Maximal violation of lifted Bell inequalities and its implications in self-testing

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Overview

1. Bell inequalities and Motivation
2. Maximal violation of outcome-lifted Bell inequality
3. Maximal violation of party-lifted Bell inequality
4. Concluding Remarks

Bell inequalities

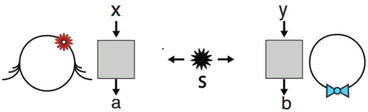

Figure: Bell scenario [Source: Brunner *et. al.* , RMP. 86, 419 (2014)].

- We denote the set of conditional probabilities characterizing the Bell experiment—dubbed a *correlation*— by the vector $\vec{P} := \{P(ab|xy)\}$.

$\vec{P}$ satisfies the non-signaling constraints:

$$\sum_a P(ab|xy) = P(b|y) \quad \forall \quad b, x, y, \qquad \sum_b P(ab|xy) = P(a|x) \quad \forall \quad a, x, y$$

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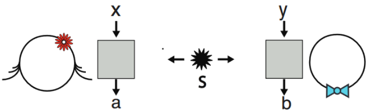

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- A linear Bell inequality takes the form of

$$\mathcal{I}_2(\vec{P}) := \sum_{a,b,x,y} B_{a,b,x,y} P(ab|xy) \overset{\mathcal{L}}{\leq} \beta_{\mathcal{L}} \quad (1)$$

Any correlation that satisfies this inequality can be written in the form [1]:

$$P(ab|xy) = \sum_{\lambda} q_{\lambda} P(a|x,\lambda) P(b|y,\lambda), \quad \forall \quad a,b,x,y \quad (2)$$

with weights q_{λ} satisfying $q_{\lambda} \geq 0$, $\sum_{\lambda} q_{\lambda} = 1$.

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Non-signaling polytope

A correlation is

- Bell non-local if it violates a Bell inequality
- Quantum if $P(ab|xy) = \mathrm{Tr}\left(\rho_{12} M^1_{a|x} \otimes M^2_{b|y}\right)$.
- There are non-local correlations which are not quantum.

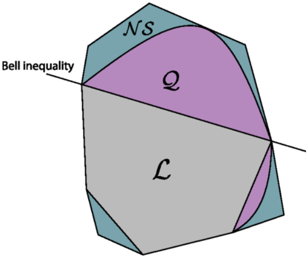

Figure: Non-signaling polytope [2].

[2] Source: Brunner *et. al.* , Rev. Mod. Phys. 86, 419 (2014)

Motivation

- Bell nonlocality is a resource for numerous quantum information and communication tasks:
  - in quantum key distribution involving untrusted devices,
  - in the reduction of communication complexity,
  - in the expansion of trusted random numbers,
  - in certifying the Hilbert space dimension of physical systems,
  - in self-testing of quantum devices
  - and in witnessing and quantifying (multipartite) quantum entanglement using untrusted devices, etc.
- Lifting:[3] a procedure to derive facet-defining Bell inequalities for more complicated Bell scenarios starting from existing ones.

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Motivation

- The facet-preserving nature of Pironio's lifting is well-known
- What about the violation of lifted Bell inequalities?
- There are Bell inequalities whose maximal quantum violation can be used for the task of self-testing.
- Question: Is the self-testing property of a Bell inequality is preserved through the lifting operation:
  - input-lifting
  - output-lifting
  - party-lifting.

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Outcome-lifting of a Bell inequality

Consider a 2-partite Bell inequality,

$$I_2(\vec{P}) := \sum_{a,b,x,y} B_{a,b,x,y} P(ab|xy) \overset{\mathcal{L}}{\leq} \beta_{\mathcal{L}}. \tag{3}$$

Outcome-lifting operation:

Replace $P(ab'|xy')$ (for some fixed outcome b' of the measurement y') by $P(ab'|xy') + P(au|xy')$, where u is the added outcome.

The above operation on the Bell inequality [Eq. (3)] implies the following new Bell inequality:

$$I_2^{LO} := \sum_{a,b,x,y\neq y'} B_{a,b,x,y} P(ab|xy) + \sum_{a,b\neq u,x} B_{a,b,x,y'} P(ab|xy') + \sum_{a,x} B_{a,b',x,y'} P(au|xy') \overset{\mathcal{L}}{\leq} \beta_L \tag{4}$$

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Preservation of quantum and non-signaling violation

- Under grouping/splitting the outcomes, the new Bell expression is equivalent to the original Bell expression.
- Any $\vec{P}$ obtained from a quantum (non-signaling) correlation by grouping/splitting the outcomes is still quantum (non-signaling [4]).

By using these two properties, we have demonstrated the following.

Proposition

Lifting of outcomes preserves the quantum bound and the non-signaling bound of any Bell inequality

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Implications on self-testing

Self-testing

- Given $\vec{P} = P(ab|xy) = \mathrm{Tr}\left(M^{(1)}_{a|x} \otimes M^{(2)}_{b|y} \rho_{12}\right)$, estimate ρ_{AB}, $\{M^{(1)}_{a|x}\}_{a,x}$, $\{M^{(2)}_{b|y}\}_{b,y}$
- $\vec{P}$ self-tests the reference (entangled) state $\psi'_{12} = |\psi'_{12}\rangle\langle\psi'_{12}|$ and the reference POVM $\{\tilde{M}^{(1)}_{a|x}\}_a$, $\{\tilde{M}^{(2)}_{b|y}\}_b$ if there exists a local isometry $\Phi = \Phi_1 \otimes \Phi_2$ such that

$$\Phi\left(M^{(1)}_{a|x} \otimes M^{(2)}_{b|y}\rho_{12}\right)\Phi^\dagger = \left(\tilde{M}^{(1)}_{a|x} \otimes \tilde{M}^{(2)}_{b|y}\psi'_{12}\right) \otimes \rho_{aux} \quad (5)$$

where ρ_{aux} is an auxiliary state.

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- There are Bell inequalities whose maximal quantum violation alone is sufficient to self-test.
- If ρ_{12} maximally violates an outcome-lifted Bell inequality, then ρ_{12} can also violate the original inequality maximally.

Corollary

If the original inequality self-tests some reference state $|\psi'_{12}\rangle$, then any inequality obtained from it by outcome-lifting also self-tests $|\psi'_{12}\rangle$.

- Question: Can the maximal quantum violation of an outcome-lifted Bell inequality can self-test both the state and measurements?

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- If a correlation self-tests both the quantum state and measurements, then it must be an extremal point [5].
- The correlation that gives the maximal quantum violation of an outcome-lifted Bell inequality is not unique.

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- The Clauser-Horne-Shimony-Holt (CHSH) [6] inequality can be written as

$$\sum_{x,y,a,b=0,1} (-1)^{xy+a+b} P(ab|xy) \overset{\mathcal{L}}{\leq} 2 \tag{6}$$

— self-tests $|\phi^+\rangle = (|00\rangle + |11\rangle)/\sqrt{2}$ and the corresponding Pauli observables giving the maximal quantum violation.

- Consider a Bell scenario obtained from the above by allowing a third outcome $b = 2$ for all of Bob's measurements.

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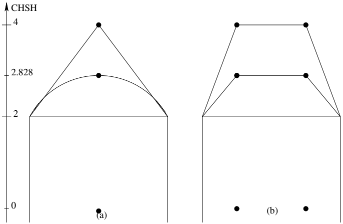
$$\sum_{x,y,a=0,1} (-1)^{xy+a} \left[\sum_{b=0,1} (-1)^{b} P(ab|xy) - P(a2|xy) \right] \overset{\mathcal{L}}{\leq} 2 \qquad (7)$$



Party-lifting of a Bell inequality

$$I_2 := \sum_{a,b,x,y} B_{a,b,x,y} P(ab|xy) \overset{\mathcal{L}}{\leq} 0 \overset{\text{Party-lifting}}{\Longrightarrow}$$

$$I_2^{\text{LP}} := \sum_{a,b,x,y} B_{a,b,x,y} P(abc'|xyz') \overset{\mathcal{L}}{\leq} 0 \tag{8}$$

$P(abc'|xyz')$ — joint conditional probabilities corresponding to our 3-partite Bell scenario with some *fixed* but *arbitrary* input-output pair c', z'.

Preservation of quantum and non-signaling violation

Note that

$$\beta_{\mathcal{Q}}^{\text{LP}} = \max_{\{P(abc'|xyz')\} \in \mathcal{Q}_3} \sum_{a,b,x,y} B_{a,b,x,y} P_{c'|z'}(ab|xy) P(c'|z') \leq \beta_{\mathcal{Q}} \tag{9}$$

where

$$P_{c'|z'}(ab|xy) := P(abc'|xyz')/P(c'|z'). \tag{10}$$

$\vec{P}_{c'|z'} := \{P_{c'|z'}(ab|xy)\}$— a legitimate correlation in the Bell scenario corresponding to the (original) bipartite Bell inequality.

Observation

Lifting of parties preserves the quantum bound and the non-signaling bound of any Bell inequality

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When the party-lifted Bell inequality is violated maximally to its quantum bound, the tripartite correlation that gives this quantum bound must factorize as follows:

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Concluding Remarks

We showed that lifting of Bell inequalities preserves

- the maximal quantum/non-signaling value of an outcome-lifted Bell inequality
  - Achievable with distinct quantum strategies
  - Gives 1st example where quantum violation can be used to self-test the underlying state but not the measurements
- partially the self-testing property of an outcome-lifted Bell inequality (state but not the measurements)
- the maximal quantum/non-signaling value of a party-lifted Bell inequality
- partially the self-testing property of a party-lifted Bell inequality (state and measurement for a subset of parties)

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