Quantum Shockwaves:
Classical and Quantum Channel Capacities

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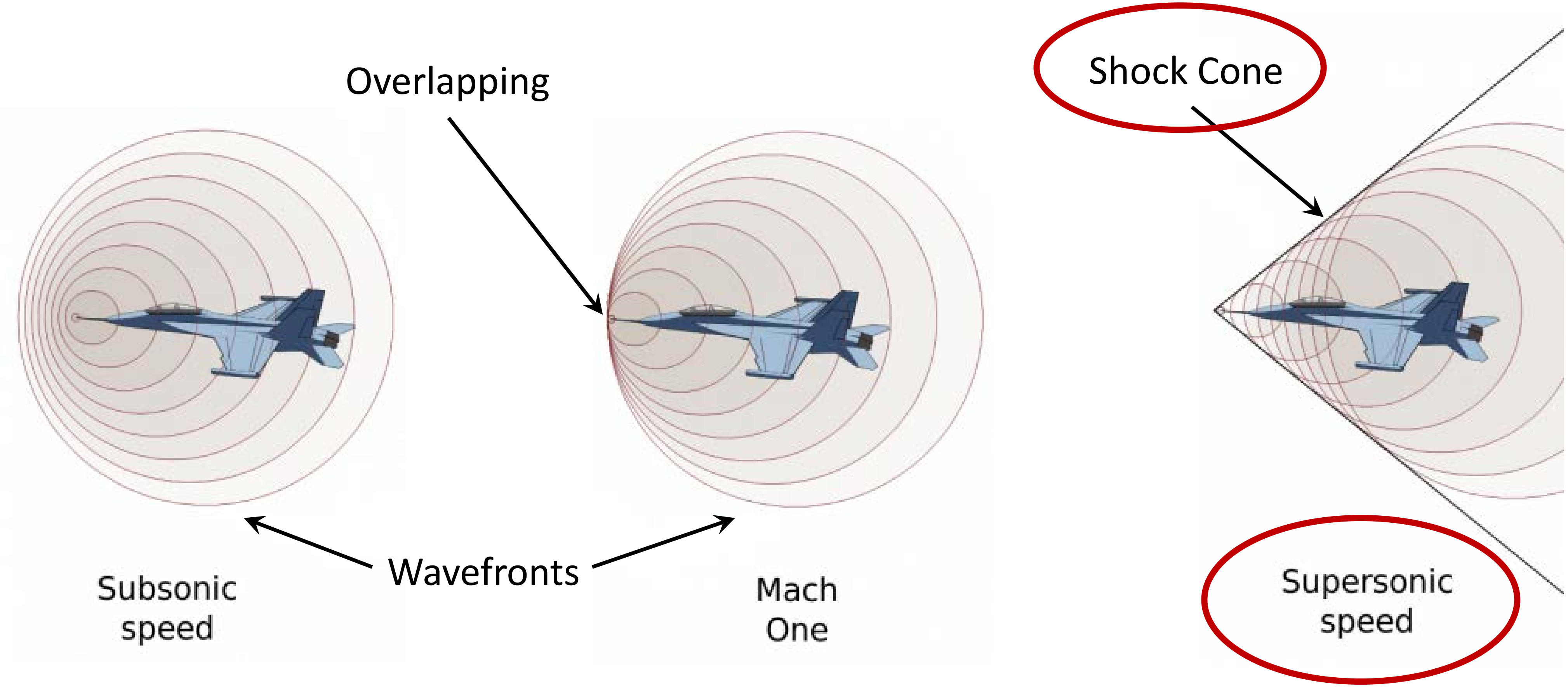
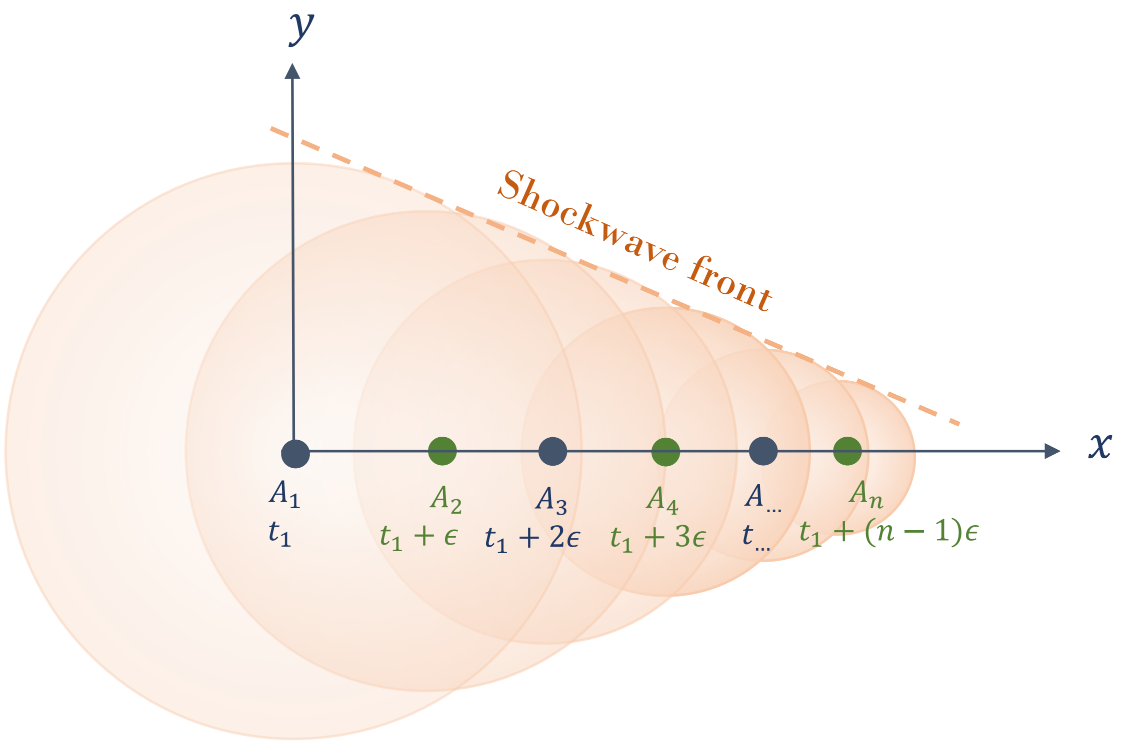
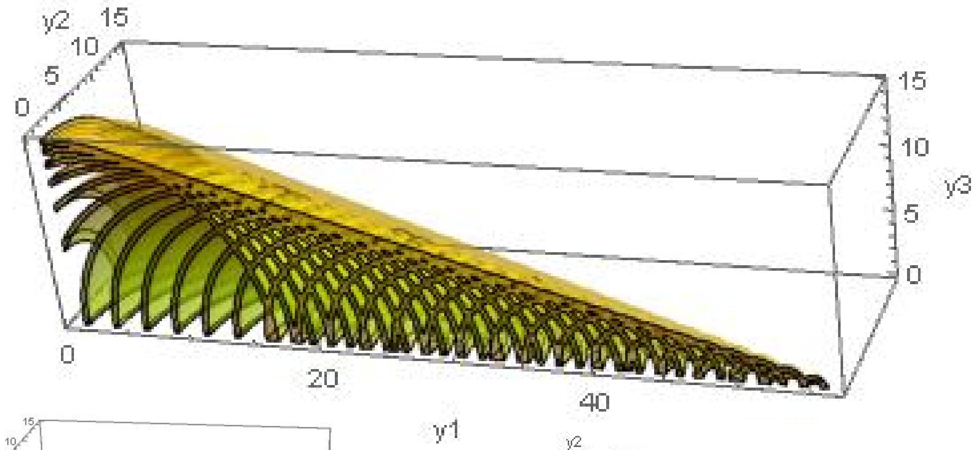
Classical sonic shockwave

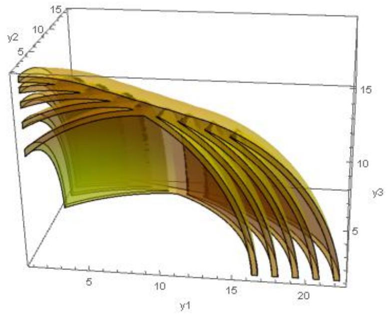


Can we create a shockwave in the vacuum? Yes!

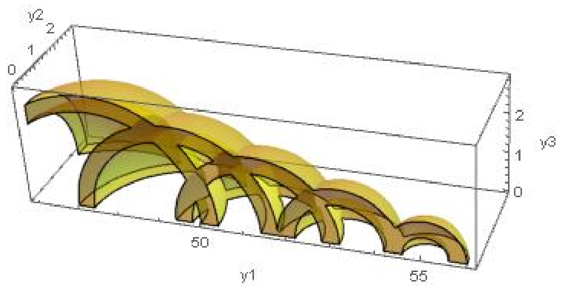
- **Pre-timed emitters (Alices):** Unruh-DeWitt detectors $[A_1, \dots, A_n]$ with a spherical spatial profile
- **Superluminal:** Spacelike separated emitters: $\Delta x^2 > \Delta t^2$
- **Non-perturbative method:** Delta in time coupling to the field







Early emissions



Late emissions

How to communicate with shockwave?

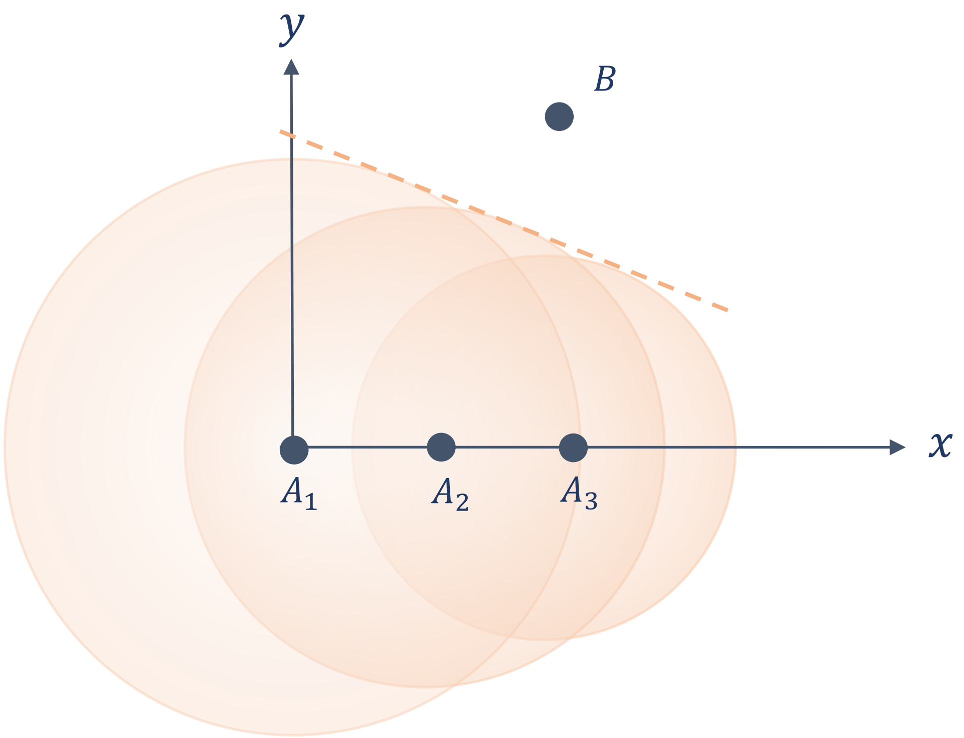
❑ **Receiver Bob is timelike to Alices**

❑ **Alices encode their bit**

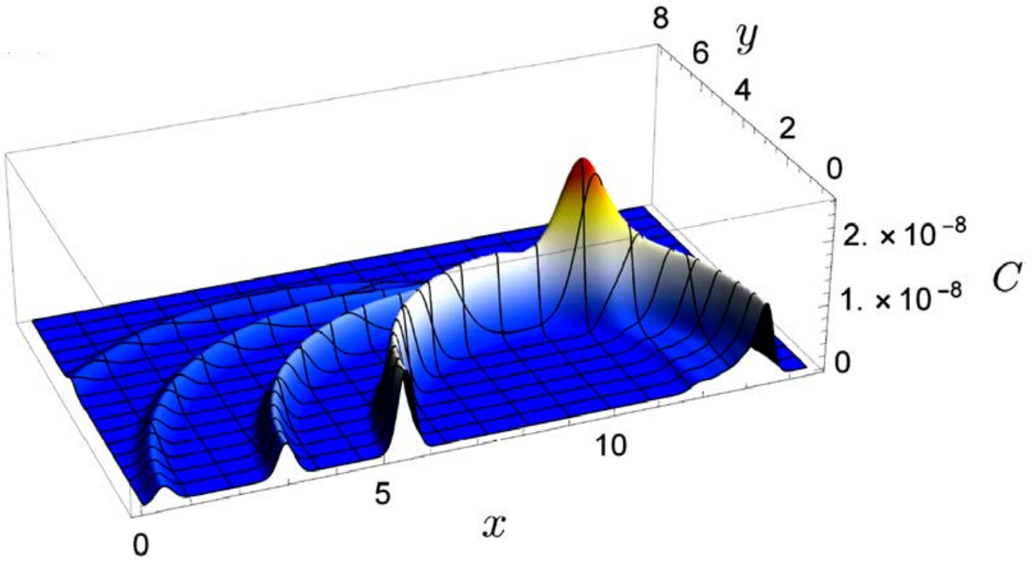
- **Protocol:**
  - **"0"** by not coupling ($\lambda_A = 0$)
  - **"1"** by coupling ($\lambda_A \neq 0$)

❑ Bob couples to the field and measure his state. He **decodes:**

  - "1" if he measures $|e\rangle$
  - "0" if he measures $|g\rangle$



Classical channel capacity



Example:

❑ **Initial state:** 4 emitters (1 excited) – 1 receiver

- **Incoherent superposition:**

$$\hat{\rho}_A = \frac{1}{4}\left(|eggg\rangle\langle eggg| + |gegg\rangle\langle gegg| + |ggeg\rangle\langle ggeg| + |ggge\rangle\langle ggge|\right)$$

- **Coherent superposition:**

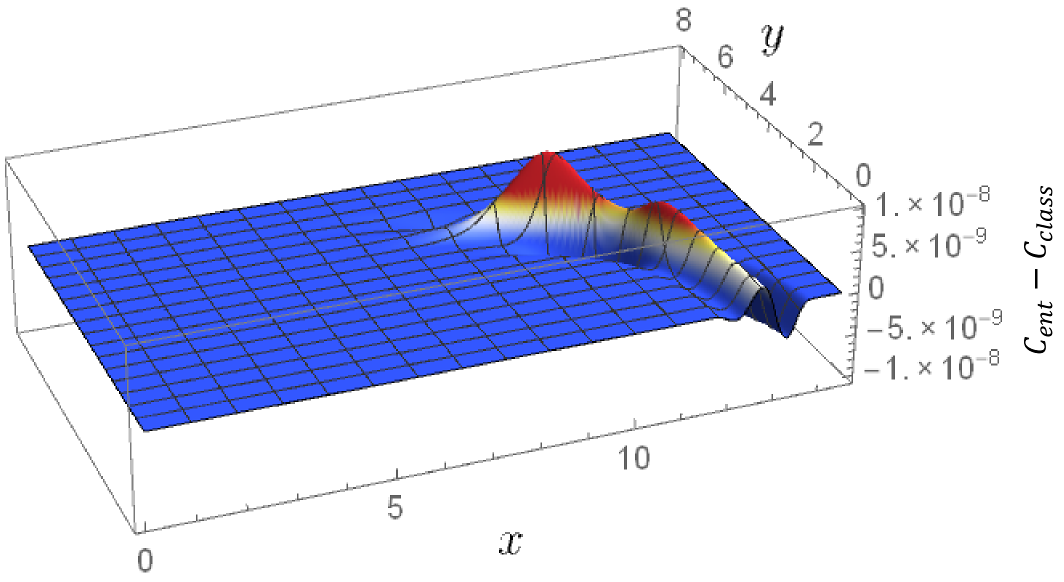
$$|\varphi\rangle = \frac{1}{\sqrt{4}}\left(\mathrm{e}^{\mathrm{i}\,\theta_1}|eggg\rangle + \mathrm{e}^{\mathrm{i}\,\theta_2}|gegg\rangle + \mathrm{e}^{\mathrm{i}\,\theta_3}|ggeg\rangle + \mathrm{e}^{\mathrm{i}\,\theta_4}|ggge\rangle\right)$$

Classical channel capacity

arXiv:1811.10606
Ahmadzadegan
et al.

❑ **Result:**

Entangling the emitters can be used to locally enhance the classical channel capacity of the shockwave



Quantum channel capacity: What is the fundamental mechanism?

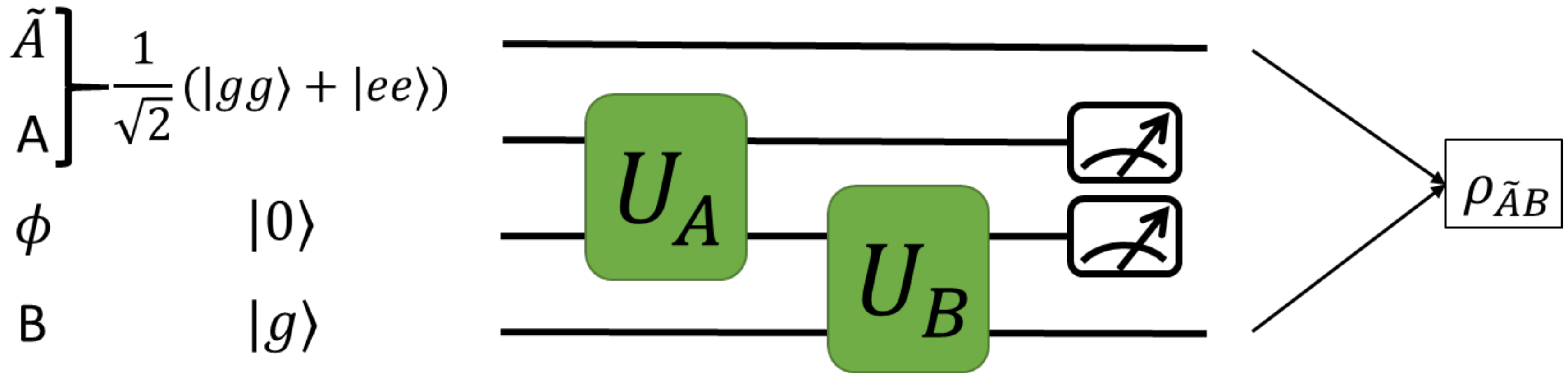
- **Quantum channel capacity:**
  - Number of qubits that A can send B per use of channel
- Prerequisite of sending quantum information: **Sending entanglement**
- Lower bound of QCC: **Coherent information**

$$QCC \geq C_I = \max\{0, S[\rho_B] - S[\rho_{\tilde{A}B}]\}$$

  - $S[\rho] = -\mathrm{Tr}\,\rho \log_2 \rho$ Von-Neumann entropy

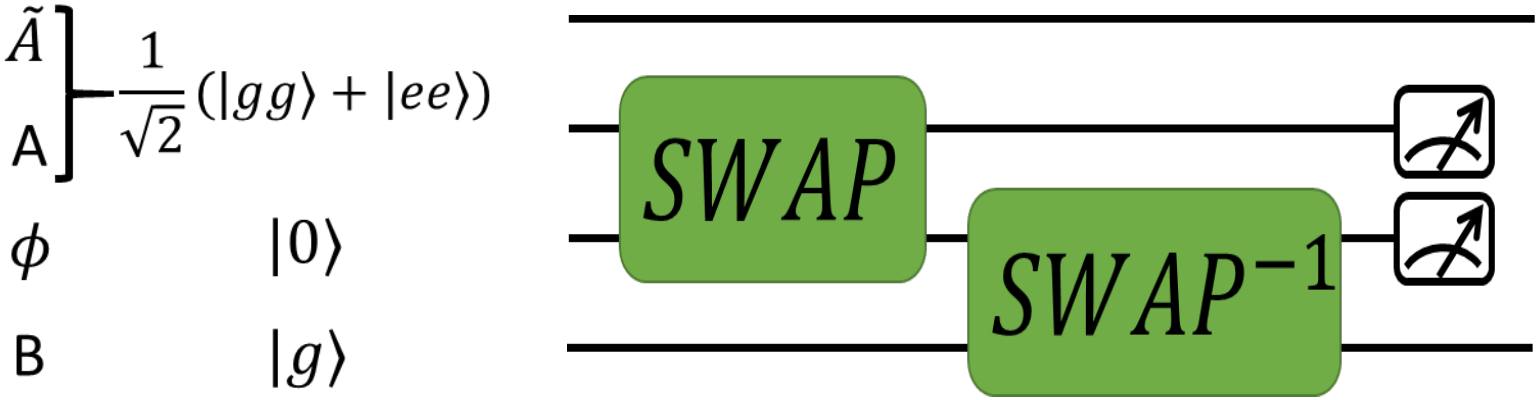
Constructing a quantum channel

- Setup:



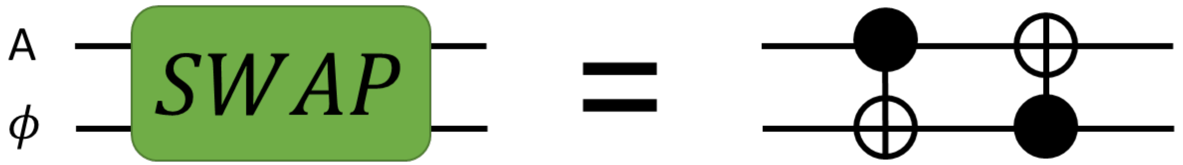
SWAP gate between a qubit and a field?

- SWAP:



- How do we implement SWAP gate between qubit and field?

Qubit to Field SWAP gate



$$\approx \; e^{i\sigma_x \pi_A} \, e^{i\sigma_z \phi_A}$$

$$\phi_A \coloneqq \lambda_\phi \int d^d x \, F(x)\phi(x, t_A)$$
$$\pi_A \coloneqq \lambda_\pi \int d^d x \, F(x)\pi(x, t_A)$$

- ❑ If $\lambda_\phi \gg 1$, first CNOT entangles A with almost orthogonal coherent field states:

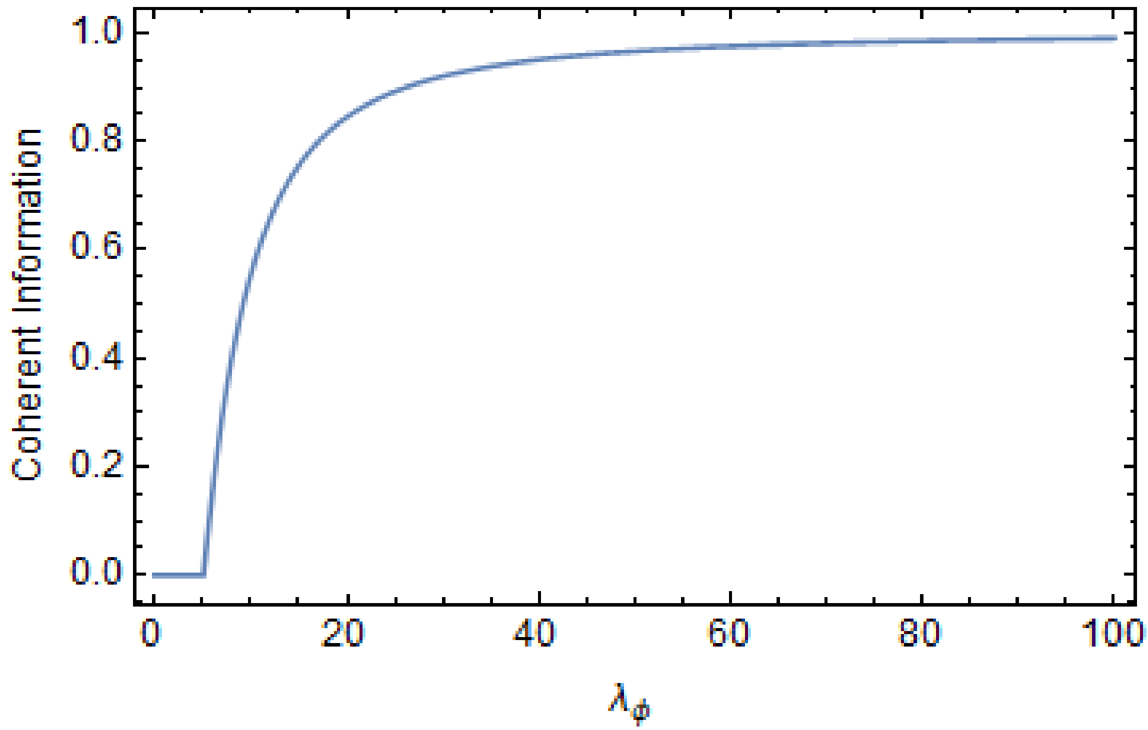
$$\frac{1}{\sqrt{2}}(|g\rangle + |e\rangle)\,|0\rangle \longmapsto \frac{1}{\sqrt{2}}(|g\rangle| + \alpha\rangle + |e\rangle| - \alpha\rangle)$$

- ❑ If λ_π is tuned so that $\pi_A|\pm\alpha\rangle \approx \pm\frac{\pi}{4}| \pm \alpha\rangle$, then second CNOT disentangles A:

$$\frac{1}{\sqrt{2}}(|g\rangle| + \alpha\rangle + |e\rangle| - \alpha\rangle) \longmapsto \frac{1}{\sqrt{2}}|+_y\rangle(|+\alpha\rangle - i| - \alpha\rangle)$$

Effect of the approximate SWAP gate

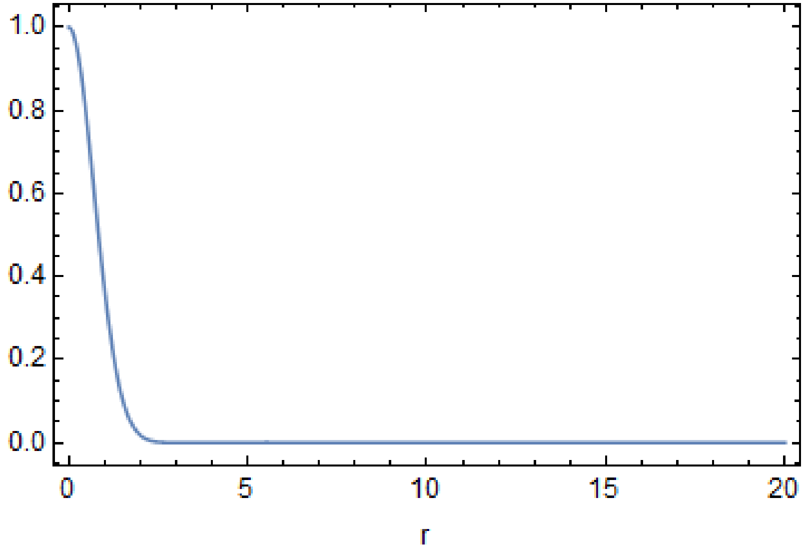
- ❑ We need $\lambda_\phi \gg 1$ in order to get field into almost orthogonal superposition.



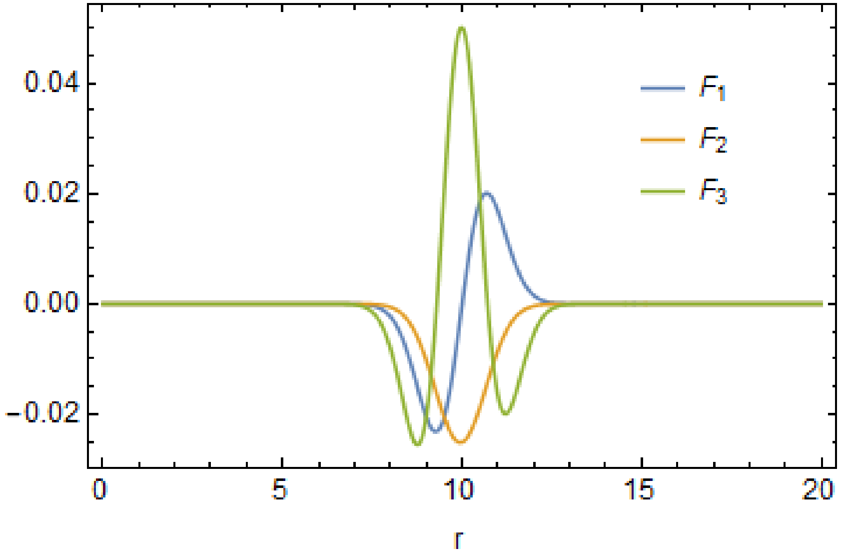
Need strong coupling to transmit quantum information

Where does quantum information propagate to in 3+1D?

Alice's smearing at $t = t_A$

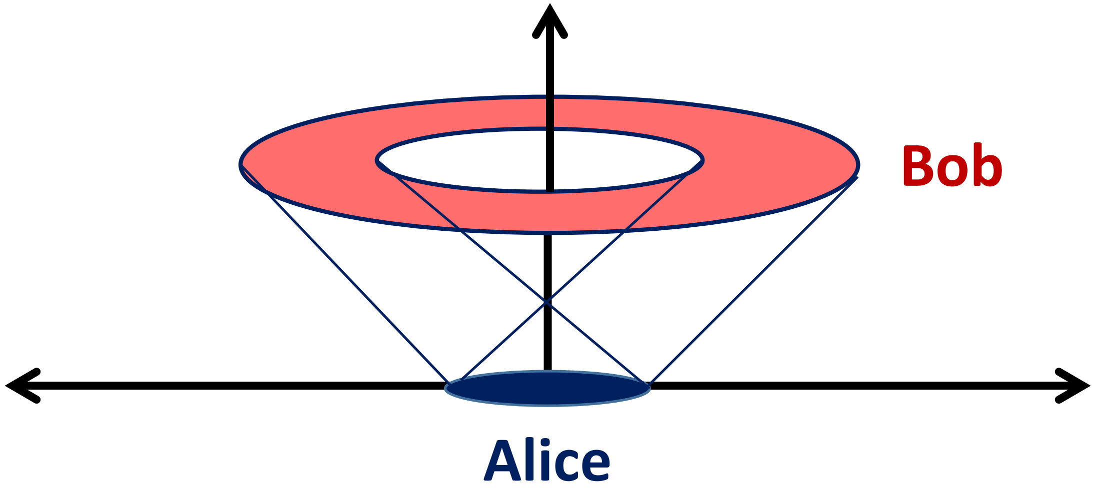


Bob's smearing at $t_B = t_A + 10$



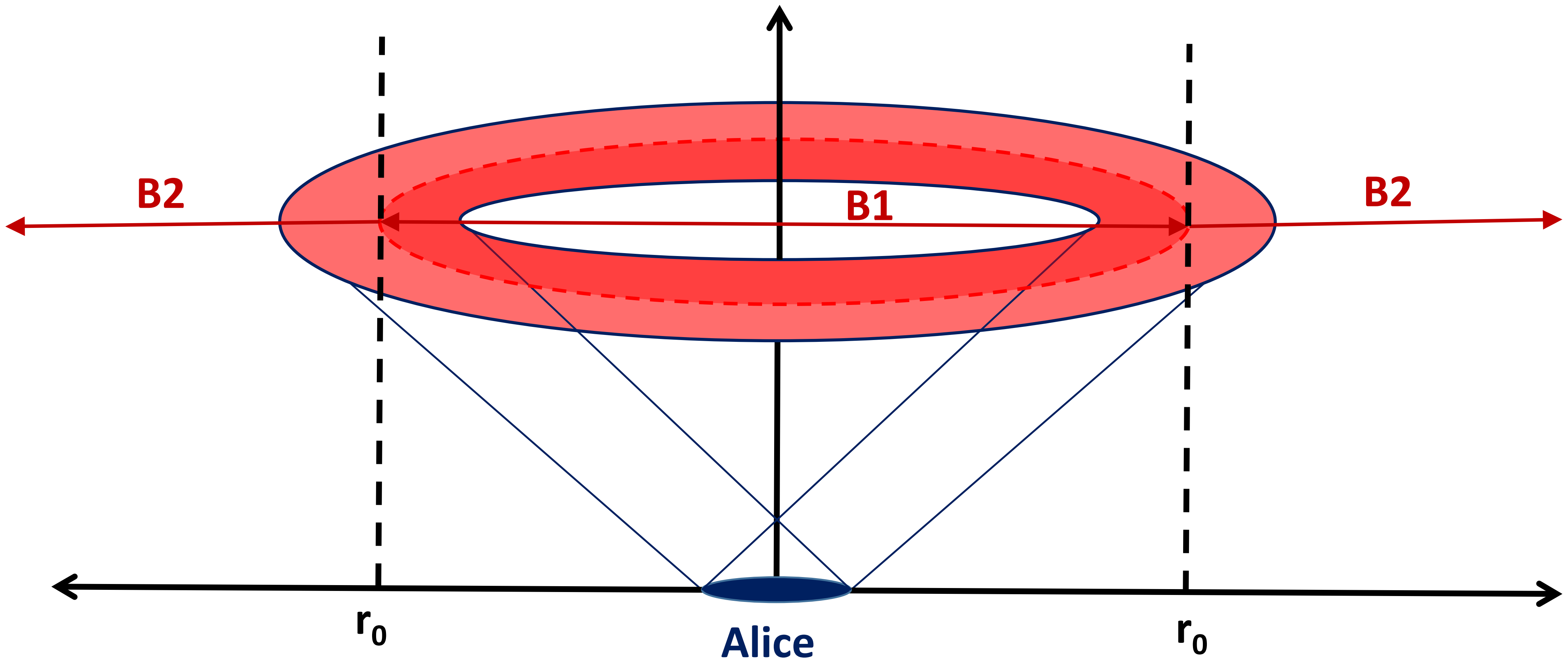
Quantum information propagates on light-cone (strong Huygens principle).

$$\phi[F](t_A) = \phi[F_1](t_B) + \pi[F_2](t_B),$$
$$\pi[F](t_A) = \phi[F_3](t_B) + \pi[F_1](t_B),$$



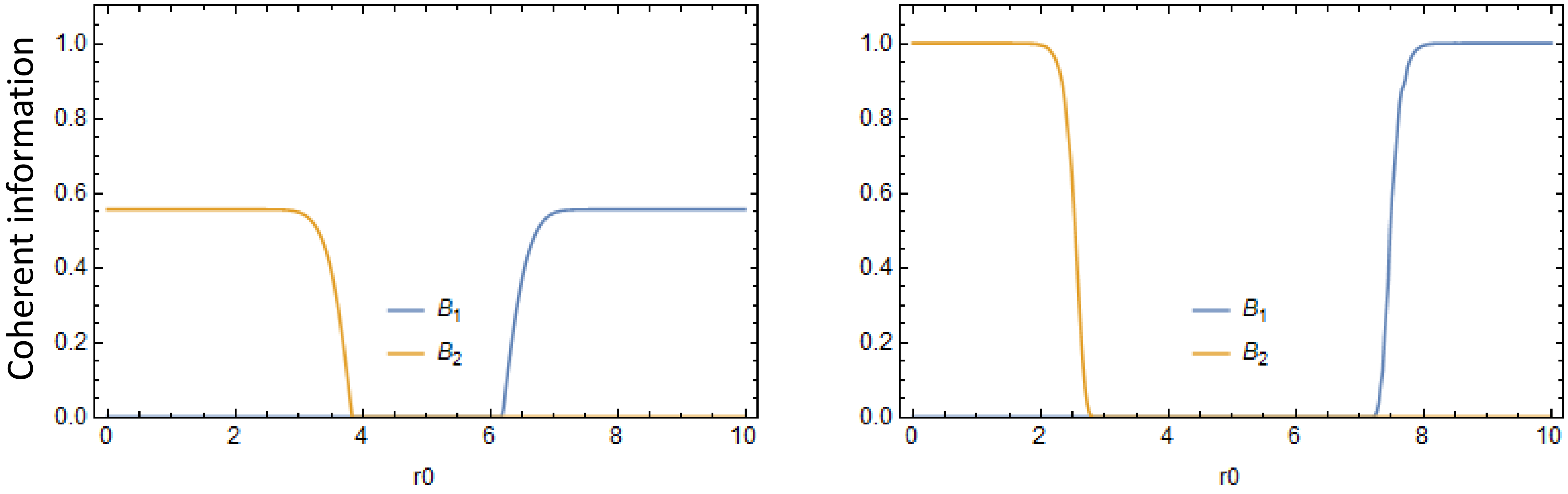
Quantum information broadcasting?

- No cloning theorem: quantum state cannot be cloned.
- Can Alice broadcast a *small* amount of quantum info to multiple **nonidentical Bobs**?



Quantum information broadcasting?

$\lambda_A = 10$ $\lambda_A = 1000$



Stronger coupling: perfect channel ($C_I = 1$) but need large receiver.

Conclusions

- **Classical channel capacity :**
  - Shockwaves concentrate the signal
  - Signal modulated by Alices’ entanglement
- **Quantum channel capacity:**
  - Use approximate SWAP gates to get information in and out of field
  - Nonperturbative calculations
  - Obtained lower bound from coherent information
  - No-cloning constraint requires beam shaping!
- **Outlook: Beam shaping** using **quantum shockwaves** and **quantum MIMO**

An algebraic QFT result

Definition: $\phi[F](t) \coloneqq \int d^d x\, F(x)\phi(x,t)$ and $\pi[F](t) \coloneqq \int d^d x\, F(x)\pi(x,t)$

We prove the following claim for **any free field in any spacetime dimension**:
Claim:

$$\phi[F](t_A) = \phi[F_1](t_B) + \pi[F_2](t_B),$$
$$\pi[F](t_A) = \phi[F_3](t_B) + \pi[F_1](t_B),$$

where

$$\widetilde{F_1}(k) = \tilde{F}(k)\cos(\Delta|k|)\,,$$
$$\widetilde{F_2}(k) = \tilde{F}(k)\,\mathrm{sinc}(\Delta|k|)\,(-\Delta),$$
$$\widetilde{F_3}(k) = \tilde{F}(k)\sin(\Delta|k|)|k|.$$

Here, $\Delta \coloneqq t_B - t_A$ and ~ denotes Fourier transform.

Corollary: We can write SWAP^{-1} in terms of observables at $t = t_B$. Problem solved!

How to calculate the shockwave?

❑ **Energy flow of the shockwave:** $\langle\psi|\hat{U}^{\dagger}(t){:}\hat{T}_{00}{:}\hat{U}(t)|\psi\rangle$

- **Initial state :**

$$|\psi\rangle = |\varphi\rangle_A |0\rangle_f; \qquad \mathrm{H}_A = \otimes_{i=1}^{n} H_{A_i}$$

- **Time evolution operator:**

$$\hat{U}(t) \coloneqq \mathrm{e}^{-i\,\hat{H}_I^{(n)}(t_{(n)})\theta(t-t_{(n)})} \ldots \mathrm{e}^{-i\,\hat{H}_I^{(1)}(t_{(1)})\theta(t-t_{(1)})}$$

Beyond Shockwaves

- **Time reversed quantum shockwave:**
  - Super focused emission becomes super focused reception
    - Super resolution power
      Example: telescope or microscope
- **Quantum MIMO**
  - **MIMO (Multiple In, Multiple Out) antennas**
    - Directional emission (beam forming)
    - Improved channel capacity

Entanglement can increase both directionality and capacity beyond the classical MIMO

Classical channel capacity: Mathematical origin of larger capacity?

- **Two contributions to the channel capacity:**
  - Sensitive term to the initial states of Alices
  - Insensitive term: result of switching

$$\prod_{i=1}^{n} (\underbrace{\cosh(C_{B\,i})}_{\text{switching}} + \underbrace{\hat{\mu}_i \sinh(C_{B\,i})}_{\text{Alice's initial state}}) \propto f(C) + C_1 \hat{\mu}_n + \cdots + C_2 \hat{\mu}_1 \hat{\mu}_n + \cdots + C_3 \hat{\mu}_1 \ldots \hat{\mu}_n$$

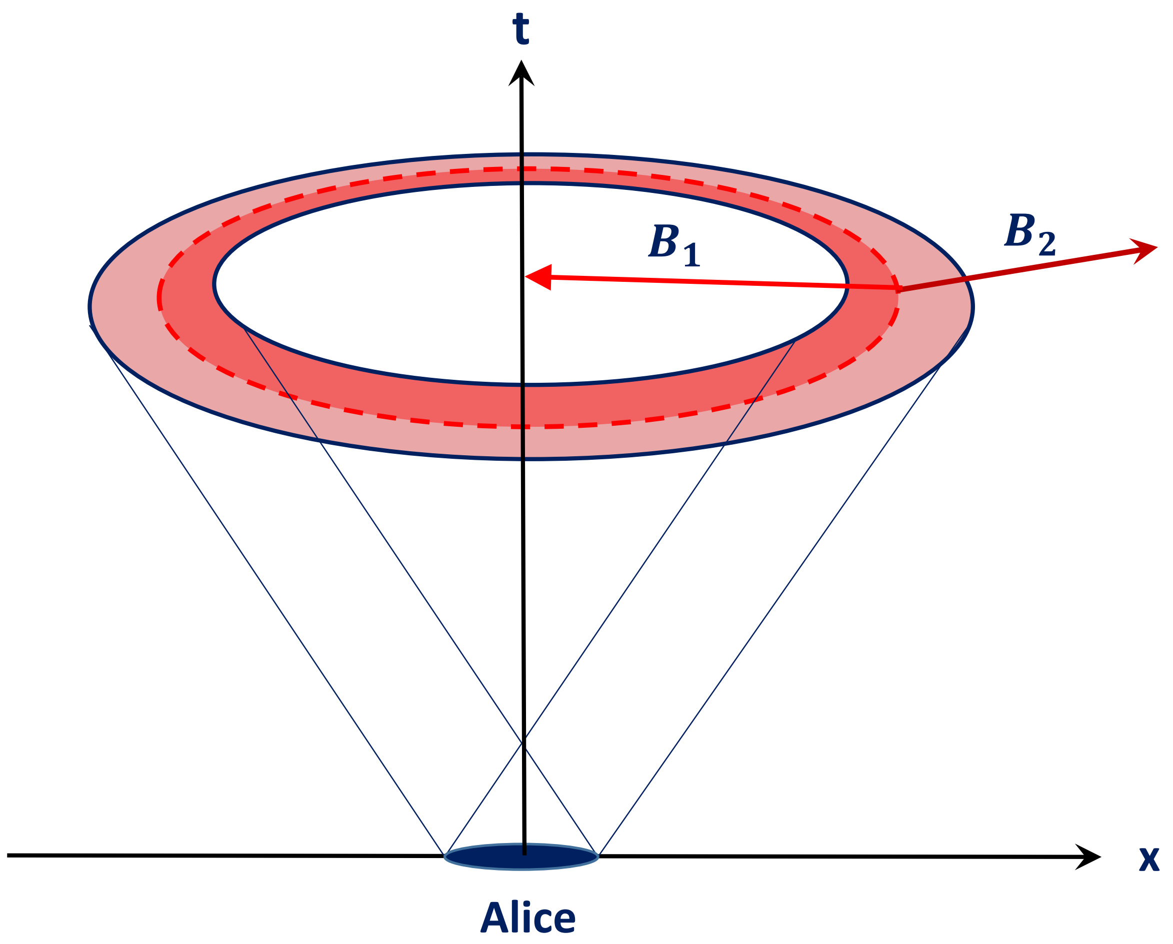
$$C_{B\,i} = \lambda_B \, \lambda_i \, \theta(t - t_i) \int d^3k \, (\alpha_{s\,B} \alpha^*_{s\,i} - \alpha^*_{s\,B} \alpha_{s\,i})$$

Number-valued function

What is the mathematical origin of concentrated signal?

- **Two contributions to the shockwave:**
  - Sensitive term to the initial states of Alices
  - Insensitive term: result of switching

$$\hat{U}^{\dagger}(t){:}\hat{T}_{00}{:}\hat{U}(t) \propto \sum_{i=1}^{n} \Big(\underbrace{f(C)}_{\text{switching}} + \underbrace{C_1\hat{\mu}_n + C_2\hat{\mu}_1\hat{\mu}_n + C_3\hat{\mu}_2\hat{\mu}_n + \cdots}_{\text{Alices' initial state}}\Big)$$


t
B_1
B_2
x
Alice

Can we create a shockwave in the vacuum?

- **Yes**

  Sequence of pre-timed emitters that go off faster than speed of light

  Classical Shockwave

- **Can it get stronger using a quantum effect?**

  Entangling the emitters

❑ Comparison:

- Choice of the emitters‘ entanglement modulates the energy density
- In some areas the amplitude is enhanced and more concentrated