Multipartite Bell-inequality violation using randomly chosen triads

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The experimental demonstration of non-Bell-local (hereafter abbreviated as nonlocal) correlations between measurement outcomes is not only crucial for a test of local causality, but is also the basis for device-independent quantum information. Typically, such a demonstration requires the spatially separated experimenters to perform local measurements in certain specifically chosen bases. This, in turn, requires the experimenters to share a global reference frame.

Inspired by the work of [1, 2], we investigate the possibility of demonstrating nonlocal correlations by each party performing three randomly chosen, but mutually-unbiased qubit measurements on the n-qubit Greenberger-Horne-Zeilinger state. Following [2], we refer to each of these three mutually unbiased qubit measurement bases as a triad, as they define three mutually orthogonal vectors on the Bloch sphere.

Importantly, in the work of [2] (where they considered the specific case of $n = 2$), the probability of finding a Bell-inequality-violating pair of triads—apart from a set of measure zero—is unity. Here, we aim to investigate the extent to which this holds in the multipartite case, for n up to 8, as well as the likelihood of revealing beyond 2-body entanglement by the observed correlations alone.

More specifically, for $n = 3, \ldots, 8$, we randomly generate, for each party, a certain number of triads N_n on the Bloch sphere according to the Haar measure. For each chosen set of triads, we calculate the largest Bell value (for a few different two-setting Bell inequalities) by considering all possible combinations of two (out of three) settings per party and check if it is greater than the various k-producible bounds [3]. If the Bell value is greater than the corresponding k-producible bound ($k \leq n-1$) then it indicates that the underlying quantum state is, at least, $(k+1)$-body entangled. From the number of instances where a certain k-producible bound is violated, we can then estimate the probability of certifying, in a device-independent manner, that the underlying state has at least $(k+1)$-body entanglement.

The probability of successfully demonstrating at least, 2, 3, and n-body entanglement using the MABK [4–7] Bell inequality are summarized in Table I. In contrast with previous studies for $n \leq 5$ [8], our results suggest that the chance of witnessing nonlocality by the MABK inequality is unity in all but the tripartite case, whereas the chance of witnessing 3-partite entanglement is unity for all scenarios with more than 4 parties. Among others, we have also investigated (not shown) the the robustness of these protocols in the presence of white noise.

Number of Parties (n)	2	3	4	5	6	7	8
Number of Simulations N_n	N/A	4×10^6	5×10^6	2×10^6	4×10^5	3×10^5	1.05×10^5
Probability of Bell violation	100%	99.99%	100%	100%	100%	100%	100%
Probability of revealing 3-partite entanglement	N/A	45.89%	99.11%	100%	100%	100%	100%
Probability of revealing n-partite entanglement	100%	45.89%	22.54%	8.83%	2.86%	0.81%	0.22%
Sufficiency to consider MABK	Yes	No	Yes	Yes	Yes	Yes	Yes

TABLE I. Numerical estimation of the probability of Bell violations and the probability of revealing 3-body and n-body entanglement for correlations obtained by measurements performed in random chosen triads for $n = 3, \ldots, 8$.

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