Do black holes store negative entropy?

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We present an equation alternative to the Bekenstein equation for black holes, to solve an inconsistency among the first law of black-hole mechanics, a microscopic pair-creation picture of Hawking radiation, Bekenstein's argument, and the unitarity of quantum mechanics. Our equation argues that the area of a black hole should be proportional to the coherent information—which is *minus* the conditional entropy, defined only in the quantum regime—from the outside, to positive-energy particles inside it. This suggests that black holes store purely quantum information, rather than classical information.

About 50 years, Bekenstein [2] pointed out that, to make black-hole physics compatible with the second law of thermodynamics, black holes should have entropy proportional to their areas. Hawking [3, 4] strengthened this argument by showing that black holes emit thermal radiation [5] from Schwarzschild black holes. But, the heuristic picture of the microscopic process for this Hawking radiation is the creation of pairs in pure states between positive-energy particles outside a black hole and negative-energy particles inside it. This leads to an inconsistency among the first law of black-hole mechanics, Bekenstein's argument, and the unitarity of quantum mechanics. In particular, while the negative-energy particles play the role of reducing the energy of the black hole and thus decreasing its area according to the first law of black-hole mechanics, they appear inside the black hole with positive entropy (like normal particles) which should contribute to an increase in the area according to Bekenstein's argument [6]. A proposal of Parikh and Wilczek [7]—which treats Hawking radiation as tunneling of particles in a dynamical geometry, rather than pair creation—is free from this inconsistency. However, it does not explain the pure thermality of the Hawking radiation [7].

In this presentation [1], we propose an equation alternative to Bekenstein's, using arguments from the viewpoint of quantum information, rather than thermodynamics. Our alternative argues that the area of a black hole is proportional to the coherent information [8] from the outside of the black hole to positive-energy particles inside the black hole. This coherent information is *minus* the conditional entropy [9], which is defined only in the quantum regime. Our equation coincides with Bekenstein's as long as the effect of the Hawking radiation is small. Nevertheless, it includes a novel argument that negative-energy particles inside a black hole behave as if they have *negative* entropy, which provides complete consistency without changing Hawking's original proposal, in contrast to Bekenstein's. These ideas suggest that black holes store purely quantum information, rather than classical information.

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