Bell violations for entangled qudit pairs from random mutually unbiased bases

Gelo Noel Tabia,[1,2] Shih-Xian Yang,[2] Pei-Sheng Lin,[2] and Yeong-Cherng Liang[3]

[1] *Center for Quantum Technology, National Tsing Hua University, Hsinchu 300, Taiwan*

[2] *Department of Physics, National Cheng Kung University, Tainan 701, Taiwan*

[3] *Department of Physics and Center for Quantum Frontiers of Research & Technology (QFort), National Cheng Kung University, Tainan 701, Taiwan*

(Dated: April 29, 2019)

Quantum theory allows for correlations between measurement outcomes that are inconsistent with any local causal model, which is exhibited by the violation of a Bell inequality. Typically, to realize such nonlocal correlations experimentally, it is important to choose the measurement settings that yield the strongest correlations possible, which requires well-calibrated devices and a shared reference frame for distant parties.

These are nontrivial requirements in any practical implementation so it is desirable if they can be relaxed. In fact, several studies have indicated that they are not necessary for revealing Bell nonlocality with entangled qubits. Liang *et al.* [1] observed the probability of finding a CHSH violation is around 41% if each party selects a random pair of mutually unbiased bases (MUBs). Shadbolt *et al.* [2] demonstrated that the CHSH inequality can always be violated if each party measures a randomly chosen set of three MUBs.

Motivated by these results for qubits, we explore Bell violations from random MUBs in higher dimensions. Here it is natural to consider the Collins-Gisin-Linden-Massar-Popescu (CGLMP) inequality, where each party picks a uniformly random pair of MUBs to apply to a maximally entangled state. To estimate the probability of violation, for every trial we compute the CGLMP value for all possible relabeling or inputs and outputs, and for all ways Alice and Bob can pick a pair of MUBs from $k = 2, \ldots, d+1$ MUBs. The largest violation obtained from these CGLMP values is taken as the value of that trial. The relative frequency of violations is then our estimate for the probability of CGLMP violation.

We also estimate the probability of violation without referring to any specific Bell inequality by computing the white-noise visibilities of the correlations. This involves a linear program that computes for each correlation P the largest visibility v such that the mixture $vP + (1-v)P_u$ remains inside the local polytope, where P_u is the uniform distribution. If $v < 1$ then it follows that P is Bell nonlocal.

Our results show that there is a dramatic increase in the probability of violation when Alice and Bob measure more MUBs. However, in contrast with the qubit case, CGLMP violation is not guaranteed even if a complete set of MUBs is used. The values for Vis% do indicate that there must be some other Bell inequality that can achieve local violations for certain choices of MUBs. In fact, our numerical results show guaranteed violation for a complete set of MUBs for qutrits and ququarts. It is well-known that the probability of violation increases with more observables for each party so it is plausible that this observation for a complete set of MUBs will persist beyond the ququart case.

This work is supported by the Ministry of Science and Technology, Taiwan (Grants No. 104-2112-M-006-021-MY3, No. 107-2112-M-006-005-MY2, and 107-2627-E-006-001).

d	Alice #MUBs	Bob #MUBs	CGLMP %	Vis %	CGLMP #trials	Vis #trials
3	2	2	7.67	31.14	10^6	10^5
	3	3	47.01	98.36	10^6	10^4
	4	4	68.24	100	10^6	10^3
	2	3	21.63		10^6	
	2	4	40.85		10^6	
	3	4	60.98		10^6	
4	2	2	1.87	14.48	10^4	10^4
	3	3	14.97	77.75	10^4	10^3
	4	4	40.0	99.91	10^3	10^3
	5	5	61.1	100	10^3	10^3
	2	3	6.05		10^4	
	2	4	10.69		10^4	
	2	5	16.8		10^3	
	3	4	25.7		10^3	
	3	5	35.8		10^3	
	4	5	52.0		10^3	

TABLE I. Numerical estimates for the probability of CGLMP and Bell violation for random MUBs in $d = 3, 4$. Vis refers to visibility from picking 2 out of k MUBs for both parties.

[1] Y.-C. Liang, N. Harrigan, S. D. Bartlett, and T. Rudolph, Phys. Rev. Lett. **104**, 050401 (2010).

[2] P. Shadbolt, T. Vértesi, Y.-C. Liang, C. Branciard, N. Brunner, and J. L. O'Brien, Sci. Rep. **2**, 470 (2012).