Quantumness and Entanglement: New approaches

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The quantum states of a spin-j can be considered "classical" if they can be represented as a convex sum of SU(2)-coherent states. These are pure (i.e.~rank-1) states with minimal uncertainty of the angular-momentum operators. At the same time, these "classical" states correspond to separable multi-qubit states, symmetric under particle exchange. Spin-j states can be represented by a "Bloch-tensor" that generalizes the well-know Bloch-vector of a spin-1/2.

Eigenvalues of tensors where introduced in the mathematical literature only recently. We first examine to what extent the so-called Z-eigenvalues of the Bloch tensor provide information about the entanglement of the state.

We then show that the Bloch-tensor when partitioned into a matrix, is similar to the partial transpose of the density matrix in the computational basis, enabling an immediate sufficient condition for entanglement on the basis of the positive partial transposition (PPT) criterion. More generally, we show that the entanglement problem can be mapped to the "truncated moment problem" studied in mathematics. It gives rise to a hierarchy of extensions of a semi-definite program with polynomial constraints and the PPT criterion as a first step that generalizes and unifies on a more abstract level previous approaches. Flat extensions play a crucial role and are a new systematic ingredient that allows us to prove separability of a state and obtain its explicit deomposition into a convex sum of product states.