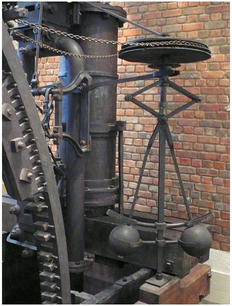
Feedback Control and Counting Statistics in Quantum Transport

Tobias Brandes (Institut für Theoretische Physik, TU Berlin)

- Quantum Transport
  - Example: particle counting.
  - Moments, cumulants, generalized density operators.
  - Quantum dots.
- Feedback control
  - Introduction, various kinds of feedback control.
  - Wiseman-Milburn control.
  - Information and thermodynamics, Maxwell demon control.
- (Talk on Tuesday)
  - Time-versus-number feedback control.
  - A recent experiment with quantum dots.

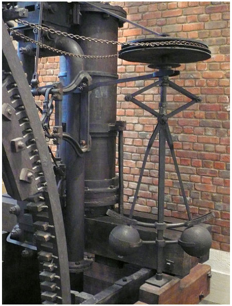
Feedback control

Feedback control



Centrifugal governor, Boulton and Watt

Feedback control



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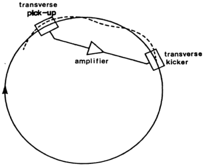
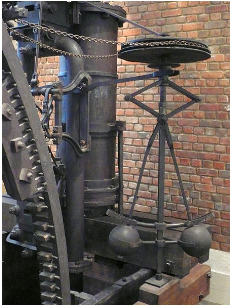


Stochastic cooling of particle collider beam, S. van der Meer (1972), Nobel Prize (1984) - discovery of W and Z bosons.

Feedback control



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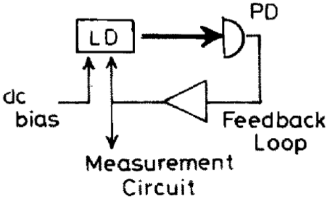
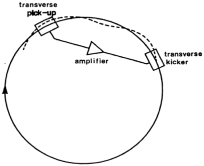


Photodetector signal corrects laser diode pump current, S. Machida, Y. Yamamoto (1986).



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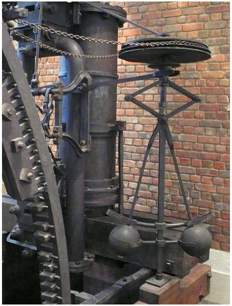

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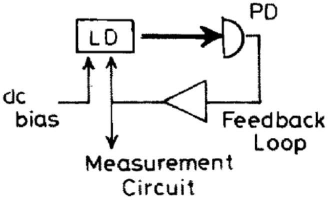

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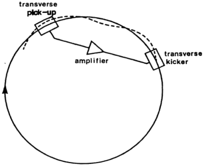

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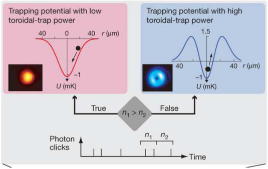


Feedback control of a single-atom trajectory, A. Kubanek, M. Koch, C. Sames, A. Ourjoumtsev, P. W. H. Pinkse, K. Murr and G. Rempe (2009).

Feedback control

Challenge for Quantum Systems

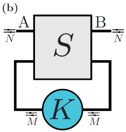
- Unitary dynamics versus measurement process.

- How to model the feedback loop?

H. M. Wiseman, G. J. Milburn, *Quantum measurement and Control (2009)*

Passive versus active feedback

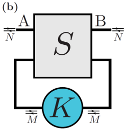
- (Passive) Coherent feedback control of quantum transport



C. Emary, J. Gough, PRB **90**, 205436 (2014).

Passive versus active feedback

- (Passive) Coherent feedback control of quantum transport



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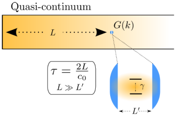
- (Passive) Cavity in waveguide



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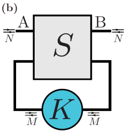

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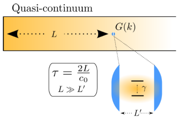

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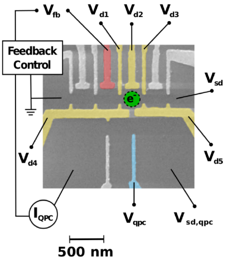
- (Active): feedback control of electron counting statistics.

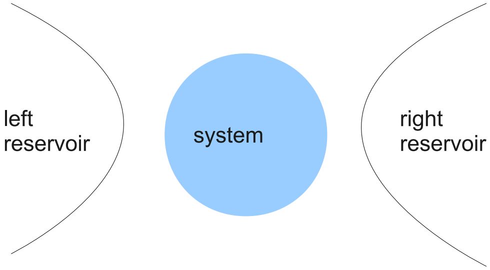


TB; Phys. Rev. Lett. **105**, 060602 (2010); T. Wagner, P. Strasberg, J. C. Bayer, E. P. Rugeramigabo, TB, R. J. Haug; nature nanotechnology (2016).

Active feedback master equation

General setup

- Open system Hamiltonian. $\mathcal{H} = \mathcal{H}_S + \mathcal{H}_{\rm res} + \mathcal{H}_T$.
  - $\mathcal{H}_S$ system.
  - $\mathcal{H}_{\rm res}$ reservoir.
  - $\mathcal{H}_T$ system-reservoir coupling.



G. Schaller, *Open Quantum Systems Far From Equilibibrium (2014)*

Active feedback master equation

Quantum jumps

- n-resolved density operator $\rho(t) \equiv \sum_{n=0}^{\infty} \rho^{(n)}(t)$
- Stochastic trajectories $\{x_n\} = x_n, ..., x_1$: jumps of type l_i at time t_i.
- 'path integral'

$$\rho^{(n)}(t) = \sum_{l_1=1,...,l_n=1}^{M} \int_0^t dt_n ... \int_0^{t_2} dt_1 \rho(t|\{x_n\}).$$

Active feedback master equation

Without feedback

- n-resolved density operator $\rho(t) \equiv \sum_{n=0}^{\infty} \rho^{(n)}(t)$
- Stochastic trajectories $\{x_n\} = x_n, ..., x_1$: jumps of type l_i at time t_i.
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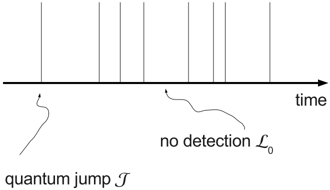
$$\rho^{(n)}(t) = \sum_{l_1=1,...,l_n=1}^{M} \int_0^t dt_n ... \int_0^{t_2} dt_1 \rho(t|\{x_n\}).$$

Without feedback:

- $\rho(t|\{x_n\}) \equiv$ $S_{t-t_n} \mathcal{J}_{l_n} S_{t_n - t_{n-1}} \mathcal{J}_{l_{n-1}} ... \mathcal{J}_{l_1} S_{t_1} \rho_{\rm in}$.
- Reduced density matrix $\dot{\rho}(t) = \mathcal{L}\rho(t)$, $\mathcal{L} \equiv \mathcal{L}_0 + \mathcal{J}$, $S_t \equiv e^{\mathcal{L}_0 t}$.



Moelmer, Zoller, Hegerfeldt, Carmichael,... 1980s

Active feedback master equation

With feedback

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- Stochastic trajectories $\{x_n\} = x_n, ..., x_1$: jumps of type l_i at time t_i.
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$$\rho^{(n)}(t) = \sum_{l_1=1,...,l_n=1}^{M} \int_0^t dt_n ... \int_0^{t_2} dt_1 \rho(t|\{x_n\}).$$

- With feedback:

$$\begin{aligned} \rho(t|\{x_n\}) &= S(t|\{x_n\})\mathcal{J}_{l_n}(t_n|\{x_{n-1}\})S(t_n|\{x_{n-1}\})... \\ &\times \mathcal{J}_{l_2}(t_2|\{x_1\})S(t_2|\{x_1\})\mathcal{J}_{l_1}(t_1)S(t_1)\rho_{\rm in}. \end{aligned}$$

  - Future time evolution conditioned upon full trajectory.

Active feedback master equation

Feedback protocols

0. Open loop control,

$$\boxed{\mathcal{J}_l(t|\{x_n\}) = \mathcal{J}_l(t).}$$

1. Feedback conditioned on the previous jump,

$$\boxed{\mathcal{J}_l(t|\{x_n\}) = \mathcal{J}_{ll_n}(t - t_n).}$$

   - Wiseman-Milburn quantum feedback control, $\mathcal{J}_{ll_n}(t - t_n) = e^{\mathcal{K}_l}\mathcal{J}_l$. Wiseman, Milburn ('90s); Pöltl, Emary TB (2011); Schaller, Kiesslich, Emary, TB (2011); Kiesslich, Emary, Schaller, TB (2012); Daryanoosh, Wiseman, TB (2016).
   - Waiting time feedback control TB, Emary (2016).
   - Delayed quantum control, Emary (2013).

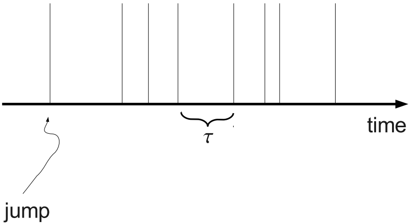
2. Time-versus-number feedback,

$$\boxed{\mathcal{J}_l(t|\{x_n\}) = \mathcal{J}_l(t, n).}$$

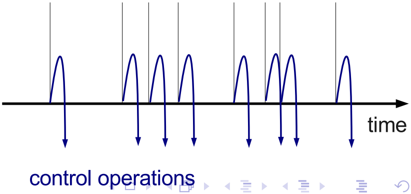
Wiseman-Milburn feedback

Wiseman-Milburn feedback

The idea

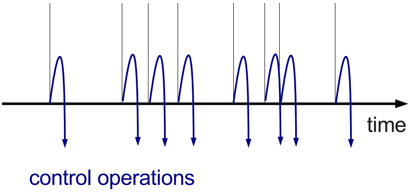


- Dissipation originates from quantum jumps.
- Fight dissipation by acting immediately after each jump: *feedback control*.



Wiseman-Milburn feedback

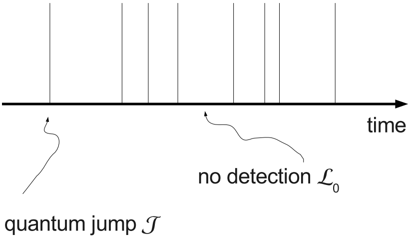
Counting fields and feedback



- $\dot{\rho} = (\mathcal{L}_0 + e^{i\chi}\mathcal{J})\rho$, counting field χ. (R. J. Cook 1981).
- FEEDBACK: complex number $e^{i\chi} \to$ superoperator $e^{\mathcal{K}}$ (Wiseman,... 1990s).

Wiseman-Milburn feedback

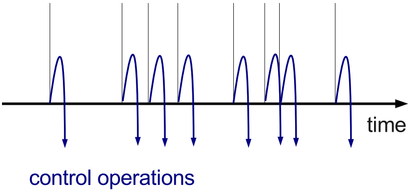
Stabilization: the effective Hamiltonian (C. Emary, 2011)



- Quantum jump $\rho \to \mathcal{J}\rho$.
- Non-jump $\rho \to e^{\mathcal{L}_0\tau}\rho$. Mixes up everything.

Wiseman-Milburn feedback

Stabilization: the effective Hamiltonian



- Quantum jump $\rho \to \mathcal{J}\rho$.
- Non-jump $\rho \to e^{\mathcal{L}_0 \tau}\rho$. Mixes up everything.
- FEEDBACK IDEA: No mixing if $\mathcal{L}_0 \rho_k = \lambda_k \rho_k$.
- Compensate quantum jumps by rotation into ρ_k via $e^{\mathcal{K}}\mathcal{J}\rho = \rho_k$.

$\rightsquigarrow$ eigenvalue problem to determine $\mathcal{K}$ in $\mathcal{L}_0 e^{\mathcal{K}}\mathcal{J}\rho_k = \lambda_k \rho_k$.

Wiseman-Milburn feedback

Stabilization: the effective Hamiltonian

- Effective, non-Hermitian 'Hamiltonian' $\mathcal{H}_{\rm eff}$ via

$$\mathcal{L}_0\rho = -i(\mathcal{H}_{\rm eff}\rho - \rho\mathcal{H}_{\rm eff}^{\dagger}).$$

- Eigenstates

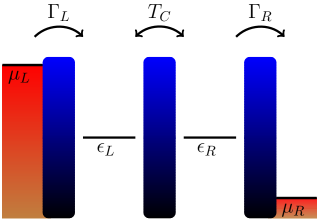
$$\rho_k = |\Psi_k\rangle\langle\tilde{\Psi}_k|$$

  of $\mathcal{L}_0$ are pure density matrices.
- The $|\Psi_k\rangle$ are (right) eigenstates of $\mathcal{H}_{\rm eff}$.

(Non-hermitian part of $\mathcal{H}_{\rm eff}$ contains the back-action of the measurement process.)

Wiseman-Milburn feedback: charge qubit

C. Pöltl, C. Emary, TB; Phys. Rev. B **84**, 085302 (2011).



- Effective Hamiltonian

$$\mathcal{H}_{\text{eff}} \equiv \mathcal{H}_S - \frac{i\Gamma_L}{2}|0\rangle\langle 0| - \frac{i\Gamma_R}{2}|R\rangle\langle R|$$

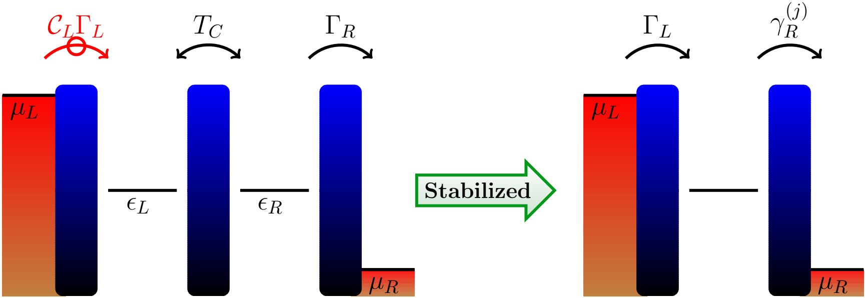
- Pure eigenstates $|0\rangle\rangle\langle\langle\tilde{0}|, |\psi_\pm\rangle\rangle\langle\langle\tilde{\psi}_\pm|$

Wiseman-Milburn feedback: charge qubit

Control Stabilization

Electron tunnels into $|L\rangle\rangle$ which is instantaneously rotated into $|\psi_+\rangle\rangle$ (alternatively, $|\psi_-\rangle\rangle$).

$\rightsquigarrow$ effective *single* level system, decay rate $\gamma_R^{(\pm)} \equiv -2\Im\varepsilon_\pm$



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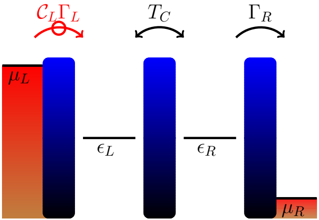


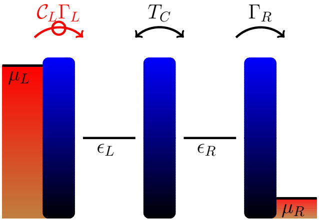
Controlled master equation

$$\dot{\rho} = (\mathcal{L}_0 + \mathcal{C}_L \mathcal{J}_L + \mathcal{J}_R)$$

$$\mathcal{C}_L \equiv e^{-2i\theta_C \boldsymbol{n}_0 \cdot \boldsymbol{\Sigma}}, \quad \boldsymbol{\Sigma}\rho \equiv [\boldsymbol{\sigma}, \rho]$$

- Rotation in Liouville-space about angle θ_C around direction $\boldsymbol{n}_0 = (\sin\theta, 0, \cos\theta)$, Pauli matrix vector $\boldsymbol{\sigma}$.
- Every qubit rotation as possible control operation, J. Wang and H. M. Wiseman, Phys. Rev. A **64**, 063810 (2001).

Wiseman-Milburn feedback: charge qubit



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- Project out empty state ($\Gamma_L \to \infty$).
- Bloch representation of stationary state $\rho_{\text{stat}} = (\langle\sigma_x\rangle, \langle\sigma_y\rangle, \langle\sigma_z\rangle)$.

Wiseman-Milburn feedback: charge qubit

Right eigenvalues of H_{eff}:

$$\varepsilon_0 = -i\frac{\Gamma_L}{2}, \quad \varepsilon_\mp = \frac{1}{4}\left(-i\Gamma_R \mp \sqrt{16T_C^2 - \Gamma_R^2}\right)$$

Eigenstates of the free Liouvillian

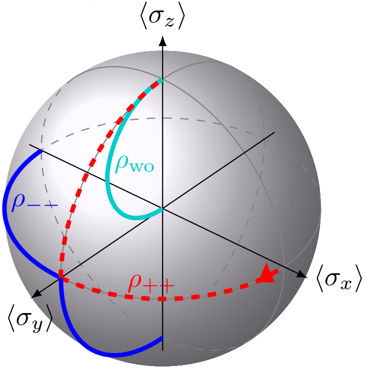
For $\Gamma_R > 4T_C$:

$$\langle\sigma_x\rangle = 0, \quad \langle\sigma_y\rangle = \frac{4T_C}{\Gamma_R}, \quad \langle\sigma_z\rangle = \mp\frac{\sqrt{\Gamma_R^2 - 16T_C^2}}{\Gamma_R}$$

For $\Gamma_R < 4T_C$:

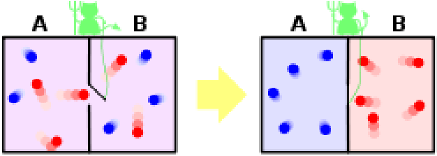
$$\langle\sigma_x\rangle = \mp\frac{\sqrt{16T_C^2 - \Gamma_R^2}}{4T_C}, \quad \langle\sigma_y\rangle = \frac{\Gamma_R}{4T_C}, \quad \langle\sigma_z\rangle = 0$$

Bloch sphere



Maxwell demon type feedback

Introduction



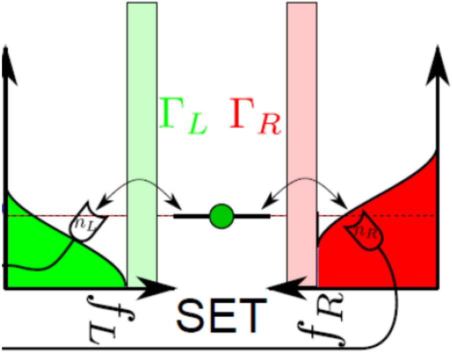
- Maxwell's thought experiment
  - Two volumes of gas in equilibrium separated by sliding door.
  - Demon opens/closes door, separates fast from slow gas molecules.
  - Decrease of entropy. "... to show that the second law of thermodynamics has only a statistical certainty".
- Concept of a transport device that acts like Maxwell's demon.

  Colloquium: The physics of Maxwell's demon and information; K. Maruyama, F. Nori, and V. Vedral, Rev. Mod. Phys. **81**, 1 (2009).

Maxwell demon type feedback

Single electron transistor

- Modify tunnel rates, e.g. Γ_R, depending on dot occupation.

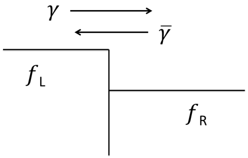


G. Schaller, C. Emary, G. Kießlich, TB; Phys. Rev. B **84**, 085418 (2011).

Maxwell demon type feedback

Single junction

- Integrate out the dot for $\Gamma_L \gg \Gamma_R \equiv \Gamma$.
- Rates $\gamma = \Gamma f_L(1 - f_R)$, $\bar{\gamma} = \bar{\Gamma} f_R(1 - f_L)$.
- $p_n(t)$ probability for n charges transfered to right reservoir.



$$\dot{p}_n = \gamma p_{n-1} + \bar{\gamma} p_{n+1} - (\gamma + \bar{\gamma}) p_n$$

Maxwell demon type feedback

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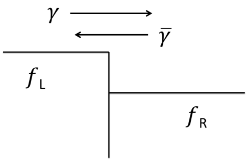

$$\dot{p}_n = \gamma p_{n-1} + \bar{\gamma} p_{n+1} - (\gamma + \bar{\gamma}) p_n$$

Without feedback:

- Identical microscopic forward and backward rates $\Gamma = \bar{\Gamma}$.
- Local **detailed balance** condition with affinity $\mathcal{A}$,

$$\frac{\gamma}{\bar{\gamma}} = e^{\mathcal{A}}, \quad \mathcal{A} \equiv \frac{\mu_L - \mu_R}{k_B T}$$

Maxwell demon type feedback

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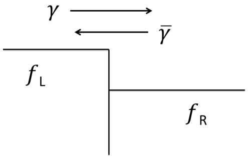

$$\dot{p}_n = \gamma p_{n-1} + \bar{\gamma} p_{n+1} - (\gamma + \bar{\gamma}) p_n$$

With feedback:

- Different forward and backward rates $\Gamma \neq \bar{\Gamma}$ ('by hand')
- Violates local detailed balance condition,

$$\frac{\gamma}{\bar{\gamma}} = e^{\mathcal{A} + \ln \frac{\Gamma}{\bar{\Gamma}}}, \quad \mathcal{A} \equiv \frac{\mu_L - \mu_R}{k_B T}$$

Maxwell demon type feedback

Single junction

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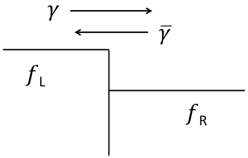
- Rates $\gamma = \Gamma f_L(1 - f_R)$, $\bar{\gamma} = \bar{\Gamma} f_R(1 - f_L)$.
- $p_n(t)$ probability for n charges transfered to right reservoir.



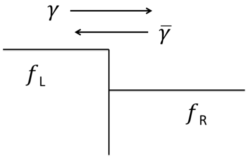
$$\dot{p}_n = \gamma p_{n-1} + \bar{\gamma} p_{n+1} - (\gamma + \bar{\gamma}) p_n$$

- Elevate (violated) local detailed balance to modified *exchange fluctuation theorem*

$$\frac{p_n(t)}{p_{-n}(t)} = e^{\left(\mathcal{A} + \ln \frac{\Gamma}{\bar{\Gamma}}\right) n}, \quad \mathcal{A} \equiv \frac{\mu_L - \mu_R}{k_B T}.$$

Maxwell demon type feedback

Single junction: Interpretation



- Rates $\gamma = \Gamma f_L(1 - f_R)$, $\bar{\gamma} = \bar{\Gamma} f_R(1 - f_L)$.

$\frac{p_n(t)}{p_{-n}(t)} = e^{\left(\mathcal{A} + \ln \frac{\Gamma}{\bar{\Gamma}}\right)n}, \quad \mathcal{A} \equiv \frac{\mu_L - \mu_R}{k_B T}.$

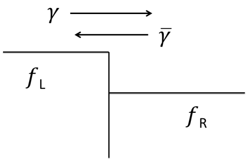
- Stationary charge current $\mathcal{J} = \gamma - \bar{\gamma}$ (set $-e = 1$).
- With feedback $\Gamma \neq \bar{\Gamma} \rightsquigarrow$ finite $\mathcal{J}$ even for zero voltage drop.

Maxwell demon type feedback

Single junction: Interpretation

- Rates $\gamma = \Gamma f_L(1-f_R)$, $\bar{\gamma} = \bar{\Gamma} f_R(1-f_L)$.

$\frac{p_n(t)}{p_{-n}(t)} = e^{\left(\mathcal{A}+\ln\frac{\Gamma}{\bar{\Gamma}}\right)n}, \quad \mathcal{A} \equiv \frac{\mu_L-\mu_R}{k_B T}.$



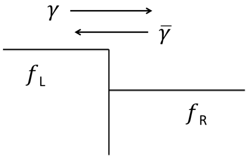
- Only n (number of transfered charges) thermodyn. relevant.
- Shannon entropy $S \equiv -\sum_n p_n \ln p_n$.
- Decompose $\dot{S} = \dot{S}_e + \dot{S}_i$ with $\dot{S}_i \geq 0$, J. Schnakenberg, Rev. Mod. Phys. **48**, 571 (1976); M. Esposito, C. Van den Broek, Phys. Rev. E **82**, 011143 (2010).

$$\lim_{t\to\infty} \frac{p_n(t)}{p_{-n}(t)} = e^{\dot{S}_i t}, \quad \dot{S}_i = \mathcal{A}\mathcal{J} + \ln\frac{\Gamma}{\bar{\Gamma}}\mathcal{J}, \quad \mathcal{J} \equiv \gamma - \bar{\gamma}$$

$\dot{S}_i$ = dissipated electric power per $k_B T$ plus *information current*.

Maxwell demon type feedback

Single junction: Interpretation



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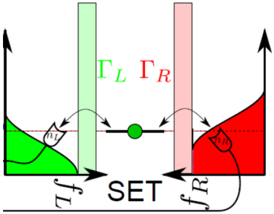
- Information gain via feedback modifies exchange fluctuation relation.
- General scenario T. Sagawa and M. Ueda ('08), J. M. P. Parrondo ('11); D. Abreu, U. Seifert ('12); T. Munakata, M. L. Rosinberg ('12); H. Tasaki ('13); J. M. Horowitz, M. Esposito ('14); D. Hartich, A. C. Barato, U. Seifert ('14); J. M. Horowitz ('15); J. M. Horowitz, K. Jacobs ('15)
- Example $\langle e^{-\beta(W - \Delta F) - I} \rangle = 1$ modified Jarzynski relation.

Feedback controlled tunnel barrier

Single electron transistor

- Rate equation $\dot{\rho} = \mathcal{L}\rho$, $\rho = (\rho_0, \rho_1)^T$.
- Explicitely break local detailed balance:

$$\mathcal{L} = \sum_{\alpha=L,R} \begin{pmatrix} -\Gamma_\alpha f_\alpha & \bar{\Gamma}_\alpha (1-f_\alpha) e^{i\chi} \\ \Gamma_\alpha f_\alpha & -\bar{\Gamma}_\alpha (1-f_\alpha) \end{pmatrix}$$



G. Schaller, C. Emary, G. Kießlich, TB; Phys. Rev. B **84**, 085418 (2011).

- Fluctuation relation (affinity $\mathcal{A} \equiv V/k_B T$, voltage $V \equiv \mu_L - \mu_R$)

$$\lim_{t\to\infty} \frac{p_n(t)}{p_{-n}(t)} = e^{\left(\mathcal{A} + \ln\frac{\Gamma_R}{\bar{\Gamma}_R} + \ln\frac{\bar{\Gamma}_L}{\Gamma_L}\right)n}.$$

M. Esposito, G. Schaller; EPL **99**, 30003 (2012).

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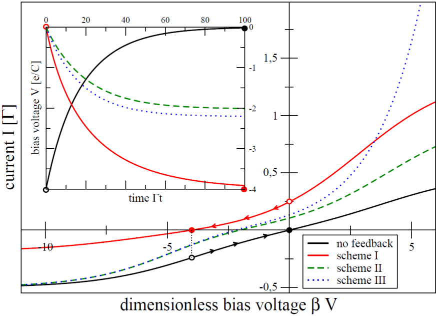
M. Esposito, G. Schaller; EPL **99**, 30003 (2012).

- Term $\ln\frac{\Gamma_R}{\bar{\Gamma}_R}+\ln\frac{\bar{\Gamma}_L}{\Gamma_L} = -V^*/k_B T$ as offset-voltage

G. Schaller, C. Emary, G. Kießlich, TB; Phys. Rev. B **84**, 085418 (2011).



Hardwiring the demon

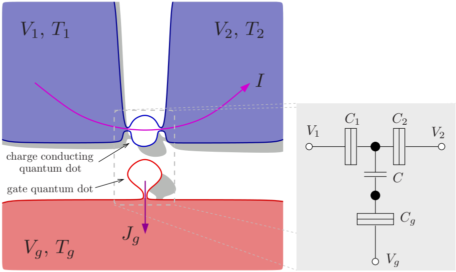
Key idea

- Microscopic model for larger system : SET + detector.
- Reduced SET dynamics described by effective model as above.

Hardwiring the demon

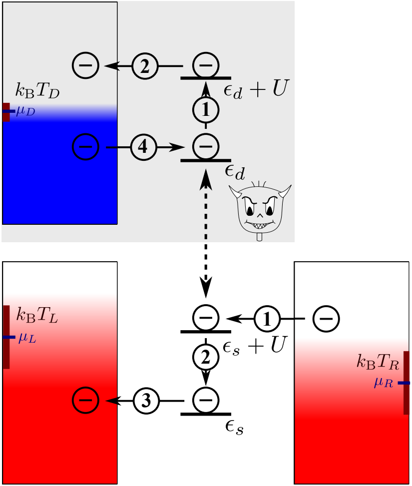
Thermoelectric device

- Energy to current converter.
- Different temperatures in different parts of the system.
- Energy-dependent tunnel rates.



R. Sánchez, M. Büttiker, Phys. Rev. B **83**, 085428 (2011); Europhys. Lett. **100**, 47008 (2012).

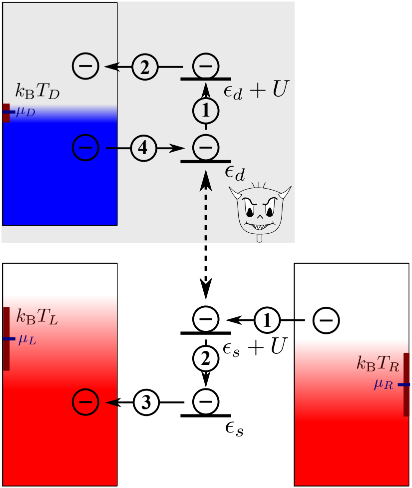
Hardwiring the demon

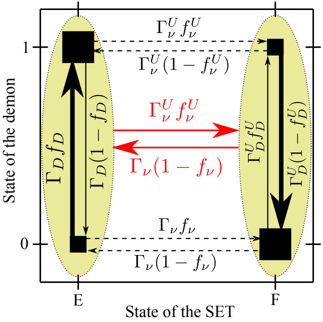


P. Strasberg, G. Schaller, TB, M. Esposito, Phys. Rev. Lett. **110**, 040601 (2013).

- Single level SET (bottom, two reservoirs L/R) and detector (top, one reservoir).
- States $|0E\rangle\rangle$, $|0F\rangle\rangle$, $|1E\rangle\rangle$, $|1F\rangle\rangle$.
- Energies 0, ϵ_s, ϵ_d, $\epsilon_s + \epsilon_d + U$.
- Energy dependent rates.

Hardwiring the demon





P. Strasberg, G. Schaller, TB, M. Esposito, Phys. Rev. Lett. **110**, 040601 (2013).

- Detector requirements:
  - Fast $\Gamma_D, \Gamma_D^U \gg \Gamma_\alpha, \Gamma_\alpha^U$.
  - Precise $U \gg k_B T_D$.
- SET requirements:
  - Spatial asymmetry $\Gamma_R^U \gg \Gamma_L^U$, $\Gamma_L \gg \Gamma_R$.

Maxwell demon limit

Reduced SET dynamics

- Full rate equation $\dot{\rho}_{ij} = \sum_{i'j'} W_{ij,i'j'} \rho_{i'j'}$
- Separation of time scales for fast detector $\Gamma_D, \Gamma_D^U \gg \Gamma_\alpha, \Gamma_\alpha^U$.
  - Technical trick: use conditional stationary probabilities M. Esposito, Phys. Rev. E **85**, 041125 (2012).
- $\rightsquigarrow$ SET rate equation $\dot{\rho}_i = \sum_{i'} V_{ii'} \rho_{i'}$.

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Reduced fluctuation theorem for SET with information current I

$$\lim_{t\to\infty} \frac{p_n(t)}{p_{-n}(t)} = \exp\left[\mathcal{A}n + I \times t\right], \quad I \times t \equiv \ln \frac{f_L^U f_R \Gamma_L^U \Gamma_R}{f_R^U f_L \Gamma_R^U \Gamma_L} n, \quad \mathcal{A} \equiv \frac{\mu_L - \mu_R}{k_B T}.$$

Maxwell demon limit

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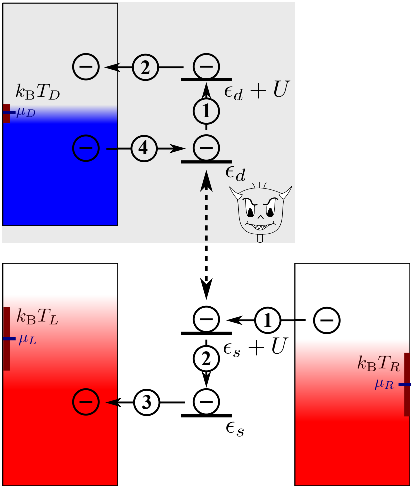
- Reduction to previous model ('feedback by hand'):

$$f_\alpha^U / f_\alpha = 1 \rightsquigarrow I \times t = (\ln \Gamma_L^U \Gamma_R / \Gamma_R^U \Gamma_L) n$$

Maxwell demon limit

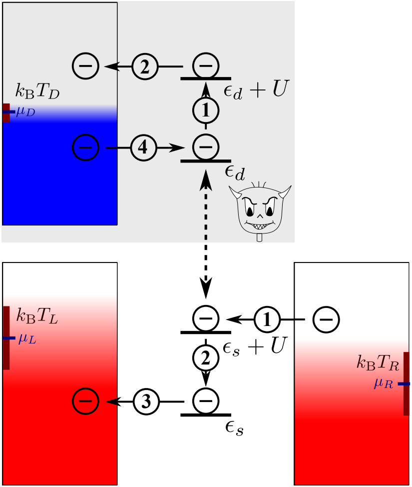
Energetics: first law



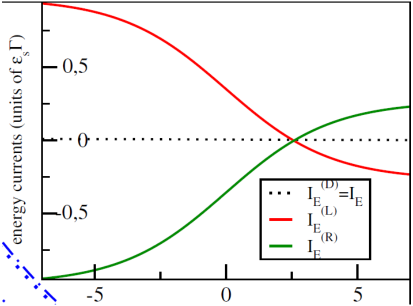
- Cycle between system energies: $\epsilon_d \to \epsilon_s + \epsilon_d + U \to \epsilon_s \to 0 \to \epsilon_d$.
- Net energy U transferred from SET to detector.

Maxwell demon limit

Energetics: first law



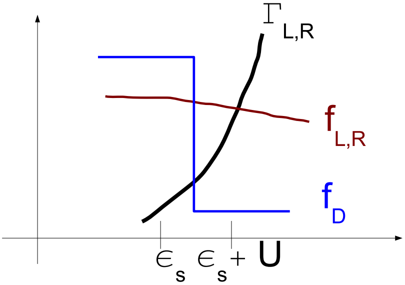
- For $\epsilon_S \gg U$, modification of **first law** $I_L^E + I_R^E = -I_D^E \approx 0$ negligible.



Maxwell demon limit

Summary of demon conditions

- Separation of time scales:
  - Fast demon $\Gamma_D, \Gamma_D^U \gg \Gamma_\alpha, \Gamma_\alpha^U$.
- Separation of energy scales:
  - No back-action, precision: $k_B T \gg U \gg k_B T_D$.
  - Almost no work done: $\epsilon_S \gg U$.
- Spatial/ energy lever condition
  - $\Gamma_\alpha \neq \Gamma_\alpha^U$, $\Gamma_L \neq \Gamma_R^{(U)}$, $\alpha =$ L,R.



$$\rightsquigarrow \lim_{t\to\infty} \frac{p_n(t)}{p_{-n}(t)} \approx \exp\left[\left(\mathcal{A} + \ln \frac{\Gamma_L^U \Gamma_R}{\Gamma_R^U \Gamma_L}\right) n\right], \quad \mathcal{A} \equiv \frac{\mu_L - \mu_R}{k_B T}.$$

Maxwell demon limit

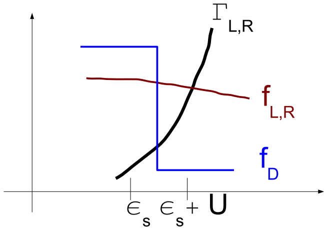
Where is the demon?

Maxwell demon limit

Where is the demon?

Maxwell's demon

A feedback mechanism that shuffles particles against a chemical or thermal gradient by adjusting barriers without requiring work.



- 'Hardwiring' of the feedback mechanism.
- 'Information' is really physical: rates in term $\ln\left(\Gamma_L^U \Gamma_R / \Gamma_R^U \Gamma_L\right)$.

Summary

- Transport master equations
  - Example: particle counting.
  - Moments, cumulants, generalized density operators.
  - Quantum dots.
- Feedback control
  - Introduction, various kinds of feedback control.
  - Wiseman-Milburn control.
  - Information and thermodynamics, Maxwell demon control.

Co-workers: G. Schaller (Berlin); C. Emary (Newcastle); C. Pöltl (Strasbourg); M. Esposito, P. Strasberg (Luxembourg)