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Do black holes store negative entropy?

Koji Azuma[1]

Collaborators:

Sathyawageeswar Subramanian[2]

arXiv: 1807.06753

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Do black holes store negative entropy?

Today, we apply quantum information theory to black-hole physics, to solve a paradoxical nature given by this equation.

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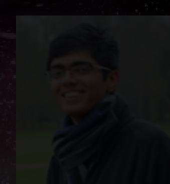
KA & S. Subramanian, arXiv: 1807.06753.



arXiv: 1807.06753

Brief review on black holes

KA & S. Subramanian, arXiv: 1807.06753.

a) Consideration by Bardeen, Carter & Hawking

Com. Math. Phys. **31**, 161 (1973).

Under general relativity, Bardeen *et al.* derived the 1st law of a black hole B:

$$\mathrm{d}E_B = \frac{\kappa_B}{8\pi}\mathrm{d}A_B + \Omega_B \mathrm{d}J_B + \phi_B \mathrm{d}Q_B,$$

as well as the 2nd law:

$$\mathrm{d}A_B \geq 0,$$

which holds for any “classical” process (except for the Hawking radiation).

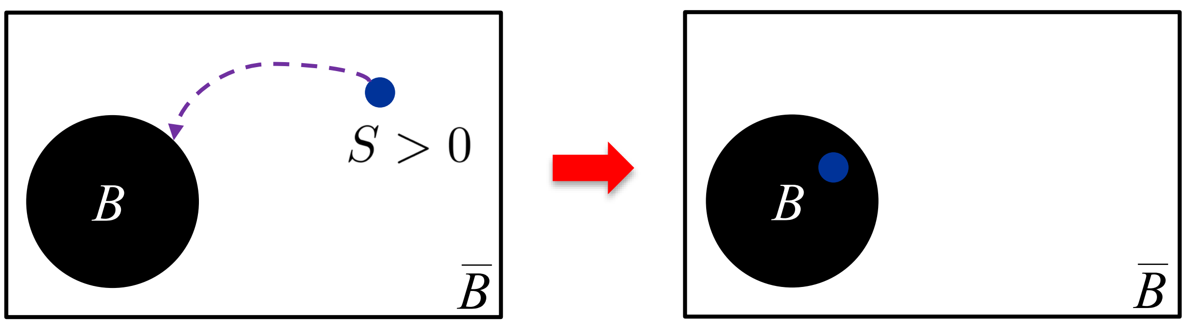
$$\boxed{T_B S(B) = \frac{\kappa_B}{8\pi} A_B \,?}$$

But, the temperature should be 0, because black holes seem to absorb only.

b) Consideration by Bekenstein

Phys. Rev. D **7**, 2333 (1973).

If B has no entropy, entropy disappears from our universe, which is incompatible with the second law of thermodynamics.



Hence, he argued

$$\boxed{T_B S(B) = \frac{\kappa_B}{8\pi} A_B}$$

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c) Hawking radiation

Com. Math. Phys. **43**, 1999 (1975).

For a Schwarzschild black hole,

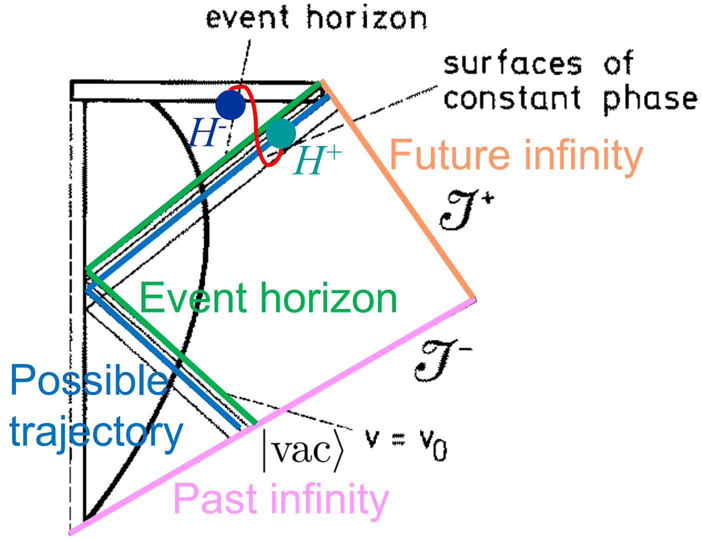
$$dE_B = \frac{\kappa_B}{8\pi} dA_B \quad \text{(1st law)}$$

Hawking expands semi-classical theory.

For a possible trajectory passing near the event horizon, Hawking finds that this trajectory creates a pair:

$$|\chi\rangle_{H^+H^-} := \exp[r_\omega(\hat{a}_k^\dagger \hat{b}_{-k}^\dagger - \hat{a}_k \hat{b}_{-k})]|\text{vac}\rangle$$

$$= \frac{1}{\cosh r_\omega} \sum_{n=0}^{\infty} \tanh^n r_\omega |n\rangle_{H^+} |n\rangle_{H^-}$$



H^+ : Positive-energy particles

H^- : Negative-energy particles

Always, the positive-energy particles H^+ go to the future infinity.

$$\hat{\chi}_{H^+} := \text{Tr}_{H^-}[|\chi\rangle\langle\chi|_{H^+H^-}] = \frac{1}{\cosh^2 r_\omega} \sum_{n=0}^{\infty} \tanh^{2n} r_\omega |n\rangle\langle n|_{H^+} = \frac{e^{-\beta_H \omega \hat{n}_{H^+}}}{Z_{\beta_H}}$$

This is a thermal state with the Hawking temperature $T_B = \beta_H^{-1} = \kappa_B/(2\pi)$.

$dE_B < 0.$ → $\boxed{dA_B < 0 \text{ from the 1st law.}}$ → Bekenstein Eq. $\boxed{S(B) = \frac{A_B}{4}}$

Innovative R&D by NTT

Inconsistency with quant. mech.

KA & S. Subramanian, arXiv: 1807.06753.

c) Hawking radiation

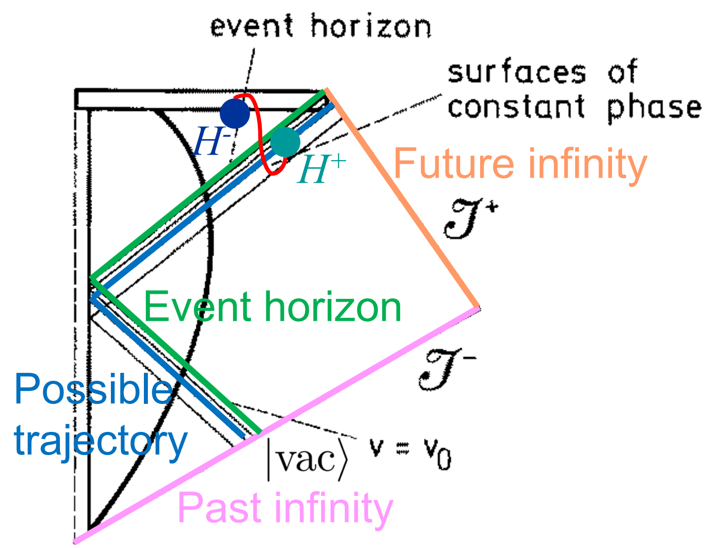
However, quantum mechanics suggests that the negative-energy particles H^- falling into the black hole B should have

$$S(H^-) = S(H^+) \geq 0$$

which should increase $S(B)$, and thus,

$$\boxed{\mathrm{d}A_B \geq 0}$$

from the Bekenstein equation.



H^+ : Positive-energy particles

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$$|\chi\rangle_{H^+H^-} := \exp[r_\omega(\hat{a}_k^\dagger \hat{b}_{-k}^\dagger - \hat{a}_k \hat{b}_{-k})]|\mathrm{vac}\rangle$$

These are incompatible.

S. L. Braunstein *et al.*, Phys. Rev. Lett. **107**, 071302 (2011); Phys. Rev. Lett. **110**, 101301 (2013).

Always, the ... infinity.

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Key observations

KA & S. Subramanian, arXiv: 1807.06753.

b) Consideration by Bekenstein

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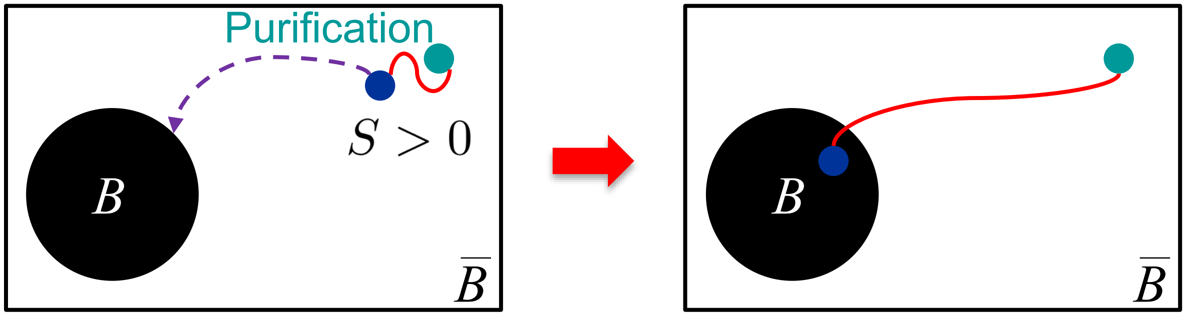

$$\mathrm{d}A_B \geq 0$$

c) Hawking radiation

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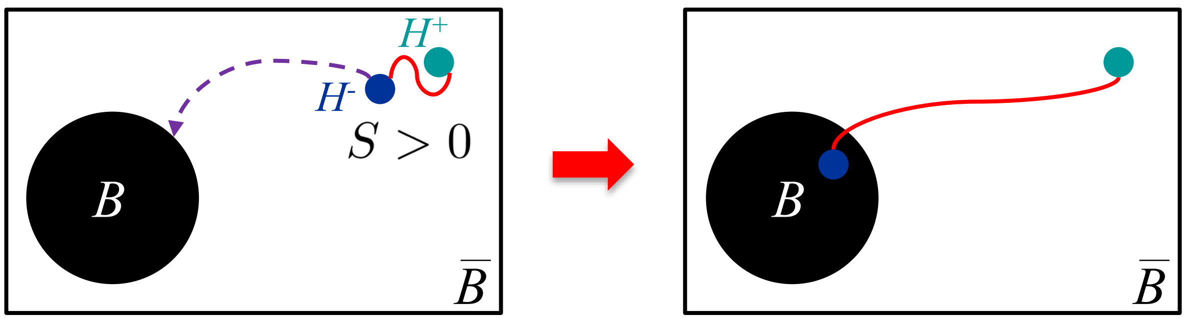


$$\mathrm{d}A_B \leq 0$$

Observations:

The microscopic views are the same. But…

What does make the difference in the area change?

Hawking radiation is special in the sense that it uniquely generates unusual negative-energy particles inside the black hole.

Bekenstein Eq.

$$\boxed{S(B) = \frac{A_B}{4}}$$

Our main argument

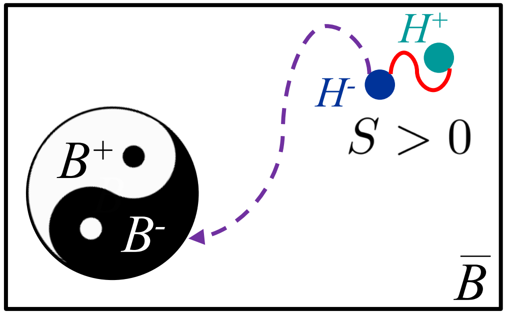
KA & S. Subramanian, arXiv: 1807.06753.

Assumptions:

1. Black hole B is composed of positive energy particles B^+ and negative energy particles B^-.

$$B = B^+B^-$$

2. The purification of the black hole B exists in its outside $\bar{B}$. Therefore, $B^+B^-\bar{B}$ is in a pure state.



Our main argument:

The area of the black hole is proportional to the coherent information:

$$\frac{A_B}{4} = I(\bar{B}\rangle B^+) := -S(\bar{B}|B^+).$$

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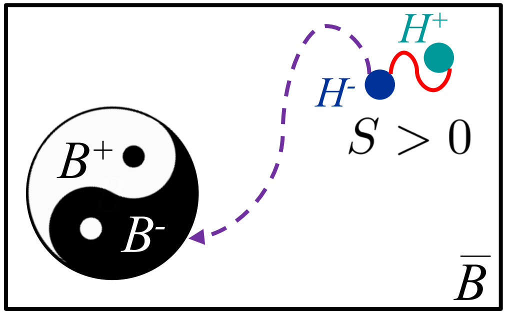

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This is consistent with typically extremely low Hawking temperatures.

Equivalently, from $I(\bar{B}\rangle B^+) = -S(\bar{B}|B^+) = S(B^+) - S(\bar{B}B^+) = S(B^+) - S(B^-)$,

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This is consistent with the original picture of Hawking radiation.

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Bekenstein Eq. $\cancel{S(B) = \frac{A_B}{4}}$

Consistency with Hawking's picture

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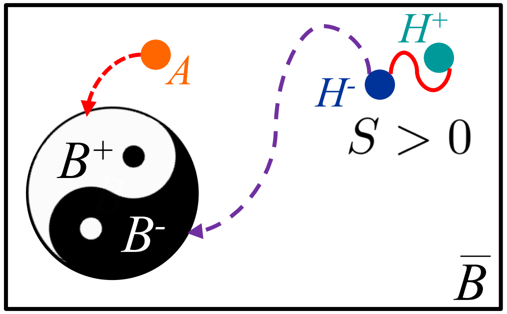
Consider a process where a black hole B absorbs a system A from the outside, while it emits a Hawking pair H^+H^-. Then, the changes of black hole's energy and coherent info are



$$\mathrm{d}E_B = E_A - E_{H^+},$$

$$\mathrm{d}I(\bar{B}\rangle B^+) = S(A) - S(H^-).$$

Equilibrium state:

If system A is in the thermal state with the Hawking temperature T_B,

$$\begin{cases} E_A = E_{H^+}, \\ S(H^+) = S(A). \end{cases} \Rightarrow \begin{cases} \mathrm{d}E_B = 0, \\ \mathrm{d}I(\bar{B}\rangle B^+) = 0. \end{cases} \Rightarrow \mathrm{d}A_B = 0$$

Either from the 1st law or from our equation.

Black hole B surrounded by thermal systems with the Hawking temperature T_B is exactly in an equilibrium state, consistent with Hawking's argument.

Perturbation from the equilibrium state:

A small perturbation to the equilibrium state leads to

$$\mathrm{d}I(\bar{B}\rangle B^+) = \mathrm{d}S(A) - \mathrm{d}S(H^+) = \frac{1}{T_B}(\mathrm{d}E_A - \mathrm{d}E_{H^+}) = \frac{\mathrm{d}E_B}{T_B} = \frac{2\pi}{\kappa_B}\mathrm{d}E_B$$

|| From our equation.

$$\frac{\mathrm{d}A_B}{4}$$

This is equivalent to the first law for Schwarzschild black holes.

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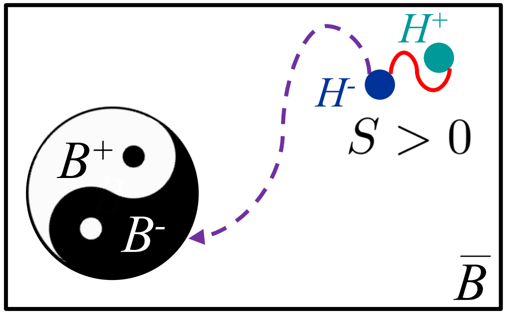

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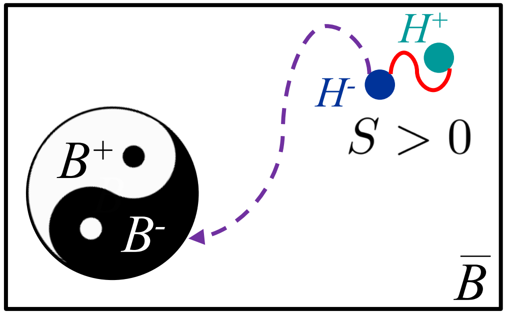

Our main argument: No violation with the unitarity of quantum mechanics.

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Reproducing the generalized 2nd law.

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Bekenstein Eq.

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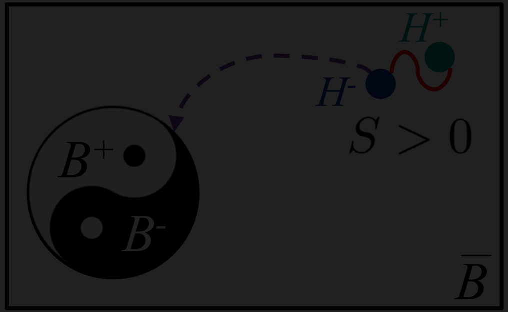

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Related with entanglement.

Negative only in the quantum realm.

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Reproducing the generalized 2nd law.

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Hence, the black holes may store negative information, implying that they are purely quantum objects.

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