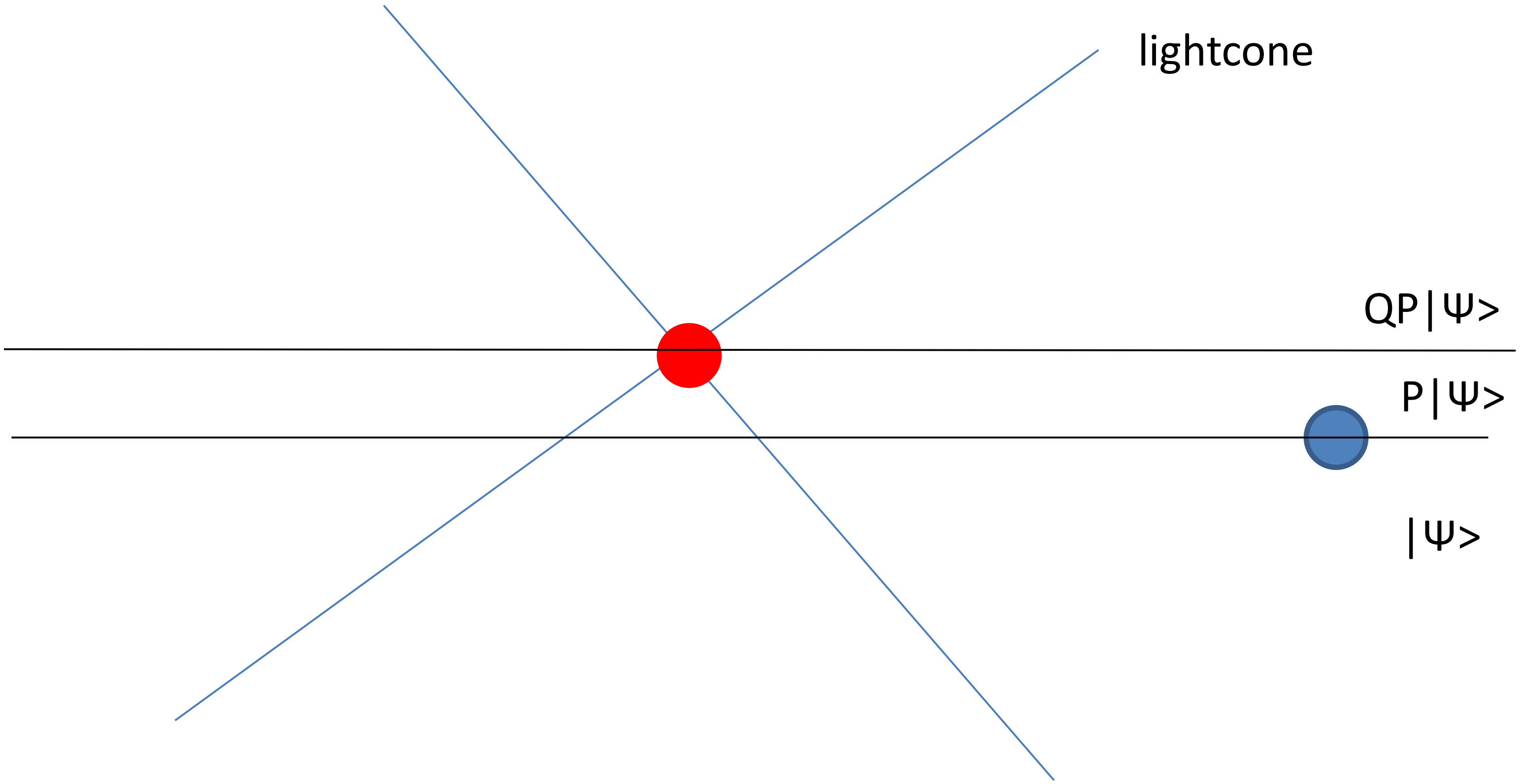
2. Argue that histories-based theories are the most appropriate to define causal and covariant observables.

3. Present a formulation of relativistic quantum measurements based on (i) QFT modeling of system-apparatus interaction, and (ii) the inclusion of temporal observables.

4. Applications to relativistic time of arrival, multi-time correlations, Hawking radiation. Implications for the information `paradox’ in black holes.

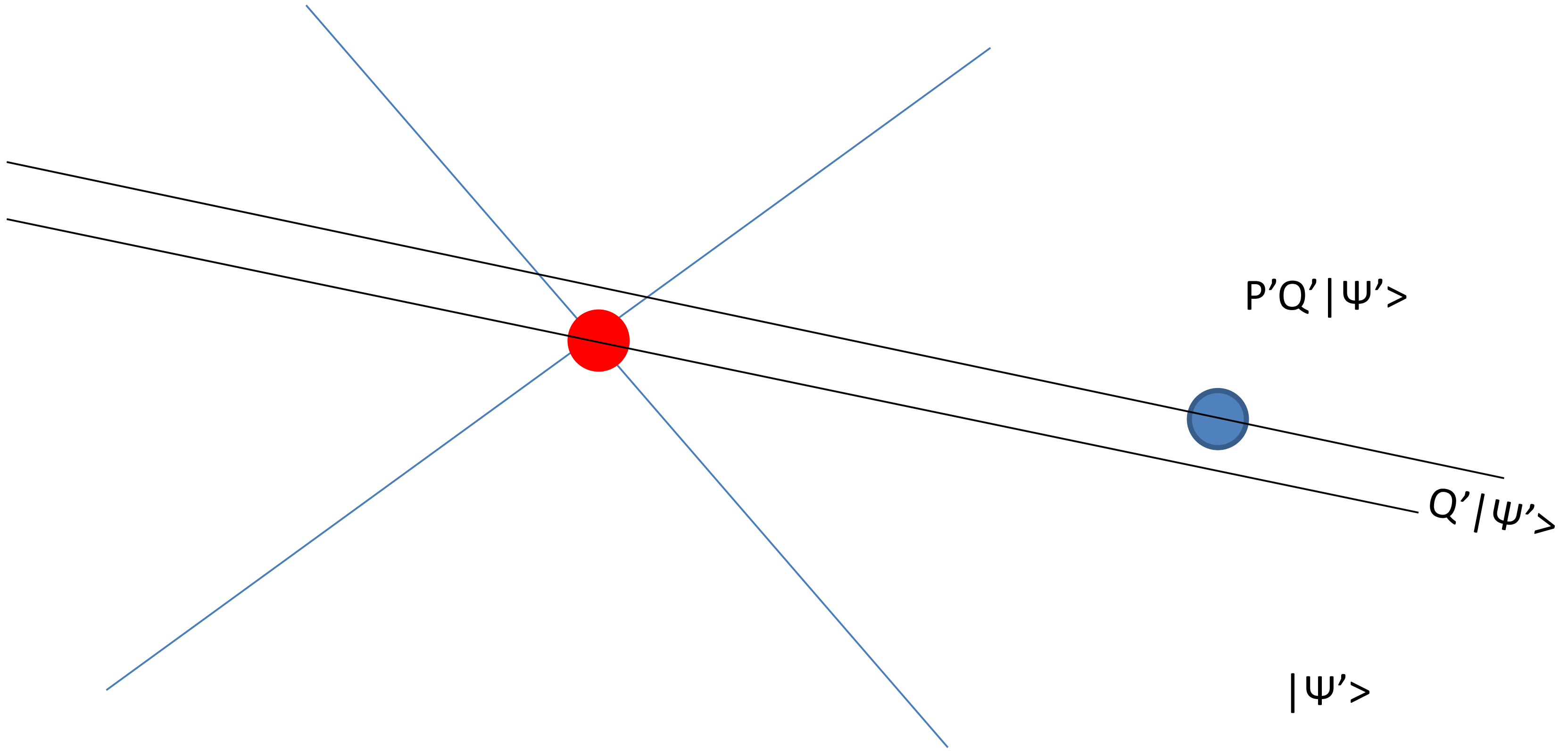
Why quantum measurements are different in relativistic theories.

P and **Q** are spacelike separated measurement events represented by projectors



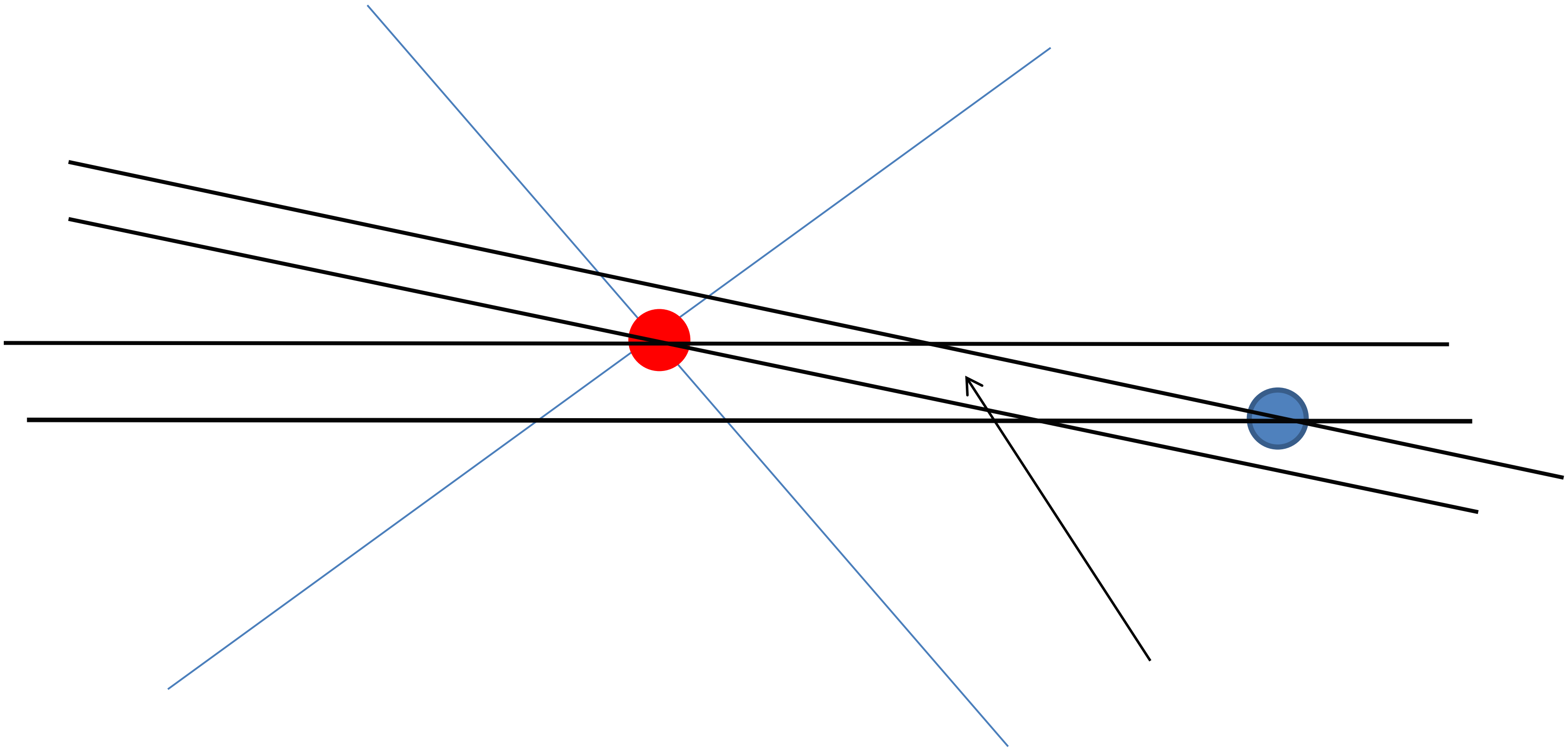
Reference frame 1: **P prior to Q**

P and Q are spacelike separated measurement events



Reference frame 2: **Q’ prior to P’**

P and Q are spacelike separated measurement events



P|Ψ> is not related to Q'|Ψ'> by a Lorentz transformation

Reference frame 2: Q prior to P

The challenge of covariance

- The change in the quantum state after a measurement is genuinely different when recorded from different Lorentz frames.

  I. Bloch, Phys. Rev. 156, 137 (1967).

  Y. Aharonov and D. Z. Albert, Phys. Rev. D4, 359 (1981);Phys. Rev. D 29, 228 (1984).

- **Standard resolution:** The quantum state is not objective w.r.t. different relativistic observers, only probabilities are.

> *"the state vector is only a shorthand expression of that part of our information concerning the past of the system which is relevant for predicting (as far as possible) the future behavior thereof. We also recognize that the laws of quantum mechanics only furnish probability connections between results of subsequent observations carried out on a system."*
>
> E Wigner, Am. J. Phys. 31, 6 (1963).

The challenge of covariance

- How does one define Lorentz-covariant information-theoretic quantities (e.g. entanglement) in set-ups that involve multiple measurements?
- Reduction rule describes a global change of the quantum state at a single moment of time. Find a different rule?

  For example, K. E. Hellwig and K. Kraus, Phys. Rev. D 1, 566 (1970).

- Or work solely at the level of probabilities for all possible sequences of measurements?

  R. M. F. Houtapel, H. van Dam and E. Wigner, Rev. Mod. Phys. 37, 595 (1965).

The challenge of localization

- How can we formalize the notion of particle localization (essential for the interpretation of any particle-physics experiment), in a way that is compatible with causality?

- **Newton-Wigner**. Fix a Lorentzian frame. Define momentum wave functions such that $\int d^3p \, |\psi(p)|^2 = 1$, and position operator as $i\, {}^{\partial}/_{\partial p}$.

  T. D. Newton and E. P. Wigner, Rev. Mod. Phys. 21, 400 (1949).

- NW operator is not part of a spacetime covariant 4-vector.

- NW-localized wave-functions evolve super-luminally.

  B. Rosenstein and M. Usher, Phys. Rev. D 36, 2381 (1987).

Several proposals for other relativistic position operators.

The challenge of localization

- **Malament's theorem:** Sharp localization (i.e., position operators) is not compatible with relativistic causality.

  D. Malament, in "Perspectives on Quantum Reality", ed. R. Clifton (Kluwer, Dordrecht 1996).

- **Hegerfeldt's theorems:** Even unsharp localization leads to superluminal transmission of information.

  G. C. Hegerfeldt, Annalen der Physik 7, 716 (1998).

What is **L**OCC in relativistic systems?

Inadequacy of single-time ideal measurements

- Does it make sense to talk about particle localization at a single moment of time?
- A single-time position measurement refers to alternatives defined over all three-space.
- However, any measurement apparatus **interacts within a finite local region** with the quantum field. How can it be compatible with alternatives defined over a Cauchy surface?
- General arguments that ideal (i.e., projective) measurements in QFT lead to contradictions with causality.

R. Sorkin, in Directions in General Relativity, edited by L. Hu and T. A. Jacobson (Cambridge University Press, 1993).

In absence of projective measurements, how do we define **relativistic qubits**?

Inadequacy of single-time measurements

- Of course we can always express measurements in terms of **POVMs** (Positive-Operator-Valued Measures).

- **POVM:** Assigns an alternative q in some set Γ to a positive operator Π(q) on the system's Hilbert space.

Histories-based theories

- Fundamental object is not the single-time state or the single-time property, but the history.
- The simplest example of a **history** is a sequence of properties (measurements) of a physical system at different moments of time.
- Prime example: **Decoherent histories** framework.

R. B. Griffiths, Consistent Quantum Theory (Cambridge University Press, 2003).

R. Omnés, The Interpretation of Quantum Mechanics, (Princeton University Press, 1994); Understanding Quantum Mechanics (Princeton University Press, 1999).

M. Gell-Mann and J. B. Hartle, in ‘Complexity, Entropy, and the Physics of Information’, ed. By W. Zurek, (Addison Wesley, Reading 1990); Phys. Rev. D47, 3345 (1993).

Decoherent histories

A discrete-time history α will then correspond to a string $\hat{P}_{t_1}, \hat{P}_{t_2}, \ldots \hat{P}_{t_n}$ of projectors, each labelled by an instant of time. From them, one can construct the class operator

$$\hat{C}_\alpha = \hat{U}^\dagger(t_1)\hat{P}_{t_1}\hat{U}(t_1)\ldots\hat{U}^\dagger(t_n)\hat{P}_{t_n}\hat{U}(t_n) \tag{4. 1}$$

where $\hat{U}(s) = e^{-i\hat{H}s}$ is the time-evolution operator. The probability for the realisation of this history is

$$p(\alpha) = Tr\left(\hat{C}_\alpha^\dagger \hat{\rho}_0 \hat{C}_\alpha\right), \tag{4. 2}$$

where $\hat{\rho}_0$ is the density matrix describing the system at time $t = 0$.

Key point: Set of histories equipped with a logical structure. We can define

$$\alpha \vee \beta \ (OR), \qquad \alpha \wedge \beta \ (AND), \qquad -\alpha \ (NOT)$$

The class operators (amplitudes) are additive:

$$C_{\alpha \vee \beta} = C_\alpha + C_\beta \text{ for } \alpha \wedge \beta = \emptyset$$

Decoherent histories

However, Kolmogorov additivity condition is not satisfied

$$p(\alpha \vee \beta) \neq p(\alpha) + p(\beta).$$

Introduce **decoherence functional** $d(\alpha, \beta) = \mathrm{Tr}(\hat{C}_\alpha \hat{\rho}_0 \hat{C}_\beta)$.

Probabilities for histories are well defined if a **decoherence condition** is satisfied. An exhaustive and exclusive set of histories is decoherent if

$$d(\alpha, \beta) = 0, \qquad \text{for } a \neq \beta$$

This condition is common for **highly coarse-grained histories**, that describe pointer variables of measurement apparatuses.

The pointer variables then behave classically. Appropriate for measurement theory of **history observables**.

Other versions of quantum histories

- **Quantum measure theory**: emphasis on the representation of histories via path integrals rather than Hilbert space variables. No need for single-time quantum states.

  R. D. Sorkin, Mod. Phys. Lett. A9, 33, 3119 (1994); J. Phys. A: Math. Gen. 40 (2007).

- **Correlation Histories**: coarse-grained histories expressed in terms of QFT correlation functions, appropriate for the emergence of thermodynamic behavior.

  E. Calzetta and B. L. Hu, in Directions in General Relativity, vol II: Brill Festschrift, eds B.L Hu and T. A. Jacobson (Cambridge University Press, 1994) [gr-qc/9302013]; in Heat Kernel Techniques and Quantum Gravity, ed. S. A. Fulling (Texas AM Press, 1995).

Other versions of quantum histories

- **History Projection Operator Theory:** mathematically rigorous, allows for the description of continuous-time and spacetime- extended histories, rich spacetime symmetries).

  C.J. Isham, J. Math. Phys. 35, 2157 (1994); C.J. Isham and N. Linden, J. Math. Phys. 35, 5452 (1994); J. Math. Phys. 36, 5408 (1995).

  N. Savvidou, J. Math. Phys. 40, 5657 (1999); 43, 3053 (2002).

- **Quantum Temporal Probabilities:** histories-based measurement theory, an <u>algorithmic</u> procedure for defining probabilities of <u>temporal</u> variables.

  C. A. and N. Savvidou, J. Math. Phys. 47, 122106 (2006); Phys. Rev. A86, 012111 (2012); Phys. Rev. A95, 032105 (2017); J. Math. Phys. 60, 0323301(2019).

- Originates from a proposal of N. Savvidou about a resolution of the problem of time in quantum gravity. Review in ”Approaches to Quantum Gravity”, ed. D. Oriti (Cambridge University Press, 2009)

Time in quantum theory

- Asymmetry between space and time in quantum theory.
- **Elementary QM**: $|\psi(x,t)|^2$ is a probability density **w.r.t.** position x, **at** time t.
- x is a random variable, t is an external parameter. We normalize by integrating over x, not over t.
- Time t is a **parameter** of Schrodinger's equation, not an observable.
- **Pauli's theorem:** There is no time operator T compatible with unitary evolution for Hamiltonians bounded from below.

  W. Pauli, The Principles of Quantum Mechanics, in Encyclopedia of Physics, ed. S. Flugge, Vol. 5/1 (Springer, Berlin, 1958)

The problem of time in quantum theory

No unique prescription for defining quantum probabilities with respect to time.

Time of arrival problem

Given an initial wave function $|\psi_0\rangle$ for a particle, centered around x = 0 and with positive mean momentum, find the probability **P(t)δt** that the particle is detected at distance **x = L** at some moment between t and t+δt.

There is no canonical answer even to such an elementary question.

Tunneling time problem

How long does it take for a particle to tunnel through a barrier?

Several proposals exist---starting from 1930.

Some tests are now possible in atto-second laser dynamics.
Landsmann + Keller, Phys. Rep. 547, 1(2015).

Relativistic Quantum measurement models

- Local, causal, unitary and Poincare invariant interactions **can only be expressed in terms of quantum fields**.

Hence,

- **Causality** in signal propagation requires QFT interactions between system and apparatus.

Prototype QFT measurement theory

Glauber's photo-detection theory. Unnormalized probabilities for photodetection by dipole detectors at specific spacetime points.

R. J. Glauber, Phys. Rev. 130, 2529 (1963);131, 2766 (1963).

$$w_1(\boldsymbol{r},t) = \mathrm{Tr}[E^{(+)}(\boldsymbol{r},t)\rho_0 \mathrm{E}^{(-)}(\mathbf{r},\mathrm{t})]$$ (single detector)

$$w_2(\boldsymbol{r}_1,t_1;\boldsymbol{r}_2,t_2) = \mathrm{Tr}[E^{(+)}(\boldsymbol{r}_2,t_2)\, E^{(+)}(\boldsymbol{r}_1,t_1)\rho_0 \mathrm{E}^{(-)}(\boldsymbol{r}_1,t_1)\mathrm{E}^{(-)}(\boldsymbol{r}_2,t_2)]$$

(two detectors)

N detectors = 2N QFT correlation function

However, splitting a field into positive and negative frequencies is not a local operation. Problematic if retarded propagation effects are important.

General field-apparatus coupling

QFT Hilbert space ← $$H = F \otimes K$$ → Apparatus Hilbert space

We assume a factorized initial condition, standard in quantum measurement theory.

Fundamentally problematic in QFT due to Haag's theorem, but suffices to lowest order in detector-field interaction.

$$H_I = \int d^4X \; O_a(X) \otimes J^a(X)$$

C. A. and N. Savvidou, Phys. Rev. A86, 012111 (2012).

O_a (X) is a local composite operator for the quantum field

J_a(X) is a current operator for the apparatus

J_a(X) has support on the apparatus's world-tube

Derive QTP probability formula

POVM: $$P(X) = \int d^4Y d^4Y' R_X^{ab}(Y,Y') G_{ab}(Y,Y'),$$

Kernel R depends on spacetime sampling (i.e., coarse-graining).

in terms of the correlation function

$$G_{ab}(Y,Y') = Tr\left[\hat{O}_a(Y)\hat{\rho}_0\hat{O}_b^\dagger(Y')\right].$$

Probability of detection at spacetime point X is a <u>linear functional of a two-point function</u> for the composite operator O.

By deconvolution, obtain sampling-independent POVM

$$P(X) = \int d^4Y\, S^{ab}(Y)\, G_{ab}(X + \tfrac{1}{2}Y, X - \tfrac{1}{2}Y)$$

$$S^{ab}(Y) \coloneqq \langle\Omega|J^a(0)e^{-iP\cdot Y}J^b(0)|\Omega\rangle$$

|Ω> initial state of ***inertial*** detector

C. A. and N. Savvidou, J. Math. Phys. 60, 0323301 (2019).

P^μ generators of spacetime translations in detector's Hilbert space.

Elementary derivation via Unruh-Dewitt detectors

- Shrink the support of J(X) to a single spacetime path X(τ).
- Introduce a switching function $g_\sigma(\tau)$ of finite width in the interaction. (Simulates temporal sampling without histories formalism)

$$\hat{V}(s) = \lambda \int d\tau g_\sigma(s-\tau) \int d^4X \hat{O}(X) \delta^4(X - X(\tau)) \hat{m}(\tau) \qquad g_\sigma(s) = e^{-\frac{s^2}{2\sigma^2}},$$

Calculate transition probability for the detector to lowest order in perturbation theory

The total probability of detection $P(\tau)$ is obtained after a summation over all detector energies, $P(\tau) = \sum_E P(E,\tau)$. We obtain after deconvolution

$$P(\tau) = \int ds g_\sigma(\frac{s}{2}) \alpha(s) G^{(2)}[X(\tau - \frac{s}{2}), X(\tau + \frac{s}{2})], \qquad (8)$$

where $\alpha(s) = \sum \alpha_E e^{-iEs} = \lambda^2 \langle 0 | \hat{m} e^{-i\hat{h}s} \hat{m} | 0 \rangle$ is proportional to a correlation function of the detector.

Divide by the duration $T = \sqrt{2\pi}\sigma$ of the switching to obtain a probability density w.r.t. time.

Clarification of the term "detector"

In quantum optics, the dipole coupling $-\boldsymbol{d} \cdot \boldsymbol{E}(\mathrm{X})$ applies to either

- **Atoms:** → theory of atom-field interactions. Consistent dynamics of atoms at all times requires the derivation of a master equation.
  We describe **atomic processes or features**: absorption, spontaneous emission, entanglement dynamics, asymptotic states, thermalization and so on.

or

- **Macroscopic (mesoscopic) dipoles:** → Glauber's photodetection theory. Detection probabilities are determined by leading-order terms in perturbation theory + (Rotating Wave Approximation).
  Incorporates logical irreversibility of the measurement process.
  We describe properties **of the EM field**: photon-number probabilities, field coherences, coincidences and so on.

Clarification of the term "detector"

In a general QFT, the UdW coupling $m \otimes \; O(X)$ applies to either

- **Microscopic systems:** Consistent dynamics at all times requires the derivation of a master equation (open quantum systems). Perturbative calculation valid at early times, or if switching interaction can be justified.

  Focus **on properties of the "detectors"**: asymptotic states, thermalization, fluctuation-dissipation relation, entanglement creation, harvesting, backreaction and so on.

  See review: B. L. Hu, S-Y Lin, J. Louko, Class. Quantum Grav. 29, 224005 (2012).

or

- **Macroscopic apparatuses:** Detection probabilities are determined by leading order terms in perturbation theory.

  Incorporates logical irreversibility of the measurement process.

  We construct POVMs for the **field degrees of freedom** as captured by the pointer variables of the apparatus.

Clarification of the term "detector"

- Landau and Peierls gave the first discussion of quantum field measurability. They assumed a microscopic "detector" (ion or electron).
  L. Landau and R. Peierls, Zeit. Phys. 69, 56 (1931).

- Bohr and Rosenfeld corrected this assumption. Showed that a test body that measures the quantum EM field accurately must have charge Q >>e, and that it must be much larger than the atomic scale.
  N. Bohr and L. Rosenfeld, Mat.-fys. Medd. Dan. Vid. Selsk.12, (1933).

- *"…in measurements of field quantities one must be able to adjust the charge of the* ***test bodies*** *to an extent which conflicts with [the presupposition that the radiation reaction is small compared to the ponderomotive forces exerted on the particles] if one considers these bodies as point charges…. these difficulties disappear if one uses test bodies whose linear extensions are chosen sufficiently large compared to atomic dimensions, so that their charge density can be considered approximately constant over the whole body."*

Rigorous proof that detectors (as measuring apparatuses) must be `**coarse-grained**'
H. Araki and M. Yanase, Phys. Rev., 120, 666 (1961).

Scalar field couplings

For a QFT with a single scalar field, possible couplings are

(i) $\hat{O}(X) = \hat{\phi}(X)$. This coupling characterizes Unruh-DeWitt detectors [39]. It is the scalar-field analogue of the EM dipole coupling. It describes particle detection by absorption.

(ii) $\hat{O}(X) = \hat{\phi}^{(+)}(X)$. The detector couples to the positive frequency part $\hat{\phi}^{(+)}(X)$ of the quantum field. This is an analogue of the Glauber EM field-detector coupling in quantum optics [40]. A drawback of this coupling is that the splitting of the field into positive and negative frequency parts is a non-local operation; hence, it could lead to causality violation in set-ups that involve multiple measurements.

(iii) $\hat{O}(X) =: \hat{\phi}^2(X) :$. This coupling describes particle detection by scattering through a scalar interaction. The double dots denote normal ordering.

(iv) $\hat{O}_\mu(X) =: \hat{\phi}(X)\partial_\mu\hat{\phi}(X) :$. This coupling describes particle detection by scattering through a vector interaction.

Time of a arrival for a scalar particle

$$P(X) = \int d^4Y\, S(Y)\, G(X + \tfrac{1}{2}Y, X - \tfrac{1}{2}Y)$$

G is correlation function for a single particle state.

For a **free field** and any **scalar** composite operator, we derive
the **single-particle probability distribution** for the time of arrival.

$$P_0(t,x) = \int \frac{dp\,dp'}{2\pi} \tilde{\rho}(p,p') \sqrt{v_p v_{p'}} L(p,p') e^{i(p-p')x - i(\epsilon_p - \epsilon_{p'})t},$$

POVM Normalization

$$\int_{-\infty}^{\infty} dt\, P_0(t,x) = 1,$$

Localisation operator

The localization operator L(p, p') depends on the physics of the apparatus.

$$\langle p|\hat{L}|p'\rangle := \frac{\tilde{S}\left(\frac{p+p'}{2}, \frac{\epsilon_p+\epsilon_{p'}}{2}\right)}{\sqrt{\tilde{S}(p,\epsilon_p)\tilde{S}(p',\epsilon_{p'})}}.$$

L describes the irreducible spread of any record due to the physics of the apparatus.

Maximal localization: L(p, p') = 1.
Obtain Leon's POVM for relativistic particles.
J. León, J. Phys A: Math. Gen. 30, 4791 (1997).
Reduces to Kijowski's POVM in non-relativistic limit:
J. Kijowski, Rep. Math. Phys. 6, 361 (1974).

$$S(t,x) = \frac{1}{(1+\frac{it}{\tau})(1-\frac{ix}{\delta})},$$

Probability distributions for different L are distinguished only in the **highly quantum regime** λ~D,
λ: de Broglie wavelength of initial state
D: source-detector separation

Covariance

The probability densities above do not transform as spacetime scalars.
There are two reasons:

1. The probabilities depend on the initial state of the apparatus, which, in general, is not Poincare invariant. It is natural to assume **translational invariance**, but **not Lorentz invariance**. Only a QFT vacuum is Poincare invariant.

$$P(X)= \int d^4Y \, S(Y) \, G(X + \frac{1}{2}Y, X - \frac{1}{2}Y)$$

2. We normalize the POVM over the sub-ensemble of particles recorded by a detector at point x. This normalization breaks Lorentz symmetry. (In QFT, there is always non-zero **no-detection probability**).

$$\int_{-\infty}^{\infty} dt P_0(t, x) = 1,$$

Time-energy uncertainty relation

Evaluate the variance of the time of arrival

$$(\Delta t)^2 \quad = \quad (\Delta T_c)^2 + \frac{m^4}{4}\langle\frac{1}{\epsilon_p^2 p^4}\rangle + \langle\frac{\sigma^2(p)}{v_p^2}\rangle,$$

σ(p) is the **irreducible detection spread** contained in the localization operator

$$\hat{T}_c = \frac{1}{2}\left[(x - \hat{x})\hat{v}^{-1} + \hat{v}^{-1}(x - \hat{x})\right].$$

Like its non-relativistic counterpart, the operator $\hat{T}_c$ is symmetric but not self-adjoint [48]. However, its domain $D_{\hat{T}_c}$ includes all vectors with *strictly positive* momentum content, i.e., functions $\psi(p)$ with positive momentum that vanish faster than any power of p as $p \to 0$.

Time-energy uncertainty relation

For all states in the domain of $\widehat{T}_c$, $\Delta \mathrm{T}_c \Delta \mathrm{H} \geq \frac{1}{2}$. Since $\sigma^2(p) \geq 0$, we obtain

$$(\Delta t)^2 \geq \frac{1}{4(\Delta \mathrm{H})^2} + \left\langle \frac{m^4}{4(m^2 + p^2)p^4} \right\rangle$$

This uncertainty relation holds for all states with positive momentum and for all detectors.

Δt is the mean deviation of the time-of-arrival, i.e., a statistical quantity.
It is NOT the speed of quantum evolution, as in Mandelstam-Tamm inequality.

Additional terms for particles with spin: work in progress.

Time-energy uncertainty relation

Sharp value on r.h.s. is larger, possibly ½ .

Non relativistic limit: $\langle E_p \rangle \Delta t > \frac{1}{4}$,

$$E_p = \frac{p^2}{2m}$$

Analogous expressions:

L. Landau and R. Peierls, Zeit. Phys. 69, 56 (1931).

A. D Baute, R. Sala Mayato, J. P. Palao, J. G. Muga, and I. L. Egusquiza, Phys. Rev. A61, 022118 (2000).

N. Margolus and L. B. Levitin, Physica D120, 188 (1998).

Ultra-relativistic limit: $\langle \hat{H} \rangle^3 \Delta t > \frac{m^2}{2}$.

Time-energy uncertainty relation

The additional term crucial for states with low momentum or with $\Delta\mathrm{H} \to \infty$.

The Levy distribution has $\Delta\mathrm{H} = \infty$.

$$P(E) = \sqrt{\frac{c_E}{2\pi}} \frac{e^{-\frac{c_E}{2(E-m)}}}{(E-m)^{3/2}},$$

$$\Delta t \geq \begin{cases} \frac{\sqrt{3}}{2c_E}, & c_E \ll m \\ 51.0\,\frac{m^2}{{c_E}^3}, & c_E \gg m \end{cases}$$

Proposal

Define **particle localization** in terms of temporally extended observables, like the time of arrival.

R. Werner, J. Math. Phys. 27, 793 (1986).

For example, use the r.h.s. of the uncertainty relation as a localization measure.

$$(\Delta t)^2 \geq \frac{1}{4(\Delta \mathrm{H})^2} + \left\langle \frac{m^4}{4(m^2+p^2)p^4} \right\rangle$$

Benefits:

i. Operational definition, in terms of observable quantities.

ii. Get around the restrictions of Malament's and Hegerfeldt's theorems.

iii. Definition involves the 2-pt functions of QFT. Expecting to have no problems with causality. A proof must still be provided.

iv. In principle, applies to any QFT (including interacting ones) as probability depends only on the 2-pt functions.

Two-point correlations (coincidences)

Recall Glauber's formula for intensity correlations.

$$w_2(\boldsymbol{r}_1, t_1; \boldsymbol{r}_2, t_2) = \mathrm{Tr}[E^{(+)}(\boldsymbol{r}_2, t_2)\, E^{(+)}(\boldsymbol{r}_1, t_1)\rho_0 \mathrm{E}^{(-)}(\boldsymbol{r}_1, t_1)\mathrm{E}^{(-)}(\boldsymbol{r}_2, t_2)]]$$

Used in order to determine second-order coherence:
Hanbury-Brown-Twiss effect, photon anti-bunching.

QTP generalization for a general QFT (after deconvolution)

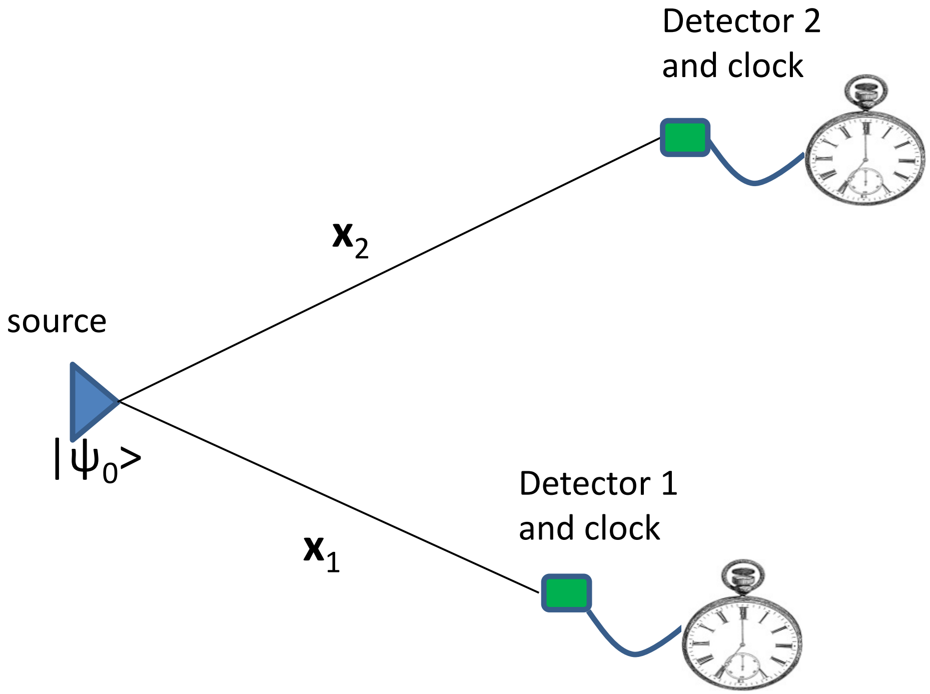
$$P(X_1, X_2) = \int d^4Y_1 \; d^4Y_2 S(Y_1)S(Y_2)G^{(4)}\left(X_1 + \frac{1}{2}Y_1, X_2 + \frac{1}{2}Y_2, X_1 - \frac{1}{2}Y_1, X_2 - \frac{1}{2}Y_2\right)$$

$$G^{(4)}(X, Y, X', Y') = \langle\Psi|\bar{\mathbf{T}}[\mathrm{O(X)O(Y)}]\mathbf{T}[\mathrm{O(X')O(Y')}]|\Psi\rangle$$

Mixed time-ordered and anti-time-ordered correlation functions; **in-in QFT formalisms**.

Phys. Rev. A86, 012111 (2012)

Two-time correlation



i-th emitted pair of particles, recorded at times s_{1i} and s_{2i} respectively.
Assume sampling functions $\chi_t(s)$

$$C(t_1, t_2) = \frac{1}{N}\sum_i \chi_{t_1}(s_{1i})\chi_{t_2}(s_{2i}) - \frac{1}{N^2}\sum_i \chi_{t_1}(s_{1i})\sum_j \chi_{t_2}(s_{2i})$$

Probability density for a double detection event

For a bipartite system

$$P_{id}^{(2)}(L_1, t_1; L_2, t_2) = Tr\left[\hat{\rho}_0^{(2)} \hat{\Pi}_{L_1}(t_1) \otimes \hat{\Pi}_{L_2}(t_2)\right],$$

$$\langle p | \widehat{\Pi}_L(t) | p' \rangle = \mathrm{L}(\mathrm{p}, \mathrm{p}') \sqrt{v_p v_{p'}} e^{i(p-p')L - i\left(E_p - E_{p'}\right)t}$$

Define **coherence function** $$C^{(2)}(L_1, t_1; L_2, t_2) = \frac{P^{(2)}(L_2, t_2; L_1, t_1)}{P^{(1)}(L_1, t_1) P^{(1)}(L_2, t_2)},$$

Coincidence function: $c^{(2)}(L, t) := C^{(2)}(L, t; L, t)$

> 1, enhanced simultaneous detection

< 1, suppressed simultaneous detection

Entanglement

- Time-of-arrival correlations define **entanglement witnesses.**

- Factorized states satisfy $c^{(2)}(L, t) \geq 1$, and

$$C^{(2)}(L_1, t_1; L_2, t_2) \leq \sqrt{c^{(2)}(L_1, t_1) c^{(2)}(L_2, t_2)}.$$

Same inequalities are satisfied by
(i) classical (stochastic) theories and
(ii) quantum theories that attempt to describe time of arrival in terms of probability currents (rather than POVMs).

To consider:
Can we define temporal quantum resources?

Examples

Consider states $$|\Psi_0\rangle = \frac{1}{\sqrt{2}}\left(|\psi_1\rangle \otimes |\psi_2\rangle \pm |\psi_2\rangle \otimes |\psi_1\rangle\right),$$

For fermionic states (-), $c^{(2)}(L, t)$ vanishes identically.

For bosonic states (+), again strong differences between quantum and classical theories

Example

Consider states $$|\Psi_0\rangle = \frac{1}{\sqrt{2}}\left(|\psi_1\rangle \otimes |\psi_2\rangle \pm |\psi_2\rangle \otimes |\psi_1\rangle\right),$$

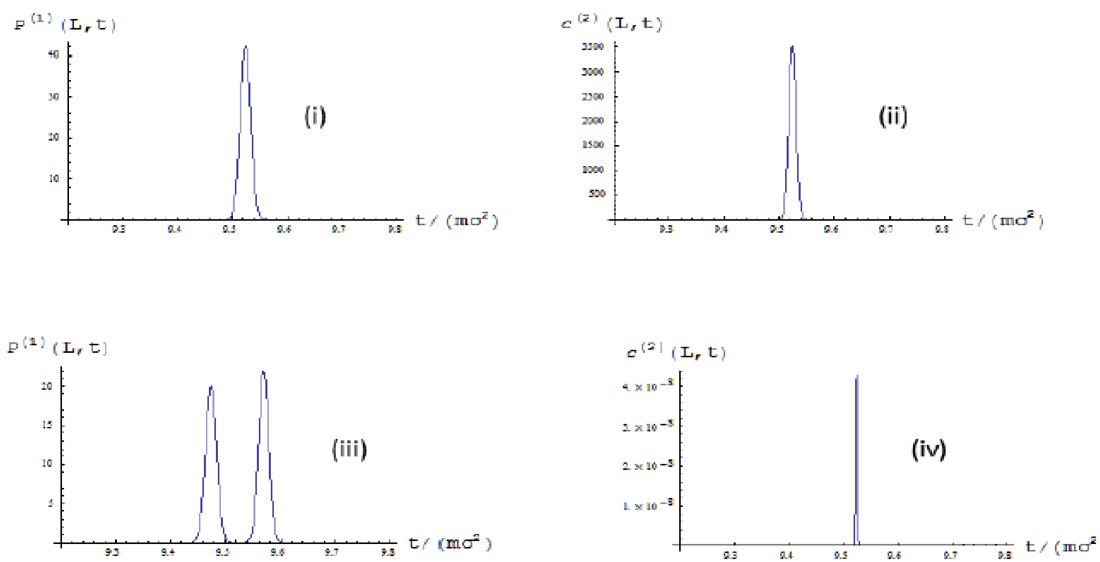
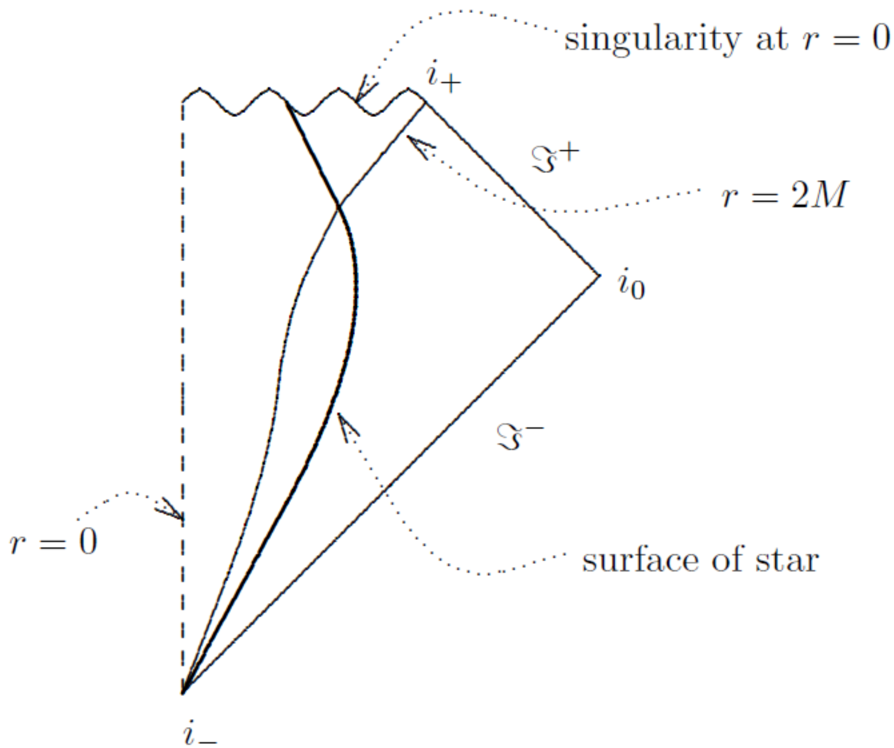


Figure 1: Single-time probability densities and coincidence functions for an initial state of the type (72), where $\psi_i(x) = \phi(x - x_i)e^{ip_i x}$, for some constants $x_i, p_i, i = 1, 2$. We choose for $\phi(x)$ a Gaussian $\phi(x) = (2\pi\sigma_x^2)^{-1/4}exp[-x^2/(4\sigma_X^2)]$. For $|x_2 - x_1| >> \sigma_X$, ψ_1 and ψ_2 are orthogonal. The mean time of arrival for each wave-packet is $\bar{t}_i = m(L - x_i)/p_i$. We have chosen $L/\sigma_X = 1000, p_1\sigma_X = 100, p_2\sigma_X = 110$. In plots (i) and (ii), $\bar{t}_1 = \bar{t}_2$. The superposition cannot be identified at the level of the probability density $P_{id}^{(1)}(L,t)$ of Plot (i). Plot (ii) describes the coincidence function $c^{(2)}(L,t)$ as a function of $t/(m\sigma_X^2)$ for bosons. Bunching or anti-bunching behavior is time-dependent. Plots (iii) and (iv) are the same as (i) and (ii) only with $\bar{t}_1 = 0.99\bar{t}_2$. There are two distinguishable peaks in the probability density, and the peak in $c^{(2)}(L,t)$ is much lower and narrower.

Unequal-time correlations in Hawking radiation

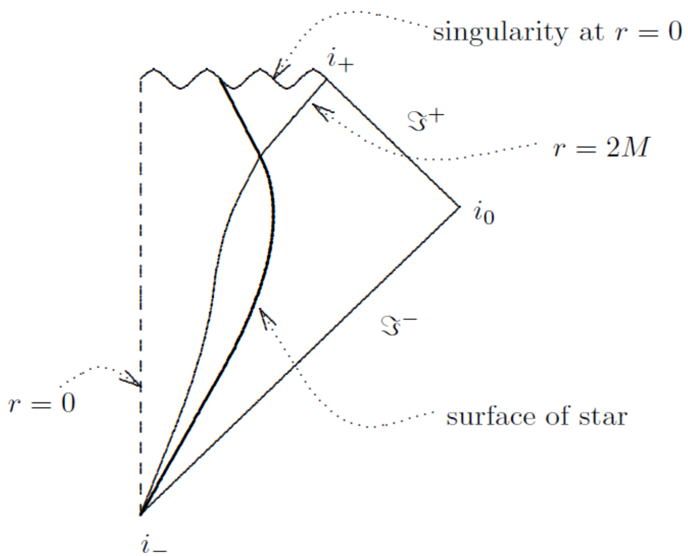


Single time response of detectors on black hole spacetime:

W. G. Unruh, Phys. Rev. D14, 870 (1976)
P. Candelas, Phys. Rev. D21, 2185 (1981)

What about multi-time correlations?

Hawking-Wald theorem



Any field measurement at I^+ is equivalent to a measurement of a field at a Gibbsian state with temperature T_H.

Field Hilbert space splits as $H = H_{I^+} \otimes H_{H^+}$

<u>Reduced density matrix at H_{I^+}</u> is Gibbsian.

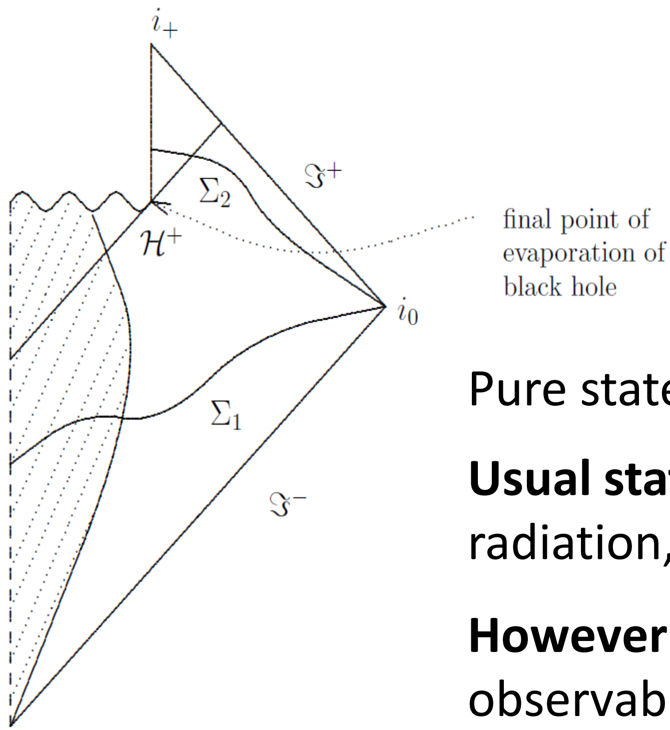
R.M. Wald, Comm. Math. Phys. 45, 9 (1975)
S. W. Hawking, Comm. Math. Phys. 43, 19 (1975).

$$\hat{\rho}_1 = \frac{1}{Z} \sum_{\{n_i\}} e^{-\sum_i n_i \omega_i / T_H} |\{n_i\}\rangle\langle\{n_i\}|$$

No information about the system prior to the collapse reaches I^+.

Information paradox(?)

Suppose BH evaporates



Part of the information available at Σ_1 does not arrive at Σ_2.

Non-unitary evolution?

Or the notion of single-time quantum state, defined at a Cauchy surface, fails in quantum gravity.

Pure state at I^- but mixed state at I^+.

Usual statement: Information cannot be hidden in radiation, by the HW theorem.

However: the HW theorem refers only to single-time observables.
Does the reduced density matrix at I^+ contain all available information outside the horizon?

Information paradox(?)

The reduced density matrix of an open system at a moment of time contains all information about the system **IF AND ONLY IF** the reduced dynamics is Markovian.

For non-Markovian dynamics, **the reduced density matrix misrepresents the probabilities for history observables**, i.e., observables defined at more than one moment of time.
H. P. Paz and W. H. Zurek, Phys.Rev.D48, 2728 (1993)

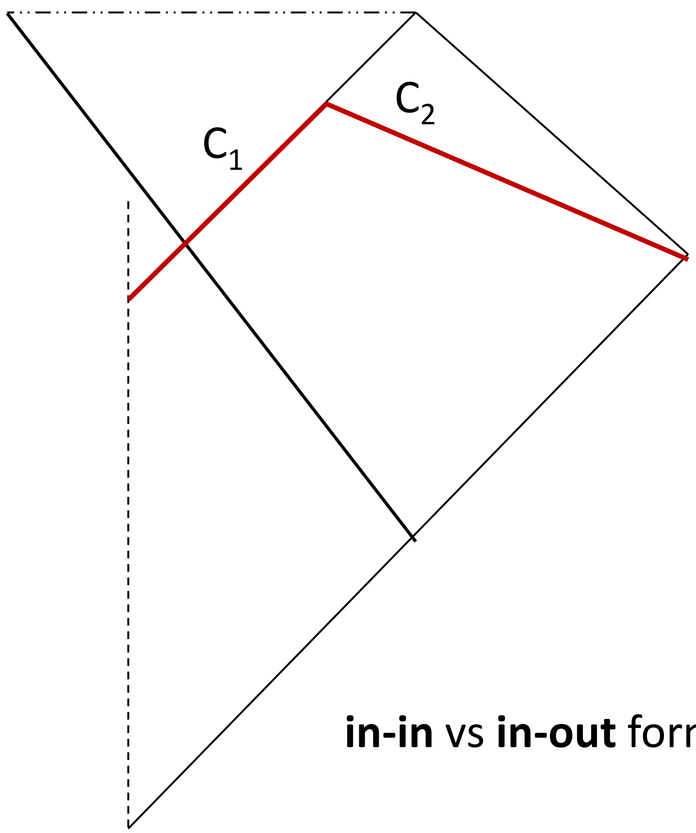
Part of the definition of non-Markovianity

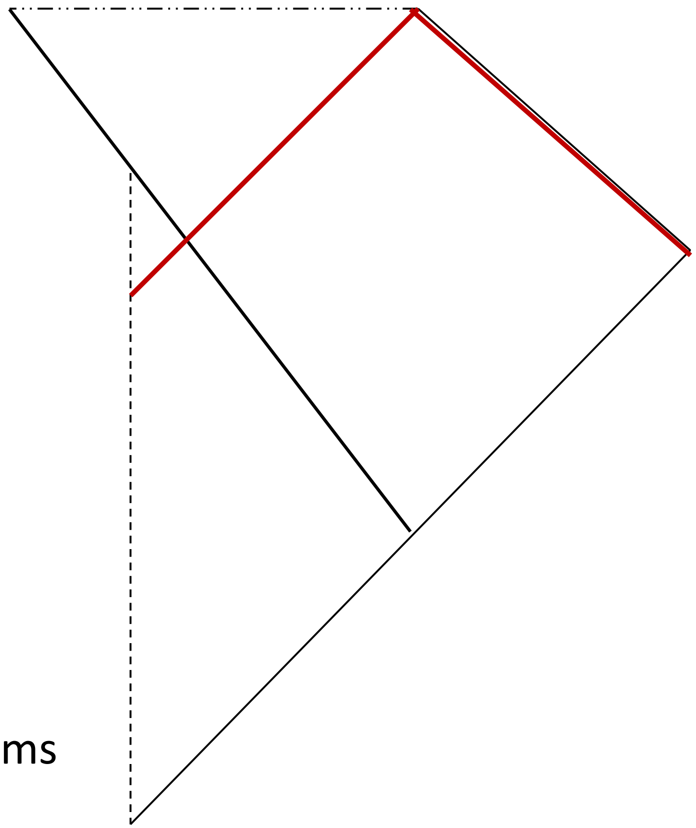
F. A. Pollock, C. Rodríguez-Rosario, T. Frauenheim, M. Paternostro, K. Modi, Phys. Rev. A 97, 012127 (2018)

Cauchy surfaces

After end of the collapse

Asymptotic: used in H-W theorem



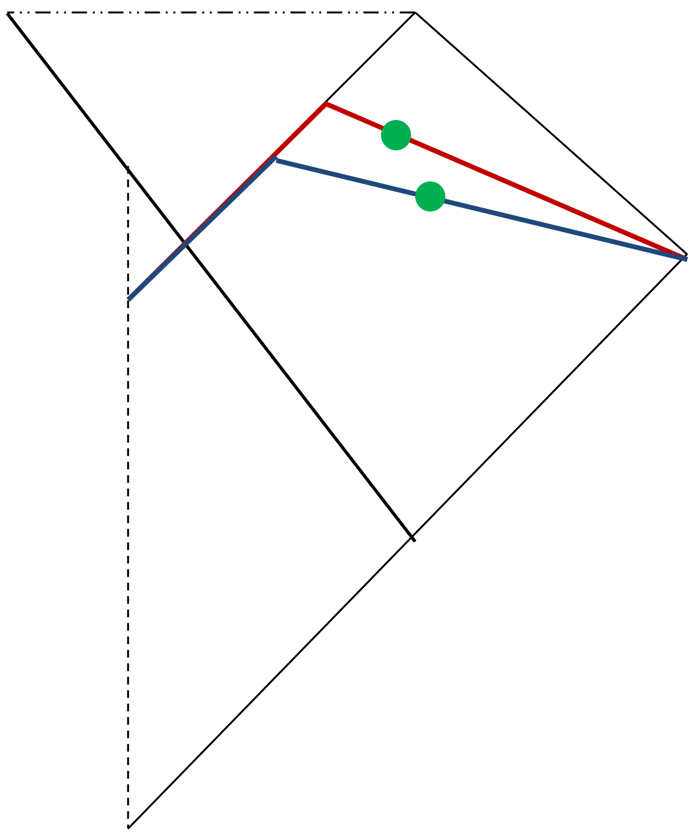


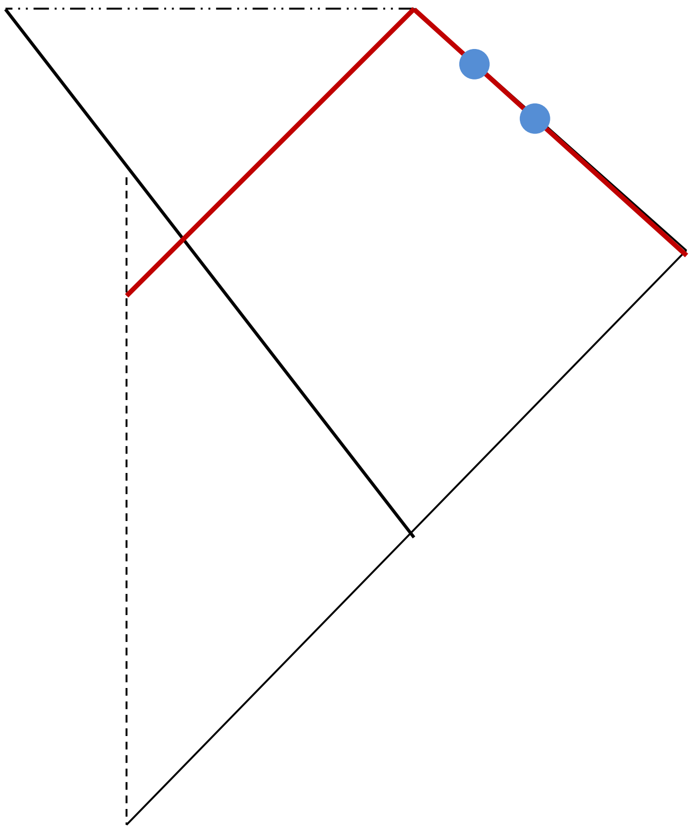
in-in vs **in-out** formalisms

If a Cauchy surface Σ splits as $\Sigma = C_1 \cup C_2$, the Fock space splits as $H(C_1) \otimes H(C_2)$.,
where $H(C_1)$ is constructed from field modes that vanish on C_2.

Cauchy surfaces

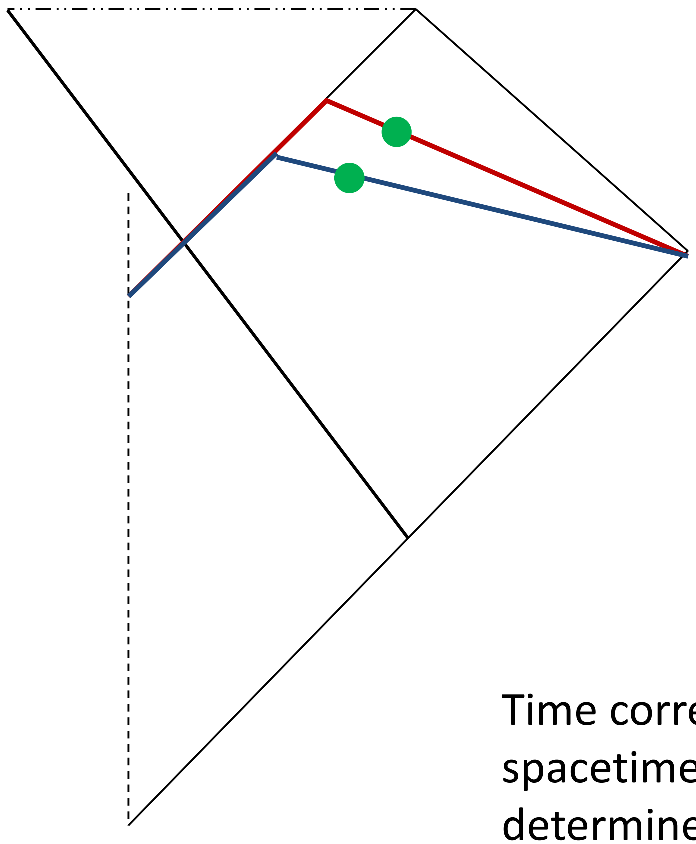
A correlation measurement





Cannot collapse a correlation measurement into measurements at null infinity.
Only a single time parameter can be taken to “infinity” in a Penrose diagram.

Correlation measurements



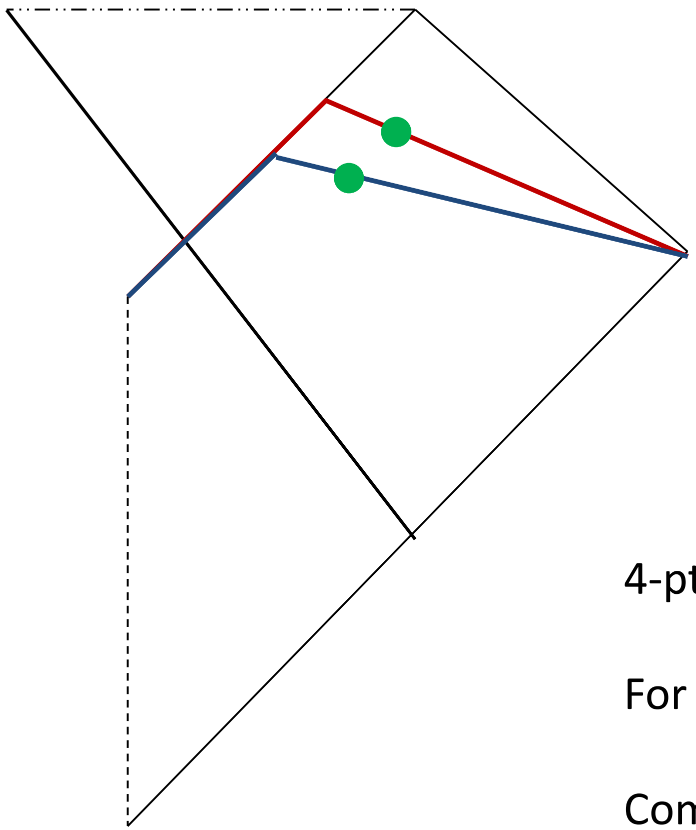
HW theorem does not imply that multi-time correlations at late times are Gibbsian.

Hawking radiation may involve non-thermal multi-time correlations.

Of course, all single-time correlations are asymptotically Gibbsian.

Time correlation of separated accelerated detectors in Minkowski spacetime is non thermal, i.e., different from correlations determined by static detectors in a field Gibbsian state.
Correlations of accelerated detectors (in 3+1) drop with e^{-ar}
Thermal correlations drop with r^{-2}
CA and N. Savvidou, J. Math. Phys. 53, 012107 (2012).

Correlation measurements



Aim: evaluate two-time correlations in an eternal black hole at the Unruh vacuum.

Use the point-like detector approximation, to evaluate

$$P(E_1, \tau_1; \mathrm{E}_2, \tau_2)$$

Coincidence function

C$(E_1, \tau_1; \mathrm{E}_2, \tau_2) \coloneqq P(E_1, \tau_1; \mathrm{E}_2, \tau_2)$-$P(E_1, \tau_1) P(\mathrm{E}_2, \tau_2)$

4-pt function of the field.

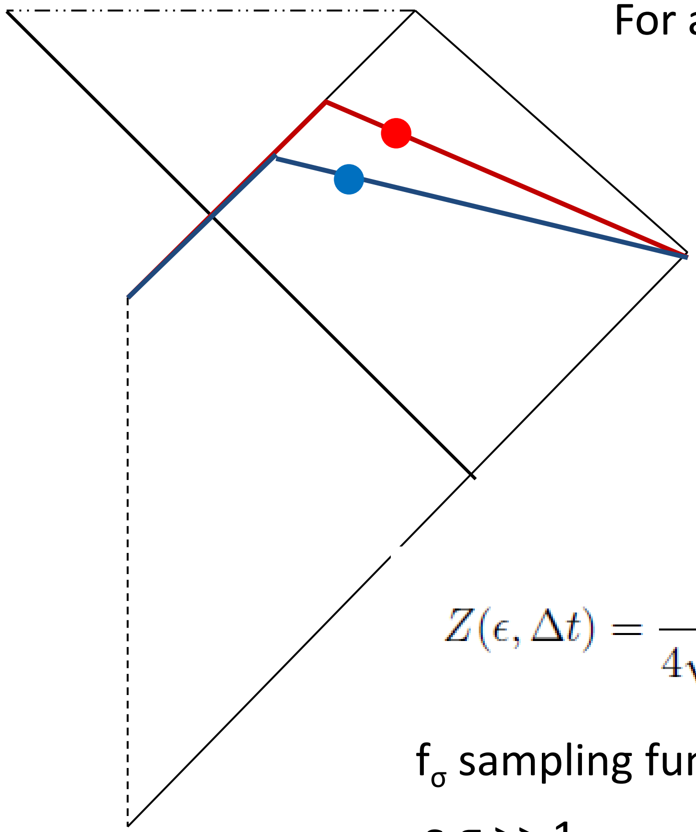
For free field, suffices to find the Wightman function.

Comparable calculation: stress-energy tensor correlations
N. G. Phillips and B. L. Hu, Phys. Rev. D63, 104001 (2001); D67, 104002 (2003).

Note: No X→X' limit is taken as in stress-tensor calculations. No need for renormalization.

Toy model: ignore backscattering

Equivalent to a 2d black hole.



For a pair of static detectors, use Killing time t.

Unruh vacuum

Detection probability: $$P(\epsilon,t)=\frac{\alpha_\epsilon}{2\epsilon r^2}\frac{1}{e^{\frac{2\pi\epsilon}{\kappa}}-1}$$

Coincidence function:

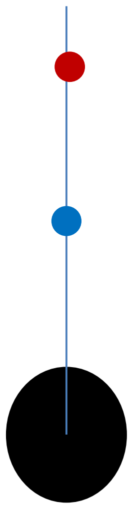
$$C(\epsilon_1,\epsilon_2;\Delta t)=4\pi\alpha_{\epsilon_1}\alpha_{\epsilon_2}\delta(\epsilon_1-\epsilon_2)Z(\epsilon_1,\Delta t),$$

$$Z(\epsilon,\Delta t)=\frac{\sqrt{\pi}}{4\sqrt{2}\epsilon^2r^2r'^2}\left[\frac{f_\sigma(\frac{1}{2}\Delta u)}{(1-e^{-\frac{2\pi\epsilon}{\kappa}})^2}+f_\sigma(\frac{1}{2}\Delta v)+2\cos(2\epsilon\Delta r_*)\frac{f_\sigma(\frac{1}{2}\Delta t)}{1-e^{-\frac{2\pi\epsilon}{\kappa}}}\right]$$

f_σ sampling function, peaked at 0, becomes delta function as $\sigma \to 0$.

$\epsilon\,\sigma >> 1$

Three terms



$$\frac{f_\sigma(\frac{1}{2}\Delta u)}{(1-e^{-\frac{2\pi\epsilon}{\kappa}})^2}$$

First blue, then red (outgoing).

Enhanced by factor $(1+<n>)^2$

$$f_\sigma(\frac{1}{2}\Delta v)$$

First red, then blue (incoming)

$$2\cos(2\epsilon\Delta r_*)\frac{f_\sigma(\frac{1}{2}\Delta t)}{1-e^{-\frac{2\pi\epsilon}{\kappa}}}$$

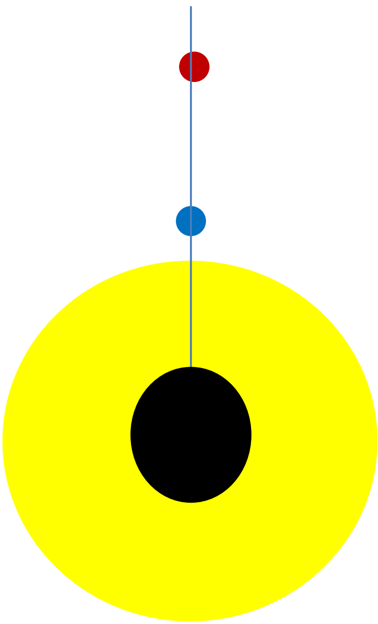
\`Non-local' term (in the sense of Bell).
Fast oscillations if distinguishable:
$\Delta r_* > \sigma \rightarrow \varepsilon\, \Delta r_* >> 1$

Does non-local term persist if backscattering is included? Plausible only if $\Delta r_* \sim M$.

Are there strong non-local temporal correlations in Hawking radiation near the horizon?

Deep quantum regime: $\lambda \sim \Delta r_*$.

Three terms



$$\frac{f_\sigma(\frac{1}{2}\Delta u)}{(1-e^{-\frac{2\pi\epsilon}{\kappa}})^2}$$

First blue, then red (outgoing).

Enhanced by factor
$(1+\langle n\rangle)^2$

$$f_\sigma(\frac{1}{2}\Delta v)$$

First red, then blue (incoming)

$$2\cos(2\epsilon\Delta r_*)\frac{f_\sigma(\frac{1}{2}\Delta t)}{1-e^{-\frac{2\pi\epsilon}{\kappa}}}$$

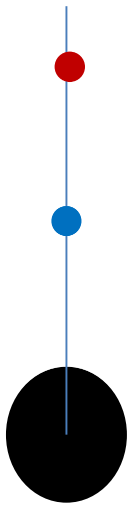
`Non-local' term.
Fast oscillations if distinguishable:
$\Delta r_* > \sigma \rightarrow \varepsilon\, \Delta r_* \gg 1$.

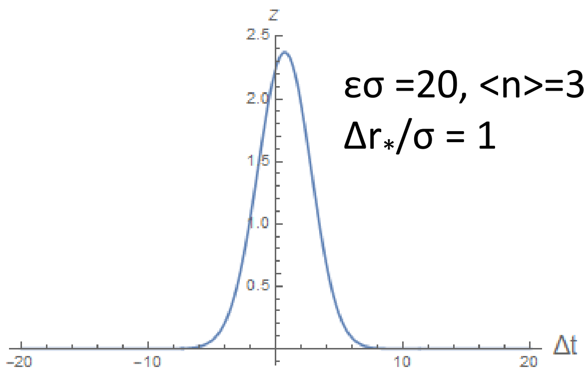
Does non-local term persist if backscattering is included? Plausible only if $\Delta r_* \sim M$.

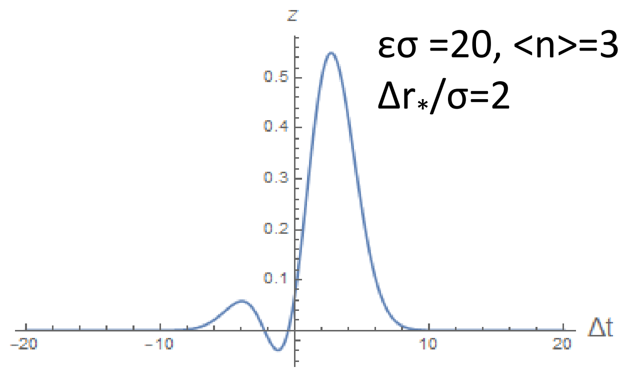
Are there strong non-local temporal correlations in Hawking radiation near the horizon?

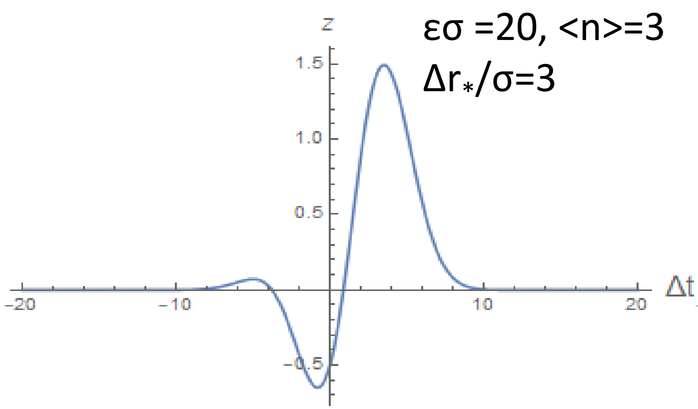
Deep quantum regime: $\lambda \sim \Delta r_*$.

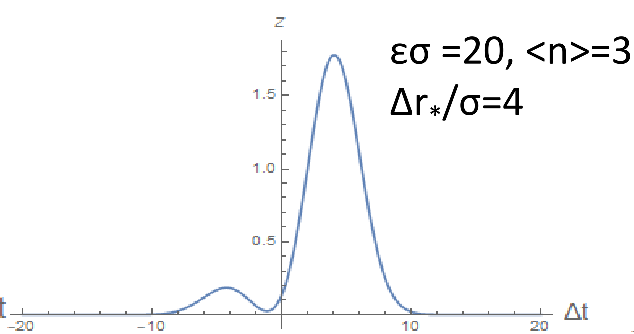
Correlation

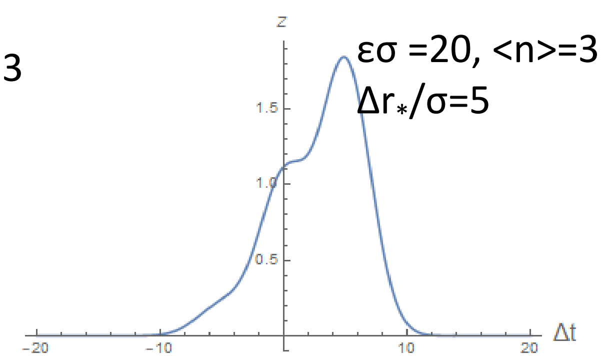












Can we recover pre-collapse information at late times?

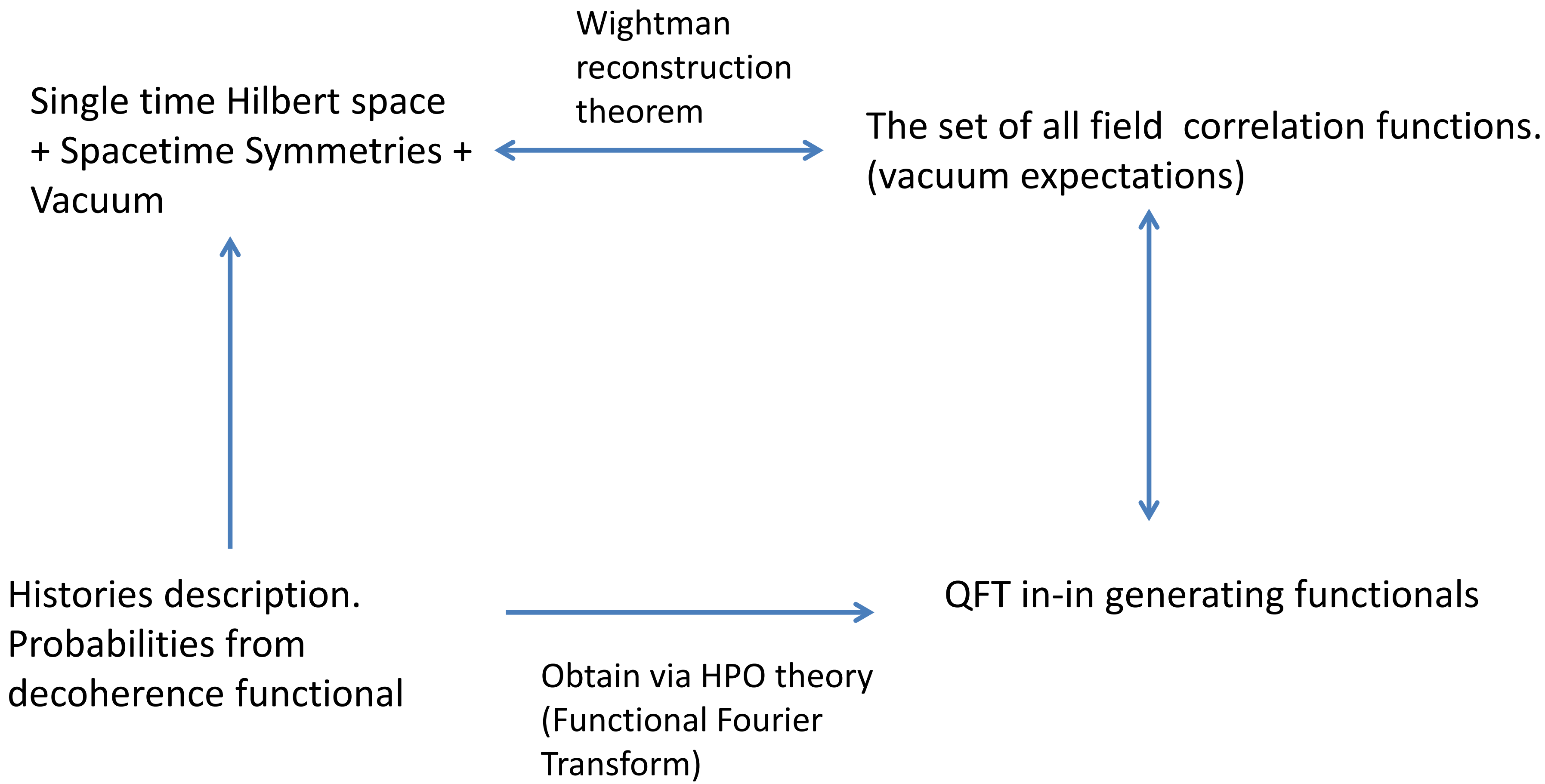
Wightman function at late times depends on parameters that characterize the collapse.
Can we extract this information in the coincidence function?

Implications for information paradox

- Thermodynamics emerges at the level of **two-point functions** (Boltzmann coarse-graining), i.e., single-time measurements. Covered by HW theorem.
- Black hole thermodynamics is unaffected!! (Indeed, the consideration of multi-time correlations serves to highlight the correspondence between BH thermodynamics and ordinary thermodynamics. Both emerge at the same level of coarse-graining.)
- There is certainly some information stored in multi-time correlations that is inaccessible from the single-time quantum state. Plausible that some pre-collapse info available from correlation measurements near the horizon already at the QFTCS level.
- Would this information be sufficient to resolve the information paradox?

Implications for information paradox

- **My opinion: There is no paradox to resolve.** The issue is not unitarity, but the inadequacy of a quantum gravity description in terms of single-time states.
- We can live happily with `non-unitarity' or its mathematical equivalents in generalized quantum theories that are based on histories.
- With or without paradoxes, the quantum information balance of BH formation and evaporation is an intriguing physical problem. The information from multi-time correlations must be explicitly taken into account.
- A resolution requires a covariant characterization of quantum information. Use QFT correlation functions rather than single-time quantum states?


Wightman reconstruction theorem
Single time Hilbert space + Spacetime Symmetries + Vacuum
The set of all field correlation functions. (vacuum expectations)
Histories description. Probabilities from decoherence functional
QFT in-in generating functionals
Obtain via HPO theory (Functional Fourier Transform)

Conclusion

A spacetime-covariant theory of quantum information must treat temporal correlations in the same footing with spatial correlations.

Conclusion

A consistent and practicable theory of QFT measurements is important

1. As a **foundational issue** for QFT.

2. In order to define **quantum informational** notions in a **relativistic** setting.

3. In relation to **Quantum Gravity.**

THANK YOU